Mathematics

AIMS Mathematics, 11(5): 14172–14210.
DOI: 10.3934/math.2026582
Received: 28 January 2026
Revised: 07 April 2026
Accepted: 27 April 2026
Published: 19 May 2026

https://www.aimspress.com/journal/Math

Research article

Development of the CRITIC-MARCOS method under triangular (p,q)-fuzzy numbers and its application in modulation technique selection

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Abstract: Selecting the most suitable modulation technique in communication systems is challenging due to multiple alternatives and inherent uncertainty. Existing approaches often fail to simultaneously capture uncertainty and criteria interdependencies, and provide a reliable ranking, highlighting a clear research gap. This study proposes a novel criteria importance through intercriteria correlation (CRITIC) measurement of alternatives and ranking according to compromise solution (MARCOS) framework under triangular (p, q)-fuzzy numbers (T(p,q)-FNs), extending the (p,q)-rung orthopair fuzzy number, to address this problem. The methodology introduces new operational laws and a weighted average aggregation operator based on Dombi t-norms to effectively aggregate fuzzy information. The CRITIC method is used for objective criteria weighting, while MARCOS ranks alternatives to identify the optimal modulation technique. The framework is validated through a communication modulation selection case study and compared with existing methods. The results demonstrate that the proposed approach provides more accurate, realistic, and flexible decision-making, clearly highlighting its novelty and practical advantages.

Keywords: triangular (p,q)-fuzzy number; Dombi t-norms; modulation technique selection; CRITIC-MARCOS method; multi criteria decision making model
Mathematics Subject Classification: 03E72, 90B50, 90C70

1. Introduction

The process of delivering an analog baseband signal (information) [1] via a band-pass media by modifying the characteristics of a sinusoidal wave is known as modulation. Digital modulation, in which a digital bit stream from the transmitter is conveyed over an analog information channel (medium) to the receiver or receivers, is another method of modulation. Analog modulation involves sending an analogue signal over an analog medium. Modulation techniques are used in almost all communication via radio channels. Transmissions in the modern era are digital and have a limited amount of usable spectrum. Therefore, using modulation techniques is crucial to making the most of the bandwidth that is accessible for high service quality.

News, broadcasting, and other activities use telecommunications as a means of communication. Radio stations or persons can communicate with one another by using agreed-upon language and signals. In order to maximize signal quality and enhance tone, frequency, and efficiency, modulation and multichannel techniques are used to transmit signals from one station to another. The signal from the source is transformed by the sensor into an electrical signal, which is subsequently prepared for routing [2,3]. The transmitter generates electromagnetic waves within a specific bandwidth and wavelength range, thereby defining the transmission frequency. To improve clarity, modulation compresses the signal while it is propagating. Through the use of a transducer, a receiver transforms these waves into an electrical signal that is audible to humans [4]. Systems for communicating use modulation techniques to collect and evaluate signals from various sources and send them to receivers. To reduce disturbances like extinction, interference, and distortion in media signal transmission, modulation is crucial. Information is sent from a transmitter to a receiver via a medium in a network of communications. Sensors change the kinds of energy signals, while processors prepare them for effective transmission and reception.

The modulation technique selection problem can be seen as a multi criteria decision making (MCDM) problem that considers different criteria for different modulation techniques because of their peculiarities. Additionally, it is critical for specialists to develop a framework for choosing modulation techniques in an uncertain environment in order to enhance expert decision-making due to the abrupt and unexpected nature of decision-making situations.

1.1. Background of the study

Two popular techniques for choosing the optimal alternative are MCDM [5] and multiple attribute group decision-making (MAGDM). Numerous disciplines, including engineering technology and economic management, have thoroughly developed the theory and methodology [6, 7]. The fuzzy set (FS) idea was put forward by Zadeh [8] to handle ambiguous data. Subsequently, other studies on the FS concept's generalizations were carried out. It is thought to have several applications, both theoretical and practical, in many fields, such as computer science, engineering, social science, natural sciences, physical sciences, and health sciences. FS theory has been the subject of extensive research in [9,10]. As described by Atanassov [11], intuitionistic fuzzy sets (IFSs) are an intriguing and broadly applicable generalization of fuzzy sets. The decision-maker may, however, specify the membership function (MF) and non-membership function (NMF) of a particular characteristic in several ways that result in a sum greater than 1. To better handle uncertain situations, Yager [12] developed the idea of a Pythagorean fuzzy set (PyFS) as a generalization of IFSs. Compared with IFSs designs, PyFS designs are more accurate and useful for explaining unclear data. For instance (3, 2)-FS is an alternative form of generalized PyFS that was described by Ibrahim et al. [13]. Senapati et al. [14] not only investigated Fermatean fuzzy set (FFS) but also integrated basic operations into the FFS. Murad et al. [15] introduced the (3,4)-FS, which handles more confusing scenarios than the (3, 2)-FS. For handling informational uncertainty, a q-Rung orthopair fuzzy set (q-ROFS) [16] is among the most successful generalizations of FSs. (p,q)-ROFS is a novel FSs modification type introduced by Ibrahim and Alshammari [17].

Fuzzy numbers can result in a more flexible decision-making environment than FSs. Triangular fuzzy numbers (TFNs) are frequently used in many real applications [18,19] because they can describe the evaluation information more precisely in some applications [20,21]. A novel approach to addressing problems with decision-making was made possible by Wan et al.'s [22] MAGDM method, which is based on the triangular intuitionistic fuzzy number (TIFN). The Frank aggregation operator was developed by Fahmi et al. [23] using the TIFN. Triangular Pythagorean fuzzy numbers (TPFNs) were established by Sharma et al. [24] and applied to selecting the location of photovoltaic solar cells. Akram et al. proposed the Fermatean triangular fuzzy number in combination with fuzzy sets in [25]. Additionally, the triangular Fermatean fuzzy evaluation based on distance from average solution (EDAS) model was established by Khan et al. [26] and applied to remote patient monitoring. For the weighted average linear programming technique for multidimensional analysis of preference (LINMAP) group decision-making approach, Wan et al. [27] established q-rung orthopair triangular fuzzy numbers (q-ROTFNs).

1.2. Related work

1.2.1. Dombi operators for different MCDM issues

To develop comprehensive assessments for a range of scenarios, numerous new types of aggregating operators (AOs) have been employed. In the field of research, AOs are intriguing and important, attracting more and more attention. In system analysis, decision-analysis, mathematical simulation, and forecasting applications, the Dombi t-norm and t-conorm [28] are frequently employed to create smoothness in information fusion under ambiguity and uncertainty. The MCDM problem on IFS was resolved by Liu et al. [29] using the Dombi Bonferroni mean operator. Dombi aggregation operators for PFSs were developed by Akram et al. [30]. Numerous researchers have developed Dombi operators further [31–34]. The PyF soft Einstein aggregation operator was developed by Ali et al. [35]. Fractional fuzzy soft sets were developed by Ullah et al. [36] using the Hamacher aggregation method. Sugeno Weber aggregation operator for triangular Fermatean fuzzy numbers was developed by Khan et al. [26]. Recent advances in fuzzy MCDM methods demonstrate the growing interest in flexible aggregation operators and their applications across diverse decision-making contexts. For example, Dhumras et al [37] explored T-spherical fuzzy soft Dombi aggregation operators integrated with an EDAS-based MCDM framework to evaluate mitigation strategies for debris flow hazards in mountainous regions, highlighting the effectiveness of parameterized Dombi aggregation under high uncertainty conditions. Another relevant development applies fuzzy decision-making to renewable energy evaluation problems [38], where T-spherical fuzzy soft Dombi-based models were used to address the complex criteria involved in energy source selection and assessment.

1.2.2. The CRITIC technique

The CRITIC method is an objective weighting approach that determines the importance of criteria by combining contrast intensity and inter-criteria correlation. First, the decision matrix is normalized to ensure comparability. The contrast intensity of each criterion is measured using its standard deviation, where higher variability indicates greater discriminative power. Then correlation coefficients among the criteria are computed to assess redundancy; highly correlated criteria provide less unique information, while less correlated ones are more informative. Finally, the weights are calculated by integrating the standard deviation with the degree of conflict (1 − correlation), assigning higher importance to criteria with greater variability and lower correlation, thus ensuring a reliable and objective weighting process.

In decision-making processes, selecting the optimal alternative is crucial. Consequently, the MAGDM problem requires the development of a novel approach. Information from previous decades is combined using several MAGDM procedures [39–41], with CRITIC and MARCOS being two of the most advanced techniques in this field. All qualities are not taken to be equally important in MAGDM situations like the method in [42]. Previous research has described a variety of criterion weighting techniques. There are two main types of attribute weights: Objective and subjective. The CRITIC approach is a weighting model that uses the correlation coefficient (CC) and standard deviation (SD) to quantify the value of each standard in order to determine the weights of characteristics in the MAGDM process. The CRITIC approach for attribute weight analysis was proposed by Diakoulaki et al. [43]. Žižović et al. [44] adapted the CRITIC method. Wu et al. [45] improved the CRITIC technique with a practical application in the roadway infrastructure system. In a system with heavy traffic, Pan et al. [46] used the CRITIC method at a traditional intersection. One could argue that standards having higher levels of conflict with other standards and higher dynamic contrast intensity are given higher weights under the CRITIC method. The CRITIC approach has been implemented in many real-life situations as a result of this characteristic. According to previous research and investigation, the CRITIC method is often combined with other subjective or objective empowerment methods to evaluate the weight. For instance, Yalcin and Ünlü [47] established the CRITIC viekriterijumsko kompromisno rangiranje (VIKOR) approaches to evaluate the ranking of initial public offerings. The CRITIC combined compromise solution (CoCoSo) approaches were presented in a fuzzy decision-making (DM) context by Peng and Huang [48]. In the 5G sector, Peng et al. [49] developed the PyF CRITIC-CoCoSo techniques. The technique of order preference similarity to the ideal solution (TOPSIS) CRITIC approach to supply chain management was presented by Rostamzadeh et al. [50]. Dutta et al. [51] defined the TOPSIS for the performance evaluation of foreign players in Indian premier league.

1.2.3. The MARCOS methodology

The goal of multi criteria group decision-making (MCGDM) is to offer a way to choose the best option from a potential collection of possibilities by ranking them according to different criteria [52,53]. Due to the prevalence of MCGDM problems in everyday life, numerous fields have made extensive use of their theories and applications [54,55]. When making decisions with incomplete, ambiguous, or fuzzy information, MCGDM techniques based on fuzzy sets have been used [55]. Stevic et al. [56] developed the MARCOS approach based on the interaction between alternatives and reference points. It provides a method to define utility functions with their aggregate and calculates the utility degree (UD) from reference point solutions. This approach uses the compromise solution to measure and rank the possibilities. By maintaining the constancy of approach with increased precision and steadiness, it may be applied to vast collections of options and criteria. The MARCOS approach has been developed and implemented from a variety of angles because of its ease of use and adaptability [55–58].

1.3. Research gap

In the existing literature, a Bonferroni mean operator combined with the complex proportional assessment (COPRAS) method has been defined for T(p,q)-fuzzy environments [59], providing a foundation for fuzzy MCDM. However, Dombi-based aggregation operators and the CRITIC-MARCOS method have not yet been developed or applied within the T(p,q)-fuzzy framework. This represents a clear research gap, as current approaches lack the flexibility, parametric adaptability, and integration of objective weighting with compromise ranking that these methods can offer. To address this gap, the present study proposes a novel Dombi-based aggregation operator combined with the CRITIC-MARCOS methodology specifically for T(p,q)-FNs. The framework is then applied to the selection of modulation techniques in communication systems, offering robust, flexible, and reliable decision-making under uncertainty. This contribution bridges both theoretical and practical limitations in complex MCDM problems, extending the applicability of fuzzy MCDM frameworks.

1.4. Motivation

Consequently, the following is a summary of the research motivations for this study.

Selecting the optimal modulation technique under uncertainty is a challenging task due to multiple conflicting criteria and the inherent vagueness in expert judgments. Existing fuzzy frameworks such as intuitionistic, Pythagorean, and Fermatean FSs [11,12,14] are widely used in MCDM, but they are limited in capturing higher-order hesitancy and providing sufficient flexibility to represent nuanced expert opinions [60,61]. Triangular (p,q)-fuzzy numbers (T-(p,q)-FNs) offer a more precise and adaptable representation of uncertainty, allowing adjustable parameters that better reflect partial membership and hesitation. Despite these advantages, T-(p,q)-FNs have not yet been applied to MCDM problems in selecting modulation [62].

Effective aggregation of expert inputs is critical, and the Dombi [32] aggregation operator provides a flexible t-norm that unifies multiple common operations and adapts to different decision contexts. However, it has not been extended to the T-(p,q)-FN framework. Additionally, ranking alternatives with interacting criteria requires both objective weighting and robust ranking, which MARCOS alone cannot fully provide. By integrating the CRITIC method with MARCOS under the T-(p,q)-FN framework, this study develops a comprehensive, novel, and systematic approach that accurately captures uncertainty, considers the criteria's interdependencies, and delivers reliable, realistic outcomes for selecting the optimal modulation technique.

1.5. Contributions

In order to improve the rationality of modulation technique selection outcomes, this research proposes a synthetic decision model-based selection framework (see Figure 1). As a result, this work helps to clarify the modulation technique selection issues in the context of uncertainty and complexity. The key contributions are enumerated below.

i. In the context of T(p,q)-FNs, we provide a novel MCDM method. In order to overcome the constraints of (p,q)-ROFS and to give decision-makers a more flexible way to represent their cognitive knowledge, T(p,q)-FNs have been utilized to assist decision-makers in expressing their information more fully. Additionally, the score functions of the T(p,q)-FNs and the related fundamental operational laws based on Dombi t-norms are defined.
ii. Due to the large degree of operational parameter diversity, the Dombi aggregation operator offers greater flexibility in information aggregation and decision-making. To better aggregate the data supplied by the decision-makers, the Dombi aggregation operator is expanded in the T(p,q)-FN environment.
iii. The CRITIC and MARCOS approaches were integrated using T(p,q)-FNs to provide a synthetic decision approach to choosing the best modulation strategy in complex and uncertain situations. In the context of inter-correlation between these factors, this approach may offer a more precise and useful way to handle the problem of selecting a modulation technique. Furthermore, the modulation technique selection problem has never been solved using the T(p,q)-FN-based CRITIC-MARCOS framework before.
iv. To demonstrate the robustness of the suggested decision framework, a sensitivity analysis was carried out using a range of parameter values. In order to evaluate the viability and superiority of the suggested T(p,q)-FNs decision framework, a comparison research of the current MCDM approaches based on modulation method selection frameworks was carried out concurrently.

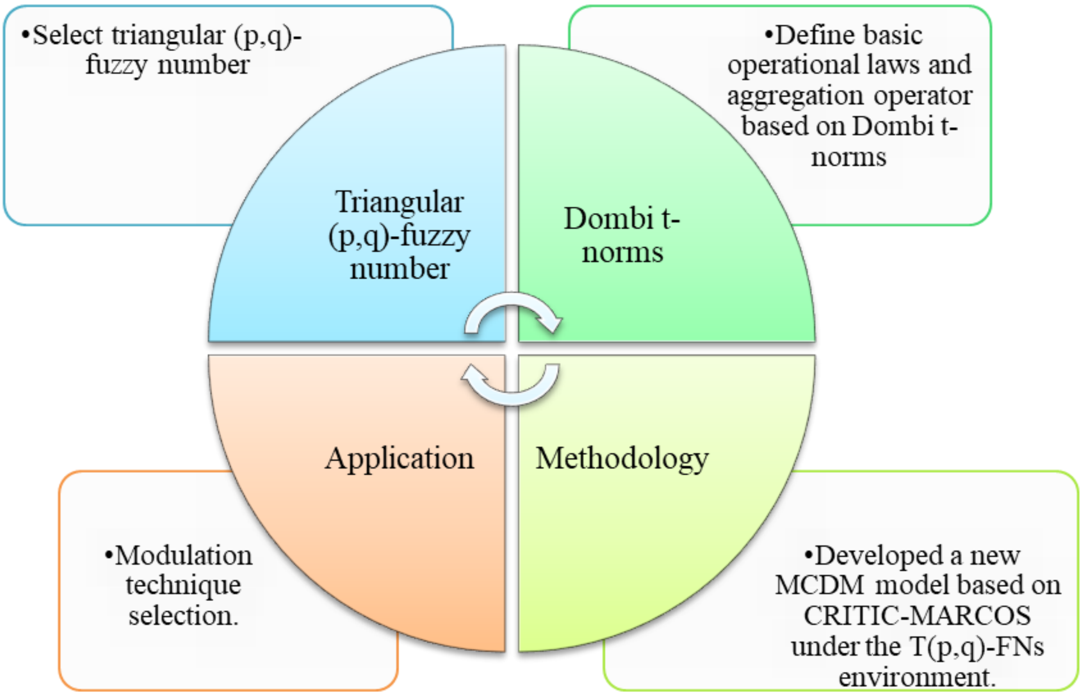

Figure 1. Contribution of the proposed work.

1.6. Novelties of this study

The key novelties of this research are as follows:

i. Formulation of new foundational operation laws and a Dombi-based aggregation operator, offering parametric flexibility and generalization of classical fuzzy operators.
ii. Integration of the CRITIC method for objective and data-driven weighting of criteria.
iii. Construction of an advanced CRITIC-MARCOS MCDM methodology tailored to the T(p,q)-FN environment.
iv. Practical validation of the proposed framework through the selection of optimal modulation techniques in communication systems, demonstrating robustness, stability, and superior decision-making performance compared with existing approaches.

This paper is structured as follows. The fundamental ideas of T(p,q)-FN are introduced in Section 2. The T(p,q)-FN concept and its fundamental scoring function are introduced in Section 3. Basic operational laws and an aggregation operator based on Dombi t-norms are presented in Section 4. The CRITIC-MARCOS approach for the T(p,q)-FN context is described in detail in Section 5. A real-world example of choosing the best modulation method is discussed in Section 6. In Section 7, the numerical results and the suggested model are presented through a comparison with current techniques. The study is finally concluded in Section 8. Figure 2 provides the paper's structure of. To aid readers in understanding the terms used in this article, Table 1 offers comprehensive explanations of the acronyms. The table provides the reader with a quick reference for improved reading and understanding, and these acronyms are crucial for simplifying the technical terminology.

