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THE WORK OF GLENN F. WEBB

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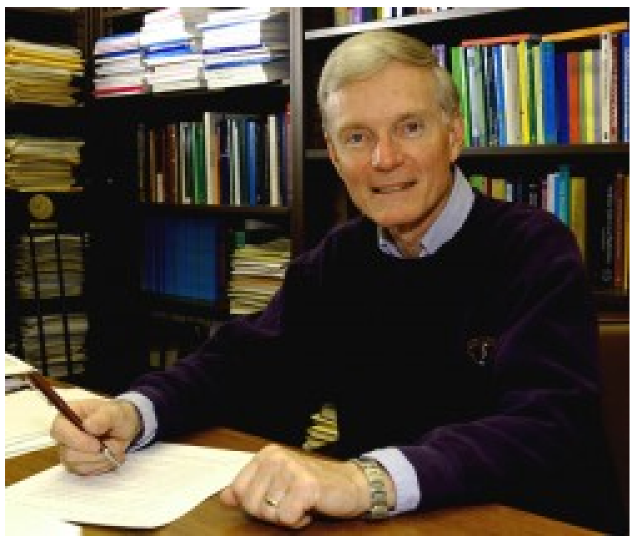

Glenn F. Webb (Neil Brake/Vanderbilt)

It is my distinct pleasure to introduce this volume honoring the 70th birthday of Professor Glenn F. Webb. The existence of this compiled volume is in itself a testimony of Glenn's dedication to, his pursuit of, and his achievement of scientific excellence. As we honor Glenn, we honor what is excellent in our profession. Aristotle clearly articulated his concept of excellence. "*We are what we repeatedly do. Excellence, then, is not an act, but a habit.*" As we look over the course of his career we observe ample evidence of Glenn Webb's habitual practice of excellence. Beginning with Glenn's first paper [1], one observes a constant stream of productivity and high impact work. Glenn has authored or co-authored over 160 papers, written one research monograph, and co-edited six volumes. He has delivered plenary lectures, colloquia, and seminars across the globe, and he serves on the editorial boards of 11 archival journals. He is a Fellow of the American Mathematical Society. Glenn's scientific career chronicles an evolution of scientific work that began with his interest in nonlinear semigroup theory and leads up to his current activity in biomedical mathematics. At each stage we see seminal contributions in the areas of nonlinear semigroups, functional differential equations, infinite dimensional dynamical systems, mathematical population dynamics, mathematical biology and biomedical mathematics. Glenn's work is distinguished by a clarity and accessibility of exposition, a precise identification and description of the problem or model under consideration, and thorough referencing. He uses elementary methods whenever possible but couples this with an ability to employ power abstract methods when necessitated by the problem.

Glenn was one of the pioneers in the area of nonlinear accretive operator theory and nonlinear semigroups, c.f. [1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12]. In this setting, nonlinear operators generate nonlinear semigroups. Nonlinear semigroups may be viewed as solution operators or generalized solution operators for Banach space differential equations of the form, $u'(t) + Au(t) = 0; u(0) = u_0$, where X is a Banach space, $u_0 \in X$, and A is a nonlinear operator (usually unbounded) mapping a subset of X to itself. The solution typically is represented by the exponential limit formula; $u(t) = \lim_{n\to\infty} \left(I + \frac{t}{n}A)\right)^{-n} u_0$ which may be interpreted as statement of convergence of the backward difference scheme. Banach space differential equations are function space realizations of partial differential equations and other distributed parameter systems. The abstract approach allows one to obtain a well-posedness framework for generalized solutions of these nonlinear systems and to extract qualitative information about these solutions. Typically the abstract solution provided by a semigroup coincides with the notion of a weak solution. One particularly important paper of this period of Glenn's career is [5], where he shows that continuous accretive nonlinear perturbations of m-accretive operators produce m-accretive operators. This seemingly abstract result continues to be applied to guarantee the existence of solutions to large classes of nonlinear elliptic, parabolic, and hyperbolic, as well as some higher order partial differential equations. Over the course of his career, Glenn has frequently returned to an abstract approach. However, he has always used abstract methods to simplify and illuminate, rather than introducing abstraction for the sake of abstraction.

