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Research article

Bayesian analysis of a novel sophisticated stochastic volatility model: A comprehensive empirical application to stock and crude oil markets price returns

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Abstract: This study primarily aimed to apply a novel, advanced stochastic volatility model incorporating leverage and an asymmetrically heavy-tailed distribution (ASV-GHskw) to 18 financial asset returns, including 10 Asia-Pacific and six European stock market returns and two crude oil market returns. The study employed a generalized hyperbolic skew student's *t*-distribution using the Bayesian approach. The study's secondary aim was to concentrate on one-day-ahead downside market risk measurement of the related financial asset returns under Basel IV regulations based on the ASV-GHskw model. For comparison, an asymmetric stochastic volatility model with a symmetric student's *t*-distribution (ASV-st) was also considered. The findings indicated that the proposed Bayesian Markov chain Monte Carlo (MCMC) approach was sufficiently efficient, especially when estimating the ASV-GHskw model parameters. Further, the ASV-GHskw model provided a better fit for all financial asset returns examined than its competing ASV-st model. Moreover, the results revealed that the financial institutions taking positions in the examined financial markets will be required to hold additional regulatory capital for market risk exposures under the latest Basel capital regulations. Lastly, the findings give valuable information for risk-averse investors interested in the examined financial markets, as they expect to receive higher-than-average returns by holding less-risky assets.

Keywords: stock markets; crude oil markets; Bayesian approach; generalized hyperbolic skew Student's *t*-distribution; asymmetric stochastic volatility
Mathematics Subject Classification: 62F15, 91B70, 91Gxx, 91G70

1. Introduction

Understanding the dynamics of stock market return volatility is crucial for making accurate and efficient financial decisions in areas such as asset pricing, hedging, market risk measurement, derivative pricing, and asset allocation [1,2]. Additionally, to achieve the desired financial stability and implement efficient economic and financial policies, policymakers must have a deep understanding of volatility behavior, including the frequency and magnitude of fluctuations in stock market returns [3]. However, global stock market volatility has increased significantly, especially in recent decades, due to changing global liquidity and financial conditions [4], US dollar fluctuations [5], rising speculative trading [6,7], substantial fluctuations in energy markets [8], the COVID-19 outbreak [9], and the Russia-Ukraine war [10], among others. Hence, modeling and forecasting volatility of stock market returns has drawn considerable attention among researchers and practitioners.

Generalized autoregressive conditional heteroskedasticity (GARCH) and stochastic volatility (SV) models are popular competing models used in the literature to capture the volatility dynamics of various financial asset returns. However, because of their theoretically attractive frameworks, the SV models can more flexibly and accurately capture the fundamental empirical properties that financial asset returns exhibit in real financial markets [11,12]. For instance, whereas GARCH-type models describe conditional volatility as a deterministic consequence of past data and model parameters, SV models identify conditional volatility more realistically as a latent stochastic process resulting solely from flow of new information into financial markets [13,14]. GARCH models consider only one error term, which is placed in the conditional mean equation to only consider return shocks, whereas SV models additionally consider separate error terms for the conditional mean and conditional variance equations to account for volatility shocks [15]; therefore, SV models are more flexible [16,17]. As an essential extension of this flexibility, they do not need any moment restrictions, suggesting that SV models display a more adequate forecasting performance compared to their competing GARCH-type models [18–20]. Therefore, as notably reported in the related literature, the SV models can more adequately capture the complex volatility dynamics of financial asset returns compared to their counterpart GARCH-type models [21,22].

For example, Kim et al. [22] find that the basic SV model fits exchange rate return data better than GARCH-type models [22]. Tiwari et al. [23] indicate that SV models tend to outperform GARCH-type models for Bitcoin and Litecoin return data [23]. Cassagnes et al. [24] show that SV models better align option prices with market realities [24]. Therefore, studies increasingly examine the performance of alternative SV models across various financial variables, including stock markets in the Asia-Pacific region and Europe, and crude oil markets [25–27].

The promising performance of SV models has also spurred the development of more general, flexible, and advanced SV models [28–31]. For example, Shi et al. [32] introduce a new fractional SV model and applied it to the US stock markets. Bank et al. [33] propose a non-Markovian local SV model and apply it to price a European put option. Otto et al. [34] emphasize the methodologically appealing frameworks of spatial and spatiotemporal SV models. Lastly, Yang et al. [35] introduce a new valuation method for rough SV models, corresponding to a new type of SV model.

However, a Bayesian analysis of SV models shows that extending them to incorporate asymmetric heavy-tailed distribution is difficult, especially using generalized hyperbolic skew student's *t*-distribution (GHskw). Unlike the GARCH-type models, which are estimated using conventional techniques such as maximum likelihood or its close variants such as quasi-maximum likelihood, in a SV framework, its parameters are difficult to identify because of its many latent variables [36,37]. However, as GHskw distribution possesses considerable properties such as effectively capturing considerably heavy-tailed and skewed return data, a promising feature to better model the volatility dynamic of financial return series [38,39], several studies, based on empirical application of GHskw distribution, examine the extent to which it improves model fit to observed data [40–42].

Accordingly, Nakajima and Omori [37] introduce an asymmetric SV model (ASV-GHskw) [37], incorporating leverage and asymmetrically heavy-tailed distribution using GHskw distribution. Nakajima and Omori [37] propose a Bayesian framework for the ASV-GHskw model by adopting a novel MCMC sampler. They employ an expanded version of Omori and Watanabe's [43] multi-move-sampler approach for sampling unobservable latent logarithmic volatility in the ASV-GHskw model. They define the GHskw distribution as a normal variance-mean mixture of the generalized inverse gaussian (GIG) distribution to develop a novel MCMC algorithm that fits the ASV-GHskw model. Regarding the ASV-GHskw model performance, Nakajima and Omori [37] provide strong evidence that the proposed model performs better than the ASV-st model that they applied in their study, both for TOPIX (Tokyo Stock Price Index) and S&P500 (Standard and Poor's 500) data [37].

In this context, this study primarily aims to apply the ASV-GHskw model as proposed by Nakajima and Omori [37] to 18 financial asset returns. Especially after the recent period of global financial crisis, which began in the U.S. in 2007 as a banking crisis, well-known traditional market risk modeling techniques have faced increasing criticism for their failure to estimate market risk accurately [44,45]. Therefore, considering recent debates on market risk measurement approaches, the Basel Committee proposed replacing traditional Value-at-Risk (VaR) estimates with expected shortfall (ES) estimates, as a new market risk measurement method under the Basel IV regulations to more appropriately address heavy-tailedness [46,47]. The ES approach is also defined as a coherent risk measure in the financial econometrics literature [48,49]. Consequently, as a new technique for calculating market risk of various financial variables, we add the one-day-ahead downside VaR and ES estimates based on the ASV-GHskw model. This is because, first, the Bayesian MCMC estimation scheme fully and coherently incorporates both parameter and model uncertainties, and its VaR and ES estimations are more efficient and coherent than those generated by GARCH-type models estimated based on conventional techniques such as maximum likelihood or its close variants such as quasi-maximum likelihood estimation [36,50]. Second, compared to the traditional VaR estimates, although ES estimates allow heavy-tailedness more adequately, the distribution's tail shape affects ES estimates significantly, especially when substantial observations are made at the tail end of the distribution, because these estimates simply average the losses beyond the VaR for a given significance level [51,52]. Therefore, the tails of distributions should be modeled adequately to obtain more accurate ES estimates. As aforementioned, as GHskw distribution can account for the more substantially skewed heavy-tailedness observed in financial assets returns innovations, the SV models consolidation based on the Bayesian MCMC estimation scheme and GHskw distribution can provide risk managers of financial institutions, their regulators, and the policy makers with the desirable properties needed to more adequately model the volatility dynamics of financial asset returns, thus enabling more accurate market risk measurement. Hence, the secondary aim of this study is to estimate the one-day-ahead downside market risk of related financial markets returns based on the ASV-GHskw model considering ES estimates under Basel IV regulations. For comparison, VaR estimates based on the ASV-GHskw model under Basel III regulations are also presented.

This study's contributions are as follows: First, to the best of our knowledge, the ASV-GHskw model is applied for the first time to the 18 financial asset returns, including relevant stock market returns and crude oil market returns. Therefore, we could explore whether the MCMC algorithm that they introduce works well for related financial asset returns. Second, the analysis provides new insights into the volatility characteristics of the related financial asset returns, thus providing essential information not only about their return generating mechanism but also for better financial risk management applications, which are crucial for risk managers of financial institutions and international investors and policy makers [53,54]. This is because an adequate and accurate volatility parameter is needed for various efficient financial analysis, as well as, for properly gauging the consequences of the economic policy actions taken by policy makers, particularly those aimed at managing economic fluctuations, thus ensuring economic and financial stability. Third, in this study, the asymmetric SV model with symmetric heavy-tailed student's *t*-distribution (ASV-st) is also included to investigate whether the ASV-GHskw model makes any further improvement on the ASV-st model, because of student's *t*-distribution as commonly applied in the related literature (see, e.g., [55–58]. Fourth, as this is the first study that applies the ASV-GHskw model to related financial markets returns to measure the market risk under Basel IV regulations, therefore, we can determine whether Basel III and Basel IV regulations have significant differences concerning capital requirements for the market risks originating from the positions in related financial markets. This issue is crucial in practice, especially for banks, as the analysis would provide more evidence regarding whether Basel IV regulations will lead them to dedicate more capital to fulfill their obligations. The analysis also gives valuable information for rational risk-averse investors interested especially in equity markets, as they expect to receive higher-than-average returns by holding less risky assets, which in turn requires having more accurate estimates of the actual market risk level of the relevant equity markets. Fifth, scholars such as Saranya and Prasanna [59] highlight that a high degree of empirical studies on SV modeling only considers the developed equity markets. However, many macroeconomic and financial indicators may exhibit different behaviors in developed and emerging equity markets. Thus, considering both developed and emerging stock markets, as done in this study, may further provide beneficial information about to what extent the SV properties of emerging equity markets returns may be different than those of developed equity markets returns, particularly important for investors and policy makers [60,61]. Sixth, in the first step, we apply the ASV-GHskw model and ASV-st to 10 Asia-Pacific stock market returns, including five emerging and five developed stock market returns. However, for more robust, and, hence reliable results, we not only apply these models to the six European stock market return data, including three emerging European stock markets – the BUX index (Budapest Stock Exchange) (Hungary), the PX index (Prague Stock Exchange Index) (Czechia), and the WIG index (The Warsaw Stock Exchange Index) (Poland) – as well as three developed European stock markets – the DAX (Deutschen Aktienindex) performance index (Germany), the CAC 40 (Cotation Assisté en Continu 40) index (France), and the IBEX-35 index (Índice Bursátil Español) (Spain) but also to crude oil markets including West Texas Intermediate (WTI) crude oil daily spot returns and Brent crude oil daily spot returns.

We focus on energy commodity returns mainly because global economic activity is mostly driven by fossil fuels –including coal, petroleum, and natural gas consumption, as well as their derivatives –; energy commodities can be defined as the primary inputs for global economic activity [62,63]. Fluctuations in energy commodity prices may also lead to a significant increase in inflation uncertainty at the international level through the production cost channel [64]. Therefore, an in-depth understanding of the volatility of energy commodity returns will assist energy producers in the organization of their energy production capacity and the quantity of their investments [65]. This is important because unexpected high volatility in energy commodity markets may cause inefficient resource allocation and an inability to develop energy-efficiency technologies [66].

Energy commodities should not solely be considered as energy products but also as financial assets, providing important information for investors in such areas as portfolio management, market risk measurement, and hedging strategies [67]. Therefore, increasing volatility in energy commodity markets may adversely affect the value of energy commodity-based contingents claims, derivative valuation, and asset allocation [68].

