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Research article

Generating topologies for attribute-value tables using lattices and neighborhoods

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Abstract: The values in information tables are responsible for dividing the objects of space into partition of object space, which may be topologically. This partition forms a base for topology in complete information system and a subbase for incomplete information systems depending on the arrangement data. The values in information system are responsible for generated different topologies according to the relationship between objects and attributes, since the value is the connection between them. The effectiveness of values in information tables is that the values generate the equivalence classes for a Pawlak approximation space in complete information system, where every equivalence class is a subset of objects and has the same properties. Moreover, attribute values are the primary method for forming these equivalence classes of the approximation space, and the collecting data may take a lot of time, money, and power. If the data does not provide a good analysis, then the effort is wasted and we lose money and time. Therefore, we want to benefit of the data, and change the type of values into another form to generalize of the approximate space. The main aim of this paper was to solve some problems related to values in information system tables by unimportant data reduction due to the structure and types of topologies. On the other hand, the formation of objects lattice for space elements through topological concepts was used. In the new approximation space, the undefinable rough sets changed into definable rough sets depending on new equivalence classes for a Pawlak approximation space through changing the values in information table.

Keywords: neighborhood; topological spaces; rough set theory; information system
Mathematics Subject Classification: 54D05, 54F15, 54F50, 37B10

1. Introduction and preliminaries

Pawlak introduced the rough set theory [1–3] which consists rough set approximations and information tables. Rough set approximations generated by an equivalence relation interpreted any

objects are equivalent if we cannot distinguish them by using information table. This means that the ability for discern objects is difficult and cannot observe individual objects . Information systems [4] are very important to the operation and management of organization. Managers who invest large amounts of money and other resources in information systems often do not know which information systems applications will benefit the organization. Management used information tables to support decision-making [5]. The decision to proceed from an initial information table evaluation to a costly, in-depth analysis must be based upon the value of using the information table. Avoid objective evaluation of an information systems because many of the benefits are intangible and imprecise data can be used to provide an objective evaluation of the benefits of an information table. Such an evaluation may serve as a "gate " to screen out unjustifiable information table before a costly analysis is performed. The value in information table is the connection between the object and attribute, the value that is given by the intersection of the row labeled by an object x, and the column labeled by the attribute. It is also interesting to notice that in information systems, for the map attaching to each attribute set, its equivalence relation has an adjoint, and therefore it determines a connection between objects and attributes. The values in information systems must be satisfying the following concepts:
Goal attainment means that the values cover the goals of research and helps the realization of these goals.
Adaptation holds when a value is able to interact successfully and its serves as a research subject. Facilitators of adaptation will include values, attributes, and incentives that make change acceptable and understandable.
Integration is when goals have been attained and involves bringing the different shapes of values and structure together into reasonably stable relationship. Successful integration implies a positive relationship between the new relationship between objects and attributes.
Model Correction is a new system that has been introduced. The essential function of model correction is to maintain its stability by making it acceptable to the values and attributes associated with it.

The concept of rough sets describes approximation knowledge of subsets of a given universe. In some sense, this theory can be considered as a generalization of classical theories [6–8]. One of the applications in the field of rough sets is the knowledge discovery [9] which is deep-seated in the classification abilities of human beings and other species. For example, knowledge about the environment is primarily manifested as an ability to classify a variety of situations from the point of view of survival in the real world, and by using the knowledge base we can understand the approximation space. Changes of topologies [10, 11] play an important role in the field of general topology [12, 13] and its applications [14, 15], and this is connected to topologies expanding. In topological space, a neighborhood system [16–18] is a mathematical structure used to describe the local behavior of points in a topological space and plays a key role in every branch of mathematics. The neighborhood system of a point x is the collection of all neighborhoods of x and has different meaning in mathematics. A neighborhood of a point in a topological space is an important concept. It is closely defined by the concepts of sets which are essential in obtaining interior and closure regions in topological spaces. The formation of objects lattice [19–21] for space elements through topological concepts will be used. In the new approximation space, the undefinable rough sets change into definable rough sets depending on new equivalence classes for a Pawlak approximation space through changing the values in information table. In this paper, some problems related to values in information system tables by unimportant data reduction due to the structure and types of topologies will be solved.

Let U be a finite set and R be an equivalence relation on U. The pair $\mathcal{A}_S = (U, R)$ is called an approximation space. Some subsets of U are definable in the given approximation space and others are said to be undefinable. $X \subseteq U$ is exact (definable) if it is a union of some equivalence classes, otherwise the set X is undefinable. the rough set (undefinable set) is defined approximately in any approximation space which depends on two exact sets referred to as lower approximation (the union of all definable sets contained in a given set) and upper approximation (the union of all definable sets containing a given set).

Definition 1.1. *[22] Let $I_S = (U, R)$ be an information table, where U is a nonempty set of finite objects (called a universe set) and A is a nonempty finite set of attributes such that $a : U \to V$ for every attribute a. V_a is the set of values that attributes from V. In other words, the information table simply assigns a value V_a to each attribute a of each object in universe U.*

Definition 1.2. *[22] Let $I_S = (O, A, R, f, V)$ be information system, then the equivalence class of each object in approximation space is defined by the following equation: $[x] = \{y \in O : R_a(x) = R_a(y) = v_i, v_i \in V and a \in A\}$.*

2. The uncertainty in attribute-value tables

The information table consists of two important concepts: Objects or space and condition, and decision attributes and the connection between them in a table by values. The values are the important part in information table and may appear in several forms such as numerical values and others. Values is the basic method to forming equivalence classes which is the base for approximation space. Problem: There are undefinable sets with respect to approximation space which generated from information table that we cannot judge and give a decision, and we cannot determine lower approximation and upper approximation always in the universe and empty subsets. Any set in approximation space has four possibilities with respect to topological vision, which depend on two open sets of topological space. The first kind is rough definable: If we can determine the lower and upper bound; in other words, if we can determine all objects in or out of the set. Whether or not we know the objects in the set is the second kind, which is called internally undefinable and externally undefinable. The third kind is if we cannot determine the objects out of the set. Finally, if we cannot know about objects, this is the fourth type. The solution to solving this problem must be reorder, modification, and collecting the values in information table to give new approximation space is that bigger than the old. As in Figure 1.

