MULTIPLAYER GAMES AND HIV TRANSMISSION VIA CASUAL ENCOUNTERS

Stephen Tully

Department of Mathematics & Statistics
University of Guelph, Guelph ON Canada N1G 2W1, Canada

Monica-Gabriela Cojocaru

Department of Mathematics & Statistics
University of Guelph, Guelph ON Canada N1G 2W1, Canada

Chris T. Bauch

Department of Applied Mathematics & Statistics
University of Waterloo, Waterloo ON Canada, Canada

(Communicated by Abba Gumel)

Abstract. Population transmission models have been helpful in studying the spread of HIV. They assess changes made at the population level for different intervention strategies. To further understand how individual changes affect the population as a whole, game-theoretical models are used to quantify the decision-making process. Investigating multiplayer nonlinear games that model HIV transmission represents a unique approach in epidemiological research. We present here 2-player and multiplayer noncooperative games where players are defined by HIV status and age and may engage in casual (sexual) encounters. The games are modelled as generalized Nash games with shared constraints, which is completely novel in the context of our applied problem. Each player's HIV status is known to potential partners, and players have personal preferences ranked via utility values of unprotected and protected sex outcomes. We model a player's strategy as their probability of being engaged in a casual unprotected sex encounter (USE), which may lead to HIV transmission; however, we do not incorporate a transmission model here. We study the sensitivity of Nash strategies with respect to varying preference rankings, and the impact of a prophylactic vaccine introduced in players of youngest age groups. We also study the effect of these changes on the overall increase in infection level, as well as the effects that a potential prophylactic treatment may have on age-stratified groups of players. We conclude that the biggest impacts on increasing the infection levels in the overall population are given by the variation in the utilities assigned to individuals for unprotected sex with others of opposite HIV status, while the introduction of a prophylactic vaccine in youngest age group (15-20 yr olds) slows down the increase in HIV infection.

1. **Introduction.** Since the beginning of the HIV epidemic, it is estimated that 75 million people have been infected with HIV and 36 million have died [40]. HIV spans the globe, affecting every country, although some have fared worse than others. This is especially apparent in sub-Saharan Africa, with an estimated 5% of all adults infected [40]. HIV is not restricted to certain age groups, and, despite prevention programs and awareness campaigns, HIV incidence continues to increase, particularly amongst men who have sex with other men (MSM).

Fortunately, there have been numerous medical advancements since AIDS and HIV were defined in 1982–85. In 1986, the first antiretroviral therapy (ART) was introduced to prolong the life of individuals affected with HIV and reduce its spread. This changed the perspective of HIV to that of a chronic disease. Questions are now raised as to why the incidence rate continues to increase amongst certain population groups. Numerous surveys attempt to explain an individuals' reasoning, but the results have been varied, showing evidence of increased risky behaviour as well as decreased or no change in behaviour. [12, 22, 28, 32]. The imminence of prophylactic HIV vaccines (such as one being developed at The University of Western Ontario [39]) would have an enormous impact worldwide, but also raises some new interesting questions, chief among them being how to asses its impact on individuals contemplating whether to engage in casual sex, specifically in unprotected casual sex.

Population models have been useful in understanding and predicting the spread of HIV [20, 36, 18, 31, 19]. To better understand how one individual can affects change at population level game-theoretical models have been used [30, 33] These models illuminate a feedback mechanism where individual's choices may affect the population, which in turn affect the choices an individual is likely to make.

Game theory dates back to late 1940s with the works of von Neumann and Morgenstern [37] and later those of Nash [26] in the early 1950s. A game is a mathematical framework to describe decision-making by individuals engaged in competitive situations, where they can behave noncooperatively or cooperatively. Noncooperative game theory is nowadays widely used in applied areas such as economics, engineering, operations research, evolutionary biology and social sciences (psychology and cognitive sciences) see [4, 15, 16] and many references therein. The question of existence and computation of Nash strategies for a given game can be tackled with various methods, such as the reaction-curves method, optimization techniques, variational inequalities, computational methods (such as genetic algorithms, evolutionary computation), or a replicator-dynamics equilibrium, etc. [4, 10, 21, 3, 11].

Generalized Nash games (GN) with finite dimensional strategy sets were first studied by Arrow and Debreu in [1], followed by [29, 23, 34, 27], with a recent review in [14]. The formulation of the generalized Nash game as a variational inequality problems dates back to Bensoussan [5], while [34] gives first equivalence results for finite-dimensional GN games and quasivariational inequalities.

In this paper, we model casual (sexual) encounters as a noncooperative generalized Nash game between 2, 3, or 4 players, where each player's HIV status is known to both one's self, and to the player they choose to interact with. We do not model in this paper a population-level transmission process. All players have personal preferences ranked in utility of unprotected and protected sex outcomes, and they are given expected utilities of the casual encounter, depending on possible outcomes: unprotected sex outcome (USE), or protected/no-sex outcome (PSE). We model a player's strategy vector as their probabilities of being engaged in USE

with other players. We further introduce multiplayer GN games to analyze casual encounters between players belonging to different age groups, where partner choice is closely tied to a player's age. This age-stratified game model is introduced here to investigate the possibility of offering a prophylactic HIV vaccine to the youngest age group, perhaps before they become sexually active (similar to the HPV vaccine). Then we study the effects of individual choices on HIV transmission where different age groups interact and have differing levels of access to treatment options.

In our previous work on the topic, we modelled a similar setup of casual encounters with an agent-based model of the population [33, 35], and we analyzed how groups can emerge from coevolution of HIV spread with partner choices and risk perception, where we also assumed that a player's true HIV status is only known to themselves. In this model, unlike our previous simulation-based work, we construct a theoretical model to identify and analyze Nash equilibria with respect to the decisions players make. This allows us to better understand: a) the impact of personal preferences for unprotected sex; and b) the impact of heterogeneity of players (division in age groups) and initial HIV age composition both in presence and absence of a prophylactic vaccine.

To the best of our knowledge, modelling HIV transmission with age-stratified multiplayer GN games models is absolutely novel in the literature. There are a handful of examples of multiplayer GN games in applied problems (the River Basin problem, electricity markets [25], cap-and-trade agreements [14], voluntary vaccination models [11]), so our work here is unique in pushing the boundaries of modelling using GN games.

The structure of the paper is as follows: In Section 2, we present a 2-player game while in Section 3 we formulate the multiplayer games of casual encounters between players in age groups. Throughout, we investigate the sensitivity of players' Nash strategies and of HIV transmission when changes in utility rankings, efficacy of prophylactic treatment and group-specific initial HIV age composition are taken into account. We close with a few conclusions and future work.

2. **Casual encounter games as generalized Nash games.** In general, a multiplayer game involves a finite number of players, denoted here by $N > 0$. A generic player $i \in \{1, ..., N\}$ is thought to have a strategy set $S_i \subset \mathbb{R}^{n_i}$, whose strategies are vectors $x_i \in S_i$, and a payoff function $f_i : S_i \to \mathbb{R}$. A Nash equilibrium of a multiplayer game is defined as follows:

Definition 2.1. Assume each player is rational and wants to maximize their payoff. Then a Nash equilibrium is a vector $x^* \in K := S_1 \times ... \times S_N$ which satisfies the inequalities:

$$\forall i, \ f_i(x_i^*, x_{-i}^*) \geq f_i(x_i, x_{-i}^*), \ \forall x_i \in S_i$$

where $x_{-i} := (x_1, ..., x_{i-1}, x_{i+1}, ..., x_N)$.

