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Research article

Stationary oscillation of delayed complex dynamical networks under impulsive control involving delays

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Abstract: This paper focused on the stationary oscillation problems for delayed complex dynamical networks, with the full consideration of time delays in impulsive actions. With the help of inequality-based techniques, several novel sufficient conditions that guarantee the existence of stationary oscillations for error system between complex dynamical network and its isolated node were derived. Our presented method, unlike the existing results, demonstrated that such an error system can exhibit stationary oscillations under impulsive control strategies involving time delays, even when the system was initially divergent or unstable. Finally, two illustrative examples along with their simulation results were put forward for demonstrating the efficacy and effectiveness of the developed control method.

Keywords: impulsive control; complex dynamical networks; time delays; stationary oscillation; inequality technique

1. Introduction

Complex dynamical networks are large-scale networks composed of numerous interrelated nodes that exchange state information through specific topological configurations [1–3]. Due to the fact that many practical problems can be addressed by modeling as complex dynamical networks, including applications in secure communication, parallel image processing, and biological medicine, complex dynamical networks have garnered significant research interest in recent years. Note that when all nodes in the networks exhibit identical dynamic behaviors as those specified by the designed network protocols, then it is regarded as achieving synchronization within complex dynamical networks. As a significant dynamic characteristic of complex dynamical networks, synchronization has drawn considerable attention and has been successfully implemented across diverse application domains; see [4–6] and references therein.

Time delays frequently arise in numerous scientific and engineering systems, originating from different mechanistic sources. For example, in analog neural network implementations, time delays naturally occur in signal transmission and response mechanisms as a result of the inherent limitations in amplifier switching speeds [7–9]. In specific nonnegative and compartmental systems, the exchange for information, energies, and material between compartments is not transient, and the past states should be consistently incorporated to accurately capture the realistic dynamics of such a system [10–12]. Importantly, these ubiquitous delays are not only incidental but they are often the source of system instability or performance degradation. Consequently, the issues on stabilization and stability for time delay systems have been two fundamental problems, which have been extensively studied [13–15].

As we all know, impulsive phenomena are commonly observed across various domains, such as frequency-modulated signal processing systems, pathological models of bursting rhythms, telecommunication, biological neural networks, and information science [16–18]. The main feature of those phenomena is characterized by the sudden change in system states occurring at some discrete instants, which cannot be sufficiently captured by purely continuous or purely discrete modeling approaches. Generally, such phenomena can be modeled as impulsive systems and have been extensively researched in the last few decades [19–21]. Moreover, impulsive control, as an effective control strategy, is characterized by low implementation cost, high confidentiality, and enhanced robustness; see [22, 23] and references therein. Therefore, it has been utilized in a broad range of intriguing domains, including synchronization in chaotic secure communication systems, optimal control of economic systems, and orbital transfer of satellites [24–26]. The main idea behind it involves regulating the states of continuous dynamical system by applying discontinuous control signals applied at some discrete instants. In practical applications, there are numerous scenarios where impulsive control demonstrates superior performance compared to continuous control [27, 28]. Thus, studying and developing the fundamental theory of impulsive systems holds significant theoretical and practical importance. To date, a substantial body of research addressing fundamental theoretical issues such as existence and uniqueness, controllability, differentiability, oscillation, and stability issues on impulsive systems have been documented within existing studies; see [29–31] and references therein. Specifically, the existence of periodic solutions for impulsive differential equations was investigated in [29] by integrating the contraction mapping principle with Lyapunov's direct method. Notably, the analysis therein adopts a dual perspective, i.e., treating impulsive effects both as perturbations and as control actions, thereby enhancing the applicability of the results. Subsequently, this framework has been extended to fractional-order impulsive systems [30], enabling the establishment of existence criteria for anti-periodic solutions to nonlinear fractional boundary value problems involving the Atangana–Baleanu derivative of order $\zeta \in (1, 2)$. However, all aforementioned works [29, 30] assume instantaneous impulses and thus neglect time delays in the system dynamics. To address this limitation, [31] recently studied the existence, uniqueness, and global exponential stability of stationary oscillations in nonlinear delayed systems subject to impulsive effects, but still under the assumption that impulses themselves occur instantaneously, i.e., without delay. In contrast, delayed impulses, where the impulsive action is triggered by or depends on past states, are widely recognized as a more realistic and effective modeling paradigm for numerous physical and engineering processes. For example, in impulsive synchronization-based secure communication systems, sampling delays inevitably arise during signal acquisition, causing impulsive control actions to depend on delayed state measurements [32]. Consequently, a rigorous treatment of delayed impulses, rather than merely delayed continuous dynamics, is essential. Further development of more methods and tools is required to deal with this issue.

To solve it, this paper investigates the problem of stationary oscillations in delayed complex dynamical networks via delayed impulsive control. In contrast to the existing findings [29–31], from the perspective of delayed impulsive control, several novel results guaranteeing the existence of stationary oscillations are established with the help of the application of inequality-based techniques. It indicates that the control mechanism primarily depends on the impulse weights, not the time delays in impulsive actions. However, the design of impulsive signals depends on the magnitude of time delays. The structure of the paper is organized as follows. Some essential mathematical preliminaries have been introduced in Section 2. In Section 3, we give the primary findings, and in Section 4 numerical examples are put forward. Finally, the concluding remarks will be presented in Section 5.

Notations. Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}_+$ the set of nonnegative real numbers, $\mathbb{Z}_+$ the set of positive integers, and $\mathbb{N}$ the set of nonnegative integers. Denote $\mathbb{R}^{n\times m}$ as the $n \times m$-dimensional real spaces and $\mathbb{R}^n$ as the n-dimensional real spaces equipped with the norm $\|x\| = \sum_{i=1}^{n} |x_i|$. For two integers $a < b$, let $\mathcal{I}[a, b] = \{a, a + 1, \dots, b\}$. For any interval $J \subseteq \mathbb{R}$ and $S \subseteq \mathbb{R}^k (1 \le k \le Nn)$, $PC(J, S) = \{\varphi : J \to S$ is continuous everywhere except at finite number of points t, at which $\varphi(t^+)$, $\varphi(t^-)$ exist and $\varphi(t^+) = \varphi(t)\}$.