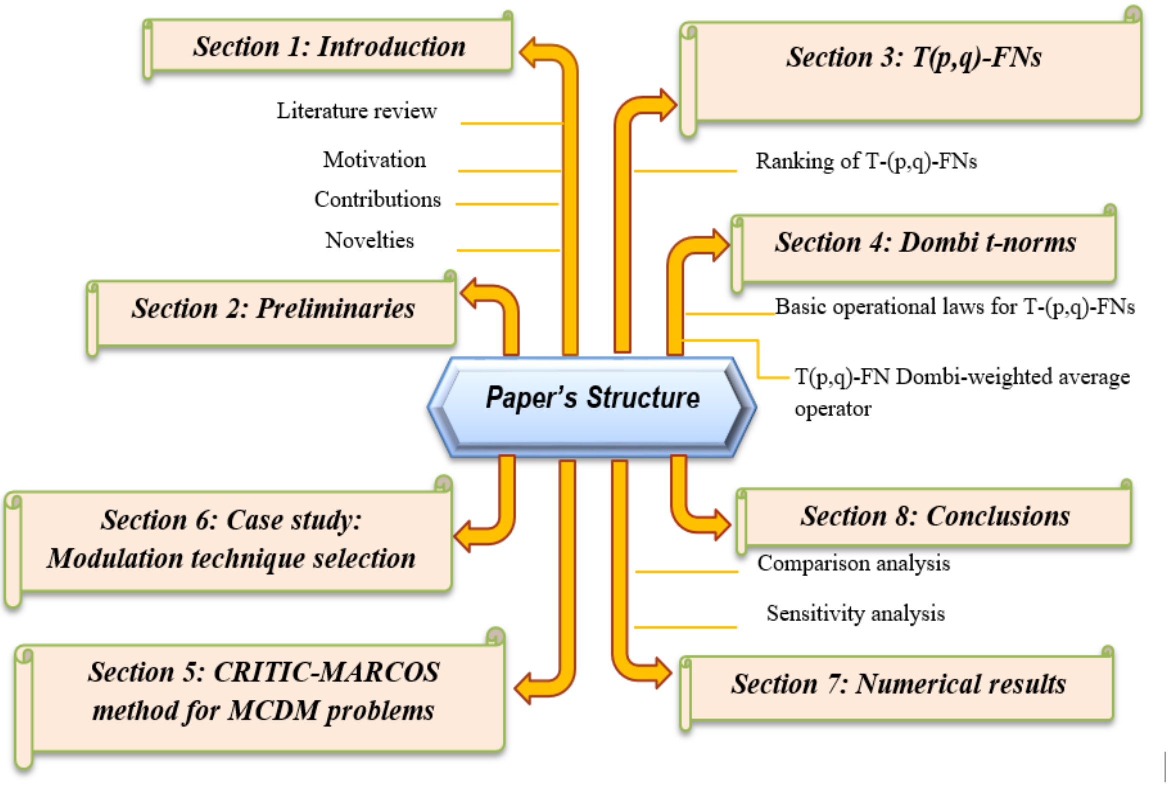

Figure 2. The outline of this paper.

Table 1. Description of the acronyms.

Acronyms	Description
MCDM	Multi-criteria decision-making
T(p,q)-FN	Triangular (p,q)-fuzzy number
MARCOS	Measuring alternatives and ranking according to the compromise solution
CRITIC	Criteria importance through intercriteria correlation
IFS	Intuitionistic fuzzy sets
PyFS	Pythagorean fuzzy set
q-ROFS	q-Rung orthopair fuzzy set
TFNs	Triangular fuzzy numbers
MF	Membership function
NMF	Non-membership function
AO	Aggregation operator
T(p,q)-FNDWA	Triangular (p,q)-fuzzy number Dombi weighted average
BPSK	Binary phase shift keying
QPSK	Quadrature phase shift keying
QAM	Quadrature amplitude modulation
OFDM	Orthogonal frequency-division multiplexing
FSK	Frequency shift keying

Table 2 presents all the symbols and notations used in the article, providing clarity and improving the readability of the mathematical expressions and discussions.

Table 2. Description of the symbols.

Symbols	Description
L	Universal set
$\breve{\mathfrak{D}}$	Intuitionistic fuzzy set
$\widehat{\mathfrak{U}}$	Pythagorean fuzzy set
$\tilde{\widehat{Q}}$	q-Rung orthopair fuzzy set
$\overline{\overline{\mathfrak{Y}}}$	(p,q)- Rung orthopair fuzzy set
$\breve{\Psi}$	Triangular (p,q)-fuzzy number
$\mathcal{Z}$	Membership function
$\mathcal{W}$	Non-membership
$\hat{\alpha}$	Lower bound of triangular fuzzy numbers
$\hat{\beta}$	Mode of triangular fuzzy numbers
$\hat{\gamma}$	Upper bound of triangular fuzzy numbers
$\mathbb{S}(\breve{\Psi})$	Score function of triangular (p,q)-fuzzy numbers
$\mathbb{A}(\breve{\Psi})$	Accuracy function of triangular (p,q)-fuzzy numbers
$£^N(Ʉ, Ƿ)$	Dombi t-norm
$£^C(Ʉ, Ƿ)$	Dombi t-conorm
Θ_i	Criteria weights
$\hbar_j$	Criteria
$Ŧ_i$	Alternatives
$\widetilde{ꞕ}_{ij}$	Normalized score matrix
Φ_j	Standard deviation
$Ꝑ_{jt}$	Correlation
ζ^+	Ideal
ζ^-	Anti-ideal
$\Gamma(\bar{Й})$	Utility function

2. Preliminaries

Definition 1. [11] The IFS $\breve{\mathfrak{D}}$ is defined by Eq (1) if L is a universal set

$$\breve{\mathfrak{D}} = \{\langle l, \mathcal{Z}_{\breve{\mathfrak{D}}}(l), \mathcal{W}_{\breve{\mathfrak{D}}}(l)\rangle \mid l \in L\}. \tag{1}$$

In this context, $\mathcal{Z}_{\breve{\mathfrak{D}}}(l)$ and $\mathcal{W}_{\breve{\mathfrak{D}}}(l)$ denote the MF and NMF of $l \in L$, respectively, where $\mathcal{Z}_{\breve{\mathfrak{D}}}(l), \mathcal{W}_{\breve{\mathfrak{D}}}(l) \in [0, 1]$, and the condition $0 \leq \mathcal{Z}_{\breve{\mathfrak{D}}}(l) + \mathcal{W}_{\breve{\mathfrak{D}}}(l) \leq 1$ holds. For $l \in L$, the degree of hesitancy is determined by $\bar{\pi}_{\breve{\mathfrak{D}}} = 1 - \mathcal{Z}_{\breve{\mathfrak{D}}}(l) - \mathcal{W}_{\breve{\mathfrak{D}}}(l)$.

Definition 2. [12] The PFS $\widehat{\mathfrak{U}}$ is defined by Eq (2) if L is a universal set

$$\widehat{\mathfrak{U}} = \{\langle l, \mathcal{Z}_{\widehat{\mathfrak{U}}}(l), \mathcal{W}_{\widehat{\mathfrak{U}}}(l)\rangle \mid l \in L\}. \tag{2}$$

In this context, $\mathcal{Z}_{\widehat{\mathfrak{U}}}(l)$ and $\mathcal{W}_{\widehat{\mathfrak{U}}}(l)$ denote the MF and NMF of $l \in L$, respectively, where $\mathcal{Z}_{\widehat{\mathfrak{U}}}(l), \mathcal{W}_{\widehat{\mathfrak{U}}}(l) \in [0,1]$, and the condition $0 \leq \left(\mathcal{Z}_{\widehat{\mathfrak{U}}}(l)\right)^2 + \left(\mathcal{W}_{\widehat{\mathfrak{U}}}(l)\right)^2 \leq 1$ holds. For $l \in L$, the degree of hesitancy is determined by $\bar{\pi}_{\widehat{\mathfrak{U}}} = \sqrt{1 - \left(\mathcal{Z}_{\widehat{\mathfrak{U}}}(l)\right)^2 - \left(\mathcal{W}_{\widehat{\mathfrak{U}}}(l)\right)^2}$.

Definition 3. [16] The q-ROFS $\widetilde{\widehat{\mathcal{Q}}}$ is defined by Eq (3) if L is a universal set

$$\widetilde{\widehat{\mathcal{Q}}} = \left\{\langle l, \mathcal{Z}_{\widehat{\mathcal{Q}}}(l), \mathcal{W}_{\widehat{\mathcal{Q}}}(l)\rangle \mid l \in L\right\}. \tag{3}$$

In this context, $\mathcal{Z}_{\widehat{\mathcal{Q}}}(l)$ and $\mathcal{W}_{\widehat{\mathcal{Q}}}(l)$ denote the MF and NMF of $l \in L$, respectively, where $\mathcal{Z}_{\widehat{\mathcal{Q}}}(l), \mathcal{W}_{\widehat{\mathcal{Q}}}(l) \in [0,1]$, and the condition $0 \leq \left(\mathcal{Z}_{\widehat{\mathcal{Q}}}(l)\right)^q + \left(\mathcal{W}_{\widehat{\mathcal{Q}}}(l)\right)^q \leq 1$ holds. For $l \in L$, the degree of hesitancy is determined by $\bar{\pi}_{\widehat{\mathcal{Q}}} = \sqrt[q]{1 - \left(\mathcal{Z}_{\widehat{\mathcal{Q}}}(l)\right)^q - \left(\mathcal{W}_{\widehat{\mathcal{Q}}}(l)\right)^q}$.

Definition 4. [17] The (p,q)-ROFS $\overline{\overline{\mathfrak{Y}}}$ is defined by Eq (4) if L is a universal set

$$\overline{\overline{\mathfrak{Y}}} = \left\{\langle l, \mathcal{Z}_{\overline{\overline{\mathfrak{Y}}}}(l), \mathcal{W}_{\overline{\overline{\mathfrak{Y}}}}(l)\rangle \mid l \in L\right\}. \tag{4}$$

In this context, $\mathcal{Z}_{\overline{\overline{\mathfrak{Y}}}}(l)$ and $\mathcal{W}_{\overline{\overline{\mathfrak{Y}}}}(l)$ denote the MF and NMF of $l \in L$, respectively, where $\mathcal{Z}_{\overline{\overline{\mathfrak{Y}}}}(l), \mathcal{W}_{\overline{\overline{\mathfrak{Y}}}}(l) \in [0,1]$, and the condition $0 \leq \left(\mathcal{Z}_{\overline{\overline{\mathfrak{Y}}}}(l)\right)^p + \left(\mathcal{W}_{\overline{\overline{\mathfrak{Y}}}}(l)\right)^q \leq 1$ holds. For $l \in L$, the degree of hesitancy is determined by $\bar{\pi}_{\overline{\overline{\mathfrak{Y}}}} = \sqrt[pq]{1 - \left(\mathcal{Z}_{\overline{\overline{\mathfrak{Y}}}}(l)\right)^p - \left(\mathcal{W}_{\overline{\overline{\mathfrak{Y}}}}(l)\right)^q}$.

Figure 3 presents the space comparison of different FSs.

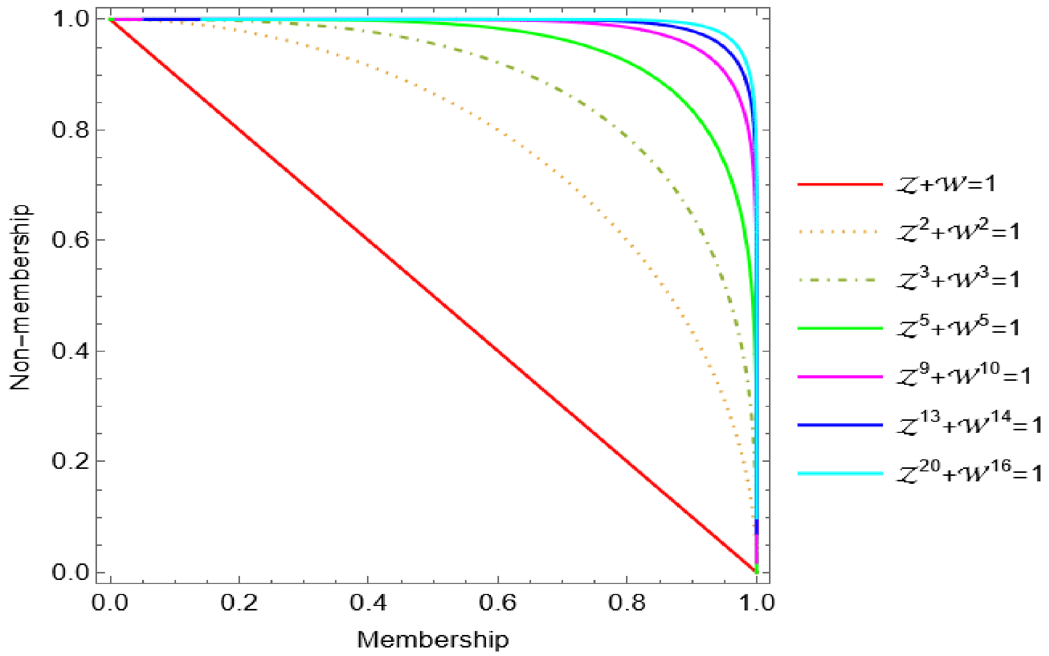

Figure 3. Space comparison of different fuzzy sets.

3. Triangular (p,q)-fuzzy numbers

A T(p,q)-FN [59] extends the classical TFN with the exponents p and q controlling the left and right spreads. It defines membership, non membership, and hesitation, enabling flexible modeling of asymmetric and higher-order uncertainty for complex decision-making.

Definition 5. [59] For a T(p,q)-FN, let $\breve{\Psi} = \left((\hat{\alpha}, \hat{\beta}, \hat{\gamma}); \mathcal{Z}_{\breve{\Psi}}(l), \mathcal{W}_{\breve{\Psi}}(l)\right)$. Equations (5) and (6) provide its membership function and non membership function;

$$\mathcal{Z}_{\breve{\Psi}}(l) = \begin{cases} \frac{(l-\hat{\alpha})\mathcal{Z}_{\breve{\Psi}}}{(\hat{\beta}-\hat{\alpha})}, & if\ \hat{\alpha} \le l < \hat{\beta}, \\ \mathcal{Z}_{\breve{\Psi}}, & if\ l = \hat{\beta}, \\ \frac{(\hat{\gamma}-l)\mathcal{Z}_{\breve{\Psi}}}{(\hat{\gamma}-\hat{\beta})}, & if\ \hat{\beta} < l \le \hat{\gamma}, \\ 0, & if\ l < \hat{\alpha}\ or\ l > \hat{\gamma}, \end{cases} \tag{5}$$

$$\mathcal{W}_{\breve{\Psi}}(l) = \begin{cases} \frac{(\hat{\beta}-l)+\mathcal{W}_{\breve{\Psi}}(l-\hat{\alpha})}{(\hat{\beta}-\hat{\alpha})}, & if\ \hat{\alpha} \le l < \hat{\beta}, \\ \mathcal{W}_{\breve{\Psi}}, & if\ l = \hat{\beta}, \\ \frac{(l-\hat{\beta})+\mathcal{W}_{\breve{\Psi}}(\hat{\gamma}-l)}{(\hat{\gamma}-\hat{\beta})}, & if\ \hat{\beta} < l \le \hat{\gamma}, \\ 1, & if\ l < \hat{\alpha}\ or\ l > \hat{\gamma}. \end{cases} \tag{6}$$

In this case, the membership is represented by $\mathcal{Z}_{\breve{\Psi}}(l)$ and the non-membership by $\mathcal{W}_{\breve{\Psi}}(l)$. $\mathcal{Z}_{\breve{\Psi}}(l), \mathcal{W}_{\breve{\Psi}}(l) \in [0,1]$, $0 \le \left(\mathcal{Z}_{\breve{\Psi}}(l)\right)^p + \left(\mathcal{W}_{\breve{\Psi}}(l)\right)^q \le 1$, and $\hat{\alpha} < \hat{\beta} < \hat{\gamma}$ in Eqs (5) and (6). For $l \in$

L, the degree of hesitancy is determined by $\bar{\pi}_{\breve{\Psi}} = \sqrt[pq]{1-\left(\mathcal{Z}_{\breve{\Psi}}(l)\right)^p-\left(\mathcal{W}_{\breve{\Psi}}(l)\right)^q}$. Figure 4 shows the geometrical representation of the T(p,q)-FNs.

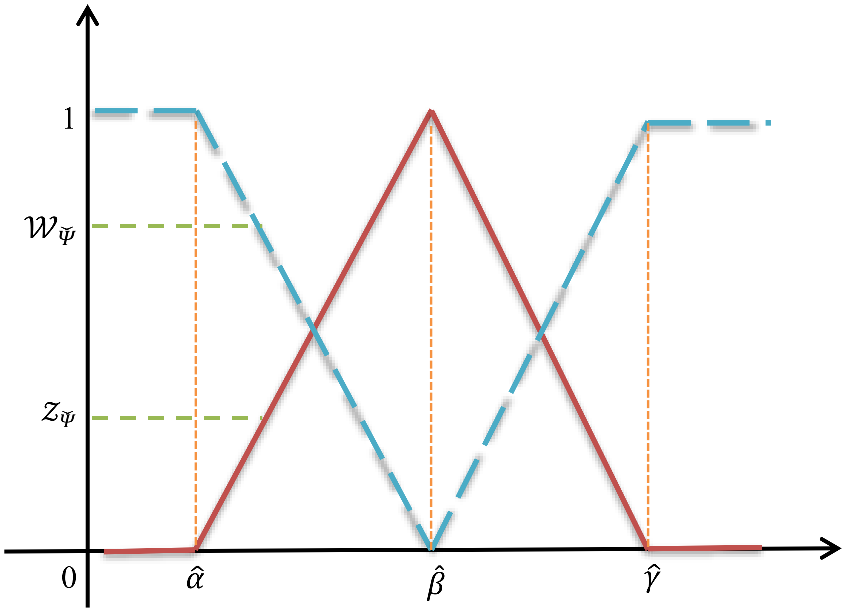

Figure 4. Geometrical representation of T(p,q)-FNs.