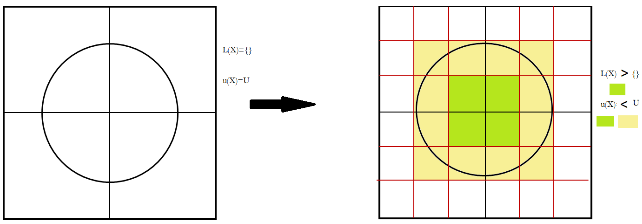

Figure 1. Change of approximation space.

Definition 2.1. *Let $A_v = (O, A, R, V)$ be an attribute-value table where O is the set of objects and A is the set of attributes, then a neighborhood of every object is given by:*
$N_i^A(x) = \{y : a(x) = a(y) = v_i, \forall v_i \in V\}$*, where $i \in A$.*

Proposition 2.2. *Let $A_v = (O, A, R, V)$ be an attribute-value table, then the set of all equivalence classes of objects is O/R_{v_a}, a base for a quasi discrete topological object space.*

In Proposition 2.3, $\leq$ means every class of $O/R_{\grave{v}}$ is included in the class of O/R_{v_a}. It is easy to prove it and so we omit it.

Proposition 2.3. *Let $A_v = (O, A, R, V)$ be an attribute-value table. If for some $v_i = \{v_1, v_2, v_3, ..., v_n\} \in V$ such that $\bigcup_i v_i = v_a$ for $a \in A$, then, the following are equivalent:*

(a) $O/R_{\grave{v}a} \leq O/R_{v_a} \forall a \in A$.

(b) The topology generated by $O/R_{\grave{v}a}$ is finer than the topology generated by O/R_{v_a}.

Remark 2.4. *Let $A_v = (O, A, R, f, V)$ be an attribute-value table, where O is objects of approximation space, A is a set of attributes, R is a equivalence relation and V is the set values between objects and attributes, then:*

(a) The bases $O/R_{\grave{v}a}, O/R_{v_a}$ form two partitions of object.

(b) Quasi discrete topology of attribute-value table based on (depended on) domain of value for every attribute and distribution of value data.

(c) The set of all object equivalence classes for every attribute is a base for unique rough topological space.

(d) The attribute-value table defines multi-rough topological spaces where every attribute defines rough topological space. $A_v = (O, A, R, f, V) = (O, \tau_i(A))$.

Example 2.5. *Let U is the group of 12 students. There are two studies about students by two different ways, as follows (see Table 1):*

Table 1. Numerical and categorical GPA in attribute-value table of students subjects.

Student	Major	GPA
1	art	3.5
2	science	2.6
3	art	3.7
4	science	3.9
5	science	3.3
6	science	2.7
7	science	3.5
8	science	3.4
9	art	3.8
10	science	2.9
11	science	3.2
12	art	3.9

Student	Major	GPA
1	art	excellent
2	science	average
3	art	excellent
4	science	excellent
5	science	good
6	science	average
7	science	excellent
8	science	good
9	art	excellent
10	science	average
11	science	good
12	art	excellent

From Table 1, in the first table, the equivalence classes of condition attribute Major (base for condition topology) are: $O/M = \{\{1, 3, 9, 12\}, \{2, 4, 5, 6, 7, 8, 10, 11\}\}$. *The equivalence class of decision attribute GPA (base for decision topology) is:* $O/GPA = \{\{1, 7\}, \{2\}, \{3\}, \{4, 12\}, \{5\}, \{6\}, \{8\}, \{9\}, \{10\}, \{11\}\}$. *We note that all equivalence classes of decision attribute are totally roughly undefinable and accuracy of all classes is zero and we cannot know any information.*

In the second table, the equivalence classes of condition attribute Major (base for condition topology) are: $O/M = \{\{1, 3, 9, 12\}, \{2, 4, 5, 6, 7, 8, 10, 11\}\}$. *The equivalence classes of decision attribute GPA (base for decision topology) are* $O/GP = \{\{1, 3, 4, 7, 9, 12\}, \{2, 6, 10\}, \{5, 8, 11\}\}$. *We note that the accuracy measure of class (excellent) is* 0.33 *and the status of class transforms (change) from totally undefinable into undefinable.*

3. Generation topological spaces from attribute-value table

To generate topological spaces from the attribute-value table, $A_v = (O, A, R, f, V)$ must be obtained on the equivalence classes of every attribute by collecting all objects that have the property which forms the base for topological space of this attribute. The reduction of attribute-value table$A_v = (O, A, R, f, V)$ is given via the following methods: First, if all objects have the same value for a given attribute, then this attribute in the information table generated the indiscrete topological space which is not useful in our study because all objects have only the universal sets in the equivalence class and cannot obtain information to classify objects. So we debate the attribute that consists of the one value. That means the attribute does not offer any information in discerning between any records, and this attribute does not have any information to classify the approximation space and gives no information in our problem classification. The second method found, if every object has a unique value that differs from other objects generated by discrete topological space and is given huge information classification, then every rough set become an exact set. So, we delete the attribute that consists of the power of the universe. In the third method, we can obtain on equivalent topologies in the information table via tow methods once any attribute has the same values for the same objects and gives us the same equivalence classes which generates the same topological space; orany attribute has the same classification with respect to different values and gives us the same topological space. So, we will keep one attribute and delete the other because all attributes will generate the same approximation space. Lastly, from Proposition 2.3 the expansion and finer topology of the family of topologies contains all topologies or these topologies contained in this topology, so, we choose the expansion and cancel the others.

