Mathematical Biosciences and Engineering

http://www.aimspress.com/journal/MBE

MBE, 16(6): 7195–7216.
DOI: 10.3934/mbe.2019361
Received: 02 June 2019
Accepted: 16 July 2019
Published: 07 August 2019

Research article

Forecasting volatility using combination across estimation windows: An application to S&P500 stock market index

Davide De Gaetano [1,2,*]

[1] Department of Economics, University of Roma Tre, Via Silvio D'Amico 77, Rome, 00145, Italy
[2] SOSE - Soluzioni per il Sistema Economico Spa, Via Mentore Maggini 48C, Rome, 00143, Italy

* **Correspondence:** Email: ddegaetano@sose.it.

Abstract: The paper focuses on GARCH-type models for analysing and forecasting S&P500 stock market index. The aim is to empirically evaluate and compare alternative forecast combinations across estimation windows for directly dealing with possible structural breaks in the observed time series. In the in-sample analysis, alternative conditional volatility dynamics, suitable to account for stylized facts, have been considered along with different conditional distributions for the innovations. Moreover, an analysis of structural breaks in the unconditional variance of the series has been performed. In the out-of-sample analysis, for each model specification, the proposed forecast combinations have been evaluated and compared in terms of their predictive ability through the model confidence set. The results give evidence of the presence of structural breaks and, as a consequence, of parameter instability in S&P500 series. Moreover, averaging across volatility forecasts generated by individual forecasting models estimated using different window sizes performs well, for all the considered GARCH-type specifications and for all the implemented conditional distributions for the innovations and it appears to offer a useful approach to forecasting S&P500 stock market index.

Keywords: forecast combinations; structural breaks; volatility forecasting; parameter instability; financial time series

1. Introduction

In finance, knowledge of the stochastic process underlying stock returns is essential for making correct investment decisions as it provides essential information about the riskiness of investments. Many of the recent theories are concerned with the conditional variance, or volatility, which is a measure of the intensity of unpredictable changes in the returns, and so it could be interpreted as a random variable that follows a stochastic process.

Forecasting volatility is an important issue in research and application area and, to this aim, over the past decades, much effort has been addressed to develop and improve forecasting models. Traditional approaches are based on the univariate GARCH-type class of models which have been intensely studied in the literature as risk models of many financial time series. The key underlying assumption of most financial models is the stability of the model but, unfortunately, this assumption is typically not satisfied as financial returns may contain structural breaks due to legislative, institutional or technological changes as well as shifts in economic policy, or large macroeconomic shocks. The presence of breaks may affect the performance of forecasting due to the varying nature of the dynamics with time.

To accommodate potential structural breaks, an approach based on the adjustment of the estimation window for the forecasting model can be used. In this case, instead of using all available observations, only a fixed number of the most recent data are used to estimate the parameters of the forecasting model. However, the forecasting performance of this scheme is sensitive to the choice of the window size i.e. the number of the recent observations used in estimation [1]. For GARCH-type models the selection of a single estimation window remains an open issue and in many empirical studies, it is arbitrarily determined. For example, in [2, 3], different specifications for this parameter have been proposed; in particular, in the analysis, the estimation window size is fixed to one-half and one-quarter of the length of the in sample period. An alternative approach, which avoids the uncertainty surrounding the identification of the optimal window size, is based on combinations of multiple individual forecasts. In the last decades, there has been a growing interest, in the econometric literature, for the combinations of forecasts which, nowadays, are a well-established and well-tested approach for improving forecasting accuracy. Generally, this approach is obtained by estimating a number of alternative models over the same sample period [4] as a way for improving the forecasting accuracy focusing, in particular, on model misspecification, instability and estimation error. When dealing with structural breaks, it is more suitable to refer to forecast combinations across estimation windows. This approach, proposed in [5] in a regression framework [6, 7] , has been fruitfully applied in macroeconomic forecasting, in particular in the context of vector autoregressive models with weakly exogenous regressors [8, 9], in the context of GDP growth on the yield curve [10] and in the context of HAR-type models [11]. In [12] the theoretical advantages of using such combinations have been discussed, considering random walks with breaks in the drift and volatility and a linear regression model with a break in the slope parameter. It has been shown that averaging forecasts over different estimation windows leads to a lower bias and root mean square forecast error than forecasts based on a single estimation window for all but the smallest breaks. Similar results are reported in [13] in which it has been highlighted that, in presence of structural breaks, averaging forecasts obtained by using all the observations in the sample and forecasts obtained by using a window can be useful for forecasting.

All the discussed approaches are feasible for linear regression models and moderate sample size. However, when dealing with GARCH-type models, they became not suitable because of the estimation of thousands models just to obtain a single forecast. In this context, some forecast combination schemes have been recently proposed. For example, in [2], two simple methods, the mean and the trimmed mean, for combining the forecasts obtained by considering different and fixed number of the most recent data have been analysed and compared in the case of a GARCH (1,1) with Normal distribution. The usefulness of these combinations in forecasting under structural breaks has been empirically highlighted, in the case of BRICS daily returns, also in [14]. This approach has been extended to GJR-GARCH (1,1) with Normal distribution for the errors, in [3].

More recently, in [15], in the context of a GARCH (1,1) with Normal distribution, the individual forecasts entering into the combination are obtained by expanding the length of an initial estimation window backwards of a fixed number of observations v. The length of the initial window is introduced to allow a convergent estimation of the GARCH model whereas the parameter v, governing the number of individual forecasts, is determined by simulation. Simulation results show that forecast combinations with high values of v are able to perform better then alternative schemes proposed in the literature.

The aim of this paper is to empirically verify that the approach proposed in [15] can be extended to the general GARCH-type class of models. In particular, using daily returns for the S&P500 stock market index, alternative conditional volatility dynamics, suitable to account for stylized facts, usual observed in financial time series, have been considered along with different conditional distributions for the innovations. For each model specification, some forecast combinations have been evaluated and compared in terms of their predictive ability through the model confidence set, proposed in [16]. The paper is organized as follows. Section 2 briefly describes the class of GARCH-type models and provides a quick overview of some forecast combinations. Section 3 presents and discusses the empirical results on S&P500 stock market index and shows the better performance of the proposed methodology. Some final remarks close the paper.

2. Forecast combinations in GARCH-type models

Let $\{a_t\}$ be the one dimensional stochastic process of the daily returns and assume that its dynamic can be modelled by using a GARCH-type model defined as:

$$\begin{aligned} a_t &= \mu + \sigma_t \epsilon_t \qquad t \in N \\ \sigma_t^2 &= h(\sigma_{t-1}^2, \ldots, \sigma_{t-q'}^2, \varepsilon_{t-1}^2, \ldots, \varepsilon_{t-p'}^2, \psi_\sigma), \end{aligned} \tag{2.1}$$

where μ is the unconditional mean of $\{a_t\}$, $\{\epsilon_t\}$ is a sequence of independent and identically distributed random variables such that $\mathbb{E}(\epsilon_t) = 0$ and $\mathbb{E}(\epsilon_t^2) = 1$ and σ_t is the conditional standard deviation of ε_t. The function $h(\cdot)$ refers to one of the ARCH-type dynamics and the vector ψ_σ contains all the conditional variance dynamic parameters, its specification depending on the structure in the data, such as leverage effects and asymmetry. We assume that the conditional variance is well defined, i.e. σ_t^2 is positive, and the process ϵ_t is stationary and ergodic. Sufficient conditions under which it is guaranteed can be derived by specifying the conditional volatility function $h(\cdot)$.

Let a_t a time series of daily returns observed at times $t = 1, \ldots, T$. Generally, in the estimation procedure of the GARCH process 2.1, assumptions about stability in volatility have been explicitly or implicitly made in the vast majority of studies. However, because financial markets are closely linked to regime shifting in the economy, structural breaks or parameter shifts can occur causing instability of parameters. In the context of GARCH-type models, the presence of structural breaks could distort the persistence estimation toward a spurious value, as pointed out in [17] and than confirmed, among others, in [18] and [19]. Moreover, lack of awareness of structural breaks makes forecasts inaccurate and constitutes one of the major reasons for poor out-of-sample forecasting performance. A way for improving the forecasting accuracy is to consider an approach based on averaging across volatility forecasts generated by individual models estimated using different window sizes.

This approach allows not to deal with the uncertainty surrounding the identification of a single estimation window size which, generally, is arbitrarily determined and could be not pertinent for the size and the location of the last break. Moreover, it incorporates the trade-off between the bias and the variance of forecasting errors since windows of earlier data are commonly used in the combinations.

In [2], in the context of GARCH (1,1) model with Normal distribution, two simple methods for combining the forecasts have been considered. The first combines four individual forecasts obtained using different estimation windows: an expanding window which uses all the available observations, two windows with size equal to one-quarter and one-half of the length of the in-sample period and a window considering the observations after the identified last break. It is defined as:

$$\widehat{\sigma}^2_{T+1} = \frac{1}{4}\left[\widehat{\sigma}^{2\ [1:T]}_{T+1} + \widehat{\sigma}^{2\ [0.75T:T]}_{T+1} + \widehat{\sigma}^{2\ [0.50T:T]}_{T+1} + \widehat{\sigma}^{2\ [T_b:T]}_{T+1}\right] \tag{2.2}$$

where $\widehat{\sigma}^{2\ [\tau:T]}_{T+1}$ denotes the one-step-ahead forecast obtained by using the observations from τ to T and T_b is the last break date. Note that, in this combination, the last individual forecast is based on a window containing the observations after the identified last break. It does not necessarily outperform combinations that use no information on break dates. This is because estimated break dates can be imprecise, and the use of inaccurate estimates of the break dates can harmful rather than helpful when choosing estimation windows. The second combination used in [2] is a trimmed version of the previous one obtained excluding the highest and lowest forecasts trying to avoid, in this way, possible implausible forecasts.

In [3], the same forecast combinations have been analysed and discussed in the context of GJR-GARCH (1,1) model, with the same distribution on the errors. More recently, in [15], the individual forecasts entering into the combination are obtained by expanding the length of an initial estimation window backwards of a fixed number of observations v.

In this scheme, the forecast combination at time $T + 1$ is obtained by averaging across forecasts generated by increasing recursively, of a fixed number v, the minimum estimation window ω. More precisely, it is:

$$\widehat{\sigma}^2_{T+1} = \sum_{\tau=0}^{k-1} c_\tau \widehat{\sigma}^{2\ [T-w-\tau v:T]}_{T+1} \tag{2.3}$$

where $\widehat{\sigma}^{2\ [T-w-\tau v:T]}_{T+1}$ is the one-step-ahead forecast obtained by using the observations from $(T - w - \tau v)$ to T; c_τ are combination weights and

$$k = \left\lceil \frac{T-\omega}{v} \right\rceil \tag{2.4}$$

being $\lceil x \rceil$ the smallest integer greater than or equal to x.

In equation (2.3), the last ω observations are used in all the forecasts, whereas the observations at the beginning of the sample are used less.

The proposed forecast combination scheme depends on the parameters ω and v and on the weights c_τ.

The parameter ω is the size of the minimum estimation window which should be fixed in such a way that allows the parameter estimation of the forecasting model to converge. For example, when using a GARCH (1,1), the estimates of the parameters are significantly negatively biased in small samples and, in many cases, converged estimates are not possible with non-negativity conditions, as showed in [20]. In this context, considering the size of biases and convergence errors, ω should be setting at least equal to 500.

The parameter v controls the number of observations which are added to the minimum estimation window ω and, as a consequence, to the number of individual forecasts which enters in the combination. The lower the value of v, the more individual forecasts enter in the combination. Moreover if the location of the last break is near to the end of the sample, the higher the value of v, the less the number of windows containing many pre-break observations are in the combination scheme. This could be advantageous since the forecasts generated by using many pre-break observations could be biased especially when the size of the breaks is high. The logic behind this approach is similar to that proposed in [5] in a regression context. In this latter case, the parameter v is set equal to 1 since the number of observations is generally not so high and the linearity of the models ensures the feasibility, in term of computational costs, of the generation of the many individual forecasts involved. In the case of GARCH-type models, the choice of $v = 1$ is unrealistic; in this context, a value for v should guarantee the effectiveness of the forecast combination in accounting for possible structural breaks in the series and, at the same time, it should ensure not too high computational costs.