In the early 1970's, Glenn began to consider nonlinear functional differential equations of the form, $\dot{u}(t) = F(u_t); u_0(\theta) = \varphi(\theta)$, with either discrete or distributed delay. Solution operators for functional differential equations of this genre are viewed as mappings of function spaces to function spaces and consequently the mathematical understanding of these operators necessitates an infinite dimensional methodology. Glenn [9] was the first to recognize how nonlinear semigroup theory could be applied to extend the linear theory of functional differential equations to nonlinear functional differential equations. A sequence of papers concerning the well-posedness, qualitative properties, and approximation of functional differential equations, Volterra integral equations and integrodifferential equations followed [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24]. It was at this time that Glenn began a very successful collaboration with Curtis Travis [13, 15, 19, 27, 26]. Of particular note were fundamental works on semilinear partial differential equations of parabolic type with delays in the nonlinear terms [13, 30].

In 1974 under the auspices of the prestigious National Research Council Research Fellowship for Foreign Mathematicians, Glenn visited the Institute of Mathematics of the University of Rome. This visit provided him with an opportunity to meet the powerful Italian applied analysis and partial differential equations research community. Toward the end of his visit he met Rosanna Villella-Bressan, beginning his ongoing collaboration and friendship with her and Janet Dyson [50, 89, 93, 103, 104, 105, 106, 108, 116, 117, 121, 122, 123, 138, 139, 140].

At the beginning of the second decade of his career Glenn broadened his interest and began to investigate equations with second order time derivatives [25, 27, 30, 31, 32, 33, 35, 36, 37]. Glenn, working alone and in collaboration with Curtis Travis, soon established leadership in the theory of cosine and sine operators, and its application to inhomogeneous and semilinear abstract differential equations and integrodifferential equations with second time derivatives. This work on abstract inhomogeneous and semilinear hyperbolic and higher order equations employed the familiar second order variation of parameters formalism paralleled the semigroup/variation of parameters approach to semilinear equations of parabolic type.

Throughout the 1960's and 1970's Glenn's work may be described as applicable mathematics as opposed to applied mathematics. His work focused on generic infinite dimensional systems and not specific equations or applications. In the 1980's Glenn's interest in biology became apparent and his work became more applied. With the publication of two papers [38, 39] on diffusive epidemic models in 1981 one observes a shift to applications, in particular mathematical biology. These papers on epidemic models formed a basis for Glenn's subsequent interest in the spread of infectious disease [42, 43, 80, 81, 87, 91, 92, 96, 120, 127, 134, 136], some of which was in collaboration with individuals such as M.E Parrott, W. Fitzgibbon, J. Morgan, and M. Langlais. Early epidemic models frequently used delays to describe periods of disease incubation and latency. A more accurate and natural means of introducing periods of latency or incubation from both the mathematical and modeling points of view is age structure. Age structured population models became and have remained at the core of Glenn's research effort. Population growth became a central theme to Glenn's research, be it populations of cells, viruses, animals, human populations, tumors, infectious agents, genotypes, bacteria, or prions.

The basic equation modeling the growth of an age structured population assumes the form $\rho_t + \rho_a = -\lambda(a)\rho$, with initial condition, $\rho(a, 0) = \varphi(a)$, and nonlocal age boundary condition, $\rho(0, t) = B(t) = \int_0^\infty b(a)\rho(a, t)da$. The independent variables a and t present age and elapsed time and the dependent variable $\rho(a, t)$ is a time dependent age density with $P(t) = \int_0^\infty \rho(a, t)da$ giving the time dependent population. The age dependent terms, $\lambda(a)$ and $b(a)$ represent age dependent mortality and birth rates respectively. $B(t)$ is commonly known as the birth function. This linear age transport population equation is a simple first order hyperbolic equation with a nonlocal age boundary condition which can be readily analyzed via straightforward characteristics arguments. If the birth rate and the mortality rate are constant, the age transport equation may be formally integrated on the age interval $[0, \infty)$ to produce the elementary Malthusian equation.

