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Research article

A new method to investigate almost periodic solutions for an Nicholson's blowflies model with time-varying delays and a linear harvesting term

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Abstract: In this paper, a delayed Nicholson's blowflies model with a linear harvesting term is investigated. By transforming the model into an equivalent integral equation, and applying a fixed point theorem in cones, we establish a sufficient condition which ensure the existence of positive almost periodic solutions for the Nicholson's blowflies model. The results of this paper are completely new and complement those of the previous studies. The approach is new.

Keywords: Nicholson's blowflies model; almost periodic solution; exponential dichotomy; time delay

1. Introduction

In recent years, various Nicholson's blowflies models have been extensively studied by many scholars due to their theoretical and practical significance in biology. In 1954 Nicholson [1] and in 1980 Gurney et al. [2] proposed the following Nicholson's blowflies model

$$\dot{x}(t) = -\delta x(t) + px(t-\tau)e^{-ax(t-\tau)}, \delta, p, \tau, a \in (0, \infty) \tag{1.1}$$

to describe the population of the Australian sheep-blowfy lucilia cuprina. Here $x(t)$ denotes the size of population at time t, p denotes the maximum per capita daily egg production rate, δ denotes the per capita daily adult death rate, $\frac{1}{a}$ denotes the size at which the blowfly population reproduces at its maximum rate, τ denotes the generation rate. Since then, model (1.1) and its revised version have been extensively investigated. For example, Kulenovic et al. [3] considered the global attractivity of model (1.1), So and Yu [4] analyzed the stability and uniform persistence of the discrete model of (1.1), Ding and Li [5] discussed the stability and bifurcation of numerical discretization model (1.1). For more details, we refer the readers to [6–21].

In 2011, assuming that a harvesting function is the delayed estimate of the true population, Berezansky et al. [22] presented an overview of the results on the classical Nicholson's blowflies models and established the following Nicholson's blowflies model with time-varying delay and a linear harvesting term

$$\dot{x}(t) = -\delta x(t) + px(t-\tau)e^{-ax(t-\tau)} - Hx(t-\sigma), \delta, p, \tau, a, H, \sigma \in (0, \infty), \tag{1.2}$$

where $Hx(t-\sigma)$ is the linear harvesting term, $x(t)$ denotes the size of population at time t, p denotes the maximum per capita daily egg production rate, δ denotes the per capita daily adult death rate, $\frac{1}{a}$ denotes the size at which the blowfly population reproduces at its maximum rate, τ denotes the generation rate. Berezansky et al. [22] also proposed an open problem: How about the dynamic behaviors of model (1.2)?

It is well known that the varying environment plays an important roles in many biological and ecological dynamical systems [23–35]. Inspired by the viewpoint, Duan and Huang [36] proposed the following Nicholson's blowflies model with varying coefficients and a linear harvesting term

$$\dot{x}(t) = -\delta(t)x(t) + p(t)x(t-\tau(t))e^{-a(t)x(t-\tau(t))} - Hx(t-\sigma(t)), \tag{1.3}$$

where $\delta(t), p(t), a(t), H(t) : R \to (0, +\infty), \tau(t), \sigma(t) : R \to [0, +\infty)$ are continuous functions. Applying the fixed point theorem and the properties of pseduo almost periodic function and Lyapunov functional method, Duan and Huang [36] obtained some sufficient conditions on the existence and convergence dynamics of positive pseudo almost periodic solution of (1.3).

Here we would like to point out that in real natural world, the almost periodic phenomenon usually more frequent than periodic ones. Moreover, a great deal of almost periodic phenomenon often appear in applied science such as physics, mechanics and engineering techqique fields. In recent years, there are numerous results on the existence of almost periodic solutions to Nicholson's blowflies models(see, e.g., [25,37–39]). To the best of knowledge, up to now, there is no paper that deal with the almost periodic solution of (1.3). Motivated by this discussion, we will investigated the almost periodic solution of model (1.3). For the sake of simplification, we assume that $a(t) = a$ is a constant and $\sigma(t) = \tau(t)$, then system (1.3) takes the form as follows

$$\dot{x}(t) = -\delta(t)x(t) + p(t)x(t-\tau(t))e^{-ax(t-\tau(t))} - H(t)x(t-\tau(t)), t \in R. \tag{1.4}$$

The main aim of this article is to discuss the existence of almost periodic solutions of (1.4). By transforming the model into an equivalent integral equation, and applying a fixed point theorem in cones, we obtain a set of sufficient condition which guarantees the existence of positive almost periodic solutions for the Nicholson's blowflies model (1.4).