The selection of an optimal value of v is an open issue. However, in [15] it has been proved, by simulations, that weighting scheme based on an high value of v ($v = 800$ or $v = 900$) outperforms all the other forecast combinations with low or moderate values of v.

As it is usual in the literature of forecast combinations, the weights $c_0, c_1, \ldots, c_{k-1}$ are assumed to verify the following constrains:

$$c_0 \geq c_1 \geq \ldots \geq c_{k-1} \geq 0 \qquad \text{and} \qquad \sum_{\tau=0}^{k-1} c_\tau = 1 \tag{2.5}$$

being c_τ the weight associated to the forecast obtained by using the observations from $(T - w - \tau v)$ to T. Although all the sequences satisfying the previous assumptions can be used in the equation 2.3, we have considered two well known choices. The first one is based on fixing equal weights for all the individual forecasts entering the combination. In this case, it is:

$$c_\tau = \frac{1}{k} \qquad \tau = 0, 1, \ldots, k-1 \tag{2.6}$$

As a consequence, the proposed forecast combination (2.3) is defined as :

$$\widehat{\sigma}^2_{T+1} = \frac{1}{k} \sum_{\tau=0}^{k-1} \widehat{\sigma}^{2\,[T-w-\tau v:T]}_{T+1} \tag{2.7}$$

This scheme is easy to compute and often has performance as good as more complicated schemes, also when there is uncertainty about the presence of structural breaks in the data.

Another possible choice is to use constant weights proportional to the location of τ in the sample. In this case, it is:

$$c_\tau = \frac{k-\tau}{\sum_{\tau=0}^{k-1}(k-\tau)} = \frac{2(k-\tau)}{k(k+1)} \qquad \tau = 0, 1, \ldots, k-1 \tag{2.8}$$

As a consequence, the proposed forecast combination (2.3) is defined as:

$$\widehat{\sigma}^2_{T+1} = \frac{1}{\sum_{\tau=0}^{k-1}(k-\tau)} \sum_{\tau=0}^{k-1}(k-\tau)\widehat{\sigma}^{2\ [T-w-\tau v:T]}_{T+1} \tag{2.9}$$

In this scheme, the weights are heavier for forecasts that use more recent information.

In [15], it has been verified that forecast combinations with location weights and with an high value of v seem to better perform also with respect to some alternative forecast combinations proposed in the literature. This result is particularly evident when the location of the structural break is near the end of the sample.

3. Empirical analysis on S&P500 stock market index

An empirical analysis has been performed on the volatility of the S&P500 stock market index. This index includes 500 stocks of US companies with the largest capitalization; it is widely diversified and actively traded on the markets. The data set covers the period from 03-01-2000 to 04-12-2017; the plot of the series is reported in Figure 1 and summary statistics in Table 1. The data presents the usual well-known stylized facts about financial time series, that is a small mean, a large standard deviation and evidence of non normality as pointed out by the Jarque-Bera test. This feature is essentially due to an excess of kurtosis that highlights the presence of a large number of significant shocks.

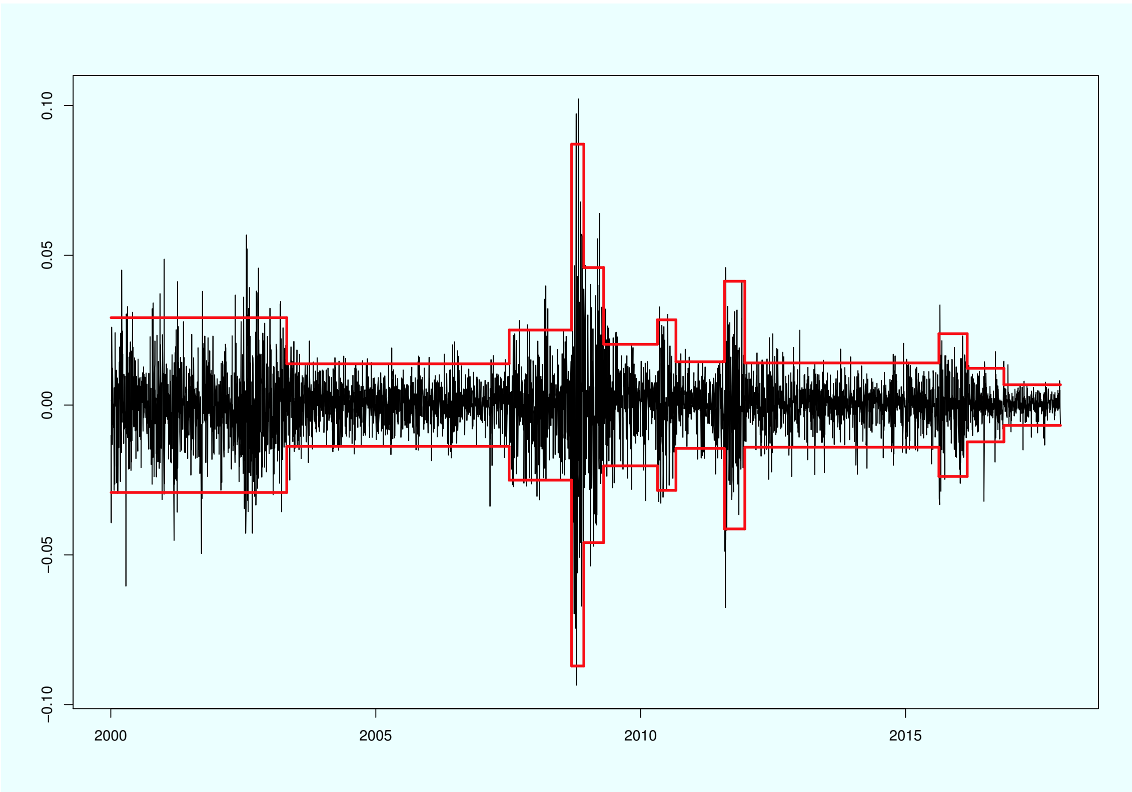

Figure 1. S&P500 daily returns from 03-01-2000 to 04-12-2017 and two-standard-deviation bands for the regimes defined by the structural breaks identified by the binary segmentation with K_2 test.

Table 1. Summary statistics of S&P500 daily returns from 03-01-2000 to 04-12-2017

Summary statistics			
Min	- 0,09351	Std.Dev	0,01159
Mean	0,00010	Skewness	- 0,17478
Median	0,00052	Kurtosis	8,14877
Max	0,10220	JB Test	12430 [2.2E-16]

3.1. In-sample analysis

In order to capture the stock market volatility and the different conditional heteroskedasticity patterns of the S&P 500 stock market index, three models belonging to the GARCH-type class of models (2.1) have been considered. The simplest conditional volatility dynamics is the GARCH (1, 1) specification, introduced in [21] for analysing financial time series:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2. \tag{3.1}$$

with $\alpha_0 > 0$, $0 < \alpha_1 < 1$ and $0 < \beta < 1$. The persistence measure for the GARCH (1,1) model is given by $\alpha_1 + \beta$ and the condition $\alpha_1 + \beta < 1$ ensures that the model is covariance-stationary. The GARCH (1,1) model assumes that the conditional variance responds symmetrically to positive and negative shocks, so that only the size, and not the sign, of the shock matters. However, it is well established that stock returns and volatility are negatively correlated. In order to allow for asymmetry in the response of conditional volatility to return shocks, several non linear generalizations of this model have been introduced in the literature.

Here, we focus on two of the most used specifications which could be suitable to account for the stylised facts usual observed in S&P500 stock market index. The first one is the EGARCH (1, 1) model, introduced in [22]. It assumes the following conditional volatility dynamics:

$$\log(\sigma_t^2) = \alpha_0 + [\alpha_1 z_{t-1} + \gamma(|z_{t-1}| - \mathbb{E}|z_{t-1}|)] + \beta \log(\sigma_{t-1}^2), \tag{3.2}$$

where $z_t = \varepsilon_t / \sigma_t$.

The exponential nature of the model ensures that the conditional variance is always positive even if the parameter values are negative, thus no positivity constraints on the parameters are required. The parameter γ captures the asymmetric effect. If $\gamma = 0$, the model is symmetric; when $\gamma < 0$, positive shocks (good news) generate less volatility than negative shocks (bad news). When $\gamma > 0$, it implies that positive innovations are more destabilizing than negative innovations. The condition for the EGARCH (1,1) to be covariance-stationary is $\beta < 1$, where β is the persistence measure. The second GARCH generalization we consider is the GJR-GARCH (1, 1) model [23] which assumes that the conditional volatility dynamics is:

$$\sigma_t^2 = \alpha_0 + (\alpha_1 + \gamma I_{\{\varepsilon_{t-i}<0\}}) a_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{3.3}$$

where $\mathbb{I}_{\{\varepsilon_{t-i}<0\}}$ assumes value one if $\varepsilon_{t-1} < 0$ and zero otherwise. This specification accounts for the leverage effect by including a dummy variable which discriminates positive and negative lagged shocks. When $\gamma = 0$, it is clear that positive and negative shocks have symmetric effects on volatility and, in this case, (3.3) reduces to (3.1). The persistence measure for the GJR-GARCH (1,1) model is given by $(\alpha_1 + \beta + \gamma/2)$ and the condition $(\alpha_1 + \beta + \gamma/2) < 1$ ensures that the model is covariance-stationary.

Moreover, for each model specification, several conditional distributions for the innovations have been considered. Valid choices are the standardized Normal distribution, the standardized Student$-T$ distribution with ν degrees of freedom and the standardized Generalized Error Distribution (GED) with shape parameter κ [24]. The last two distributions might better characterize the error process for S&P500 stock market index which presents very high and very low returns suggesting a fatter-tailed distribution then the Standard Normal. Besides fat tails, empirical distributions of financial time series may also be skewed. To handle this additional characteristic of S&P500 stock market index, we have also considered the asymmetric skewed extensions of the previous distributions, that is the skew Normal, the skew Student-T and the skew GED distribution. They are obtained by introducing a parameter $Skew$ regulating the skewness to the corresponding distributions. Therefore in our analysis we have considered three GARCH specifications and six error distributions, for a total of eighteen models.

In Table 2 the estimation results are reported. The log-likelihood (LL) and the Bayesian Information Criterion (BIC) do not provide evidence about the best model to fit the data. Moreover, the persistence of all the models are very close to one. This is related to the presence of structural breaks, as pointed out in [17], and, as a consequence, of parameter instability. This feature is confirmed by the Nyblom test [25] which provides a means of testing for parameter instability for models estimated by methods other than OLS. It is a Lagrange multiplier test in which the distribution of the test statistic is non-standard. Critical values, computed by simulation in [26], are reported in parenthesis. It is evident that, for all the models and for all the considered distributions for the innovations, the hypothesis of a stable model is always rejected. Note that the test is not informative about the date or type of structural changes.

3.2. Structural breaks analysis

In order to confirm whether a non stable GARCH-type process governs the conditional volatility of S&P500, a CUSUM of squares test, designed for testing for variance changes, has been considered. The variance is, in fact, a functional of GARCH-type parameters and, as a consequence, by examining the existence of the variance change, it is possible to detect parameter changes.

In particular, we have implemented a test proposed in [27] which takes into account both the fourth order moment of the process and persistence in the variance. The test is based on the following statistic:

$$K_2 = \sup_k |T^{-1/2} G_k| \tag{3.4}$$

where $G_k = \widehat{\omega}_4^{-1/2}[C_k - (k/T)C_T]$, being $C_k = \sum_{t=1}^{k} a_t^2$ for $k = 1, \dots, T$ the cumulative sum of squares of a_t, and $\widehat{\omega}_4$ is a consistent estimator of ω_4, the long-run fourth order moment of a_t.

The statistic K_2 makes adjustments to the classical IT statistic proposed in [28] and allows a_t to obey a wide class of dependent processes, including GARCH processes, under the null.