For several decades, there exists, in the game theoretic literature, the concept of a *generalized Nash (GN) game* [1, 29], which in brief is a game such as above, where however each player's strategy set S_i is in fact dependent on the vector of strategies of other players [14]. It is known that in general a GN game has entire sets of Nash equilibria (as defined above), and it is also known that very few solution methods exist to date to compute all these for a given game. In particular, the subclass of GN games with *shared constraints (SC)* [27, 13] has developed more than its generic GN games counterparts. These are GN games where each player has an individual

constraint set (applied only to their vector of strategies, such as $S_i := S_i(x_i)$ here), however all players in a game have to also obey a shared constraint, i.e., a set of inequalities involving other players' strategies. This class of games is detailed in [27, 13, 14] and most recent work by the first author developed a theory and a computational method for finding the entire solutions set of a GNSC [9].

In this paper, we need to use the framework of GN theory, as our player interactions lead to casual sexual encounters in a closed population, thus the encounters have to obey a counting rule from both an $HIV+$ and and $HIV-$ player viewpoint. This constraint is in fact a shared constraint by all players, which naturally leads us to consider multiplayer GN games.

There are results asserting existence of generalized Nash equilibria for the type of games we model (see for instance [13, 34, 29] and references therein). One such approach is that of finding a small subset of solutions of a GN game by reformulating the GN game into a variational problem as below in Definition 2.2. Once the VI is proven to have solutions, computational methods are employed to find its solution set.

Definition 2.2. Given a set $K \subset \mathbb{R}^n$, closed and convex, and given $F : K \to \mathbb{R}^n$ a continuous function, the variational inequality (VI) problem is to find a vector x^* $\in K$ such that

$$\langle F(x^*), y - x^* \rangle \geq 0, \ \forall y \in K.$$

The existence of a solution to the VI problem in Definition 2.2 can be shown in many mathematical contexts (see [24]) but, specifically for our cases here, we use the results in [13]. Specifically, we solve for generalized Nash strategies of players where $F(x) := (-\frac{\partial f_1(x)}{\partial x_1}, ..., -\frac{\partial f_N(x)}{\partial x_N})$ and where $K = \{S_1 \times ... \times S_N \mid g(x) \leq 0\}$, where $g(x)$ is the shared constraint.

To solve the VI problem we compute its solution set as the set of critical points of a projected dynamical system (see [2, 8]) given by

$$\frac{dx}{d\tau} = P_{T_K(x(\tau))}(F(x(\tau))), \ x(0) \in K, \tag{1}$$

where $F(x) := (\nabla_{x_1} f_1, ..., \nabla_{x_N} f_N)$. The advantages are three-fold: we can assert existence of solutions to the game (given known results of existence of solutions to (1); we can use computational methods developed for projected systems to analyze and compute Nash equilibria for the game; and we can check if/when uniqueness of Nash strategies can be asserted.

It is known that the system (1) is well-defined if F is Lipschitz continuous on K, where K is a closed and convex set. Under these assumptions, solutions to this system exist and are unique through each initial point $x(0) \in K$. A projection-type algorithm can be used to compute its trajectories and its stationary points such as the ones in [2, 8]. To answer the uniqueness question, we numerically explore the set of initial conditions of system (1) and study how many (and what values of) Nash strategies we uncover.

2.1. **Two-player casual encounter game.** Let us consider now a casual encounter between two individuals from a general population of individuals aged 15 and over. A player can have one of two statuses: HIV negative ($HIV-$) or HIV positive ($HIV+$). Let us then denote by ϵ_-, respectively ϵ_+, the proportion of HIV negative, respectively HIV positive individuals so that $\epsilon_- + \epsilon_+ = 1$. Now, let us define by P_1 and P_2 respectively two players such that their statuses are $s_1 := HIV+$

and $s_2 := HIV-$. Define by $x^i \in [0,1]^2, x^i = (x^i_-, x^i_+), i \in \{1, 2\}$ a player's probability vector of having unprotected sex, where x^i_- is probability of unprotected sex (USE) with an $HIV-$ player and x^i_+ is the probability of USE with an $HIV+$ player. We try to find out, via our game model, what are the probabilities of having unprotected sex as a result of a casual encounter, based on their preference of sex outcomes.

Utility ranking of preferences of each player is given based on the HIV status of the individual they might engage with, which is considered known to them, as well as based on the type of sexual outcome they may find themselves in, namely USE, or PSE (where by PSE we amalgamate preferences of either protected sex or no sex outcomes).

Specifically, we assign as a numerical value for the utility of a sexual encounter the range of $[0,1]$. We consider throughout the paper that $USE(+,+) = 1$, $USE(-,-) = 1$ and that, as in [33], $USE(+,-) < 1$ and $USE(-,+) < 1$. This is to say that players are rational, and prefer unprotected sexual encounters with players of same status over all other outcomes.

Let us define the expected utilities for P_i, $i \in \{1,2\}$ out of a casual encounter as: $E^i_- :=$ expected utility from interacting with an $HIV-$ player and $E^i_+ :=$ expected utility from interacting with an $HIV+$ player. For P_1, who is $HIV+$, these are:

$$E^1_- = \rho[x^1_- USE(+,-) + (1 - x^1_-)PSE(+,-)] \text{ and}$$

$$E^1_+ = \rho[x^1_+ USE(+,+) + (1 - x^1_+)PSE(+,+)]$$

where ρ is the activity parameter of P_1. This represents a multiplicative factor controlling overall sexual activity level. It is outlined further in Section 3.

Then the overall expected utility of the encounter for P_1 is:

$$E^1(x^1_-, x^1_+) = \epsilon_+ E^1_- + (1 - \epsilon_+)E^1_+.$$

Similarly, for P_2, whose status is $HIV-$, we have:

$$E^2_- = \rho[x^2_- USE(-,-) + (1 - x^2_-)PSE(-,-)],$$

$$E^2_+ = \rho[x^2_+ USE(-,+) + (1 - x^2_+)PSE(-,+)].$$

$$\text{Thus } E^2(x^2_-, x^2_+) = \epsilon_+ E^2_- + (1 - \epsilon_+)E^2_+.$$

Now let us recall that ϵ_+ depends on the probabilities of players P_1 and P_2 to have unprotected sex. Consequently, we express ϵ_+ as:

$$\epsilon_+ = \epsilon_+(0) + [x^1_- \epsilon_+(0) + x^2_+ \epsilon_-(0)]\tau \tag{2}$$

where $\epsilon_+(0)$ is the initial (before the game) fraction of $HIV+$ people in the population, and $\tau = 0.02$ is the known transmission probability of HIV [17].

Last but not least, we need to make sure that the number of possible sexual encounters that lead to transmission is the same whether counted from the P_1 or $P2$ perspective. This leads us to the constraint:

$$\epsilon_- \sum_{i \in HIV_-} x^i_+ = \epsilon_+ \sum_{i \in HIV+} x^i_- \Leftrightarrow (1 - \epsilon_+)x^2_+ = \epsilon_+ x^1_-,$$

with ϵ_+ as in (2).

Players P_1 and P_2 want to maximize their expected utilities $E^1(x^1, x^2)$ and $E^2(x^1, x^2)$ subject to (x^1, x^2) in the set

$$K := \{S_1 \times S_2 \mid (1 - \epsilon_+)x^2_+ = \epsilon_+ x^1_-\}$$

where
$$S_i = \{x^i = (x^i_-, x^i_+) \in [0,1]^2,\, 0 \leq x^i_- + x^i_+ \leq 1,\, i = 1, 2\}.$$
Due to expression (2) we see that these utilities have actual dependencies on the other player's choices, so our model is a 2-player game with nonlinear payoffs.

