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IMPULSIVE ECOLOGICAL CONTROL OF A STAGE-STRUCTURED PEST MANAGEMENT SYSTEM

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Abstract. The dynamics of a stage-structured pest management system is studied by means of autonomous piecewise linear systems with impulses governed by state feedback control. The sufficient conditions of existence and stability of periodic solutions are obtained by means of the sequence convergence rule and the analogue of the Poincaré criterion. The attractive region of periodic solutions is investigated theoretically by qualitative analysis. The bifurcation diagrams of periodic solutions are obtained by using the Poincaré map, as well as the chaotic solution generated via a cascade of period-doubling bifurcations. The superiority of the state feedback control strategy is also discussed.

1. **Introduction.** Many systems in physics, chemistry, biology, and information science have impulsive dynamical behavior that result from abrupt jumps or controls at certain instants during the evolving processes. These can be modelled by impulsive differential equations. The theory of impulsive differential systems has been developed by numerous mathematicians; see [1, 2, 3]. There are three kinds of typical impulsive differential equations(DEs): (1) systems with impulses at fixed time, (2) systems with impulses at variable time, and (3) autonomous systems with impulses. In recent years, most of the research on IDEs concerns systems of types 1 and 2, while only little has addressed systems of type 3. This work focuses on IDEs of type 3.

The results about biological dynamical systems described by continuous differential equations are very rich. Various stage-structure models have been proposed and studied for populations [4, 5, 6]. As for the application of IDEs to ecology, systems like type 1 are used to model practical problems in most cases, for example, impulsive vaccination of disease [7, 8, 9], chemo-therapeutic treatment [10, 11] birth

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pulse [12, 13], and population growth [14, 15, 16]. Since the impulses in type 1 occur at fixed time, the discretized system is obtained easily and then may be used to discuss the dynamics of the impulsive systems under consideration. Much has been written recently about the bifurcation theory of ordinary differential equations or smooth dynamical systems [17, 18], however, little is known about the bifurcation theory of IDEs. For example, Lakmeche, A. transformed the problem of periodic solution into a fixed-point problem and obtained only the conditions of existence of trivial solution and positive period-1 solution in [11]; Tang, S.Y. [13] obtained the complete expression for the period-1 solution and discussed the bifurcation of periodic solutions numerically by using the discrete dynamical system determined by the stroboscopic map. Because the discrete map is not easily derived from autonomous systems with impulses, there are fewer methods to discuss the dynamics of systems of type 3. The paper introduces a new method to study type 3.

Pest control is very important in agriculture. Out-of-control pests can wreak havoc. For example, many countries in the world suffer deeply from plagues of locusts each year. In pest management, the pest population can be controlled by many methods, among which spraying pesticides is common. Usually, the pesticide is abruptly sprayed at fixed time (for example, the season of pest growth or spread) to diminish the pest population. This measure of pest management is called fixed-time control strategy, modeled by IDEs of type 1. However, this measure has some shortcomings, regardless of the growth rules of the pest and the cost of management. Another measure based on the state feedback control strategy is proposed in which the pesticide is sprayed only when the observed pest population reaches a certain threshold size. The latter measure is obviously more reasonable and suitable for pest control.

In this paper, instead of the pest management systems with impulses at fixed times, an autonomous piecewise linear system with impulses modeling the state feedback control is considered. The dynamical behavior of this system is discussed by means of both theoretical and numerical ways. Comparing with the impulsive fixed-time control strategy, the superiority of the impulsive state feedback control strategy is also shown by an example.

An outline of this paper is as follows: An autonomous piecewise linear system with impulses, modeling the state feedback control strategy, is introduced in section 2. The existence and stability of a period solution are obtained by using the sequence convergence rule and the analogue of the Poincaré criterion in section 3. In section 4, we qualitatively analyze the existence of a periodic solution of a general model and get a Poincaré map for bifurcation analysis. The numerical results about the bifurcation of stable periodic solutions are presented in an example. In section 5, the superiority of impulsive state feedback control strategy is discussed. Finally, section 6 presents the conclusion.

2. **Model description.** In [19], a general stage-structured population model is given as follows:

$$\begin{cases} \dot{N}_i(t) = -D_i(t) - W(t) + B(t), \\ \dot{N}_m(t) = a(t)W(t) - D_m(t), \end{cases} \tag{2.1}$$

where $N_i(t)$ and $N_m(t)$ denote the immature and mature pest populations, respectively, and $(x,\ y) \in \Omega = \{(x,\ y) | x \geq 0,\ y \geq 0\}$ for ecological practice. Now we give a concrete model of equation (2.1). Let $x(t)$ and $y(t)$ represent the densities of the immature and mature pests, respectively. We consider the following pest population model:

$$\begin{cases} \dot{x} = -ax + by, \\ \dot{y} = cx - dy, \end{cases} \tag{2.2}$$

where a, b, c, and d are positive constants; b and d denote the birth rate of the immature pest population and the death rate of the mature pest population, respectively; and c denotes the rate of an immature population turning into a mature population. It is obvious that $a > c$ and $a = c_1 + c$; c_1 denotes the death rate of the immature population. The dynamics of system (2.2) is very simple. The equilibrium point of system (2.2) is $O(0, 0)$, and the characteristic equation is

$$\lambda^2 + (a + d)\lambda + ad - bc = 0. \tag{2.3}$$

In the case of $ad-bc > 0$, equation (2.3) has two negative roots, and the variables x and y of system (2.2) all tend to zero as time $t \to +\infty$; that is, both the mature and immature pest populations tend to zero, and then no pest control measure is needed in this case. But in the case of $ad-bc < 0$, equation (2.3) has a positive root and a negative root, $O(0, 0)$ is a saddle with a separatrix l: $y = \frac{c}{d+\lambda_1}x$. So the sum of the mature and immature pest population will tend to positive infinity as time $t \to +\infty$. Hence, effective measures are necessary to control the pest population in this case.

The solution of system (2.2) is continuous, so the number of pests changes continuously. When the pesticide is sprayed, it is natural to assume that both the mature and immature pest populations diminish abruptly; that is, impulse effects exist. It is obvious that system (2.2) is invalid in this case, and IDEs are needed to model the above problem.

An impulsive state feedback control strategy for pest management is proposed here, rather than the usual impulsive fixed-time control strategy. When the sum of the immature and mature pest population reaches a threshold value h (that is $x(t(h)) + y(t(h)) = h$), the pesticide is sprayed and the immature and mature pest population turns abruptly to $(1 - p)x(t(h))$ and $(1 - q)y(t(h))$, respectively. Now we build the following model:

$$\begin{cases} \left.\begin{aligned} \dot{x} &= -ax + by, \\ \dot{y} &= cx - dy, \end{aligned}\right\} & x + y \neq h, \\ \left.\begin{aligned} \triangle x &= -px, \\ \triangle y &= -qy, \end{aligned}\right\} & x + y = h, \end{cases} \tag{2.4}$$

where a, b, c, d, $h > 0$, p and $q \in (0, 1)$, $\triangle x(t) = x(t^+)-x(t)$, $x(t^+) = \lim_{\tau\to 0^+} x(t+\tau)$, $\triangle y(t) = y(t^+)-y(t)$, and $y(t^+) = \lim_{\tau\to 0^+} y(t+\tau)$. The phase portrait of system (2.4) is shown in Figure 1a.