Consequently, efficient and accurate estimates and forecasts of the volatility of energy commodity markets are among the main concerns among policymakers, risk managers, and international investors. Further, we primarily focus directly on daily WTI and Brent crude oil spot returns among alternative energy commodity returns, because as a fundamental input to international production, both WTI and Brent serve as an essential oil benchmark price internationally. Thus, remarkable volatility fluctuations in WTI and Brent returns could destabilize both financial and macroeconomic systems worldwide [69]. Indeed, Zhang et al. [70] and Caporin et al. [71] report that, historically, price and volatility dynamics that occurred in WTI and Brent crude oil markets demonstrate a close relationship. Moreover, recent research by Mensi et al. [72], which investigates volatility spillovers among six different energy commodity markets, including Brent and WTI crude oil, natural gas, gas oil, heating oil, and gasoline, shows strong evidence that the WTI crude oil market is the largest net contributor of volatility spillovers to the other energy commodity markets studied. This finding highlights the central role that WTI crude oil price fluctuations play in the energy commodity markets.

2. Data and methodology

2.1. *Data*

Initially, the ASV-GHskw and ASV-st models are applied to daily stock market returns, including five emerging and five developed Asia-Pacific stock markets. The former comprise the SSE (Shanghai Stock Exchange) Composite Index (People's Republic of China, China hereafter), the Nifty 50 Index (India), the TSEC (Taiwan Stock Exchange Capitalization) Weighted Index (Chinese Taipei, Taiwan hereafter), the SET (Stock Exchange of Thailand) Index (Thailand), and the IDX Indonesia Stock Exchange) Composite Index (Indonesia). The latter comprise the NIKKEI (Nihon Keizai Shimbun) 225 Index (Japan), the S&P/ASX (Australian Securities Exchange) 200 Benchmark Index (Australia), the FTSE Straits Times Index (Singapore), the Hang Seng Index (Hong Kong), and the KOSPI (Korea Composite Stock Price Index) data (The Republic of Korea, Korea hereafter). (The stock markets are classified within the Financial Times Stock Exchange (FTSE) equity country classification process as of 2023. For more detailed information, see https://research.ftserussell.com/ products/downloads/ FTSE-Country-Classification-Update_latest.pdf). All stock market indices examined are expressed in their respective local currencies.

We focus primarily on Asia-Pacific stock markets because Asian stock markets are rapidly growing, especially since the mid-1990s, owing to developments such as rising financial liberalization, technological progress, international capital flow, advanced electronic trading systems, and high economic activity [73–77]. Therefore, in today's global financial ecosystem, Asian stock exchanges have become among the world's largest capital markets –because they have the highest total market value and because of the substantial capital raised through initial public offerings (IPOs) [78]. IPO is a financial operation conducted by listed firms aimed at raising substantial equity capital for future investments through issuing new shares to public investors for the first time. Further, Park [79] shows that stock market integration in the Asia-Pacific region has strengthened, especially after the recent global financial crisis [79]. However, such developments could intensify the possibility of cross-border spillover and contagion of regional financial instabilities, leading to a significant increase in the volatility of Asia-Pacific equity markets and widespread effects in global financial markets [80–83]. Hence, understanding the volatility behavior as well as estimating and forecasting the volatility of Asia-Pacific stock markets efficiently and accurately are among the main concerns among policy makers, investors, and risk managers of financial institutions.

As of the end of 2022, the study had covered the last 10 years, namely from 4 January 2013 to 30 December 2022, using daily data. This period can also be defined as highly volatile. Different crises, successive financial market stress episodes, and extreme events occurred. These include the Taper Tantrum of 2013; collapse in oil prices (from mid-2014 to early 2016); Chinese stock market crash (between mid-June 2015 and early January 2016); COVID-19 outbreak (in March 2020); Russia-Ukrainian war (in February 2022), gradually rising tensions between the US and China over economic, technology, and defense issues; and North Korea's intensifying missile tests. All the stock market data are extracted from the Finance Yahoo (https://finance.yahoo.com/world-indices/) and The Wall Street Journal (https://www.wsj.com/market-data/stocks/asia/indexes) databases.

As a robustness check, we change not only the stock markets studied but also the sample period considered. Specifically, both models are applied to daily European stock market returns, including three emerging European stock markets—the BUX index (Hungary), PX index (Czechia), and WIG index (Poland)—and three developed European stock markets—the DAX Performance index (Germany), CAC 40 index (France), and IBEX-35 index (Spain). They are applied for the period from 5 June 2013 to 5 June 2023, corresponding to the 10-year time horizon. Other periods of significantly high volatility stem from important developments, particularly for European countries, such as the UK Brexit referendum (in June 2016); Russia's decision to cut off the natural gas supply to Europe (after Russia-Ukrainian war began); the European Union ban on Russian crude oil (from December 2022) and petroleum products (from February 2023); and as a monetary policy reaction to high inflation, the announcement made especially by the ECB (European Central Bank) (in December 2022), indicating the end of quantitative easing and the beginning of quantitative tightening. All the stock market data is extracted from the investing.com (https://www.investing.com/indices/) and marketwatch.com (https://www. marketwatch.com/) databases. All stock market indices examined are expressed in their own local currencies.

Lastly, for similar reasons, the ASV-GHskw and ASV-st models are also applied to daily WTI and Brent crude oil returns over the 10-year time horizon, spanning from January 2015 to December 2024. This period includes further episodes of geopolitical and economic disruption, such as the Israel-Hamas War (in October 2023), contributing to the gradually rising tensions and instability, especially in the Middle East. All data is extracted from the U.S. Energy Information Administration (https://www.eia.gov/dnav/ pet/pet_pri_spt_s1_d.htm).

Log returns y_t of the relevant financial variables are calculated as follows:

$$y_t = lnP_t - lnP_{t-1}, \tag{1}$$

where P_t is the closing price on day t.

2.2. *Methodology*

2.2.1. ASV-GHskw model structure

The ASV-GHskw model is given by Nakajima and Omori [37].

$$y_t = \exp\left(\frac{h_t}{2}\right)\{\beta(z_t - \mu_z) + \sqrt{z_t}\,\varepsilon_t\}, t = 1,2, \dots \dots, T, \quad (2)$$

$$h_{t+1} = \mu + \phi(h_t - \mu) + \eta_t, t = 1,2, \dots . . T - 1, \quad (3)$$

$$z_t \sim IG\left(\frac{\nu}{2}, \frac{\nu}{2}\right), \quad (4)$$

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \sim N(0, \Sigma.), \text{ with } \Sigma = \begin{pmatrix} 1 \; \rho\sigma \\ \rho\sigma \; \sigma^2 \end{pmatrix}, \quad (5)$$

where y_t denotes the studied financial variables' daily returns; h_t stands for time-varying measure of logarithmic stochastic volatility, which is assumed to display a stationary first-order autoregressive; that is, AR (1), process with $|\phi| < 1$, where ϕ captures the persistence level of shocks to stochastic volatility. Thus, it is known as a volatility persistence parameter. z_t represents the latent, that is, unobservable, variable series necessary for practical implementation of a novel MCMC algorithm. ν refers to the unknown degrees of freedom parameter that should be estimated by imposing ν > 4 restriction. β corresponds to a skewness parameter, a measure of degree of asymmetry of a concerned distribution assumption. IG is the representative of inverse gamma distribution with scale (0.025) and shape (2.5) parameters, a useful distribution for a dataset that exhibits nonnegative right-skewed distribution characteristics. σ_η stands for the volatility-of-volatility parameter of underlying financial variable returns. μ refers to the unconditional mean of unobservable latent stochastic volatility. ϵ_t and η_t show the stochastic disturbance terms; ρ corresponds to the correlation between them; thus, when ϵ_t and η_t are found to be correlated, this means that the observation equation is affected by these two disturbance terms, and when ϵ_t and η_t are negatively correlated disturbance terms ($i.e., \rho < 0$), this indicates leverage effect. In other words, a negative shock to concerned financial market return series at time t due to bad news will increase volatility at time $t + 1$, but a positive shock to stock market return series of the same magnitude at time t due to good news will lead to a smaller volatility at time $t + 1$ [84]. Further, to implement the ASV-st model, β in the ASV-GHskw model is set to $\beta = 0$.

Consistent with mainstream literature [20,37,39], we set the values of β, μ, ν, σ, ρ and ϕ equal to -0.5, -9, 15, 0.15, -0.5 and 0.95, respectively, to initialize the MCMC sampler. Furthermore, prior distributions we consider for the estimation process can be expressed as follows: β~N (0, 1), μ~N (−10, 1), ν~Gamma (16, 0.8) I (ν > 4), $\sigma^2 \sim$ Inverse-Gamma (2.5, 0.025), $\rho \sim$ ∪ (−1, 1), and $(\phi + 1)/2 \sim$ Beta (20, 1.5). Accordingly, Table 1 also presents all this essential information in a more convenient visual way.

Table 1. Initial values and associated prior distributions considered in the MCMC implementation.

Initial values	Prior distributions
$\phi = 0.95$	$(\phi + 1)/2$~Beta (20, 1.5)
$\sigma = 0.15$	$\sigma^2 \sim$ Inverse-Gamma (2.5, 0.025)
$\rho = -0.5$	$\rho \sim \cup$ (−1, 1)
$\mu = -9$	μ~N (−10, 1)
$\beta = -0.5$	β~N (0, 1)
$\upsilon = 15$	υ~Gamma (16, 0.8) I (υ > 4)

The reasonable initial settings (values) and prior distributions considered in the estimation process of the ASV-GHskw model parameters are consistent with both empirical findings and typical characteristics of high-frequency financial return data. They are therefore frequently used in similar empirical studies (see, [37,39,42]). These typical characteristics include mean reverting volatility behavior, high volatility persistence, noticeable asymmetric heavy-tailed distribution patterns, and a notable leverage effect (see, [20,22,85]). In other words, since these initial values and prior distributions are both empirically validated and reflect the stylized facts of high-frequency financial time series data (see, [37]), as mentioned above, they are widely adopted by researchers for various high-frequency financial time series data, ranging from traditional financial asset returns, such as stock price returns and exchange rate returns, to non-traditional financial asset returns, such as Bitcoin and Ethereum returns. Therefore, Nakajima and Omori [37], who examine the stochastic volatility dynamics of S&P 500 and TOPIX stock returns, adopt the same initial settings (values) and prior distributions for both stock markets. Furthermore, although Lafosse and Rodríguez [39] consider different emerging and developed stock market returns together, including the Argentine stock exchange, the Mexican stock exchange, the Brazilian stock exchange, the Chilean stock exchange, and the US stock market, they use the same initial settings (values) and prior distributions for all these stock market returns. Moreover, a similar approach is evident in studies such as Kim et al. [22], Nakajima and Omori [29], Omori et al. [28], and Yu [20], among others.

In that regard, based on Nakajima and Omori's [37] methodological approach, the steps of the MCMC simulation technique are as follows (for more detailed information on this issue, please see, [37]):

Let $\theta = (\phi, \sigma, \rho, \mu, \beta, \nu), y = (y_t)_{t=1}^n, h = (h_t)_{t=1}^n$, and $z = (z_t)_{t=1}^n$. For the prior distributions of μ and β, assume that $\mu \sim N(\mu_0, v_0^2)$ and $\beta \sim N(\beta_0, \sigma_0^2)$, and let $\pi(\vartheta)$, $\pi(\phi)$, and $\pi(\upsilon)$ express the prior probability densities of $\vartheta \equiv (\sigma, \rho)'$, ϕ, and υ, respectively. Using the MCMC method for the ASV-GHskw model, random samples are drawn from the posterior distribution of (θ, h, z) given y, as follows:

1) Initialize θ, h and z;
2) Generate $\phi \mid \sigma, \rho, \mu, \beta, \nu, h, z, y$;
3) Generate $(\sigma, \rho) \mid \phi\ \mu, \beta, \nu, h, z, y$;
4) Generate $\mu \mid \phi, \sigma, \rho, \beta, \nu, h, z, y$;
5) Generate $\beta \mid \phi, \sigma, \rho, \mu, \nu, h, z, y$;
6) Generate $\nu \mid \phi, \sigma, \rho, \mu, \beta, h, z, y$;
7) Generate $z \mid \theta, h, y$;
8) Generate $h \mid \theta, z, y$;
9) Go to 2.