2. Preliminaries

Consider the following complex dynamical network with coupling delays:

$$\dot{x}_i(t) = A(t)x_i(t) + C(t)g(x_i(t)) + c \sum_{j=1}^{N} b_{ij}\Gamma x_j(t - \tau) + \mathcal{U}_i(t) + \mathcal{J}(t),\ t \ge t_0, \tag{2.1}$$

where $x_i = (x_{i1}, x_{i2}, \dots, x_{in})^T \in \mathbb{R}^n$ is the state vector related to the i-th node with $i \in \mathcal{I}[1, N]$; $\Gamma = \operatorname{diag}\{\gamma_1, \gamma_2, \dots, \gamma_n\}$ is the inner coupling matrix with $\gamma_i > 0$, $i \in \mathcal{I}[1, n]$; $c, \tau > 0$ are two constants associated with the coupling strengths and coupling delays, respectively; $g(s) = (g_1(s_1), g_2(s_2), \dots, g_n(s_n))^T \in \mathbb{R}^n$ is corresponding to the activation function; $A(t) = (a_{ij}(t))_{n\times n} \in \mathcal{PC}(\mathbb{R}_+, \mathbb{R}^{n\times n})$ and $C(t) = (c_{ij}(t))_{n\times n} \in \mathcal{PC}(\mathbb{R}_+, \mathbb{R}^{n\times n})$; $\mathcal{U}_i \in \mathbb{R}^n$ represents the control input; and $\mathcal{J} \in \mathbb{R}^n$ denotes the same external input vector applied to each node throughout this paper. Let $B = (b_{ij})_{N\times N} \in \mathbb{R}^{N\times N}$ denote the outer coupling matrix, with b_{ij} defined as follows: $b_{ij} > 0$ if node i connects j $(i \neq j)$; otherwise $b_{ij} = 0$ $(i \neq j)$, and the diagonal elements are denoted by $b_{ii} = -\sum_{j=1, i\neq j}^{N} b_{ij}$. Set the initial condition of system (2.1) as $x_i(t_0 + s) = \chi_i(s) \in \mathcal{PC}([-\tau, 0], \mathbb{R}^n)$. In addition, we denote $\|\chi_i\|_\tau = \sup_{-\tau\le s\le 0} \|\chi_i(s)\|$.

Consider an isolated node characterized by

$$\dot{y}(t) = A(t)y(t) + C(t)g(y(t)),\ t \ge t_0, \tag{2.2}$$

where $y = (y_1, y_2, \dots, y_n)^T \in \mathbb{R}^n$, and the initial condition of isolated node (2.2) is denoted by $y(t_0) = y_0 \in \mathbb{R}^n$. Let the error signal between complex dynamical network (2.1) and isolated node (2.2) be $e_i(t) = x_i(t) - y(t)$, of which the dynamic is described by

$$\dot{e}_i(t) = A(t)e_i(t) + C(t)G(e_i(t)) + c \sum_{j=1}^{N} b_{ij}\Gamma e_j(t - \tau) + \mathcal{U}_i(t) + \mathcal{J}(t),\ t \ge t_0, \tag{2.3}$$

where, for a fixed reference trajectory $y(t)$, the function G is defined by

$$G(t, e_i) = g(e_i + y(t)) - g(y(t)).$$

For each fixed t, $G(t, \cdot) : \mathbb{R}^n \to \mathbb{R}^n$ depends on e_i only, while its time dependence arises solely through the reference solution $y(t)$. For notational simplicity, we may write $G(e_i)$ instead of $G(t, e_i)$ when no confusion arises. In addition, $e_i(t_0 + s) = \varphi_i(s) = \chi_i(s) - y_0 \in \mathcal{PC}([-\tau, 0], \mathbb{R}^n)$.

As mentioned in Section 1, impulsive control not only lowers the costs associated with the control actions and decreases the frequency of information exchange, but also demonstrates considerable robustness and enhanced confidentiality. In this regard, the control input $\mathcal{U}_i(t)$ takes a form involving the Dirac function herein. Consider the ubiquity of time delays, then $\mathcal{U}_i(t)$ is designed by (see Figure 1)

$$\mathcal{U}_i(t) = \sum_{k\in\mathbb{Z}_+} (D^k e_i(t-\tau) - e_i(t))\delta(t-t_k),\ i \in \mathcal{I}[1,N], \tag{2.4}$$

with impulsive control gain $D^k \in \mathbb{R}^{n\times n}$, $k \in \mathbb{Z}_+$, and impulse instants t_k satisfying $0 \le t_0 < t_1 < \cdots < t_k \to \infty$ as $k \to \infty$. In this paper, we consider the control input with $U_i(t) = \sum_{k\in Z_+}(D^k e_i(t-\tau) - e_i(t))\delta(t-t_k)$, not $U_i(t) = \sum_{k\in Z_+}(D^k e_i(t-\tau))\delta(t-t_k)$. From an engineering perspective, the controller $U_i(t) = \sum_{k\in Z_+}(D^k e_i(t-\tau))\delta(t-t_k)$ using only $e_i(t-\tau)$ is easier to implement since it requires only one buffer to store the delayed state and no synchronization between current and delayed measurements is needed. Despite the higher implementation complexity for $U_i(t) = \sum_{k\in Z_+}(D^k e_i(t-\tau)-e_i(t))\delta(t-t_k)$, it offers two potential benefits. First, by incorporating the current state $e_i(t_k^-)$, it can respond more promptly to deviations, whereas the alternative controller acts solely on delayed information. Second, the inclusion of $e_i(t_k^-)$ may help compensate for delay-induced perturbations, potentially enhancing robustness against delay mismatches. Based on control input (2.4), error system (2.3) is rewritten as

$$\begin{cases} \dot{e}_i(t) = A(t)e_i(t) + C(t)G(e_i(t)) + c\sum_{j=1}^{N} b_{ij}\Gamma e_j(t-\tau) + \mathcal{J}(t),\ t \ge t_0, \\ e_i(t_k) = D^k e_i(t_k^- - \tau),\ k \in \mathbb{Z}_+, \\ e_i(t_0+s) = \varphi_i(s),\ s \in [-\tau, 0]. \end{cases} \tag{2.5}$$

To study the stationary oscillation of error system (2.5), the assumptions should first be introduced.

(H_1) There exists $\omega > 0$ such that functions $A(t)$, $C(t)$, and $\mathcal{J}(t)$ are periodic with period ω;

(H_2) There exists $q \in \mathbb{Z}_+$ such that $D^{k+q} = D^k$ and $t_{k+q} = t_k + \omega$, $k \in \mathbb{Z}_+$, with $\omega > 0$;

(H_3) The function $g : \mathbb{R}^n \to \mathbb{R}^n$ is globally Lipschitz, i.e., there exist constants $g_i > 0$ such that for all $u_i, v_i \in \mathbb{R}$,

$$|g_i(u_i) - g_i(v_i)| \le g_i|u_i - v_i|,\ i \in \mathcal{I}[1,n].$$

Consequently, for a fixed y, the function $G(e_i) = g(e_i + y) - g(y)$ satisfies

$$|G_i(s_1) - G_i(s_2)| \le g_i|s_1 - s_2|,\ \forall s_1, s_2 \in \mathbb{R}.$$

(H_4) $c_{ij}^\star = \sup_{t\in[0,\omega]} |c_{ij}(t)|$, $a_{ii}^\star = \sup_{t\in[0,\omega]} a_{ii}(t)$, and $a_{ij}^\star = \sup_{t\in[0,\omega]} |a_{ij}(t)|(i \ne j)$, $i, j \in \mathcal{I}[1,n]$.

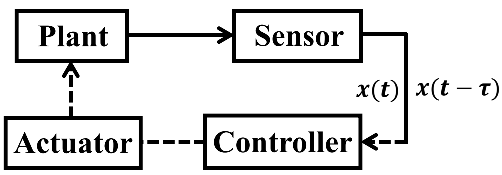

Figure 1. The control diagram for system (2.3) under (2.4).