In this section, we use topological spaces as a tool to reduce attribute-value tables to have the core of reduction. Therefore, we rewrite the steps of Example 3.7 in the Algorithm 3.2.

Example 3.1. *Let the objects of approximation space be* $O = \{p1, p2, p3, p4, p5, p6\}$ *with eight conditions and decision attributes: Headache, muscle pain, temperature, sex, stomach pain, runny nose, and flu.*

From Table 2, the reduction attributes steps of information system is given as follows,

1. *Equivalence classes of Age attribute give the base for discreet topology as the following:* $U/R = \{\phi, \{p1\}, \{p2\}, \{p3\}, \{p4\}, \{p5\}, \{p6\}\}$.
2. *Equivalence classes of Sex attribute give the base for indiscreet topology as:* $U/R = \{\phi, \{p1, p2, p3, p4, p5, p6\}\}$.

3. *Equivalence classes for equivalent topologies are given by following methods:*
(a) *The bases Muscle pain and Runny Nose are equal with respect to object values:* $U/R = \{\phi, \{p1, p3, p6\}, \{p2, p4, p5\}\}$.
(b) *The bases Headache and Stomach pain are equal with respect to classification* $U/R = \{\phi, \{p1, p2, p3\}, \{p4, p5, p6\}\}$.
4. *Equivalence classes of temperature include equivalence classes of headache as* $U/R_T = \{\{p1, p2\}, \{p3\}, \{p4\}, \{p5, p6\}\} \leq U/R_H = \{\{p1, p2, p3\}, \{p4, p5, p6\}\}$.
5. *From Remark 2.4, reduction of approximation space is given by the following table after debating the pervious topologies and only measuring with respect to temperature and muscle pain. Now compute the accuracy and quality of flu approximation with respect to temperature and muscle pain approximation:*

Table 2. Conditions and decision attribute-value table for flu patients.

	Age	Headache	Muscle Pain	Temperature	Sex	Stomach Pain	Runny Nose	Flu
p1	22	Yes	Yes	Normal	Male	No	Yes	No
p2	55	Yes	No	Normal	Male	No	No	Yes
p3	19	Yes	Yes	High	Male	No	Yes	Yes
p4	31	No	No	Cool	Male	Yes	No	No
p5	60	No	No	Very High	Male	Yes	No	No
P6	9	No	Yes	Very High	Male	Yes	Yes	Yes

From Table 3, we note that, the accuracy measure and equality of flu with respect to muscle pain is $\alpha(F)_M = 0, \gamma(F)_M = 0$. *Also, the accuracy measure and equality of flu with respect to temperature are* $\alpha(F)_T = 1/5 = 0.20, \gamma(F)_T = 1/3 = 0.3333$. *Then, the core of the given approximation information system is temperature.*

Table 3. The Reduction table for conditions and decision attribute-value table.

	Muscle Pain	Temperature	Flu
p1	Yes	Normal	No
p2	No	Normal	Yes
p3	Yes	High	Yes
p4	No	Cool	No
p5	No	Very High	No
p6	Yes	Very High	Yes

In Algorithm 3.2, we give procedures to determine the core of reduction of the attribute value table.

Algorism 3.2. ***The reduction of attribute value table***
Input: *Attribute value table* $A_v = (O, A, R, f, V)$.
Output: *Topological spaces, reduct, core.*
Step 1: *Delete the base for indiscrete topology.*
Step 2: *Cancel the base for discrete topology .*
Step 3: *Choose one of the equivalent topologies.*

Step 4: *Take the finer topology by using inclusion property.*
Step 5: *Construct the reduction of $A_v = (O, A, R, f, V)$ from the remaining attributes.*
Step 6: *Compute the accuracy measure to choose the core of $A_v = (O, A, R, f, V)$.*

The Algorithm 3.2 can be given through Figure 2.

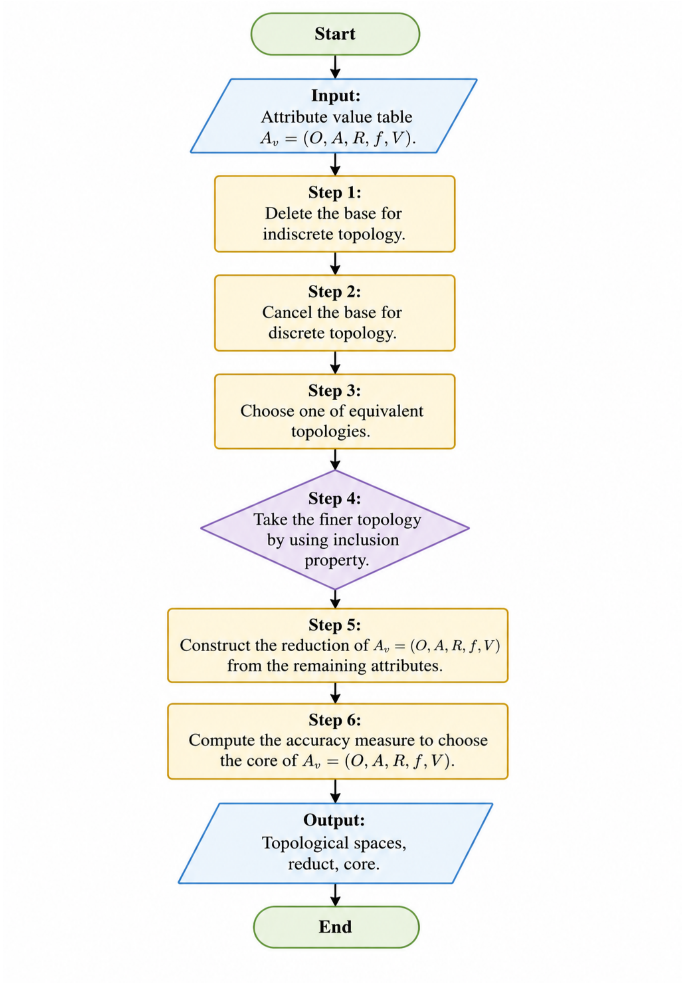

Figure 2. Process flow of topological computation.