A more realistic population model was introduced by Mort Gurtin and Richard MacCamy. The Gurtin-MacCamy model incorporates birth rates and mortality rates which depend nonlinearly on the population density $\rho(a, t)$. The Gurtin-McCamy equation is a mathematically challenging nonlinear hyperbolic equation. An understanding and appreciation of the nonlinear population equation can be obtained via application of sophisticated functional analytic techniques of nonlinear semigroups and evolution operators. It is fair to say that Glenn's monograph, *Theory of Nonlinear Age Dependent Dynamics* by Marcel Dekker in 1985, Glenn remains the definitive treatment of the mathematical analysis of this genre of equations.

Glenn has frequently modeled epidemics by a system of differential equations with leading equation representing dynamics of the population of susceptible class and the second equation being a linear or nonlinear age transport equation describing the dynamics of the population density (with respect to the age of the infection) of the infective class. The birth function is calculated using the loss term from the susceptible class created by the infection. Various stages of the infection such as periods of latency, incubation, low infectivity, high infectivity, recovery, etc., are obtained through integration with respect to age over appropriate segments of the age interval. Introducing age structure also allows one to assess the effect of intervention during the course of the infection.

During the 1980's and up through the 1990's, Glenn focused on nonlinear population models, cf. [42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 86]. The major portion of his work was classical mathematical biology – it dealt with epidemic models, cell populations, and tumor growth as well the mathematical theory of general population models. Many of his papers during this time frame were singly authored. Others were written in collaboration with individuals such as K. Kunish, W. Schappacher, W. Desch, E. Sinestrari, M. Gyllenberg, and Annette Grabosh. With the exception of the collaboration with Doug Hardin and P. Takac and a paper on the spread of anthrax and another on the periodicity of cicada emergences, Glenn's attention has been continuous models as opposed to discrete models.

Glenn's work with D. Kirschner on the immune response and HIV [94, 95, 97, 99, 102, 119] can be considered pivotal by virtue of not only impact but perhaps more importantly the fact that this signals Glenn's transition to the realm of the mathematics of medicine or biomedical mathematics from more traditional mathematical biology. Here one sees an emphasis on specific diseases such as HIV, anthrax, SARS, drug resistant bacteria, and a variety of cancers. It involves a variety of topics including the immune response, agents of infection, treatment strategies, tumor growth, genetic structure, the spread of infectious disease and toxic agents. It is data driven frequently requiring the specification and estimation of parameters and the validation of the modeling by comparison of simulations with actual medical data. In addition to collaborating with applied mathematicians (including P. Magal, M. Kimmel, L. Pujo-Menjouet, M.A. Horn, E. Sanchez, S. Ruan, J. Pruss, H. Engler, B. Ayati, H. Zhu, S. Ardal, J. Wu, P. van den Driessche, L. Wang and O. Arino), Glenn has engaged the community of medical scientists and physicians through his collaborations with M. Blaser, E. D' Agata, M. Cloyd, S. Barjara, E. Wang, N. Bryce, and C. Arteaga.

Glenn's impact extends far beyond the impressive corpus of his work. Glenn revived Vanderbilt's program in analysis and differential equations and has taught courses at all levels of the curriculum from elementary calculus to advanced graduate courses. In addition to directing the dissertations of eighteen graduate students and working with ten postdoctoral fellows, he has served as a mentor to a large number of individuals, including undergraduate students, graduate students, and mid-career mathematicians. His presence has produced a steady stream of visitors that continues to come to Vanderbilt

In a profession populated with some gargantuan egos, Glenn's message has always been about his science and never about himself. He has always striven to communicate and collaborate rather than dictate and dominate.

On the occasion of his 70th birthday it is appropriate for us as friends and colleagues of Glenn Webb not only to commend him on his accomplishments and achievements, but to eagerly anticipate additional success as he passes this important milestone. A recent article in the *Financial Times* quotes researchers at the Max Planck Institute as saying the human longevity has increased so rapidly that 72 has become the new 30. So Glenn, according to the Max Planck Institute you are barely out of graduate school – we expect a lot more.

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