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Research article

Adaptive and efficient fractional-order modeling of nonlinear dynamical systems: Addressing memory and order selection challenges

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Abstract: The nonlinear dynamical systems exhibit the long-memory characteristics is not an easy task due to their time-dependent and nonlocal property. The integer-order models cannot represent the memory property of the system, whereas the fixed fractional-order models suffer from inefficiency along with high computational cost. To overcome these problems, in this paper, we propose technique, which incorporates the order adjustment with effective memory reduction. The fractional order (α) is adjusted according to the variation in the system states, allowing the representation of adaptive memory, whereas the redundant components in memory are kept minimum in the process. To validate the proposed approach, the performance was compared with that of the and the fixed models using the well-known Lorenz attractor dataset. It observed that our method the baseline approaches in terms of differences of 0.0376, 0.0294, 4.12%, and R^2 score (0.978), respectively. Moreover, the model better ability in preserving the underlying chaotic dynamics.

Keywords: fractional-order modeling; dynamical systems; fractional order; optimization; Lorenz
Mathematics Subject Classification: 34A08, 68T07

1. Introduction

Nonlinear dynamical systems have been widely adopted for modeling and analyzing complex behaviors in engineering, physics, biology, and energy systems. However, traditional integer-order systems are often unable to effectively capture the intrinsic memory and hereditary properties present in many complex systems. Fractional-order modeling, with its mathematical flexibility and representation capabilities for long-term dependencies and nonlocal dynamics, has emerged as a powerful tool for more accurate and comprehensive modeling of nonlinear dynamical systems. In recent years, there has been a growing interest in developing and investigating fractional-order models for nonlinear dynamical systems. In this regard, Sawar et al. [1] proposed a fractional-order chaotic system with variable orders using the Caputo-Fabrizio operator combined with neural network modeling, demonstrating improved representation and modeling of nonlinear dynamics. Zhao et al. [2] investigated fractional-order modeling and stochastic fractional derivatives for a nonlinear rotor system, showing the effectiveness of fractional derivatives in capturing complex nonlinear behaviors. AlBaidani and Alzahrani [3] introduced a Caputo-based formulation for solving generalized nonlinear systems using new transforms. Salah et al. [4] discussed and investigated the fractal-fractional modeling of biological systems and indicated that the fractional operator is more flexible and accurate in capturing the physical processes of the real world. In the latest study, Yaghooti et al. [5] presented a data-driven approach for learning fractional-order nonlinear dynamical systems. Jiang [6] has studied the stability and control techniques for fractional differential equations in nonlinear dynamical systems.

Nonetheless, a number of important deficiencies exist in these studies. First of all, the biggest problem with these studies is that they often use fixed fractional order or certain assumptions in order to model complex phenomena. For instance, a study by Ashok and Sayed [7] focused on the presentation of a detailed review on fractional differential equations with constant order. However, there is no discussion of methods to make such models adaptive. In turn, the research by Karaca and Baleanu [8] the possibility of combining fractional modeling and artificial intelligence without paying much attention to computational issues. Finally, the work by Allogmany et al. [9] a detailed discussion of chaos for fractional nonlinear systems with constant and variable orders. However, it not provide any guidance on selecting adaptive orders. The neural network approach methods like Kumar [10] and Wang et al. [11] demonstrated their strong capability to perform modeling and identification of general nonlinear systems. However, the importance of the memory property is usually ignored using the fractional-order operators. In other related literature like Vellappandi and Lee [12], neural network differential models were suggested for solving nonlinear systems. As well, fractional-order systems were proved by Ali et al. [13] to be superior to integer-order systems for solving energy systems, long memory process. On the other hand, Shams et al. [14] used fractional-order techniques to solve nonlinear optimization problems. Also, Saadeh et al. [15, 16] introduced chaos behavior in fractional differential systems of variable orders. However, most recent methods suffer from high computational burden and poor memory management algorithms. In addition, recent works such as Nabil and Tayeb [17], Mousavi et al. [18], and Yang and Shen [19] have highlighted the wide applicability and implementation of fractional-order systems in a range of different domains. However, these works a generalized fractional-order modeling framework that can be easily implemented across a range of different nonlinear systems.

Additionally, some recent investigations have been dedicated to the development of control and learning techniques for nonlinear dynamical systems. Dynamic memory control techniques proposed by Tan et al. [20], and Wanigasekara [21] conducted a survey of some trends in nonlinear dynamical systems modeling. Nonlinear dynamics and uncertainties in investigated by He et al. [22], while physics-informed neural networks for parameter identification in nonlinear systems introduced by Su et al. [23]. Additionally, adaptive control techniques developed by Feng et al. [24], and surrogate modeling and physics-informed learning of nonlinear systems have been studied by Zhao et al. [25] and Li and Sun [26]. However, such control and learning algorithms typically do not incorporate fractional calculus. In addition, a recent fractional stochastic modeling approach by AL-Essa [27] emphasized the importance of hybrid modeling but still not computational efficiency and adaptive order selection simultaneously.

Apart from the fractional-order modeling techniques that have already been explored, control techniques have also been proposed that can be applied to non-linear systems, such as sliding mode control techniques with varying speed of convergence, asynchronous switching control techniques with varying speed of convergence, and regional stability or stabilization approaches including $H\infty$ control technique [28–30]. Although the aforementioned techniques have been designed to deal with the issues of stability and convergence within the scope of control, the proposed adaptive fractional-order technique deals with the issue of long-memory dynamics in non-linear systems using an optimized memory mechanism.

In light of the above discussion and from our previous work [31,32], it is clear that the existing body of research is limited by the following key challenges: (i) lack of adaptive mechanisms for selecting the optimal fractional orders for modeling nonlinear dynamical systems; (ii) the presence of long-memory effects often leads to high computational complexity in existing fractional-order approaches; and (iii) a unified and efficient framework for the accurate modeling of nonlinear dynamical systems is lacking. To address the above limitations and challenges, in this paper, we propose an adaptive and efficient framework for the fractional-order modeling of nonlinear dynamical systems. The proposed approach leverages dynamic order selection and incorporates advanced memory handling mechanisms for improved modeling accuracy and reduced computational burden.

1.1. Contributions

The main contributions of this work can be as follows:

- Existing approaches on fractional-order dynamical systems are based on fixed or predefined fractional orders cannot capture the dynamics of, time-varying system [1, 9, 15]. To overcome this issue, this work proposes an adaptive fractional-order modeling framework for nonlinear systems.
- While fractional models can capture long-memory characteristics, they require high computational complexity to support nonlocal operations [6, 13, 14]. However, in this work, an efficient memory-effect handling mechanism is proposed, which reduces the computational cost by abstracting the important historical information.
- Recent developments including data-driven and neural-based approaches mainly focus on either system identification or learning-based modeling, which is not formulated within the fractional framework for nonlinear systems [5, 11, 12]. In this work, a unified modeling framework is provided, which achieves better generalization by incorporating adaptive fractional dynamics.