Theorem 3.3. *Let $I_S = (O, A, R, f, V)$ be the information table generating the multi- topological spaces $(O, \tau_i(A))$ and $H \in O/R_2$, which is a union of some members of O/R_1; then the approximation space generated by O/R_1 is expansion of the approximation space generated by O/R_2.*

Proof. Let $O/R_1 \leq O/R_2$ where O/R_1 and O/R_2 are bases of equivalence relations of R_1, $R_2 \in R$ respectively. We want to prove that the topology τ_{R_1} generated by R_1 is expansion of topology τ_{R_2} generated by R_2. Let $H \in \tau_{R_1}$ be the union of members of O/R_2 that means $H = \bigcup_{i=1}^{n}\{B_i2, B_i2 \in O/R_2\}$ but every member of O/R_2 be the union of members of O/R_1. So, $B_i2 = \bigcup_{j=1}^{m}\{B_i1, B_i1 \in O/R_1\}$. Then $H = \bigcup_{i=1}^{n}\bigcup_{j=1}^{m}\{B_i1, B_i1 \in O/R_1\}$. Thus, $H \in \tau_{R_1}$. Therefore, τ_{R_1} is an expansion of τ_{R_2}. □

Remark 3.4.

1- In the information table, $R_1 \subseteq R_2$, the approximation space generated by R_1, is larger than the approximation space generated by R_2. On the other hand, $O/R_1 \leq O/R_2$ means that if every member of O/R_1 exists in O/R_2 or is a subset of many members of O/R_2 (or, if any largest member of O/R_2 is a union of some members of O/R_1), then the approximation space generated by R_1 is larger than the approximation space generated by R_2.

2- Let $I_S = (O, A, R, f, V)$ be the information table generating the multi- topological spaces $(O, \tau_i(A))$ and $\tau_{R_1}\tau_{R_2}$ be two rough topological spaces of R_1 and R_2 where $R_1 \subset R_2$, then rough topological space of R_1 is an expansion of rough topological space R_2.

The expansion of topologies is used in the proof of Theorem 3.5. So, we omit it.

Theorem 3.5. *Let $I_S = (O, A, R, f, V)$ be the information table generating the multi-topological spaces $(O, \tau_i(A))$ and τ_{R_1}, τ_{R_2} be two rough topological spaces of R_1 and R_2 where $R_1 \subset R_2$. Then,*

(a) If H is roughly definable or undefinable in τ_{R_2}, then H is also roughly definable in τ_{R_1}.

(b) If H is externally undefinable (internally undefinable and totally undefinable) in τ_{R_2}, then H is not externally undefinable (is not internally undefinable and not totally undefinable) in τ_{R_1}.

(c) The lower approximation of H in τ_{R_1} is greater than the lower approximation of H in τ_{R_2}; also, the upper of H in τ_{R_2} is greater than the lower approximation of H in τ_{R_1}. So, the accuracy measure of H in τ_{R_1} is greater than the accuracy measure of H in τ_{R_2}.

(d) All basic properties of Pawlak approximation space are similar to the basic properties in its expansions.

Remark 3.6. *Let $I_S = (O, A, R, V)$ be the attribute-value table and the set of all equivalence classes of objects is O/Ra base for topological object space generating from the Pawlak approximation space.*

1- The base of every attribute defines a unique topological space.

2- The intersection of any number of attributes gives the base for topological space.

3- The topological space of intersection between two attributes is expansion of the other.

Example 3.7. ***Application attributes-values table***

Consider the objects of approximation space $O = \{p1, p2, p3, p4, p5, p6\}$ with four conditions and decision attributes, headache, muscle pain, temperature, and flu.

From Table 4, the equivalence classes of condition attributes are $O/H = \{\{1, 2, 3\}, \{4, 5, 6\}\}$, $O/M = \{\{1, 3, 6\}, \{2, 4, 5\}\}$, $O/T = \{\{1, 2\}, \{3, 4, 5, 6\}\}$. The equivalence class of the decision attribute is $O/F = \{\{1, 4, 5\}, \{2, 3, 6\}\}$. Note that all equivalence classes of the decision attribute are totally roughly undefinable, the accuracy of all classes is zero, and we cannot not know any information with respect to all conditions attributes.

Table 4. Information table for the flu of 6 patients-A.

	Headache	Muscle Pain	Temperature	Flu
p1	Yes	Yes	Normal	No
p2	Yes	No	Normal	Yes
p3	Yes	Yes	High	Yes
p4	No	No	High	No
p5	No	No	High	No
p6	No	Yes	High	Yes

Moreover, in Table 5, the equivalence classes of condition attributes are $O/H = \{\{1,2,3\},\{4,5,6\}\}$, $O/M = \{\{1,3,6\},\{2,4,5\}\}$, $O/T = \{\{1,2\},\{3,6\},\{4,5\}\}$. The equivalence class of the decision attribute is $O/F = \{\{1,4,5\},\{2,3,6\}\}$. From condition attribute temperature, the set of patients who do or do not have the flu become roughly definable and have accuracy of 50% as $L(flu = yes) = L(\{2,3,6\}) = \{3,6\}$, $u(flu = yes) = u(\{2,3,6\}) = \{1,2,3,6\}$, $L(flu = no) = L(\{1,4,5\}) = \{4,5\}$, $u(flu = no) = u(\{1,4,5\}) = \{1,2,4,5\}$.

Table 5. Information table for the flu of 6 patients-B.