A consistent estimator of the long-run fourth order moment of a_t, has been obtained by using a non

Table 2. Quasi-maximum likelihood estimation results for GARCH-type models. Standard errors are given in round brackets. Critical values of the Nyblom test in square brackets.

	μ	α_0	α_1	β	γ	ν	κ	$Skew$	Persistence	LL	BIC	Nyb. Test
GARCH (1,1) NORM	0,00040	1,07E-06	0,09724	0,89505					0,99229	14740,90	- 6,57472	105,66
	(0.0001)	(1,03E-06)	(0, 0173)	(0, 0170)								[1, 24]
GARCH (1,1) STD	0,00054	5,88E-07	0,09765	0,90135		6,71025			0,99900	14832,97	- 6,61395	219,43
	(9,97E-05)	(6,42E-07)	(0, 0120)	(0, 0111)		(0, 5435)						[1, 47]
GARCH (1,1) GED	0,00054	7,17E-07	0,09968	0,89865			1,32512		0,99833	14840,87	- 6,61748	187,93
	(0, 0001)	(7,91E-07)	(0, 0150)	(0, 0140)			(0, 0296)					[1, 47]
GARCH (1,1) SNORM	0,00032	9,30E-07	0,09747	0,89569				0,86683	0,99316	14772,47	- 6,58694	147,24
	(0, 0001)	(1,07E-06)	(0, 0187)	(0, 0183)				(0, 0158)				[1, 47]
GARCH (1,1) SSTD	0,00038	5,70E-07	0,09927	0,89973		7,10138		0,89925	0,99900	14847,84	- 6,61872	225,07
	(0, 0001)	(6,37E-07)	(0, 0123)	(0, 0113)		(0, 6029)		(0, 0175)				[1, 68]
GARCH (1,1) SGED	0,00031	6,74E-07	0,09960	0,89834			1,35145	0,90030	0,99793	14858,49	- 6,62347	202,67
	(0, 0001)	(7,62E-07)	(0, 0144)	(0, 0135)			(0, 0309)	(0, 0162)				[1, 68]
EGARCH (1,1) NORM	0,00010	- 0,18700	- 0,13689	0,97990	0,12791				0,97990	14844,47	- 6,61909	2,09
	(9,00E-05)	(0, 0020)	(0, 0076)	(7,20E-05)	(0, 0042)							[1, 47]
EGARCH (1,1) STD	0,00029	- 0,13009	- 0,14202	0,98673	0,12703	7,42558			0,98673	14917,88	- 6,64999	4,74
	(9,00E-05)	(0, 0045)	(0, 0102)	(0, 0005)	(0, 0132)	(0, 9195)						[1, 68]
EGARCH (1,1) GED	0,00032	- 0,15217	- 0,13905	0,98451	0,12937		1,40065		0,98451	14917,59	- 6,64987	4,52
	(9,93E-05)	(0, 0024)	(0, 0081)	(1,44E-05)	(0, 0085)		(0, 0402)					[1, 68]
EGARCH (1,1) SNORM	4,57E-06	- 0,18274	- 0,14014	0,98024	0,12411			0,84100	0,98024	14887,80	- 6,63656	2,44
	(8,64E-05)	(0, 0022)	(0, 0074)	(4,62E-05)	(0, 0043)			(0, 0158)				[1, 68]
EGARCH (1,1) SSTD	0,00007	- 0,15350	- 0,14937	0,98383	0,12912	8,09396		0,86362	0,98383	14943,84	- 6,65971	4,69
	(2,17E-05)	(0, 0064)	(0, 0096)	(0, 0010)	(0, 0266)	(1, 1151)		(0, 0165)				[1, 90]
EGARCH (1,1) SGED	0,00004	- 0,17404	- 0,14763	0,98167	0,12974		1,43205	0,86517	0,98167	14948,67	- 6,66186	4,10
	(9,67E-05)	(0, 0027)	(0, 0080)	(1,52E-05)	(0, 0069)		(0, 0436)	(0, 0158)				[1, 90]
GJR-GARCH (1,1) NORM	0,00010	1,46E-06	4,98E-08	0,89814	0,17253				0,98440	14826,91	- 6,61125	49,91
	(8,36E-05)	(5,87E-07)	(0, 0027)	(0, 0063)	(0, 0111)							[1, 47]
GJR-GARCH (1,1) STD	0,00033	8,57E-07	8,54E-08	0,90452	0,17571	7,29426			0,99237	14900,44	- 6,64221	193,99
	(9,74E-05)	(6,24E-07)	(0, 0024)	(0, 0124)	(0, 0086)	(0, 6320)						[1, 68]
GJR-GARCH (1,1) GED	0,00035	1,07E-06	7,82E-10	0,90099	0,17571		1,38851		0,98778	14902,60	- 6,64317	146,85
	(9,91E-05)	(1,71E-06)	(0, 0103)	(0, 0309)	(0, 0086)		(0, 0251)					[1, 68]
GJR-GARCH (1,1) SNORM	0,00001	1,39E-06	5,27E-08	0,89824	0,17571			0,84883	0,98284	14866,44	- 6,62702	50,53
	(8,74E-05)	(4,84E-07)	(0, 0029)	(0, 0068)	(0, 0085)			(0, 0158)				[1, 68]
GJR-GARCH (1,1) SSTD	0,00010	9,94E-07	1,02E-08	0,90028	0,17571	7,85673		0,86753	0,98915	14925,16	- 6,65137	160,36
	(0, 0001)	(1,16E-06)	(0, 0081)	(0, 0225)	(0, 0086)	(0, 3629)		(0, 0172)				[1, 90]
GJR-GARCH (1,1) SGED	0,00006	1,19E-06	6,17E-08	0,89733	0,17571		1,41704	0,87047	0,98470	14931,54	- 6,65422	107,67
	(0, 0001)	(1,40E-06)	(0, 0098)	(0, 0212)	(0, 0403)		(0, 0528)	(0, 0158)				[1, 90]

Note: NORM, STD and GED indicate the standardized Normal distribution, the standardized Student$-T$ distribution and the standardized Generalized error distribution for the innovations, respectively. SNORM, SSTD and SGED are the asymmetric skewed extensions.

parametric approach based on the Bartlett kernel [27]. In particular it is:

$$\hat{\omega}_4 = \hat{\gamma}_0 + 2\sum_{l=1}^{m}[1 - l(m+1)^{-1}]\hat{\gamma}_l \tag{3.5}$$

where

$$\hat{\gamma}_l = T^{-1}\sum_{t=l+1}^{T}(\epsilon_t^2 - \hat{\sigma}^2)(\epsilon_{t-1}^2 - \hat{\sigma}^2) \tag{3.6}$$

and $\hat{\sigma}^2 = T^{-1}C_T$. The bandwidth m has been obtained by using the procedure in [29]. Under quite general conditions, in [27] it has shown that:

$$K_2 \xrightarrow{A} sup_r|W^*(r)| \tag{3.7}$$

where $W^* = W(r) - rW(1)$ is a Brownian bridge and $W(r)$ is a standard Brownian motion. Finite-sample critical values for the test can be determined by simulation.

The statistic k_2 has been applied in a sequential manner to identify and test for multiple volatility changes of S&P500 stock market index. In particular, we have used a binary segmentation in which, in the first step, the entire sample is tested for the presence of a single break in volatility. If a significant structural break is detected, the data are split into two segments and the detection method is applying to each of them. The procedure continues until no further change-points are detected. To avoid the identification of spurious break points, due to the presence of extreme observations which can be erroneously interpreted as being change points [30], a final step has been included in the binary segmentation procedure. It consists of a re-evaluation and a re-estimation of the break points, in the spirit of the ICCS algorithm proposed in [28].

Moreover, to avoid the problem of using the asymptotic critical value for any segments, which may distort the performance of the iterative procedure especially when the length of the subsamples can quickly become quite small, we have used the finite sample critical values estimated through simulation using the response surfaces methodology [31].

Figure 1 shows the two-standard-deviation bands for each of the regimes defined by the structural breaks whose dates are reported in Table 3.

Table 3. Structural break dates identified by the binary segmentation with K_2 test.of S&P500 daily returns from 03-01-2000 to 04-12-2017

Structural breaks dates			
28-04-2003	09-07-2007	12-09-2008	05-12-2008
21-04-2009	26-04-2010	01-09-2010	01-08-2011
20-12-2011	19-08-2015	01-03-2016	09-11-2016

All of the identified structural breaks correspond to significant changes in the unconditional variance across regimes and they are related not only to global but also to specific financial, economic, social and political events. It is evident, for example, the impact on the analysed series of events such as the "internet bubble bursting", in the first years of 2000; the subprime mortage crisis in mid 2007, which lead to the global financial crisis of 2008–2009; the "August 2011 stock market fall" due to fears of the contagion of the European Sovereign debt crisis; the "2015–2016 stock market sell-off" linked to the Chinese stock market turbulence, the fall in petroleum prices, the Greek debt default, the effect to the end of quantitative easing and the "Brexit referendum".

3.3. Out-of-sample analysis

In this section we have analysed and compared the forecasting performances of the forecast combinations proposed in Section 2, by looking at their ability to predict future values of the series. We have divided the analysis in two steps. In the first, we have compared the forecast combinations for each the considered GARGH-type model, in order to understand which of them, produce better results than a stable model. In the second step, we have compared all the models and the forecast combinations, in order to deduce if it is possible, in the considered dataset, to identify which model and which forecast combination is better able to predict future values of S&P500 stock market index.

In the analysis,we have divided the observations into an estimation period, composed of the data from $t = 1$ to $t = R$, and an evaluation period. The models and the forecast combinations are estimated using the first R intra-day observations and, for each of them, the one-step-ahead out-of-sample forecast is produced. The sample is increased by one, the model is re-estimated and one-step-ahead forecasts are produced. The procedure continues until the end of the available out-of-sample period. In the following R has been fixed such that the number of out-of-sample observations is 500.

In order to evaluate the effect of different values of the parameter v on the proposed forecast combinations, the following different values $v \in \{100, 200, 300, 400, 500, 600, 700, 800, 900\}$ have been considered. For each value of v, the proposed forecast combinations with equal weights, defined in (2.7) and with location weights defined in (2.9) have been considered. Moreover, when it is reasonable, a trimmed version, obtained by excluding the individual forecasts with the lowest and the highest value respectively has been also included in the analysis. Moreover, to assess the effectiveness of the proposed combinations, a benchmark forecasting method has been introduced in the analysis. It is the expanding window method, which uses all the available observations in the estimation sample to produce the forecasts. As pointed out in [5], this choice is optimal in situations in which no breaks are present in the sample and, as a consequence, it is appropriate for forecasting when the data is generated by a stable model. For each class of models, this method produces out-of-sample forecasts using a recursive expanding estimation window. Moreover the forecast combination proposed in [2] and reported in (2.2) and its trimmed version have also been considered. Table 4 lists the considered forecasting schemes, together with the used acronyms.

We assess the statistical significance of differences in the forecasting performance of the models by using the Model Confidence Set (MCS) proposed in [16]. This procedure consists of a sequence of tests which allows the construction of a Set of Superior Models (SSM), where the null hypothesis of equal predictive ability is not rejected at a specified confidence level α. The test statistic can be evaluated for any arbitrary loss function whose choice depends on the nature of the candidate models. Here we have considered, for $t = R + 1, \ldots, T$ the following five loss functions:

$$QLIKE_t = \left[\frac{\tilde{\sigma}_t^2}{\widehat{\sigma}_t^2} - \log\left(\frac{\tilde{\sigma}_t^2}{\widehat{\sigma}_t^2}\right) - 1\right] \tag{3.8}$$

$$MSE_t = (\tilde{\sigma}_t^2 - \widehat{\sigma}_t^2)^2 \tag{3.9}$$

$$MAE_t = \left|\tilde{\sigma}_t^2 - \widehat{\sigma}_t^2\right| \tag{3.10}$$

$$MAD_t = (\tilde{\sigma}_t - \widehat{\sigma}_t)^2 \tag{3.11}$$

$$MSD_t = \left|\tilde{\sigma}_t - \widehat{\sigma}_t\right| \tag{3.12}$$

where $\tilde{\sigma}_t^2$ is some volatility measure and $\widehat{\sigma}_t^2$is the punctual volatility forecast. In this context, squared returns have been used as a proxy for the latent volatility.