The remainder of the paper is organized as follows. In section 2, we introduce some notations and assumptions, which can be used to check the existence of almost periodic solution of system (1.4). In section 3, we present a sufficient condition for the existence of almost periodic solution of (1.4). An example is given to illustrate the effectiveness of the obtained results in section 4. A brief conclusion is drawn in section 5.

2. Preliminaries

For convenience, in this section, we would like to introduce some notations, definitions and assumptions which are used in what follows.

Definition 2.1. [40–41] *Let $f(t) : R \to R^n$ be continuous in t. $f(t)$ is said to almost periodic on R, if for any $\varepsilon > 0$, the set $T(f, \varepsilon) = \{\delta : |f(t + \delta) - f(t)| < \varepsilon, \forall t \in R\}$ is relatively dense, i.e., for $\forall \varepsilon > 0$, it is possible to find a real number $l = l(\varepsilon) > 0$, for any interval with length $l(\varepsilon)$, there exists a number $\delta = \delta(\varepsilon)$ in this interval such that $|f(t + \delta) - f(t)| < \varepsilon$, for $\forall \in R$.*

Definition 2.2. *Let $z \in R^n$ and $Q(t)$ be a $n \times n$ continuous matrix defined on R. The linear system*

$$\frac{dz}{dt} = Q(t)z(t) \tag{2.1}$$

is said to admit an exponential dichotomy on R if there exist constants $k, \lambda > 0$, projection P and the fundamental matrix $Z(t)$ of (3.1) satisfying

$$\|Z(t)PZ^{-1}(s)\| \leq ke^{-\lambda(t-s)}, for\ t \geq s, \|Z(t)(I - P)Z^{-1}(s)\| \leq ke^{-\lambda(t-s)}, for\ t \leq s.$$

In this paper, we denote by $AP(R)$ the set of such function. Let $BC(R, R)$ denote the set of bounded continuous functions from R to R, $\|.\|$ denote the supremum norm $\|f\| = \sup_{t \in R} |f(t)|$.

Lemma 2.1. [41–42] *If the linear system (3.1) admits an exponential dichotomy, then the following almost periodic system*

$$\frac{dz}{dt} = Q(t)z(t) + g(t) \tag{2.2}$$

has a unique almost periodic solution $z(t)$ and

$$z(t) = \int_{-\infty}^{t} Z(t)PZ^{-1}(s)g(s)ds - \int_{t}^{+\infty} Z(t)(I - P)Z^{-1}(s)g(s)ds.$$

Lemma 2.2. [41–42] *Let $a_i(t)$ be an almost periodic function on R and $a_i(t) > 0$. Then the system*

$$\frac{dz}{dt} = diag(-a_1(t), -a_2(t), \cdots, -a_n(t))z(t) \tag{2.3}$$

admits an exponential dichotomy.

Remark 2.1. *It follows from Lemma 3.2 that system (3.3) has a unique almost periodic solution $z(t)$ which takes the form*

$$z(t) = \int_{-\infty}^{t} Z(t)Z^{-1}(s)g(s)ds = \left(\int_{-\infty}^{t} e^{-\int_s^t a_1(u)du} g_1(s)ds, \cdots, \int_{-\infty}^{t} e^{-\int_s^t a_n(u)du} g_n(s)ds\right).$$

Lemma 2.3. [43] *Let C be a normal and solid cone in a real Banach space X, and $\phi : C^0 \to C^0$ be a nondecreasing operator, where C^0 is the interior of C. Suppose that there exists a function ϕ : $(0,1) \times C^0 \to (0, +\infty)$ such that for each $\lambda \in (0,1)$ and $x \in C^0, \phi(\lambda, x) > \lambda, \phi(\lambda, .)$ is nondecreasing in C^0, and $\Phi(\lambda x) \geq \phi(\lambda, x)\Phi(x)$. Assume, in addition, there exists $z \in C^0$ such that $\Phi(z) \geq z$. Then Φ has a unique fixed point x^* in C^0. Moreover, for any initial $x_0 \in C^0$, the iterative sequence*

$$x_n = \Phi(x_{n-1}), n \in N, \tag{2.4}$$

satisfies

$$\|x_n - x^*\| \to 0 (n \to +\infty). \tag{2.5}$$

Throughout this paper, denote

$$h^+ = \sup_{t \in R} h(t), h^- = \inf_{t \in R} h(t), a^+ = \max_{t \in R}\{a(t)\},$$

where $h(t)$ is a bounded continuous function on R. Denote

$$f(x) = \begin{cases} xe^{-ax} - \frac{H(t)x}{p(t)}, 0 \leq x \leq \frac{1}{a}, \\ \frac{1}{ae} - \frac{H^-}{ap^+}, \quad x > \frac{1}{a}. \end{cases} \tag{2.6}$$

For convenience, we make the following assumptions.