	Headache	Muscle Pain	Temperature	Flu
p1	Yes	Yes	Normal	No
p2	Yes	No	Normal	Yes
p3	Yes	Yes	High	Yes
p4	No	No	Very High	No
p5	No	No	Very High	No
p6	No	Yes	High	Yes

We note that the roughly undefinable sets become roughly definable with respect to temperature attribute after changing, so we can cancel (reduce) other attributes. Boundary region of a subset is contracted with respect to our idea and the degree of accuracy of approximation increases.
$O/(H \bigcap M) = \{\{1,3\},\{4,5\},\{2\},\{6\}\}, \mu(yes) = 50, \mu(no) = 50$
$O/(H \bigcap T) = \{\{1,2\},\{4,5\},\{3\},\{6\}\}, \mu(yes) = 50, \mu(no) = 50$
$O/(T \bigcap M) = \{\{1\},\{4,5\},\{2\},\{3,6\}\}, \mu(yes) = 66, \mu(no) = 66$
$O/(H \bigcap M \bigcap T) = \{\{1\},\{3\},\{4,5\},\{2\},\{6\}\}, \mu(yes) = 100, \mu(no) = 66.$
So, the reduction attributes of the given attribute-value table are $\{T, M\}$.

4. Particular object topological spaces of attribute-value table

Definition 4.1. *Let $I_S = (O, A, R, f, V)$ be an attribute-value table then R is the equivalence relation generated by f and the subbase for patient P_i is given by: $\delta_P = \{\phi, [P_i]_a \subset U$ for all $a \in A\}$, where the neighborhood object $[P_i]_a$ of an attribute $a \in A$ is defined by:*
$[P_i]_a = \{P_j \in O : f(P_i, a) = f(P_i, a) = v : v \in V_a\}$ where $a \in A$.

Definition 4.2. *Let $I_S = (O, A, R, f, V)$ be an information table, then conditions are accurately measured given by the following equation: $\mu_{E \in U/D}(P_i) = \frac{|\sum_{i=1}^{n}[P_i]_a \bigcap E|}{|\sum_{i=1}^{n}[P_i]_a|}$, where n is the number of neighborhoods for every object P_i with respect to attribute $a \in A$.*

Example 4.3. *Let the objects of approximation space be* $O = \{p1, p2, p3, p4, p5, p6\}$ *with seven conditions and decision attributes: Headache, muscle pain, temperature, stomach pain, runny nose, and flu.*

From Table 6, the subbase of $P1$ with respect to all condition's attributes in the information table: $\delta_{P1} = \{\phi, \{P1\}, \{P1, P2, P3\}, \{P1, P3, P6\}, \{P1, P2\}\}$. The base of $P1$ with respect to all condition's attributes in the information table: $\beta_{P1} = \{\phi, \{P1\}, \{P1, P2\}, \{P1, P3\}, \{P1, P2, P3\}, \{P1, P3, P6\}\}$. The topological space of $P1$ with respect to all condition's attributes in the information table: $\tau_{P1} = \{U, \phi, \{P1\}, \{P1, P2\}, \{P1, P3\}, \{P1, P2, P3\}, \{P1, P3, P6\}, \{P1, P2, P3, P6\}\}$. The accuracy of $P1$ of the equivalence class of the decision attribute is: $\mu(No = \{P1, P4, P5\}) = 0.17$ and $\mu(Yes = \{P2, P3, P6\}) = 0$ The condition's accuracy measure of decision attributes $N = \{P1, P4, P5\}$ and $Y = \{P2, P3, P6\}$ are: $\mu_{N \in U/D}(P1) = \frac{1+1+1+1}{1+3+3+2} = 0.44$ and $\mu_{Y \in U/D}(P1) = \frac{0+2+2+1}{1+3+3+2} = 0.56$. See Figure 3.

Table 6. Condition and decision of attributes-value table for flu patients.

	Age	Headache	Muscle Pain	Temperature	Stomach Pain	Runny Nose	Flu
P1	22	Yes	Yes	Normal	No	Yes	No
P2	55	Yes	No	Normal	No	No	Yes
P3	19	Yes	Yes	High	No	Yes	Yes
p4	31	No	No	Cool	Yes	No	No
p5	60	No	No	Very High	Yes	No	No
P6	9	No	Yes	Very High	Yes	Yes	Yes

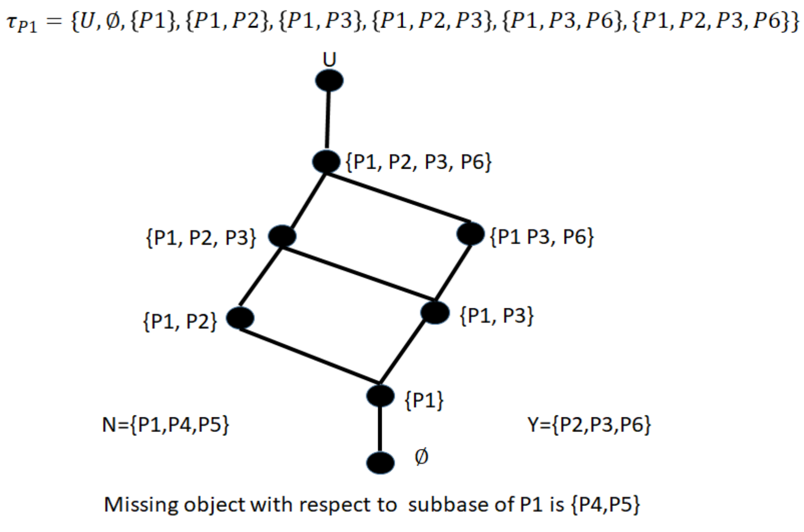

Figure 3. The lattices of topologies for decision attribute flu=No of patient P1.