Definition 1. *We declare that the map $e_i(t) : \mathbb{R}_+ \to \mathbb{R}^n$ with $i \in \mathcal{I}[1,N]$ is the ω-periodic solution of error system (2.5) provided that* *(i)* $e_i(t)$ *meets* $e_i(t_k+\omega^+) = e_i(t_k^+)$ *for* $k \in \mathbb{Z}_+$ *and* $e_i(t+\omega) = e_i(t)$ *for* $t \neq t_k$*;*

(ii) $e_i(t)$ *meets (2.5) and denotes a piecewise continuous map possessing first-kind discontinuities.*

Definition 2. *Let* $e_i^* = e_i^*(t, t_0, \varphi_i^*)$ *be a solution of error system (2.5) with initial condition* (t_0, φ_i^*)*,* $i \in \mathcal{I}[1, N]$*. Then, we state that the solution* e_i^* *with* $i \in \mathcal{I}[1, N]$ *is a stationary oscillation for error system (2.5) provided that*

(i) For any other solution $e_i = e_i(t, t_0, \varphi_i)$ *regarding error system (2.5) through* (t_0, φ_i)*,*

$$|e_i - e_i^*| \to 0 \text{ as } t \to \infty, \ i \in \mathcal{I}[1, N];$$

(ii) e_i^* *is a unique* ω*-periodic solution regarding error system (2.5).*

Lemma 1. *[33] Let* $u \in PC(\mathbb{R}, \mathbb{R}_+)$ *and* λ_1*,* λ_2*,* υ_k*,* $\tau > 0$ *satisfy*

$$\begin{cases} D^+u(t) \leq \lambda_1 u(t) + \lambda_2 u(t-\tau), \ t \geq t_0, \ t \neq t_k, \\ u(t_k) \leq \upsilon_k u(t_k^- - \tau), \ k \in \mathbb{Z}_+. \end{cases} \tag{2.6}$$

Suppose that a sequence $\{\mu_k\}_{k\in\mathbb{N}}$ *exists such that*

$$\mu_0 = 0 \text{ and } t_k - \tau \leq \mu_k + t_{k-1}, \ k \in \mathbb{Z}_+, \tag{2.7}$$

constants $0 < \upsilon \leq \kappa < 1$ *and* $\eta > 0$*, with* $\mu_k \leq \eta$*,* $\upsilon_k \geq \upsilon$*,* $k \in \mathbb{Z}_+$*, such that*

$$\exp[\mu_k(\lambda_1 + \frac{\lambda_2}{\upsilon})]\upsilon_k \leq \kappa, \ k \in \mathbb{Z}_+.$$

Then, (2.6) has the following estimate

$$u(t) \leq \bar{u}(t_0)\exp\{(\lambda_1 + \frac{\lambda_2}{\upsilon})(\tau + \eta)\}\kappa^n, \ t \in [t_n, t_{n+1}), \ n \in \mathbb{N},$$

with $\bar{u}(t_0) = \sup_{t_0-\tau\leq s\leq t_0} u(s)$*.*

Remark 1. Let $\beta_3 = 0$ and $\varsigma_k = \tau$, then we can conclude from Proposition 2 in [33] that Lemma 1 holds. Consider a fact that $\kappa \in (0, 1)$, which implies that u tends to zero as t approaches infinity. In other words, the solution to (2.6) can achieve asymptotic stability through impulsive effects that incorporate time delays. In addition, it should be noted that although the constraints proposed in Lemma 1 do not depend on the delay effects in impulse terms, the design of impulse signals and the degree of convergence are affected by the size of delays.

3. Main results

Some sufficient results for the stationary oscillation of error system (2.5) will be proposed in this section, by means of impulsive control involving delays.

Theorem 1. *Under assumptions* $(H_1) - (H_4)$*, error system (2.5) exhibits a stationary oscillation. Suppose that there exist sequence* $\{\mu_k\}_{k\in\mathbb{N}}$ *satisfying (2.7), constants* $0 < \upsilon \leq \kappa < 1$ *and* $\eta > 0$*, with* $\mu_k \leq \eta$*,* $k \in \mathbb{Z}_+$*, such that*

$$\sum_{j=1}^{n} \max_{m\in\mathcal{I}[1,n]} |D_{jm}^k| \geq \upsilon, \ k \in \mathbb{Z}_+, \tag{3.1}$$

$$\exp(\mu_k\Omega)\cdot\sum_{j=1}^{n}\max_{m\in\mathcal{I}[1,n]}|D^k_{jm}|\le\kappa,\ k\in\mathbb{Z}_+,\tag{3.2}$$

where

$$\Omega=\max_{i\in\mathcal{I}[1,n]}a^\star_{ii}+\sum_{j=1}^{n}\max_{m\in\mathcal{I}[1,n],m\neq j}a^\star_{jm}+\sum_{j=1}^{n}\max_{m\in\mathcal{I}[1,n]}c^\star_{jm}g_m+\frac{c\max_{j\in\mathcal{I}[1,n]}\gamma_j\cdot\sum_{i=1}^{N}\max_{\theta\in\mathcal{I}[1,N]}|b_{i\theta}|}{\upsilon}.$$

Proof. Let $\bar{e}_i=\bar{e}_i(t,t_0,\phi_i)$ and $\hat{e}_i=\hat{e}_i(t,t_0,\varphi_i)$ be two arbitrary solutions of error system (2.5) with initial conditions (t_0,ϕ_i) and (t_0,φ_i), respectively, where $\bar{e}_i=(\bar{e}_{i1},\dots,\bar{e}_{in})^T\in\mathbb{R}^n$, $\hat{e}_i=(\hat{e}_{i1},\dots,\hat{e}_{in})^T\in\mathbb{R}^n$, and $\phi_i,\varphi_i\in\mathcal{PC}([-\tau,0],\mathbb{R}^n)$, $i\in\mathcal{I}[1,n]$. For later use, we denote $\bar{e}=(\bar{e}_1^T,\dots,\bar{e}_N^T)^T\in\mathbb{R}^{Nn}$, $\hat{e}=(\hat{e}_1^T,\dots,\hat{e}_N^T)^T\in\mathbb{R}^{Nn}$, $\phi=(\phi_1^T,\dots,\phi_N^T)^T\in\mathcal{PC}([-\tau,0],\mathbb{R}^{Nn})$, $\varphi=(\varphi_1^T,\dots,\varphi_N^T)^T\in\mathcal{PC}([-\tau,0],\mathbb{R}^{Nn})$. Consider the following auxiliary function

$$\mathcal{V}(t)=\|\bar{e}(t)-\hat{e}(t)\|=\sum_{i=1}^{N}\|\bar{e}_i(t)-\hat{e}_i(t)\|=\sum_{i=1}^{N}\sum_{j=1}^{n}|\bar{e}_{ij}(t)-\hat{e}_{ij}(t)|,$$

implying $\mathcal{V}\in\mathcal{PC}(\mathbb{R},\mathbb{R}_+)$. According to the continuous dynamic of error system (2.5), we obtain that

$$\begin{aligned}\dot{\bar{e}}_{ij}(t)-\dot{\hat{e}}_{ij}(t)=&\sum_{m=1}^{n}a_{jm}(t)[\bar{e}_{im}(t)-\hat{e}_{im}(t)]+\sum_{m=1}^{n}c_{jm}(t)[G_m(\bar{e}_{im}(t))-G_m(\hat{e}_{im}(t))]\\&+c\gamma_j\sum_{\theta=1}^{N}b_{i\theta}[\bar{e}_{\theta j}(t-\tau)-\hat{e}_{\theta j}(t-\tau)],\ t\in[t_{k-1},t_k).\end{aligned}$$