The first two loss functions are the most popular and used in the econometric literature since they provide robust ranking of the models in the context of volatility forecasts [32]. The QLIKE loss is a simple modification of the Gaussian log-likelihood in such a way that the minimum distance of zero is obtained when $\sigma_t^2 = \widehat{\sigma}_t^2$. Moreover, according to [32], it is able to better discriminate among models and it is less affected by the most extreme observations in the sample. The MSE loss function has the tendency to penalize large forecast errors more severely than other common accuracy measures and therefore is considered as the most appropriate measure to determine which methods avoid large errors.

Table 4. Typology of the empirical specifications used in the paper.

Acronym	Forecasting methods
Expanding Wind	Expanding Window forecast. This method uses all the available observations.
Mean Wind E v	Mean window forecast combination with equal weights and tuning parameter v (Eq (2.7)).
Mean Wind L v	Mean window forecast combination with location weights and tuning parameter v (Eq (2.9)).
Mean Wind T v	Trimmed mean window forecast combination with location weights and tuning parameter v.
RS Mean	Forecast combination which averages individual forecasts obtained with four different estimation windows (Eq (2.2)).
RS Mean Trim	Trimmed version of the RS Mean obtained excluding the highest and lowest forecast.

Tables 5–10 show the results of the model confidence set for each considered GARCH-type specification. For each method, the mean loss of the expanding window method, which has been considered as a benchmark, has been reported along with the ratio of the mean loss for each combination to the mean loss for the expanding window method. Obviously, a value of the ratio below unit indicates that the forecasting method beats the benchmark according to the loss function metric. Bold entries denote the forecast combinations in the superior set of models at a confidence level $\alpha = 0.15$. The asterisk is in correspondence of the smallest mean loss among the forecasting methods.

For the GARCH (1,1) with symmetric error distributions (Table 5), it is evident that almost all the considered combinations have a ratio of the mean loss to the mean loss for the expanding window method less then one, indicating a better forecasting performance with respect to the benchmark. Moreover, the expanding window is always excluded from the SSM while the mean window combination with $v = 900$ and location weights enters the SSM, for all the distributions and for all the five considered loss functions. In same cases, also the mean window combinations with location weights and high values of v, i.e. $v = 700$ and $v = 800$, are included in SSM. The results for GARCH (1,1) with Normal innovations confirm the findings in [15]. Note that for all the distributions and for all the loss functions, the smallest mean loss value is in correspondence of the mean window combination with $v = 900$ and location weights. When the distribution of the errors are skewed (Table 6), similar results hold. The only exception is the GARCH (1,1) with skew Student$-T$ distribution in which, the combination "RS Mean" is in the SSM but, only for MSE and MAD loss functions, it has the smallest mean loss value.

For the EGARCH (1,1) with symmetric distributions for the innovations (Table 7), the mean window combinations with $v = 700$, both with location and equal weights, are in SSM for all the five

Table 5. Out-of-sample volatility forecasting results for GARCH (1,1) with symmetric distributions: Loss functions

	GARCH (1,1) NORM					GARCH (1,1) STD					GARCH (1,1) GED				
	QLIKE	MSE	MAE	MAD	MSD	QLIKE	MSE	MAE	MAD	MSD	QLIKE	MSE	MAE	MAD	MSD
Expanding Wind	0,6315	0,2095	0,3732	0,1305	0,3028	0,6098	0,2011	0,3584	0,1232	0,2916	0,6180	0,2040	0,3639	0,1259	0,2958
RS Mean	0,9911	0,9742	0,9835	0,9808	0,9883	0,9954	0,9842	0,9894	0,9906	0,9932	0,9934	0,9777	0,9859	0,9851	0,9906
RS Mean Trim	0,9971	0,9902	0,9933	0,9939	0,9957	0,9981	0,9944	0,9951	0,9975	0,9970	0,9981	0,9916	0,9942	0,9960	0,9966
Mean Wind E 100	0,9951	0,9819	0,9889	0,9877	0,9927	0,9991	0,9917	0,9948	0,9971	0,9975	0,9964	0,9850	0,9905	0,9908	0,9941
Mean Wind T 100	0,9974	0,9884	0,9928	0,9930	0,9956	1,0008	0,9978	0,9982	1,0016	0,9998	0,9983	0,9911	0,9941	0,9955	0,9967
Mean Wind L 100	0,9917	0,9736	0,9827	0,9806	0,9882	0,9978	0,9873	0,9913	0,9943	0,9953	0,9938	0,9781	0,9853	0,9853	0,9903
Mean Wind E 200	0,9946	0,9808	0,9881	0,9867	0,9921	0,9985	0,9905	0,9938	0,9960	0,9968	0,9959	0,9839	0,9897	0,9897	0,9934
Mean Wind T 200	0,9969	0,9875	0,9921	0,9922	0,9951	1,0006	0,9970	0,9977	1,0010	0,9995	0,9980	0,9903	0,9936	0,9949	0,9963
Mean Wind L 200	0,9911	0,9722	0,9816	0,9792	0,9873	0,9969	0,9855	0,9899	0,9925	0,9942	0,9930	0,9765	0,9841	0,9836	0,9894
Mean Wind E 300	0,9942	0,9797	0,9875	0,9856	0,9916	0,9982	0,9899	0,9933	0,9952	0,9964	0,9955	0,9830	0,9891	0,9888	0,9930
Mean Wind T 300	0,9967	0,9862	0,9915	0,9914	0,9947	1,0005	0,9961	0,9972	1,0005	0,9992	0,9979	0,9893	0,9930	0,9942	0,9959
Mean Wind L 300	0,9904	0,9707	0,9808	0,9777	0,9866	0,9962	0,9846	0,9891	0,9911	0,9934	0,9923	0,9754	0,9832	0,9822	0,9886
Mean Wind E 400	0,9942	0,981	0,9877	0,9862	0,9917	0,9981	0,9907	0,9935	0,9955	0,9964	0,9954	0,984	0,9892	0,9892	0,9930
Mean Wind T 400	0,9964	0,9861	0,9909	0,9909	0,9942	1,0001	0,9956	0,9966	0,9998	0,9987	0,9975	0,9890	0,9924	0,9937	0,9954
Mean Wind L 400	0,9904	0,9725	0,9811	0,9784	0,9866	0,9961	0,9858	0,9892	0,9914	0,9934	0,9922	0,9769	0,9834	0,9827	0,9886
Mean Wind E 500	0,9935	0,9786	0,9861	0,9843	0,9905	0,9976	0,9893	0,9923	0,9942	0,9955	0,9948	0,9822	0,9878	0,9875	0,9919
Mean Wind T 500	0,9958	0,9850	0,9899	0,9899	0,9934	0,9998	0,9954	0,9961	0,9995	0,9984	0,9971	0,9883	0,9917	0,9930	0,9949
Mean Wind L 500	0,9898	0,9703	0,9794	0,9765	0,9855	0,9956	0,9845	0,9879	0,9900	0,9924	0,9916	0,9752	0,9819	0,9809	0,9875
Mean Wind E 600	0,9926	0,9764	0,9849	0,9823	0,9895	0,9967	0,9869	0,9907	0,9919	0,9943	0,9939	0,9799	0,9865	0,9854	0,9909
Mean Wind L 600	0,9883	0,9668	0,9774	0,9733	0,9839	0,9938	0,9806	0,9853	0,9860	0,9903	0,9900	0,9716	0,9797	0,9775	0,9857
Mean Wind E 700	0,9909	0,9735	0,9823	0,9790	0,9874	0,9945	0,9837	0,9876	0,9879	0,9916	0,9920	0,9769	0,9838	0,9818	0,9885
Mean Wind L 700	**0,9862**	0,9633	**0,9745**	**0,9693**	**0,9813**	**0,9910**	0,9767	**0,9815**	0,9810	**0,9870**	0,9876	0,9680	0,9764	0,9731	0,9828
Mean Wind E 800	0,9907	0,9742	0,9827	0,9792	0,9875	0,9944	0,9843	0,9883	0,9882	0,9920	0,9918	0,9775	0,9843	0,9821	0,9888
Mean Wind L 800	**0,9858**	0,9636	**0,9747**	**0,9690**	**0,9813**	**0,9908**	0,9771	0,9820	0,9810	**0,9871**	0,9872	0,9682	0,9767	0,9728	0,9829
Mean Wind E 900	0,9905	0,9719	0,9822	0,9787	0,9875	0,9949	0,9828	0,9880	0,9882	0,9922	0,9918	0,9749	0,9833	0,9810	0,9883
Mean Wind L 900	**0,9858***	**0,9608***	**0,9738***	**0,9679***	**0,9810***	**0,9905***	**0,9743***	**0,9808***	**0,9795***	**0,9865***	**0,9867***	**0,9649***	**0,9752***	**0,9708***	**0,9820***

Note: The entries for the GARCH (1,1) expanding window method give the mean loss for this model. The entries for the combinations give the ratio of the mean loss for each of them to the mean loss for the GARCH (1,1) expanding window method. A bold entry denotes the models in the Superior Set of Models. The asterisk is in correspondence of the smallest mean loss among the forecasting methods.The forecasting methods are described in Table 4. NORM, STD and GED indicate the standardized Normal distribution, the standardized Student$-T$ distribution and the standardized Generalized error distribution for the innovations, respectively.

loss functions and for all the distributions. Moreover, although in the case of Student$-T$ and GED distributions, the SSM sometimes includes the expanding window method, the ratio of the mean loss with $v = 800$ and $v = 900$ to the mean loss of the expanding window method is almost always less then one. Furthermore, the smallest mean loss value is in correspondence of the mean window combinations with $v = 700$, with location or equal weights, for all the distributions and for all the loss functions. When considering EGARCH (1,1) with skewed distributions (Table 8), again, the mean window combination with $v = 700$ and location weights seems to have good forecasting performance being always in the SSM. Moreover, it always has the smallest mean loss value with the exception of the EGARCH (1,1) with skew Student$-T$ distribution for MAE and MSD loss functions.

For the GJR-GARCH (1,1) with symmetric error distribution (Table 9), the expanding window is always excluded from the SSM while the mean window combination with $v = 900$ and location weights always enters, for all the distributions and for all the five considered loss functions. In most cases, the SSM includes also the mean window combinations with $v = 700$ and $v = 800$ and location weights Note that the smallest mean loss value is in correspondence of the mean window combination with $v = 900$ and location weights with the only exception of the MSE loss function for the GJR-GARCH (1,1) with Student$-T$ distribution; in this case the smallest values is in correspondence of the mean window combination with $v = 800$ and location weights. Similar results hold for the GJR-GARCH (1,1) with skewed error distributions (Table 10). Note that, for Student$-T$ and GED distribution, in the case of MSE loss function, also the "RS-mean" combination seems to have the

Table 6. Out-of-sample volatility forecasting results for GARCH (1,1) with skewed distributions: Loss functions