Example 1. Let $\breve{\mathcal{T}} = \left((2,4,6); \mathcal{Z}_{\breve{\Psi}}, \mathcal{W}_{\breve{\Psi}}\right)$ be a T(p,q)-FN. We can obtain the MF and NMF are as follows:

$$\mathcal{Z}_{\breve{\Psi}}(l) = \begin{cases} \dfrac{(l-2)\mathcal{Z}_{\breve{\Psi}}}{(\hat{\beta}-\hat{\alpha})}, & if\ 2 \le l < 4, \\ \mathcal{Z}_{\breve{\Psi}}, & if\ l = 4, \\ \dfrac{(6-l)\mathcal{Z}_{\breve{\Psi}}}{(6-4)}, & if\ 4 < l \le 6, \\ 0, & if\ l < 2\ or\ l > 6, \end{cases}$$

$$\mathcal{W}_{\breve{\Psi}}(l) = \begin{cases} \dfrac{(4-l)+\mathcal{W}_{\breve{\Psi}}(l-2)}{(4-2)}, & if\ 2 \le l < 4, \\ \mathcal{W}_{\breve{\Psi}}, & if\ l = 4, \\ \dfrac{(l-4)+\mathcal{W}_{\breve{\Psi}}(6-l)}{(6-4)}, & if\ 4 < l \le 6, \\ 1, & if\ l < 2\ or\ l > 6. \end{cases}$$

Where $\mathcal{Z}_{\breve{\Psi}}, \mathcal{W}_{\breve{\Psi}} \in [0,1]$, the MF and NMF of T(p,q)-FNs in Example 1 are shown in Figure 5.

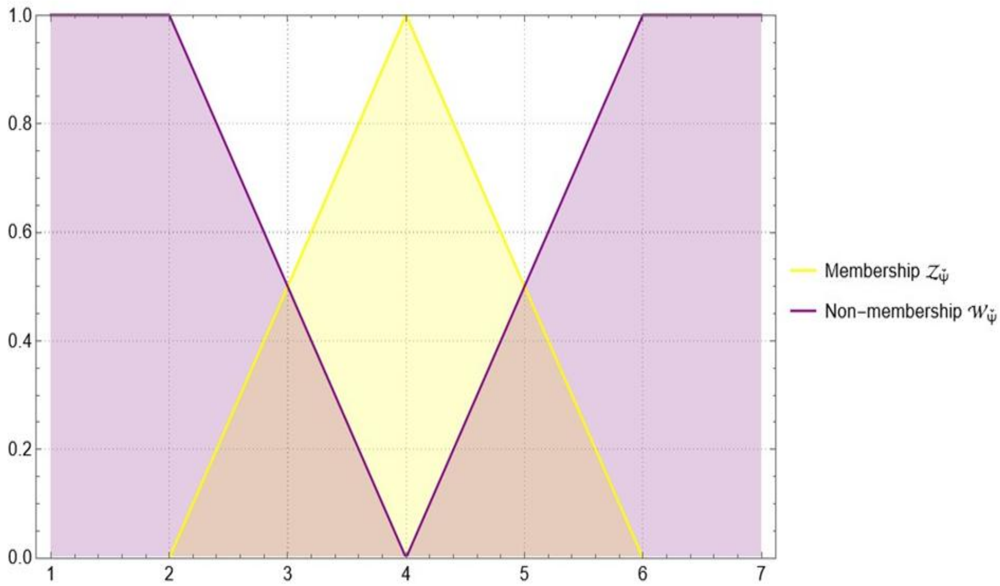

Figure 5. Graphically representation of Example 1.

3.1. Ranking of T(p,q)-FNs

Definition 6. Consider the T(p,q)-FNs denoted as $\breve{\Psi} = \left((\hat{\alpha}, \hat{\beta}, \hat{\gamma}); \mathcal{Z}_{\breve{\Psi}}, \mathcal{W}_{\breve{\Psi}}\right)$. The following is the definition of their score function:

$$\mathbb{S}(\breve{\Psi}) = \frac{\hat{\alpha}+2\hat{\beta}+\hat{\gamma}}{4}\left(\mathcal{Z}_{\breve{\Psi}}^{p} - \mathcal{W}_{\breve{\Psi}}^{q}\right). \tag{7}$$

The following is a definition of the accuracy function:

$$\mathbb{A}(\breve{\Psi}) = \frac{\hat{\alpha}+2\hat{\beta}+\hat{\gamma}}{4}\left(\mathcal{Z}_{\breve{\Psi}}^{p} + \mathcal{W}_{\breve{\Psi}}^{q}\right). \tag{8}$$

Definition 7. Let $\breve{\Psi}_1 = \left((\hat{\alpha}_1, \hat{\beta}_1, \hat{\gamma}_1); \mathcal{Z}_{\breve{\Psi}_1}, \mathcal{W}_{\breve{\Psi}_1}\right)$ and $\breve{\Psi}_2 = \left((\hat{\alpha}_2, \hat{\beta}_2, \hat{\gamma}_2); \mathcal{Z}_{\breve{\Psi}_2}, \mathcal{W}_{\breve{\Psi}_2}\right)$ be two T(p,q)-FNs. The ranking rule is defined as follows:

1) If $\mathbb{S}(\breve{\Psi}_1) < \mathbb{S}(\breve{\Psi}_2) \Rightarrow \breve{\Psi}_1 < \breve{\Psi}_2$;
2) If $\mathbb{S}(\breve{\Psi}_1) > \mathbb{S}(\breve{\Psi}_2) \Rightarrow \breve{\Psi}_1 > \breve{\Psi}_2$;
3) If $\mathbb{S}(\breve{\Psi}_1) = \mathbb{S}(\breve{\Psi}_2)$ then
   i. $\mathbb{A}(\breve{\Psi}_1) < \mathbb{A}(\breve{\Psi}_2) \Rightarrow \breve{\Psi}_1 < \breve{\Psi}_2$;
   ii. $\mathbb{A}(\breve{\Psi}_1) > \mathbb{A}(\breve{\Psi}_2) \Rightarrow \breve{\Psi}_1 > \breve{\Psi}_2$;
   iii. $\mathbb{A}(\breve{\Psi}_1) = \mathbb{A}(\breve{\Psi}_2) \Rightarrow \breve{\Psi}_1 \sim \breve{\Psi}_2$.

4. Dombi t-norms and t-conorms

Definition 8. [28] Let Ʉ and Ƥ be two real numbers. The following is the definition of the Dombi t-norm and t-conorm on Ʉ and Ƥ:

$$Ⱡ^{N}(Ʉ, Ƥ) = \frac{1}{1+\left\{\left(\frac{1-Ʉ}{Ʉ}\right)^{Ⱡ}+\left(\frac{1-Ƥ}{Ƥ}\right)^{Ⱡ}\right\}^{\frac{1}{Ⱡ}}}, \tag{9}$$

$$Ⱡ^{C}(Ʉ, Ƥ) = 1-\frac{1}{1+\left\{\left(\frac{Ʉ}{1-Ʉ}\right)^{Ⱡ}+\left(\frac{Ƥ}{1-Ƥ}\right)^{Ⱡ}\right\}^{\frac{1}{Ⱡ}}}, \tag{10}$$

where $Ⱡ \geq 1$ and $(Ʉ, Ƥ) \in [0,1] \times [0,1]$.

4.1. Triangular (p,q)-fuzzy numbers under Dombi operation

Basic operational laws and AO, and the T(p,q)-FN Dombi weighted average (T(p,q)-FNDWA) operator, are the main topics of this section.

4.1.1. Operational laws for T(p,q)-FNs based on Dombi operator

Definition 9. Let $\breve{\Psi}_1 = \left((\hat{\alpha}_1, \hat{\beta}_1, \hat{\gamma}_1); \mathcal{Z}_{\breve{\Psi}_1}, \mathcal{W}_{\breve{\Psi}_1}\right)$ and $\breve{\Psi}_2 = \left((\hat{\alpha}_2, \hat{\beta}_2, \hat{\gamma}_2); \mathcal{Z}_{\breve{\Psi}_2}, \mathcal{W}_{\breve{\Psi}_2}\right)$ are any two T(p,q)-FNs, where $\nu \in [0,\ 1]$ and $Ⱡ \geq 1$, be a real number. The following definition of operations will be used:

1) $$\breve{\Psi}_1 \oplus \breve{\Psi}_2 = \left(\begin{array}{c}(\hat{\alpha}_1 + \hat{\alpha}_2,\ \hat{\beta}_1 + \hat{\beta}_2,\ \hat{\gamma}_1 + \hat{\gamma}_2); \\ \left(\sqrt[p]{1-\frac{1}{1+\left\{\left(\frac{z^{p}_{\breve{\Psi}_1}}{1-z^{p}_{\breve{\Psi}_1}}\right)^{Ⱡ}+\left(\frac{z^{p}_{\breve{\Psi}_2}}{1-z^{p}_{\breve{\Psi}_2}}\right)^{Ⱡ}\right\}^{\frac{1}{Ⱡ}}}}, \sqrt[q]{\frac{1}{1+\left\{\left(\frac{1-w^{q}_{\breve{\Psi}_1}}{w^{q}_{\breve{\Psi}_1}}\right)^{Ⱡ}+\left(\frac{1-w^{q}_{\breve{\Psi}_2}}{w^{q}_{\breve{\Psi}_2}}\right)^{Ⱡ}\right\}^{\frac{1}{Ⱡ}}}}\right)\end{array}\right);$$

2) $$\breve{\Psi}_1 \otimes \breve{\Psi}_2 = \left(\begin{array}{c}(\hat{\alpha}_1 . \hat{\alpha}_2,\ \hat{\beta}_1 . \hat{\beta}_2,\ \hat{\gamma}_1 . \hat{\gamma}_2); \\ \left(\sqrt[p]{\frac{1}{1+\left\{\left(\frac{1-z^{p}_{\breve{\Psi}_1}}{z^{p}_{\breve{\Psi}_1}}\right)^{Ⱡ}+\left(\frac{1-z^{p}_{\breve{\Psi}_2}}{z^{p}_{\breve{\Psi}_2}}\right)^{Ⱡ}\right\}^{\frac{1}{Ⱡ}}}}, \sqrt[q]{1-\frac{1}{1+\left\{\left(\frac{w^{q}_{\breve{\Psi}_1}}{1-w^{q}_{\breve{\Psi}_1}}\right)^{Ⱡ}+\left(\frac{w^{q}_{\breve{\Psi}_2}}{1-w^{q}_{\breve{\Psi}_2}}\right)^{Ⱡ}\right\}^{\frac{1}{Ⱡ}}}}\right)\end{array}\right);$$

$$3)\ \ \nu \odot \breve{\Psi}_1 = \left(\begin{array}{c} \left(\nu.\hat{\alpha}_1,\ \nu.\hat{\beta}_1,\ \nu.\hat{\gamma}_1\right); \\ \left(\sqrt[p]{1-\frac{1}{1+\left\{\nu\left(\frac{\mathcal{Z}^p_{\breve{\Psi}_1}}{1-\mathcal{Z}^p_{\breve{\Psi}_1}}\right)^{\text{Ⱡ}}\right\}^{\frac{1}{\text{Ⱡ}}}}}, \sqrt[q]{\frac{1}{1+\left\{\nu\left(\frac{1-\mathcal{W}^q_{\breve{\Psi}_1}}{\mathcal{W}^q_{\breve{\Psi}_1}}\right)^{\text{Ⱡ}}\right\}^{\frac{1}{\text{Ⱡ}}}}} \right) \end{array}\right);$$

$$4)\ \ \left(\breve{\Psi}_1\right)^{\nu} = \left(\begin{array}{c} \left(\hat{\alpha}_1^{\nu},\ \hat{\beta}_1^{\nu},\ \hat{\gamma}_1^{\nu}\right); \\ \left(\sqrt[p]{\frac{1}{1+\left\{\nu\left(\frac{1-\mathcal{Z}^p_{\breve{\Psi}_1}}{\mathcal{Z}^p_{\breve{\Psi}_1}}\right)^{\text{Ⱡ}}\right\}^{\frac{1}{\text{Ⱡ}}}}}, \sqrt[q]{1-\frac{1}{1+\left\{\nu\left(\frac{\mathcal{W}^q_{\breve{\Psi}_1}}{1-\mathcal{W}^q_{\breve{\Psi}_1}}\right)^{\text{Ⱡ}}\right\}^{\frac{1}{\text{Ⱡ}}}}} \right) \end{array}\right).$$

4.1.2. T(p,q)-FNDWA aggregation operator

Definition 10. Let $\breve{\Psi}_i = \left(\left(\hat{\alpha}_i, \hat{\beta}_i, \hat{\gamma}_i\right); \mathcal{Z}_{\breve{\Psi}_i}, \mathcal{W}_{\breve{\Psi}_i}\right) (i = 1,2,\dots,n)$ be an assembely of T(p,q)-FNs. Then, the T(p,q)-FNDWA operator is a function $T(p,q) - FNDWA: \breve{\Psi}^n \to \breve{\Psi}$, such that:

$$T(p,q) - FNDWA\left(\breve{\Psi}_1, \breve{\Psi}_2, \dots, \breve{\Psi}_n\right) = \Theta_1\breve{\Psi}_1 \oplus \Theta_1\breve{\Psi}_2 \oplus \dots \oplus \Theta_n\breve{\Psi}_n, \tag{11}$$

where $\Theta_i > 0, \sum_{i=1}^{n}\Theta_i = 1$ represent the weights of the criteria.

Theorem 1. If $\breve{\Psi}_i = \left(\left(\hat{\alpha}_i, \hat{\beta}_i, \hat{\gamma}_i\right); \mathcal{Z}_{\breve{\Psi}_i}, \mathcal{W}_{\breve{\Psi}_i}\right) (i = 1,2,\dots,n)$ be an assembely of T(p,q)-FNs. Then, aggregated values are also T(p,q)-FNs and

$$T(p,q) - FNDWA\left(\breve{\Psi}_1, \breve{\Psi}_2, \dots, \breve{\Psi}_n\right)$$

$$= \Theta_1\breve{\Psi}_1 \oplus \Theta_1\breve{\Psi}_2 \oplus \dots \oplus \Theta_n\breve{\Psi}_n$$

$$= \left(\begin{array}{c} \left(\sum_{i=1}^{n}\Theta_i\hat{\alpha}_i, \sum_{i=1}^{n}\Theta_i\hat{\beta}_i, \sum_{i=1}^{n}\Theta_i\hat{\gamma}_i\right); \\ \left(\sqrt[p]{1-\frac{1}{1+\left\{\sum_{i=1}^{n}\Theta_i\left(\frac{\mathcal{Z}^p_{\breve{\Psi}_i}}{1-\mathcal{Z}^p_{\breve{\Psi}_i}}\right)^{\text{Ⱡ}}\right\}^{\frac{1}{\text{Ⱡ}}}}}, \sqrt[q]{\frac{1}{1+\left\{\sum_{i=1}^{n}\Theta_i\left(\frac{1-\mathcal{W}^q_{\breve{\Psi}_i}}{\mathcal{W}^q_{\breve{\Psi}_i}}\right)^{\text{Ⱡ}}\right\}^{\frac{1}{\text{Ⱡ}}}}} \right) \end{array}\right). \tag{12}$$

Proof. We apply mathematical induction to prove it.

For $n = 2$,

$$T(p,q)-FNDWA\left(\breve{\Psi}_1,\breve{\Psi}_2\right)=\Theta_1\breve{\Psi}_1\oplus\Theta_1\breve{\Psi}_2,$$

$$\Theta_1\breve{\Psi}_1=\begin{pmatrix}\left(\Theta_1.\hat{\alpha}_1,\Theta_1.\hat{\beta}_1,\Theta_1.\hat{\gamma}_1\right);\\ \left(\sqrt[p]{1-\frac{1}{1+\left\{\Theta_1\left(\frac{z^p_{\breve{\Psi}_1}}{1-z^p_{\breve{\Psi}_1}}\right)^{\text{Ł}}\right\}^{\frac{1}{\text{Ł}}}}},\sqrt[q]{\frac{1}{1+\left\{\Theta_1\left(\frac{1-w^q_{\breve{\Psi}_1}}{w^q_{\breve{\Psi}_1}}\right)^{\text{Ł}}\right\}^{\frac{1}{\text{Ł}}}}}\right)\end{pmatrix},$$

$$\Theta_2\breve{\Psi}_2=\begin{pmatrix}\left(\Theta_2.\hat{\alpha}_2,\Theta_2.\hat{\beta}_2,\Theta_2.\hat{\gamma}_2\right);\\ \left(\sqrt[p]{1-\frac{1}{1+\left\{\Theta_2\left(\frac{z^p_{\breve{\Psi}_2}}{1-z^p_{\breve{\Psi}_2}}\right)^{\text{Ł}}\right\}^{\frac{1}{\text{Ł}}}}},\sqrt[q]{\frac{1}{1+\left\{\Theta_2\left(\frac{1-w^q_{\breve{\Psi}_2}}{w^q_{\breve{\Psi}_2}}\right)^{\text{Ł}}\right\}^{\frac{1}{\text{Ł}}}}}\right)\end{pmatrix}$$

$$=\begin{pmatrix}\left(\Theta_1.\hat{\alpha}_1,\Theta_1.\hat{\beta}_1,\Theta_1.\hat{\gamma}_1\right);\\ \left(\sqrt[p]{1-\frac{1}{1+\left\{\Theta_1\left(\frac{z^p_{\breve{\Psi}_1}}{1-z^p_{\breve{\Psi}_1}}\right)^{\text{Ł}}\right\}^{\frac{1}{\text{Ł}}}}},\sqrt[q]{\frac{1}{1+\left\{\Theta_1\left(\frac{1-w^q_{\breve{\Psi}_1}}{w^q_{\breve{\Psi}_1}}\right)^{\text{Ł}}\right\}^{\frac{1}{\text{Ł}}}}}\right)\end{pmatrix},$$

$$\oplus\begin{pmatrix}\left(\Theta_2.\hat{\alpha}_2,\Theta_2.\hat{\beta}_2,\Theta_2.\hat{\gamma}_2\right);\\ \left(\sqrt[p]{1-\frac{1}{1+\left\{\Theta_2\left(\frac{z^p_{\breve{\Psi}_2}}{1-z^p_{\breve{\Psi}_2}}\right)^{\text{Ł}}\right\}^{\frac{1}{\text{Ł}}}}},\sqrt[q]{\frac{1}{1+\left\{\Theta_2\left(\frac{1-w^q_{\breve{\Psi}_2}}{w^q_{\breve{\Psi}_2}}\right)^{\text{Ł}}\right\}^{\frac{1}{\text{Ł}}}}}\right)\end{pmatrix}$$

$$=\begin{pmatrix}\left(\Theta_1.\hat{\alpha}_1+\Theta_2.\hat{\alpha}_2,\Theta_1.\hat{\beta}_1+\Theta_2.\hat{\beta}_2,\Theta_1.\hat{\gamma}_1+\Theta_2.\hat{\gamma}_2\right);\\ \left(\sqrt[p]{1-\frac{1}{1+\left\{\Theta_1\left(\frac{z^p_{\breve{\Psi}_1}}{1-z^p_{\breve{\Psi}_1}}\right)^{\text{Ł}}+\Theta_2\left(\frac{z^p_{\breve{\Psi}_2}}{1-z^p_{\breve{\Psi}_2}}\right)^{\text{Ł}}\right\}^{\frac{1}{\text{Ł}}}}},\sqrt[q]{\frac{1}{1+\left\{\Theta_1\left(\frac{1-w^q_{\breve{\Psi}_1}}{w^q_{\breve{\Psi}_1}}\right)^{\text{Ł}}+\Theta_2\left(\frac{1-w^q_{\breve{\Psi}_2}}{w^q_{\breve{\Psi}_2}}\right)^{\text{Ł}}\right\}^{\frac{1}{\text{Ł}}}}}\right)\end{pmatrix}$$

$$= \left(\begin{array}{c} \left(\sum_{i=1}^{2} \Theta_i \hat{\alpha}_i , \sum_{i=1}^{2} \Theta_i \hat{\beta}_i , \sum_{i=1}^{2} \Theta_i \hat{\gamma}_i\right); \\ \left(\sqrt[p]{1 - \frac{1}{1+\left\{\sum_{i=1}^{2} \Theta_i \left(\frac{z_{\breve{\Psi}_i}^{p}}{1-z_{\breve{\Psi}_i}^{p}}\right)^{\text{Ł}}\right\}^{\frac{1}{\text{Ł}}}}}, \sqrt[q]{\frac{1}{1+\left\{\sum_{i=1}^{2} \Theta_i \left(\frac{1-w_{\breve{\Psi}_i}^{q}}{w_{\breve{\Psi}_i}^{q}}\right)^{\text{Ł}}\right\}^{\frac{1}{\text{Ł}}}}} \right) \end{array} \right).$$

So, Eq (10) holds true when $n = 2$.