2.2.2. Model performance evaluation

Bayes factor serves as a well-known and credible Bayesian model selection criterion [86]. Therefore, it is widely employed in the relevant literature. Accordingly, as this study employs a Bayesian model comparison method, we also use the Bayes factor, which can be expressed as a ratio of two competing models' marginal likelihoods, implying that models can be directly selected based on their marginal likelihoods [87]. Therefore, we consider the typical approach adopted in the relevant literature and opt for the model with the largest marginal likelihood, rendering it the most suitable model that outperforms its counterparts [22,88,89].

The marginal likelihood $m\ (y)$ and logarithm of the marginal likelihood (log-ML) are computed as follows [37,90]:

$$m\ (y) = \frac{f\ (y|\ \Theta)\ \pi\ (\Theta)}{\pi(\Theta\ |\ y)}, \tag{6}$$

$$log\ m\ (y) = log\ f\ (y|\ \Theta) + log\ \pi\ (\Theta) - log\pi(\Theta\ |y), \tag{7}$$

where π (Θ) refers to a prior probability density, f $(y$ | Θ) indicates a likelihood, π (Θ | y) represents a posterior probability density, and Θ denotes a parameter.

2.2.3. Market risk measurement procedure

Regarding market risk calculation, as stated previously, the Basel Committee on Banking Supervision suggests replacing the VaR by the ES as a new market risk measure under the Basel IV regulations. More specifically, to fulfill the level of capital requirements, instead of using conventional VaR estimates at a significance level of $\alpha = 0.01$, Basel Committee on Banking Supervision advises applying ES estimates at a significance level of $\alpha = 0.025$ [91]. Thus, based on the ASV-GHskw model, the conventional VaR estimates under Basel III regulations can be obtained through the solution of the Eq (8) [53,92].

$$\alpha = \int_{-\infty}^{VaR_\alpha} f(r) dr, \tag{8}$$

where $1 - \alpha$ is the confidence level, r represents the returns of a financial variable, and $f(r)$ denotes the probability density function of the relevant financial variable return distribution.

Conventional VaR estimates can also be calculated via the Eq (9) [93],

$$VaR_\alpha = F^{-1}(\alpha), \tag{9}$$

where $F^{-1}(\alpha)$ shows the inverse cumulative distribution function.

Following Artzner et al. [48], the ES can be defined as the conditional expectation of the magnitude of the potential losses, provided that they surpass VaR estimates for a given confidence level $1 - \alpha$. Thus, ES values under Basel IV regulations can be formulated as follows [94]:

$$ES_{(\alpha)} = \frac{1}{\alpha} \int_0^\alpha VaR\ (u)\ du. \tag{10}$$

3. Results

Table 2 presents the descriptive statistics, and Figures 1 and 2 show normal and student's *t* QQ-plots (Quantile-Quantile-plots) for all Asia-Pacific stock markets return examined, respectively. Results in Table 2 indicate that all coefficients of skewness and excess kurtosis are statistically significant at the 5% significance level, except the coefficient of skewness for Hong Kong's stock market. Further, the findings confirm that all the stock markets return series are skewed to the left, except for Hong Kong's stock market, which is skewed to the right, implying that all the stock market returns considered exhibit an asymmetrical distribution feature. The excess kurtosis is in the range of 3.793 to 18.662 across the stock market returns examined, inferring heavy-tailed distribution characteristics.

Table 2. Descriptive statistics (%).

Stock markets	Mean	Standard deviation	Skewness	Excess kurtosis	Minimum	Maximum
Japan	0.0365 %	1.3309 %	-0.244*[0.000]	4.057*[0.000]	-8.253 %	7.731 %
Hong Kong	-0.0067 %	1.2603 %	0.0307[0.533]	3.793*[0.000]	-6.567 %	8.693 %
Singapore	0.0006 %	0.8261 %	-0.524*[0.000]	8.969*[0.000]	-7.637 %	5.895 %
Korea	0.0039 %	0.9853 %	-0.246*[0.000]	8.114*[0.000]	-8.767 %	8.251 %
Australia	0.0159 %	0.9754 %	-1.032*[0.000]	11.958*[0.000]	-10.203 %	6.766 %
China	0.0126 %	1.3365 %	-1.031*[0.000]	7.259*[0.000]	-8.873 %	6.039 %
India	0.0452 %	1.1194 %	-1.092*[0.000]	15.83*[0.000]	-13.904 %	8.400 %
Indonesia	0.0187 %	1.0247 %	-0.372*[0.000]	5.856*[0.000]	-6.805 %	8.395 %
Taiwan	0.0245 %	0.9504 %	-0.594*[0.000]	5.204*[0.000]	-6.521 %	6.173 %
Thailand	0.0070 %	0.9886 %	-1.389*[0.000]	18.662*[0.000]	-11.428 %	7.653 %

(*, denotes 5% significance level. P-values from *t*-tests are shown in square brackets.)

Regarding QQ-plots, the findings based on normal QQ-plots support that all the stock markets' returns distributions accommodate heavier tails than those of the normal distribution, and the level of heavy-tailedness may vary across the right and left tails, meaning they depart from normal distribution assumption. However, although the student's *t* QQ-plots confirm that this distribution assumption is more appropriate compared to the normal distribution assumption for the relevant daily stock market returns, student's *t* QQ-plots also reveal a lack of fit, especially in tails, supporting the application of SV models using GHskw distribution assumption to obtain more favorable tail fits and to overcome the possible misspecification problems.

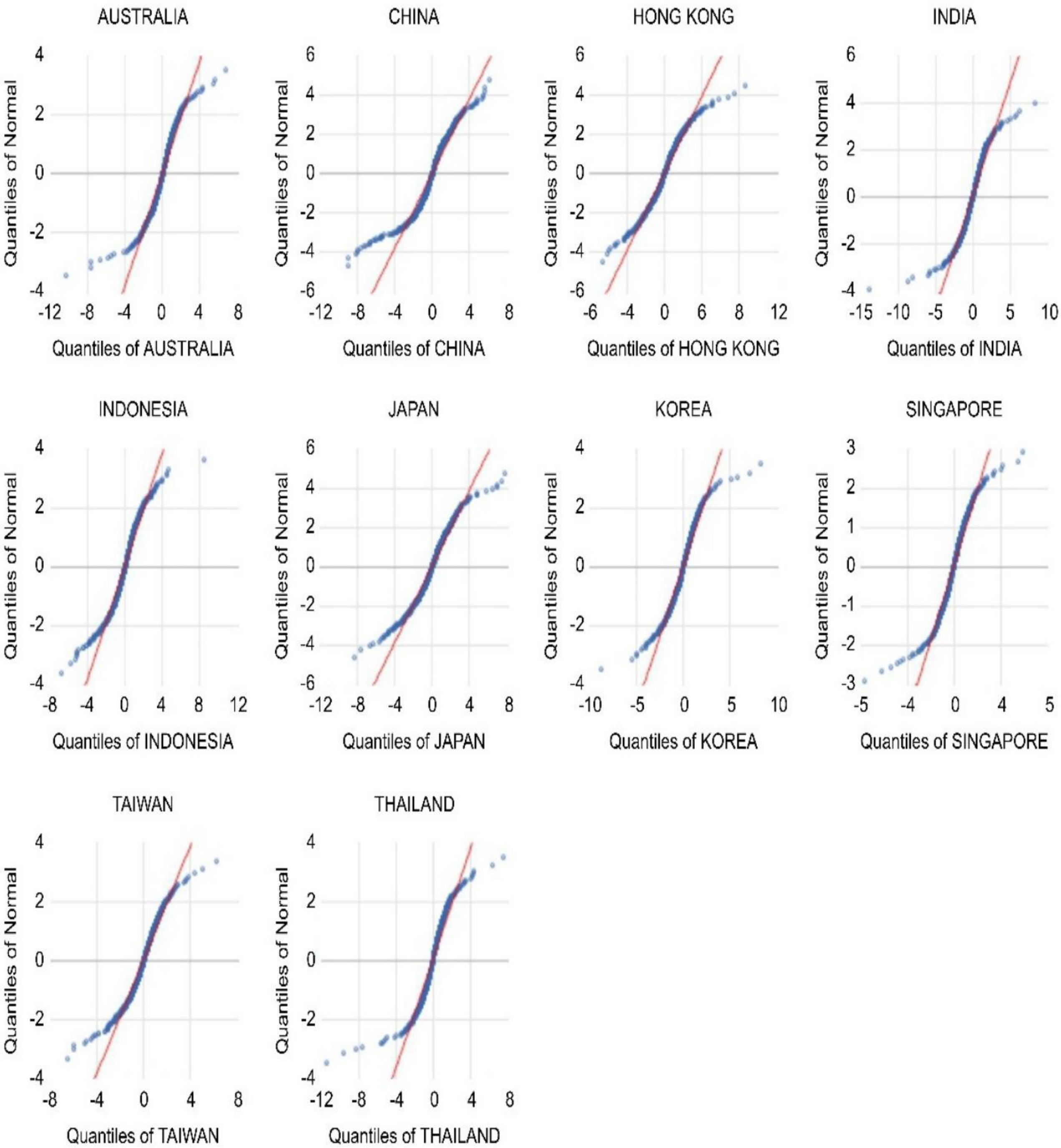

Figure 1. Normal QQ-plots for daily returns of the stock market data.

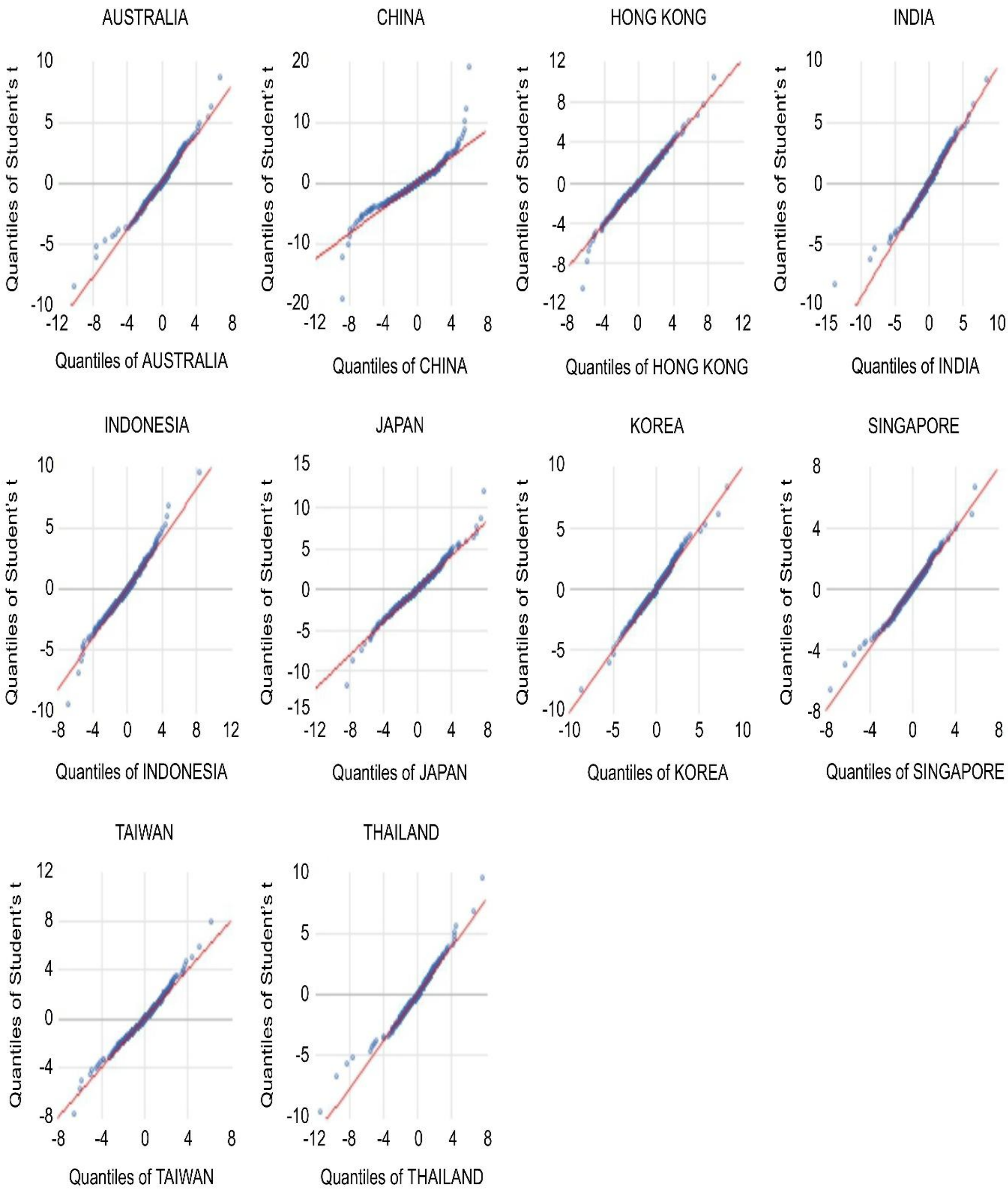

Figure 2. Student's *t* QQ-plots for daily returns of the stock market data.