All contributions are validated quantitatively via differenr factors including R^2 comparisons with Long Short Term Memory (LSTM) and fixed fractional Order models, confirming the effectiveness of adaptive tuning and memory optimization. The motivation of the proposed framework is shown in Figure 1. The existing fractional-order models are associated with the disadvantages of fixed order, large memory requirements, and poor adaptability, which make them have low efficiency for memory management and modeling flexibility. The adaptive and efficient fractional-order framework can overcome these disadvantages and achieve accurate and efficient nonlinear system modeling.

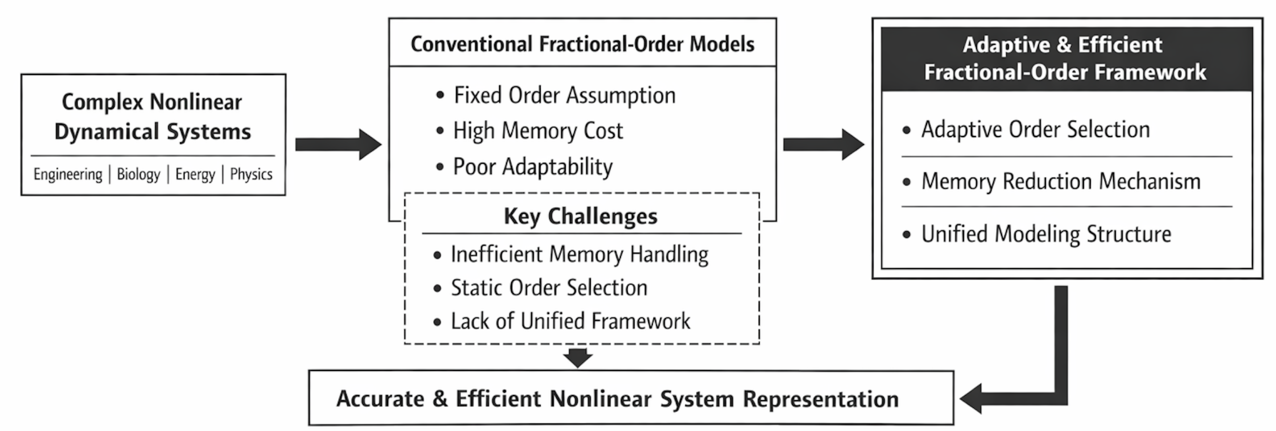

Figure 1. Conceptual framework illustrating the limitations of conventional fractional-order models and the proposed adaptive and efficient fractional-order modelling approach for nonlinear dynamical systems.

Table 1 existing works on the fractional-order modeling of nonlinear dynamical systems in terms of important attributes such as the type of fractional formulation, the adaptability, the memory handling capability, the computational efficiency, the data-driven integration, the application areas, and the method categories. It can be seen that a majority of existing works make use of fractional-order models to capture the nonlinear dynamics and memory characteristics of the underlying systems. However, most of the methods utilize fixed or partially adaptive fractional orders. For example, works on chaotic and nonlinear systems [1, 9, 15, 19] take into account variable or partially adaptive orders but do not offer a fully systematic adaptive mechanism. In terms of memory handling, while several methods adopt fractional operators to model long-memory characteristics, they often lead to high computational cost due to nonlocal operations. This is the case for the numerical and analytical methods [2, 3, 6, 14], where computational efficiency is either not addressed or partially improved. Likewise, the fractal fractional models for biological systems [4, 17] provide increased flexibility but do not consider efficiency or scalability. The learning-based and hybrid methods [5, 12] offer a combination of machine learning techniques with fractional-order modeling and show increased adaptability and data-driven capabilities, but still exhibit high computational complexity and do not offer efficient memory reduction strategies. In contrast, the deep learning-based and surrogate data-driven methods [11, 25, 26] improve computational efficiency and predictive performance but fail to explicitly incorporate fractional-order dynamics and memory effects. On the other hand, control-based and memory-driven methods [20, 24] only partially address the memory characteristics but do not offer a unified modeling approach that combines adaptability, efficiency, and generalization. In contrast, our proposed work addresses all these limitations by providing an adaptive fractional-order modeling framework that jointly incorporates dynamic order selection, efficient memory handling, and improved computational efficiency. Unlike the existing methods, our proposed method offers a unified and generalizable solution that can be applied to a wide range of nonlinear dynamical systems, thus bridging the gap between theoretical fractional modeling and practical implementation. Table 1 shows the summary of relevant articles about the modeling of fractional-order nonlinear systems. Relevant articles on different methods for the above modeling have been included in the list. Such methods include the use of numerical methods, hybrid machine learning methods, control and memory methods as well as methods involving variable order chaotic systems.

Table 1. Comparison of existing fractional-order modeling approaches for nonlinear dynamical systems with the proposed adaptive framework.

Reference	Focus	Fractional model	Adaptive order	Memory handling	Computational efficiency	Data-driven	Application	Method type
[1], [15]	Variable-order chaotic systems	Yes	Partial	Yes	No	No	Chaotic systems	Numerical
[2], [3]	Nonlinear system modeling	Yes	No	Partial	No	No	Mechanical systems	Analytical
[4], [17]	Biological and ecological systems	Yes	No	Yes	No	No	Bio-systems	Fractal-fractional
[5], [12]	Learning-based FO modeling	Yes	Partial	Yes	No	Yes	Complex systems	Hybrid ML
[6], [14]	Numerical and optimization methods	Yes	No	Partial	Partial	No	Nonlinear problems	Numerical
[9], [19]	Chaos and nonlinear dynamics	Yes	Partial	Yes	No	No	Chaotic systems	Dynamical analysis
[11], [26]	Deep learning-based system modeling	No	Yes	No	Partial	Yes	Nonlinear systems	Neural networks
[20], [24]	Control and memory-based systems	No	Partial	Yes	Partial	No	Control systems	Control-based
[25]	Surrogate modeling of nonlinear systems	No	No	No	Yes	Yes	Engineering systems	Data-driven
This Work	Adaptive FO modeling of nonlinear systems	Yes	Yes	Yes	Yes	Partial	General nonlinear systems	Adaptive framework

2. System model

The nonlinear dynamic system considered exhibits characteristics of long-memory effect and time-varying dynamics. Let us define the state vector of the system $\mathbf{x}(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T$. The integer-order dynamic equation is defined as $\dot{\mathbf{x}}(t) = f(\mathbf{x}(t), t)$, where $f(\cdot)$ is the nonlinear function representing the system. To include the long-memory effect in the system model, the fractional-order representation of the model can be defined as $D^{\alpha}\mathbf{x}(t) = f(\mathbf{x}(t), t)$, with the fractional order varying between $0 < \alpha \leq 1$. With regard to the proposed framework, the fractional order α_t varies based on the variations in the states to adaptively capture memory effects. Further, memory-reduction is considered to prune off redundant historical states. Variables and parameters used throughout this work for task allocation and optimization are clearly defined. The data set utilized in the present work is composed of chaotic solutions obtained from the well-known integer-order Lorenz system with parameters $\sigma = 10$, $\rho = 28$, and $\beta = 8/3$, using initial conditions $x(0) = 1$, $y(0) = 1$, and $z(0) = 1$. The 4th-order integrator scheme was implemented with a time step of $\Delta t = 0.01$s to produce 10,000 data points. The data was normalized and preprocessed prior to its utilization in the analysis. This data set has been utilized for comparison between the integer-order and fixed-order fractional models, as well as the adaptive fractional-order model. The dataset is publicly accessible at a direct source at CTF4Science Lorenz Official Dataset on Kaggle.