Moreover, system (2.4) can be generalized to a model with more complicated impulsive effects:

$$\begin{cases} \left.\begin{aligned} \dot{x} &= -ax + by, \\ \dot{y} &= cx - dy, \end{aligned}\right\} & x + y \neq h, \\ \left.\begin{aligned} \triangle x &= px + \tau_1, \\ \triangle y &= qy + \tau_2, \end{aligned}\right\} & x + y = h, \end{cases} \tag{2.5}$$

where τ_1 and τ_2 are real constants. Under the condition $ad - bc < 0$, the main purpose of this paper is to find periodic solutions and effective control measures for the pest population of system (2.4) or (2.5).

3. **The dynamics of system** (2.4)**.** In this section, we discuss the properties of the solutions of system (2.4) under the condition (H): $a,\ b,\ c,\ d>0,\ ad-bc<0$, and $p,\ q\in(0,\ 1)$. As is well known, the general solution of the first and second equations of system (2.4) with the initial condition $x(0)=x_0$, $y(0)=y_0$ is given by

$$\begin{cases} x(t)=\frac{d+\lambda_1}{\lambda_1-\lambda_2}(x_0-\frac{d+\lambda_2}{c}y_0)\exp(\lambda_1 t)+\frac{d+\lambda_2}{\lambda_2-\lambda_1}(x_0-\frac{d+\lambda_1}{c}y_0)\exp(\lambda_2 t), \\ y(t)=\frac{c}{\lambda_1-\lambda_2}(x_0-\frac{d+\lambda_2}{c}y_0)\exp(\lambda_1 t)+\frac{c}{\lambda_2-\lambda_1}(x_0-\frac{d+\lambda_1}{c}y_0)\exp(\lambda_2 t), \end{cases} \tag{3.1}$$

where

$$\lambda_1=\frac{-a-d+\sqrt{(a-d)^2+4bc}}{2}>0,\quad \lambda_2=\frac{-a-d-\sqrt{(a-d)^2+4bc}}{2}<0. \tag{3.2}$$

Assume that system (2.4) has a periodic solution $(\xi(t),\ \eta(t))$ with period T. Denote $\xi(0)=\xi_0$, $\eta(0)=\eta_0$, $\xi(T)=\xi_1$, and $\eta(T)=\eta_1$. From the T-periodicity of the solution, we have $\xi(T^+)=\xi_0$, $\eta(T^+)=\eta_0$; that is,

$$(1-p)\xi_1=\xi_0\ ,\quad (1-q)\eta_1=\eta_0, \tag{3.3}$$

namely,

$$\begin{cases} \xi(T)=\frac{d+\lambda_1}{\lambda_1-\lambda_2}(\xi_0-\frac{d+\lambda_2}{c}\eta_0)\exp(\lambda_1 T)+\frac{d+\lambda_2}{\lambda_2-\lambda_1}(\xi_0-\frac{d+\lambda_1}{c}\eta_0)\exp(\lambda_2 T)=\frac{\xi_0}{1-p}, \\ \eta(T)=\frac{c}{\lambda_1-\lambda_2}(\xi_0-\frac{d+\lambda_2}{c}\eta_0)\exp(\lambda_1 T)+\frac{c}{\lambda_2-\lambda_1}(\xi_0-\frac{d+\lambda_1}{c}\eta_0)\exp(\lambda_2 T)=\frac{\eta_0}{1-q}. \end{cases}$$

It is easy to calculate that

$$\exp(\lambda_1 T)=\frac{\frac{\xi_0}{1-p}-\frac{d+\lambda_2}{c}\frac{\eta_0}{1-q}}{\xi_0-\frac{d+\lambda_2}{c}\eta_0},\quad \exp(\lambda_2 T)=\frac{\frac{\xi_0}{1-p}-\frac{d+\lambda_1}{c}\frac{\eta_0}{1-q}}{\xi_0-\frac{d+\lambda_1}{c}\eta_0}. \tag{3.4}$$

First, a lemma is given for further use.

LEMMA3.1. (*Analogue of the Poincare criterion* [20].) *The T-periodic solution $x=\xi(t), y=\eta(t)$ of the system*

$$\begin{cases} \frac{dx}{dt}=P(x,y), \frac{dy}{dt}=Q(x,y), & if\ \phi(x,y)\neq 0, \\ \triangle x=\alpha(x,y), \triangle y=\beta(x,y), & if\ \ \phi(x,y)=0, \end{cases} \tag{3.5}$$

is orbitally asymptotically stable if the Floquet multiplier μ_2 satisfies the condition

$$|\mu_2|<1, \tag{3.6}$$

where

$$\mu_2=\prod_{k=1}^{q}\triangle_k\exp\left[\int_0^T\left(\frac{\partial P}{\partial x}(\xi(t),\eta(t))+\frac{\partial Q}{\partial y}(\xi(t),\eta(t))\right)dt\right], \tag{3.7}$$

$$\triangle_k=\frac{P_+\left(\frac{\partial\beta}{\partial y}\frac{\partial\phi}{\partial x}-\frac{\partial\beta}{\partial x}\frac{\partial\phi}{\partial y}+\frac{\partial\phi}{\partial x}\right)+Q_+\left(\frac{\partial\alpha}{\partial x}\frac{\partial\phi}{\partial y}-\frac{\partial\alpha}{\partial y}\frac{\partial\phi}{\partial x}+\frac{\partial\phi}{\partial y}\right)}{P\frac{\partial\phi}{\partial x}+Q\frac{\partial\phi}{\partial y}},$$

and $P, Q, \frac{\partial\alpha}{\partial x}, \frac{\partial\alpha}{\partial y}, \frac{\partial\beta}{\partial x}, \frac{\partial\beta}{\partial y}, \frac{\partial\phi}{\partial x}$, *and* $\frac{\partial\phi}{\partial y}$ *are calculated at the point* $(\xi(\tau_k),\eta(\tau_k))$, *and* $P_+=P(\xi(\tau_k^+),\ \eta(\tau_k^+))$ *and* $Q_+=Q(\xi(\tau_k^+),\ \eta(\tau_k^+))$. *Also,* $\phi(x,\ y)$ *is a sufficiently smooth function such that* $\mathrm{grad}\phi(x,\ y)\neq 0$, *and* $\tau_k(k\in N)$ *are the times for the jumps.*

In what follows, we discuss the dynamics of system (2.4) in the cases of $p=q$ and $p\neq q$.

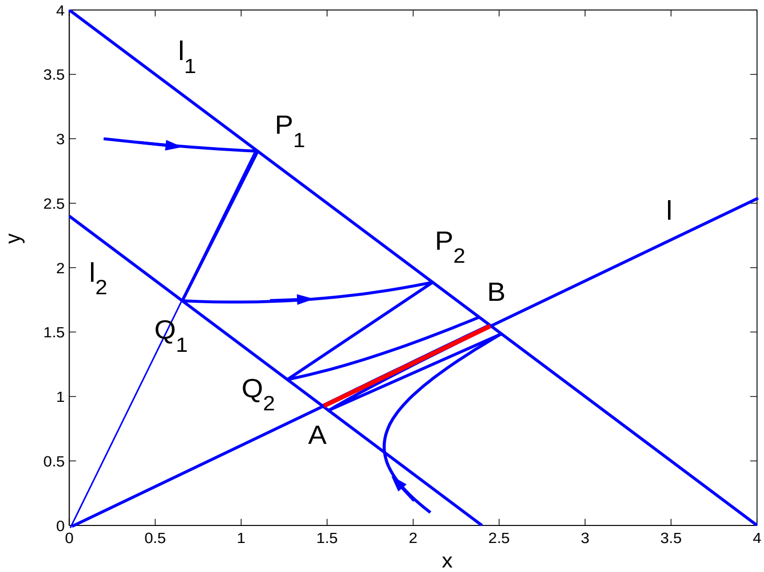


FIGURE 1. The periodic solution (red line) and its attractive region in the case of $p = q = 0.4$ for system (2.4) with $h = 4$.