2.2. **Base case scenarios, parameter values and uniqueness of Nash strategies.** At this stage, a discussion of parameter values we define as our "base case scenario" throughout the rest of the manuscript is needed. We outlined above our starting assumptions on the utility ranking values for $USE(+,+)$ and $USE(-,-)$. Furthermore, we also assume $USE(+,-) = 0$ and $USE(-,+) = 0$ and $PSE(s_1, s_2) = 0$, for any status values $s_1 \neq s_2 \in \{+,-\}$ and $PSE(-,-) = USE(+,+) = 0.25 > 0$. The latter values mean that a protected sex outcomes or no sex outcome is less preferred than USE by all players, but a PSE outcome with a partner of same status has a positive utility for all players.

The base values used in our simulations, unless otherwise noted, are listed in Table 1.

P_1	P_2	Utility for USE	Utility for PSE	Range
HIV+	HIV+	$USE(+,+) = 1$	$PSE(+,+)$=0.25	$[0,1]$
HIV+	HIV–	$USE(+,-) = 0$	$PSE(+,-)$=0	$[0,1]$
HIV–	HIV+	$USE(-,+) = 0$	$PSE(-,+)$=0	$[0,1]$
HIV–	HIV–	$USE(-,-) = 1$	$PSE(-,-)$=0.25	$[0,1]$

TABLE 1. This table outlines the base case preferences for different sexual acts given a players' status.

In general, it is expected that a Nash game will have multiple equilibria. We took a numerical approach to investigate the type of Nash equilibria we get in the game above. We first set the rest of our game parameters as described in Table 2 below. We then vary the initial conditions of the game reformulated as in equation (1), us-

Term	Definition	Baseline value	Range
τ	Probability of HIV spread from an $HIV+$ player to an $HIV-$ player through USE	0.02	–
$\epsilon_+(0)$	Initial proportion of $HIV+$ individuals in the population.	0.05	5% of population
$\epsilon_-(0)$	Initial proportion of $HIV-$ individuals in the population.	0.95	95% of population

TABLE 2. Parameter definitions and parameter values for baseline scenario. Here τ is a fixed probability of transmission per contact.

ing experiments with uniformly distributed points from K. We ran 100 simulations, each starting with 50 uniformly distributed initial values $(x^1_-, x^1_+, x^2_-, x^2_+) \in K$, and the only resulting Nash equilibrium strategies are the ones pictured in Figure 1. This specifically shows a (unique) Nash point $(0, 1, 1, 0)$ regardless of the initial conditions, i.e., players have USE with others from same status groups only. How-
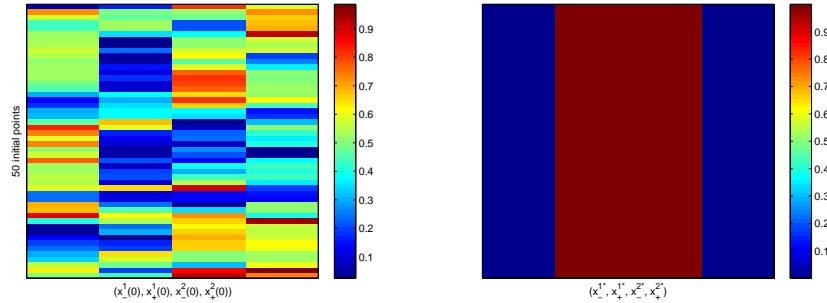
FIGURE 1. Heat map for 2-player game showing $(x_-^1, x_+^1, x_-^2, x_+^2) = (0, 1, 1, 0)$ equilibrium values for the respective initial conditions.

ever, this base case, though ideal in the sense of the type of behaviour suggested, is not realistic, since under Nash strategies $(0, 1, 1, 0)$ no one would have unprotected sex with a positive partner, thus HIV would die out in the population.

In the next section, we investigate the sensitivity of these results with respect to changes in utilities for casual sex with players of opposite status.

2.2 Analysis of the 2-player encounter game. In this section we are interested in investigating the effects of varying base utilities on the equilibrium strategies of both players to have unprotected sex in a casual encounter, namely on x_-^{1*} and x_+^{2*}. This question was not investigated in our previous works [33, 35]. We then track the effect of these changes on the overall fraction of infected individuals, ϵ_+. We use Table 2 to describe the values of our parameters. We compute the general Nash equilibrium points as described in the previous Section 2.1. Figure 2 shows the impact that changing utilities has on x_-^{1*}, x_+^2, as these strategies relate to HIV spread. We plotted 3-dimensional surfaces that observes these changes while varying both $USE(-, +)$ and $USE(+, -)$ over the range $[0, 1]$. We also plotted the change in ϵ_+ according to the same changes in utilities.

We see a change in the equilibrium solution for both x_-^{1*} and x_+^{2*} as we vary $USE(+, -)$ values, and a similar impact on values of ϵ_+. In contrast, varying $USE(-, +)$ values has no effect on the equilibrium values of players' strategies. A new set of Nash strategies, obtained for instance for a value of $USE(+, -) = 0.5$ is $(x_-^1, x_+^1, x_-^2, x_+^2) = (1, 0, 0.9463, 0.0537)$ as seen in Figure 2, whereas the equilibrium strategies for instance for $USE(+, -) = 0.08$ are $(x_-^{1*}, x_+^{1*}, x_-^{2*}, x_+^{2*}) = (0.4282, 0.5718, 0.9773, 0.0227)$.

3. **Multiplayer game.** We extend next the 2-player game presented in Section 2.2 to a multiplayer game, to capture interactions between players belonging to different age groups, as a possibly important factor in HIV transmission [7]. We consider here a population with 5 age cohorts, 15–20, 20–30, 30–40, 40–50 and 50+. Age group 1 (G_1), representing 15–20 year-old individuals, interacts with individuals from their age group plus with individuals of the 20–30 age cohort (G_2). Age group 2 (G_2), representing 20–30 year-old individuals, interacts with individuals from their own age cohort, plus with individuals in their adjacent age groups, i.e. G_1 and 30–40 year-olds (G_3). This continues for the 3rd and 4th age groups, with
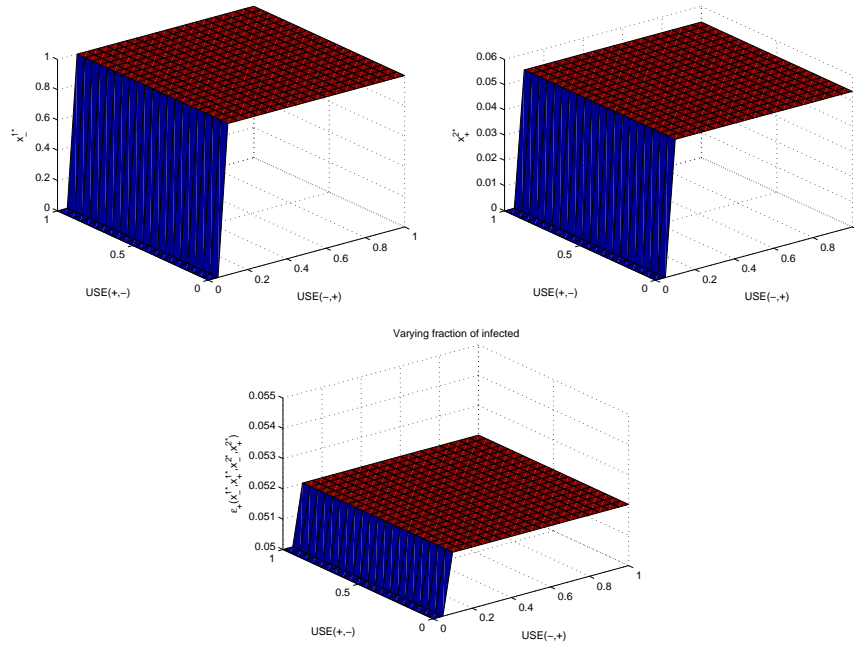
FIGURE 2. The 2-player game showing x_-^{1*}, x_+^{2*} and ϵ_+ varying $USE(-,+)$ and $USE(-,+)$.

the 5th age group (G5) interacting with themselves and the 40– 50 (G_4) age cohort. Surveys have shown that HIV prevalence varies greatly by age group, with youth accounting for a substantial amount of infections [6]. In general, interactions can be modeled in a variety of ways, subject to differing assumptions. We guided our groups interactions here following our previous model [35].