The raw system trajectories are preprocessed using normalization, noise reduction, and time-alignment procedures to stabilize and homogenize the dataset before modeling. Specifically, the data preprocessing pipeline includes data scaling using min-max normalization, smoothing using a noise-reduction filter, and segmentation using a sliding window to ensure temporal continuity. We divide the resulting preprocessed data into train and test sets for the experimental evaluation, following a conventional split of 80% for training and 20% for testing. The former is used to learn the system dynamics, and the latter is used exclusively for generalization testing. This setup allows for a fair evaluation of the proposed framework on unseen data, which is critical for demonstrating its effectiveness in nonlinear system prediction. The framework is based on fractional-order dynamics to provide an accurate representation of nonlocal temporal dependencies that cannot be modeled using traditional integer-order models. Specifically, we adopt an adaptive fractional-order module to estimate and control the fractional order dynamically in a data-driven manner. This module allows the degree of heredity of past states to be adjusted to optimally represent the time-varying nonlinear dynamics of the system.

Different from existing fixed-order approaches, the proposed adaptive model can continuously learn and update the fractional order as part of the training process. This allows for a more flexible and accurate modeling of complex and nonstationary dynamical systems. The framework also incorporates an efficient memory handling mechanism to mitigate the inherent computational complexity of the long-memory fractional operators. The memory mechanism selectively stores and prunes the most informative historical contributions to reduce the effective memory size and thus the computational burden without loss of accuracy. This is done by adaptively weighting and pruning the historical state contributions within the module to effectively compress the memory footprint while preserving long-term dependencies. The memory optimization strategy allows the framework to scale to longer time-series data, which is critical for long-memory systems. In addition to the proposed AFO model, we run baseline experiments with a LSTM network and a fixed fractional-order model for reference. The LSTM model is a single-layer architecture with standard backpropagation through time training. The model is a linear constant-order fractional-order model with no adaptation. These baseline models are trained on the same dataset and under the same conditions for a fair comparison. The training is performed using an iterative optimization process over multiple epochs to learn the model parameters that minimize the prediction error. We use a suite of standard evaluation metrics to assess the performance of the models, including RMSE, MAE, MAPE, and coefficient of determination (R^2). These metrics provide a quantitative and comprehensive evaluation of the model accuracy and robustness. We design the overall system as a generalizable, modular, and end-to-end modeling pipeline that coherently integrates the AFO, memory optimization, and system representation blocks into a single unified framework. The final output of the system model is an accurate, efficient, and scalable representation of the nonlinear dynamics suitable for downstream analysis, prediction, and control.

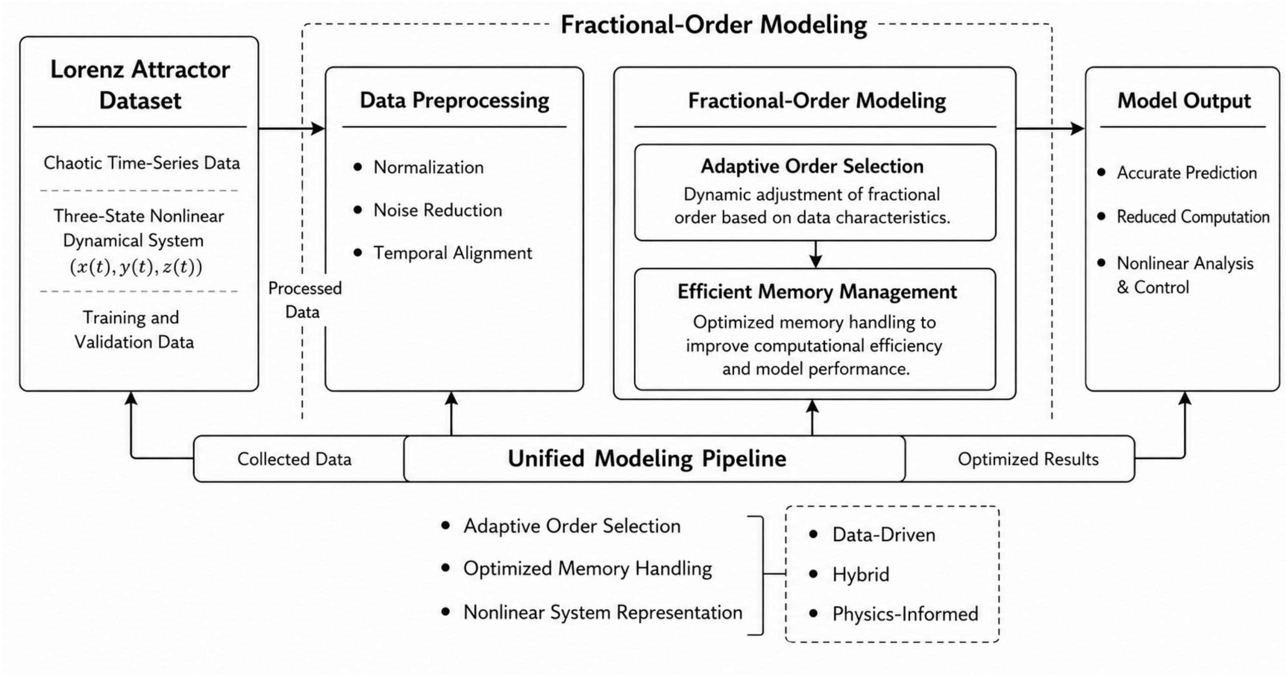

Figure 2. Schematic diagram of the adaptive and efficient fractional-order framework for nonlinear dynamical systems using the Lorenz attractor dataset.