3.1. The case of $p = q$.

THEOREM 3.1. *Assume that condition* (H_1): $a,\ b,\ c, \delta > 0$, $ad - bc < 0$, *and* $p = q \in (0,\ 1)$ *holds, then system* (2.4) *has an orbitally asymptotically stable periodic solution, which is unique in the attractive region* $\Omega_0 = \{(x,\ y)|x \geq 0,\ y \geq 0,\ x + y < h\}$.

Proof. Set the initial point $A(x_0,\ y_0)$ on the separatrix l; then, the solution of system (2.4) passing through the point A is $y(t) = \frac{c}{d+\lambda_1}x(t)$. The trajectory of this solution intersects the line $l_1:\ x + y = h$ at a point $B(x_1,\ y_1)$. It is seen from (3.3) that if $x_0 = (1-p)x_1$ and $y_0 = (1-q)y_1$, then system (2.4) has a periodic solution. Now we choose $x_0,\ x_1$ satisfying $x_0 = (1-p)x_1$. We can calculate that the positions of the points A and B are

$$A = (\frac{(1-p)(d+\lambda_1)}{c+d+\lambda_1}h,\ \frac{c(1-p)}{c+d+\lambda_1}h),\quad B = (\frac{d+\lambda_1}{c+d+\lambda_1}h,\ \frac{c}{c+d+\lambda_1}h).$$

Taking account of $p = q$, we have

$$y_0 = \frac{c(1-p)}{c+d+\lambda_1}h) = (1-p)y_1 = (1-q)y_1.$$

This means that system (2.4) has a periodic solution on the separatrix l for $p = q$. From (3.4), the period T is given by

$$\begin{aligned} T &= \frac{1}{\lambda_1}\ln(\frac{\frac{\xi_0}{1-p} - \frac{d+\lambda_2}{c}\frac{\eta_0}{1-q}}{\xi_0 - \frac{d+\lambda_2}{c}\eta_0}) \\ &= \frac{1}{\lambda_1}\ln(\frac{\frac{1}{1-p}(\xi_0 - \frac{d+\lambda_2}{c}\eta_0)}{\xi_0 - \frac{d+\lambda_2}{c}\eta_0}) \\ &= \frac{1}{\lambda_1}\ln(\frac{1}{1-p}). \end{aligned} \tag{3.8}$$

Next, we prove the uniqueness of this periodic solution. As shown in Figure 1, set line l_2: $x+y=(1-p)h$. The line l_1 is parallel to the line l_2. The line l_1 intersects the x-axis and y-axis at the points $(h,\ 0)$ and $(0,\ h)$, respectively, while the l_2 intersects the x-axis and y-axis at the points $((1-p)h,\ 0)$ and $(0,\ (1-p)h)$, respectively. Consider a trajectory originating from any initial point in the region $\Omega_0=\{(x,\ y)|x\geq 0,\ y\geq 0,\ x+y<h\}$. Without loss of generality, we assume the initial point is above the separatrix l (see Figure 1). The trajectory reaches the point $P_1(x_1,\ y_1)$ on the line l_1, next jumps to the point $Q_1((1-p)x_1,\ (1-p)y_1)$ on the line l_2, and then reaches the point P_2 on the line l_1, and then jumps to the point Q_2 on the line l_2 again, and so on. Then we obtain two sequences $\{P_n(x_n,\ y_n)\}$ and $\{Q_n((1-p)x_n,\ (1-p)y_n)\}$. In view of $\frac{y_n}{x_n}=\frac{(1-p)y_n}{(1-p)x_n}$, the points P_n, Q_n lie on the same line passing through the origin O, and then the correspondence between the points P_n on $\overline{EF}$ and Q_n on $\overline{GH}$ is one to one. So we obtain two sequences $\{|\overline{BP_n}|\}$ and $\{|\overline{AQ_n}|\}$ with increasing time t.

It follows from $l_1 \parallel l_2$ that $\frac{|\overline{OA}|}{|\overline{OB}|}=\frac{(1-p)h}{h}=1-p$; then $\frac{|\overline{AQ_1}|}{|\overline{BP_1}|}=\frac{|\overline{OA}|}{|\overline{OB}|}=1-p$ and $|\overline{AQ_1}|=(1-p)|\overline{BP_1}|$. Furthermore, $|\overline{AQ_n}|=(1-p)|\overline{BP_n}|$ for $n\in N$. The distance from the point $Q_1(x_0,\ y_0)$ to the separatrix $l: y=\frac{c}{d+\lambda_1}x$ is $\frac{|cx_0-(d+\lambda_1)y_0|}{\sqrt{c^2+(d+\lambda_1)^2}}$. The point $P_2(x(t),\ y(t))$ is on the trajectory originating from the initial point $Q_1(x_0,\ y_0)$. Taking account of (3.1), the distance from the point $P_2(x(t),\ y(t))$ to the separatrix l is $\frac{|cx(t)-(d+\lambda_1)y(t)|}{\sqrt{c^2+(d+\lambda_1)^2}}=\frac{|cx_0-(d+\lambda_1)y_0|\exp(\lambda_2 t)}{\sqrt{c^2+(d+\lambda_1)^2}}$. From the fact that $\exp(\lambda_2 t)<1$ for $t>0$, the distance from the point P_2 to the separatrix l is less than the distance from the point Q_1 to the separatrix l. It follows that, together with $l_1\parallel l_2$, $|\overline{BP_2}|<|\overline{AQ_1}|$. So $|\overline{BP_2}|<|\overline{AQ_1}|=(1-p)|\overline{BP_1}|<|\overline{BP_1}|$, namely, $|\overline{BP_2}|<|\overline{BP_1}|$, while $|\overline{AQ_2}|=(1-p)|\overline{BP_2}|<(1-p)|\overline{AQ_1}|<|\overline{AQ_1}|$, namely, $|\overline{AQ_2}|<|\overline{AQ_1}|$. Similarly,

$$|\overline{BP_n}|<(1-p)|\overline{BP_{n-1}}|,\ |\overline{AQ_n}|<(1-p)|\overline{AQ_{n-1}}|.$$

Then we get

$$|\overline{BP_n}|<(1-p)^{n-1}|\overline{BP_1}|,\ |\overline{AQ_n}|<(1-p)^{n-1}|\overline{AQ_1}|,$$

$$\begin{aligned} &|\overline{BP_1}|>|\overline{BP_2}|>\cdots>|\overline{BP_n}|>\cdots>0,\\ &|\overline{AQ_1}|>|\overline{AQ_2}|>\cdots>|\overline{AQ_n}|>\cdots>0, \end{aligned} \tag{3.9}$$

which imply that

$$0\leq \lim_{n\to+\infty}|\overline{BP_n}|\leq \lim_{n\to+\infty}|(1-p)^{n-1}|\overline{BP_1}|=0,$$

$$0\leq \lim_{n\to+\infty}|\overline{AQ_n}|\leq \lim_{n\to+\infty}|(1-p)^{n-1}|\overline{AQ_1}|=0.$$

This leads to

$$\lim_{n\to+\infty}P_n=B,\ \ \lim_{n\to+\infty}Q_n=A. \tag{3.10}$$

Therefore, the trajectory originating from any initial point in region Ω_0 tends to a unique periodic solution on the section $\overline{AB}$ as $t\to+\infty$, and the attractive region of the periodic solution is the region Ω_0.