The autocorrelation functions (top), the sample paths (middle), and the posterior densities (bottom) of the related parameters are shown in Figures 3–6 for Japanese, Australian, Chinese, and Indian stock markets for illustrative purposes (however, all results are available upon request). The findings reveal that the novel MCMC estimation scheme proposed by Nakajima and Omori [37] is sufficiently efficient [37]. This can be attributed to the determination that sample paths exhibit a stable pattern and sample autocorrelation functions decay rapidly in all cases except for parameters ν for Korean, Indonesian, and particularly Chinese, Taiwanese and Australian stock markets. We show parameter ϕ for Chinese and Australian stock markets, and parameter σ for the Chinese stock market, in which results indicate a relatively slow converge and high autocorrelation for these parameters. This means that the Chinese stock market is the worst case, followed by the Australian stock market, because Chinese stock markets exhibit poor mixing and high autocorrelation for ϕ, σ, and ν parameters while Australia stock markets have similar problems to Chinese stock markets especially for ϕ and ν parameters.

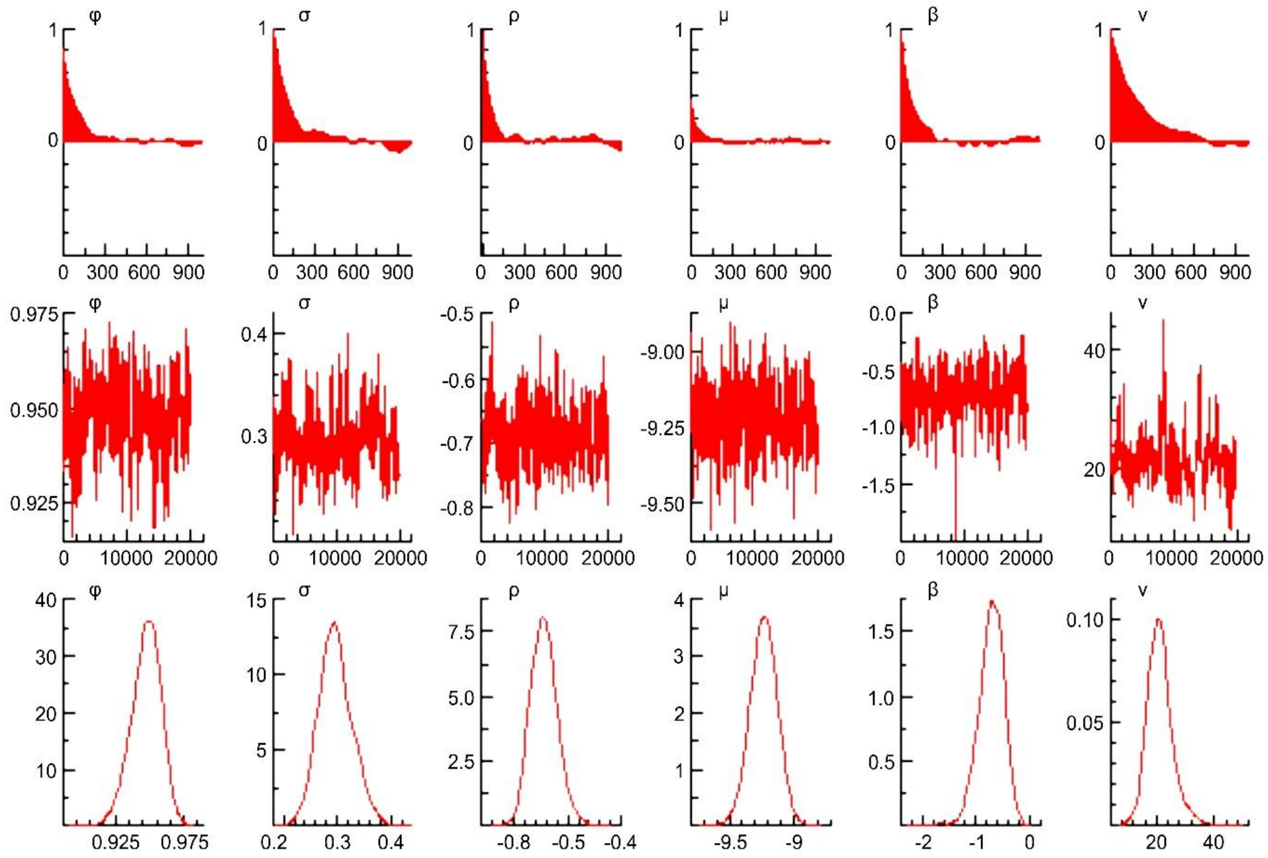

Figure 3. Graphical illustration of the efficiency of the proposed MCMC simulation technique of ASV-GHskw model for Japanese stock market data.

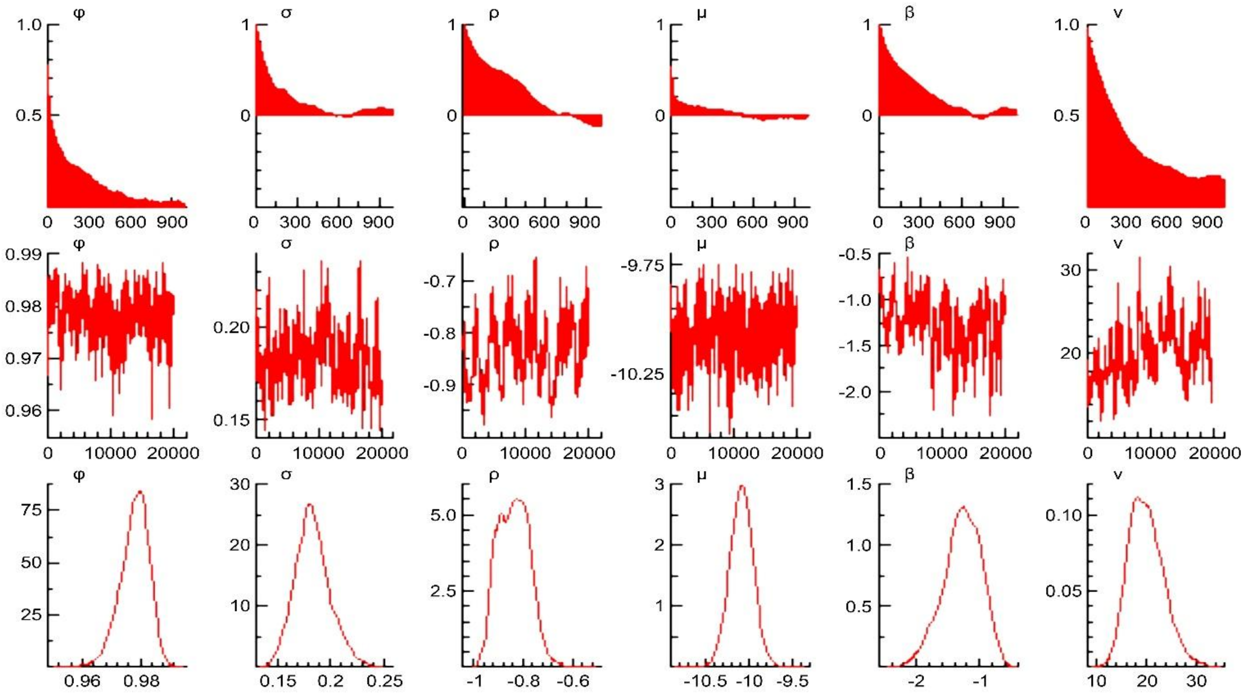

Figure 4. Graphical illustration of the efficiency of the proposed MCMC simulation technique of ASV-GHskw model for Australian stock market data.

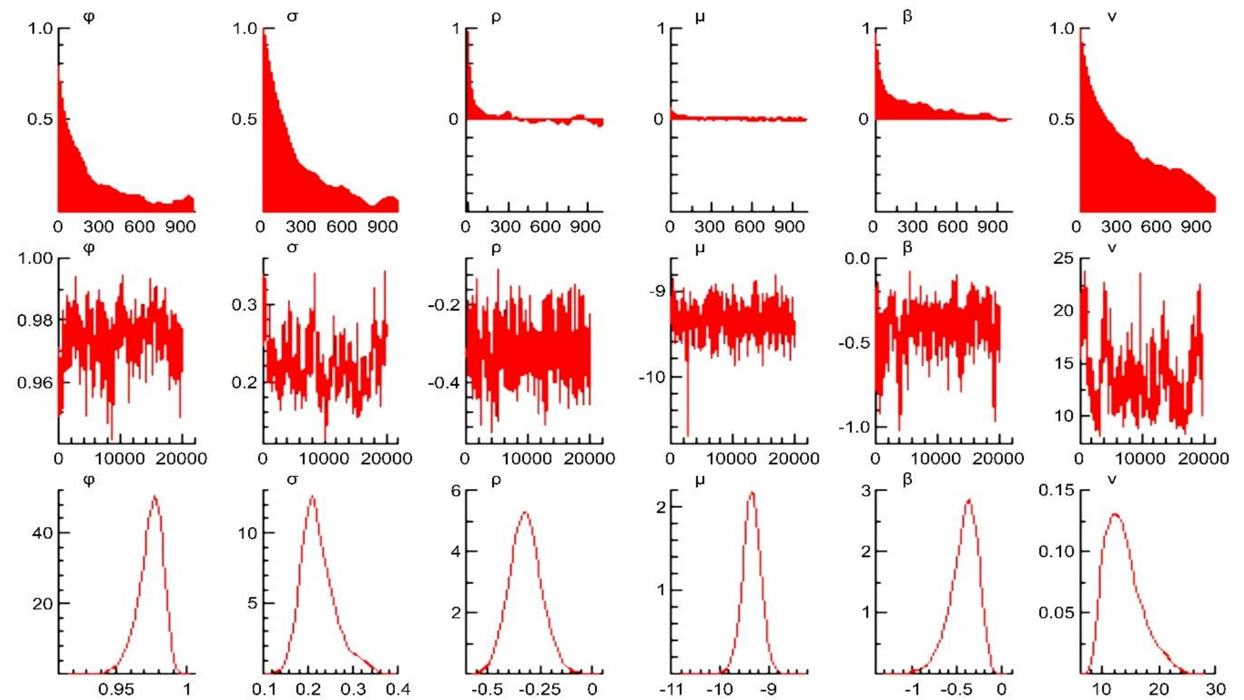

Figure 5. Graphical illustration of the efficiency of the proposed MCMC simulation technique of ASV-GHskw model for Chinese stock market data.