The subbase of $P2$ with respect to all condition's attributes in the information table: $\delta_{P2} = \{\phi, \{P2\}, \{P1, P2\}, \{P1, P2, P3\}, \{P2, P4, P5\}\}$ The base of $P2$ with respect to all condition's attributes in the information table: $\beta_{P2} = \{\phi, \{P2\}, \{P1, P2\}, \{P1, P2, P3\}, \{P2, P4, P5\}\}$. The topological space of $P2$ with respect to all condition's attributes in the information table: $\tau_{P2} = \{U, \phi, \{P2\}, \{P1, P2\}, \{P1, P2, P3\}, \{P2, P4, P5\}, \{P1, P2, P4, P5\}, \{P1, P2, P3, P4, P5\}\}$. The accuracy of $P2$ of equivalence class of the decision attribute is $\mu(No = \{P1, P4, P5\}) = 0$ and $mu(Yes =$

$\{P2, P3, P6\}) = 0.17$. The condition's accuracy measure of decision attributes $N = \{P1, P4, P5\}$ and $Y = \{P2, P3, P6\}$ are: $\mu_{N\in U/D}(P2) = \frac{0+1+1+2}{1+2+3+3} = 0.44$ and $\mu_{Y\in U/D}(P2) = \frac{1+2+2+1}{1+3+3+2} = 0.56$. As in Figure 4.

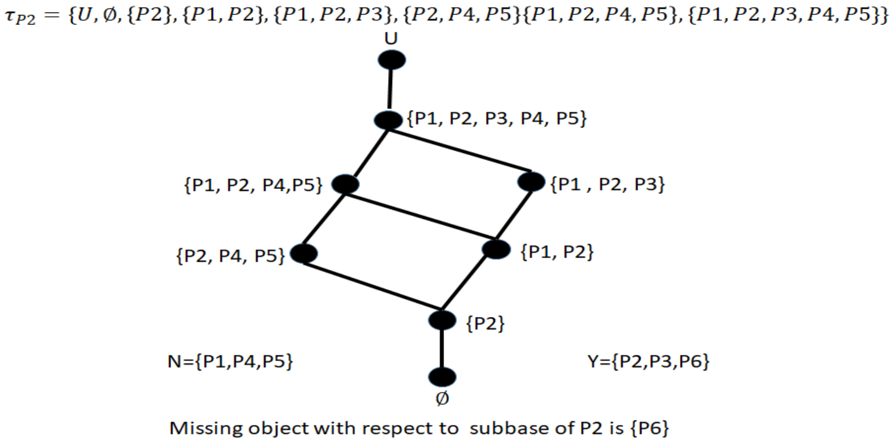

Figure 4. The lattices of topologies for decision attribute Flu=Yes of patient P2.

The subbase of $P3$ with respect to all condition's attributes in information table: $\delta_{P3} = \{\phi, \{P3\}, \{P1, P2, P3\}, \{P1, P3, P6\}\}$. The base of $P3$ with respect to all condition's attributes in the the information table: $\beta_{P3} = \{\phi, \{P3\}, \{P1, P3\}, \{P1, P2, P3\}, \{P1, P3, P6\}\}$. The topological space of $P3$ with respect to all condition's attributes in the information table: $\tau_{P3} = \{U, \phi, \{P3\}, \{P1, P3\}, \{P1, P2, P3\}, \{P1, P3, P6\}, \{P1, P2, P3, P6\}\}$. The condition's accuracy measure of decision attributes $N = \{P1, P4, P5\}$ and $Y = \{P2, P3, P6\}$ are $\mu_{N\in U/D}(P3) = \frac{0+1+1}{1+3+3} = 0.29$ and $\mu_{Y\in U/D}(P3) = \frac{1+2+2}{1+3+3} = 0.71$. See Figure 5.

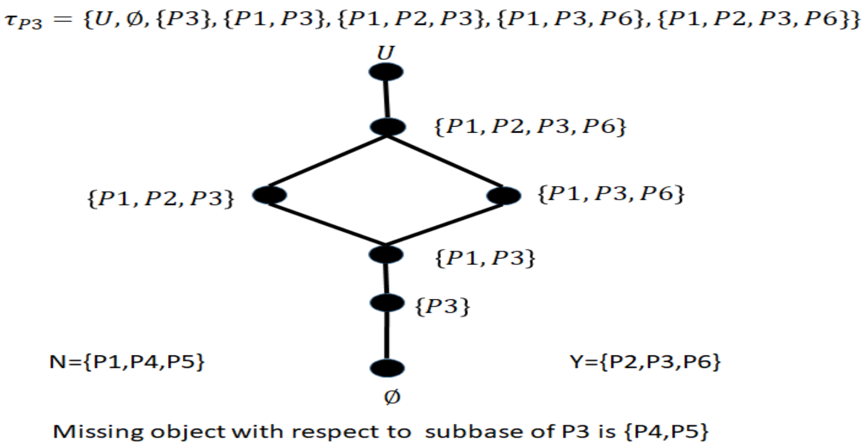

Figure 5. The lattices of topologies for decision attribute flu=Yes of patient P3.

The subbase of $P4$ with respect to all condition's attributes in the information table: $\delta_{P4} = \{\phi, \{P4\}, \{P4, P5, P6\}, \{P2, P4, P5\}\}$. The base of $P4$ with respect to all condition's attributes in the information table: $\beta_{P4} = \{\phi, \{P4\}, \{P4, P5\}, \{P4, P5, P6\}, \{P2, P4, P5\}\}$. The topological space of $P4$ with respect to all condition's attributes in the information table: $\tau_{P4} = \{U, \phi, \{P4\}, \{P4, P5\}, \{P4, P5, P6\}, \{P2, P4, P5\}, \{P2, P4, P5, P6\}\}$. The condition's accuracy measure of decision attributes $N = \{P1, P4, P5\}$ and $Y = \{P2, P3, P6\}$ are $\mu_{N\in U/D}(P4) = \frac{1+2+2}{1+3+3} = 0.71$ and $\mu_{Y\in U/D}(P4) = \frac{0+1+1}{1+3+3} = 0.29$. See Figure 6.