Lastly, we discuss the stability of this periodic solution by using Lemma 3.1. In this case, $P(x,y)=-ax+by$, $Q(x,y)=cx-dy$, $\phi(x,y)=x+y-h$, $\alpha(x,y)=-px$, $\beta(x,y)=-qy$, $(\xi(T),\eta(T))=(\frac{d+\lambda_1}{c+d+\lambda_1}h,\ \frac{c}{c+d+\lambda_1}h))$, and $(\xi(T^+),\eta(T^+))=$

$(\frac{(1-p)(d+\lambda_1)}{c+d+\lambda_1}h,\ \frac{c(1-p)}{c+d+\lambda_1}h)$; then

$$\frac{\partial P}{\partial x} = -a, \quad \frac{\partial Q}{\partial y} = -d, \quad \frac{\partial \alpha}{\partial x} = -p, \quad \frac{\partial \alpha}{\partial y} = 0,$$
$$\frac{\partial \beta}{\partial x} = 0, \quad \frac{\partial \beta}{\partial y} = -q, \quad \frac{\partial \phi}{\partial x} = 1, \quad \frac{\partial \phi}{\partial y} = 1,$$

$$\begin{aligned}
\triangle_1 &= \frac{P_+\left(\frac{\partial\beta}{\partial y}\frac{\partial\phi}{\partial x} - \frac{\partial\beta}{\partial x}\frac{\partial\phi}{\partial y} + \frac{\partial\phi}{\partial x}\right) + Q_+\left(\frac{\partial\alpha}{\partial x}\frac{\partial\phi}{\partial y} - \frac{\partial\alpha}{\partial y}\frac{\partial\phi}{\partial x} + \frac{\partial\phi}{\partial y}\right)}{P\frac{\partial\phi}{\partial x} + Q\frac{\partial\phi}{\partial y}} \\
&= \frac{(1-p)(P(\xi(T^+),\eta(T^+)) + Q(\xi(T^+),\eta(T^+)))}{P(\xi(T),\eta(T)) + Q(\xi(T),\eta(T))} \\
&= \frac{(1-p)(1-p)(P(\xi(T),\eta(T)) + Q(\xi(T),\eta(T)))}{P(\xi(T),\eta(T)) + Q(\xi(T),\eta(T))} \\
&= (1-p)^2;
\end{aligned}$$

$$\begin{aligned}
\mu_2 &= \triangle_1 \exp\left[\int_0^T \left(\frac{\partial P}{\partial x}(\xi(t),\eta(t)) + \frac{\partial Q}{\partial y}(\xi(t),\eta(t))\right) dt\right] \\
&= (1-p)^2 \exp\left[\int_0^T (-a-d)dt\right] \\
&= (1-p)^2 \exp(\frac{-a-d}{\lambda_1}\ln(\frac{1}{1-p})) \\
&= (1-p)^{2+\frac{a+d}{\lambda_1}}.
\end{aligned}$$

Taking account of $p \in (0,\ 1)$, $a,\ d > 0$ and $\lambda_1 > 0$, we have $0 < \mu_2 < 1$. From Lemma 3.1, the periodic solution of system (2.4) is orbitally asymptotically stable in the case of $p = q$. The proof is then completed.

3.2. **The case of** $p \neq q$**.** In view of (3.2), $\lambda_2 < 0$. Then $0 < \exp(\lambda_2 T) < 1$, and it follows from (3.4) that the initial value for a periodic solution $(\xi(t),\ \eta(t))$ satisfies

$$\frac{1-p}{1-q}\cdot\frac{d+\lambda_1}{c}\eta_0 < \xi_0 < \frac{1-p}{1-q}\cdot\frac{q}{p}\cdot\frac{d+\lambda_1}{c}\eta_0, \tag{3.11}$$

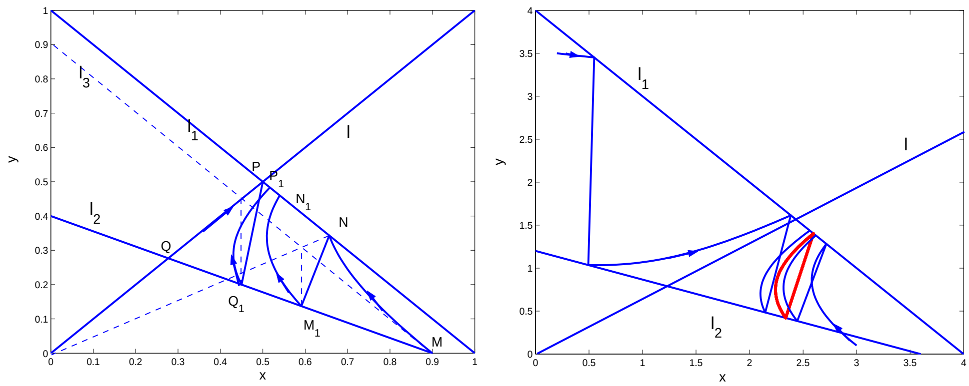
or

$$\frac{1-p}{1-q}\cdot\frac{q}{p}\cdot\frac{d+\lambda_1}{c}\eta_0 < \xi_0 < \frac{1-p}{1-q}\cdot\frac{d+\lambda_1}{c}\eta_0. \tag{3.12}$$

It is seen that inequality (3.11) is valid under the condition $\frac{q}{p} > 1$. In this case, since $\frac{1-q}{1-p} < 1$, we have $\frac{\eta_0}{\xi_0} < \frac{1-q}{1-p}\cdot\frac{c}{d+\lambda_1} < \frac{c}{d+\lambda_1}$, which means that the point $(\xi_0,\ \eta_0)$ is under the separatrix l. So the periodic solution $(\xi(t),\ \eta(t))$ is under the separatrix l for $\frac{q}{p} > 1$. Similarly, inequality (3.12) leads to the condition $\frac{q}{p} < 1$, and the periodic solution $(\xi(t),\ \eta(t))$ is above the separatrix l. In what follows, we verify the result qualitatively in the case of $\frac{q}{p} > 1$, which is well explained in Figure 2a. The result for the case $\frac{q}{p} < 1$ can be similarly obtained.

THEOREM 3.2. *Assume that condition* (H_2): $a,\ b,\ c,\ d > 0,\ ad - bc < 0,$ *and* $p,\ q \in (0,\ 1)$ *holds, then system (2.4) has periodic solutions inside the region between the lines* $l_1:\ x + y = h$ and $l_2:\ \frac{x}{1-p} + \frac{y}{1-q} = h$. *If the periodic solution is unique, then the attractive region is* $\Omega_0 = \{(x,\ y)|x \geq 0,\ y \geq 0,\ x + y < h\}$.