Equation (10) is assumed to apply for $n = k$.

$$T(p,q) - FNDWA\left(\breve{\Psi}_1, \breve{\Psi}_2, \ldots, \breve{\Psi}_k\right) = \Theta_1 \breve{\Psi}_1 \oplus \Theta_1 \breve{\Psi}_2 \oplus \ldots \oplus \Theta_k \breve{\Psi}_k$$

$$= \left(\begin{array}{c} \left(\sum_{i=1}^{k} \Theta_i \hat{\alpha}_i , \sum_{i=1}^{k} \Theta_i \hat{\beta}_i , \sum_{i=1}^{k} \Theta_i \hat{\gamma}_i\right); \\ \left(\sqrt[p]{1 - \frac{1}{1+\left\{\sum_{i=1}^{k} \Theta_i \left(\frac{z_{\breve{\Psi}_i}^{p}}{1-z_{\breve{\Psi}_i}^{p}}\right)^{\text{Ł}}\right\}^{\frac{1}{\text{Ł}}}}}, \sqrt[q]{\frac{1}{1+\left\{\sum_{i=1}^{k} \Theta_i \left(\frac{1-w_{\breve{\Psi}_i}^{q}}{w_{\breve{\Psi}_i}^{q}}\right)^{\text{Ł}}\right\}^{\frac{1}{\text{Ł}}}}} \right) \end{array} \right).$$

Now, for $n = k + 1$, we have

$$T(p,q) - FNDWA\left(\breve{\Psi}_1, \breve{\Psi}_2, \ldots, \breve{\Psi}_k, \breve{\Psi}_{k+1}\right)$$

$$= \Theta_1 \breve{\Psi}_1 \oplus \Theta_1 \breve{\Psi}_2 \oplus \ldots \oplus \Theta_k \breve{\Psi}_k \oplus \Theta_{k+1} \breve{\Psi}_{k+1}$$

$$= \left(\begin{array}{c} \left(\sum_{i=1}^{k} \Theta_i \hat{\alpha}_i , \sum_{i=1}^{k} \Theta_i \hat{\beta}_i , \sum_{i=1}^{k} \Theta_i \hat{\gamma}_i\right); \\ \left(\sqrt[p]{1 - \frac{1}{1+\left\{\sum_{i=1}^{k} \Theta_i \left(\frac{z_{\breve{\Psi}_i}^{p}}{1-z_{\breve{\Psi}_i}^{p}}\right)^{\text{Ł}}\right\}^{\frac{1}{\text{Ł}}}}}, \sqrt[q]{\frac{1}{1+\left\{\sum_{i=1}^{k} \Theta_i \left(\frac{1-w_{\breve{\Psi}_i}^{q}}{w_{\breve{\Psi}_i}^{q}}\right)^{\text{Ł}}\right\}^{\frac{1}{\text{Ł}}}}} \right) \end{array} \right)$$

$$\oplus \left(\begin{array}{c} \left(\Theta_{k+1}.\hat{\alpha}_{k+1}, \Theta_{k+1}.\hat{\beta}_{k+1}, \Theta_{k+1}.\hat{\gamma}_{k+1}\right); \\ \left(\sqrt[p]{1 - \frac{1}{1+\left\{\Theta_{k+1} \left(\frac{z_{\breve{\Psi}_{k+1}}^{p}}{1-z_{\breve{\Psi}_{k+1}}^{p}}\right)^{\text{Ł}}\right\}^{\frac{1}{\text{Ł}}}}}, \sqrt[q]{\frac{1}{1+\left\{\Theta_{k+1} \left(\frac{1-w_{\breve{\Psi}_{k+1}}^{q}}{w_{\breve{\Psi}_{k+1}}^{q}}\right)^{\text{Ł}}\right\}^{\frac{1}{\text{Ł}}}}} \right) \end{array} \right)$$

$$=\left(\begin{array}{c}\left(\sum_{i=1}^{k+1}\Theta_i\hat{\alpha}_i,\sum_{i=1}^{k+1}\Theta_i\hat{\beta}_i,\sum_{i=1}^{k+1}\Theta_i\hat{\gamma}_i\right);\\ \left(\sqrt[p]{1-\frac{1}{1+\left\{\sum_{i=1}^{k+1}\Theta_i\left(\frac{z_{\breve{\Psi}_i}^p}{1-z_{\breve{\Psi}_i}^p}\right)^{Ł}\right\}^{\frac{1}{Ł}}}},\sqrt[q]{\frac{1}{1+\left\{\sum_{i=1}^{k+1}\Theta_i\left(\frac{1-w_{\breve{\Psi}_i}^q}{w_{\breve{\Psi}_i}^q}\right)^{Ł}\right\}^{\frac{1}{Ł}}}}\right)\end{array}\right).$$

Example 2. Assume that $\breve{\Psi}_1=\left((2,3,4);0.8,0.7\right)$, $\breve{\Psi}_2=\left((1,2,3);0.9,0.5\right)$, and $\breve{\Psi}_3=\left((1,3,5);0.5,0.6\right)$ are any three T(p,q)-FNs, then we can aggregate these three T(p,q)-FNs using the T(p,q)-FNDWA operator. If we let $Ł=2$, $p=3$, $q=4$, and $\Theta=\{0.2,0.3,0.5\}^T$, then the aggregated T(p,q)-FN is as follows:

$$T(p,q)-FNDWA\left(\breve{\Psi}_1,\breve{\Psi}_2,\breve{\Psi}_3\right)=\left((1.2,2.7,4.2);0.8471,0.5543\right).$$

The T(p,q)-FNDWA operator possesses several important properties, which are outlined below.

Property 1. Idempotency. If all T(p,q)-FNs are identical, i.e, $\breve{\Psi}_i=\left(\left(\hat{\alpha}_i,\hat{\beta}_i,\hat{\gamma}_i\right);z_{\breve{\Psi}_i},w_{\breve{\Psi}_i}\right)(i=1,2,\dots,n)=\left(\left(\hat{\alpha},\hat{\beta},\hat{\gamma}\right);z_{\breve{\Psi}},w_{\breve{\Psi}}\right)$ for all i, $\breve{\Psi}_i=\breve{\Psi}$ then;

$$T(p,q)-FNDWA\left(\breve{\Psi}_1,\breve{\Psi}_2,\dots,\breve{\Psi}_n\right)=\breve{\Psi}=\left(\left(\hat{\alpha},\hat{\beta},\hat{\gamma}\right);z_{\breve{\Psi}},w_{\breve{\Psi}}\right). \tag{13}$$

Proof.

$$\begin{aligned}&T(p,q)-FNDWA\left(\breve{\Psi}_1,\breve{\Psi}_2,\dots,\breve{\Psi}_n\right)\\&=\Theta_1\breve{\Psi}_1\oplus\Theta_1\breve{\Psi}_2\oplus\dots\oplus\Theta_n\breve{\Psi}_n\\&=\left(\begin{array}{c}\left(\sum_{i=1}^{n}\Theta_i\hat{\alpha}_i,\sum_{i=1}^{n}\Theta_i\hat{\beta}_i,\sum_{i=1}^{n}\Theta_i\hat{\gamma}_i\right);\\ \left(\sqrt[p]{1-\frac{1}{1+\left\{\sum_{i=1}^{n}\Theta_i\left(\frac{z_{\breve{\Psi}_i}^p}{1-z_{\breve{\Psi}_i}^p}\right)^{Ł}\right\}^{\frac{1}{Ł}}}},\sqrt[q]{\frac{1}{1+\left\{\sum_{i=1}^{n}\Theta_i\left(\frac{1-w_{\breve{\Psi}_i}^q}{w_{\breve{\Psi}_i}^q}\right)^{Ł}\right\}^{\frac{1}{Ł}}}}\right)\end{array}\right),\end{aligned}$$

where

$$\sum_{i=1}^{n}\Theta_i=1=\left(\begin{array}{c}(\hat{\alpha},\hat{\beta},\hat{\gamma});\\ \left(\sqrt[p]{1-\frac{1}{1+\left\{\left(\frac{\mathcal{Z}_{\breve{\Psi}}^{p}}{1-\mathcal{Z}_{\breve{\Psi}}^{p}}\right)^{Ł}\right\}^{\frac{1}{Ł}}}},\sqrt[q]{\frac{1}{1+\left\{\left(\frac{1-\mathcal{W}_{\breve{\Psi}}^{q}}{\mathcal{W}_{\breve{\Psi}}^{q}}\right)^{Ł}\right\}^{\frac{1}{Ł}}}}\right)\end{array}\right)$$

$$=\left((\hat{\alpha},\hat{\beta},\hat{\gamma});\mathcal{Z}_{\breve{\Psi}},\mathcal{W}_{\breve{\Psi}}\right)=\breve{\Psi}.$$

Thus, $T(p,q)-FNDWA(\breve{\Psi}_1,\breve{\Psi}_2,\dots,\breve{\Psi}_n)=\breve{\Psi}$ holds.

Property 2. Monotonicity. Let $\breve{\Psi}_i''=\left((\hat{\alpha}_i'',\hat{\beta}_i'',\hat{\gamma}_i'');\mathcal{Z}_{\breve{\Psi}_i}'',\mathcal{W}_{\breve{\Psi}_i}''\right)$ and $\breve{\Psi}_i=\left((\hat{\alpha}_i,\hat{\beta}_i,\hat{\gamma}_i);\mathcal{Z}_{\breve{\Psi}_i},\mathcal{W}_{\breve{\Psi}_i}\right)$ be two sets of T(p,q)-FNs with $(i=1,2,\dots,n)$ such that $\hat{\alpha}_i\le\hat{\alpha}_i''$, $\hat{\beta}_i\le\hat{\beta}_i''$, $\hat{\gamma}_i\le\hat{\gamma}_i''$, $\mathcal{Z}_{\breve{\Psi}_i}''\le\mathcal{Z}_{\breve{\Psi}_i}$, and $\mathcal{W}_{\breve{\Psi}_i}\ge\mathcal{W}_{\breve{\Psi}_i}''$.

$$T(p,q)-FNDWA(\breve{\Psi}_1,\breve{\Psi}_2,\dots,\breve{\Psi}_n)\le T(p,q)-FNDWA(\breve{\Psi}_1'',\breve{\Psi}_2'',\dots,\breve{\Psi}_n''). \quad (14)$$

Proof. Let $\breve{\Psi}_i''=\left((\hat{\alpha}_i'',\hat{\beta}_i'',\hat{\gamma}_i'');\mathcal{Z}_{\breve{\Psi}_i}'',\mathcal{W}_{\breve{\Psi}_i}''\right)$ and $\breve{\Psi}_i=\left((\hat{\alpha}_i,\hat{\beta}_i,\hat{\gamma}_i);\mathcal{Z}_{\breve{\Psi}_i},\mathcal{W}_{\breve{\Psi}_i}\right)$ be two sets of T(p,q)-FNs with $(i=1,2,\dots,n)$ such that $\breve{\Psi}_i''\le\breve{\Psi}_i$ which implies that $\hat{\alpha}_i\le\hat{\alpha}_i''$, $\hat{\beta}_i\le\hat{\beta}_i''$, $\hat{\gamma}_i\le\hat{\gamma}_i''$, $\mathcal{Z}_{\breve{\Psi}_i}''\le\mathcal{Z}_{\breve{\Psi}_i}$, and $\mathcal{W}_{\breve{\Psi}_i}\ge\mathcal{W}_{\breve{\Psi}_i}''$. In this case, we have

$$\hat{\alpha}_i\le\hat{\alpha}_i'',\ \hat{\beta}_i\le\hat{\beta}_i'',\ \hat{\gamma}_i\le\hat{\gamma}_i''.$$

For membership, $\mathcal{Z}_{\breve{\Psi}_i}\le\mathcal{Z}_{\breve{\Psi}_i}''\Rightarrow\mathcal{Z}_{\breve{\Psi}}^{p}\le\mathcal{Z}_{\breve{\Psi}}''^{p}$, and because $p\ge 1$,

$$\Rightarrow\frac{\mathcal{Z}_{\breve{\Psi}}^{p}}{1-\mathcal{Z}_{\breve{\Psi}}^{p}}\le\frac{\mathcal{Z}_{\breve{\Psi}}''^{p}}{1-\mathcal{Z}_{\breve{\Psi}}''^{p}}$$

$$\Rightarrow\left(\frac{\mathcal{Z}_{\breve{\Psi}_i}^{p}}{1-\mathcal{Z}_{\breve{\Psi}_i}^{p}}\right)^{Ł}\le\left(\frac{\mathcal{Z}''^{p}_{\breve{\Psi}_i}}{1-\mathcal{Z}''^{p}_{\breve{\Psi}_i}}\right)^{Ł}$$

$$\Rightarrow\sum_{i=1}^{n}\Theta_i\left(\frac{\mathcal{Z}_{\breve{\Psi}_i}^{p}}{1-\mathcal{Z}_{\breve{\Psi}_i}^{p}}\right)^{Ł}\le\sum_{i=1}^{n}\Theta_i\left(\frac{\mathcal{Z}''^{p}_{\breve{\Psi}_i}}{1-\mathcal{Z}''^{p}_{\breve{\Psi}_i}}\right)^{Ł}$$

$$\Rightarrow\left\{\sum_{i=1}^{n}\Theta_i\left(\frac{\mathcal{Z}_{\breve{\Psi}_i}^{p}}{1-\mathcal{Z}_{\breve{\Psi}_i}^{p}}\right)^{Ł}\right\}^{\frac{1}{Ł}}\le\left\{\sum_{i=1}^{n}\Theta_i\left(\frac{\mathcal{Z}''^{p}_{\breve{\Psi}_i}}{1-\mathcal{Z}''^{p}_{\breve{\Psi}_i}}\right)^{Ł}\right\}^{\frac{1}{Ł}}$$

$$\Rightarrow 1+\left\{\sum_{i=1}^{n}\Theta_i\left(\frac{\mathcal{Z}_{\breve{\Psi}_i}^{p}}{1-\mathcal{Z}_{\breve{\Psi}_i}^{p}}\right)^{Ł}\right\}^{\frac{1}{Ł}}\le 1+\left\{\sum_{i=1}^{n}\Theta_i\left(\frac{\mathcal{Z}''^{p}_{\breve{\Psi}_i}}{1-\mathcal{Z}''^{p}_{\breve{\Psi}_i}}\right)^{Ł}\right\}^{\frac{1}{Ł}}$$

$$\Rightarrow \frac{1}{1+\left\{\sum_{i=1}^{n}\Theta_i\left(\frac{z_{\breve{\Psi}_i}^{p}}{1-z_{\breve{\Psi}_i}^{p}}\right)^{Ⱡ}\right\}^{\frac{1}{Ⱡ}}} \geq \frac{1}{1+\left\{\sum_{i=1}^{n}\Theta_i\left(\frac{z_{\breve{\Psi}_i}^{\prime\prime p}}{1-z_{\breve{\Psi}_i}^{\prime\prime p}}\right)^{Ⱡ}\right\}^{\frac{1}{Ⱡ}}}$$

$$\Rightarrow 1-\frac{1}{1+\left\{\sum_{i=1}^{n}\Theta_i\left(\frac{z_{\breve{\Psi}_i}^{p}}{1-z_{\breve{\Psi}_i}^{p}}\right)^{Ⱡ}\right\}^{\frac{1}{Ⱡ}}} \leq 1-\frac{1}{1+\left\{\sum_{i=1}^{n}\Theta_i\left(\frac{z_{\breve{\Psi}_i}^{\prime\prime p}}{1-z_{\breve{\Psi}_i}^{\prime\prime p}}\right)^{Ⱡ}\right\}^{\frac{1}{Ⱡ}}}$$

$$\Rightarrow \sqrt[p]{1-\frac{1}{1+\left\{\sum_{i=1}^{n}\Theta_i\left(\frac{z_{\breve{\Psi}_i}^{p}}{1-z_{\breve{\Psi}_i}^{p}}\right)^{Ⱡ}\right\}^{\frac{1}{Ⱡ}}}} \leq \sqrt[p]{1-\frac{1}{1+\left\{\sum_{i=1}^{n}\Theta_i\left(\frac{z_{\breve{\Psi}_i}^{\prime\prime p}}{1-z_{\breve{\Psi}_i}^{\prime\prime p}}\right)^{Ⱡ}\right\}^{\frac{1}{Ⱡ}}}}.$$

Similarly, for non-membership, we have

$$\sqrt[q]{\frac{1}{1+\left\{\sum_{i=1}^{n}\Theta_i\left(\frac{1-w_{\breve{\Psi}_i}^{q}}{w_{\breve{\Psi}_i}^{q}}\right)^{Ⱡ}\right\}^{\frac{1}{Ⱡ}}}} \leq \sqrt[q]{\frac{1}{1+\left\{\sum_{i=1}^{n}\Theta_i\left(\frac{1-w_{\breve{\Psi}_i}^{\prime\prime q}}{w_{\breve{\Psi}_i}^{\prime\prime q}}\right)^{Ⱡ}\right\}^{\frac{1}{Ⱡ}}}}.$$

Thus,

$$T(p,q)-FNDWA\left(\breve{\Psi}_1,\breve{\Psi}_2,\ldots,\breve{\Psi}_n\right) \leq T(p,q)-FNDWA\left(\breve{\Psi}_1^{\prime\prime},\breve{\Psi}_2^{\prime\prime},\ldots,\breve{\Psi}_n^{\prime\prime}\right).$$

Property 3. Boundedness. If $\breve{\Psi}_i=\left(\left(\hat{\alpha}_i,\hat{\beta}_i,\hat{\gamma}_i\right);\mathcal{Z}_{\breve{\Psi}_i},\mathcal{W}_{\breve{\Psi}_i}\right)(i=1,2,\ldots,n)$ be set of T(p,q)-FNs. Let $\breve{\Psi}^{-}=min\left(\breve{\Psi}_1,\breve{\Psi}_2,\ldots,\breve{\Psi}_n\right)$ and $\breve{\Psi}^{+}=max\left(\breve{\Psi}_1,\breve{\Psi}_2,\ldots,\breve{\Psi}_n\right)$. Then

$$\breve{\Psi}^{-} \leq T(p,q)-FNDWA\left(\breve{\Psi}_1,\breve{\Psi}_2,\ldots,\breve{\Psi}_n\right) \leq \breve{\Psi}^{+}.$$

The proof is similar to that of Property 2.