Based on this, one deduces from (H_3) that

$$\begin{aligned}D^+\mathcal{V}(t)=&\sum_{i=1}^{N}\sum_{j=1}^{n}[\dot{\bar{e}}_{ij}(t)-\dot{\hat{e}}_{ij}(t)]\cdot\operatorname{sgn}(\bar{e}_{ij}(t)-\hat{e}_{ij}(t))\\=&\sum_{i=1}^{N}\sum_{j=1}^{n}\Big\{\sum_{m=1}^{n}a_{jm}(t)[\bar{e}_{im}(t)-\hat{e}_{im}(t)]+\sum_{m=1}^{n}c_{jm}(t)[G_m(\bar{e}_{im}(t))-G_m(\hat{e}_{im}(t))]\\&+c\gamma_j\sum_{\theta=1}^{N}b_{i\theta}[\bar{e}_{\theta j}(t-\tau)-\hat{e}_{\theta j}(t-\tau)]\Big\}\cdot\operatorname{sgn}(\bar{e}_{ij}(t)-\hat{e}_{ij}(t))\\\le&\sum_{i=1}^{N}\Big\{\sum_{j=1}^{n}a_{jj}(t)|\bar{e}_{ij}(t)-\hat{e}_{ij}(t)|+\sum_{j=1}^{n}\sum_{m=1,m\neq j}^{n}|a_{jm}(t)|\cdot|\bar{e}_{im}(t)-\hat{e}_{im}(t)|\\&+\sum_{j=1}^{n}\sum_{m=1}^{n}|c_{jm}(t)|\cdot|G_m(\bar{e}_{im}(t))-G_m(\hat{e}_{im}(t))|+c\sum_{j=1}^{n}\gamma_j\sum_{\theta=1}^{N}|b_{i\theta}||\bar{e}_{\theta j}(t-\tau)-\hat{e}_{\theta j}(t-\tau)|\Big\}\\\le&\sum_{i=1}^{N}\Big\{\sum_{j=1}^{n}a_{jj}(t)|\bar{e}_{ij}(t)-\hat{e}_{ij}(t)|+\sum_{j=1}^{n}\sum_{m=1,m\neq j}^{n}|a_{jm}(t)|\cdot|\bar{e}_{im}(t)-\hat{e}_{im}(t)|\\&+\sum_{j=1}^{n}\sum_{m=1}^{n}g_m|c_{jm}(t)|\cdot|\bar{e}_{im}(t)-\hat{e}_{im}(t)|+c\sum_{j=1}^{n}\gamma_j\sum_{\theta=1}^{N}|b_{i\theta}||\bar{e}_{\theta j}(t-\tau)-\hat{e}_{\theta j}(t-\tau)|\Big\},\end{aligned}$$

which, together with assumption (H_4), it can be further estimated that for $t \in [t_{k-1}, t_k)$, $k \in \mathbb{Z}_+$,

$$\begin{aligned}
D^+\mathcal{V}(t) \le & \sum_{i=1}^{N}\Big\{\max_{i\in\mathcal{I}[1,n]} a_{ii}^\star \cdot \sum_{j=1}^{n}|\bar{e}_{ij}(t)-\hat{e}_{ij}(t)| + (\sum_{j=1}^{n}\max_{m\in\mathcal{I}[1,n],m\neq j} a_{jm}^\star)\cdot\sum_{j=1}^{n}|\bar{e}_{ij}(t)-\hat{e}_{ij}(t)| \\
& +(\sum_{j=1}^{n}\max_{m\in\mathcal{I}[1,n]} c_{jm}^\star g_m)\cdot\sum_{j=1}^{n}|\bar{e}_{ij}(t)-\hat{e}_{ij}(t)| + c\max_{j\in\mathcal{I}[1,n]}\gamma_j\cdot\max_{\theta\in\mathcal{I}[1,N]}|b_{i\theta}|\cdot\sum_{\theta=1}^{N}\sum_{j=1}^{n}|\bar{e}_{\theta j}(t-\tau)-\hat{e}_{\theta j}(t-\tau)|\Big\} \\
\le & \Big\{\max_{i\in\mathcal{I}[1,n]} a_{ii}^\star + \sum_{j=1}^{n}\max_{m\in\mathcal{I}[1,n],m\neq j} a_{jm}^\star + \sum_{j=1}^{n}\max_{m\in\mathcal{I}[1,n]} c_{jm}^\star g_m\Big\}\mathcal{V}(t) + \Big(c\max_{j\in\mathcal{I}[1,n]}\gamma_j\cdot\sum_{i=1}^{N}\max_{\theta\in\mathcal{I}[1,N]}|b_{i\theta}|\Big)\mathcal{V}(t-\tau).
\end{aligned}$$

In addition, one derives

$$\begin{aligned}
\mathcal{V}(t_k) &= \sum_{i=1}^{N}\sum_{j=1}^{n}|\bar{e}_{ij}(t_k)-\hat{e}_{ij}(t_k)| \\
&= \sum_{i=1}^{N}\sum_{j=1}^{n}|\sum_{m=1}^{n} D_{jm}^k \bar{e}_{im}(t_k^- - \tau) - \sum_{m=1}^{n} D_{jm}^k \hat{e}_{im}(t_k^- - \tau)| \\
&\le \sum_{i=1}^{N}\sum_{j=1}^{n}\sum_{m=1}^{n}|D_{jm}^k|\cdot|\bar{e}_{im}(t_k^- - \tau)-\hat{e}_{im}(t_k^- - \tau)| \\
&\le \sum_{j=1}^{n}\max_{m\in\mathcal{I}[1,n]}|D_{jm}^k|\cdot\mathcal{V}(t_k^- - \tau),\ k\in\mathbb{Z}_+.
\end{aligned}$$

Then, using Lemma 1, we can calculate from conditions (3.1) and (3.2) that

$$\mathcal{V}(t) \le \bar{\mathcal{V}}(t_0)\exp\{(\lambda_1 + \frac{\lambda_2}{\upsilon})(\tau+\eta)\}\kappa^n,\ t\in[t_n, t_{n+1}),\ n\in\mathbb{N}, \tag{3.3}$$

where $\bar{\mathcal{V}}(t_0) = \sup_{t_0-\tau\le s\le t_0}\mathcal{V}(s)$,

$$\lambda_1 = \max_{i\in\mathcal{I}[1,n]} a_{ii}^\star + \sum_{j=1}^{n}\max_{m\in\mathcal{I}[1,n],m\neq j} a_{jm}^\star + \sum_{j=1}^{n}\max_{m\in\mathcal{I}[1,n]} c_{jm}^\star g_m,$$

and

$$\lambda_2 = c\max_{j\in\mathcal{I}[1,n]}\gamma_j\cdot\sum_{i=1}^{N}\max_{\theta\in\mathcal{I}[1,N]}|b_{i\theta}|.$$