Proof. Suppose that the separatrix l intersects with the lines $l_1:\ x+y = h$ and $l_2:\ \frac{x}{1-p} + \frac{y}{1-q} = h$ at the points $P(\frac{d+\lambda_1}{c+d+\lambda_1}h,\ \frac{c}{c+d+\lambda_1}h)$ and Q, respectively. Consider a



(a) (b)

FIGURE 2. For system (2.4) with $\frac{q}{p} > 1$, (a) the proof for the existence of periodic solutions; (b) a unique periodic solution (red dashed curve) with its attractive region.

trajectory originating from the initial point Q on the separatrix l. After reaching the point P on the line l, first it jumps to the point $Q_1(\frac{d+\lambda_1}{c+d+\lambda_1}(1-p)h,\ \frac{c}{c+d+\lambda_1}(1-q)h)$ on the line l_2, and then reaches the point P_1 on the line l_1, and so on (see Fig. 2a). So we obtain two sequences $\{|\overline{PP_n}|\}$ and $\{|\overline{QQ_n}|\}$ with increasing time t, where $\overline{PP_n}$ denotes the section between the points P and P_n and similar for $\overline{QQ_n}$.

For any point $P_n(x_n,\ y_n)$ on the line $l_1 : x_n, +y_n = h$, we search its corresponding point $Q_{n+1}((1-p)x_n,\ (1-q)y_n)$ on the line l_2 by using the following method. Suppose that OP_n intersects the line l_3 : $x + y = (1-p)h$ at the point $A_n((1-p)x_n,\ (1-p)y_n))$. Then the line passing the point A_n parallel to the y-axis intersects with the line l_2 : $\frac{x}{1-p} + \frac{y}{1-q} = h$ at the point $Q_{n+1}((1-p)x_n,\ (1-q)y_n)$.

Set the point $M = ((1-p)h,\ 0)$, and the trajectory originating from the point M firstly intersects the line l_1 at the point N. The point Q_1 is under the separatrix l in view of $\frac{q}{p} > 1$, so it is between the points Q and M on the line l_2. Thus the abscissa $x_{Q_1} > x_Q$. It follows from the autonomous property of system (2.1) that the trajectory $\widehat{Q_1P_1}$ is between $\overline{QP}$ and the trajectory $\widehat{MN}$. Then the point P_1 is between the points P and N on the line l_1, and then $x_{P_1} > x_P$. So $(1-p)x_{P_1} > (1-p)x_P$; that is, $x_{Q_2} > x_{Q_1}$. This means that the point Q_2 is between the points Q_1 and M and similarly leads to the point P_2 appearing between the points P_1 and N. In general, the point Q_n is between the points Q_{n-1} and M, and the point P_n is between the points P_{n-1} and N, so we have two bounded increasing sequences $\{|\overline{PP_n}|\}$ and $\{|\overline{QQ_n}|\}$:

$$\begin{aligned} &0 < |\overline{PP_1}| < |\overline{PP_2}| < \cdots < |\overline{PP_n}| < \cdots < |\overline{PN}|, \\ &0 < |\overline{QQ_1}| < |\overline{QQ_2}| < \cdots < |\overline{QQ_n}| < \cdots < |\overline{QM}|. \end{aligned} \tag{3.13}$$

Then we have the limits

$$\lim_{n\to\infty} P_n = P_0,\ \lim_{n\to\infty} Q_n = Q_0. \tag{3.14}$$

This means that the trajectory with the initial point $Q_0(x_0,\ y_0)$ intersects the line l_1 at $P_0(\frac{x_0}{1-p},\ \frac{y_0}{1-q})$, and then jumps back to Q_0 again. Hence, it is a periodic solution of system (2.4) with period T, located between the lines l_1 and l_2. From the above deduction, we have $\frac{(d+\lambda_1)(1-p)h}{c+d+\lambda_1} < x_0 < (1-p)h$ and $y_0 = (1-q)(h - \frac{x_0}{1-p})$. In view of the first of (3.4), we obtain the period

$$T = \frac{1}{\lambda_1} \ln\left(\frac{\frac{cx_0}{1-p} - (d+\lambda_2)(h - \frac{x_0}{1-p})}{cx_0 - (d+\lambda_2)(1-q)(h - \frac{x_0}{1-p})}\right). \quad (3.15)$$

Moreover, note that the trajectory originating from the point $M = ((1-p)h,\ 0)$ firstly intersects the line l_1 at N. Then it jumps to the point M_1 on the line l_2, then reaches the point N_1 on the line l_1 again, and so on (see Fig. 2a). As in the above discussion, we can obtain two bounded increasing sequences $\{|\overline{MM_n}|\}$ and $\{|\overline{NN_n}|\}$ such that

$$\lim_{n\to\infty} M_n = M_0,\ \lim_{n\to\infty} N_n = N_0.$$

This means that the trajectory with the initial point N_0 intersects the line l_1 at M_0, and then jumps back to N_0 again. Hence, it is also a periodic solution of system (2.4). Especially, if $P_0 = M_0$ and $Q_0 = N_0$, then there is a unique periodic solution of system (2.4), located between the lines l_1 and l_2. In this case, it is obvious from the above discussion that this unique periodic is orbitally asymptotically stable and its attractive region is $\Omega_0 = \{(x,\ y)|x \geq 0,\ y \geq 0,\ x+y < h\}$. As a numerical result, this unique periodic solution with its attractive region Ω_0 is shown in Fig. 2b.

We can explain the existence of periodic solutions of system (2.4) further. Taking account of the periodic trajectory passing through the point $Q_0(x_0, y_0)$, in view of the fact that $(\exp(\lambda_1 T))^{\lambda_2} = (\exp(\lambda_2 T))^{\lambda_1}$ and the T-periodicity condition (3.4), we have

$$\left[\frac{\frac{cx_0}{1-p} - (d+\lambda_2)(h - \frac{x_0}{1-p})}{cx_0 - (d+\lambda_2)(1-q)(h - \frac{x_0}{1-p})}\right]^{\lambda_2} = \left[\frac{\frac{cx_0}{1-p} - (d+\lambda_1)(h - \frac{x_0}{1-p})}{cx_0 - (d+\lambda_1)(1-q)(h - \frac{x_0}{1-p})}\right]^{\lambda_1}. \quad (3.16)$$

We set $F(x) = F_1(x) - F_2(x)$, where

$$F_i(x) = \left[\frac{\frac{cx}{1-p} - (d+\lambda_i)(h - \frac{x}{1-p})}{cx - (d+\lambda_i)(1-q)(h - \frac{x}{1-p})}\right]^{\lambda_i},\ (i = 1,\ 2).$$

Since $\frac{cx}{1-p} - (d+\lambda_1)(h - \frac{x}{1-p}) > 0$, for $x \in]\frac{(d+\lambda_1)(1-p)h}{c+d+\lambda_1},\ (1-p)h]$ and $\frac{cx}{1-p} - (d+\lambda_2)(h - \frac{x}{1-p}) > 0$, for $x \in [0,\ (1-p)h]$, it is easy to calculate that $F_1(\frac{(d+\lambda_1)(1-p)h}{c+d+\lambda_1}) = 0$, $F_2(\frac{(d+\lambda_1)(1-p)h}{c+d+\lambda_1}) > 0$, $F_1((1-p)h) = (\frac{1}{1-p})^{\lambda_1}$, and $F_2((1-p)h) = (\frac{1}{1-p})^{\lambda_2}$. Then $F(\frac{(d+\lambda_1)(1-p)h}{c+d+\lambda_1}) = 0 - F_2(\frac{(d+\lambda_1)(1-p)h}{c+d+\lambda_1}) < 0$ and $F((1-p)h) = (\frac{1}{1-p})^{\lambda_1} - (\frac{1}{1-p})^{\lambda_2} > 0$. In view of the fact that the function $F(x)$ is continuous in the interval $[\frac{(d+\lambda_1)(1-p)h}{c+d+\lambda_1},\ (1-p)h]$, there exists $x_0 \in]\frac{(d+\lambda_1)(1-p)h}{c+d+\lambda_1},\ (1-p)h[$ such that (3.16) holds. The solvability of x_0 also confirms the existence of periodic solutions of system (2.4). The proof of Theorem 3.2 is completed.