A game is defined by choosing one $HIV+$ player (always taken to be Player 1) in one of the groups at a time; the other players in a game will have an $HIV-$ status and will belong to the age cohorts allowed to interact with the age cohort P_1 belongs to. Following the age-group interactions allowed above, if P_1 belongs to G_1 or G_5, then the $HIV-$ players will be from the same or adjacent groups; thus we model these interactions as a 3-player game. Whenever P_1 is chosen in one of the G_2, G_3, or G_4, then we model their interactions as a 4-player game.

Let us denote by $game_i, i \in \{1, ..., 5\}$ one of the games describes above, such that

$$P_1 \text{ in } game_i \text{ belongs to } G_i.$$

We assume first that $HIV+$ individuals are spread among the five groups; thus each age group has a subgroup of $HIV+$ individuals of size $\epsilon_+^{G_i}(0)$, $i \in \{1, ..., 5\}$ so that $\sum_{i=1}^{5} \epsilon_+^{G_i}(0) = \epsilon_+(0) = 5\%$ as in Section 2. We start first by assuming an even spread of $HIV+$ individuals among age groups: $\epsilon_+^{G_i}(0) = 0.01$.

Finally, each age group has a differing activity parameter $\rho_i, i \in \{1, ..., 5\}$ given by: $\rho_1 = 0.5, \rho_2 = 1, \rho_3 = 0.8, \rho_4 = 0.6, \rho_5 = 0.3$, representing a multiplicative factor controlling overall sexual activity level relative to the second age group, by

age. Each P_i has an expected utility E^i of unprotected sex in a casual encounter, assuming they are aware of their partner status:

$$E^i := \epsilon_+ E^i_+(x) + (1 - \epsilon_+)E^i_-(x), \ \forall i \in \{1, 2, 3\}.$$

3.1. **3-player game.** We start by setting up the game concerning age cohort 15–20, where we choose P_1 to be $HIV+$ in G_1. Then P_2 and P_3 are $HIV-$ players from age groups G_1, G_2 respectively. The vector of strategies of player P_i is $x^i = (x^i_{1-}, x^i_{1+}, x^i_{2-})$, where by x^i_{j-} we denote the probability of P_i to have unprotected sex with an $HIV-$ individual in G_j, and by x^i_{j+} the probability of unprotected sex with an $HIV+$ individual in G_j.

We define their expected utilities as:

$$E^1_- = \rho_1 \Big[(x^1_{1-} + x^1_{2-})USE(+, -) + (1 - (x^1_{1-} + x^1_{2-}))PSE(+, -) \Big]$$

$$E^1_+ = \rho_1 \Big[(x^1_{1+})USE(+, +) + (1 - x^1_{1+})PSE(+, +) \Big]$$

Then P_1's strategies have to satisfy: $0 \leq x^1_{1-}, x^1_{1+}, x^1_{2-} \leq 1$ and that $x^1_{1-} + x^1_{1+} + x^1_{2-} = 1$.

Similarly for P_2 an $HIV-$ individual $\in G_1$, and P_3 an $HIV-$ individual $\in G_2$, we get (for $j = \{2, 3\}$):

$$E^j_- = \rho_{j-1} \Big[(x^j_{1-} + x^j_{2-})USE(-, -) + (1 - (x^j_{1-} + x^j_{2-}))PSE(-, -) \Big]$$

$$E^j_+ = \rho_{j-1} \Big[(x^j_{1+})USE(-, +) + (1 - x^j_{1+})PSE(-, +) \Big]$$

Each player strategies have to satisfy ($j \in \{2, 3\}$): $0 \leq x^j_{1-}, x^j_{1+}, x^j_{2-} \leq 1$ and $x^j_{1-} + x^j_{1+} + x^j_{2-} = 1$.

As a consequence of the interaction between players, the fractions of $HIV+$ individuals in G_1 and G_2 change now as follows (note that $\epsilon^{G_j}_+(game_1) = \epsilon^{G_j}_+(0), \ j \in \{3, 4, 5\}$):

$$\epsilon^{G_1}_+(game_1) = \epsilon^{G_1}_+(0) + \tau \Big[x^1_{1-}\epsilon^{G_1}_+(0) + x^2_{1+}\epsilon^{G_1}_-(0) \Big], \ \text{ and}$$

$$\epsilon^{G_2}_+(game_1) = \epsilon^{G_2}_+(0) + [x^1_{2-}\epsilon^{G_1}_+(0) + x^3_{1+}\epsilon^{G_2}_-(0)]\tau$$

Then we compute

$$\epsilon_+(game_1) := \sum_{i=1}^{5} \epsilon^{G_i}_+(game_1). \tag{3}$$

Last but not least, we need to impose the shared constraint that the number of possible sexual encounters that lead to transmission is the same when counted from each of the $+$ and $-$ players' perspective. This leads us to the constraint:

$$(1 - \epsilon_+(game1))(x^2_{1+} + x^3_{1+}) = \epsilon_+(game1)(x^1_{1-} + x^1_{2-}),$$

with $\epsilon_+(game1)$ as in (3).

We investigate uniqueness of solutions for the generalized Nash equilibrium strategies computed as in Section 2.2. Figure 3 shows a non-unique Nash point while varying initial conditions of system (1). We ran 100 simulations each starting with 50 uniformly distributed initial values and the compiled results are always as in Figure 3. As expected, the equilibrium strategies are not unique, however, the equilibrium strategies for players engaging in USE with players of opposite status
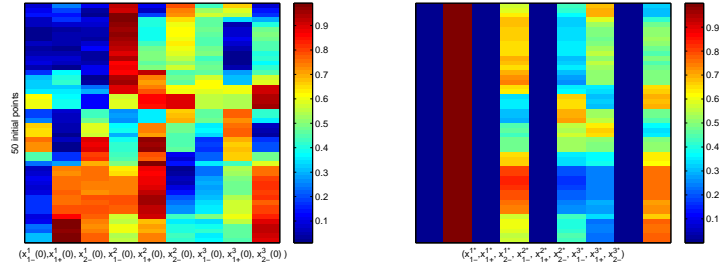
FIGURE 3. Heat map for 3-player game showing $(\underline{x}^{1*}, \underline{x}^{2*}, \underline{x}^{3*})$ equilibrium values for a uniform spread of initial conditions.

(namely x_{1-}^{1*}, x_{2-}^{1*} for P_1 and respectively x_{1+}^{2*} for P_2 and x_{1+}^{3*} for P_3) are always unique, and all equal to 0.