According to the definition of sequence $\{\mu_k\}_{k\in\mathbb{N}}$, one has for $k\in\mathbb{Z}_+$ that it holds $t_k - t_{k-1} \le \tau+\eta$. To this end, (3.3) can be further deduced from $\kappa\in(0,1)$ such that

$$\mathcal{V}(t) \le \kappa^{-1}\bar{\mathcal{V}}(t_0)\exp\{(\lambda_1+\frac{\lambda_2}{\upsilon})(\tau+\eta)\}\cdot\exp\{\frac{\ln\kappa}{\tau+\eta}(t-t_0)\},\ t\ge t_0,$$

i.e.,

$$\|\bar{e}(t)-\hat{e}(t)\| \le \|\phi-\varphi\|_\tau\cdot\kappa^{-1}\exp\{(\lambda_1+\frac{\lambda_2}{\upsilon})(\tau+\eta)\}\cdot\exp\{\frac{\ln\kappa}{\tau+\eta}(t-t_0)\},\ t\ge t_0.$$

Considering the fact that $\frac{\ln\kappa}{\tau+\eta} < 0$, there exists $T \geq t_0$ such that

$$\|\bar{e}(t) - \hat{e}(t)\| \leq \frac{1}{2}\|\phi - \varphi\|_\tau, \ t \geq T. \tag{3.4}$$

Construct the following operator

$$\mathbf{F} : \hat{\phi} \to e_{t_0+\omega}(t_0, \hat{\phi}),$$

with $\hat{\phi} \in \mathcal{PC}([-\tau, 0], \mathbb{R}^{Nn})$. In view of $e_{t_0+\omega}(t_0, \hat{\phi}) \in \mathcal{PC}([-\tau, 0], \mathbb{R}^{Nn})$, one knows that operator $\mathbf{F}$ maps $\mathcal{PC}([-\tau, 0], \mathbb{R}^{Nn})$ into itself. Let k be sufficiently large such that $t_0 + k\omega - \tau \geq T$. Considering the fact that $\mathbf{F}^k\hat{\phi} = e_{t_0+k\omega}(t_0, \hat{\phi})$ for $k \in \mathbb{Z}_+$, it then follows from (3.4) that

$$\|\mathbf{F}^k\hat{\phi} - \mathbf{F}^k\phi\| = \|e_{t_0+k\omega}(t_0, \hat{\phi}) - \bar{e}_{t_0+k\omega}(t_0, \phi)\| \leq \frac{1}{2}\|\hat{\phi} - \phi\|_\tau.$$

Therefore, $\mathbf{F}^k$ is a contraction mapping on the Banach space $\mathcal{PC}([-\tau, 0], \mathbb{R}^{Nn})$ with contraction constant $\frac{1}{2}$. By the Banach fixed point theorem, there exists a unique $\phi^* \in \mathcal{PC}([-\tau, 0], \mathbb{R}^{Nn})$ such that $\mathbf{F}^k\phi^* = \phi^*$. Moreover, since $\mathbf{F}^k(\mathbf{F}\phi^*) = \mathbf{F}(\mathbf{F}^k\phi^*) = \mathbf{F}\phi^*$, the uniqueness of the fixed point of $\mathbf{F}^k$ implies $\mathbf{F}\phi^* = \phi^*$. Hence, ϕ^* is also the unique fixed point of $\mathbf{F}$. Therefore, we obtain that error system (2.5) has exactly an ω-periodic solution which is globally exponentially stable, i.e., error system (2.5) admits a stationary oscillation, thus completing the proof. □

Remark 2. Theorem 1 establishes some sufficient conditions for the existence of a stationary oscillatory solution for error system (2.5), involving several unknown parameters D_{jm}^k, μ_k, υ, κ, η, and Ω. Specifically, Ω encodes structural information of the considered system, and D_{jm}^k characterizes the impulse identity information. Then, the remaining parameters are auxiliary parameters for applying Lemma 1, which should be further illustrated in the Examples section.

Especially when $A(t)$ takes the form of a diagonal matrix, the result can be directly obtained.

Corollary 1. *Under assumptions $(H_1) - (H_4)$, error system (2.5) exhibits a stationary oscillation. Suppose that there exist sequence $\{\mu_k\}_{k\in\mathbb{N}}$ satisfying (2.7), constants $0 < \upsilon \leq \kappa < 1$ and $\eta > 0$, with $\mu_k \leq \eta$, $k \in \mathbb{Z}_+$, such that*

$$\sum_{j=1}^{n} \max_{m\in\mathcal{I}[1,n]} |D_{jm}^k| \geq \upsilon, \ k \in \mathbb{Z}_+,$$

$$\exp(\mu_k\Omega) \cdot \sum_{j=1}^{n} \max_{m\in\mathcal{I}[1,n]} |D_{jm}^k| \leq \kappa, \ k \in \mathbb{Z}_+,$$

where

$$\Omega = \max_{i\in\mathcal{I}[1,n]} a_{ii}^\star + \sum_{j=1}^{n} \max_{m\in\mathcal{I}[1,n]} c_{jm}^\star g_m + \frac{c \max_{j\in\mathcal{I}[1,n]} \gamma_j \cdot \sum_{i=1}^{N} \max_{\theta\in\mathcal{I}[1,N]} |b_{i\theta}|}{\upsilon}.$$

Remark 3. One may observe from the estimate of $D^+\mathcal{V}$ provided in Theorem 1 that error system (2.5) may not achieve stationary oscillation and could diverge in the absence of impulsive control. Nevertheless, the conditions established in Theorem 1 and Corollary 1 indicate that appropriately impulsive control schemes can promote periodic behavior in the system and ensure existence of stationary oscillations, with further verification provided in Section 4.

Remark 4. Recently, [31] established some conditions of stationary oscillation for nonlinear delay system with the help of impulsive control. It should be pointed out that the results in [31] were based on impulsive actions without delays and, moreover, the system under study was a general nonlinear system, not a more complex networked structure. In contrast, in this paper, Theorem 1 presents some conditions of stationary oscillation for error system (2.5) between complex dynamical network (2.1) and isolated node (2.2). Moreover, the time delays in impulsive actions are also fully considered in this paper. It shows that the design of impulsive signals is dependent on the magnitude of time delays.

4. Examples

Two examples and their simulations are employed in this section for showing the validity for the obtained results.