4. **The dynamics of system** (2.5)**.**

4.1. **Existence of periodic solution.**

The periodic solution shown in section 3 is called a period-1 solution in the sense that it has one jump per period. In this section, we discuss the periodic solutions of the more general system

$$\left\{\begin{array}{ll}\left.\begin{array}{l}\dot{x}=-ax+by,\\ \dot{y}=cx-dy,\end{array}\right\} & x+y\neq h,\\ \left.\begin{array}{l}\triangle x=px+\tau_1,\\ \triangle y=qy+\tau_2,\end{array}\right\} & x+y=h.\end{array}\right. \tag{4.1}$$

Set lines $l_1:\ x+y=h$ and $l_2:\ \frac{x}{1+p}+\frac{y}{1+q}=h+\frac{\tau_1}{1+p}+\frac{\tau_2}{1+q}$. Now l_1 intersects the x-axis and y-axis at $E(h,\ 0)$ and $F(0,\ h)$, respectively, while l_2 intersects the x-axis and y-axis at $G((1+p)(h+\frac{\tau_1}{1+p}+\frac{\tau_2}{1+q}),\ 0)$ and $H(0,\ (1+q)(h+\frac{\tau_1}{1+p}+\frac{\tau_2}{1+q}))$, respectively. When a trajectory intersects l_1 at the point $(x,\ y)$, it jumps to the point $((1+p)x+\tau_1,\ (1+q)y+\tau_2)$ according to the impulsive effect. So $E(h,\ 0)$ and $F(0,\ h)$ jump to $\bar{E}((1+p)h+\tau_1,\ \tau_2)$ and $\bar{F}(\tau_1,\ (1+q)h+\tau_2)$, respectively. The correspondence of the points on $\overline{EF}$ and $\overline{\bar{E}\bar{F}}$ is one to one; however, the points $\bar{E},\ \bar{F}$ may be out of region Ω_0.

If l_1 intersects l_2 in region Ω_0, then one of the following conditions is true:

$$(1+p)(h+\tfrac{\tau_1}{1+p}+\tfrac{\tau_2}{1+q})\geq h,\quad 0<(1+q)(h+\tfrac{\tau_1}{1+p}+\tfrac{\tau_2}{1+q}))<h, \tag{4.2}$$

or

$$0<(1+p)(h+\tfrac{\tau_1}{1+p}+\tfrac{\tau_2}{1+q})<h,\quad (1+q)(h+\tfrac{\tau_1}{1+p}+\tfrac{\tau_2}{1+q}))\geq h. \tag{4.3}$$

1. As shown in Figure 3, assume that l_1 intersects l_2 at a point A under condition (4.3), then we obtain

$$A(\frac{(1+p)(qh+\tau_2)+(1+q)\tau_1}{q-p},\ \frac{(1+q)(ph+\tau_1)+(1+p)\tau_2}{p-q}).$$

Since the point A is on l_2, the corresponding point A_1 on l_1 is given by

$$(\frac{1}{1+p}(\frac{(1+p)(qh+\tau_2)+(1+q)\tau_1}{q-p}-\tau_1),\ \frac{1}{1+q}(\frac{(1+q)(ph+\tau_1)+(1+p)\tau_2}{p-q}-\tau_2)).$$

If A_1 is above A on line l_1, that is,

$$\frac{1}{1+p}(\frac{(1+p)(qh+\tau_2)+(1+q)\tau_1}{q-p}-\tau_1)<\frac{(1+p)(qh+\tau_2)+(1+q)\tau_1}{q-p}, \tag{4.4}$$

then we can prove that system (4.1) has a periodic solution in region Ω_0 under conditions (4.3) and (4.4), similar to that seen in the case of $p\neq q$ in section 3 (see Fig. 3).

If condition (4.2) holds but condition (4.4) doesn't hold, system (4.1) has a periodic solution in region Ω_0. The analysis is similar and omitted here.

2. If condition (4.3) holds but condition (4.4) does not hold, any trajectory with an initial point in Ω_0 will leave the region Ω_0 eventually, and then system (4.1) has no periodic solution. This is also true under conditions (4.2) and (4.4).

3. If the following condition

$$(1+p)(h+\tfrac{\tau_1}{1+p}+\tfrac{\tau_2}{1+q})\geq h,\quad (1+q)(h+\tfrac{\tau_1}{1+p}+\tfrac{\tau_2}{1+q}))\geq h \tag{4.5}$$

is valid, line l_2 is out of region $\Omega_0=\{(x,\ y)|x\geq 0,\ y\geq 0, x+y<h\}$, and the trajectory with an initial point in Ω_0 will leave region Ω_0 forever. Hence, system (4.1) has no periodic solution under conditions $ad-bc<0$ and (4.5).

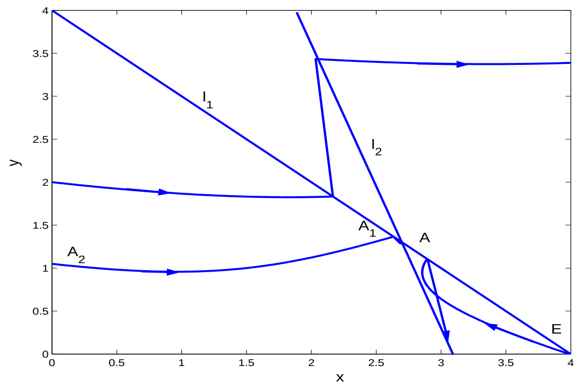


FIGURE 3. The existence of the periodic solution of system (4.1).

To summarize the above results, we give the following theorem:

THEOREM 4.1. *Assume that* $ad - bc < 0$.

1. *If both conditions* (4.3) *and* (4.4) *hold or condition* (4.2) *holds but* (4.4) *does not hold, then system* (4.1) *has a periodic solution in region* Ω_0.
2. *If both conditions* (4.2) *and* (4.4) *hold or condition* (4.3) *holds but* (4.4) *does not hold, then system* (4.1) *has no periodic solution.*
3. *If condition* (4.5) *holds, then system* (4.1) *has no periodic solution.*