3.2. Results & discussion: 3-player game. Similar to studying the 2-player game, we study the effects of varying $USE(-,+)$ and $USE(+,-)$ on the choices of players and on the fraction ϵ_+ of the population, with baseline values of Table 2.
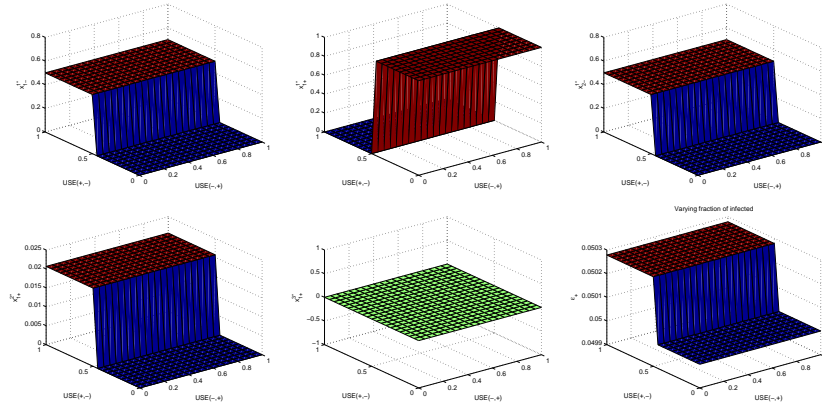

FIGURE 4. Results for 3-player game. The three upper panels show the $(x_{1-}^{1*}\ x_{1+}^{1*},\ x_{2-}^{1*})$ choices of P_1, whereas the lower left panels show the x_{1+}^{2*} choice of P_2, x_{1+}^{3*} for P_3 dependent on $USE(-,+)$ and $USE(+,-)$. Lower right panel shows the $\epsilon_+(game_1)$ variation.

We know from our previous section that equilibrium strategies found for this game are not unique, thus initial conditions for the computation of equilibrium strategies under varying parameters are important. In order to derive our analysis, we use as initial conditions one of the equilibrium points computed in the subsection above:

$$\underline{x}^* = \Big((0,1,0),(0.6369,0,0.3631),(0.62,0,0.38)\Big), \tag{4}$$

relying on the fact that all equilibrium strategies for players engaging in USE with players of opposite status in the baseline scenario are 0. The new equilibrium strategies pictured above are:

$$\underline{x}^* = \Big((0.5,0,0.5),(0.62,0.02,0.36),(0.62,0,0.38)\Big).$$

Figure 4 shows more refined results than those in the 2-player scenario. Choices for P_3 interacting with the $HIV+$ player are not shown, as they are not changed from the initial values. Varying $USE(+,-)$ affects the strategies of P_1 interacting with both P_2 and P_3 (upper left and upper right panels of Figure 4), while varying $USE(-,+)$ affects the strategies of P_2 interacting with P_1 (upper middle panel of Figure 4). We see a switch in strategy of P_2 as $USE(-,+)$ increases from 0 to 1. With increase in both utilities of USE with players of opposite status, P_1's choices move to unprotected sex with both P_2, while P_2, engages with P_1, though not exclusively (they maintain nonzero strategies of unprotected sex with players of $HIV-$ status as well). The impact on the infection levels from such a game alone is shown to have an increasing direction. The infected fraction in the population (lower left panel of Figure 4) increases from the baseline value of 0.05% to $\epsilon_+(game1) = 0.05028\%$, most affected by values of $USE(-,+)$ beyond a threshold of ≈ 0.5.

3.2. **4-player game.** We describe next the 4-player game arising from choosing for instance P_1 as an $HIV+$ player in G_2 (the game will be identical for a choice of P_1 in either G_3 or G_4).

We denote P_1 as $HIV+$ from G_2 and P_2, P_3 and P_4 as $HIV-$ players from G_1, G_2 and G_3 respectively. The vectors of strategies are as follows:

$$P_1: \quad \underline{0} \le (x_{1-}^1, x_{2-}^1, x_{3-}^1, x_{2+}^1) \le \underline{1} \text{ s.t} x_{1-}^1 + x_{2-}^1 + x_{3-}^1 + x_{2+}^1 = 1$$

$$P_2: \quad \underline{0} \le (x_{1-}^2, x_{2-}^2, x_{2+}^2) \le \underline{1} \text{ s.t. } x_{1-}^2 + x_{2-}^2 + x_{2+}^2 = 1$$

$$P_3: \quad \underline{0} \le (x_{1-}^3, x_{2-}^3, x_{3-}^3, x_{2+}^3) \le \underline{1} \text{ s.t. } x_{1-}^3 + x_{2-}^3 + x_{3-}^3 + x_{2+}^3 = 1$$

$$P_4: \quad \underline{0} \le (x_{2-}^4, x_{3-}^4, x_{2+}^4) \le \underline{1} \text{ s.t. } x_{2-}^4 + x_{3-}^4 + x_{2+}^4 = 1$$

The expected utilities for these individuals are listed below, starting with P_1 representing $HIV+$ individuals from G_2:

$$E_-^1 = \rho_2 \Big[(x_{1-}^1 + x_{2-}^1 + x_{3-}^1)USE(+,-) + (1 - (x_{1-}^1 + x_{2-}^1 + x_{3-}^1))PSE(+,-) \Big]$$

$$E_+^1 = \rho_2 \Big[x_{2+}^1 USE(+,+) + (1 - x_{2+}^1)PSE(+,+) \Big]$$

Similarly for P_2 who is an $HIV-$ individual $\in G_1$ we have:

$$E_-^2 = \rho_1 \Big[(x_{1-}^2 + x_{2-}^2)USE(-,-) + (1 - (x_{1-}^2 + x_{2-}^2))PSE(-,-) \Big]$$

$$E_+^2 = \rho_1 \Big[x_{2+}^2 USE(-,+) + (1 - x_{2+}^2)PSE(-,+) \Big]$$

For P_3 an $HIV-$ individual $\in G_2$:

$$E_-^3 = \rho_2 \Big[(x_{1-}^3 + x_{2-}^3 + x_{3-}^3)USE(-,-) + (1 - (x_{1-}^3 + x_{2-}^3 + x_{3-}^3))PSE(-,-) \Big]$$

$$E_+^3 = \rho_2 \Big[x_{2+}^3 USE(-,+) + (1 - x_{2+}^3)PSE(-,+) \Big]$$

Finally, for P_4, an $HIV-$ individual $\in G_3$ we have:

$$E_-^4 = \rho_3 \Big[(x_{2-}^4 + x_{3-}^4)USE(-,-) + (1 - (x_{2-}^4 + x_{3-}^4))PSE(-,-) \Big]$$

$$E_+^4 = \rho_3 \Big[x_{2+}^4 USE(-,+) + (1 - x_{2+}^4)PSE(-,+) \Big]$$

As a result of interactions allowed in this game, the fraction of the infected individuals changes in groups 1,2,3 (note that $\epsilon_+^{G_j}(game_2) = \epsilon_+^{G_j}(0)$, $j \in \{4,5\}$):

$$\epsilon_+^{G_j}(game_2) = \epsilon_+^{G_j}(0) + \tau \Big[x_{1-}^j \epsilon_+^{G_2}(0) + x_{2+}^{j+1} \epsilon_-^{G_j}(0) \Big], j \in \{1,2,3\}$$

Then we compute

$$\epsilon_+(game_2) := \sum_{i=1}^{5} \epsilon_+^{G_i}. \tag{5}$$

Last but not least, we need to impose the shared constraint that the number of possible sexual encounters that lead to transmission is the same when counted from each of the $+$ and $-$ players' perspective. This leads us to the constraint:

$$(1 - \epsilon_+(game2))(x_{2+}^2 + x_{2+}^3 + x_{2+}^4) = \epsilon_+(game2)(x_{1-}^1 + x_{2-}^1 + x_{3-}^1),$$

with $\epsilon_+(game2)$ as in (5).