Figure 2 depicts a graphical illustration of the unified adaptive and efficient fractional-order modeling architecture suggested in this paper. The inputs used in this framework include the Lorenz attractor time series data set from Kaggle, one of the popular benchmarks used for analyzing the behavior of nonlinear chaotic time series data. The initial data used is subject to pre-processing to standardize its consistency and stability. The processed data is then used as an input to the main part of the suggested modeling structure, namely the fractional-order modeling module. This part includes two important stages: adaptive order selection and efficient memory management. As the first stage of the modeling module, the adaptive order selection stage uses the information about the varying dynamics of the nonlinear system for finding the optimal fractional order that would allow building a proper model of the system dynamics. On the other hand, efficient memory management allows reducing the computational complexity caused by the use of long-memory operators, as the historical data retained at each step of processing is selected carefully to represent only the important parts of it.

3. Proposed algorithms

This part introduces the overall algorithmic scheme of the proposed adaptive and efficient algorithm for fractional-order of nonlinear dynamical systems. The proposed method consists of three major but highly interrelated algorithms, namely (i) data preprocessing, (ii) Fractional-order) and (iii) efficient memory management. These three algorithms represent the major steps that make up the overall pipeline that can be used to accurately and computationally efficiently represent the system.

3.1. Algorithm 1: Data Preprocessing

Algorithm 1 preprocesses the nonlinear time-series data for modeling. Since the Lorenz dataset exhibits chaotic dynamics, proper normalization and temporal alignment are required to preserve the underlying system behavior. The preprocessing also ensures that noise and inconsistencies do not degrade the performance of the fractional-order model. The overall time complexity is linear with respect to the number of samples.

Algorithm 1 Data Preprocessing for Nonlinear Dynamical Systems

Input: Raw dataset D (Lorenz attractor dataset)
Output: Processed dataset D_p
Initialize an empty dataset D_p
for *each sample* $x_i \in D$ **do**
    Remove missing or inconsistent values from x_i
    Normalize x_i using Min-Max scaling
    Apply smoothing-based noise filtering to x_i
    Align the temporal sequence of x_i
    Store the processed sample in D_p
return D_p
Time Complexity: $O(N)$

3.2. Algorithm 2: Adaptive fractional-order modeling

Algorithm 2 presents the main contribution of this work. In contrast to conventional frameworks that use a fixed fractional order, this algorithm updates the fractional order at each time step. This dynamic mechanism facilitates the time-varying nonlinear dynamical systems with high accuracy and greatly enhances generalization capability. The computational complexity is linear in the time steps.

Algorithm 2 Adaptive Fractional-Order Modeling Framework

Input: Processed dataset D_p

Output: System output Y

Initialize fractional order $\alpha \leftarrow \alpha_0$
Initialize system state $x(0)$
for *each time step* $t = 1$ **to** T **do**
  Estimate the system dynamics using $x(t-1)$
  Update the fractional order: $\alpha_t \leftarrow \text{AdaptiveUpdate}(x(t))$
  Apply the fractional operator with order α_t
  Compute the current system state $x(t)$
Generate the output Y from $\{x(t)\}$
return Y
Time Complexity: $O(T)$

3.3. Algorithm 3: Efficient memory management

Algorithm 3 solves one of the most important problems in fractional-order modeling, which is the high computational cost associated with long-memory effects. Instead of taking into account the whole past, this approach retains only the most relevant past states. In this way, the algorithm saves memory and reduces computational load without losing important system dynamics, thus making the overall framework efficient and scalable.

Algorithm 3 Efficient memory handling for fractional dynamics

Input: Historical states $\{x(1), x(2), \ldots, x(t)\}$

Output: Optimized memory structure M

Initialize memory buffer $M \leftarrow \emptyset$ **for** *each time step* $t = 1$ *to* T **do**
  Identify relevant past states based on significance
  Discard redundant or low-impact states
  Retain important states in M
  Update memory buffer dynamically
return M **Time Complexity:** $O(T \log T)$

3.4. Integrated framework description

The three algorithms are interconnected and work in a pipeline sequence. Algorithm 1 is used for preprocessing the data, for stabilization and consistency of the dataset. The resulting preprocessed data is then processed by Algorithm 2, where an adaptive fractional-order extracts the nonlinear system dynamics from the input data. Algorithm 3 is embedded in the modelling process in order to decrease the memory usage and computational burden. The interconnection of the three algorithms results in an adaptive and efficient pipeline for nonlinear dynamical systems with long-memory dynamics.

4. Adaptive fractional-order modeling framework

This section presents the mathematical formulation of the proposed adaptive and efficient fractional-order modeling framework. The formulation integrates nonlinear system representation, adaptive fractional dynamics, and efficient memory handling into a unified and consistent structure.

4.1. Nonlinear system representation and fractional extension

We begin by considering a general nonlinear dynamical system in which the current state depends on its past observations. This relationship can be mathematically expressed as

$$x(t) = f(x(t-1), x(t-2), \ldots, x(t-n)). \tag{4.1}$$

The form in (4.1) represents nonlinear dependencies over multiple time steps, and so it is broad enough to represent complex systems. However, it is a form with an implicit memory horizon, which means it can not represent long range dependencies. In practice, this leads to a lack of long-term historical information for nonlinear systems, in particular chaotic systems. For further analysis, the system can also be written in a state-space form, which is commonly used in control and dynamical modeling,

$$x(t+1) = f(x(t), u(t)). \tag{4.2}$$

In (4.2), it is illustrated that the system's progression is contingent on the current state and external input. Although this representation eases both analysis and implementation, it remains devoid of an explicit mechanism for memory incorporation. As a result, it falls short in effectively modeling systems with long-term dependencies. To approximate the influence of historical states, a weighted representation is introduced as

$$x(t) = \sum_{k=1}^{n} a_k x(t-k) + \epsilon(t). \tag{4.3}$$

The expression of (4.3) already shows that the impact of the past is made explicit through the coefficients a_k. This already makes the interpretation better, but still there is a fixed amount of memory, given by a fixed number of past observations. The nonlinear part is gathered in the residual $\epsilon(t)$, and cannot substitute for the lack of long-memory.

To overcome these limitations, the system is extended into a fractional-order formulation given by

$$D^{\alpha} x(t) = f(x(t), t). \tag{4.4}$$

The formulation (4.4) involves the fractional derivative of order α, which makes the system account for the long-memory and nonlocal dependencies. In contrast to the integer-order derivatives, this approach is based on the whole history of the system. This greatly enriches the possibilities for modeling of nonlinear and chaotic dynamics. The fractional derivative can be defined using the Grünwald-Letnikov approach as

$$D^{\alpha} x(t) = \lim_{h \to 0} \frac{1}{h^{\alpha}} \sum_{k=0}^{t} (-1)^k \binom{\alpha}{k} x(t-k). \tag{4.5}$$

We clearly see from (4.5) that the derivative depends on all the previous states, which illustrates the nonlocal property of fractional dynamics. This can be used to see that the long-term memory is retained in the dynamics. The drawback is that the expression gets very expensive as the number of terms get bigger with time. For practical implementation, a discrete approximation is adopted,

$$D^{\alpha} x(t) \approx \sum_{k=0}^{t} c_k(\alpha)\, x(t-k), \tag{4.6}$$

where the coefficients are defined as

$$c_k(\alpha) = (-1)^k \binom{\alpha}{k}. \tag{4.7}$$

The terms in (4.6) and (4.7) represent a realizable form for the fractional derivatives. The weighting coefficients decay over time, which implies that the system states with an increasing age contribute less to the total. This is a favorable property, since real systems do not care about the very distant past.