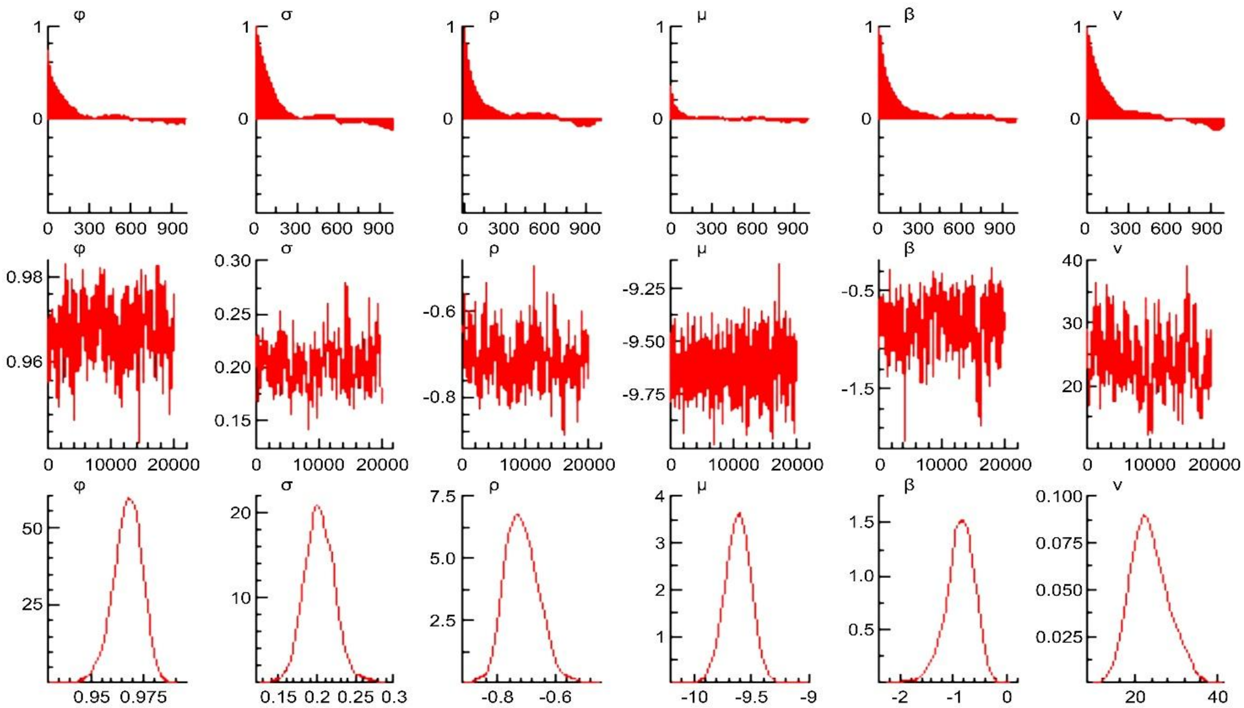

Figure 6. Graphical illustration of the efficiency of the proposed MCMC simulation technique of ASV-GHskw model for Indian stock market data.

Table 3 presents the posterior means, inefficiency factors, standard deviations, and 95% credible intervals for the ASV-GHskw model for all daily stock market returns considered. Posterior means of ϕ are found to range between 0.9473 and 0.9781 across the daily stock market returns, with the highest estimate from the Australian stock market (0.9781), followed closely by the Thai stock market (0.9780), and with the lowest estimate from the Japanese stock market (0.9473), followed by the Indonesian stock market (0.9619). This suggests mean reverting volatility behavior and high persistence of volatility. For example, based on the traditional formula [i.e., $\frac{-LN\,(2)}{LN(\phi)}$], half-life durations of the shocks are found to range between 13 days and 31 days across the log-volatilities of the stock market returns examined. Additionally, in all cases, the volatility persistence parameters ϕ are found to be statistically significant as their 95% credible intervals do not include zero and/or negative values.

Table 3. ASV-GHskw model estimation results.

Parameter	Mean	Stdev.	95% interval	Inefficiency
Japan				
ϕ	0.9473	0.0110	[0.9238, 0.9667]	125.33
σ	0.2972	0.0316	[0.2387, 0.3626]	188.98
ρ	-0.6889	0.0496	[-0.7779, -0.5839]	102.52
μ	-9.2286	0.1073	[-9.4456, -9.0236]	24.29
β	-0.7107	0.2390	[-1.1988, -0.3010]	136.62
v	21.2950	4.5911	[13.4999, 32.0396]	290.87

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Parameter	Mean	Stdev.	95% interval	Inefficiency
Hong Kong				
ϕ	0.9728	0.0066	[0.9588, 0.9845]	128.52
σ	0.1653	0.0210	[0.1260, 0.2069]	214.49
ρ	-0.6673	0.0626	[-0.7822, -0.5311]	113.51
μ	-9.2093	0.1104	[-9.4277, -8.9921]	15.85
β	-0.6344	0.2302	[-1.1638, -0.2576]	140.94
ν	20.3345	4.5885	[12.9639, 30.5811]	283.06
Singapore				
ϕ	0.9724	0.0073	[0.9555, 0.9844]	110.45
σ	0.1754	0.0214	[0.1397, 0.2243]	179.99
ρ	-0.5462	0.0720	[-0.6770, -0.3974]	77.79
μ	-10.1187	0.1236	[-10.3642, -9.8759]	10.62
β	-0.5946	0.2290	[-1.1098, -0.2017]	102.37
ν	22.3325	4.1780	[15.4335, 31.8461]	204.62
Korea				
ϕ	0.9702	0.0082	[0.9519, 0.9838]	190.99
σ	0.1948	0.0243	[0.1513, 0.2455]	286.68
ρ	-0.6076	0.0626	[-0.7242, -0.4772]	108.51
μ	-9.8506	0.1286	[-10.1133, -9.6060]	15.73
β	-0.8672	0.2436	[-1.4087, -0.4573]	138.81
ν	19.8840	3.7196	[13.6493, 27.8232]	265.50
Australia				
ϕ	0.9781	0.0050	[0.9672, 0.9866]	173.44
σ	0.1838	0.0165	[0.1538, 0.2198]	231.12
ρ	-0.8360	0.0621	[-0.9407, -0.7153]	394.56
μ	-10.0865	0.1354	[-10.3556, -9.8218]	65.60
β	-1.2775	0.3024	[-1.9265, -0.7518]	330.79
ν	20.0302	3.5160	[14.1016, 27.7101]	389.07
China				
ϕ	0.9742	0.0088	[0.9539, 0.9882]	201.02
σ	0.2230	0.0387	[0.1603, 0.3196]	314.62
ρ	-0.3198	0.0773	[-0.4662, -0.1618]	71.16
μ	-9.3432	0.1917	[-9.7293, -8.9707]	13.82
β	-0.4240	0.1576	[-0.7919, -0.1793]	174.14
ν	13.7374	3.2124	[9.0474, 21.4083]	384.71
India				
ϕ	0.9674	0.0068	[0.9527, 0.9794]	110.37
σ	0.2033	0.0200	[0.1657, 0.2440]	180.94
ρ	-0.7165	0.0581	[-0.8171, -0.5970]	154.54
μ	-9.6174	0.1109	[-9.8416, -9.4008]	23.48
β	-0.8681	0.2725	[-1.4771, -0.4040]	157.23
ν	23.5969	4.5818	[15.5634, 33.3149]	209.25

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Parameter	Mean	Stdev.	95% interval	Inefficiency
Indonesia				
ϕ	0.9619	0.0099	[0.9404, 0.9788]	194.61
σ	0.2255	0.0298	[0.1716, 0.2860]	272.29
ρ	-0.5401	0.0701	[-0.6688, -0.3953]	94.10
μ	-9.7467	0.1209	[-9.9812, -9.5063]	10.11
β	-0.7556	0.2338	[-1.2473, -0.3609]	114.23
v	19.9838	3.8599	[13.3567, 28.2475]	280.52
Taiwan				
ϕ	0.9645	0.0083	[0.9468, 0.9787]	169.65
σ	0.1955	0.0231	[0.1530, 0.2421]	240.21
ρ	-0.7477	0.0526	[-0.8421, -0.6365]	167.11
μ	-9.9119	0.1060	[-10.1223, -9.7053]	55.84
β	-1.1544	0.3101	[-1.8825, -0.6588]	316.06
v	20.1799	3.6926	[13.9378, 28.2702]	402.92
Thailand				
ϕ	0.9780	0.0059	[0.9653, 0.9883]	121.93
σ	0.1927	0.0213	[0.1522, 0.2371]	227.19
ρ	-0.5665	0.0647	[-0.6894, -0.4362]	115.16
μ	-9.9736	0.1698	[-10.3162, -9.6380]	10.07
β	-0.7676	0.2367	[-1.2921, -0.3720]	91.58
v	20.6823	4.2448	[13.6911, 30.7240]	227.85

(Note: We draw the last 20,000 samples after the first 2000 samples are removed as a burn-in.)

The posterior means of ρ are found to fluctuate within the range of −0.3198 to −0.8360. In absolute value, it is found to be the highest for the Australian stock market returns (−0.8360), followed by the Taiwanese stock market returns (−0.7477), and Indian stock market returns (−0.7165). It is found to be the lowest for the Chinese stock market returns (−0.3198), followed by Indonesian stock market returns (−0.5401), and Singaporean stock market returns (−0.5462). Further, because the 95% credible intervals do not include zero and/or positive values in any case, this situation provides strong evidence for the presence of leverage effect in all Asia-Pacific stock market returns examined. This in turn suggests that a negative shock to the related stock market returns generates stronger effects on volatility compared to a positive shock to the related stock market returns of the same magnitude.

The posterior means of σ is found to be the largest for Japan's stock market (0.2972), followed by Indonesia's stock market (0.2255), China's stock market (0.2230), India's stock market (0.2033), Taiwan's stock market (0.1955), Korea's stock market (0.1948), Thailand's stock market (0.1927), Australia's stock market (0.1838), Singapore's stock market (0.1754), and Hong Kong's (0.1653) stock market. This means that the shocks in the log volatility of Japanese, Indonesian, and Chinese stock market returns can lead to more variability, compared to those in the remaining daily stock market returns examined. Further, it is also noteworthy to indicate that σ parameters are statistically significant because their 95% credible intervals include only positive values in all cases.

The posterior means of β are found to be negative in all cases, as expected, and take values ranging from −0.4240 to −1.2775 for the stock markets data considered. In absolute value, the posterior means of β are highest by far for the Australian stock market with $\beta = -1.2775$, the Taiwanese stock market with $\beta = -1.1544$, and lowest for the Chinese stock market with $\beta = -0.4240$, followed by the Singaporean stock market with $\beta = -0.5946$. This supports that all the Asia-Pacific stock market returns under study exhibit an asymmetrical distribution characteristic, even though their degree of skewness may differ considerably. As is known, a smaller skewness parameter β means both a more pronounced left-skewness and heavier tails for the ASV-GHskw model when v fixed [37,39]. Thus, the findings indicate that, compared to other stock market returns under study, Australian and Taiwanese stock market returns by far exhibit notable skewness with v fixed. Further, skewness parameters β are found to be statistically significant as their 95% credible intervals do not contain zero in any case, presenting strong evidence for the presence of negative skewness in all the Asia-Pacific stock market returns examined.

The posterior means of v are found to vary from 13.7374 to 23.5969 across the daily Asia-Pacific stock market returns under study, with the highest estimate from the Indian stock market returns (23.5969), followed closely by the Singaporean (22.3325), Japanese (21.2950), and Thai (20.6823) stock market returns. The lowest estimate by far is from the Chinese stock market returns (13.7374), followed by Korean stock market returns (19.8840) and Indonesian stock market returns (19.9838), both of which have similar estimates. For the ASV-GHskw model, with β fixed, the lower magnitude of parameter v is the indicator of a higher level of heavy-tailedness and skewness [37,39]. Thus, the outcomes reveal that the Chinese stock market returns have the heaviest tails, whereas the Indian and the Singaporean stock market returns have lighter tails with β fixed. The overall findings, based on both parameter β and parameter v, suggest that all Asia-Pacific stock market returns considered exhibit characteristics of the left skewed and heavy-tailed distribution.