We again investigate uniqueness of solutions as we did in Section 2.2. Figure 5 shows non-unique Nash points while varying initial conditions of (1). We ran 100 simulations each starting with 50 uniformly distributed initial values and the compiled results are always as in Figure 5. As expected, the equilibrium strategies are not unique, however, the equilibrium strategies for players engaging in US with players of opposite status (namely $x_{1-}^{1*}, x_{2-}^{1*}, x_{3-}^{1*}$ for P_1 and respectively x_{2+}^{2*} for P_2, x_{2+}^{3*} for P_3 and x_{2+}^{4*} for P_4) are always unique, and all equal to 0.
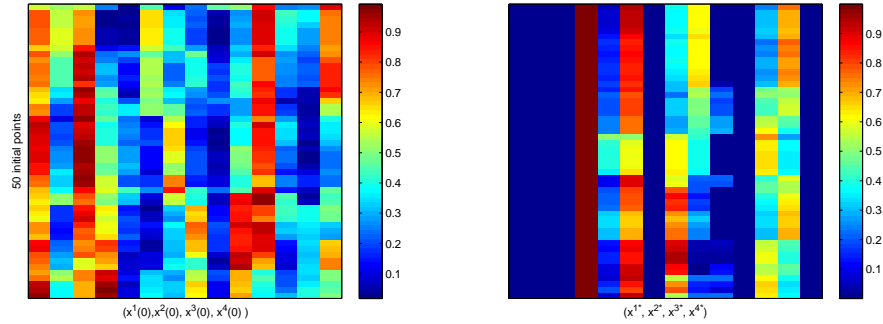

FIGURE 5. Heat map for 4-player game showing (x^1, x^2, x^3, x^4) equilibrium values for the respective initial conditions.

3.4. Results & discussion: 4-player game. Similar to studying the 2-player and 3-player games, we vary $USE(+, -)$ and $USE(-, +)$ and study the Nash choices of players, as well as the evolution of the $\epsilon_+(game_2)$ fraction. From our previous section, equilibrium strategies found for this game are not unique, thus initial conditions for the computation of equilibrium strategies under varying parameters are important. In order to derive our analysis, we use as initial conditions one of the equilibrium points computed in the subsection above

$$\underline{x}^* = \Big((0, 0, 0, 1), (0.2799, 0.7201, 0), (0.3174, 0.3651, 0.3175, 0), (0.5339, 0.4661, 0)\Big), \tag{6}$$

relying on the fact that all equilibrium strategies for players engaging in USE with players of opposite status in the baseline scenario are 0.

Figure 6 shows results similar to the 3-player scenario, where $HIV-$ individuals engage with $HIV+$ individuals as $USE(-, +)$ increases. Choices for P_1 become $x_{2+}^{1*} = 0$, $x_{1-}^{1*} = x_{2-}^{1*} = x_{3-}^{1*} \approx 0.334$, thus P_1 engages in USE with opposite status HIV players only, beyond a threshold of $USE(-, +) \approx 0.1$ (note that P_1 here is in

G_2, which is the most active group). Choices of P_2 vary slightly into USE with P_1, with $x_{2+}^{2*} \approx .0.032$, while maintaining positive strategies with P_3, P_4. Choices of P_3 and P_4 for USE with P_1 do not change from 0 (not shown).

The impact on the infection levels from such a game alone is shown to have an increasing direction. The infected fraction in the population (lower left panel of Figure 6) increases from the baseline value of 0.05% to $\epsilon_+(game2) = 0.5082\%$, most affected by positive values of $USE(-,+)$.
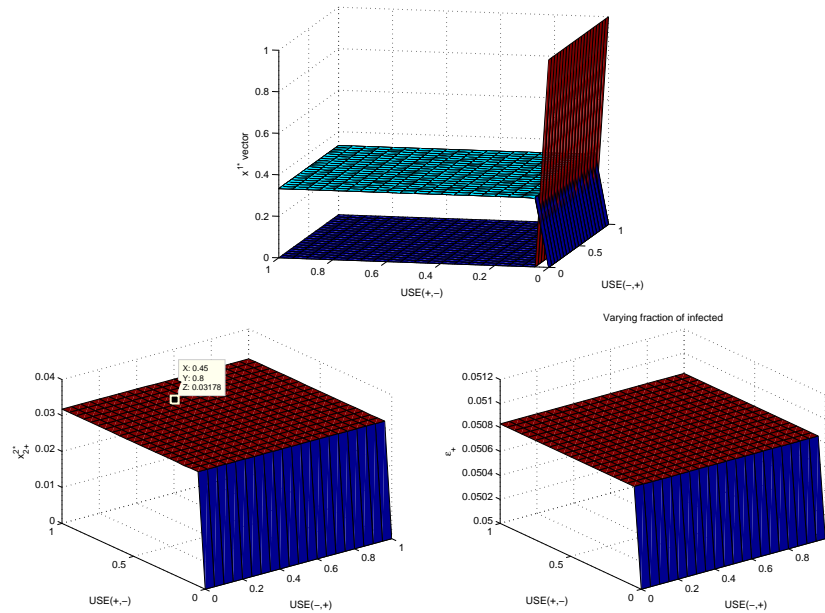

FIGURE 6. 3-dimensional results for 4-player game2 showing choices for varying $USE(+,-)$ and $USE(-,+)$ utilities. The upper panels show the change in equilibrium strategies of P_1: the upper left panel shows the strategies $x_{2+}^{1*} = 0$, $x_{1-}^{1*} = x_{2-}^{1*} = x_{3-}^{1*} \approx 0.334$. The likelihoods of P_2 to engage in USE with P_1 are shown in lower left panel, while $x_{2+}^{3*} = x_{2+}^{4*} = 0$ are not shown. The effect on the infected fraction due to this game is shown in lower right panel.

3.3. **Compounded effect of multiplayer games on HIV transmission.** Recall that by $game_i, i \in \{1, ..., 5\}$ we denoted the game where P_1 who is $HIV+$ belongs to G_i. We let $p_i, i \in \{1, ..., 5\}$ be the size of the age group G_i in the population (which was taken to be 20% for all groups in previous sections) so that $\epsilon_+^{gi}(0) := p_i\epsilon_+(0) = p_i \cdot 0.01, i \in \{1, ..., 5\}$. We denote by γ_i the probability of $game_i$ taking place. Given that each game leads to possible unprotected sex among $HIV+$ and $HIV-$ players, we estimate the infected fraction of individuals in each group, after interactions occur, to be described as follows:

$$\epsilon_+^{G_i} = \sum_{k=1}^{5}\gamma_k\epsilon_+^{G_i}(game_k) \text{ where for each } i,k \in \{1, ..., 5\} \text{ we have that } \epsilon_+^{G_i}(game_k)$$

are defined as in Section 3. The overall fraction of infected individuals in the population is obtained by adding the fractions above, after compiling the results for

each game: $\epsilon_+ = (\sum_{i=1}^{5} \epsilon_+^{G_i})$. For two quick examples, we plot ϵ_+ in Figure 7 using $p = (0.086, 0.173, 0.159, 0.170, 0.412)$ the estimated size of the age groups according to U.S. census [40, 38], and $p = (0.21, 0.313, 0.18, 0.12, 0.18)$ using Zimbabwe census data [41], both assuming $\gamma_k = 1, \forall k$.