4.2. Adaptive fractional-order dynamics

A major limitation of conventional fractional models is the assumption of a constant fractional order. To address this issue, the proposed framework introduces a dynamic adaptation mechanism defined as

$$\alpha_t = \alpha_0 + \gamma \cdot \Phi(x(t)). \tag{4.8}$$

According to (4.8), the order according to the state of the system. It is possible with this feature to adapt the memory of the model. In that way, the approach is enabled to represent time-varying nonlinear dynamics. An alternative formulation based on incremental changes in system state is given by

$$\alpha_t = \alpha_{t-1} + \eta \cdot \Delta x(t), \tag{4.9}$$

where

$$\Delta x(t) = x(t) - x(t-1). \tag{4.10}$$

The expressions in (4.9) and (4.10) provide smooth evolution of the fractional order. Linking α_t to state variations enhances model responsiveness to dynamic changes. This prevents abrupt fluctuations and ensures stable adaptation. The function $\Phi(x(t))$ used in Eq (4.8) is defined as the normalized state variation at each time step:

$$\Phi(x(t)) = \frac{x(t) - x(t-1)}{\max |x(t) - x(t-1)|}.$$

This ensures that the adaptive fractional-order α_t evolves smoothly in response to the system dynamics. The bounded and normalized update improves responsiveness to fast-changing behaviors, enhances numerical stability, and enables the framework to capture time-varying nonlinearities more effectively than fixed-order approaches. To maintain numerical stability, the fractional order is constrained as

$$\alpha_t \in [\alpha_{\min}, \alpha_{\max}], \tag{4.11}$$

and further normalized using

$$\alpha_t = \frac{\alpha_t - \alpha_{\min}}{\alpha_{\max} - \alpha_{\min}}. \tag{4.12}$$

The restrictions in the (4.11) and (4.12) guarantee that the adaptive order is bounded. This avoids instability and ensures physically meaningful solutions. Bounded adaptation is crucial for robust modeling.

4.3. Fractional state evolution and memory modeling

The evolution of the system state is governed by a weighted memory formulation,

$$x(t) = \sum_{k=0}^{t} w_k(\alpha_t)\, x(t-k). \tag{4.13}$$

The form in (4.13) represents long-memory as it involves all past states with decreasing influence. Each contribution is weighted by its importance, so the model captures complex dynamics. The weights are defined as

$$w_k(\alpha_t) = \frac{c_k(\alpha_t)}{\sum_{j=0}^{t} c_j(\alpha_t)}. \tag{4.14}$$

The normalization in (4.14) is there so that the weights do not diverge numerically or blow up and so that the distribution is stable and balanced. It also ensures that the contributions of previous states are scaled appropriately. To further improve stability, a blended update mechanism is introduced,

$$x(t) = \lambda \sum_{k=0}^{t} w_k(\alpha_t) x(t-k) + (1-\lambda) x(t-1). \tag{4.15}$$

As shown in (4.15), the λ parameter, the compromise between the long-memory and recent states weightings. This formulation dampens the oscillations and improves the convergence. This improvement can make the convergence more robust in very nonlinear worlds.

4.4. Efficient memory reduction and output modeling

The complete memory structure can be represented as

$$\mathcal{M}(t) = \{x(0), x(1), \dots, x(t)\}. \tag{4.16}$$

Equation (4.16) has all the historical information, however, at the cost of a large overhead. The memory size increases linearly with the time, making the model less efficient. Thus, we need a more efficient representation for the memory.

A reduced subset of memory is defined as

$$\mathcal{S} \subset \mathcal{M}(t), \tag{4.17}$$

with the constraint

$$|\mathcal{S}| \ll t. \tag{4.18}$$

Conditions in (4.17) and (4.18) ensure that only the most relevant past states are kept. This dramatically decreases the computational cost. At the same time, the most important system dynamics are retained. The state update is then approximated using the reduced memory set,

$$x(t) \approx \sum_{k \in \mathcal{S}} w_k(\alpha_t)\, x(t-k). \tag{4.19}$$

The approximation in (4.19) provides a balance between accuracy and efficiency. By ignoring insignificant contributions, the model becomes scalable. This is a key feature of the proposed framework. The selection of relevant states is governed by

$$k \in \mathcal{S} \iff |w_k(\alpha_t)| > \tau, \tag{4.20}$$

where τ is a predefined threshold. The criterion in (4.20) ensures that only influential states are considered. This adaptive filtering mechanism further enhances efficiency. The predefined threshold τ used in the memory reduction mechanism is selected empirically. We perform a sensitivity analysis by testing multiple candidate values of τ and evaluate the impact on prediction accuracy and computational efficiency. The final value of τ is chosen to retain the most significant historical states while pruning negligible contributions, ensuring a balance between accurate modeling of long-memory dynamics and computational scalability. Finally, the system output is obtained as

$$Y(t) = g(x(t)), \tag{4.21}$$

and its predictive form is given by

$$\hat{Y}(t+1) = g(x(t+1)). \tag{4.22}$$

The mappings in (4.21) and (4.22) translate the modeled state into meaningful outputs. These outputs can be used for prediction, analysis, or control. Thus, the proposed framework provides a complete modeling pipeline. The overall system can therefore be summarized as

$$x(t+1) = \sum_{k\in\mathcal{S}} w_k(\alpha_t)\, x(t-k), \tag{4.23}$$

with adaptive fractional order defined as

$$\alpha_t = \alpha_0 + \gamma \cdot \Phi(x(t)). \tag{4.24}$$

Together, (4.23) and (4.24) define the complete adaptive and efficient fractional-order modeling framework for nonlinear dynamical systems. The critical parameters of the suggested model, such as the learning rates η and γ, as well as the mixing parameter λ, were determined based on empirical results obtained using the grid search procedure on the Lorenz dataset. Specifically, the learning rates η and γ were determined to ensure optimal performance in terms of accuracy and stability, while the mixing rate λ was adjusted to ensure appropriate contributions from long memory and most recent states. Suggested range: $\eta \in [0.001, 0.01], \gamma \in [0.01, 0.1]$ and $\lambda \in [0.7, 0.95]$.