The posterior means of μ reveal that the Hong Kong's stock market has the highest value of mean with $\mu = -9.2093$, followed by Japan's stock market with $\mu = -9.2286$, and China's stock market with $\mu = -9.3432$. The lowest mean value belongs to Singapore's stock market with $\mu = -10.1187$, followed by the Australia's stock market with $\mu = -10.0865$, and Thailand's stock market with $\mu = -9.9736$. This provides evidence that, on average, the stock markets in Hong Kong, Japan, and China have higher levels of volatility compared to the remaining stock markets considered. Further, in all cases, all posterior means of the estimates parameter are found to be statistically significant because their 95% credible intervals never include zero.

Overall, the findings confirm that all daily Asia-Pacific stock market returns considered exhibit a mean reverting SV process, strong leverage effect, and a negatively skewed heavy-tailed distribution feature as well as high volatility persistence.

Regarding inefficiency factors of the estimated parameters, we observe that the highest inefficiency factors tend to belong to σ and v parameters whereas the smallest ones pertain to the μ parameter, inferring that the estimation process of σ and v parameters display higher inefficiency levels compared to the remaining parameters examined, which is generally consistent with Lafosse and Rodríguez and Nakajima and Omori [37,39].

Table 4 presents the posterior means, inefficiency factors, standard deviations, and 95% credible intervals for the ASV-st model for all daily Asia-Pacific stock market returns examined in the study. In general, the findings indicate similar results.

Table 4. ASV-st model estimation results.

Parameter	Mean	Stdev.	95% interval	Inefficiency
Japan				
ϕ	0.9405	0.0126	[0.9129, 0.9626]	143.59
σ	0.3078	0.0370	[0.2411, 0.3876]	198.74
ρ	-0.6223	0.0511	[-0.7148, -0.5163]	66.84
μ	-9.1167	0.0963	[-9.3040, -8.9252]	12.15
ν	19.7625	4.5128	[12.1852, 29.8452]	218.15
Hong Kong				
ϕ	0.9710	0.0099	[0.9420, 0.9840]	425.02
σ	0.1629	0.0286	[0.1245, 0.2444]	509.75
ρ	-0.6030	0.0794	[-0.7349, -0.4214]	257.65
μ	-9.1381	0.1072	[-9.3471, -8.9253]	16.82
ν	16.6605	3.9241	[10.5170, 25.8445]	286.20
Singapore				
ϕ	0.9487	0.0097	[0.9283, 0.9661]	42.97
σ	0.2544	0.0224	[0.2159, 0.3025]	94.59
ρ	-0.3365	0.0615	[-0.4531, -0.2113]	36.91
μ	-10.0164	0.1047	[-10.2169, -9.8038]	10.07
ν	24.3034	4.9991	[15.8700, 35.4470]	209.01
Korea				
ϕ	0.9622	0.0098	[0.9403, 0.9789]	127.51
σ	0.2115	0.0269	[0.1611, 0.2721]	193.16
ρ	-0.5102	0.0640	[-0.6267, -0.3753]	94.65
μ	-9.7050	0.1115	[-9.9219, -9.4841]	12.27
ν	18.7839	4.6615	[11.8180, 29.4637]	274.83
Australia				
ϕ	0.9732	0.0059	[0.9602, 0.9834]	65.22
σ	0.1850	0.0172	[0.1538, 0.2232]	116.80
ρ	-0.6560	0.0674	[-0.7717, -0.5095]	123.05
μ	-9.7727	0.1160	[-10.0002, -9.5384]	4.89
ν	19.6537	4.3818	[13.1559, 30.2970]	202.64
China				
ϕ	0.9634	0.0195	[0.9062, 0.9866]	477.22
σ	0.2742	0.0888	[0.1633, 0.5519]	547.92
ρ	-0.2512	0.0683	[-0.3780, -0.1085]	45.45
μ	-9.2328	0.1818	[-9.6012, -8.8761]	56.94
ν	16.2047	5.2812	[8.8254, 29.1423]	460.08
India				
ϕ	0.9677	0.0066	[0.9534, 0.9793]	91.40
σ	0.1945	0.0183	[0.1616, 0.2324]	185.30
ρ	-0.6665	0.0535	[-0.7593, -0.5512]	94.09
μ	-9.4933	0.1030	[-9.6946, -9.2917]	7.54
ν	21.8761	4.4073	[14.5243, 31.7012]	174.43

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Parameter	Mean	Stdev.	95% interval	Inefficiency
Indonesia				
ϕ	0.9577	0.0117	[0.9308, 0.9768]	210.03
σ	0.2344	0.0354	[0.1734, 0.3212]	318.85
ρ	-0.4441	0.0722	[-0.5827, -0.2977]	102.28
μ	-9.6293	0.1157	[-9.8547, -9.4010]	16.80
v	20.2802	5.2706	[11.7970, 31.7847]	321.46
Taiwan				
ϕ	0.9564	0.0107	[0.9323, 0.9747]	185.24
σ	0.2034	0.0265	[0.1550, 0.2622]	261.90
ρ	-0.6359	0.0553	[-0.7379, -0.5188]	120.57
μ	-9.7156	0.0884	[-9.8851, -9.5371]	22.02
v	16.2458	3.8027	[10.6257, 25.2694]	353.35
Thailand				
ϕ	0.9759	0.0062	[0.9623, 0.9869]	114.25
σ	0.1959	0.0227	[0.1532, 0.2461]	238.71
ρ	-0.4875	0.0700	[-0.6153, -0.3378]	76.96
μ	-9.8421	0.1591	[-10.1540, -9.5236]	6.54
v	19.8946	4.7554	[13.2310, 31.6885]	214.16

(Note: We draw the last 20,000 samples after the first 2000 samples are removed as a burn-in.)

For instance, posterior means of ϕ vary from 0.9405 to 0.9759 for all daily Asia-Pacific stock market returns; the Japanese stock market returns have the lowest estimate (0.9405) and the Thai stock market returns have the highest (0.9759), followed closely by Australian stock market returns (0.9732). This indicates the presence of a high volatility persistence in the related daily Asia-Pacific stock market returns.

The posterior means of ρ reveal that, in absolute value, the Indian stock market exhibits the most significant leverage effect with $\rho = -0.6665$, followed by the Australian stock market with $\rho = -0.6560$ and the Taiwanese stock market with $\rho = -0.6359$. Meanwhile, the Chinese stock market demonstrates the lightest leverage effect with $\rho = -0.2512$, followed by the Singaporean stock market with $\rho = -0.3365$, implying that, compared to remaining stock markets under study, the leverage effect is stronger for Indian, Australian, and Taiwanese stock markets.

The posterior means of σ ranges between 0.1629 and 0.3078 across the stock market returns considered, whereas Japan's stock market has the largest value and Hong Kong's stock market has the lowest value, suggesting that shocks in the log volatility of the Japanese stock market can lead to more variability compared to other daily stock markets under study.

The posterior means of v are found to fluctuate, ranging between 16.2047 and 24.3034, and it is found to be the highest for Singapore's equity market and the lowest for China's equity market, indicating that Singapore's equity market has the lightest tails whereas China's equity market has the heaviest tails.

The posterior means of μ indicate that Japan's stock market data exhibits the largest value of mean with $\mu = -9.1167$, followed by Hong Kong's stock market data with $\mu = -9.1381$. The lowest mean belongs to Singapore's stock market data with $\mu = -10.0164$. This indicates that, compared to other stock markets data considered, on average, Japan and Hong Kong's stock market data has the highest volatility levels whereas Singapore's stock market data has, on average, the lowest volatility level.

Furthermore, in all cases, all posterior means of estimates parameter are found to be statistically significant because their 95% credible intervals do not contain zero under any circumstances. Regarding the inefficiency factor, similar to the results obtained from the ASV-GHskw model, the outcomes delivered from the ASV-st model also indicate that, almost always, the highest values belong to the σ and ν parameters while the smallest ones pertain to the μ parameter.

Regarding the model performance evaluation, the findings, presented in Table 5, show that, compared to the ASV-st model, the ASV-GHskw model has the largest log-ML values for all the Asia-Pacific stock markets return series considered. This suggests that the ASV-GHskw model performs better than the ASV-st model in all cases, thus providing clear evidence that the ASV-GHskw model can improve upon the commonly used ASV-st model in the related literature.

Table 5. Estimated Log-ML results for Asia-Pacific stock market returns.

	Likelihood (s.e)	Prior	Posterior (s.e)	Log-ML (s.e)
ASV-GHskw				
Japan	7445.718 (0.859)	-5.393	8.306 (0.284)	**7432.019 (0.905)**
Hong Kong	7569.915 (0.790)	-1.888	8.597 (0.477)	**7559.429 (0.923)**
Singapore	8881.555 (0.348)	-2.061	7.990 (0.280)	**8871.504 (0.447)**
Korea	8271.332 (0.863)	-2.610	9.068 (0.295)	**8259.654 (0.913)**
Australia	8665.702 (1.244)	-2.801	9.922 (0.374)	**8652.979 (1.299)**
China	7572.349 (1.432)	-3.982	8.354 (0.432)	**7560.013 (1.496)**
India	8064.407 (0.265)	-3.301	9.485 (0.229)	**8051.621 (0.350)**
Indonesia	8089.065 (0.868)	-3.361	8.303 (0.176)	**8077.402 (0.885)**
Taiwan	8249.599 (1.775)	-2.897	9.703 (0.307)	**8236.999 (1.801)**
Thailand	8388.006 (0.546)	-2.581	8.373 (0.286)	**8377.053 (0.616)**
ASV-st				
Japan	7436.928 (0.580)	-4.440	6.384 (0.224)	7426.104 (0.622)
Hong Kong	7563.501 (0.584)	-0.786	7.298 (0.288)	7555.417 (0.652)
Singapore	8868.826 (0.432)	-3.362	6.761 (0.151)	8858.703(0.458)
Korea	8256.887 (0.329)	-1.757	6.643 (0.284)	8248.487 (0.435)
Australia	8631.536 (0.618)	-1.054	6.972 (0.322)	8623.510 (0.696)
China	7555.935 (0.699)	-3.753	4.584 (0.604)	7547.589 (0.924)
India	8053.055 (0.387)	-1.570	7.670 (0.330)	8043.814(0.509)
Indonesia	8076.518 (0.538)	-2.432	5.503 (0.269)	8068.583 (0.601)
Taiwan	8226.868 (0.805)	-1.656	7.349 (0.214)	8217.863 (0.833)
Thailand	8373.849 (0.401)	-1.401	6.535 (0.265)	8365.913 (0.481)

(Note: Bold figures denote the most appropriate SV models selected by Bayes factor.)

A time series plot of the log volatility series generated by the ASV-GHskw model, together with log returns for the stock markets considered (these figures are not presented here, but are available upon request), indicates that stylized facts of financial asset returns volatility, such as volatility clustering and mean reverting behavior, are readily apparent. Additionally, it seems that high log volatility also leads to large fluctuations in the log returns and vice versa. Further, the ASV-GHskw model well captures the effects of historically experienced economics and financial developments on the related stock markets volatility behavior. This is because of the collapse in oil prices between mid-2014 and early 2016; the COVID-19 outbreak in March 2020, with the first huge adverse consequences experienced globally; and the Russo-Ukrainian war that started February 2022, which caused significant volatility increases in all the Asia-Pacific stock market returns, despite their magnitude and duration exhibiting differences across the stock market returns considered.

The results have important implications for financial institutions and rational risk-averse investors in the Asia-Pacific equity markets. For instance, for all daily Asia-Pacific stock market data examined, we also provide market risk analysis based on the ASV-GHskw model under both Basel III and Basel IV regulations to elucidate the potential differences between two regulations. In this regard, the findings presented in Table 6 and the visualizations of these results, also presented in Figure 7 for the purpose of visual emphasis, show that market risk levels calculated based on the ASV-GHskw model under Basel IV regulations are higher than those calculated under Basel III regulations, especially for the Chinese stock market, followed by the Japanese and Singaporean stock markets.