As we have stated in previous sections, in both analyses of $game1$ and $game2$, we are dependent on initial meaningful states in order to evolve them while varying $USE(+, -)$ and $USE(-, +)$. For consistency of all simulations presented here, we use as initial data for the compounded games the points $\underline{x}^*$ in (4) for the 3player games, and the point $\underline{x}^*$ in (6) for the 4player games. Figure 7 presents our results, where clearly we see that the level of infection will be higher in the Zimbabwe population, as it contains more numerous younger age groups.
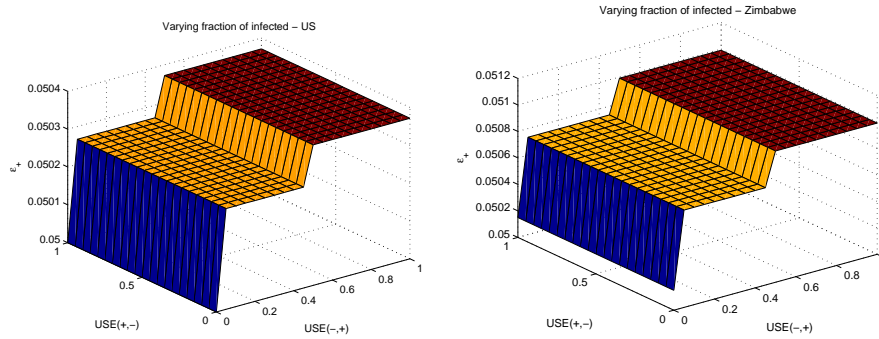

FIGURE 7.   Compounded ϵ_+ using U.S. census data (left) over 5 games vs. the same using Zimbabwe data (right)

4. **Prophylactic vaccine in youngest group.** In this section we investigate implementing a theoretical prophylactic vaccine within the population, with efficacy μ. Consequently, we adjust some of the baseline utilities as follows: a) the utility for an unvaccinated $HIV-$ individual interacting with an $HIV+$ individual, i.e., $USE(-, +)$ is increased from 0 to 0.25 due to the assumption that unvaccinated individuals aware of a vaccination program would place higher utility for USE given an increased level of protection through treatment optimism [12][1]; b) we define $USE(-, +, vacc) = 0.5$ as the utility for $HIV-$ vaccinated individuals engaging in USE with $HIV+$; c) we define $(1 - \mu)\tau$ to be the relative reduction in transmission. This will affect the variation of $\epsilon_+(game1)$ and $\epsilon_+(game2)$, as we assume P_2 in $game1$ and P_2 in $game2$ are now $HIV-$ and vaccinated.

The population sizes are adjusted to fit $p = (0.086, 0.173, 0.159, 0.170, 0.412)$ with $p_i, i \in \{1, ..., 5\}$ the estimated size of the age group according to U.S. census [40, 38] and $p = (0.21, 0.313, 0.18, 0.12, 0.18)$ using Zimbabwe census data [41].

We investigate what impact these changes have on HIV transmission if $HIV-$ players in youngest age class are vaccinated. We run the compounded games of Section 3 for both U.S. and Zimbabwe populations assuming that $\gamma_k = 1$ for all $k \in \{1, ..., 5\}$. We now have:

game 1: $P_2 \in G_1$ is $HIV-$ vaccinated, $P_3 \in G_2$ is $HIV-$ unvaccinated

---

[1]This is an assumption only; treatment optimism can in fact increase HIV transmission, as we show in [35].

game 2: $P_2 \in G_1$ is $HIV-$ vaccinated, P_3, P_4 are $HIV-$ unvaccinated.
Games 3, 4, 5 do not involve players in G_1, by the rules of our interactions, hence there is no vaccinated $HIV-$ player in these games.

Results presented are dependent on varying the vaccine efficacy $USE(+, -) \in [0, 1]$ and $USE(-, +) \in [0.25, 1]$. Figure 8 illustrates the compounded results for the U.S. and Zimbabwe data. We see that the fraction ϵ_+ show similar behaviour for both populations, with Zimbabwe at higher overall prevalence. It is also notable that if values of $USE(-, +) > 0.4$, the infection level reaches the same value as in the unvaccinated population. Thus, too much confidence in the efficacy of a vaccine leads $HIV-$ unvaccinated players to raise their preference for USE with $HIV+$ players, thus leading in fact to a nondecrease in HIV infection.
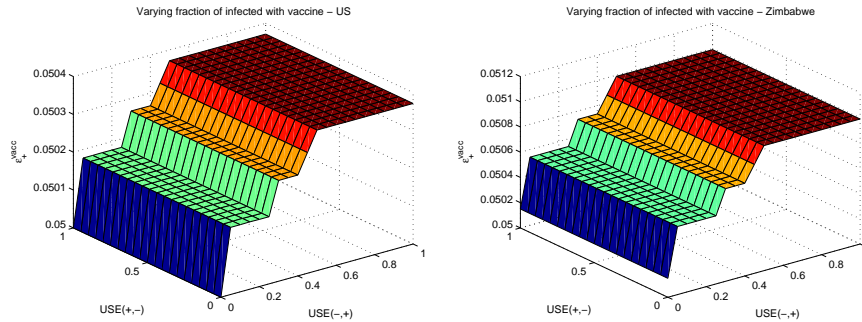

FIGURE 8. Compounded ϵ_+ using U.S. (left panel) and Zimbabwe (right panel) census data comparing $USE(+, -) \in [0, 1]$ and $USE(-, +) \in [0.25, 1]$ values, while with $U(USE, -, +, vacc) = 0.5$ and $\mu = 0.75$.

The increase in ϵ_+ remains less than 1% as a result of initial HIV prevalence, transmission probability, as well as implementing a one-off game. In cases where the games are repeated, then every repetition brings a small possible increase in the value of ϵ_+, which over time leads to potentially more significant increases.

5. **Conclusions.** The previous sections outlined a one-off casual sexual encounters game for 2, 3 and 4 player variations dependent on status and age. Our most interesting conclusions are that preferences of $HIV-$ players towards unprotected sex have the largest impact on HIV transmission in the population. In games with age group interactions, and assumed treatment introduced for the youngest group, populations with higher proportion of youth see a decrease in HIV prevalence through a possible implementation of a theoretical prophylactic vaccine in these age groups similar to the HPV case.

We showed that generalized Nash equilibria exist and can be computed for these types of games. Moreover, we demonstrated the sensitivity of GN equilibria with respect to varying players' utilities of unprotected sex with partners of opposite HIV status, giving an appreciation for how changes in individual decisions may contribute to an increase in HIV transmission.

As we expanded from 2-player to 3- and 4-player games, we in fact refined the interactions among the individuals in a population, previously regarded as one-on-one positive–negative outcomes. Given that the sizes of groups and activity parameter

values matter, we saw that the increase in transmission due to a single instance of a 3-player or a 4-player game is smaller than in a 2-player game. However, the biggest contributing factor in changing HIV transmission is found to be due to the $HIV-$ players' ranking of USE with an $HIV+$ player. Compounding over the age groups and adjusting for HIV prevalence in the population, we showed that a higher increase in transmission arises in populations with a larger youth composition.

We need to stress again here that the value of this modelling framework depends on the initial conditions of a population under observation, especially in the case of multiplayer groups. Moreover, knowing specific equilibrium strategies within the population of players can lead to sensitivity analyses having these specific equilibrium strategies as initial values. Last but not least, the fact that mathematically a generalized Nash game has in fact sets of equilibria gives strength to this modelling paradigm, in the sense that differing initial conditions in a players' population can give rise to different outcomes, which seems more appropriate to model real life encounter outcomes.

As future work, it would be interesting to include incorporation of ART for $HIV+$ individuals, which could show how adjusting their utilities may affect transmission. It would also be interesting to conduct a repeated game in order to better see how transmission evolves and how this evolution affects the overall risk assessment and spread of HIV.

The authors would like to thank the anonymous referees for their excellent suggestions that led to a clearer presentation of our work. It is through their suggestions that we have come to realize the naturally arising shared constraint which pushed our models into the generalized Nash game applications presented here.

REFERENCES

[1] K. J. Arrow and G. Debreu, Existence of an equilibrium for a competitive economy, *Econometrica,* **22** (1954), 265–290.

[2] J. P. Aubin and A. Cellina, *Differential Inclusions: Set-Valued Maps and Viability Theory*, Springer-Verlag New York, 1984.