(H1) $\delta^- > 0, p^- > 0$ and $\tau^- > 0$.

(H2) $0 < \frac{p^+}{\delta^-}\left(\frac{1}{ae} - \frac{H^-}{ap^+}\right) \leq \frac{1}{a}$.

(H3) $\frac{p^- - \frac{H^+}{a}}{\delta^+} > 1$.

Remark 2.2. *In model (1.4), $Hx(t - \tau(t))$ is the linear harvesting term, $x(t)$ denotes the size of population at time t, $p(t)$ denotes the maximum per capita daily egg production rate at time t, $\delta(t)$ denotes the per capita daily adult death rate at time t, $\frac{1}{a(t)}$ denotes the size at which the blowfly population reproduces at its maximum rate at time t, $\tau(t)$ denotes the generation rate at time t. Thus all the conditions (H1)–(H3) have practical significance of neural networks. If these variables in model (1.4) satisfy an appropriate condition, then model (1.4) has a unique almost periodic solution. From this viewpoint, all the assumptions (H1)–(H3) represent some problem of applied nature.*

3. Existence of almost periodic solution

In this section, we will establish sufficient conditions on the existence of almost periodic solutions of (1.4). Now we are in a position to state our main results on the existence of almost periodic solution for system (1.4).

Lemma 3.1. *Suppose that (H1) and (H2) hold. Then, in the sense of almost periodic nonnegative solution, system (1.4) is equivalent to the following integral equation:*

$$x(t) = \int_{-\infty}^{t} e^{-\int_s^t \delta(\theta)d\theta}[p(s)f(x(s - \tau(s)))]ds, t \in R. \tag{3.1}$$

That is to say, every almost periodic nonnegative solution φ of system (1.4) is an almost nonnegative solution of (3.1), and vice versa.

Proof. Let φ be an almost periodic nonnegative solution of (1.4). Notice that $\tau(t)$ is almost periodic, we can easily obtain

$$\varphi(\cdot - \tau(\cdot)) \in AP(R).$$

Then

$$p(\cdot)\varphi(\cdot - \tau(\cdot))e^{-ax(\cdot-\tau(\cdot))} - H(\cdot)\varphi(\cdot - \tau(\cdot)) \in AP(R).$$

Since $\delta^- > 0$, it follows from Lemma 2.1 that

$$\varphi(t) = \int_{-\infty}^{t} e^{-\int_s^t \delta(\theta)d\theta}[p(s)f(\varphi(s - \tau(s))]ds, t \in R.$$

By (H2), we have

$$\begin{aligned}
\varphi(t) &\leq \int_{-\infty}^{t} e^{-\delta^-(t-s)}\left[p^+\left(\frac{1}{a^-e} - \frac{H^-}{a^+p^+}\right)\right]ds \\
&= \frac{p^+}{\delta^-}\left(\frac{1}{a^-e} - \frac{H^-}{a^+p^+}\right) \\
&\leq \frac{1}{a^+}, t \in R.
\end{aligned}$$

Then

$$p(s)\varphi(s - \tau(s))e^{-a\varphi(s-\tau(s))} - H(s)\varphi(s - \tau(s)) = f(s - \tau(s)), s \in R.$$

Thus

$$\varphi(t) = \int_{-\infty}^{t} e^{-\int_s^t \delta(\theta)d\theta}[p(s)f(x(s - \tau(s))]ds, t \in R.$$

So, φ is an almost periodic solution of system (3.1). Similar to the above proof, for every almost periodic nonnegative solution ψ of system (3.1), we can easily to prove that ψ is an almost periodic solution of system (1.4). The proof of Lemma 3.1 is completed.

Now we will state our main result.

Theorem 3.1. *Suppose that (H1)–(H3) are satisfied. Then (1.4) has exactly one almost periodic solution x^* with a positive infimum. Moreover, for any initial $x_0 \in AP(R)$ with positive infimum, the iterative sequence*

$$x_k(t) = \int_{-\infty}^{t} e^{-\int_s^t \delta(\theta)d\theta}[p(s)x_{k-1}(s - \tau(s))e^{-ax_{k-1}(s-\tau(s))} - H(s)x_{k-1}(s - \tau(s))]ds, k = 1, 2, \cdots \quad (3.2)$$

satisfies

$$\|x_k - x^*\|_{AP(R)} \to 0, k \to +\infty. \quad (3.3)$$

Proof. Let

$$C = \{x \in AP(R) | \ x(t) \geq 0 \text{ for all } t \in R\}.$$

It is easy to prove that C is a normal and solid cone in $AP(R)$, and

$$C^0 = \{x \in AP(R) | \text{ There exists } \epsilon > 0 \text{ such that } x(t) > \epsilon \text{ for all } t \in R\}.$$