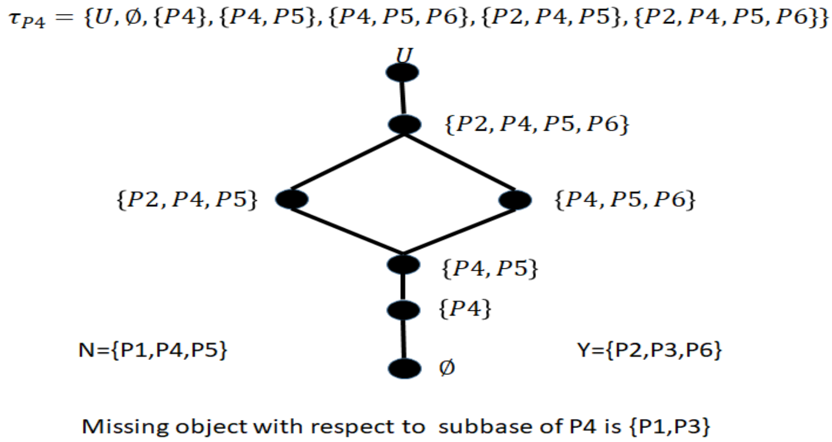

Figure 6. The lattices of topologies for decision attribute Flu=No of patient P4.

The subbase of $P5$ with respect to all condition's attributes in the information table: $\delta_{P5} = \{\phi, \{P5\}, \{P5, P6\}, \{P4, P5, P6\}, \{P2, P4, P5\}\}$. The base of $P5$ with respect to all condition's attributes in the information table: $\beta_{P5} = \{\phi, \{P5\}, \{P5, P6\}, \{P4, P5\}, \{P4, P5, P6\}, \{P2, P4, P5\}\}$. The topological space of $P5$ with respect to all condition's attributes in the information table: $\tau_{P5} = \{U, \phi, \{P5\}, \{P5, P6\}, \{P4, P5\}, \{P4, P5, P6\}, \{P2, P4, P5\}, \{P2, P4, P5, P6\}\}$. The condition's accuracy measure of decision attributes$N = \{P1, P4, P5\}$ and $Y = \{P2, P3, P6\}$ are $\mu_{N\in U/D}(P5) = \frac{1+1+2+2}{1+2+3+3} = 0.67$ and $\mu_{Y\in U/D}(P5) = \frac{0+1+1+1}{1+2+3+3} = 0.33$.See Figure 7.

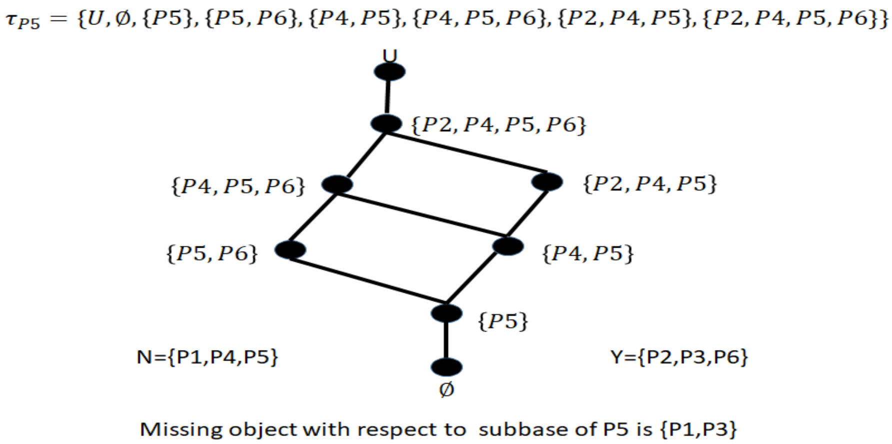

Figure 7. The lattices of topologies for decision attribute flu=No of patient P5.

The subbase of $P6$ with respect to all condition's attributes in the information table: $\delta_{P6} = \{\phi, \{P6\}, \{P5, P6\}, \{P4, P5, P6\}, \{P1, P3, P6\}\}$. The base of $P6$ with respect to all condition's attributes in the information table: $\beta_{P6} = \{\phi, \{P6\}, \{P5, P6\}, \{P4, P5, P6\}, \{P1, P3, P6\}\}$. The topological space of $P6$ with respect to all condition's attributes in the information table: $\tau_{P6} = \{U, \phi, \{P6\}, \{P5, P6\}, \{P4, P5, P6\}, \{P1, P3, P6\}, \{P1, P3, P4, P5, P6\}\}$. The condition's accuracy measure of decision attributes $N = \{P1, P4, P5\}$ and $Y = \{P2, P3, P6\}$ are $\mu_{N\in U/D}(P6) = \frac{0+1+2+1}{1+2+3+3} = 0.44$

and $\mu_{Y\in U/D}(P6) = \frac{1+1+1+2}{1+2+3+3} = 0.56$.As in Figure 8.

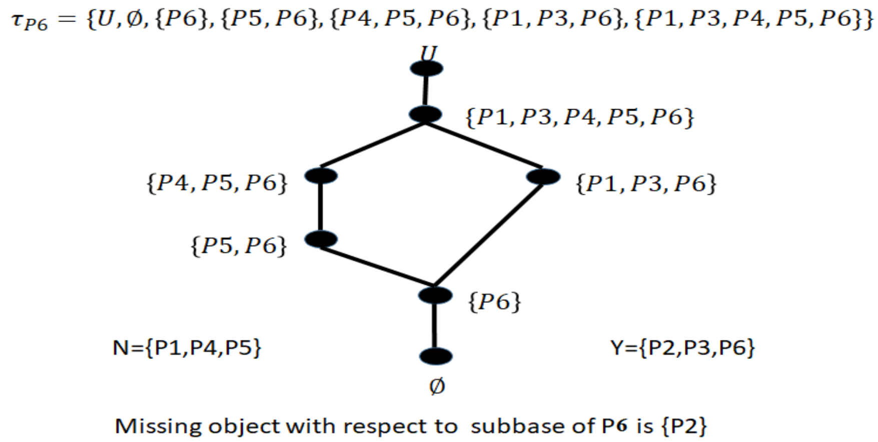

Figure 8. The lattices of topologies for decision attribute flu=Yes of patient P6.