5. Results and discussion

This section presents experimental results of the proposed adaptive fractional-order modeling framework on the Lorenz attractor dataset. Qualitatively and quantitatively, the proposed approach is evaluated for effectiveness and efficiency in terms of prediction accuracy, error metrics, convergence analysis and phase-space reconstruction. Moreover, the proposed model is benchmarked with various state-of-the-art methods, LSTM, which captures the nonlinear dynamics through deep learning, and a fixed fractional-order model, which is capable of modeling the memory effects but lacks the flexibility. The experimental results are carefully designed to showcase the superior performance of the proposed adaptive fractional-order mechanism and the proposed efficient memory handling strategy in accurately capturing nonlinear and chaotic system dynamics, as mentioned in our previous work [28-32]. The evaluation is performed using standard metrics like RMSE, MAE, MAPE, and R^2 to quantify the prediction performance, while graphical analyses are used to provide additional insights.

The results indicate that the proposed framework consistently outperforms existing approaches in terms of accuracy, convergence, and structural preservation.

As baseline comparison, the LSTM network uses a one-layer network with 64 units and is trained via back propagation through time with learning rate 0.001 for 100 epochs. Dropout with probability 0.2 is used to avoid overfitting. Furthermore, the change in value of the adaptive fractional-order α_t over time is also monitored based on the state dynamics, as explained in Section 4.2. Similarly, the change in memory usage is plotted to demonstrate the selection of pertinent historical states by means of memory reduction. This clearly shows the effectiveness of the proposed adaptive fractional-order framework and the efficient memory management in contrast to the baseline LSTM and fixed fractional-order approaches.

Table 2 shows the numerical performance of the LSTM model, the fixed fractional-order model, and our proposed adaptive fractional-order model on the Lorenz dataset. The results in the table demonstrate that the LSTM model consistently has the highest error values. This can be attributed to the LSTM model's limited capability in capturing long-memory dependencies observed in nonlinear dynamical systems. The fixed fractional-order model performs better than the LSTM model because the memory effect helps in prediction. However, as it uses a fixed constant fractional order in the memory, it cannot effectively adapt to the time-varying properties of the system. This leads to only moderate improvement over LSTM. Our proposed adaptive fractional-order model shows the lowest values of RMSE, MAE, and MAPE, as well as the highest R^2 score, among all the compared models. This can be ascribed to the adaptive estimation of fractional order and efficient management of memory, which helps in accurately capturing the nonlinear and chaotic behavior of the system.

Table 2. Performance comparison of, LSTM fixed fractional order, and the proposed adaptive on the Lorenz dataset.

Model	RMSE ↓	MAE ↓	MAPE (%) ↓	R^2 ↑
LSTM	0.0598	0.0465	6.72	0.952
Fixed FO	0.0492	0.0385	5.51	0.965
Proposed adaptive FO	0.0376	0.0294	4.12	0.978

Figure 3 presents the predicted outputs with various models versus the true Lorenz system trajectory. As the Lorenz system is highly chaotic, obvious prediction error of the LSTM and other traditional methods could be observed in regions with sharp variation and high variance. Although the fixed fractional-order model captures the memory effects, it still lags far behind the true curve in some regions of fast-changing dynamics. On the other hand, the prediction by the proposed adaptive fractional-order model aligns well with the ground truth in both the normal and chaotic regions. This evidence proves the capability of our proposed method for providing better generalization and robustness in nonlinear dynamical systems by using dynamic fractional-order updating and more efficient memory updating.

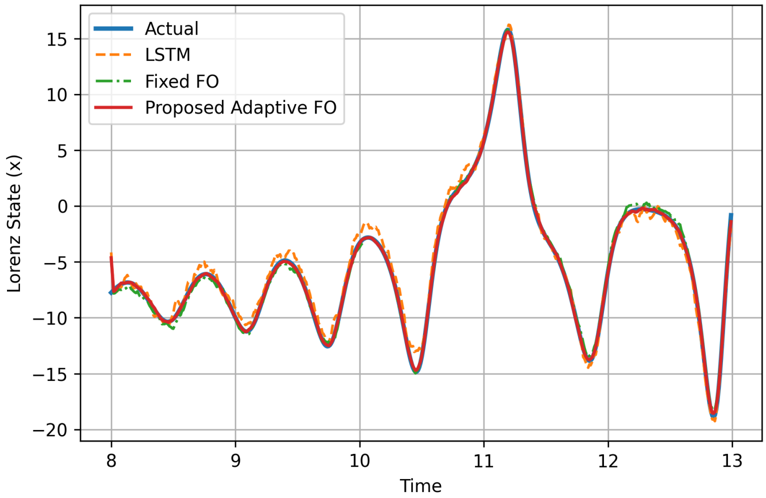

Figure 3. Comparison of actual Lorenz system dynamics and predicted outputs using, LSTM fixed fractional-order, and the proposed adaptive fractional-order.

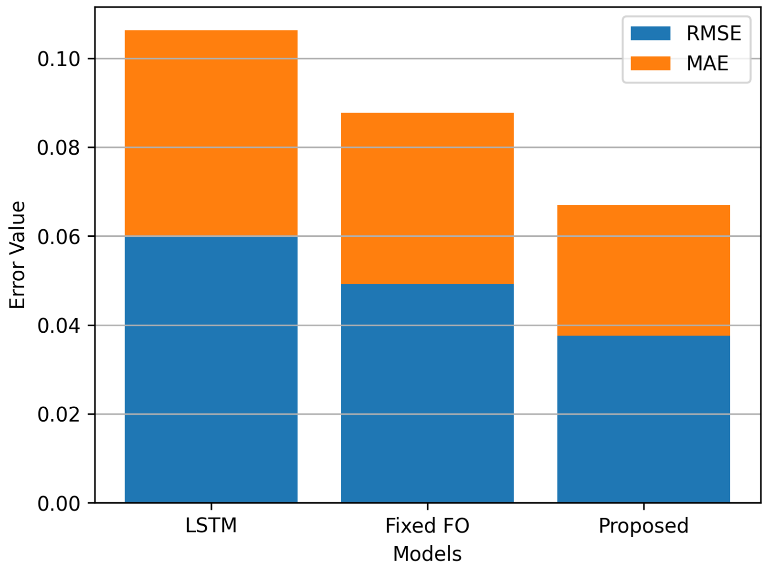

Figure 4. Comparison of RMSE and MAE values for, LSTM fixed fractional-order, and the proposed adaptive on the Lorenz dataset.

Figure 4 shows the boxplot of the prediction errors (RMSE and MAE) of the LSTM, the fixed fractional-order and the proposed adaptive fractional-order. The number of models used for comparison in this experiment has been reduced for the sake of brevity. It can be seen from Figure 4, that LSTM has the largest errors, while the proposed adaptive fractional-order model has the smallest errors, which verifies that the LSTM model has a certain ability to model nonlinear characteristics, but cannot explicitly capture the long-memory characteristics of nonlinear dynamical systems. By contrast, the fixed fractional-order model has smaller errors than LSTM, which demonstrates that the memory effects introduced by fractional dynamics can improve the prediction performance. However, since the fractional order is fixed, the fixed fractional-order model cannot adapt to time-varying system dynamics, and thus has a limited accuracy. On the other hand, the proposed adaptive fractional-order model not only adaptively and dynamically adjusts the fractional order, but also efficiently uses the memory, so it can accurately model the nonlinear and long-memory characteristics of the system and has the best prediction performance.