Define a nonlinear operator Φ on C^0 as follows

$$\Phi(x)(t) = \int_{-\infty}^{t} e^{-\int_s^t} \delta(\theta) d\theta [p(s) f(x(s - \tau(s))] ds, t \in R.$$

Next, we will prove that Φ satisfies all the assumptions in Lemma 2.3. It is not difficult to prove that Φ is a nondecreasing operator. Firstly, we show that Φ is from C^0 to C^0. Let $x_0 \in C^0$. Then there exists a $\epsilon_0 > 0$ such that $x_0(t) \geq \epsilon_0$ for all $t \in R$. Thus for all $t > R$, we have

$$\begin{aligned} \Phi(x)(t) &\geq \int_{-\infty}^{t} e^{-\delta^+(t-s)} \left[p^- \min\left\{ \epsilon_0 e^{-a\epsilon_0} - \frac{H^+}{ap^-}, \frac{1}{ae} - \frac{H^-}{ap^+} \right\} \right] ds \\ &= \frac{p^- \min\left\{ \epsilon_0 e^{-a\epsilon_0} - \frac{H^+}{ap^-}, \frac{1}{ae} - \frac{H^-}{ap^+} \right\}}{\delta^+} > 0, \end{aligned}$$

which implies that $\Phi(x) \in C^0$. By (H3), we can choose $\epsilon^* \in (0, \frac{1}{a})$ satisfying

$$\frac{p^- \epsilon^* e^{-a\epsilon^*} - \frac{H^+}{a}}{\delta^+} \geq 1.$$

Then for $t \in R$,

$$\Phi(\epsilon^*)(t) \geq \frac{p^- \epsilon^* e^{-a\epsilon^*} - \frac{H^+}{a}}{\delta^+} \geq \epsilon^*,$$

Namely, $\Phi(\epsilon^*) \geq \epsilon^*$. Next, we will show that there exists a function $\phi : (0,1) \times C^0 \to (0, +\infty)$ such that for each $\lambda \in (0, 1)$ and $x \in C^0$, $\phi(\lambda, x) > \lambda$, $\phi(\lambda, \cdot)$ is nondecreasing in C^0, and $\phi(\lambda, x) \geq \phi(\lambda, x)\phi(x)$. For $\lambda \in (0, 1)$ and $x \in (0, +\infty)$, let

$$\psi(\lambda, x) = \begin{cases} \lambda e^{(1-\lambda)ax}, & 0 \leq x \leq \frac{1}{a}, \\ \lambda \frac{1}{\frac{1}{a^- e} - \frac{H^-}{a^+ p^+}} e^{-\lambda ax}, & \frac{1}{a} < x < \frac{1}{\lambda a}, \\ 1, & x > \frac{1}{\lambda a}. \end{cases} \tag{3.4}$$

Hence, for $\lambda \in (0, 1)$ and $x \in (0, +\infty)$, we get

$$f(\lambda x) \geq \psi(\lambda, x) f(x).$$

Let

$$\phi(\lambda, x) = \psi(\lambda, \inf_{t \in R} x(t)), \lambda \in (0, 1), x \in C^0.$$

Since $\psi(\lambda, \cdot)$ is nondecreasing in $(0, +\infty)$, then $\varphi(\lambda, \cdot)$ is nondecreasing in $(0, +\infty)$. In addition, we have

$$\begin{aligned}\Phi(\lambda x)(t) &= \int_{-\infty}^{t} e^{-\int_s^t \delta(\theta)d\theta}[p(s)f(\lambda x(s-\tau(s))]ds \\ &\geq \int_{-\infty}^{t} e^{-\int_s^t \delta(\theta)d\theta}[p(s)\psi(\lambda, x(s-\tau(s))f(x(s-\tau(s))]ds \\ &\geq \int_{-\infty}^{t} e^{-\int_s^t \delta(\theta)d\theta}[p(s)\phi(\lambda, x)f(x(s-\tau(s))]ds \\ &\geq \phi(\lambda, x)\int_{-\infty}^{t} e^{-\int_s^t \delta(\theta)d\theta}[p(s)f(x(s-\tau(s))]ds \\ &\geq \phi(\lambda, x)\Phi(x)(t).\end{aligned}$$

Thus for all $\lambda \in (0, 1)$ and $x \in C^0$, we have $\Phi(\lambda x) \geq \phi(\lambda, x)\Phi(x)$.