4.2. **Poincaré map.** To analyze the bifurcation of periodic solutions numerically, we resort to a Poincaré map. Set $P_k(x_k,\ h - x_k)$ on l_1, then we have $P_k^+((1+p)x_k + \tau_1,\ (1+q)(h-x_k)+\tau_2)$ on l_2. The trajectory with the initial point P_k^+ first intersects l_2 at $P_{k+1}(x_{k+1},\ h - x_{k+1})$. It follows from (3.1) that

$$\begin{cases} \frac{d+\lambda_1}{\lambda_1-\lambda_2}((1+p)x_k + \tau_1 - \frac{d+\lambda_2}{c}((1+q)(h-x_k)+\tau_2))\exp(\lambda_1 T) \\ +\frac{d+\lambda_2}{\lambda_2-\lambda_1}((1+p)x_k + \tau_1 - \frac{d+\lambda_1}{c}((1+q)(h-x_k)+\tau_2))\exp(\lambda_2 T) = x_{k+1}, \\ \frac{c}{\lambda_1-\lambda_2}((1+p)x_k + \tau_1 - \frac{d+\lambda_2}{c}((1+q)(h-x_k)+\tau_2))\exp(\lambda_1 T) \\ +\frac{c}{\lambda_2-\lambda_1}((1+p)x_k + \tau_1 - \frac{d+\lambda_1}{c}((1+q)(h-x_k)+\tau_2))\exp(\lambda_2 T) = h - x_{k+1}, \end{cases}$$

which means

$$\begin{cases} \exp(\lambda_1 T) = \frac{x_{k+1}-(h-x_{k+1})\frac{d+\lambda_2}{c}}{(1+p)x_k+\tau_1-\frac{d+\lambda_2}{c}((1+q)(h-x_k)+\tau_2)}, \\ \exp(\lambda_2 T) = \frac{x_{k+1}-(h-x_{k+1})\frac{d+\lambda_1}{c}}{(1+p)x_k+\tau_1-\frac{d+\lambda_1}{c}((1+q)(h-x_k)+\tau_2)}. \end{cases}$$

Then we have

$$\left(\frac{x_{k+1}-(h-x_{k+1})\frac{d+\lambda_2}{c}}{(1+p)x_k+\tau_1-\frac{d+\lambda_2}{c}((1+q)(h-x_k)+\tau_2)}\right)^{\lambda_2} = \left(\frac{x_{k+1}-(h-x_{k+1})\frac{d+\lambda_1}{c}}{(1+p)x_k+\tau_1-\frac{d+\lambda_1}{c}((1+q)(h-x_k)+\tau_2)}\right)^{\lambda_1}. \tag{4.6}$$

From the above relation between x_k and x_{k+1}, we obtain the Poincaré map $P : x_k \rightarrow x_{k+1}$. A fixed point of the Poincaré map P corresponds to one of the periodic solutions of system (4.1).

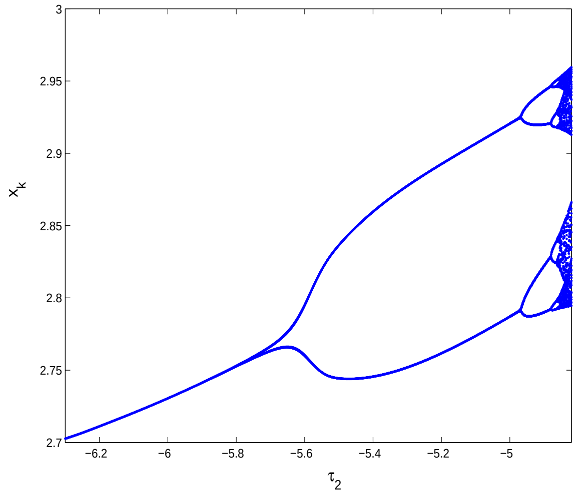


FIGURE 4. The bifurcation diagram of periodic solutions of system (4.7) with $q = 3.9$ and $\tau_2 \in (-6.3,\ -4.82)$.

4.3. **Numerical simulation.** We give the numerical results about the bifurcation of periodic solutions through the following example:

$$\begin{cases} \left.\begin{array}{l} \dot{x} = -0.2x + 0.6y, \\ \dot{y} = 0.1x - 0.1y, \end{array}\right\} & x + y \neq 4, \\ \left.\begin{array}{l} \triangle x = 0.4x - 1, \\ \triangle y = qy + \tau_2, \end{array}\right\} & x + y = 4. \end{cases} \tag{4.7}$$

In this case, $a = 0.2,\ b = 0.6,\ c = 0.1,\ d = 0.1,\ p = 0.4,\ \tau_1 = -1,\ h = 4$. We have $\lambda_1 = 0.1,\ \lambda_2 = -0.4$.

Viewing τ_2 as a parameter, the bifurcation diagram is given for system (4.7) with $q = 3.9$ and $\tau_2 \in (-6.3,\ -4.82)$ in Figure 4. There exists a period-1 solution of system (4.7), and a period-doubling bifurcation leads to chaotic solutions. The period-1 solution is stable for $\tau_2 \in (-6.3,\ -5.83)$ and unstable for $\tau_2 \in (-5.83,\ -4.82)$. A period-2 solution bifurcates from the period-1 solution at $\tau_2 = -5.83$. The stability of these periodic solutions is shown in Figure 5.

Taking $\tau_2 = -5, q \in (3.4,\ 4.02)$ and viewing q as a parameter in system (4.7), there is a route to chaos via period-doubling bifurcation in Figure 6. The stable period-1, period-2, and period-4 solutions, respectively, are given in Figure 7 for different q.

Set $q = 4.01$ and the initial point $(2.9,\ 0.8)$, the phase portrait of the chaotic solution of system (4.7) is shown in Figures 8a. The time series of y and x are shown in Figures 8b and 8c, respectively.

5. **Discussion of impulsive state feedback control strategy.** We know that the equilibrium point $O(0,\ 0)$ of system (2.2) is a saddle under the fact that $a > c$ and $ad - bc < 0$. In the case without pest control, we have $d < \frac{c}{a}b < b$, which means the birth rate of the immature pest is larger than the death rate of the mature pest, and the sum of the immature and mature pest population will tend to positive infinity as $t \to +\infty$. In this paper, when the sum of the pest population

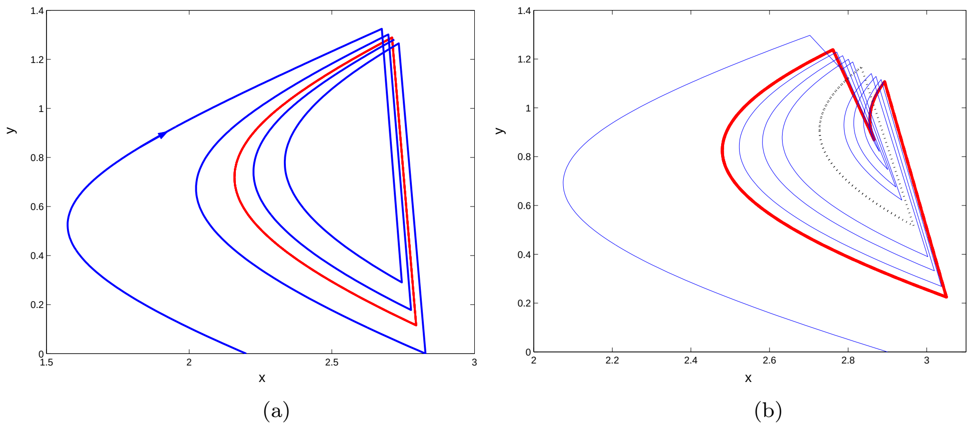


FIGURE 5. For system (4.7) with $q = 3.9$, (a) the stable period-1 solution (red dashed line) for $\tau = -6.2$; (b) the stable period-2 solution (red dashed line) and unstable period-1 solution (black dotted line) for $\tau = -5.2$.