	GARCH (1,1) SNORM					GARCH (1,1) SSTD					GARCH (1,1) SGED				
	QLIKE	MSE	MAE	MAD	MSD	QLIKE	MSE	MAE	MAD	MSD	QLIKE	MSE	MAE	MAD	MSD
Expanding Wind	0,6253	0,2066	0,3688	0,1283	0,2996	0,6104	0,2015	0,3587	0,1234	0,2919	0,6158	0,2032	0,3623	0,1252	0,2947
RS Mean	0,9889	0,9705	0,9804	0,9762	0,9856	**0,9903**	**0,9755***	**0,9830**	**0,9805***	**0,9876**	0,9891	0,9711	0,9805	0,9768	0,9858
RS Mean Trim	0,9956	0,9879	0,9913	0,9908	0,9939	0,9941	0,9881	0,9904	0,9897	0,9928	0,9956	0,9876	0,9910	0,9907	0,9937
Mean Wind E 100	0,9947	0,9826	0,9886	0,9875	0,9922	0,9994	0,9950	0,9959	0,9990	0,9981	0,9961	0,9866	0,9907	0,9912	0,9939
Mean Wind T 100	0,9973	0,9894	0,9930	0,9933	0,9955	1,0017	1,0016	0,9999	1,0042	1,0010	0,9985	0,9931	0,9949	0,9966	0,9970
Mean Wind L 100	0,9911	0,9747	0,9822	0,9804	0,9874	0,9977	0,9911	0,9921	0,9959	0,9955	0,9928	0,9796	0,9848	0,9848	0,9895
Mean Wind E 200	0,9941	0,9816	0,9878	0,9865	0,9916	0,9988	0,9938	0,9949	0,9978	0,9973	0,9955	0,9855	0,9898	0,9901	0,9932
Mean Wind T 200	0,9969	0,9885	0,9923	0,9926	0,9950	1,0014	1,0007	0,9993	1,0036	1,0005	0,9981	0,9922	0,9942	0,9959	0,9966
Mean Wind L 200	0,9903	0,9733	0,9810	0,9788	0,9865	0,9967	0,9892	0,9906	0,9940	0,9943	0,9920	0,9781	0,9835	0,9832	0,9885
Mean Wind E 300	0,9937	0,9806	0,9871	0,9854	0,9910	0,9985	0,9933	0,9944	0,9971	0,9969	0,9952	0,9848	0,9892	0,9892	0,9928
Mean Wind T 300	0,9966	0,9873	0,9917	0,9917	0,9946	1,0013	1.0001	0,9989	1,0032	1,0003	0,9980	0,9913	0,9937	0,9953	0,9962
Mean Wind L 300	0,9896	0,9721	0,9801	0,9774	0,9857	0,9960	0,9885	0,9897	0,9926	0,9934	0,9913	0,9772	0,9826	0,9818	0,9877
Mean Wind E 400	0,9936	0,9817	0,9872	0,9857	0,9910	0,9983	0,9937	0,9943	0,9970	0,9967	0,9950	0,9856	0,9892	0,9894	0,9927
Mean Wind T 400	0,9962	0,9872	0,9911	0,9911	0,9940	1,0007	0,9990	0,9980	1,0021	0,9996	0,9975	0,9908	0,9930	0,9946	0,9956
Mean Wind L 400	0,9894	0,9737	0,9802	0,9777	0,9855	0,9957	0,9893	0,9896	0,9925	0,9932	0,9911	0,9785	0,9827	0,9821	0,9876
Mean Wind E 500	0,9930	0,9798	0,9858	0,9842	0,9930	0,9978	0,9926	0,9931	0,9958	0,9959	0,9944	0,9841	0,9879	0,9879	0,9917
Mean Wind T 500	0,9960	0,9866	0,9904	0,9906	0,9936	1,0007	0,9993	0,9978	1,0022	0,9995	0,9974	0,9908	0,9926	0,9944	0,9953
Mean Wind L 500	0,9889	0,9719	0,9787	0,9761	0,9845	0,9952	0,9882	0,9883	0,9911	0,9922	0,9906	0,9772	0,9813	0,9805	0,9865
Mean Wind E 600	0,9918	0,9771	0,9841	0,9816	0,9886	0,9966	0,9901	0,9914	0,9933	0,9944	0,9933	0,9816	0,9863	0,9855	0,9904
Mean Wind L 600	0,9870	0,9680	0,9762	0,9722	0,9824	0,9932	0,9840	0,9854	0,9869	0,9898	0,9887	0,9734	0,9788	0,9767	0,9844
Mean Wind E 700	0,9902	0,9746	0,9818	0,9786	0,9866	0,9945	0,9870	0,9883	0,9894	0,9918	0,9915	0,9788	0,9836	0,9821	0,9881
Mean Wind L 700	0,9849	0,9648	0,9733	0,9684	**0,9799**	**0,9905**	**0,9803**	**0,9817**	**0,9820**	**0,9866**	0,9864	0,9713	0,9756	0,9725	0,9817
Mean Wind E 800	0,9899	0,9750	0,9821	0,9785	0,9866	0,9943	0,9873	0,9887	0,9895	0,9920	0,9913	0,9793	0,9841	0,9822	0,9883
Mean Wind L 800	**0,9845**	0,9650	0,9736	0,9680	**0,9799**	**0,9902**	**0,9805**	**0,9820**	**0,9819**	**0,9867**	0,9861	0,9702	0,9759	0,9723	0,9818
Mean Wind E 900	0,9899	0,9729	0,9812	0,9778	0,9863	0,9950	0,9869	0,9888	0,9901	0,9924	0,9914	0,9776	0,9833	0,9816	0,9880
Mean Wind L 900	**0,9841***	**0,9623***	**0,9723***	**0,9665***	**0,9792***	**0,9899***	**0,9787**	**0,9810***	**0,9809**	**0,9861***	**0,9855***	**0,9676***	**0,9745***	**0,9706***	**0,9809***

Note: The entries for the GARCH (1,1) expanding window method give the mean loss for this model. The entries for the combinations give the ratio of the mean loss for each of them to the mean loss for the GARCH (1,1) expanding window method. A bold entry denotes the models in the Superior Set of Models. The asterisk is in correspondence of the smallest mean loss among the forecasting methods. The forecasting methods are described in Table 4. SNORM, SSTD and SGED indicate the standardized skew Normal distribution, the standardized skew Student$-T$ distribution and the standardized skew Generalized error distribution for the innovations, respectively.

same forecasting performance.

In the second step of the out-of-sample analysis, a comparison, in term of forecasting accuracy, among the considered GARCH-type models and the best forecast combinations identified in the previous analysis has been performed. Again, the model confidence set has been used at a confidence level $\alpha = 0.15$. The results are reported in Table 11 in which the entries give the ratio of the mean loss for each of the selected forecasting method to the mean loss for the GARCH (1,1) expanding window method with Normal error distribution. It is evident that the all the forecast combinations involving the GJR-GARCH model and the EGARCH with skewed distributions of the innovations are always excluded from the SSM. Moreover, the smallest value of the MSE and MAE loss functions is obtained in correspondence of the GARCH (1,1) Student$-T$ - mean window combination with $v = 900$ and location weights; for QLIKE and MAD loss functions, it is in correspondence of the EGARCH (1,1) Student$-T$ - mean window combination with $v = 700$ and, again, with location weights; for MSD loss function, it is in correspondence of the EGARCH (1,1) Student$-T$ - mean window combination with $v = 700$ and with equal weights. Once again, this result confirms the findings in [15] that a high value of the parameter v and a forecasting scheme with location weights seem the best choice in forecasting time series with structural breaks.

Table 7. Out-of-sample volatility forecasting results for EGARCH (1,1) with symmetric distributions: Loss functions

	EGARCH (1,1) NORM					**EGARCH (1,1) STD**					**EGARCH (1,1) GED**				
	QLIKE	MSE	MAE	MAD	MSD	QLIKE	MSE	MAE	MAD	MSD	QLIKE	MSE	MAE	MAD	MSD
Expanding Wind	0,6421	0,2151	0,3802	0,1345	0,3081	**0,6231**	**0,2075**	**0,3664**	**0,1278**	**0,2979**	**0,6310**	**0,2106**	0,3721	0,1306	**0,3021**
RS Mean	1,0055	0,9978	1,0036	1,0063	1,0050	1,0056	**1,0004**	1,0063	1,0091	1,0070	1,0051	0,9985	1,0049	1,0071	1,0058
RS Mean Trim	1,0090	1,0175	1,0111	1,0188	1,0097	1,0087	1,0187	1,0126	1,0202	1,0108	1,0087	1,0179	1,0119	1,0194	1,0102
Mean Wind E 100	1,0043	1,0051	1,0046	1,0079	1,0044	1,0057	1,0166	1,0101	1,0148	1,0078	1,0043	1,0099	1,0070	1,0101	1,0056
Mean Wind T 100	1,0051	1,0135	1,0068	1,0122	1,0055	1,0068	1,0268	1,0129	1,0197	1,0091	1,0053	1,0191	1,0096	1,0149	1,0069
Mean Wind L 100	1,0072	1,0084	1,0095	1,0126	1,0085	1,0086	1,0208	1,0166	1,0201	1,0128	1,0070	1,0135	1,0124	1,0147	1,0098
Mean Wind E 200	1,0042	1,0044	1,0044	1,0075	1,0044	1,0054	1,0153	1,0096	1,0140	1,0075	1,0041	1,0089	1,0067	1,0095	1,0054
Mean Wind T 200	1,0056	1,0141	1,0075	1,0131	1,0061	1,0071	1,0270	1,0134	1,0204	1,0096	1,0057	1,0196	1,0102	1,0157	1,0075
Mean Wind L 200	1,0068	1,0068	1,0090	1,0114	1,0081	1,0078	1,0181	1,0155	1,0180	1,0119	1,0063	1,0113	1,0115	1,0131	1,0092
Mean Wind E 300	1,0038	1,0029	1,0037	1,0065	1,0039	1,0050	1,0136	1,0088	1,0127	1,0069	1,0037	1,0074	1,0059	1,0084	1,0049
Mean Wind T 300	1,0054	1,0118	1,0068	1,0120	1,0057	1,0070	1,0245	1,0126	1,0191	1,0092	1,0056	1,0174	1,0095	1,0145	1,0071
Mean Wind L 300	1,0061	1,0049	1,0076	1,0096	1,0071	1,0069	1,0157	1,0137	1,0157	1,0106	1,0056	1,0094	1,0100	1,0112	1,0080
Mean Wind E 400	1,0044	1,0053	1,0051	1,0081	1,0048	1,0053	1,0144	1,0096	1,0135	1,0075	1,0041	1,0090	1,0069	1,0096	1,0056
Mean Wind T 400	1,0056	1,0123	1,0075	1,0124	1,0063	1,0067	1,0231	1,0123	1,0184	1,0090	1,0055	1,0170	1,0097	1,0144	1,0073
Mean Wind L 400	1,0067	1,0081	1,0094	1,0117	1,0082	1,0071	1,0166	1,0146	1,0167	1,0111	1,0060	1,0113	1,0113	1,0126	1,0088
Mean Wind E 500	1,0040	1,0010	1,0035	1,0061	1,0040	1,0051	1,0120	1,0089	1,0123	1,0072	1,0037	1,0055	1,0056	1,0078	1,0049
Mean Wind T 500	1,0061	1,0118	1,0076	1,0130	1,0066	1,0072	1,0246	1,0135	1,0197	1,0099	1,0059	1,0172	1,0102	1,0151	1,0078
Mean Wind L 500	1,0061	1,0021	1,0076	1,0086	1,0072	1,0068	1,0126	1,0131	1,0145	1,0104	1,0054	1,0062	1,0094	1,0099	1,0078
Mean Wind E 600	1,0036	0,9994	1,0025	1,0046	1,0032	1,0047	1,0095	1,0077	1,0105	1,0063	1,0034	1,0037	1,0046	1,0063	1,0042
Mean Wind L 600	1,0048	0,9986	1,0051	1,0051	1,0053	1,0053	1,0080	1,0103	1,0103	1,0082	1,0042	1,0025	1,0070	1,0063	1,0058
Mean Wind E 700	**0,9981**	**0,9881**	**0,9949***	**0,9936**	**0,9969***	**0,9991**	**0,9973**	**0,9994***	**0,9988**	**0,9994***	**0,9981**	**0,9921**	**0,9968***	**0,9953**	**0,9978***
Mean Wind L 700	**0,9979***	**0,9842***	**0,9951**	**0,9913***	**0,9970**	**0,9983***	**0,9921***	**0,9996**	**0,9954***	**0,9996**	**0,9975***	**0,9877***	**0,9970**	**0,9924***	**0,9978***
Mean Wind E 800	**0,9990**	0,9900	0,9966	0,9960	0,9983	**0,9993**	**0,9968**	**0,9999**	**0,9996**	**1,0001**	**0,9986**	**0,9928**	**0,9979**	0,9969	**0,9987**
Mean Wind L 800	0,9994	0,9879	0,9976	0,9952	0,9991	**0,9989**	**0,9932**	**1,0009**	**0,9975**	**1,0007**	**0,9985**	**0,9899**	0,9990	0,9952	0,9994
Mean Wind E 900	1,0031	0,9947	1,0023	1,0031	1,0033	1,0038	1,0028	1,0065	1,0077	1,0057	1,0027	0,9978	1,0037	1,0041	1,0038
Mean Wind L 900	1,0029	0,9913	1,0026	1,0008	1,0034	1,0023	**0,9964**	1,0060	1,0031	1,0051	1,0017	**0,9929**	1,0035	1,0005	1,0034

Note: The entries for the EGARCH (1,1) expanding window method give the mean loss for this model. The entries for the combinations give the ratio of the mean loss for each of them to the mean loss for the EGARCH (1,1) expanding window method. A bold entry denotes the models in the Superior Set of Models. The asterisk is in correspondence of the smallest mean loss among the forecasting methods. The forecasting methods are described in Table 4. NORM, STD and GED indicate the standardized Normal distribution, the standardized Student−T distribution and the standardized Generalized error distribution for the innovations, respectively.