5. CRITIC-MARCOS method for MCDM problems based on T(p,q)-FNs

The CRITIC-MARCOS method's balanced and intuitive approach makes it very helpful, particularly in MCDM problems. By allowing decision-makers to consider each alternative's performance in relation to a basic standard, this approach provides them with a more realistic and nuanced judgement.

Given a collection of criteria $\{\mathscr{k}_1,\mathscr{k}_2,\ldots,\mathscr{k}_n\}$ and a finite set of alternatives $\{Ŧ_1,Ŧ_2,\ldots,Ŧ_m\}$, let $\Theta=(\Theta_1,\ \Theta_2,\ldots,\Theta_n)^T$ be the vectors for criteria weights that satisfy the following conditions $\Theta_i\in[0,1]$, and $\sum_{i=1}^{n}\Theta_i=1$. Depending on attributes of the different kinds of data, the decision-maker presents their evaluation of each option. In this decision-making process, we consider the data to be

T(p,q)-FNs with $\breve{\Psi}_i = \left(\left(\hat{\alpha}_i, \hat{\beta}_i, \hat{\gamma}_i\right); \mathcal{Z}_{\breve{\Psi}_i}, \mathcal{W}_{\breve{\Psi}_i}\right)$ and list them in a decision matrix $\mathfrak{W} = \left(\breve{\Psi}_{ij}\right)_{m\times n}$. The advanced decision algorithm of the CRITIC-MARCOS approach has the following structure.

Step 1: Establish the decision matrix using the T(p,q)-FNs that were obtained from decision-making experts. The selection matrix that was produced is displayed as follows:

$$\mathfrak{W} = \begin{pmatrix} \breve{\Psi}_{11} & \breve{\Psi}_{12} & \cdots & \breve{\Psi}_{1n} \\ \breve{\Psi}_{21} & \breve{\Psi}_{22} & \cdots & \breve{\Psi}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \breve{\Psi}_{m1} & \breve{\Psi}_{m2} & \cdots & \breve{\Psi}_{mn} \end{pmatrix}. \tag{15}$$

Step 2: To ascertain the weights of the criteria, apply the CRITIC method. The CRITIC method's steps are given below.

Step 3: Utilizing the score function and the following Eq (16), determine the score matrix:

$$\text{Ꞔ}_{ij} = \mathbb{S}\left(\breve{\Psi}_{ij}\right) = \frac{\hat{\alpha}_{ij}+2\hat{\beta}_{ij}+\hat{\gamma}_{ij}}{4}\left(\mathcal{Z}^{p}_{\breve{\Psi}_{ij}} - \mathcal{W}^{q}_{\breve{\Psi}_{ij}}\right). \tag{16}$$

Step 4: The score matrix is transformed into standard form by using Eq (17),

$$\widetilde{\text{Ꞔ}}_{ij} = \frac{\text{Ꞔ}_{ij}-\text{Ꞔ}_j^-}{\text{Ꞔ}_j^+-\text{Ꞔ}_j^-}, \tag{17}$$

where $\text{Ꞔ}_j^+ = max_i \text{Ꞔ}_{ij}$ and $\text{Ꞔ}_j^- = min_i \text{Ꞔ}_{ij}$.

Step 5: Determine the challenges' S.D. The challenge's S.D is computed using Eq (18).

$$\Phi_j = \sqrt{\frac{\sum_{i=1}^{m}\left(\widetilde{\text{Ꞔ}}_{ij}-\overline{\widetilde{\text{Ꞔ}}}_j\right)^2}{m}}, \tag{18}$$

where $\overline{\widetilde{\text{Ꞔ}}}_j = \frac{\sum_{i=1}^{m}\widetilde{\text{Ꞔ}}_{ij}}{m}$

Step 6: Calculating the correlation between criteria. Eq (19) is used to calculate the correlation between criteria.

$$\text{Ꝑ}_{jt} = \frac{\sum_{i=1}^{m}\left(\widetilde{\text{Ꞔ}}_{ij}-\overline{\widetilde{\text{Ꞔ}}}_j\right)\left(\widetilde{\text{Ꞔ}}_{it}-\overline{\widetilde{\text{Ꞔ}}}_t\right)}{\sqrt{\sum_{i=1}^{m}\left(\widetilde{\text{Ꞔ}}_{ij}-\overline{\widetilde{\text{Ꞔ}}}_j\right)^2 \sum_{i=1}^{m}\left(\widetilde{\text{Ꞔ}}_{it}-\overline{\widetilde{\text{Ꞔ}}}_t\right)^2}}. \tag{19}$$

Step 7: Use Eq (20) to analyze the data for each criterion.

$$\Pi\text{Ꞔ}_j = \Phi_j \sum_{t=1}^{n}\left(1 - \text{Ꝑ}_{jt}\right). \tag{20}$$

Step 8: Equation (21) is used in the final stage to determine the weight of each criterion.

$$\Theta_j = \frac{\Pi\text{Ꞔ}_j}{\sum_{j=1}^{n}\Pi\text{Ꞔ}_j}. \tag{21}$$

Step 9: To assess the best options, the MARCOS approach is used. The procedure is given below.

Step 10: Determine the weighted normalized T(p,q)-FNs decision matrix by applying Eq (22).

$$\breve{\Psi}_{ij}^{N} = \left(\left(\hat{\alpha}_{ij}^{N}, \hat{\beta}_{ij}^{N}, \hat{\gamma}_{ij}^{N}\right); \mathcal{Z}_{\breve{\Psi}_{ij}}^{N}, \mathcal{W}_{\breve{\Psi}_{ij}}^{N}\right)$$

$$= \Theta_j \odot \breve{\Psi}_{ij} = \left(\begin{array}{c} \left(\Theta_j . \hat{\alpha}_{ij},\ \Theta_j . \hat{\beta}_{ij},\ \Theta_j . \hat{\gamma}_{ij}\right); \\ \left(\sqrt[p]{1 - \frac{1}{\left(1 + \left\{\Theta_j \left(\frac{\mathcal{Z}_{\breve{\Psi}_{ij}}^{p}}{1-\mathcal{Z}_{\breve{\Psi}_{ij}}^{p}}\right)\right\}^{Ł}\right)^{\frac{1}{Ł}}}}, \sqrt[q]{\frac{1}{\left(1 + \left\{\Theta_j \left(\frac{1-\mathcal{W}_{\breve{\Psi}_{ij}}^{q}}{\mathcal{W}_{\breve{\Psi}_{ij}}^{q}}\right)\right\}^{Ł}\right)^{\frac{1}{Ł}}}} \right) \end{array}\right). \quad (22)$$

Step 11: From the weighted normalized T(p,q)-FNs decision matrix, determine the ideal and anti-ideal ratings as

$$\zeta^{+} = \left(\left(\hat{\alpha}_{j}^{N+}, \hat{\beta}_{j}^{N+}, \hat{\gamma}_{j}^{N+}\right); \mathcal{Z}_{\breve{\Psi}_{j}}^{N+}, \mathcal{W}_{\breve{\Psi}_{j}}^{N+}\right) = \left(max_{i} \mathbb{S}\left(\breve{\Psi}_{ij}^{N}\right)\right), \quad (23)$$

$$\zeta^{-} = \left(\left(\hat{\alpha}_{j}^{N-}, \hat{\beta}_{j}^{N-}, \hat{\gamma}_{j}^{N-}\right); \mathcal{Z}_{\breve{\Psi}_{j}}^{N-}, \mathcal{W}_{\breve{\Psi}_{j}}^{N-}\right) = \left(min_{i} \mathbb{S}\left(\breve{\Psi}_{ij}^{N}\right)\right). \quad (24)$$

Step 12: Using Eq (7), the weighted normalized T(p,q)-FNs decision matrix's score rating is estimated.

Step 13: The utility degree of the alternative is calculated as follows:

$$\bar{Й}_{i}^{+} = \frac{Ф_{i}}{Ф^{+}}, \quad (25)$$

$$\bar{Й}_{i}^{-} = \frac{Ф_{i}}{Ф^{-}}, \quad (26)$$

where $Ф^{+}$ and $Ф^{-}$ represent the total score values of the ideal and anti-ideal ratings from the weighted normalized T(p,q)-FNs decision matrix, respectively, and where $Ф_{i}$ is defined as follows: $Ф_{i} = \sum_{j=1}^{n} \mathbb{S}\left(\breve{\Psi}_{ij}^{N}\right)$.

Step 14: Determine the alternatives' utility functions using the following Eq (27):

$$\Gamma(\bar{Й}) = \frac{\bar{Й}_{i}^{+} + \bar{Й}_{i}^{-}}{1 + \frac{1-\Gamma\left(\bar{Й}_{i}^{+}\right)}{\Gamma\left(\bar{Й}_{i}^{+}\right)} + \frac{1-\Gamma\left(\bar{Й}_{i}^{-}\right)}{\Gamma\left(\bar{Й}_{i}^{-}\right)}}, \quad (27)$$

where $\Gamma(\bar{Й}_{i}^{+}) = \frac{\bar{Й}_{i}^{-}}{\bar{Й}_{i}^{+} + \bar{Й}_{i}^{-}}$ and $\Gamma(\bar{Й}_{i}^{-}) = \frac{\bar{Й}_{i}^{+}}{\bar{Й}_{i}^{+} + \bar{Й}_{i}^{-}}$.

Prioritize the selections based on the utility functions, then select the best one.

The method's step-by-step logic and decision-making process are shown depicted in Figure 6.

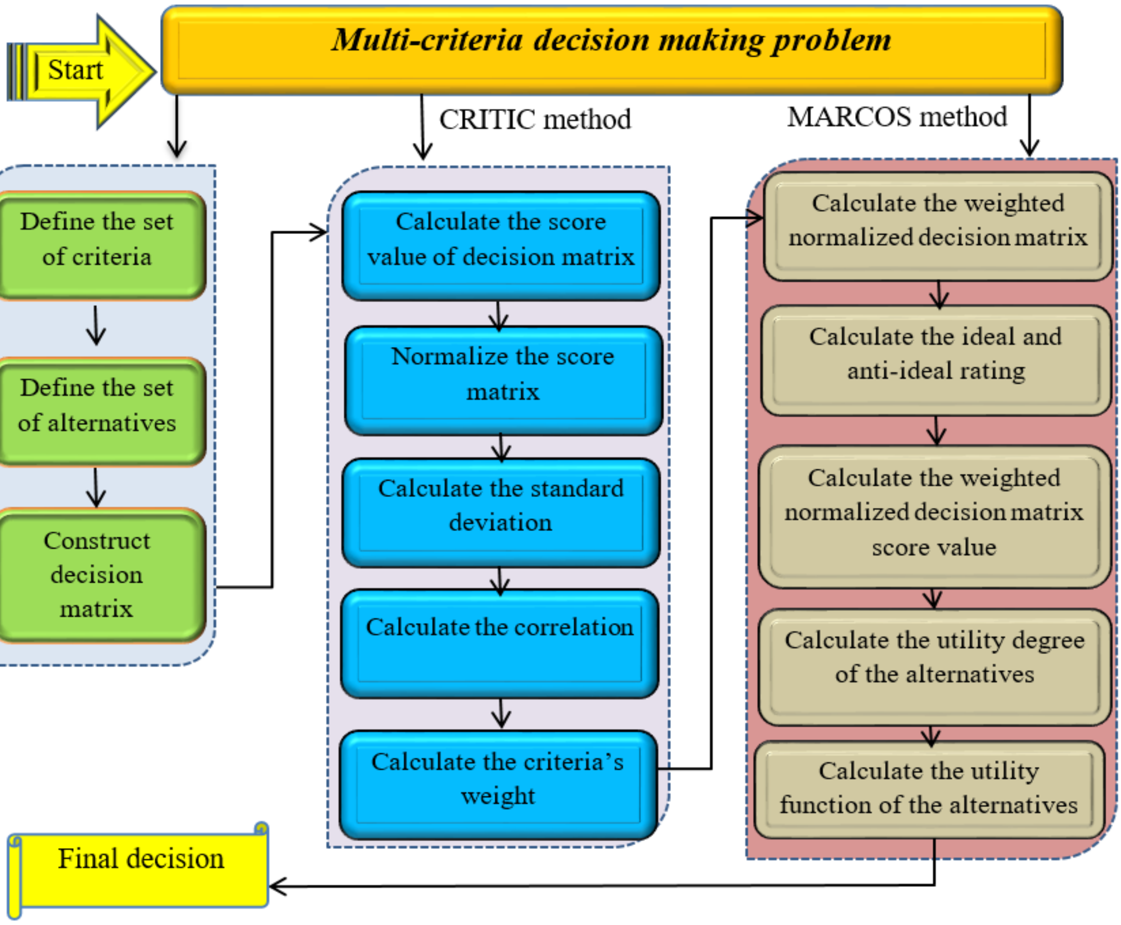

Figure 6. Flowchart of the proposed CRITIC-MARCOS model.

6. Case study: Selection of modulation techniques

In modern communication systems [51], the transmission of audio, video, and data relies on efficient signal processing techniques, where modulation plays a critical role in enabling reliable communication over noisy and bandwidth-limited channels. The selection of an appropriate modulation technique is a complex MCDM problem, as different techniques offer trade-offs in terms of performance, efficiency, and implementation requirements. In this study, the decision-making framework is based on both a literature review and expert judgment. A domain expert with relevant knowledge in wireless communication systems was consulted to evaluate the alternatives. The evaluation process was conducted systematically, and the consistency of the assigned values was carefully verified to ensure the logical coherence and reliability of the decision matrix. The considered communication scenario represents a general wireless communication environment under typical assumptions, including the presence of noise, limited bandwidth, and varying power constraints. The objective is to identify the most suitable modulation technique for efficient and reliable data transmission under such conditions.

Five widely used modulation techniques are selected as alternatives: "binary phase shift keying (BPSK) ($Ŧ_1$)", "quadrature phase shift keying (QPSK) ($Ŧ_2$)", "quadrature amplitude modulation

(QAM) ($Ŧ_3$)", "orthogonal frequency-division multiplexing (OFDM) ($Ŧ_4$)" and "frequency shift keying (FSK) ($Ŧ_5$)". These techniques are commonly applied in practical wireless and digital communication systems. The evaluation is based on five key criteria: bit error rate (BER), data rate, power efficiency, spectral efficiency, and computational complexity. These criteria are selected from the literature due to their significant impact on the system's performance. BER reflects transmission reliability, data rate indicates throughput, power efficiency measures energy consumption, spectral efficiency represents bandwidth utilization, and computational complexity captures implementation cost and processing requirements.

The decision matrix is constructed on the basis of expert evaluation, and the proposed CRITIC-MARCOS method under the T(p,q)-FN framework is applied to determine the ranking of alternatives. This approach ensures a structured, consistent, and reliable decision-making process for selecting the most appropriate modulation technique under uncertainty. These criteria are discussed in Table 3.

Table 3. Modulation techniques' criteria.

Notation	Criterion	Description
$\mathcal{k}_1$	Bit error rate (BER)	In digital communication systems, bit transmission errors are measured using a metric called the bit error rate (BER). It shows the proportion of incorrect bits received during transmission over a specific time period to all bits received during that same period.
$\mathcal{k}_2$	Data rate	This is the quantity of data sent via a network in a given amount of time. Usually, it is expressed in MBps or Mbps.
$\mathcal{k}_3$	Power efficiency	The ability of a modulation technique to maintain the quality of the digital message at low power levels is referred to as power efficiency.
$\mathcal{k}_4$	Spectral efficiency	The quantity of data transferred with the fewest possible transmission mistakes over a specific spectrum or bandwidth is referred to as spectrum efficiency.
$\mathcal{k}_5$	Computational complexity	The quantity of resources needed to execute an algorithm is known as its computational complexity, or simply complexity.

$Ŧ_1$: BPSK

One modulation method used in communication systems to send data over a communication channel is called BPSK. In BPSK, the carrier signal's phase is altered by 180 degrees for every symbol. A binary 0 is represented by a 180-degree phase shift, and a binary 1 is represented by no phase shift. Because the BPSKs modulation technique is simple and effective, it can be used in situations where noise and interference affect the communication channel.

$Ŧ_2$: QPSK

A digital modulation method called QPSK changes the carrier waveform's phase in accordance with the digital baseband signal. The carrier's phase stays constant when the input logic is 1; the phase shift happens when the logic is 0. In contrast to BPSK, which only transmits one bit per symbol, QPSK modulates two information bits simultaneously. Here, four different two-bit combinations (00, 01, 10, 11) have four carrier phase offsets with a phase difference of ±90°. In this modulation, the symbol's duration is double that of the bit.

$Ŧ_3$***: QAM***

In contemporary telecommunications, QAM refers to a family of digital modulation techniques and a related family of analogue modulation techniques that are commonly used to transport data. It transmits two analogue message signals or two digital bit streams by varying (modulating) the amplitude of the two carrier waves, using the amplitude shift keying (ASK) digital modulation scheme or amplitude modulation (AM) analogue modulation technique. When two carrier waves of the same frequency are 90° out of phase with one another, this is known as orthogonality or quadrature. By combining the two carrier waves, the transmitted signal is produced. The two waves' orthogonality allows for their coherent separation (demodulation) at the receiver. The narrowband assumption, which states that the modulations are low-frequency/low-bandwidth waveforms with relation to the carrier frequency, is another important characteristic.