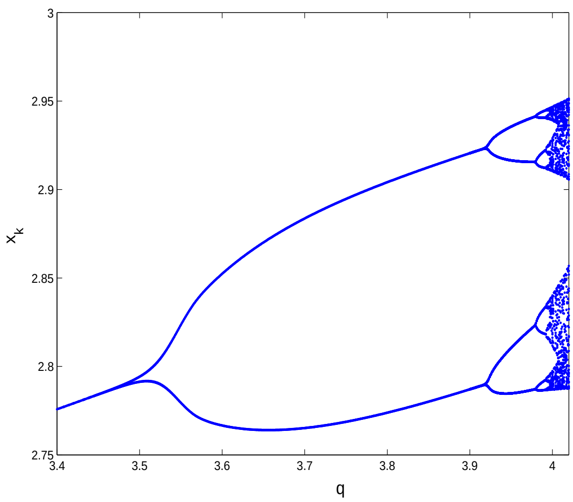
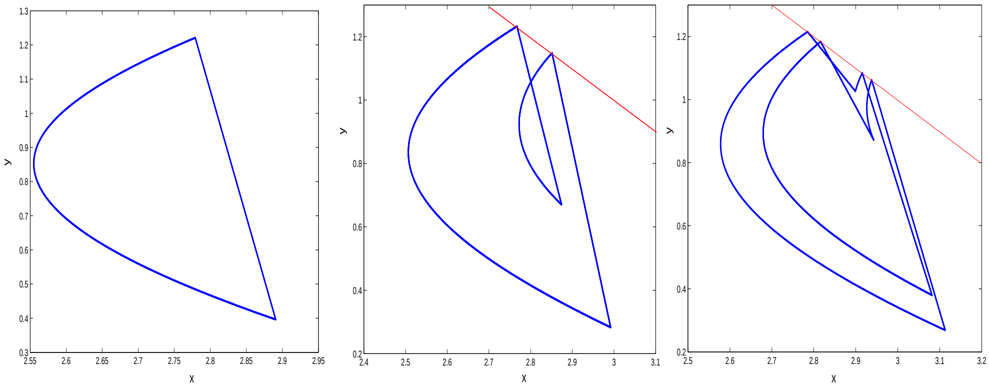


FIGURE 6. The bifurcation diagram of periodic solutions of system (4.7) with $\tau_2 = -5$ and $q \in (3.4,\ 4.02)$.

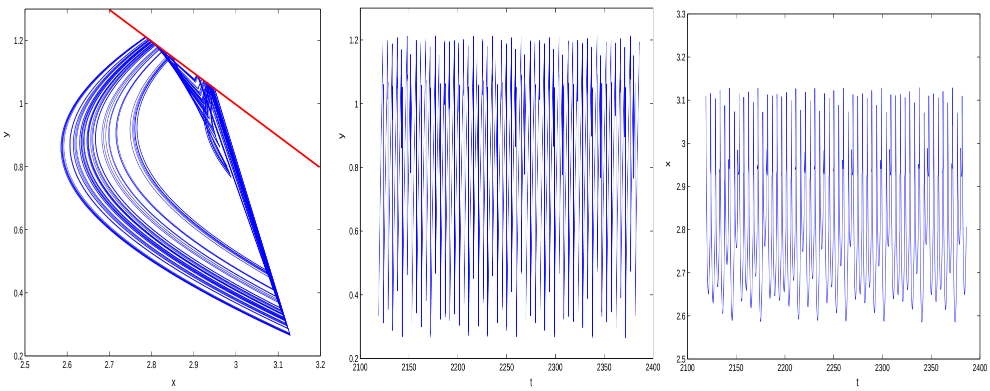
reaches an appropriate threshold value, an impulsive state feedback control measure for pesticide spraying is taken. According to the analysis in section 3, this measure is effective. The total pest population can be controlled below the threshold and the potential disaster posed by the pest can be avoided. System (2.4) has a stable periodic solution and its attractive region is region Ω_0; that is, the population of immature and mature pests will eventually vary periodically, and the biological environment can be preserved. Of course, this requires that we determine the threshold value h correctly through pest monitoring.

If we take the impulsive control measures at fixed times, for example, periodically, the effect and the cost of control may not be optimal. Now consider an example.



(a) (b) (c)

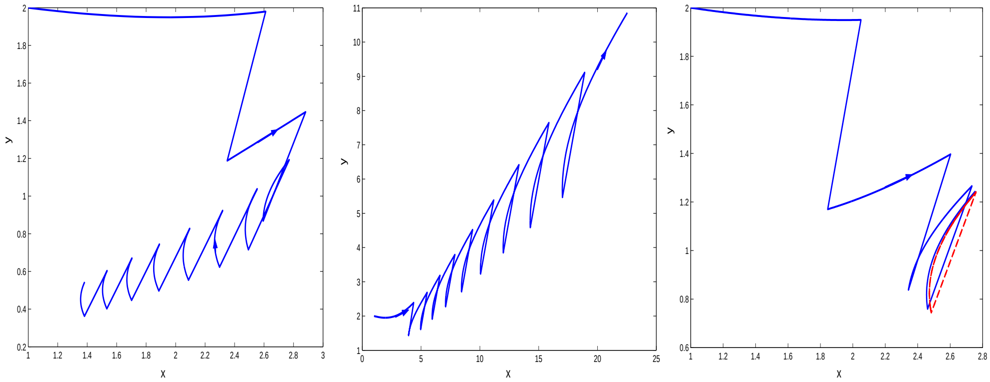
FIGURE 7. The stable periodic solutions for system (4.7) with $\tau_2 = -5$. (a) $q = 3.42$; (b) $q = 3.6$; (c) $q = 3.96$.



(a) (b) (c)

FIGURE 8. The chaotic solution for system (4.7) with the initial point (2.9, 0.8), $\tau_2 = -5$, and $q = 4.01$. (a) phase portrait; (b) time-series of y; (c) time-series of x.

Assume that the initial point is (1, 2), $p = 0.1$, $q = 0.4$, and the threshold value is 4. If the pesticide is sprayed at time $t = 2k(k = 1, 2, ...)$ (see Fig. 9a), then the pesticide is still sprayed even when the pest population is smaller than the threshold value 4 and does not affect the growth of crops at all. The cost of pest management is high, and the large amount of pesticide is harmful to the environment. If the pesticide is sprayed at time $t = 5k(k = 1, 2, ...)$ (see Fig. 9b), then the pest population cannot be controlled below the threshold value 4, and this leads to a disaster by the pest. Figure 9c, the state feedback control measure is taken when the sum of pest population reaches the threshold value $h = 4$. Only after three attempts at control does the solution approach the periodic solution (red dashed line in Fig. 9c). This example shows that the impulsive state feedback control measure is more effective.



(a) (b) (c)

FIGURE 9. The trajectories with respect to the impulsive effects $\triangle x = -0.1x, \triangle y = -0.4y$ for a = 0.2; b = 0.6; c = 0.1; and d = 0.1. (a) The initial point is (1, 2) and $t = 2k(k = 1, 2, ...)$; (b) the initial point is (1, 2) and $t = 5k(k = 1, 2, ...)$; and (c) the initial point is (1, 2) and the threshold $h = 4$.

6. **Conclusion.** To control a pest population by spraying pesticide, two models of autonomous systems with impulses were discussed in this paper. It is seen that although the dynamical property of the original system (2.2) is simple, the dynamics of the impulsive systems (2.4) and (2.5) is very complex. Sufficient conditions of the existence and stability of periodic solutions, as well as the attractive region of periodic solution, were obtained through qualitative analysis. A cascade of period-doubling bifurcations of periodic solutions led to chaotic solutions in system (2.5). The bifurcation diagrams of stable periodic solutions were obtained by using a discrete map on the Poincaré section. It was seen that the impulsive state feedback control was more effective than the impulsive fixed-time control in pest management.

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