4. Concluding Remarks

In this paper, focusing on GARCH-type class of models, we have empirically investigated the effectiveness in forecasting of some combinations able to take into account possible structural breaks in the observed time series. In particular, the combinations proposed in [15] have been evaluated and compared to some alternatives suggested in [2, 3], focusing on S&P500 stock market index. In the in-sample analysis, some alternative conditional volatility dynamics have been considered along with different conditional distributions for the innovations. The Nyblom test evidences the presence of parameter instability in S&P500 stock market index, in all the models and for all the assumed distributions for the innovations. This result has been confirmed by analysing the unconditional variance of the series; the k_2 test, implemented with a binary segmentation search, has actually identified structural breaks. In the out-of-sample analysis, for each model specification and for all the considered distributions for the innovations, the proposed forecast combinations have been evaluated and compared in terms of their predictive ability by using the model confidence set with five different loss functions. The analysis has highlighted that, in general, the combinations offer a suitable approach for forecasting in presence of structural breaks, confirming the findings in [2, 14]. Moreover, the combinations proposed in [15] seem to have better performance, especially when the parameter v assumes high values and a forecasting scheme with location weights. By comparing the considered GARCH-type models and the best forecast combinations identified in the previous

Table 8. Out-of-sample volatility forecasting results for EGARCH (1,1) with skewed distributions: Loss functions

	EGARCH (1,1) SNORM					EGARCH (1,1) SSTD					EGARCH (1,1) SGED				
	QLIKE	MSE	MAE	MAD	MSD	QLIKE	MSE	MAE	MAD	MSD	QLIKE	MSE	MAE	MAD	MSD
Expanding Wind	0,6361	0,2121	0,3756	0,1322	0,3048	**0,6225**	**0,2076**	**0,3661***	**0,1277**	**0,2976***	**0,6284**	0,2098	0,3704	0,1297	0,3008
RS Mean	1,0028	0,9945	1,0000	1,0018	1,0018	1,0032	**1,0003**	**1,0036**	1,0054	1,0041	1,0021	0,9962	1,0013	1,0020	1,0022
RS Mean Trim	1,0075	1,0151	1,0089	1,0159	1,0079	1,0072	1,0176	1,0112	1,0174	1,0092	1,0066	1,0151	1,0093	1,0151	1,0078
Mean Wind E 100	1,0032	1,0042	1,0035	1,0066	1,0034	1,0074	1,0251	1,0141	1,0200	1,0104	1,0043	1,0133	1,0081	1,0115	1,0061
Mean Wind T 100	1,0043	1,0123	1,0059	1,0111	1,0048	1,0088	1,0352	1,0175	1,0253	1,0122	1,0058	1,0225	1,0113	1,0167	1,0079
Mean Wind L 100	1,0048	1,0061	1,0068	1,0092	1,0060	1,0094	1,0296	1,0201	1,0242	1,0145	1,0057	1,0160	1,0124	1,0140	1,0090
Mean Wind E 200	1,0029	1,0032	1,0030	1,0058	1,0031	1,0070	1,0236	1,0137	1,0189	1,0101	1,0039	1,0121	1,0076	1,0106	1,0057
Mean Wind T 200	1,0046	1,0126	1,0064	1,0116	1,0052	1,0089	1,0351	1,0177	1,0256	1,0125	1,0059	1,0225	1,0116	1,0170	1,0082
Mean Wind L 200	1,0041	1,0041	1,0059	1,0075	1,0052	1,0084	1,0264	1,0185	1,0216	1,0133	1,0049	1,0135	1,0112	1,0120	1,0080
Mean Wind E 300	1,0026	1,0019	1,0024	1,0049	1,0026	1,0065	1,0221	1,0127	1,0176	1,0093	1,0036	1,0109	1,0070	1,0096	1,0052
Mean Wind T 300	1,0044	1,0102	1,0055	1,0103	1,0046	1,0087	1,0328	1,0167	1,0243	1,0119	1,0056	1,0202	1,0107	1,0157	1,0077
Mean Wind L 300	1,0034	1,0024	1,0046	1,0058	1,0042	1,0074	1,0242	1,0167	1,0193	1,0119	1,0042	1,0118	1,0098	1,0102	1,0070
Mean Wind E 400	1,0028	1,0034	1,0032	1,0057	1,0031	1,0066	1,0217	1,0131	1,0177	1,0096	1,0037	1,0115	1,0075	1,0101	1,0055
Mean Wind T 400	1,0041	1,0095	1,0054	1,0097	1,0046	1,0083	1,0303	1,0159	1,0230	1,0114	1,0053	1,0189	1,0103	1,0148	1,0074
Mean Wind L 400	1,0036	1,0044	1,0056	1,0067	1,0048	1,0073	1,0235	1,0169	1,0192	1,0120	1,0041	1,0124	1,0103	1,0106	1,0072
Mean Wind E 500	1,0028	1,0003	1,0022	1,0046	1,0028	1,0064	1,0203	1,0127	1,0170	1,0094	1,0036	1,0091	1,0068	1,0090	1,0053
Mean Wind T 500	1,0051	1,0105	1,0066	1,0115	1,0057	1,0090	1,0330	1,0178	1,0248	1,0127	1,0061	1,0204	1,0115	1,0164	1,0084
Mean Wind L 500	1,0036	0,9997	1,0045	1,0048	1,0044	1,0071	1,0203	1,0157	1,0175	1,0115	1,0040	1,0084	1,0089	1,0088	1,0066
Mean Wind E 600	1,0017	0,9973	1,0001	1,0015	1,0011	1,0056	1,0169	1,0109	1,0143	1,0080	1,0028	1,0062	1,0051	1,0064	1,0040
Mean Wind L 600	1,0013	0,9945	1,0006	0,9994	1,0012	1,0051	1,0146	1,0120	1,0122	1,0085	1,0021	1,0034	1,0056	1,0039	1,0039
Mean Wind E 700	0,9964	0,9864	**0,9927**	0,9909	0,9949	**1,0002**	**1,0045**	**1,0028**	1,0028	**1,0014**	**0,9977**	0,9946	**0,9974**	0,9957	**0,9978**
Mean Wind L 700	**0,9945***	**0,9805***	**0,9912***	**0,9859***	**0,9935***	**0,9982***	**0,9981***	**1,0015**	**0,9975***	**1,0001**	**0,9956***	**0,9883***	**0,9959***	**0,9903***	**0,9962***
Mean Wind E 800	0,9971	0,9879	0,9943	0,9931	0,9962	**1,0001**	**1,0033**	**1,0027**	1,0029	**1,0016**	**0,9979**	0,9949	0,9981	0,9967	0,9984
Mean Wind L 800	0,9960	0,9837	0,9937	0,9896	0,9955	**0,9987**	**0,9987**	**1,0025**	**0,9991**	**1,0011**	**0,9965**	0,9902	0,9975	0,9927	0,9975
Mean Wind E 900	1,0013	0,9931	1,0001	1,0005	1,0014	1,0050	1,0112	1,0103	1,0124	1,0079	1,0024	1,0012	1,0047	1,0050	1,0040
Mean Wind L 900	0,9994	0,9871	0,9986	0,9952	0,9998	1,0022	**1,0028**	**1,0076**	1,0052	1,0055	0,9998	0,9937	1,0024	0,9984	1,0018

Note: The entries for the EGARCH (1,1) expanding window method give the mean loss for this model. The entries for the combinations give the ratio of the mean loss for each of them to the mean loss for the GJR-GARCH (1,1) expanding window method. A bold entry denotes the models in the Superior Set of Models. The asterisk is in correspondence of the smallest mean loss among the forecasting methods. The forecasting methods are described in Table 4. SNORM, SSTD and SGED indicate the standardized skew Normal distribution, the standardized skew Student−T distribution and the standardized skew Generalized error distribution for the innovations, respectively.

analysis, it has been highlighted that all the forecast combinations involving the GJR-GARCH model and the EGARCH with skewed distributions of the innovations are always excluded from the SSM. Furthermore, the EGARCH (1,1) model with Student−T innovations and GARCH (1,1) model with the same error distribution, associated to forecast combinations with high values of v seem to have better forecasting performance.

In any case, several different aspects should be further explored in future research to get a better insight into the usage of the proposed forecast combinations. The procedure should be compared with a component GARCH model, such as Spline GARCH, which is able to capture lower frequency variations on the volatility like seasonality and trends, or GARCH-MIDAS, which allows to link the daily observations on stock returns with macroeconomic variables, sampled at lower frequencies, in order to examine directly the macroeconomic variables impact on the stock volatility.

Moreover, an application of the proposed methodology to forecast various risk measures such as Value-at-Risk (VaR), Conditional VaR and coherent Expected Shortfall would also be of interest.

Table 9. Out-of-sample volatility forecasting results for GJR-GARCH (1,1) with symmetric distributions: Loss functions