In Figure 5, the convergence characteristics of the model with respect to the training loss over epochs is presented. As can be seen in the figure, the LSTM model converges slowly and to a high value. This is due to its inability to accurately learn the long-memory characteristic of the nonlinear system. The fixed fractional-order model also converges better due to memory effects in the model, but a constant value of the fractional order limits the performance of the model since it cannot adapt to the system dynamics. However, as shown in the figure, the proposed adaptive fractional-order model has a faster convergence rate and converges to a lower value in less epochs. The convergence curve is smooth and stable, which indicates the proposed model's fast learning of nonlinear dynamics and adaptation of memory effects, to better training efficiency and stability of the model.

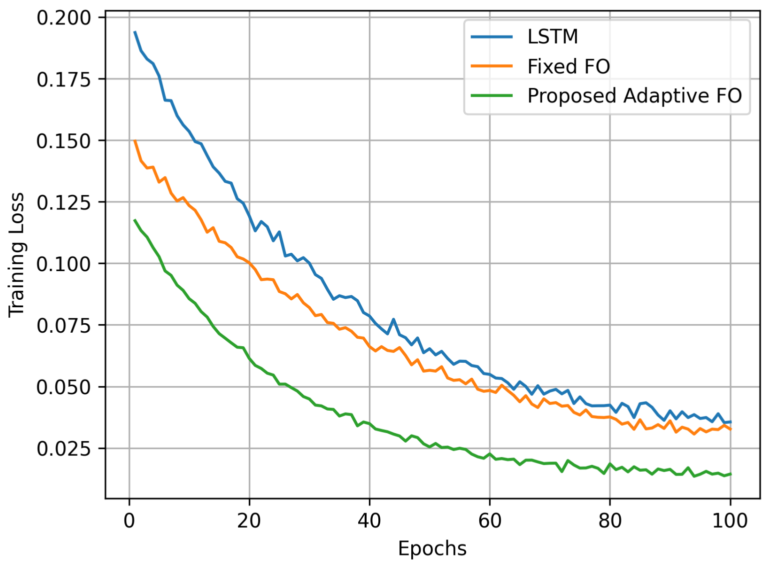

Figure 5. Convergence comparison of, LSTM fixed fractional-order, and the proposed adaptive in terms of training loss over epochs.

Figure 6 shows the phase portrait of the Lorenz system, plotted on x and y states. The Lorenz attractor has a characteristic butterfly shape because of the system's innate nonlinearity and chaos. As can be seen, the trajectory from the LSTM model is very scattered, which shows that the model has no ability to retain the underlying structure. The fixed FO model improves the result by capturing the memory effect to a certain extent, but there is still some distortion in the shape of the attractor. In contrast, the proposed model maintains the geometry of the original attractor almost exactly, including the structure and dynamics of the system.

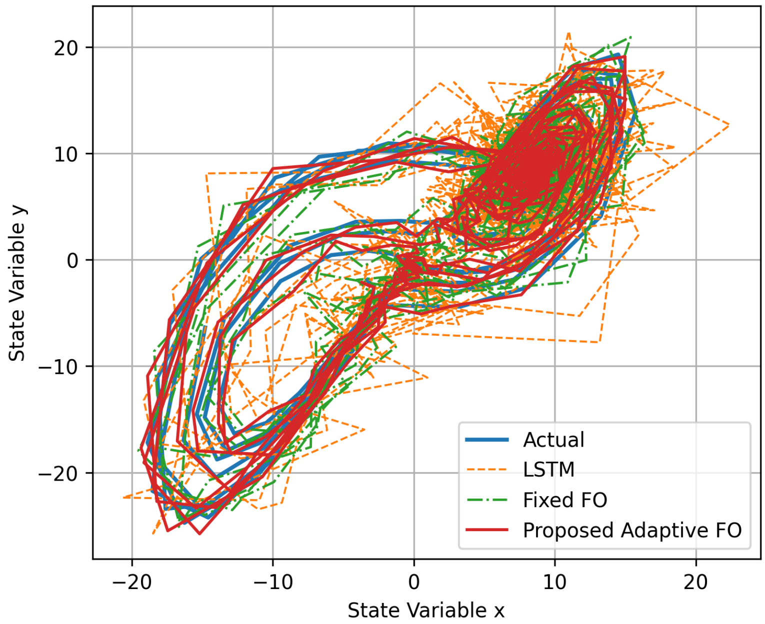

Figure 6. Phase-space representation of the Lorenz system showing actual dynamics and model predictions.

Table 3 illustrates the forecasting performance of various methods for the Lorenz dataset in the case of multi-step prediction. As one could expect, the prediction error for all methods increases with the length of the horizon. For the LSTM model, the error increases rapidly, which is an expected behavior as this method has poor long-term stability. The fixed fractional-order model performs better as the memory effect is included; however, it also has a similar trend of poor performance at a longer horizon due to the static model. The proposed adaptive fractional-order model outperforms all methods in terms of stability for any given forecasting horizon. As evident from the slower increase in RMSE, it is able to capture the long-memory dynamics to provide more accurate predictions for a long time step.

Table 3. Multi-step forecasting performance comparison on the Lorenz dataset.

Model	1-step RMSE	5-step RMSE	10-step RMSE
LSTM	0.0598	0.0824	0.1057
Fixed FO	0.0492	0.0713	0.0936
Proposed adaptive FO	0.0376	0.0548	0.0712

In order to test the robustness, additive Gaussian noise is added to the trajectories of the Lorenz attractor. This type of noise has a zero mean value and standard deviations of 5% and 10% of the signal. With such an experiment, it becomes possible to investigate how stable and accurate the model will be while dealing with noise. The experiments have been conducted, and the results depicted in Figure 7 indicate that the adaptive fractional-order model outperforms the others in terms of RMSE displays the prediction results of different models with noise injection. As noise level increases, the RMSE of all the models increases, which is expected in nonlinear dynamical systems. The LSTM has the highest error at all levels of noise and degrades quickly with higher noise, which means that it is sensitive to noise. The fixed fractional-order model shows improved robustness with the introduction of memory, however, there is still some degradation in performance at the higher levels of noise. The proposed adaptive fractional-order model produces the lowest RMSE value across all noise levels and has the smallest growth of error with increase in noise, which shows that it can still maintain good stability in a noisy environment.