Example 1. In the example, consider complex dynamical network (2.1) composed of four 2-D subnetworks, with the relevant parameters for each node defined as follows:

$$A(t) = \begin{bmatrix} -0.1 + 0.02\sin\frac{2\pi}{\omega}t & 1 + 0.01\sin\frac{2\pi}{\omega}t \\ 0.5 + 0.01\cos\frac{2\pi}{\omega}t & -0.5 + 0.02\cos\frac{2\pi}{\omega}t \end{bmatrix},\ \Gamma = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$C(t) = 0,\ D^k = \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix},\ \tau = 0.1,\ c = 0.1,\ \mathcal{J}(t) = [2\cos\tfrac{2\pi}{\omega}t, 3\sin\tfrac{2\pi}{\omega}t]^T,$$

where $\omega > 0$ and $d_1, d_2 \in (0, 1)$ (to test the controller under non-identical components of the input vector), see [34] for some more detailed information. The network topological structure of complex dynamical network (2.1) is shown in Figure 2 (it is the directed weighted network), with the coupling matrix

$$B = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -2 & 0 & 1 \\ 1 & 0 & -2 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}.$$

Note that in the absence of control input (2.4), the error trajectories between complex dynamical network (2.1) and target system (2.2) are divergent, as shown in Figure 3. In other words, error system (2.5) without control input (2.4) exhibits no stationary oscillation and is furthermore divergent. In the following, we will design appropriate impulse signals to guarantee stationary oscillation of error system (2.5). Let $t_{k+q} - t_k = \omega$ and $t_k - t_{k-1} \le \mu_k + 0.1$, then one obtains from Theorem 1 that error system (2.5) admits a stationary oscillation, where $d_1 + d_2 \ge \upsilon$ and $(d_1 + d_2)\exp\{\mu_k(1.44 + 0.06/\upsilon)\} \le \kappa$ with $0 < \upsilon \le \kappa < 1$. In simulations, let $\omega = 1.1$, $d_1 = d_2 = 0.08$, $\upsilon = 0.16$, $\kappa = 0.99$, $q = 1$, and $t_k = 1.1k,\ k \in \mathbb{Z}_+$ (see Table 1), of which the simulation is shown in Figure 4 by MATLAB's toolbox using step size $p = 0.0001$, where the initial state is selected as

$$e_0 = (5.3, 3.5, 2.8, 5.2, 3.5, 6.7, 4.5, 8.3)^T.$$

In addition, when $\omega = 11$, one may observe from Figure 5 that error system (2.5) also admits a stationary oscillation, with $t_k = 1.1k$ and $q = 10$. On the other hand, for the constraint of the inter-impulse interval $t_k - t_{k-1} \leq \mu_k + 0.1$, if the effect of delay is ignored, then it updated as $t_k - t_{k-1} \leq \mu_k$, thereby deducing the maximum upper bound of inter-impulse interval. These simulations demonstrate the validity of the obtained results.

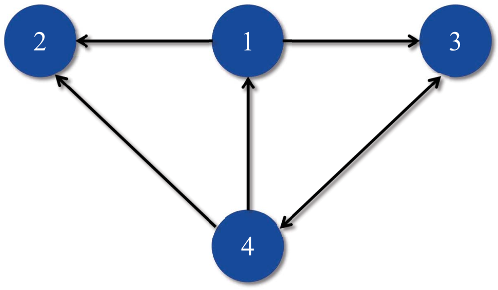

Figure 2. Network topology with four nodes.

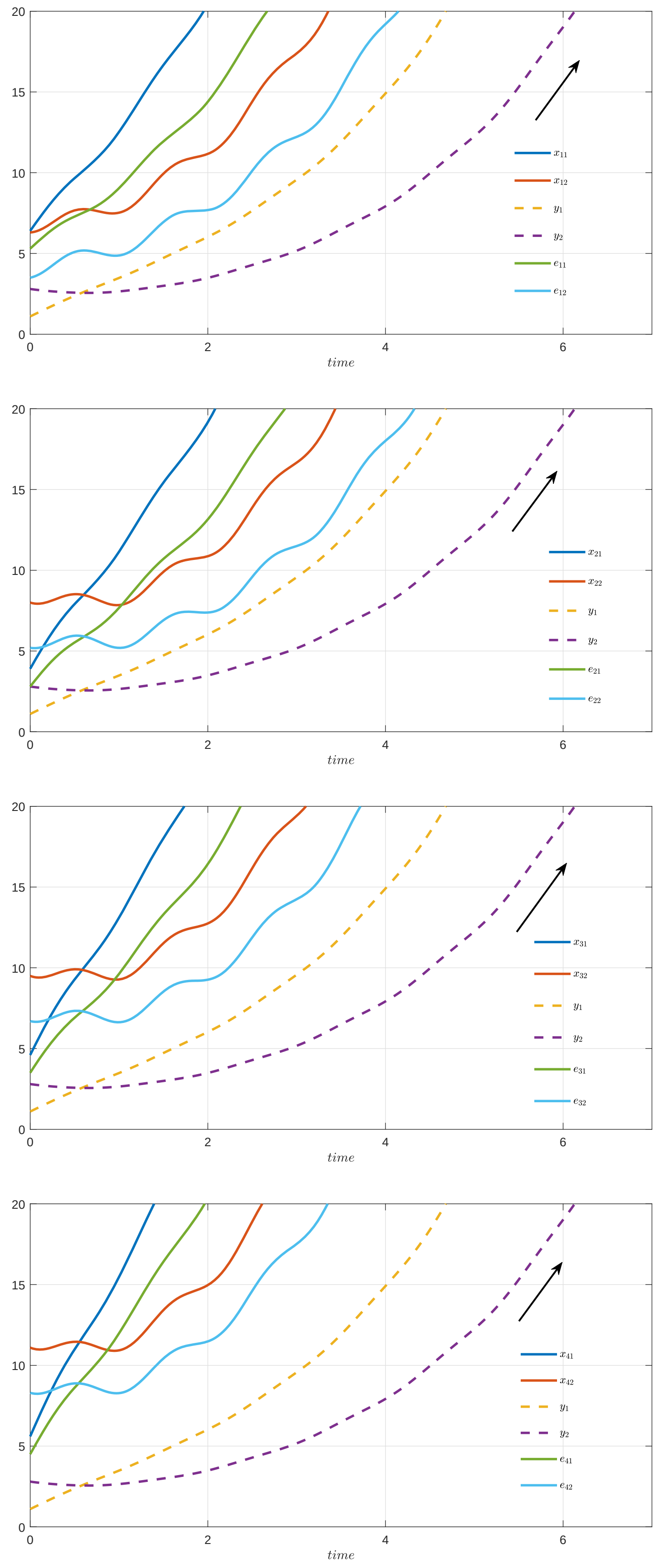

Figure 3. Trajectories of $x_{ij}(t)$, $y_j(t)$, and $e_{ij}(t)$, $i \in \mathcal{I}[1,4]$, $j \in \mathcal{I}[1,2]$ under $\omega = 1.1$, without control input.