$Ŧ_4$***: OFDM***

The digital communication method known as OFDM was first created for usage in cable television networks. The broadcasting method known as frequency division multiplexing (FDM), which uses numerous transmitters and receivers to transmit data on various frequencies over a single wire, like an electrical power connection, is similar to OFDM.

$Ŧ_5$***: FSK***

In FSK, the carrier wave is assigned a distinct frequency by each symbol in the message signal. Binary and M-ary are the two varieties of FSK. Logic 1 in binary FSK is linked to a specific carrier wave frequency, such as 50 MHz, while logic 0 is linked to a frequency other than 50 MHz, such as 25 MHz. Instead of considering one bit at a time, a group of log2M bits is taken into consideration in M-ary FSK, and the frequency is linked to this group of bits.

7. Numerical results

The following describes the process of choosing the best modulation methods for communication using the suggested T(p,q)-FN framework.

Step 1: Table 4 displays the T(p,q)-FN decision matrix that the decision-making expert provided.

Table 4. T(p,q)-FN decision matrix.

Alternatives	$\mathcal{k}_1$	$\mathcal{k}_2$	$\mathcal{k}_3$	$\mathcal{k}_4$	$\mathcal{k}_5$
$Ŧ_1$	$\begin{pmatrix}(2,3,4);\\0.6,0.7\end{pmatrix}$	$\begin{pmatrix}(1,3,5);\\0.8,0.7\end{pmatrix}$	$\begin{pmatrix}(2,3,5);\\0.8,0.8\end{pmatrix}$	$\begin{pmatrix}(3,4,5);\\0.9,0.6\end{pmatrix}$	$\begin{pmatrix}(3,4,5);\\0.9,0.7\end{pmatrix}$
$Ŧ_2$	$\begin{pmatrix}(4,5,6);\\0.5,0.9\end{pmatrix}$	$\begin{pmatrix}(3,4,5);\\0.9,0.2\end{pmatrix}$	$\begin{pmatrix}(7,8,9);\\0.6,0.3\end{pmatrix}$	$\begin{pmatrix}(1,2,3);\\0.8,0.5\end{pmatrix}$	$\begin{pmatrix}(1,2,3);\\0.7,0.4\end{pmatrix}$
$Ŧ_3$	$\begin{pmatrix}(3,4,5);\\0.9,0.4\end{pmatrix}$	$\begin{pmatrix}(4,5,6);\\0.6,0.2\end{pmatrix}$	$\begin{pmatrix}(3,4,5);\\0.7,0.7\end{pmatrix}$	$\begin{pmatrix}(4,5,6);\\0.9,0.4\end{pmatrix}$	$\begin{pmatrix}(5,6,7);\\0.9,0.5\end{pmatrix}$
$Ŧ_4$	$\begin{pmatrix}(1,2,3);\\0.7,0.6\end{pmatrix}$	$\begin{pmatrix}(1,2,3);\\0.8,0.3\end{pmatrix}$	$\begin{pmatrix}(1,2,3);\\0.5,0.4\end{pmatrix}$	$\begin{pmatrix}(2,3,4);\\0.8,0.1\end{pmatrix}$	$\begin{pmatrix}(4,5,6);\\0.5,0.2\end{pmatrix}$
$Ŧ_5$	$\begin{pmatrix}(6,7,8);\\0.8,0.5\end{pmatrix}$	$\begin{pmatrix}(2,3,4);\\0.9,0.7\end{pmatrix}$	$\begin{pmatrix}(2,3,4);\\0.8,0.7\end{pmatrix}$	$\begin{pmatrix}(1,2,3);\\0.6,0.5\end{pmatrix}$	$\begin{pmatrix}(1,2,3);\\0.9,0.1\end{pmatrix}$

Algorithm 1: CRITIC-MARCOS approach.

Step 2: The suggested CRITIC method is used in the following steps to determine the criteria's unknown weights.

Step 3: The scoring matrix is calculated as follows using Eq (16) (see Table 5).

Table 5. Decision matrix's score values.

Alternatives	$\mathscr{k}_1$	$\mathscr{k}_2$	$\mathscr{k}_3$	$\mathscr{k}_4$	$\mathscr{k}_5$
Ŧ_1	-0.0723	0.8157	0.3328	2.3976	1.9556
Ŧ_2	-2.6555	2.9096	1.6632	0.899	0.6348
Ŧ_3	2.8136	1.0720	1.2696	3.5170	3.9990
Ŧ_4	0.4268	1.0078	-0.2302	1.5357	0.6170
Ŧ_5	3.1465	1.4667	1.4592	0.3070	1.4578

Step 4: The normalized score matrix is displayed in Table 6 according to Eq (17).

Table 6. Normalized score matrix.

Alternatives	$\mathscr{k}_1$	$\mathscr{k}_2$	$\mathscr{k}_3$	$\mathscr{k}_4$	$\mathscr{k}_5$
Ŧ_1	0.4723	0.4563	0.2973	0.6513	0.3958
Ŧ_2	0	1	1	0.1844	0.0053
Ŧ_3	1	0.03376	0.6543	1	1
Ŧ_4	0.5636	0	0	0.3828	0
Ŧ_5	0.4563	0.2513	0.8943	0	0.2486

Step 5: Table 7 displays the results of the calculations made using Eq (18) to determine the standard deviation of each criterion.

Table 7. Standard deviation.

	$\boldsymbol{\Phi_1}$	$\boldsymbol{\Phi_2}$	$\boldsymbol{\Phi_3}$	$\boldsymbol{\Phi_4}$	$\boldsymbol{\Phi_5}$
Standard deviation	0.6194	0.2348	0.5963	0.4437	0.3299

Step 6: The correlation coefficients between the criteria are calculated using Eq (19) and are presented in Table 8.

Table 8. Correlation coefficients.

Correlation	$\mathscr{k}_1$	$\mathscr{k}_2$	$\mathscr{k}_3$	$\mathscr{k}_4$	$\mathscr{k}_5$
$\mathscr{k}_1$	1.0000	-0.6295	0.0437	0.2192	0.6040
$\mathscr{k}_2$	-0.6295	1.0000	0.6706	-0.5251	-0.4488
$\mathscr{k}_3$	0.0437	0.6706	1.0000	-0.2264	0.2177
$\mathscr{k}_4$	0.2192	-0.5251	-0.2264	1.0000	0.8146
$\mathscr{k}_5$	0.6040	-0.4488	0.2177	0.8146	1.0000

Steps 7 and 8: Lastly, using Eqs (20) and (21), the criteria weights were calculated; Table 9 and

Figure 7 displays the findings.

Table 9. Criteria weights.

	Θ_1	Θ_2	Θ_3	Θ_4	Θ_5
Criteria weights	0.29	0.14	0.24	0.21	0.12

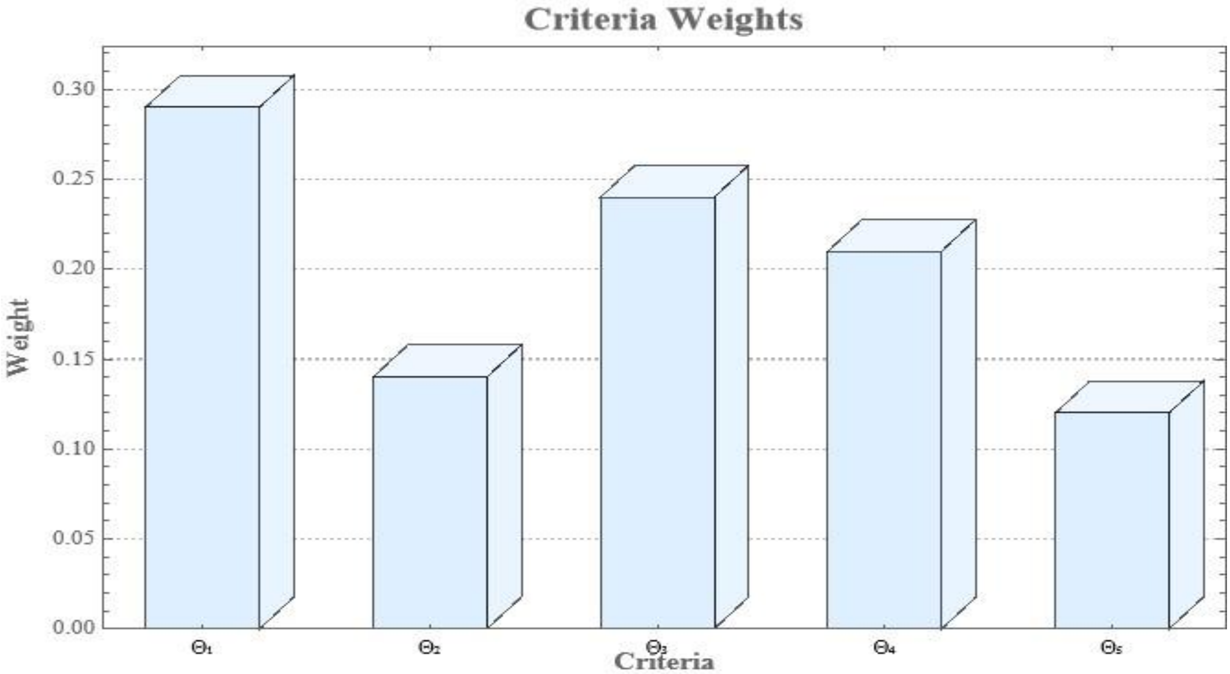

Figure 7. Criteria weight for the process of selecting a modulation technique.

Step 9: To assess the best alternatives, the MARCOS approach is used.

Step 10: The weighted normalized T(p,q)-FNs decision matrix is calculated and listed in Table 10 using Eq (22), the criteria weights, and Table 3.

Table 10. Weighted normalized T(p,q)-FNs decision matrix.

	$\hbar_1$	$\hbar_2$	$\hbar_3$	$\hbar_4$	$\hbar_5$
$Ŧ_1$	$\begin{pmatrix}(0.6,0.9,1.2);\\ 0.50,0.78\end{pmatrix}$	$\begin{pmatrix}(0.1,0.4,0.7);\\ 0.66,0.82\end{pmatrix}$	$\begin{pmatrix}(0.5,0.7,1.2);\\ 0.70,0.87\end{pmatrix}$	$\begin{pmatrix}(0.6,0.8,1.0);\\ 0.82,0.70\end{pmatrix}$	$\begin{pmatrix}(0.4,0.5,0.6);\\ 0.78,0.83\end{pmatrix}$
$Ŧ_2$	$\begin{pmatrix}(1.2,1.4,1.7);\\ 0.41,0.94\end{pmatrix}$	$\begin{pmatrix}(0.4,0.6,0.7);\\ 0.79,0.26\end{pmatrix}$	$\begin{pmatrix}(1.7,1.9,2.2);\\ 0.49,0.36\end{pmatrix}$	$\begin{pmatrix}(0.2,0.4,0.6);\\ 0.69,0.60\end{pmatrix}$	$\begin{pmatrix}(0.1,0.2,0.4);\\ 0.54,0.52\end{pmatrix}$
$Ŧ_3$	$\begin{pmatrix}(0.9,1.2,1.4);\\ 0.84,0.46\end{pmatrix}$	$\begin{pmatrix}(0.6,0.7,0.8);\\ 0.45,0.26\end{pmatrix}$	$\begin{pmatrix}(0.7,1.0,1.2);\\ 0.59,0.47\end{pmatrix}$	$\begin{pmatrix}(0.8,1.0,1.3);\\ 0.82,0.48\end{pmatrix}$	$\begin{pmatrix}(0.6,0.7,0.8);\\ 0.78,0.63\end{pmatrix}$
$Ŧ_4$	$\begin{pmatrix}(0.3,0.6,0.9);\\ 0.60,0.68\end{pmatrix}$	$\begin{pmatrix}(0.1,0.3,0.4);\\ 0.66,0.38\end{pmatrix}$	$\begin{pmatrix}(0.2,0.5,0.7);\\ 0.40,0.79\end{pmatrix}$	$\begin{pmatrix}(0.4,0.6,0.8);\\ 0.69,0.12\end{pmatrix}$	$\begin{pmatrix}(0.5,0.6,0.7);\\ 0.36,0.26\end{pmatrix}$
$Ŧ_5$	$\begin{pmatrix}(1.7,2.0,2.3);\\ 0.71,0.58\end{pmatrix}$	$\begin{pmatrix}(0.3,0.4,0.6);\\ 0.79,0.82\end{pmatrix}$	$\begin{pmatrix}(0.5,0.7,1.0);\\ 0.70,0.47\end{pmatrix}$	$\begin{pmatrix}(0.2,0.4,0.6);\\ 0.48,0.60\end{pmatrix}$	$\begin{pmatrix}(0.1,0.2,0.4);\\ 0.78,0.13\end{pmatrix}$

Step 11: When evaluating appropriate modulation techniques, use Eqs (23) and (24) in Table 10 to determine the T(p,q)-FNs' ideal and anti-ideal ratings, which are given as

$$\zeta^{+}=\left\{\begin{pmatrix}(0.9,1.2,1.4);\\0.84,0.46\end{pmatrix},\begin{pmatrix}(0.4,0.6,0.7);\\0.79,0.26\end{pmatrix},\begin{pmatrix}(0.5,0.7,1.0);\\0.70,0.47\end{pmatrix},\begin{pmatrix}(0.8,1.0,1.3);\\0.82,0.48\end{pmatrix},\begin{pmatrix}(0.6,0.7,0.8);\\0.78,0.63\end{pmatrix}\right\},$$

$$\zeta^{-}=\left\{\begin{pmatrix}(1.2,1.4,1.7);\\0.41,0.94\end{pmatrix},\begin{pmatrix}(0.1,0.4,0.7);\\0.66,0.82\end{pmatrix},\begin{pmatrix}(0.5,0.7,1.2);\\0.70,0.87\end{pmatrix},\begin{pmatrix}(0.2,0.4,0.6);\\0.48,0.60\end{pmatrix},\begin{pmatrix}(0.4,0.5,0.6);\\0.78,0.83\end{pmatrix}\right\}.$$

Step 12: Table 10 and Eq (16) are used to determine the score value for all the alternatives listed in Table 11.

Table 11. Score value of weighted normalized T(p,q)-FNs decision matrix.

Alternatives	$\mathscr{k}_1$	$\mathscr{k}_2$	$\mathscr{k}_3$	$\mathscr{k}_4$	$\mathscr{k}_5$
$Ŧ_1$	-0.2093	-0.0739	-0.1924	0.2578	0.0026
$Ŧ_2$	-1.0272	0.2785	0.1968	0.0830	0.0198
$Ŧ_3$	0.6323	0.0624	0.1467	0.5228	0.2311
$Ŧ_4$	0.0016	0.0730	-0.1568	0.2044	0.0255
$Ŧ_5$	0.5092	0.0184	0.2078	-0.0063	0.1157

Steps 13 and 14: Lastly, Eqs (25)–(27) were used to derive the utility degrees and functions. Table 12 displays each alternative's overall ranking.

Table 12. Final ranking of the MARCOS method.

Alternatives	$Ֆ_i$	$\bar{Й}_i^{+}$	$\bar{Й}_i^{-}$	$\boldsymbol{\Gamma}(\bar{Й})$	Ranking
$Ŧ_1$	-0.2152	-0.1149	0.1659	-0.0449	Fourth
$Ŧ_2$	-0.4490	-0.2398	0.3461	-0.0936	Fifth
$Ŧ_3$	1.5953	0.8520	-1.2298	0.3325	First
$Ŧ_4$	0.1477	0.0789	-0.1139	0.0308	Third
$Ŧ_5$	0.8449	0.4512	-0.6513	0.1761	Second

The modulation techniques' ranking orders, as shown in Table 12, are, $Ŧ_3 > Ŧ_5 > Ŧ_4 > Ŧ_1 > Ŧ_2$. Therefore, among the various possibilities for evaluating modulation techniques, the QAM ($Ŧ_3$) is the best option with the largest utility functions.

Algorithm 2: The T(p,q)-FNDWA operator approach

The first two steps are the same as in Algorithm 1.

Step 3: To apply the T(p,q)-FNDWA operator with $Ł = 2$ and the criteria weight vector $\Theta = \{0.29,0.14,0.24,0.21,0.12\}$, we aggregate the decision matrix (see Table 13).

Table 13. Aggregated value of the decision matrix using the T(p,q)-FNDWA operator.

Alternatives	T(p,q)-FNDWA operator
$Ŧ_1$	$\big((2.19,3.33,4.71); 0.86,0.67\big)$
$Ŧ_2$	$\big((3.59,4.59,5.59); 0.81,0.25\big)$
$Ŧ_3$	$\big((3.11,4.59,5.59); 0.88,0.25\big)$
$Ŧ_4$	$\big((1.57,2.57,3.57); 0.74,0.12\big)$
$Ŧ_5$	$\big((2.83,3.83,4.83); 0.85,0.13\big)$

Step 4: Following data aggregation, we assess the score values (see Table 14).

Table 14. Score values of the T(p,q)-FNDWA operator.

Alternatives	T(p,q)-FNDWA operator	Ranking
$Ŧ_1$	1.4389	Fourth
$Ŧ_2$	2.4269	Second
$Ŧ_3$	3.1070	First
$Ŧ_4$	1.0458	Fifth
$Ŧ_5$	2.3415	Third

From Algorithms 1 and 2 we conclude that the best modulation technique is QAM ($Ŧ_3$). Figure 8 shows the graphical representation of the rankings of Algorithms 1 and 2.