	GJR-GARCH (1,1) NORM					GJR-GARCH (1,1) STD					GJR-GARCH (1,1) GED				
	QLIKE	MSE	MAE	MAD	MSD	QLIKE	MSE	MAE	MAD	MSD	QLIKE	MSE	MAE	MAD	MSD
Expanding Wind	0,6123	0,2060	0,3627	0,1254	0,2941	0,5991	0,2028	0,3538	0,1211	0,2869	0,6046	0,2040	0,3575	0,1229	0,2899
RS Mean	0,9874	0,9793	0,9862	0,9763	0,9874	0,9917	0,9945	0,9951	0,9876	0,9936	0,9884	0,9854	0,9895	0,9801	0,9894
RS Mean Trim	0,9973	0,9992	0,9982	0,9961	0,9978	1,0002	1,0102	1,0050	1,0042	1,0024	0,9979	1,0033	1,0007	0,9986	0,9993
Mean Wind E 100	0,9892	0,9822	0,9886	0,9794	0,9895	0,9931	1,0008	0,9979	0,9912	0,9954	0,9897	0,9898	0,9918	0,9828	0,9910
Mean Wind T 100	0,9938	0,9908	0,9943	0,9886	0,9945	0,9976	1,0095	1,0038	1,0002	1,0005	0,9943	0,9981	0,9976	0,9919	0,9961
Mean Wind L 100	0,9823	0,9745	0,9817	0,9674	0,9827	0,9874	0,9998	0,9942	0,9831	0,9906	0,9829	0,9851	0,9859	0,9720	0,9847
Mean Wind E 200	0,9884	0,9809	0,9877	0,9780	0,9887	0,9922	0,9994	0,9968	0,9896	0,9945	0,9889	0,9884	0,9908	0,9813	0,9901
Mean Wind T 200	0,9930	0,9896	0,9934	0,9872	0,9936	0,9968	1,0086	1,0030	0,9989	0,9997	0,9935	0,9971	0,9967	0,9905	0,9952
Mean Wind L 200	0,9813	0,9727	0,9804	0,9654	0,9816	0,9861	0,9975	0,9925	0,9805	0,9891	0,9817	0,9831	0,9845	0,9698	0,9834
Mean Wind E 300	0,9879	0,9802	0,9872	0,9769	0,9881	0,9917	0,9994	0,9964	0,9887	0,9940	0,9883	0,9882	0,9903	0,9804	0,9896
Mean Wind T 300	0,9927	0,9887	0,9931	0,9863	0,9933	0,9966	1,0084	1,0028	0,9984	0,9995	0,9932	0,9965	0,9964	0,9898	0,9950
Mean Wind L 300	0,9806	0,9720	0,9796	0,9641	0,9807	0,9852	0,9974	0,9916	0,9792	0,9881	0,9810	0,9829	0,9837	0,9685	0,9826
Mean Wind E 400	0,9875	0,9803	0,9868	0,9763	0,9877	0,9910	0,9978	0,9954	0,9873	0,9932	0,9878	0,9874	0,9896	0,9793	0,9890
Mean Wind T 400	0,9917	0,9879	0,9921	0,9846	0,9924	0,9953	1,0056	1,0012	0,9959	0,9981	0,9921	0,9949	0,9951	0,9878	0,9938
Mean Wind L 400	0,9801	0,9721	0,9790	0,9632	0,9801	0,9843	0,9952	0,9902	0,9772	0,9870	0,9802	0,9818	0,9827	0,9671	0,9817
Mean Wind E 500	0,9867	0,9777	0,9855	0,9743	0,9867	0,9903	0,9969	0,9946	0,9859	0,9924	0,9870	0,9857	0,9885	0,9776	0,9880
Mean Wind T 500	0,9915	0,9868	0,9916	0,9840	0,9921	0,9955	1,0069	1,0015	0,9964	0,9983	0,9920	0,9951	0,9949	0,9876	0,9936
Mean Wind L 500	0,9792	0,9694	0,9776	0,9611	0,9791	0,9836	0,9943	0,9893	0,9758	0,9861	0,9794	0,9801	0,9815	0,9653	0,9807
Mean Wind E 600	0,9854	0,9753	0,9837	0,9718	0,9852	0,9892	0,9947	0,9930	0,9836	0,9910	0,9858	0,9833	0,9868	0,9752	0,9866
Mean Wind L 600	0,9774	0,9658	0,9751	0,9574	0,9769	0,9818	0,9909	0,9868	0,9722	0,9841	0,9777	0,9766	0,9791	0,9618	0,9786
Mean Wind E 700	0,9834	0,9718	0,9811	0,9679	0,9830	0,9871	0,9909	0,9903	0,9794	0,9887	0,9839	0,9801	0,9844	0,9714	0,9845
Mean Wind L 700	**0,9747**	**0,9611**	**0,9715**	**0,9524**	**0,9739**	0,9790	**0,9855**	**0,9829**	0,9665	0,9807	0,9751	**0,9719**	**0,9755**	0,9568	0,9756
Mean Wind E 800	0,9830	0,9715	0,9809	0,9675	0,9827	0,9859	**0,9892**	0,9891	0,9777	0,9876	0,9831	**0,9790**	0,9837	0,9704	0,9838
Mean Wind L 800	**0,9746**	**0,9614**	**0,9717**	**0,9526**	**0,9740**	**0,9779**	**0,9842***	**0,9819**	**0,9651**	**0,9797**	0,9744	**0,9714**	**0,9751**	0,9561	0,9751
Mean Wind E 900	0,9835	0,9720	0,9818	0,9680	0,9834	0,9865	0,9917	0,9906	0,9790	0,9885	0,9833	0,9801	0,9843	0,9706	0,9842
Mean Wind L 900	**0,9743***	**0,9607***	**0,9714***	**0,9515***	**0,9736***	**0,9775***	**0,9846**	**0,9816***	**0,9641***	**0,9792***	**0,9738***	**0,9708***	**0,9744***	**0,9546***	**0,9744***

Note: The entries for the GJR-GARCH (1,1) expanding window method give the mean loss for this model. The entries for the combinations give the ratio of the mean loss for each of them to the mean loss for the GJR-GARCH (1,1) expanding window method. A bold entry denotes the models in the Superior Set of Models. The asterisk is in correspondence of the smallest mean loss among the forecasting methods. The forecasting methods are described in Table 4. NORM, STD and GED indicate the standardized Normal distribution, the standardized Student−T distribution and the standardized Generalized error distribution for the innovations, respectively.

Table 10. Out-of-sample volatility forecasting results for GJR-GARCH (1,1) with skew distributions: Loss functions

	GJR-GARCH (1,1) SNORM					GJR-GARCH (1,1) SSTD					GJR-GARCH (1,1) SGED				
	QLIKE	MSE	MAE	MAD	MSD	QLIKE	MSE	MAE	MAD	MSD	QLIKE	MSE	MAE	MAD	MSD
Expanding Wind	0,6081	0,2041	0,3596	0,1238	0,2918	0,6001	**0,2038**	0,3547	0,1215	0,2874	0,6039	0,2044	0,3572	0,1228	0,2896
RS Mean	0,9867	0,9813	0,9861	0,9759	0,9868	0,9889	**0,9940***	0,9920	0,9836	0,9903	0,9871	**0,9867**	0,9884	0,9784	0,9879
RS Mean Trim	0,9978	1,0011	0,9992	0,9972	0,9984	0,9981	1,0092	1,0028	1,0013	1,0001	0,9976	1,0041	1,0006	0,9982	0,9989
Mean Wind E 100	0,9890	0,9853	0,9896	0,9804	0,9897	0,9947	1,0095	1,0016	0,9964	0,9977	0,9903	0,9955	0,9938	0,9857	0,9921
Mean Wind T 100	0,9944	0,9937	0,9960	0,9906	0,9956	0,9997	1,0182	1,0081	1,0061	1,0034	0,9956	1,0037	1,0003	0,9956	0,9979
Mean Wind L 100	0,9815	0,9792	0,9827	0,9683	0,9825	0,9889	1,0114	0,9983	0,9890	0,9928	0,9831	0,9927	0,9881	0,9751	0,9855
Mean Wind E 200	0,9881	0,9840	0,9886	0,9788	0,9888	0,9937	1,0081	1,0004	0,9946	0,9967	0,9894	0,9941	0,9927	0,9841	0,9911
Mean Wind T 200	0,9935	0,9925	0,9950	0,9890	0,9946	0,9987	1,0174	1,0071	1,0046	1,0024	0,9946	1,0026	0,9992	0,9940	0,9968
Mean Wind L 200	0,9803	0,9774	0,9812	0,9661	0,9812	0,9875	1,0089	0,9964	0,9863	0,9912	0,9819	0,9905	0,9865	0,9726	0,9841
Mean Wind E 300	0,9876	0,9835	0,9880	0,9778	0,9882	0,9932	1,0085	1,000	0,9939	0,9961	0,9889	0,9942	0,9923	0,9833	0,9906
Mean Wind T 300	0,9931	0,9916	0,9945	0,9880	0,9942	0,9986	1,0176	1,0071	1,0043	1,0023	0,9943	1,0023	0,9990	0,9934	0,9966
Mean Wind L 300	0,9796	0,9769	0,9803	0,9647	0,9803	0,9866	1,0091	0,9955	0,9851	0,9902	0,9811	0,9907	0,9857	0,9715	0,9832
Mean Wind E 400	0,9869	0,9832	0,9874	0,9768	0,9876	0,9924	1,0061	0,9988	0,9920	0,9951	0,9882	0,9929	0,9913	0,9818	0,9897
Mean Wind T 400	0,9919	0,9908	0,9935	0,9860	0,9931	0,9970	1,0139	1,0050	1,0011	1,0006	0,9929	1,0002	0,9973	0,9908	0,9951
Mean Wind L 400	0,9787	0,9764	0,9794	0,9633	0,9794	0,9854	1,0057	0,9936	0,9824	0,9888	0,9801	0,9887	0,9843	0,9694	0,9820
Mean Wind E 500	0,9863	0,9811	0,9862	0,9751	0,9867	0,9917	1,0056	0,9980	0,9908	0,9944	0,9875	0,9918	0,9904	0,9805	0,9890
Mean Wind T 500	0,9918	0,9901	0,9931	0,9857	0,9929	0,9973	1,0159	1,0055	1,0019	1,0009	0,9930	1,0010	0,9974	0,9910	0,9951
Mean Wind L 500	0,9780	0,9743	0,9782	0,9615	0,9785	0,9847	1,0052	0,9927	0,9810	0,9879	0,9795	0,9876	0,9833	0,9680	0,9812
Mean Wind E 600	0,9847	0,9785	0,9841	0,9721	0,9848	0,9905	1,0035	0,9964	0,9885	0,9929	0,9861	0,9891	0,9885	0,9777	0,9873
Mean Wind L 600	0,9759	0,9704	0,9752	0,9573	0,9759	0,9829	1,0018	0,9901	0,9773	0,9856	0,9776	0,9837	0,9806	0,9641	0,9788
Mean Wind E 700	0,9827	0,9752	0,9816	0,9683	0,9826	0,9883	0,9995	0,9935	0,9840	0,9905	0,9842	0,9857	0,9861	0,9739	0,9852
Mean Wind L 700	0,9731	**0,9659**	**0,9717**	**0,9523**	**0,9729**	0,9801	**0,9960**	0,9861	0,9714	0,9822	0,9750	**0,9789**	**0,9770**	0,9591	0,9758
Mean Wind E 800	0,9821	0,9747	0,9812	0,9677	0,9823	0,9870	**0,9973**	0,9921	0,9821	0,9892	0,9833	**0,9845**	0,9852	0,9727	0,9844
Mean Wind L 800	**0,9729**	**0,9661**	**0,9719**	**0,9523**	**0,9729**	**0,9788**	**0,9944**	**0,9850***	**0,9698**	**0,9812**	**0,9743**	**0,9783***	**0,9766**	**0,9583**	**0,9754**
Mean Wind E 900	0,9825	0,9757	0,9821	0,9682	0,9829	0,9880	1,0018	0,9943	0,9845	0,9906	0,9837	0,9868	0,9864	0,9736	0,9851
Mean Wind L 900	**0,9724***	**0,9657***	**0,9713***	**0,9511***	**0,9723***	**0,9786***	**0,9964**	**0,9853**	**0,9697***	**0,9811***	**0,9737***	**0,9787**	**0,9761***	**0,9572***	**0,9747***

Note: The entries for the GJR-GARCH (1,1) expanding window method give the mean loss for this model. The entries for the combinations give the ratio of the mean loss for each of them to the mean loss for the GJR-GARCH (1,1) expanding window method. A bold entry denotes the models in the Superior Set of Models. The asterisk is in correspondence of the smallest mean loss among the forecasting methods. The forecasting methods are described in Table 4. SNORM, SSTD and SGED indicate the standardized skew Normal distribution, the standardized skew Student$-T$ distribution and the standardized skew Generalized error distribution for the innovations, respectively.