Applying Lemma 2.3, we can conclude that (3.1) has exactly one almost periodic solution x^* with a positive infimum. In view of Lemma 3.1, we know that x^* is the unique almost periodic solution with a positive infimum of (1.4). By (2.4) and (2.5), we can conclude that (3.2) and (3.3) hold. The proof of Theorem 3.1 is completed.

Remark 3.1. *In [36], the authors investigated the convergence dynamics of positive pseduo almost periodic solution of Nicholson's blowflies model with varying coefficients and a linear harvesting term by applying the fixed point theorem and the properties of pseduo almost periodic function and Lyapunov functional method. They did not consider the existence of almost periodic solution. In this paper, we consider the existence of positive almost periodic solutions for the Nicholson's blowflies model. The results of this paper are completely new and complete the previous results in [36].*

Remark 3.2. *Although the model (1.1) of this paper is a special case of the model (1.1) in [44], the analysis method is different from that in [44]. In addition, check carefully, we find that the method used in this paper is similar to that in [20] and [45], but the analysis technique is quite different due to the different models.*

4. Numerical example

In this section, we will give an numerical example and its simulations to illustrate the effectiveness of our main results. Considering the following delay Nicholson's blowflies model with a linear harvesting term

$$\dot{x}(t) = -\delta(t)x(t) + p(t)x(t-\tau(t))e^{-ax(t-\tau(t))} - H(t)x(t-\tau(t)), \tag{4.1}$$

where

$$\delta(t) = 1 + \frac{|\sin t + \sqrt{3}\sin\sqrt{2}t|}{20}, p(t) = 2 + \frac{1+\sin^2 \pi t}{5},$$

$$H(t) = 0.04 + 0.02\sin^2 t, a = 1, \tau(t) = 1 + 0.02\sin t.$$

Then we have

$$\delta^- = 1, \delta^+ = 1.1, p^+ = 2.4, p^- = 2.2, H^+ = 0.06, H^- = 0.02.$$

Thus all assumptions in Theorems 3.1 are satisfied. Thus we can conclude that (4.1) has exactly one almost periodic solution with a positive infimum. The results are verified by the numerical simulations in Figure 1.

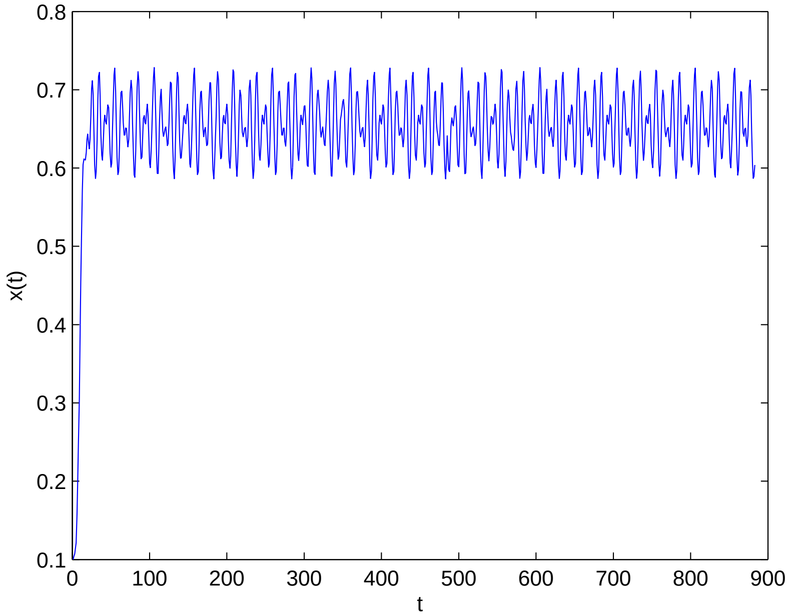

Figure 1. Time response of state variable $x(t)$.

5. Conclusion

In this paper, we study a delay Nicholson's blowflies model with a linear harvesting term. By transforming the model into an equivalent integral equation, and applying a fixed point theorem in cones, we establish some sufficient conditions for the existence of almost periodic solution of the delay Nicholson's blowflies model with a linear harvesting term. The obtained sufficient conditions are given in terms of algebraic inequalities, which is easy to check in practice. An example with its numerical simulations is given to illustrate the feasibility of the theoretical findings. The results of this article are completely new and complement those of the previous studies in [22,36].

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Conflict of interest

The authors declare no conflict of interest.

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