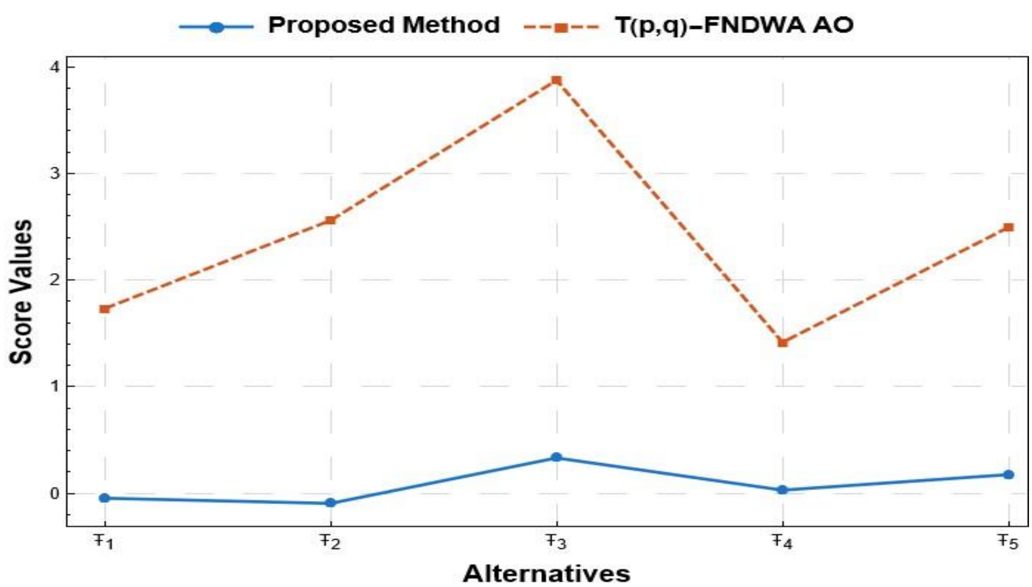

Figure 8. Graphical representation ranking of MARCOS and the T(p,q)-FNDWA operator.

The numerical results indicate that QAM ($Ŧ_3$) is the most preferred alternative among the considered techniques. This result is consistent with both the theoretical and practical expectations, as QAM offers a superior balance between a high data rate and spectral efficiency, which are critical in modern communication systems. Although QAM may involve relatively higher computational complexity, its ability to transmit more bits per symbol makes it highly efficient for bandwidth utilization. Furthermore, under the selected criteria, including BER, power efficiency, and data rate, QAM demonstrates strong overall performance compared with other techniques. Therefore, its top ranking reflects its practical suitability and widespread adoption in real-world wireless and digital communication applications.

7.1. Sensitivity analysis

The selection of the Dombi operational parameter Ł in this study is based on its role in controlling the flexibility and strictness of the aggregation operator. Specifically, the values Ł = 2, 3, 4, 5, 10, 20, 30, 40, and 50 were considered to explore a broad range of operator behaviors from more moderate aggregation (lower Ł) to more decisive or extreme aggregation (higher Ł). This range allows us to examine the sensitivity of the CRITIC-MARCOS model to Ł and provides guidance for practitioners to calibrate Ł according to the desired level of conservatism or responsiveness in MCDM under the T(p,q)-FN environment.

Stability, consistency, and robustness of the proposed T(p,q)-FNDWA-based decision-making framework. To ensure a comprehensive evaluation, the model is applied using a wide range of parameter values, namely $Ł = 2, 3, 4, 5, 10, 20, 30, 40,$ and 50. The corresponding ranking results are illustrated in Figure 9 and summarized in Tables 15 and 16. The results show that changes in Ł influence the aggregated performance values of the alternatives, reflecting the role of this parameter in controlling the aggregation behavior. However, despite these numerical variations, the overall ranking pattern remains largely stable. Two ranking orders are observed: For Ł = 1, the ranking is $Ŧ_3 > Ŧ_5 > Ŧ_2 > Ŧ_1 > Ŧ_4$, while for Ł = 2, 3, 4, 5, 10, 20, 30, 40, and 50, the ranking becomes $Ŧ_3 > Ŧ_2 > Ŧ_5 > Ŧ_1 > Ŧ_4$. This comparison clearly indicates that the most preferred alternative ($Ŧ_3$) consistently retains the top position across all parameter values, while the least preferred alternative ($Ŧ_4$) remains unchanged at the last position. This strong consistency at both ends demonstrates the robustness of the proposed model. The only variation occurs between alternatives $Ŧ_2$ and $Ŧ_5$, whose positions interchange when moving from Ł = 1 to higher values. This behavior suggests that these alternatives have very close performance scores and are therefore more sensitive to parameter changes.

Moreover, as Ł increases, the ranking stabilizes into a consistent pattern, indicating that higher values of the parameter enhance the reliability of the aggregation process by reducing sensitivity to small fluctuations in the data. Importantly, no significant rank reversal is observed beyond this minor interchange, and the final decision outcome remains unaffected. Overall, the analysis confirms that the proposed method is robust, stable, and reliable, producing consistent rankings even under varying parameter conditions.

Table 15. Sensitivity analysis.

Ł	$\mathbb{S}(Ŧ_1)$	$\mathbb{S}(Ŧ_2)$	$\mathbb{S}(Ŧ_3)$	$\mathbb{S}(Ŧ_4)$	$\mathbb{S}(Ŧ_5)$
1	1.4485	2.0626	3.6517	1.2619	2.3130
2	1.7318	2.5590	3.8719	1.4161	2.4944
3	1.9096	2.8422	3.9670	1.5070	2.6136
4	2.0262	3.0079	4.0164	1.5644	2.6930
5	2.1057	3.1125	4.0460	1.6032	2.7470
10	2.2780	3.3228	4.1041	1.6901	2.8620
20	2.3633	3.4237	4.1326	1.7357	2.9187
30	2.3911	3.4565	4.1420	1.7510	2.9373
40	2.4048	3.4727	4.1467	1.7586	2.9465
50	2.4130	3.4824	4.1495	1.7631	2.9520

Table 16. Ranking of sensitivity analysis.

Ł	Ranking	Best one
1	$Ŧ_3 > Ŧ_5 > Ŧ_2 > Ŧ_1 > Ŧ_4$	$Ŧ_3$
2	$Ŧ_3 > Ŧ_2 > Ŧ_5 > Ŧ_1 > Ŧ_4$	$Ŧ_3$
3	$Ŧ_3 > Ŧ_2 > Ŧ_5 > Ŧ_1 > Ŧ_4$	$Ŧ_3$
4	$Ŧ_3 > Ŧ_2 > Ŧ_5 > Ŧ_1 > Ŧ_4$	$Ŧ_3$
5	$Ŧ_3 > Ŧ_2 > Ŧ_5 > Ŧ_1 > Ŧ_4$	$Ŧ_3$
10	$Ŧ_3 > Ŧ_2 > Ŧ_5 > Ŧ_1 > Ŧ_4$	$Ŧ_3$
20	$Ŧ_3 > Ŧ_2 > Ŧ_5 > Ŧ_1 > Ŧ_4$	$Ŧ_3$
30	$Ŧ_3 > Ŧ_2 > Ŧ_5 > Ŧ_1 > Ŧ_4$	$Ŧ_3$
40	$Ŧ_3 > Ŧ_2 > Ŧ_5 > Ŧ_1 > Ŧ_4$	$Ŧ_3$
50	$Ŧ_3 > Ŧ_2 > Ŧ_5 > Ŧ_1 > Ŧ_4$	$Ŧ_3$

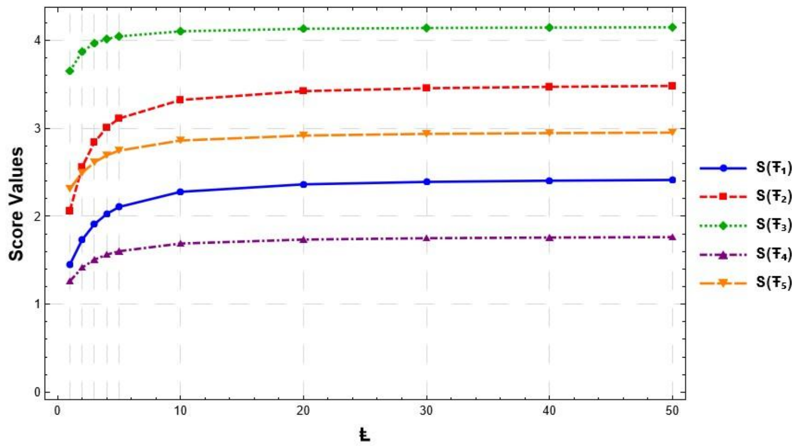

Figure 9. Graphical representation of the sensitivity analysis.

7.2. Comparative analysis

A numerical example was conducted to compare the proposed T(p,q)-FN CRITIC-MARCOS approach with several established MCDM methods, including TOPSIS [64], grey relational analysis (GRA) [65], EDAS [26], CoCoSo [63], weighted aggregated sum product assessment (WASPAS) [66], and VIKOR [45]. All approaches were applied using the same input data to ensure consistency in the evaluation process. The results of the computation are presented in Table 17 and Figure 10, while Table 18 summarizes the ranking outcomes.

The analysis shows that alternative $Ŧ_3$ consistently receives the highest priority ranking in most methods, confirming that the proposed framework produces reliable and reasonable decision results. While the best and worst alternatives are identical across all approaches, minor deviations are observed in the intermediate rankings. These deviations are attributable to the inherent differences in the aggregation and compromise mechanisms among the MCDM techniques, as well as their distinct sensitivity to criteria interactions and handling of uncertainty. For example, methods like TOPSIS and EDAS evaluate relative performance in a different way from MARCOS, which integrates compromise ranking with objective criterion weighting under the T(p,q)-FN environment.

To quantify the overall consistency, Pearson correlation coefficients were calculated for the ranking results (Figure 11). All coefficients are ≥0.80, indicating strong agreement between the proposed model and existing approaches. This confirms that the T(p,q)-FN CRITIC-MARCOS method is highly consistent, while its unique use of the Dombi-based aggregation operator enhances flexibility, robustness, and the ability to handle uncertainty, thereby providing more reliable and adaptable decision-making outcomes compared with conventional MCDM methods.

Table 17. Comparison results.

Methods	$Ŧ_1$	$Ŧ_2$	$Ŧ_3$	$Ŧ_4$	$Ŧ_5$
Proposed method	-0.0449	-0.0936	0.3325	0.0308	0.1761
T(p,q)-FNDWA AO	1.7318	2.5590	3.8719	1.4161	2.4944
TOPSIS method [64]	0.6334	0.5322	0.8274	0.5962	0.7316
GRA method [65]	0.5480	0.4920	0.6510	0.5259	0.5932
EDAS method [26]	0.2026	0.0750	0.9060	0.1031	0.5158
CoCoSo method [63]	1.6767	2.3999	2.7351	1.4301	2.1406
WASPAS method [66]	0.9860	1.8802	2.3545	0.7495	1.5113
VIKOR method [45]	0.6645	0.7803	0.8443	0.5645	0.7312

Table 18. Comparison ranking.

Methods	Ranking	Best one modulation technique
Proposed method	$Ŧ_3 > Ŧ_5 > Ŧ_4 > Ŧ_1 > Ŧ_2$	QAM ($Ŧ_3$)
T(p,q)-FNDWA AO	$Ŧ_3 > Ŧ_2 > Ŧ_5 > Ŧ_1 > Ŧ_4$	QAM ($Ŧ_3$)
TOPSIS method [64]	$Ŧ_3 > Ŧ_5 > Ŧ_1 > Ŧ_4 > Ŧ_2$	QAM ($Ŧ_3$)
GRA method [65]	$Ŧ_3 > Ŧ_5 > Ŧ_1 > Ŧ_4 > Ŧ_2$	QAM ($Ŧ_3$)
EDAS method [26]	$Ŧ_3 > Ŧ_5 > Ŧ_1 > Ŧ_4 > Ŧ_2$	QAM ($Ŧ_3$)
CoCoSo method [63]	$Ŧ_3 > Ŧ_2 > Ŧ_5 > Ŧ_1 > Ŧ_4$	QAM ($Ŧ_3$)
WASPAS method [66]	$Ŧ_3 > Ŧ_2 > Ŧ_5 > Ŧ_1 > Ŧ_4$	QAM ($Ŧ_3$)
VIKOR method [45]	$Ŧ_3 > Ŧ_2 > Ŧ_5 > Ŧ_1 > Ŧ_4$	QAM ($Ŧ_3$)

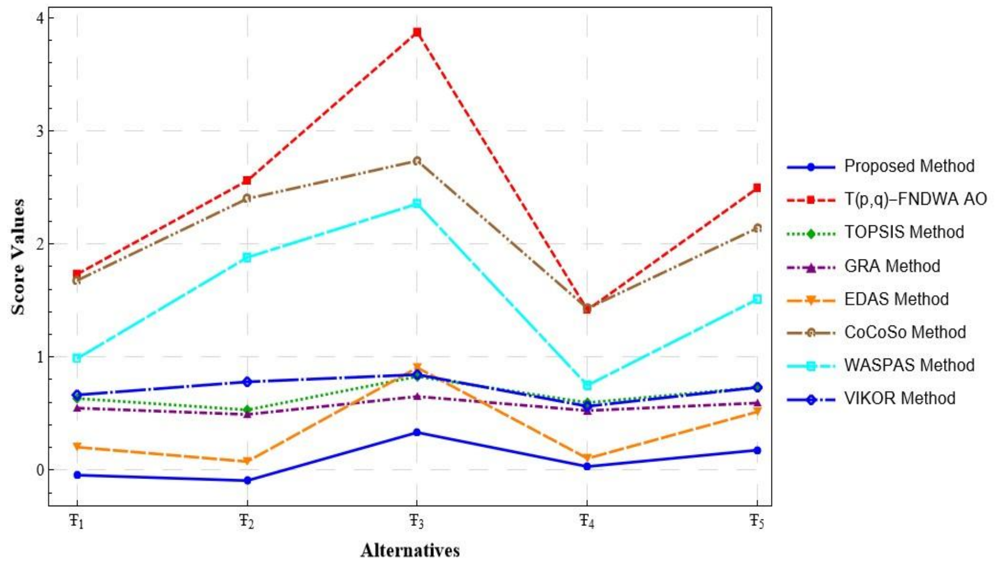

Figure 10. Comparative analysis of the proposed technique with existing methods.

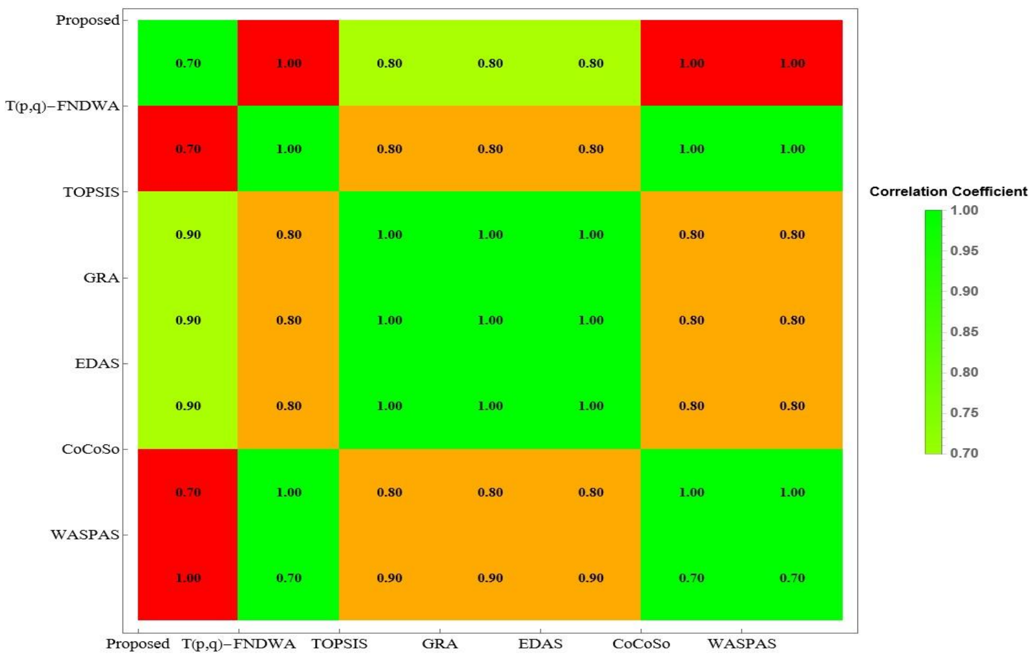

Figure 11. Pearson correlation coefficients.

7.3. Advantages of the proposed method

i. Integrates CRITIC and MARCOS under T(p, q)-FNs for a more robust decision-making framework;
ii. Effectively handles uncertainty and vagueness using T(p, q)-FN representation;
iii. Incorporates the Dombi aggregation operator to enhance flexibility in information fusion;
iv. Provides an objective weighting of criteria through the CRITIC method, reducing bias;
v. Improves ranking accuracy and reliability via the MARCOS method;

vi. Offers a systematic and efficient approach for selecting a modulation technique.

7.4. Limitations of the proposed method

i. The computational complexity increases with the number of alternatives and criteria due to the combined CRITIC-MARCOS framework under T(p,q)-FNs.
ii. The accuracy of results is sensitive to the selection of fuzzy parameters and the Dombi aggregation operator, requiring careful tuning.
iii. Reliance on expert judgments may introduce subjectivity, affecting the robustness of the decision-making process.
iv. Scalability to very large or high-dimensional datasets may be challenging without optimization techniques.
v. Interpretation of the (p, q)-FNs and aggregation results demands a certain level of expertise, potentially limiting broader practical adoption.

8. Conclusions

In order to select the best modulation approach, this research suggests an integrated selection framework. For the modulation technique selection problem, we first determined the particular criterion system and alternatives. Then, the T(p,q)-FNs are introduced to establish the scale, which solves the problem of uncertain decision information expression in the modulation technique selection process and provides a way to describe subjective preferences. Next, we presented a weighted average aggregation operator based on Dombi t-norms and new fundamental operational laws. The most suitable decision for the modulation technique was then identified by combining the MARCOS and CRITIC methods. This combined approach can show how the criteria relate to one another. Finally, the application of the proposed framework is demonstrated through a case study with selected modulation techniques, and sensitivity and comparative studies are conducted to confirm the performance of the proposed framework.

Future work will extend the proposed model to applications such as medical diagnostics, healthcare, finance, and pattern recognition. The framework can be generalized to advanced fuzzy environments, including triangular (p,q)-hesitant fuzzy sets and Type 3 fuzzy systems. Additionally, new aggregation operators (e.g., Hamacher, Einstein, Frank, Yager, Aczel–Alsina) will be explored. Integration with machine learning models, particularly neural networks [67,68], is also considered to improve scalability and performance in complex decision-making problems.

Author contributions

Muhammad Ismail: Conceptualization, methodology, formal analysis, writing original draft preparation; Darjan Karabasevic: Methodology, software, writing review and editing, supervision; Pavle Brzakovic: Software, software, validation, investigation, writing review and editing; Asghar Khan: Conceptualization, validation, formal analysis, writing original draft preparation; Tahir Zaman: Software, validation, investigation, writing review and editing. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflicts of interest

The authors declare no conflicts of interest.