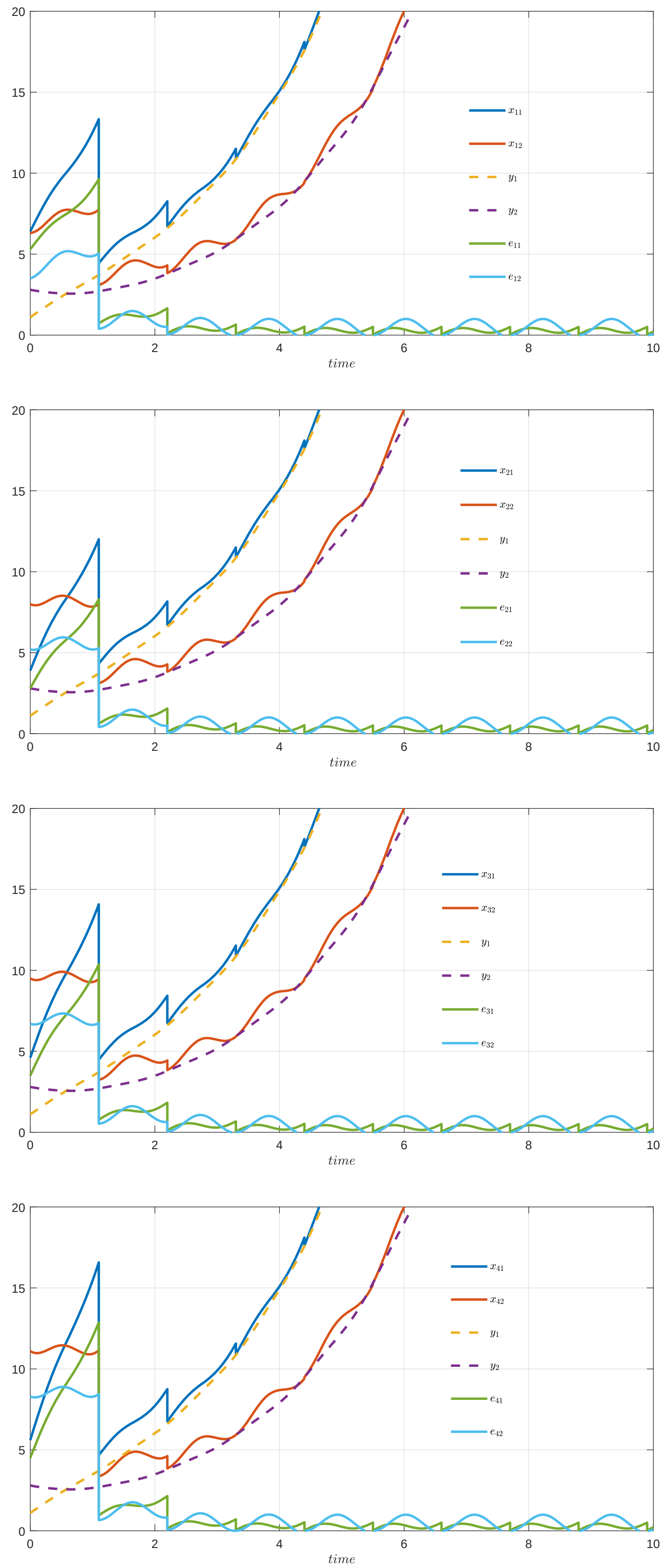

Figure 4. Trajectories of $x_{ij}(t)$, $y_j(t)$, and $e_{ij}(t)$, $i \in \mathcal{I}[1,4]$, $j \in \mathcal{I}[1,2]$ under $\omega = 1.1$.

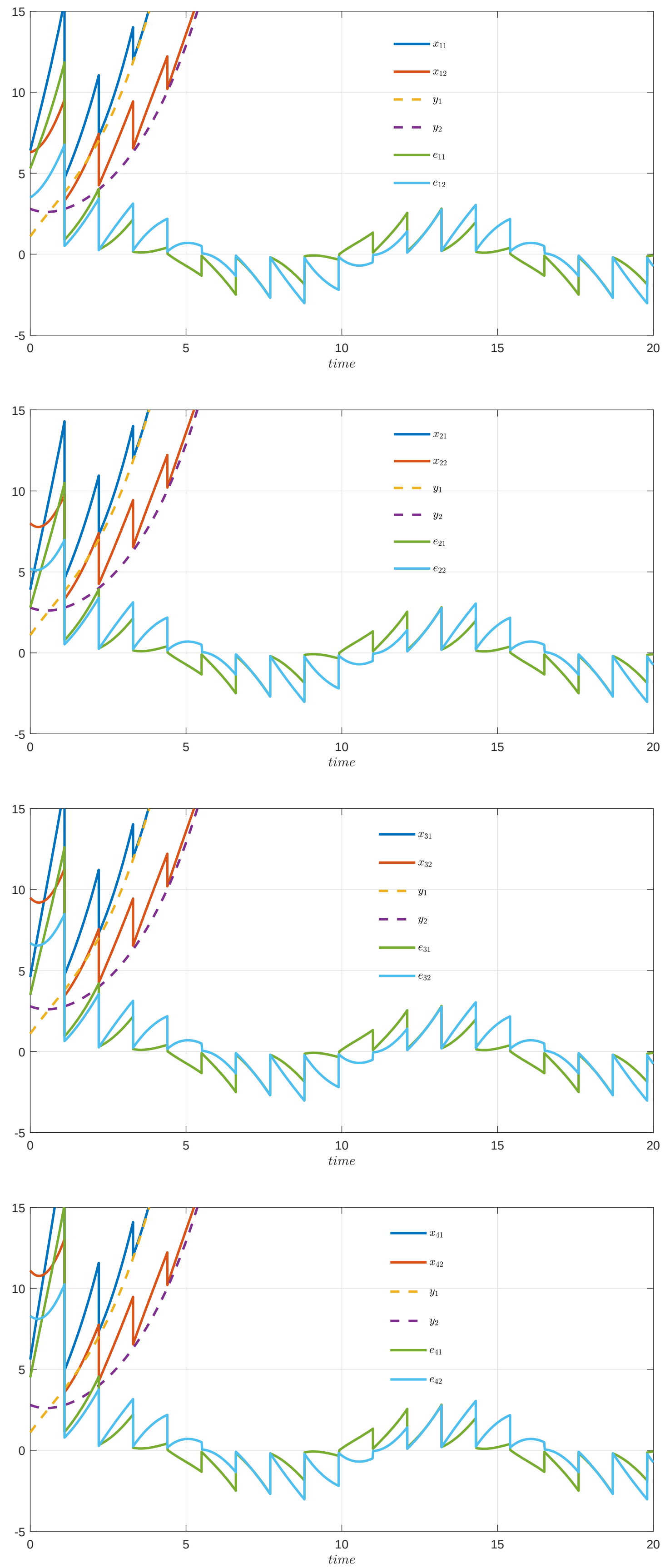

Figure 5. Trajectories of $x_{ij}(t)$, $y_j(t)$, and $e_{ij}(t)$, $i \in \mathcal{I}[1,4]$, $j \in \mathcal{I}[1,2]$ under $\omega = 11$.