The results for Example 4.3.

1. Patient P1 has a flu with respect to our classification. See Table 7.

Table 7. Measurement of particular object topology for attribute-value table.

	$\mu_{N\in U/D}(P_i)$	$\mu_{Y\in U/D}(P_i)$
P1	0.44	0.56
P2	0.44	0.56
P3	0.29	0.71
p4	0.71	0.29
p5	0.67	0.33
P6	0.44	0.56

2. The values for patients with decision flu = No are: (Headache, No), (Muscle Pain, No), (Stomach Pain, Yes) and (Runny Nose, No).
3. The values for patients with decision flu = Yes are: (Headache, Yes), (Muscle Pain, Yes), (Stomach Pain, No), (Runny Nose, Yes), and (Temperature, High); or (Headache, Yes), (Muscle Pain, Yes), (Stomach Pain, No), (Runny Nose, Yes).
4. For the reduction values of the attribute-value table, we noted that the patient does have not a flu if they have stomach pain, and the patient does have the flu if they have headache, muscle pain, and runny nose. As in Table 8.
5. Figures 9 and 10 are given the lattices for flu =No and flu =Yes , respectively.

Table 8. The rules of attribute-value table for flu patients.

	Age	Headache	Muscle Pain	Temperature	Stomach Pain	Runny Nose	Flu
P1		Yes	Yes		No	Yes	Yes
P2		Yes			No		Yes
P3		Yes	Yes	High	No	Yes	Yes
p4		No	No		Yes	No	No
p5		No	No		Yes	No	No
P6			Yes			Yes	Yes

The lattices of attribute-value table for flu patients

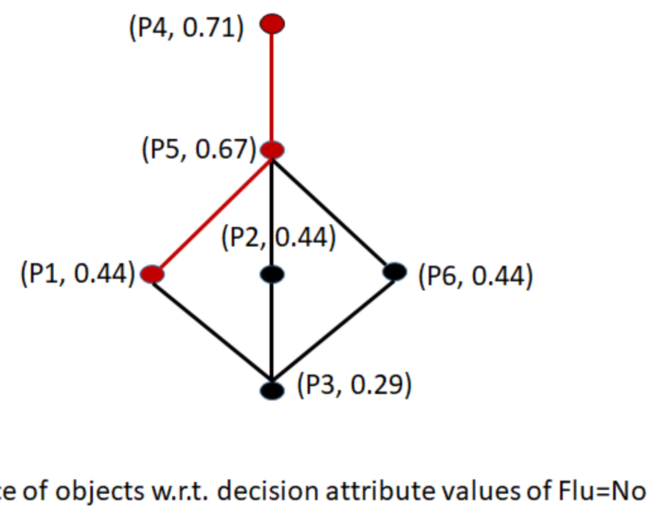

Figure 9. The lattice of patients have not flu.

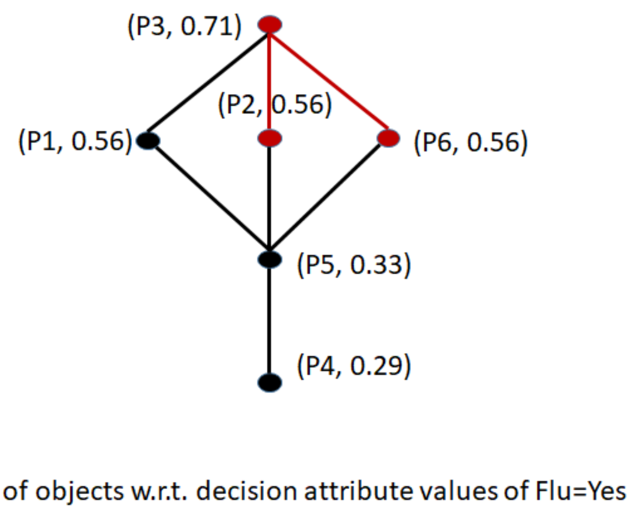

Figure 10. The lattice of patients who have flu.

5. Conclusion and future work

Information tables spend a lot of time, money, and work(fatiguing) to collect the information data. As we know, these tables consist of objects and attributes indicating the case under studying and the basic element to connect the objects and attributes as the value which is considered the basic indicator to describing the case via information tables. So, for the effectiveness of value in studying we must choose and determine the shape and size of value, which the results of our study depended totally on the value concept. Every information table expresses approximation space into subsets of objects which consider topological space and how changing the kind and shape of value will change the approximation space with respect to the information table. So when the approximation doesn't help, we found that the relationship between the case and information tables must change the value to improve the studying. The value is the basic concept to forming the final form of structure depending on the given data. As we know, giving us quasi discreet topological space when the value divided the space into partition by function from the object to attribute depended on given values which expresses equivalence relation with respect to the complete information table as in our paper and define the Pawlak approximation space. When change size of the value of equivalence classes expands or reduces, the equivalence classes have an important rule to obtain on expansion and reduction of our approximation, which improves the results of approximation for undefinable rough sets, expands the lower approximation and reduces the upper approximation, and gives us good accuracy measure of undefinable rough sets. Finally, in our research, when the size and form of the value in the information table are changed, we will have an expansion of the approximation space with important results of undefinable rough sets in the Pawlak approximation space. The examples considered in this paper are intended to illustrate the mathematical structure of the proposed model. Future work will focus on applying the framework to real-world datasets containing noisy and inconsistent information in order to evaluate its practical effectiveness and robustness.

Author contributions

All authors jointly worked on the results. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare no conflict of interest.

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