References

1. D. Wang, M. Su, Y. Ren, J. Zuo, Y. Jin, C. Xu, Phase synchronization algorithm for high-speed space optical communication modulation baseband signal based on FPGA, In: *2024 International conference on optoelectronic information and optical engineering (OIOE 2024)*, **13182** (2024), 74–81. https://doi.org/10.1117/12.3030365
2. Y. Niu, Z. Wei, L. Wang, H. Wu, Z. Feng, Interference management for integrated sensing and communication systems: A survey, *IEEE Internet Things J.*, **12** (2025), 8110–8134. https://doi.org/10.1109/JIOT.2024.3506162
3. A. O. O. Esho, T. D. Iluyomade, T. M. Olatunde, O. P. Igbinenikaro, A comprehensive review of energy-efficient design in satellite communication systems, *Int. J. Eng. Res. Updates*, **6** (2024)., 013–025. https://doi.org/10.53430/ijeru.2024.6.2.0024
4. P. Patel, S. Pampaniya, A. Ghosh, R. Raj, D. Karuppaih, S. Kandasamy, Enhancing accessibility through machine learning: A review on visual and hearing impairment technologies, *IEEE Access*, **13** (2025), 33286–33307. https://doi.org/10.1109/ACCESS.2025.3539081
5. İ. Kaya, M. Çolak, F. Terzi, Use of MCDM techniques for energy policy and decision-making problems: A review, *Int. J. Energy Res.*, **42** (2018), 2344–2372. https://doi.org/10.1002/er.4016
6. C. Zhang, P. Xia, X. Zhang, Multi-attribute decision-making method of pumped storage capacity planning considering wind power uncertainty, *J. Clean. Prod.*, **449** (2024), 141655. https://doi.org/10.1016/j.jclepro.2024.141655
7. R. M. A. Ikram, S. G. Meshram, M. A. Hasan, X. Cao, E. Alvandi, C. Meshram, et al., The application of multi-attribute decision making methods in integrated watershed management, *Stoch. Environ. Res. Risk Assess.*, **38** (2024), 297–313. https://doi.org/10.1007/s00477-023-02557-3
8. L. A. Zadeh, Fuzzy sets, *Inf. Control*, **8** (1965), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X
9. B. Oztaysi, S. C. Onar, S. Cebi, C. Kahraman, Fuzzy MCDM approaches in sustainability research: A literature review, In: *Intelligent and fuzzy systems*, Cham: Springer, 2025. https://doi.org/10.1007/978-3-031-97985-9_92
10. J. Khan, A. Ishizaka, M. Z. Babai, Enhancing multi-criteria inventory classification: Resolving boundary issues with VIKOR-fuzzy sorting, *Int. J. Prod. Econ.*, **281** (2025), 109526. https://doi.org/10.1016/j.ijpe.2025.109526
11. K. T. Atanassov, *On intuitionistic fuzzy sets theory*, Springer, 2012. https://doi.org/10.1007/978-3-642-29127-2

12. R. R. Yager, A. M. Abbasov, Pythagorean membership grades, complex numbers, and decision making, *Int. J. Intell. Syst.*, **28** (2013), 436–452. https://doi.org/10.1002/int.21584
13. H. Z. Ibrahim, T. M. Al-Shami, O. G. Elbarbary, (3, 2)-fuzzy sets and their applications to topology and optimal choices, *Comput. Intell. Neurosci.*, **2021** (2021), 1272266. https://doi.org/10.1155/2021/1272266
14. T. Senapati, R. R. Yager, Fermatean fuzzy sets, *J. Ambient Intell. Human. Comput.*, **11** (2020), 663–674. https://doi.org/10.1007/s12652-019-01377-0
15. K. H. Murad, H. Z. Ibrahim, (3, 4)-fuzzy sets and their topological spaces, *J. Math. Comput. Sci.*, **28** (2022), 158–170. http://dx.doi.org/10.22436/jmcs.028.02.04
16. R. R. Yager, Generalized orthopair fuzzy sets, *IEEE Trans. Fuzzy Syst.*, **25** (2016), 1222–1230. https://doi.org/10.1109/TFUZZ.2016.2604005
17. H. Z. Ibrahim, I. Alshammari, n, m-Rung orthopair fuzzy sets with applications to multicriteria decision making, *IEEE Access*, **10** (2022), 99562–99572. https://doi.org/10.1109/ACCESS.2022.3207184
18. S. Malik, S. C. Malik, N. Nandal, A. D. Yadav, Reliability assessment of a NSP system under constant triangular fuzzy failure rates, *Int. J. Ind. Syst. Eng.*, **49** (2025), 405–420. https://doi.org/10.1504/IJISE.2025.146064
19. X. Sun, D. Wang, Conflict analysis of disputes in livelihood vulnerability assessment of flood using fuzzy TOPSIS method and GMCR with triangular fuzzy numbers, *Sci. Rep.*, **15** (2025), 8609. https://doi.org/10.1038/s41598-025-93456-w
20. L. Wang, S. Abdullah, A. A. Rahimzai, I. Ullah, A novel fuzzy neural network approach with triangular fuzzy information for the selection of logistics service providers, *Artif. Intell. Rev.*, **58** (2025), 213. https://doi.org/10.1007/s10462-025-11209-7
21. J. Dong, S. Wan, S. M. Chen, Fuzzy best-worst method based on triangular fuzzy numbers for multi-criteria decision-making, *Inf. Sci.*, **547** (2021), 1080–1104. https://doi.org/10.1016/j.ins.2020.09.014
22. S. P. Wan, F. Wang, L. L. Lin, J. Y. Dong, Some new generalized aggregation operators for triangular intuitionistic fuzzy numbers and application to multi-attribute group decision making, *Comput. Ind. Eng.*, **93** (2016), 286–301. https://doi.org/10.1016/j.cie.2015.12.027
23. A. Fahmi, A. Hashmi, A. Khan, A. Mukheimer, T. Abdeljawad, R. Thinakaran, Triangular intuitionistic fuzzy frank aggregation for efficient renewable energy project selection, *Eur. J. Pure Appl. Math.*, **18** (2025), 6227–6227. https://doi.org/10.29020/nybg.ejpam.v18i3.6227
24. P. Sharma, M. K. Mehlawat, S. Verma, P. Gupta, Multi-attribute group decision-making in site selection of solar photovoltaic cells under triangular Pythagorean fuzzy environment, *Soft Comput.*, 2023. https://doi.org/10.1007/s00500-023-09009-8
25. M. Akram, S. M. U. Shah, M. M. A. Al-Shamiri, S. A. Edalatpanah, Extended DEA method for solving multi-objective transportation problem with Fermatean fuzzy sets, *AIMS Mathematics*, **8** (2023), 924–961. https://doi.org/10.3934/math.2023045
26. A. Khan, S. Islam, M. Ismail, A. Alotaibi, Development of a triangular Fermatean fuzzy EDAS model for remote patient monitoring applications, *Sci. Rep.*, **15** (2025), 22073. https://doi.org/10.1038/s41598-025-00914-6
27. B. Wan, R. Lu, M. Han, Weighted average LINMAP group decision-making method based on q-rung orthopair triangular fuzzy numbers, *Granul. Comput.*, **7** (2022), 489–503. https://doi.org/10.1007/s41066-021-00280-4

28. J. Dombi, A general class of fuzzy operators, the demorgan class of fuzzy operators and fuzziness measures induced by fuzzy operators, *Fuzzy Sets Syst*, **8** (1982), 149–163. https://doi.org/10.1016/0165-0114(82)90005-7
29. P. Liu, J. Liu, S. M. Chen, Some intuitionistic fuzzy Dombi Bonferroni mean operators and their application to multi-attribute group decision making, *J. Oper. Res. Soc.*, 2017. https://doi.org/10.1057/s41274-017-0190-y
30. M. Akram, W. A. Dudek, J. M. Dar, Pythagorean Dombi fuzzy aggregation operators with application in multicriteria decision-making, *Int. J. Intell. Syst.*, **34** (2019), 3000–3019. https://doi.org/10.1002/int.22183
31. J. Ali, Z. Mehmood, p, q-Quasirung orthopair fuzzy multi-criteria group decision-making algorithm based on generalized Dombi aggregation operators, *J. Appl. Math. Comput.*, **71** (2025), 69–102. https://doi.org/10.1007/s12190-024-02227-9
32. Z. Ali, Circular pq-quasirung orthopair fuzzy sets with Dombi power aggregation operators: Application to renewable natural gas, *Int. J. Res. Ind. Eng.*, **14** (2025), 466–490. https://doi.org/10.22105/riej.2025.489830.1497
33. T. Senapati, G. Chen, I. Ullah, M. S. A. Khan, F. Hussain, A novel approach towards multiattribute decision making using q-rung orthopair fuzzy Dombi–Archimedean aggregation operators, *Heliyon*, **10** (2024), 27969. https://doi.org/10.1016/j.heliyon.2024.e27969
34. N. Yaqoob, M. Gulistan, M. M. Abbas, K. Hayat, M. M. Al-Shamiri, Dombi aggregation operator in terms of complex bipolar fuzzy sets with application in decision making problems, *Complex Intell. Syst.*, **11** (2025), 483. https://doi.org/10.1007/s40747-025-02078-2
35. Z. Ali, M. Waqas, S. Moslem, T. Senapati, D. Esztergár-Kiss, Evaluation of public bus transport service quality based on circular Pythagorean fuzzy soft Einstein aggregation operators, *Complex Intell. Syst.*, **11** (2025), 276. https://doi.org/10.1007/s40747-025-01864-2
36. S. Ullah, S. Khan, A. A. Rahimzai, S. Abdullah, M. Ismail, H. Ullah, Application of fractional fuzzy soft sets using hamacher aggregation operators in agriculture robots selection, *Artif Intell Rev.*, **58** (2025), 395. https://doi.org/10.1007/s10462-025-11329-0
37. H. Dhumras, M. Kumar, R. K. Bajaj, Effectiveness of debris flow mitigation measures through T-spherical fuzzy soft Dombi aggregation operators with EDAS-based multi-criteria decision making in mountainous regions, *J. Ind. Inf. Integr.*, **48** (2025), 100968. https://doi.org/10.1016/j.jii.2025.100968
38. M. Pal, H. Dhumras, G. Garg, V. Shukla, On renewable energy source selection problem using T-spherical fuzzy soft Dombi aggregation operators, In: *Sustainable mobility: Policies, challenges and advancements*, 2024, 237–253. https://doi.org/10.1002/9781394166831.ch14
39. M. Akram, F. Ilyas, H. Garg, Multi-criteria group decision making based on ELECTRE I method in Pythagorean fuzzy information, *Soft Comput.*, **24** (2020), 3425–3453. https://doi.org/10.1007/s00500-019-04105-0
40. A. Almjally, M. Khan, M. Ismail, A. Khan, Broadband technology selection based on three-way decision-making model under the tripolar fuzzy information, *Int. J. Fuzzy Syst.*, 2025. https://doi.org/10.1007/s40815-025-02054-5
41. P. Liu, J. Shen, P. Zhang, B. Ning, Multi-attribute group decision-making method using single-valued neutrosophic credibility numbers with fairly variable extended power average operators and GRA-MARCOS, *Expert Syst. Appl.*, **263** (2025), 125703. https://doi.org/10.1016/j.eswa.2024.125703

42. Z. Ali, M. Waqas, K. Hila, Multi-attributive border approximation area comparison model based on Yager weighted aggregation operators for circular Pythagorean fuzzy information and their application in shortest path problems, *J. Supercomput.*, **81** (2025), 1308. https://doi.org/10.1007/s11227-025-07327-2
43. D. Diakoulaki, G. Mavrotas, L. Papayannakis, Determining objective weights in multiple criteria problems: The critic method, *Comput. Oper. Res.*, **22** (1995), 763–770. https://doi.org/10.1016/0305-0548(94)00059-H
44. M. Žižović, B. Miljković, D. Marinković, Objective methods for determining criteria weight coefficients: A modification of the CRITIC method, *Decis. Mak.: Appl. Manag. Eng.*, **3** (2020), 149–161. https://doi.org/10.31181/dmame2003149z
45. H. W. Wu, J. Zhen, J. Zhang, Urban rail transit operation safety evaluation based on an improved CRITIC method and cloud model, *J. Rail Transp. Plan. Manag.*, **16** (2020), 100206. https://doi.org/10.1016/j.jrtpm.2020.100206
46. B. Pan, S. Liu, Z. Xie, Y. Shao, X. Li, R. Ge, Evaluating operational features of three unconventional intersections under heavy traffic based on CRITIC method, *Sustainability*, **13** (2021), 4098. https://doi.org/10.3390/su13084098
47. N. Yalcin, U. Ünlü, A multi-criteria performance analysis of initial public offering (IPO) firms using CRITIC and VIKOR methods, *Technol. Econ. Dev. Econ.*, **24** (2018), 534–560. https://dx.doi.org/10.3846/20294913.2016.1213201
48. X. Peng, H. Huang, Fuzzy decision making method based on CoCoSo with critic for financial risk evaluation, *Technol. Econ. Dev. Econ.*, **26** (2020), 695–724. https://doi.org/10.3846/tede.2020.11920
49. X. Peng, X. Zhang, Z. Luo, Pythagorean fuzzy MCDM method based on CoCoSo and CRITIC with score function for 5G industry evaluation, *Artif. Intell. Rev.*, **53** (2020), 3813–3847. https://doi.org/10.1007/s10462-019-09780-x
50. R. Rostamzadeh, M. K. Ghorabaee, K. Govindan, A. Esmaeili, H. B. K. Nobar, Evaluation of sustainable supply chain risk management using an integrated fuzzy TOPSIS-CRITIC approach, *J. Clean. Prod.*, **175** (2018), 651–669. https://doi.org/10.1016/j.jclepro.2017.12.071
51. V. Dutta, S. Haldar, P. Kaur, Y. Gajpal, Comparative analysis of TOPSIS and TODIM for the performance evaluation of foreign players in Indian premier league, *Complexity*, **2022** (2022), 9986137. https://doi.org/10.1155/2022/9986137
52. Ö. Işık, A. Çalık, M. Shabir, A consolidated MCDM framework for overall performance assessment of listed insurance companies based on ranking strategies, *Comput. Econ.*, **65** (2025), 271–312. https://doi.org/10.1007/s10614-024-10578-5
53. F. Xiong, *Digital modulation techniques*, 2006.
54. J. Brodny, M. Tutak, Assessing the energy security of European Union countries from two perspectives–A new integrated approach based on MCDM methods, *Appl. Energy*, **347** (2023), 121443. https://doi.org/10.1016/j.apenergy.2023.121443
55. A. Saha, D. Pamucar, O. F. Gorcun, A. R. Mishra, Warehouse site selection for the automotive industry using a fermatean fuzzy-based decision-making approach, *Expert Syst. Appl.*, **211** (2023), 118497. https://doi.org/10.1016/j.eswa.2022.118497

56. Ž. Stević, D. Pamučar, A. Puška, P. Chatterjee, Sustainable supplier selection in healthcare industries using a new MCDM method: Measurement of alternatives and ranking according to COmpromise solution (MARCOS), *Comput. Ind. Eng.*, **140** (2020), 106231. https://doi.org/10.1016/j.cie.2019.106231
57. K.Ghasemi, M. Behzadfar, K. Borhani, Spatial analysis of leisure land uses in Tehran: Assessing inequity using the MARCOS method within a GIS framework, *Heliyon*, **9** (2023), e19691. https://doi.org/10.1016/j.heliyon.2023.e19691
58. S. Zeng, A. Ye, W. Su, M. Chen, C. Llopis-Albert, Site evaluation of subsea tunnels with sightseeing function based on dynamic complex MARCOS method, *Technol. Forecast. Soc. Chang.*, **199** (2024), 123041. https://doi.org/10.1016/j.techfore.2023.123041
59. S. Qu, X. Kong, A Bonferroni mean operator for p, q-Rung triangular Orthopair fuzzy environments and its application in COPRAS method, *Symmetry*, **17** (2025), 1422. https://doi.org/10.3390/sym17091422
60. M. Hajiaghaei-Keshteli, Z. Cenk, B. Erdebilli, Y. S. Özdemir, F. Gholian-Jouybari, Pythagorean fuzzy TOPSIS method for green supplier selection in the food industry, *Expert Syst. Appl.*, **224** (2023), 120036. https://doi.org/10.1016/j.eswa.2023.120036
61. I. Ullah, S. Abdullah, M. Nawaz, Analyzing the traffic light control systems based on novel fuzzy neural network under triangular fuzzy numbers, *Signal Image Video Process.*, **19** (2025), 1169. https://doi.org/10.1007/s11760-025-04697-1
62. D. Rudolph, Modulation methods, In: *Fundamentals of RF and microwave techniques and technologies*, Cham: Springer, 2023, 1297–1508. https://doi.org/10.1007/978-3-030-94100-0_14
63. M. Khan, A. Khan, M. Ismail, R. A. Ziar, A development of the circular non-linear Diophantine fuzzy set and their application in cloud services provider selection, *Int. J. Comput. Intell. Syst.*, **19** (2026), 13. https://doi.org/10.1007/s44196-025-01079-w
64. S. S. Majd, A. Maleki, S. Basirat, A. Golkarfard, Fermatean fuzzy TOPSIS method and its application in ranking business intelligence-based strategies in smart city context, *J. Oper. Intell.*, **3** (2025), 1–16. https://doi.org/10.31181/jopi31202532
65. S. Abdullah, M. Nawaz, N. Ali, S. Khan, A novel linguistic pq-rung orthopair fuzzy framework for decision-making using extended TOPSIS and enhanced GRA approaches, *Int. J. Dynam. Control*, **14** (2026), 1. https://doi.org/10.1007/s40435-025-01907-z
66. S. Abdullah, H. Ali, A. A. Rahimzai, S. Khan, Novel concept of linguistic fractional fuzzy information for effective water filtration decision-making problem based on WASPAS method, *Sci. Rep.*, **15** (2025), 33169. https://doi.org/10.1038/s41598-025-15817-9
67. M. Nawaz, S. Abdullah, I. Ullah, An integrated fuzzy neural network model for surgical approach selection using double hierarchy linguistic information, *Comput. Biol. Med.*, **186** (2025), 109606. https://doi.org/10.1016/j.compbiomed.2024.109606
68. S. Abdullah, I. Ullah, F. Ghani, Heterogeneous wireless network selection using feed forward double hierarchy linguistic neural network, *Artif. Intell. Rev.*, **57** (2024), 191. https://doi.org/10.1007/s10462-024-10826-y

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