Table 6. Market risk measurement results under different Basel regulations.

	Basel III regulation	Basel IV regulation	Difference
ASV-GHskw			
Japan	-3.657 %	-3.819 %	4.430 %
Hong Kong	-3.714 %	-3.795 %	2.181 %
Singapore	-1.608 %	-1.669 %	3.794 %
Korea	-3.623 %	-3.736 %	3.119 %
Australia	- 2.752 %	-2.781 %	1.054 %
China	-2.570 %	-2.756 %	7.237 %
India	-2.472 %	-2.544 %	2.913 %
Indonesia	-2.175 %	-2.244 %	3.172 %
Taiwan	-3.193 %	-3.251 %	1.816 %
Thailand	-1.514 %	-1.545 %	2.048 %

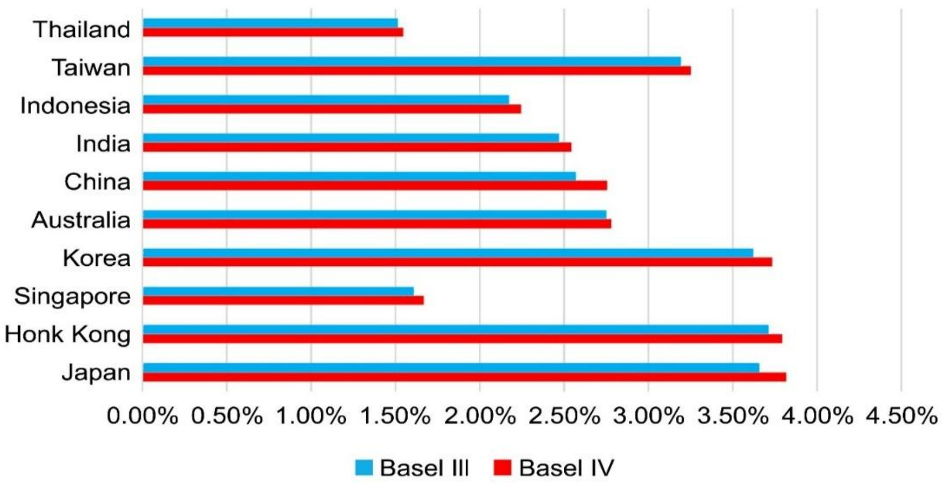

Figure 7. Market risk measurement results based on the ASV-GHskw model under Basel III and Basel IV regulations for Asia-Pacific equity markets (in absolute values).

This suggests that, in particular, fat-tail risk is more notable for these stock markets, whereas the Australian and Taiwanese stock markets exhibit lower differences between Basel IV and Basel III regulations. Moreover, the results indicate that Japan's stock market has the highest downside market risk level (−3.819%), followed by Hong Kong's stock market (−3.795%), and Korea's stock market (−3.736%), whereas Thailand's stock market has the lowest downside market risk level (−1.545%), followed by Singapore's stock market (−1.669%). Overall, the findings reveal that, compared to Basel III regulations, Basel IV regulations have increased the capital requirements of banks and other financial institutions to offset market risk arising from positions taken in the relevant Asia-Pacific stock markets, particularly for Chinese, Japanese, and Singaporean stock markets.

Further, although the Bayes factor indicates that the ASV-GHskw model is superior to the ASV-st model in all cases, we also present the results calculated with the ASV-st model to examine the difference in market risk level across the models used. The findings, shown in Table 7 and in Figure 8 for visual emphasis, reveal that, in all cases, market risk level measured using the ASV-GHskw model is remarkably different compared to those calculated using the ASV-st model, with the downside market risk level varying from 4.679% to 17.045%, emphasizing the crucial role of using more appropriate and complex models in market risk capital requirements calculations.

Table 7. Comparison of different market risk measurement results.

	Basel IV regulation ASV-GHskw	Basel IV regulation ASV-st	Difference
Japan	-3.819 %	-3.482 %	9.678 %
Hong Kong	-3.795 %	-3.560 %	6.601 %
Singapore	-1.669 %	-1.476 %	13.076 %
Korea	-3.736 %	-3.569 %	4.679 %
Australia	-2.781 %	-2.376 %	17.045 %
China	-2.756 %	-2.383 %	15.653 %
India	-2.544 %	-2.372 %	7.251 %
Indonesia	-2.244 %	-2.032 %	10.433 %
Taiwan	-3.251 %	-2.814 %	15.529 %
Thailand	-1.545 %	-1.433 %	7.816 %

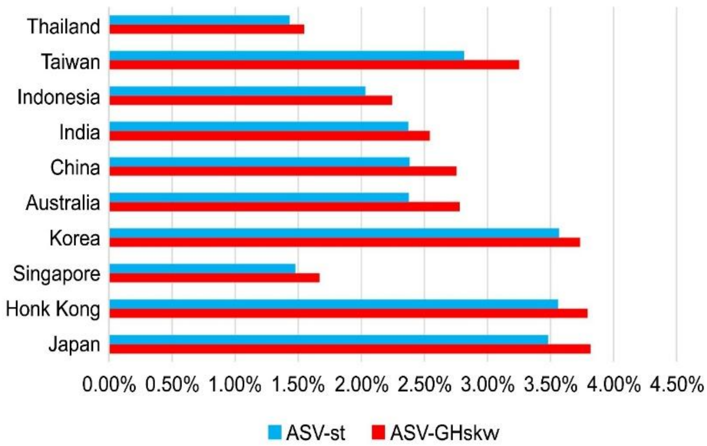

Figure 8. Comparison of market risk measurement results of the ASV-GHskw and ASV-st models under Basel IV regulations for Asia-Pacific equity markets (in absolute values).

To evaluate the results from the perspective of rational Asia-Pacific stock markets investors, for which risk-return trade-off is a fundamental investment principle, consistent with Kajtazi and Moro's [95] and Iglesias and Lagoa Varela's [96] studies [95,96], in which VaR and ES estimates are considered as representative of financial risk level, we also consider risk-adjusted returns, calculating risk based on the ASV-GHskw model under Basel IV regulations. The findings shown in Table 8 clearly indicate that the Indian stock market provides the most appropriate investment opportunity to rational risk averse investors, followed by the Indonesian and Japanese stock markets, because, among alternative investment opportunities, a risk-averse investor prefers the one that offers a larger rate of return with a lower risk level.

Table 8. Ranking of the stock markets according to their benefits to investors.

	Mean (Average return)	ES (in absolute values)	Ranking based on risk adjusted returns (ES / Mean)
Japan	0.0365 %	3.819 %	3
Hong Kong	-0.0067 %	3.795 %	10
Singapore	0.0006 %	1.669 %	9
Korea	0.0039 %	3.736 %	8
Australia	0.0159 %	2.781 %	5
China	0.0126 %	2.756 %	6
India	0.0452 %	2.544 %	1
Indonesia	0.0187 %	2.244 %	2
Taiwan	0.0245 %	3.251 %	4
Thailand	0.0070 %	1.545 %	7

However, Hong Kong's stock market appears as the least attractive stock market for investment, followed by Singapore and Korea's stock markets. This is because Hong Kong's stock market has the highest downside market risk and can only offer a negative daily average return. Similarly, Singapore and Korea's stock markets do not offer efficient investment opportunities to Asian-Pacific risk-averse stock market investors, because the returns they provide are not enough to compensate the risk that investors will be exposed to when they take the long position in these stock markets, compared to the investment opportunities that the remaining stock markets present.

3.1. Robustness check

We also perform the same econometric analyses for the six related European stock market returns and WTI and Brent crude oil returns, as we have done thus far, to obtain more robust and, thus, reliable results. However, as we have reached similar conclusions, we briefly describe the main findings to avoid repetition and thus maintain the fluidity and simplicity of the study (however, all results are available upon request). First, the results indicate that neither the normal distribution nor the student's *t*-distribution adequately fit daily stock market returns and daily crude oil returns, thereby supporting the adoption of SV models incorporating a GHskw distribution. Second, the outcomes show that the Bayesian MCMC sampling method is sufficiently efficient, particularly in the case of the ASV-GHskw model. This is supported by stable sample paths and rapidly decaying autocorrelations in most cases. Third, all daily European stock market returns and crude oil returns display a mean reverting SV process, strong leverage effect, a negatively skewed and heavy-tailed distribution feature, and high volatility persistence. Fourth, the model performance evaluation analysis shows that, in all cases, the ASV-GHskw model outperforms ASV-st model, implying that the ASV-GHskw model provides a further improvement over the widely utilized ASV-st model in the empirical analyses (see Table 9).

Table 9. Estimated Log-ML results for European stock markets and crude oil markets.

	Likelihood (s.e)	Prior	Posterior (s.e)	Log-ML (s.e)
Stock markets				
ASV-GHskw				
DAX	8011.597 (0.487)	-4.734	10.021 (0.270)	**7996.842 (0.557)**
CAC 40	8249.504 (0.973)	-4.682	10.386 (0.355)	**8234.436 (1.036)**
IBEX-35	7986.807 (1.229)	-3.539	9.528 (0.208)	**7973.740 (1.247)**
BUX	7745.900 (0.795)	-2.706	8.394 (0.255)	**7734.801 (0.834)**
PX	8525.772 (0.732)	-4.197	8.107 (0.336)	**8513.467 (0.806)**
WIG	7983.736 (0.680)	-1.885	9.246 (0.254)	**7972.605 (0.726)**
ASV-st				
DAX	7998.436 (0.663)	-4.045	7.426 (0.306)	7986.964 (0.731)
CAC 40	8231.428 (0.814)	-3.978	7.925 (0.336)	8219.525 (0.880)
IBEX-35	7972.705 (0.547)	-2.489	6.346 (0.424)	7963.870 (0.692)
BUX	7740.718 (0.597)	-1.631	6.780 (0.180)	7732.307 (0.624)
PX	8510.226 (0.263)	-3.375	6.038 (0.313)	8500.812 (0.409)
WIG	7978.800 (0.679)	-1.215	6.952 (0.231)	7970.632 (0.718)
Crude oil markets				
ASV-GHskw				
WTI	5916.584 (0.800)	-5.170	8.564 (0.284)	**5902.851 (0.848)**
Brent	6066.090(0.309)	-4.829	8.839(0.182)	**6052.422 (0.359)**
ASV-st				
WTI	5840.063 (1.206)	-4.259	6.786 (0.316)	5829.018 (1.247)
Brent	6035.114 (1.135)	-3.886	7.135 (0.222)	6024.093 (1.156)

(Note: Bold figures denote the most appropriate SV models selected by Bayes factor.)

Fourth, time series plots of the log volatility series generated by the ASV-GHskw model together with log returns for European stock market returns and crude oil markets reveal that stylized facts of financial asset returns volatility, such as volatility clustering and mean reverting behavior, can be easily observed. Further, it seems that the relevant ASV-GHskw model captures the effects of historical developments both on related European stock markets volatility and the crude oil market volatility patterns. This is because the 2007–2008 global financial crisis period, the 2010–2012 European debt crisis period, and the 2022 February Russia-Ukrainian war and its first huge adverse consequences significantly increased volatility in all the European stock market returns, despite their magnitude and duration exhibiting differences across the stock market returns considered. Similarly, the outbreak of COVID-19 in March 2020, and the huge adverse consequences experienced globally, led to significant volatility in the crude oil market returns.

The outcomes have several significant implications for financial institutions and rational risk averse investors, particularly for those interested in these financial markets. For instance, for all the European stock markets examined, we provide one-day-ahead downside market risk analysis based on the ASV-GHskw model under both Basel III and Basel IV regulations to elucidate the potential differences between the two regulations. In this regard, the findings presented in Table 10 and in Figure 9, for visual emphasis, show that market risk levels calculated based on the ASV-GHskw model under Basel IV regulations are higher than those calculated under Basel III regulations, especially for the PX and the BUX markets, followed by the WIG market. This implies that fat-tail risk is more notable for emerging European daily stock market returns compared to the developed European daily stock market returns. Further, regarding risk managers' taken positions in the European stock markets examined, the outcomes reveal that the WIG market has the highest downside market risk level (-3.676%), followed by the CAC 40 market (-3.576%). The IBEX-35 market has the lowest downside market risk level (-2.479%), followed by the DAX market (-2.812%).