Table 11. Out-of-sample volatility forecasting results for GARCH-type models: Loss functions

	QLIKE	MSE	MAE	MAD	MSD		QLIKE	MSE	MAE	MAD	MSD
GARCH (1,1) NORM - Mean Wind L 700	0,9862		0,9745	0,9693	0,9813	EGARCH (1,1) SSTD - Mean Wind E 700	0,9505	0,9771	0,9529		0,9506
GARCH (1,1) NORM - Mean Wind L 800	0,9858		0,9747	0,9690	0,9813	EGARCH (1,1) SSTD - Mean Wind L 700	0,9486	0,9708	0,9517	0,9289	0,9494
GARCH (1,1) NORM - Mean Wind L 900	0,9858	0,9608	0,9738	0,9679	0,9810	EGARCH (1,1) SSTD - Mean Wind E 800	0,9504	0,9759	0,9529		0,9508
GARCH (1,1) STD - Mean Wind L 700	0,9570		**0,9424**		0,9504	EGARCH (1,1) SSTD - Mean Wind L 800	0,9491	0,9714	0,9526	0,9304	0,9503
GARCH (1,1) STD - Mean Wind L 800	0,9568				0,9505	EGARCH (1,1) SSTD - Mean Wind L 900		0,9754	0,9575		
GARCH (1,1) STD - Mean Wind L 900	0,9565	**0,9353***	**0,9417***	**0,9244**	**0,9499**	EGARCH (1,1) SGED - Expanding Wind	0,9564				
GARCH (1,1) GED - Mean Wind L 900	0,9657	0,9395	0,9508	0,9364	0,9593	EGARCH (1,1) SGED - Mean Wind E 700	0,9542		0,9546		0,9541
GARCH (1,1) SNORM - Mean Wind L 700					0,9695	EGARCH (1,1) SGED - Mean Wind L 700	0,9522	0,9643	0,9531	0,9314	0,9526
GARCH (1,1) SNORM - Mean Wind L 800	0,9750				0,9695	EGARCH (1,1) SGED - Mean Wind E 800	0,9544				
GARCH (1,1) SNORM - Mean Wind L 900	0,9746	0,9492	0,9607	0,9501	0,9688	EGARCH (1,1) SGED - Mean Wind L 800	0,9531				
GARCH (1,1) SSTD - RS Mean	0,9572	0,9384	0,9448	**0,9273**	0,9519	GJR - GARCH (1,1) NORM - Mean Wind L 700	0,9912	0,9869	0,9897	0,9814	0,9909
GARCH (1,1) SSTD - Mean Wind L 700	0,9574	0,9430	**0,9436**	0,9288	0,9509	GJR - GARCH (1,1) NORM - Mean Wind L 800	0,9911	0,9872	0,9899	0,9816	0,9910
GARCH (1,1) SSTD - Mean Wind L 800	0,9571	0,9432	0,9439	0,9286	0,9510	GJR - GARCH (1,1) NORM - Mean Wind L 900	0,9908	0,9865	0,9895	0,9804	0,9906
GARCH (1,1) SSTD - Mean Wind L 900	0,9569	0,9415	**0,9429**	0,9277	0,9504	GJR - GARCH (1,1) STD - Mean Wind L 700		0,9762	0,9649		
GARCH (1,1) SGED - Mean Wind L 900	0,9611	0,9388	0,9460	0,9309	0,9545	GJR - GARCH (1,1) STD - Mean Wind E 800		0,9798			
EGARCH (1,1) NORM - Mean Wind E 700	0,9678	0,9716	0,9669	0,9544	0,9682	GJR - GARCH (1,1) STD - Mean Wind L 800	0,9649	0,9749	0,964	0,9454	0,9638
EGARCH (1,1) NORM - Mean Wind L 700	0,9676	0,9679	0,9671	0,9522	0,9684	GJR - GARCH (1,1) STD - Mean Wind L 900	0,9645	0,9753	0,9636	0,9444	0,9633
EGARCH (1,1) NORM - Mean Wind E 800	0,9687					GJR - GARCH (1,1) GED - Mean Wind L 700		0,9770	0,9727		
EGARCH (1,1) STD - Expanding Wind	**0,9488**	0,9679	0,9479	0,9279	**0,9475**	GJR - GARCH (1,1) GED - Mean Wind E 800		0,9841			
EGARCH (1,1) STD - RS Mean		0,9683				GJR - GARCH (1,1) GED - Mean Wind L 800		0,9764	0,9722		
EGARCH (1,1) STD - Mean Wind E 700	**0,9480**	0,9652	0,9473	**0,9267**	**0,9469***	GJR - GARCH (1,1) GED - Mean Wind L 900	0,9732	0,9759	0,9715	0,9552	0,9723
EGARCH (1,1) STD - Mean Wind L 700	**0,9472***	0,9602	0,9475	**0,9236***	**0,9471**	GJR - GARCH (1,1) SNORM - Mean Wind L 700		0,9781	0,9777	0,9647	0,9793
EGARCH (1,1) STD - Mean Wind E 800	**0,9482**	0,9648	0,9478	**0,9275**	**0,9475**	GJR - GARCH (1,1) SNORM - Mean Wind L 800	0,9801	0,9782	0,9779	0,9647	0,9793
EGARCH (1,1) STD - Mean Wind L 800	**0,9478**	0,9614	0,9488	**0,9255**	**0,9481**	GJR - GARCH (1,1) SNORM - Mean Wind L 900	0,9796	0,9778	0,9773	0,9635	0,9787
EGARCH (1,1) STD - Mean Wind L 900		0,9644				GJR - GARCH (1,1) SSTD - Expanding Wind		0,9913			
EGARCH (1,1) GED - Expanding Wind	0,9575	0,9739			0,9575	GJR - GARCH (1,1) SSTD - RS Mean		0,9853			
EGARCH (1,1) GED - Mean Wind E 700	0,9557	0,9663	0,9547	0,9372	0,9553	GJR - GARCH (1,1) SSTD - Mean Wind L 700		0,9873			
EGARCH (1,1) GED - Mean Wind L 700	0,9551	0,9619	0,9550	0,9345	0,9553	GJR - GARCH (1,1) SSTD - Mean Wind E 800		0,9886			
EGARCH (1,1) GED - Mean Wind E 800	0,9562	0,9669	0,9558		0,9562	GJR - GARCH (1,1) SSTD - Mean Wind L 800	0,9648	0,9857	0,9663	0,9489	0,9643
EGARCH (1,1) GED - Mean Wind L 800	0,9561	0,9641				GJR - GARCH (1,1) SSTD - Mean Wind L 900	0,9647	0,9877	0,9666	0,9488	0,9642
EGARCH (1,1) GED - Mean Wind L 900		0,9670				GJR - GARCH (1,1) SGED - RS Mean		0,9883			
EGARCH (1,1) SNORM - Mean Wind E 700			0,9564			GJR - GARCH (1,1) SGED - Mean Wind L 700		0,9805	0,9696		
EGARCH (1,1) SNORM - Mean Wind L 700	0,9577	0,9555	0,9550	0,9353	0,9573	GJR - GARCH (1,1) SGED - Mean Wind E 800		0,9862			
EGARCH (1,1) SSTD - Expanding Wind	0,9503	0,9727	0,9503	0,9313	0,9493	GJR - GARCH (1,1) SGED - Mean Wind L 800	0,9695	0,9800	0,9692	0,9524	0,9688
EGARCH (1,1) SSTD - RS Mean		0,973	0,9538			GJR - GARCH (1,1) SGED - Mean Wind L 900	0,9690	0,9803	0,9687	0,9513	0,9682

Note: The entries for the combinations give the ratio of the mean loss for each of them to the mean loss for the GARCH (1,1) NORM expanding window method. A bold entry denotes the models in the Superior Set of Models. The asterisk is in correspondence of the smallest mean loss among the forecasting methods. The forecasting methods are described in Table 4. NORM, STD and GED indicate the standardized Normal distribution, the standardized Student$-T$ distribution and the standardized Generalized error distribution for the innovations, respectively. SNORM, SSTD and SGED are the asymmetric skewed extensions.

Acknowledgments

The author gratefully thanks two anonymous Referees for their useful comments and suggestions.

Conflict of interest

The author declares no conflict of interest.

References

1. B. Rossi and A. Inoue, Out-of-sample forecast tests robust to the choice of window size, *J. Bus. Econ. Stat.*, **30** (2012), 432–453.
2. D. E. Rapach and J. K. Strauss, Structural breaks and GARCH models of exchange rate volatility, *J. Appl. Econom.*, **23** (2008), 65–90.
3. D. E. Rapach, J. K. Strauss and M. E. Wohar, Chapter 10 Forecasting Stock Return Volatility in the Presence of Structural Breaks, in *Forecasting in the presence of structural breaks and model uncertainty*, Emerald Group Publishing Limited, (2008), 381–416.
4. A. Timmermann, Chapter 4 Forecast combinations, in *Handbook of economic forecasting*, (2006), 135–196.
5. M. H. Pesaran and A. Timmermann, Selection of estimation windows in the presence of breaks, *J. Econometrics*, **137** (2007), 134–161.
6. J. Tian and H. M. Anderson, Forecast combinations under structural break uncertainty, *Int. J. Forecasting*, **30** (2014), 161–175.
7. M. H. Pesaran, A. Pick and M. Pranovich, Optimal forecasts in the presence of structural breaks, *J. Econometrics*, **177** (2013), 134–152.
8. K. Assenmacher-Wesche and M. H. Pesaran, Forecasting the Swiss economy using VECX models: An exercise in forecast combination across models and observation windows, *Nat. Inst. Econ. Rev.*, **203** (2008), 91–108.
9. M. H. Pesaran, T. Schuermann and L. V. Smith, Forecasting economic and financial variables with global VARs, *Int. J. Forecasting*, **25** (2009), 642–675.
10. A. Schrimpf and Q. Wang, A reappraisal of the leading indicator properties of the yield curve under structural instability, *Int. J. Forecasting*, **26** (2010), 836–857.
11. D. De Gaetano, Forecast combinations in the presence of structural breaks: Evidence from U.S. equity markets, *Math.*, **6** (2018), 34.
12. M. H. Pesaran and A. Pick, Forecast combination across estimation windows, *J. Bus. Econ. Stat.*, **29** (2011), 307–318.
13. T. E. Clark and M. W. McCracken, Improving forecast accuracy by combining recursive and rolling forecasts, *Int. Econ. Rev.*, **50** (2009), 363–395.

14. D. De Gaetano, Forecast Combinations for Structural Breaks in Volatility: Evidence from BRICS Countries, *J. Risk. Financ. Manag.*, **11** (2018), 64.

15. D. De Gaetano, Forecasting with GARCH models under structural breaks: An approach based on combinations across estimation windows, *Commun. Stat. Simulat. Comput.*, **6** (2018), 1–19.

16. P. R. Hansen, A. Lunde and J. M. Nason, The model confidence set, *Econometrica*, **79** (2011), 453–497.

17. F. X. Diebold, Modelling the persistence of conditional variance: A comment, *Economet. Rev.*, **5** (1986), 51–56.

18. T. Mikosh and C. Stărică, Nonstationarities in financial time series, the long-range dependence and the IGARCH effects, *Rev. Econ. Stat.*, **86** (2004), 378–390.

19. E. Hillebrand, Neglecting parameter changes in GARCH models, *J. Econometrics*, **129** (2005), 121–138.

20. S. Hwang and P. L. V. Pereira, Small sample properties of GARCH estimates and persistence, *Eur. J. Financ.*, **12** (2006), 473–494.

21. T. Bollerslev, Generalized Autoregressive Conditional Heteroskedasticity, *J. Econometrics*, **31** (1986), 307–332.

22. D. B. Nelson, Conditional heteroschedasticity in asset returns: A new approach, *Econometrica*, **59** (1991), 217–235.

23. L. R. Glosten, R. Jagannathan and D. E. Runkle, On the Relation between the expected value and the volatility of the nominal excess return on stocks, *J. Financ.*, **48** (1993), 1779–1801.

24. C. Forbes, M. Evans, N. Hastings, et al., *Statistical distributions*, 2^{nd} edition, John Wiley & Sons, 2011.

25. J. Nyblom, Testing for the constancy of parameters over time, *J. Am. Stat. Assoc.*, **84** (1989), 223–230.

26. B. E. Hansen, Tests for parameter instability in regressions with I (1) processes, *J. Bus. Econ. Stat.*, **20** (2002), 45–59.

27. A. Sansó, V. Arragó and J. L. Carrion-i-Silvestre, Testing for change in the unconditional variance of financial time series, *Rev. Econ. Financ.*, **4** (2004), 32–53.

28. C. Inclan and G. C. Tiao, Use of cumulative sums of squares for retrospective detection of changes in variance, *J. Am. Stat. Assoc.*, **89** (1994), 913–923.

29. W. K. Newey and K. D. West, Automatic Lag Selection in Covariance Matrix estimation, *Rev. Econ. Stud.*, **61** (1994), 631–654.

30. G. J. Ross, Modelling financial volatility in the presence of abrupt changes, *Physica. A*, **392** (2013), 350–360.

31. J. G. MacKinnon, Computing numerical distribution functions in econometrics, in *High Performance Computing Systems and Applications*, Kluwer, (2000), 455–471.

32. A. J. Patton, Volatility forecast comparison using imperfect volatility proxies, *J. Econometrics*, **160** (2011), 246–256.

AIMS Press