[3] R. M. Axelrod, The complexity of cooperation: Agent-based models of competition and collaboration, Princeton University Press, 1997.

[4] T. Basar and G. J. Olsder, *Dynamic Noncooperative Game Theory,* SIAM, 1995.

[5] A. Bensoussan, Points de Nash dans le cas de fonctionnelles quadratiques et jeux differentiels lineaires a N personnes, *SIAM J. Control,* **12** (1974), 460–499.

[6] Center for Disease Control and Prevention, *Diagnoses of HIV Infection in the United States and Dependent Areas,* HIV Surveillance Report, 2011.

[7] B. Coburn, and S. Blower, A major HIV risk factor for young men who have sex with men is sex with older partners, *J. Acquir. Immune Defic. Syndr.*, **54** (2010), 113–114.

[8] M. G. Cojocaru and L. B. Jonker, Existence of solutions to projected differential equations on Hilbert spaces, *Proceedings of the American Mathematical Society,* **132** (2004), 183–193.

[9] Cojocaru, M.-G., Wild, E., *On Describing the Solution Sets of Generalized Nash Games with Shared Constraints*, submitted to: Optimization and Engineering, 2016.

[10] M. G. Cojocaru, C. T. Bauch and M. D. Johnston, Dynamics of vaccination strategies via projected dynamical systems, *Bulletin of mathematical biology,* **69** (2007), 1453–1476.

[11] M. G. Cojocaru, Dynamic equilibria of group vaccination strategies in a heterogeneous population, *Journal of Global Optimization,* **40** (2008), 51–63.

[12] J. P. Dodds, A. Nardone, D. E. Mercey and A. M. Johnson, Increase in high risk sexual behaviour among homosexual men, London 1996-8: Cross sectional, questionnaire study, *BMI*, **320** (2000), 1510–1511.

[13] F. Facchinei, A. Fischer and V. Piccialli, On generalized Nash games and variational inequalities, *Operations Research Letters*, **35** (2007), 159–164.

[14] F. Fachinei and C. Kanzow, Generalized Nash Equilibrium Problems, *Ann Oper Res*, **175** (2010), 177–211.

[15] J. W. Friedman, Game theory with applications to economics, Oxford University Press New York, 1990.

[16] D. Gabay and H. Moulin, On the uniqueness and stability of Nash-equilibria in noncooperative games, *Applied Stochastic Control in Econometrics and Management Science,* **130** (1980), 271–293.

[17] R. H. Gray, M. J. Wawer, R. Brookmeyer, N. K. Sewankambo, D. Serwadda, F. Wabwire-Mangen, T. Lutalo, X. Li, T. VanCott and T. C. Quinn, Probability of HIV-1 transmission per coital act in monogamous, heterosexual, HIV-1-discordant couples in Rakai, Uganda, *The Lancet.*, **357** (2001), 1149–1153.

[18] D. Greenhalgh and F. Lewis, Stochastic models for the spread of HIV amongst intravenous drug users, *Stochastic Models,* **17** (2001), 491–512.

[19] T. B. Hallett, S. Gregson, O. Mugurungi, E. Gonese and G. P. Garnett, Assessing evidence for behaviour change affecting the course of HIV epidemics: a new mathematical modelling approach and application to data from Zimbabwe, *Epidemics,* **1** (2009), 108–117.

[20] Y. H. Hsieh and C. H. Chen, Modelling the social dynamics of a sex industry: Its implications for spread of HIV/AIDS, *Bulletin of Mathematical Biology,* **66** (2004), 143–166.

[21] J. Hofbauer and K. Sigmund, *Evolutionary Games and Population Dynamics*, Cambridge University Press, 1998.

[22] D. M. Huebner and M. A. Gerend, The relation between beliefs about drug treatments for HIV and sexual risk behavior in gay and bisexual men, *Annals of Behavioral Medicine,* **4** (2001), 304–312.

[23] T. Ichiishi, *Game Theory for Economic Analysis*, New York: Academic Press, New York, 1983.

[24] D. Kinderlehrer and D. Stampacchia, *An Introduction to Variational Inequalities and their Application*, Academic Press, New York, 1980.

[25] K. Nabetani, P. Tseng and M. Fukushima, *Parametrized Variational Inequality Approaches to Generalized Nash Equilibrium Problems with Shared Constraints*, (Technical Report 2008), Department of Applied Mathematics and Physics, Graduate School of Informatics, Kyoto University, Japan.

[26] J. F. Nash, Equilibrium points in n-person games, *Proceedings of the national academy of sciences,* **36** (1950), 48–49.

[27] J.-S. Pang and M. Fukushima, Quasi-variational inequalities, generalized Nash equilibria, and multi-leader-follower games, *CMS*, **2** (2005), 21–56.

[28] R. H. Remien, P. N. Halkitis, A. O'Leary, R. J. Wolitski and C. A. Gómez, Risk perception and sexual risk behaviors among HIV-positive men on antiretroviral therapy, *AIDS and Behavior,* **9** (2005), 167–176.

[29] J. B. Rosen, Existence and uniqueness of equilibrium points for concave n-person games, *Econometrica,* **33** (1965), 520–534.

[30] K. D. Schroeder and F. G. Rojas, A game theoretical analysis of sexually transmitted disease epidemics, *Rationality and Society,* **14** (2002), 353–383.

[31] R. J. Smith and S. M. Blower, Could disease-modifying HIV vaccines cause population-level perversity?, *The Lancet Infectious Diseases,* **4** (2004), 636–639.

[32] J. M. Stephenson, J. Imrie, M. M. D. Davis, C. Mercer, S. Black, A. J. Copas, G. J. Hart, O. R. Davidson and I. G. Williams, Is use of antiretroviral therapy among homosexual men associated with increased risk of transmission of HIV infection?, *Sexually Transmitted Infections,* **79** (2003), 7–10.

[33] S. Tully, M. G. Cojocaru and C. T. Bauch, Coevolution of risk perception, sexual behaviour, and HIV transmission in an agent-based model, *Journal of theoretical biology,* **337** (2013), 125–132.

[34] P. T. Harker, Generalized Nash games and quasi-variational inequalities, *European Journal of Operational Research*, **54** (1991), 81–94.

[35] S. Tully, M. G. Cojocaru and C. T. Bauch, Sexual behaviour, risk perception, and HIV transmission can respond to HIV antiviral drugs and vaccines through multiple pathways, *Scientific Reports*, **5** (2015).

[36] J. X. Velasco-Hernandez and Y. H. Hsieh, Modelling the effect of treatment and behavioral change in HIV transmission dynamics, *Journal of mathematical biology,* **32** (1994), 233–249.

[37] J. Von Neumann and O. Morgenstern, Theory of games and economic behavior, *Bulletin of American Mathematical Society,* **51** (1945), 498–504.

[38] U. S. Census Bureau, Current Population Survey: Annual Social and Economic Supplement, 2012. Available at: `https://www.census.gov/topics/population.html`, (Accessed: Aug 2013).

[39] University of Western News, HIV vaccine produces no adverse effects in trials, 2013, Available at: `http://communications.uwo.ca/western_news/stories/2013/\September/ hiv_vaccine_produces_no_adverse_effects_in_trials.html`, (Accessed: Sept 2014).

[40] Global Health Observatory, *World Health Organization*, 2014, Available at: `http://www.who. int/gho/hiv/en/`, (Accessed: Sept 2014).

[41] Index Mundi, Zimbabwe Age structure, 2007, Available at: `http://www.indexmundi.com/ zimbabwe/age_structure.html`, (Accessed: Aug 2014).

E-mail address: `TullyS@fellows.iarc.fr`
E-mail address: `mcojocar@uoguelph.ca`
E-mail address: `cbauch@uwaterloo.ca`