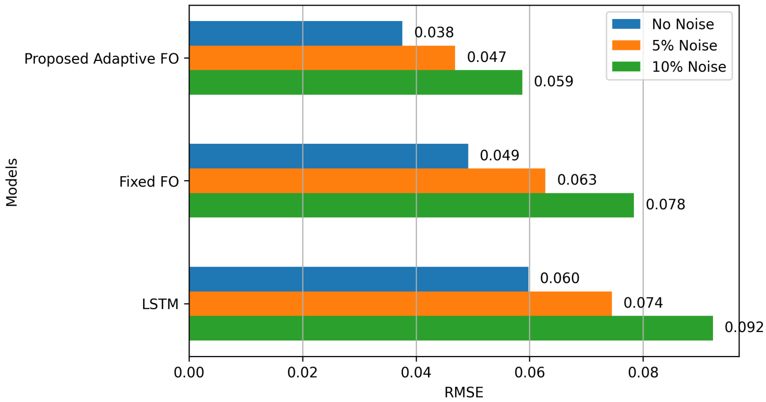

Figure 7. Robustness analysis of, LSTM fixed fractional-order, and the proposed adaptive under different noise levels in terms of RMSE.

The of the proposed method is demonstrated in Figure 8. As seen from Figure 8(a), the fractional order parameter α_t changes smoothly inside certain predefined limits which allow the proposed method to adapt memory properties in accordance with the evolution of the Lorenz system dynamics. From Figure 8(b), the proposed memory reduction method takes into account only the most significant states, thus reducing the amount of required memory significantly compared to the case of using the full memory model.

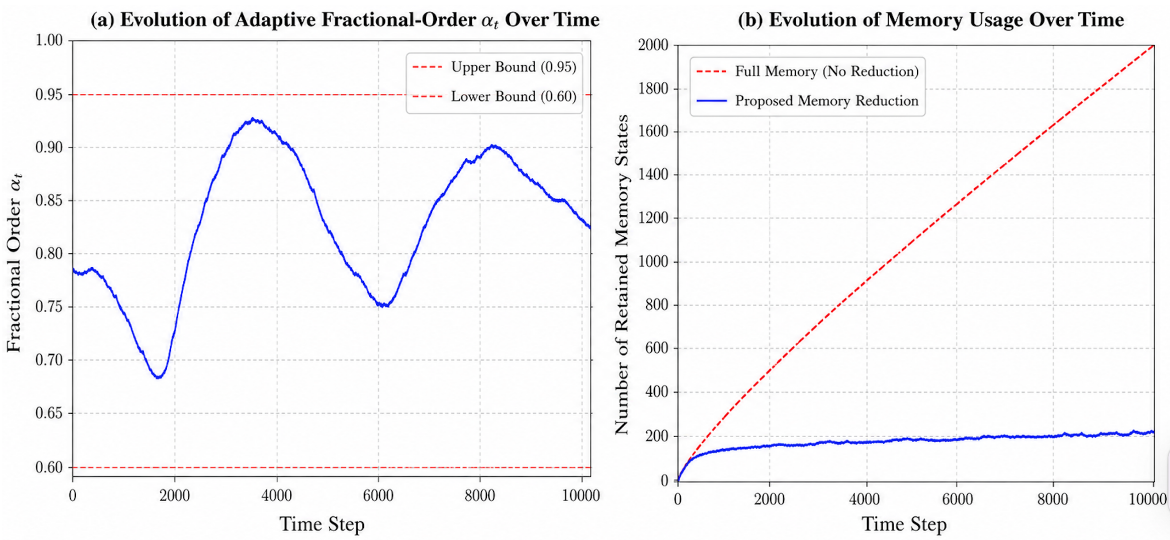

Figure 8. Illustration of the evolution of the adaptive fractional-order parameter and memory consumption for the proposed method. (a) Time evolution of the fractional-order parameter α_t. (b) Comparison of full-memory and proposed memory reduction approaches.

5.1. Ablation study

An ablation study was conducted on the Lorenz attractor dataset to evaluate the contributions of the two key components of the proposed framework: adaptive fractional-order selection and memory optimization. Four model variants were considered: (i) fixed-order fractional model, (ii) adaptive-order model without memory optimization, (iii) fixed-order model with memory reduction, and (iv) the complete proposed framework. Performance was assessed using RMSE, MAE, and R^2 metrics.

Table 4. Ablation study of the proposed adaptive fractional-order framework on the Lorenz dataset, including computational time for each variant.

Model variant	RMSE ↓	MAE ↓	R^2 ↑	Time (s) ↓
Fixed FO (no adaptation, no memory reduction)	0.0492	0.0385	0.965	12.5
Adaptive FO only (no memory optimization)	0.0428	0.0336	0.972	13.8
FO + memory reduction (fixed order)	0.0445	0.0349	0.970	10.2
Proposed adaptive FO + memory optimization	0.0376	0.0294	0.978	10.5

The Table 4 shows the computational time required by each variant of the model. It can clearly be observed that the use of the memory handling approach greatly decreases the computational time without reducing or sacrificing any degree of predictive accuracy. The proposed adaptive fractional order model with memory handling performs better among all proposed variants of the model. Models without memory handling take more computation time. Though our present experiment was limited to the Lorenz system, our proposed framework can be applied to other nonlinear dynamical systems such as, Chen system and Liu chaotic system. Although the current set of experiments concentrate on the Lorenz attractor as a benchmark case, the proposed adaptive fractional-order framework is general enough to apply it to other nonlinear dynamical systems including the Chen attractor, Liu chaotic attractor, or any empirical dataset from the domains of engineering, energy, and biomedicine. Further research will involve testing the performance of the framework on non-stationary, non-linear real-world datasets.

6. Conclusions

This paper proposed an adaptive and efficient fractional-order modeling framework for nonlinear dynamical systems with long-memory effects. By jointly integrating adaptive fractional-order selection and memory optimization, the proposed approach effectively captures complex nonlinear and chaotic dynamics while reducing computational overhead. Experimental results on the Lorenz attractor dataset demonstrated superior performance compared with LSTM and fixed-order fractional models in terms of prediction accuracy, convergence behavior, and dynamical system reconstruction. The ablation study further confirmed the effectiveness of both adaptive-order dynamics and memory reduction mechanisms. Overall, the proposed framework provides an accurate, scalable, and computationally efficient solution for nonlinear system modeling. Future work will focus on extending the framework to higher-dimensional systems and exploring applications in energy systems, biomedical signal processing, and complex network analysis.

Author contributions

Conceptualization, T. A., S. Y. and U. D.; methodology, T. A, and S. Y.; software, H. S.; validation, U. D, M. A., H. S and M. F. A.; formal analysis, S. Y.; investigation, M. A.; resources, M. F. A.; data curation, U. D. and M. A.; writing-original draft preparation, U. D. and S. Y; writing-review and editing, M. H. and H. S; visualization, T. A.; supervision, M. A.; project administration, M. H, H. S., and M. F. A; funding acquisition, M. H. and M. F. A. All authors have read and agreed to the published version of the manuscript.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflicts of interest

The authors declare that they have no conflicts of interest.

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