Example 2. Consider 3-D complex dynamical network (2.1) with $C(t) = 0$, $N = 10$, $c = 0.1$, $\tau = 0.2$,

$$\Gamma = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}, \; A(t) = \begin{bmatrix} -1 + 0.02\cos\frac{2\pi}{\omega}t & 0.2 + 0.02\cos\frac{2\pi}{\omega}t & 0.3 + 0.02\cos\frac{2\pi}{\omega}t \\ 0 & -1 + 0.03\sin\frac{2\pi}{\omega}t & 0.1 + 0.03\sin\frac{2\pi}{\omega}t \\ 0.2 + 0.05\cos\frac{2\pi}{\omega}t & 0.5 + 0.01\sin\frac{2\pi}{\omega}t & 0 \end{bmatrix},$$
$$D^k = \mathrm{diag}\{d_1, d_2, d_3\}, \; \mathcal{J}(t) = [10\cos\tfrac{2\pi}{\omega}t, 10\cos\tfrac{2\pi}{\omega}t, 10\cos\tfrac{2\pi}{\omega}t]^T \in \mathbb{R}^3,$$

where $\omega > 0$ and d_1, d_2, $d_3 \in (0, 1)$ (to test the controller under non-identical components of the input vector). The network topology of complex dynamical network (2.1) is illustrated in Figure 6, along with the corresponding coupling matrix

$$B = \begin{bmatrix} -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -2 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -2 \end{bmatrix}.$$

When $\mathcal{U}(t) = 0$, error system (2.5) is divergent, not to mention a stationary oscillation, which is shown in Figure 7(a). Let $t_{k+q} - t_k = \omega$ and $t_k - t_{k-1} \le 0.2 + \mu_k$, then one can conclude from Theorem 1 that error system (2.5) admits a stationary oscillation, where $d_1 + d_2 + d_3 \ge \upsilon$ and $(d_1 + d_2 + d_3)\exp\{\mu_k(0.96 + 0.4/\upsilon)\} \le \kappa$, with $0 < \upsilon \le \kappa < 1$. In simulations, choose $\omega = 0.6$, $d_1 = d_2 = d_3 = 0.2$, $\upsilon = 0.28$, $q = 2$, and $\kappa = 0.85$; see Table 1. Then, error system (2.5) exhibits stationary oscillations for $t_k = 0.3k$, as illustrated in Figure 7(b), by MATLAB's toolbox using step size $p = 0.0001$, where the initial state is selected as

$$\begin{aligned} e_0 = &(5.3, 3.5, 2.8, 5.2, 3.5, 6.7, 4.5, 8.3, 4.4, 3.5, 0.2, 6.7, 5.8, 8.7, 6.9, \\ &4.7, 3.4, 7.4, 6.9, 3.7, 8.4, 5.7, 8.9, 3.5, 4.2, 6.5, 8.9, 5.9, 6.3, 1.4)^T. \end{aligned}$$

From the above simulations, it can be observed that appropriate impulsive control schemes can stabilize a divergent system, thereby enabling it to achieve stationary oscillation.

Table 1. Parameter selection in Examples 1 and 2.

Example	$\sum_{j=1}^{n} \max_{m\in\mathcal{I}[1,n]} \lvert D^k_{jm}\rvert$	μ_k	υ	κ	η	Ω
Example 1	0.16	$1k$	0.16	0.99	1	$1.44 + \frac{0.06}{\upsilon}$
Example 2	0.6	$0.1k$	0.28	0.85	0.1	$0.96 + \frac{0.4}{\upsilon}$

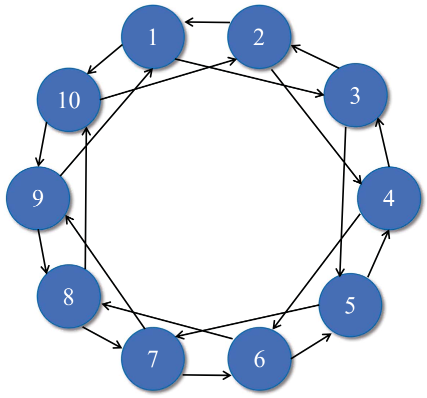

Figure 6. Network topology with ten nodes.

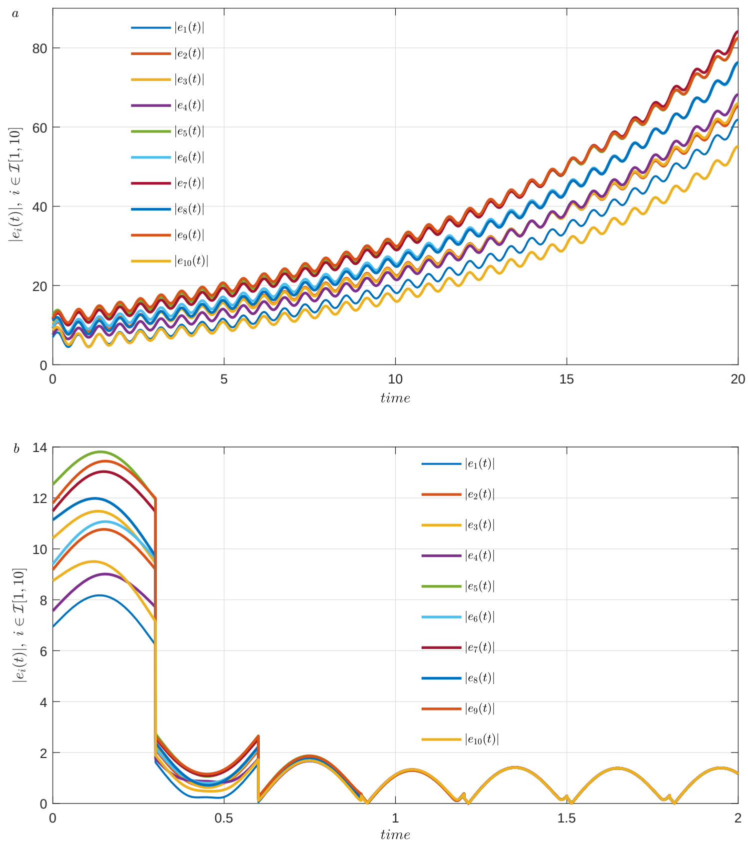

Figure 7. (a) Trajectories of error states $|e_i(t)| = \sqrt{\sum_{j=1}^{3} e_{ij}^2(t)}$, $i \in \mathcal{I}[1, 10]$ with $\omega = 0.6$, without control input. (b) Trajectories of error states $|e_i(t)| = \sqrt{\sum_{j=1}^{3} e_{ij}^2(t)}$, $i \in \mathcal{I}[1, 10]$ with $\omega = 0.6$, under control input (2.4).

5. Conclusions

This paper investigates the stationary oscillation issue for error system between the complex dynamical network and its isolated node via impulsive control. It should be emphasized that the time delays associated with impulsive effects are taken into account in the paper. By leveraging inequality-based techniques, several novel sufficient conditions are deduced for ensuring the existence of stationary oscillations. In contrast to the existing approaches reported in the literature, our findings demonstrate that the delay system can exhibit stationary oscillations through appropriately designed impulsive control strategies involving time delays, even when such system is inherently divergent or unstable. Two examples accompanied by their corresponding simulations are presented for validating the feasibility of the proposed method. In forthcoming work, we intend to investigate the approach in this paper to studying the case of unmeasurable states. Moreover, it is widely recognized that event-triggered impulsive control is an effective control strategy, which avoids the inefficient use of resources. Also, data-driven control [35, 36] not only has theoretical value, but is also attractive for situations in which system identification can be difficult or time-consuming. We will also consider event-triggered delay impulsive control combined with data-driven control to extend our results to the case where the impulse delay is allowed to be larger than the impulsive interval, or is different from the coupling time-varying delay in the future work.

Author contributions

Guixia Sui: Writing–original draft, Validation, Supervision; Jingting Hu: Methodology, Writing–review & editing.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare there is no conflict of interest.

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