Table 10. Market risk measurement results under different Basel regulations.

Stock markets	Basel III regulation	Basel IV regulation	Difference
DAX	-2.771 %	-2.812 %	1.479 %
CAC 40	-3.485 %	-3.576 %	2.6112%
IBEX-35	-2.431 %	-2.479 %	1.974 %
BUX	-2.727 %	-2.855 %	4.694 %
PX	-2.828 %	-2.990 %	5.728 %
WIG	-3.553 %	-3.676 %	3.462 %

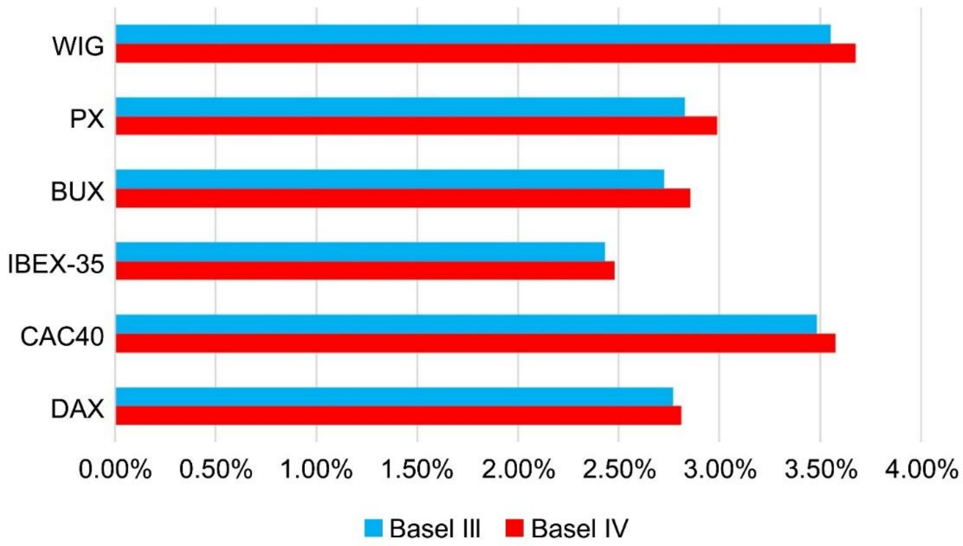

Figure 9. Market risk measurement results based on the ASV-GHskw model under Basel III and Basel IV regulations for European equity markets (in absolute values).

Moreover, to evaluate the results from the perspective of rational European stock markets investors, for which risk-return trade-off is a fundamental investment principle, we again consider risk-adjusted returns, calculating risk based on the ASV-GHskw model under Basel IV regulations, and mean returns are presented in Table 11.

Hence, the findings shown in Table 11 suggest that the BUX market provides the most appropriate investment opportunity for rational risk-averse investors, followed by the DAX market, because, among alternative investment opportunities, a risk-averse investor prefers one that offers a higher rate of return for the same risk level. However, it seems that the IBEX-35 market is the least attractive stock market. This is because although it has the lowest downside market risk, it can only offer a daily average return close to zero, implying that the rate of return it provides is not enough to compensate for the risk that the investors will be exposed to when they take a long position in this stock market.

Table 11. Ranking of the stock markets according to their benefits to investors.

Stock markets	Mean (Average return)	ES (in absolute values)	Ranking based on risk adjusted returns (ES / Mean)
DAX	0.0258 %	2.812 %	2
CAC 40	0.0237 %	3.576 %	3
IBEX-35	0.0040 %	2.479 %	6
BUX	0.0368 %	2.855 %	1
PX	0.0123 %	2.990 %	4
WIG	0.0127 %	3.676 %	5

Lastly, we present market risk measurement results based on the ES estimates derived from the ASV-GHskw model under both Basel III and Basel IV regulations for WTI and Brent crude oil returns. This is because these findings are particularly valuable for both investors interested in the energy commodity markets under study and risk managers with trading positions in these markets. As shown in Table 12 and in Figure 10 for visual emphasis, the findings indicate that market risk levels calculated based on the ASV-GHskw model under Basel IV regulations are higher than those calculated under Basel III regulations; this underscores the importance of accounting for the fat-tail risk in the WTI and Brent crude oil markets to more appropriately measure the market risk. Further, regarding risk managers, the findings show that the Brent crude oil market displays a higher downside market risk level (-4.692 %) than the WTI crude oil market (- 4.454 %).

Table 12. Market risk measurement results under different Basel regulations.

	Basel III regulation	Basel IV regulation	Difference
Crude oil markets			
WTI	-4.077 %	-4.254 %	4.341 %
Brent	-4.492 %	-4.692 %	4.454 %

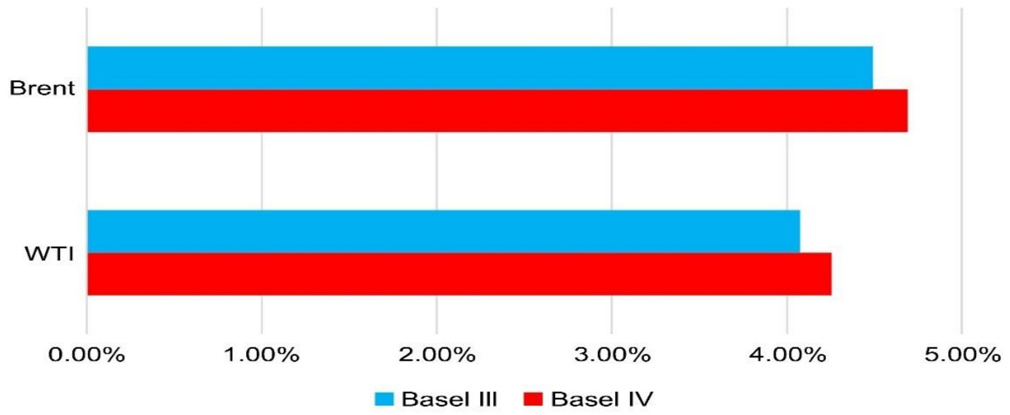

Figure 10. Market risk measurement results based on the ASV-GHskw model under Basel III and Basel IV regulations for crude oil markets (in absolute values).

Moreover, to assess the outcomes from the perspective of rational crude oil markets investors, as we have done thus far, we again consider risk-adjusted returns where risk is calculated based on the ASV-GHskw model under Basel IV regulations, and mean returns are those presented in Table 13.

Table 13. Ranking of the crude oil markets according to their benefits to investors.

Crude oil markets	Mean (Average return)	ES (in absolute values)	Ranking based on risk adjusted returns (ES / Mean)
WTI	1.2695 %	4.254 %	1
Brent	1.1723 %	4.692 %	2

Hence, the findings provide evidence that the WTI crude oil market offers a better investment opportunity to rational risk-averse investors, compared to Brent crude oil market. Together, all these findings imply that when a significant price shock—a common occurrence—is experienced in the energy commodity markets, originating from either regional and/or global factors, investors will conduct more efficient and effective investment strategies and risk managers will implement more adequate risk management practices for the positions they take in the relevant energy commodity markets when they consider the results presented in this study.

4. Conclusions

Rising market risk in international equity and crude oil markets has underscored the need for more advanced and flexible models, particularly for volatility forecasting [97]. Therefore, in this study, we first apply the ASV-GHskw model to 10 Asia-Pacific stock market returns. These include five emerging and five developed Asia-Pacific stock markets. The former comprise the IDX Composite Index (Indonesia), the S&P CNX Nifty Index (India), the TSEC Weighted Index (Taiwan), the SSE Composite Index (China), and the SET Index (Thailand). The latter comprise the NIKKEI 225 Index (Japan), the S&P/ASX 200 Benchmark Index (Australia), the FTSE Straits Times Index (Singapore), the KOSPI Composite Index (Korea), and the Hang Seng Index (Hong Kong). Furthermore, the ASV-st model is also considered to investigate whether the ASV-GHskw model improves upon the ASV-st model. The Bayes factor is employed to examine the performance of the two competing SV models. Moreover, as a new methodological approach to measuring market risk in Asia-Pacific stock markets, we also consider one-day-ahead downside ES estimates based on the ASV-GHskw model under Basel IV regulations and compare them with those calculated under Basel III regulations. Additionally, we provide investment analysis from the perspective of rational risk-averse investors in the financial markets considered, where financial risk is calculated using the ASV-GHskw model under Basel IV regulations.

Finally, as a robustness check, the models are applied to daily European stock market returns, including three emerging European stock markets—the BUX index (Hungary), PX index (Czechia), and WIG index (Poland)—and three developed European stock markets—the CAC 40 index (France), DAX Performance index (Germany), and IBEX-35 index (Spain)—from June 5, 2013 to June 5, 2023, as well as to WTI and the Brent crude oil market using daily data from January 2015 to December 2024.

The findings reveal that the proposed Bayesian MCMC algorithm is sufficiently efficient, and that the ASV-GHskw model successfully captures the stochastic volatility behavior of returns for 18 financial variables. Hence, the results provide evidence that the financial asset returns under study exhibit a mean-reverting stochastic volatility process, a strong leverage effect, and high volatility persistence. Furthermore, the Bayes factor used to evaluate the performance of two competing models, namely ASV-GHskw and ASV-st, indicates that ASV-GHskw is superior to ASV-st across all 18 cases. This means that the ASV-GHskw model better fits the tail characteristics of the relevant financial asset returns distributions than the ASV-st model. This pattern likely arises because the financial asset returns examined exhibit asymmetric (skewed), heavy-tailed distributions, contrary to the symmetric assumption of the ASV-st model. In other words, since the ASV-GHskw model allows for both heavy-tailedness and skewness simultaneously—which is more consistent with what researchers frequently report in their empirical analyses and what practitioners commonly experience in real financial markets—it appears to capture the tail behaviors of the distributions of the examined financial asset returns more appropriately than its competing ASV-st model. This supports the view that risk managers at financial institutions and policymakers should prefer the ASV-GHskw model to better capture the volatility behavior of returns in related financial markets, particularly during periods of financial turbulence.

The findings have several significant implications not only for investors and risk managers at financial institutions holding positions in these markets but also for relevant policymakers, particularly central banks. For example, as Pederzoli [11], among others, points out, adequately modeling and forecasting the volatility of financial asset returns is a fundamental step for effective and efficient financial risk management applications, particularly in areas such as market risk measurement, portfolio management, derivative pricing, hedging, and formulating more successful investment strategies [11]. This is because biases inherent in volatility estimates derived from misspecified models may lead to failures in risk modeling and investment practices, thereby reducing the resilience of banks and other financial institutions to adverse financial shocks, whether global or domestic in origin. In this regard, the findings indicate that the ASV-GHskw model should be used when risk managers at financial institutions and investors seek a more comprehensive understanding of the characteristics of the volatility of the related financial assets' price returns.

Furthermore, from the policymakers' perspective, especially central banks, an in-depth look at the volatility patterns of stock market returns is necessary to evaluate adequately the consequences of the economic policy actions taken. This is because stock markets are not only highly sensitive to macroeconomic conditions but also serve as an important transmission mechanism for monetary policy, functioning as an essential sub-channel of the asset price channel, hence further confirming the importance of accurately predicted volatility estimates [98,99]. Accordingly, an in-depth understanding of the volatility behavior of relevant stock market returns is also a fundamental requirement for understanding how their economic policy decisions and new releases of financial and economic data affect market participants' pricing behavior for financial assets. This results in a greater understanding of the consequences of economic policy decisions, as their announcements are considered a significant driver of short-term stock market fluctuations [100]. Moreover, monetary policy surprises and changes in monetary and fiscal policy making procedures also constitute a significant source of arrival of new information to stock markets that are not previously transmitted into stock market prices. Thus, a deeper understanding of stock market volatility dynamics may significantly enhance their ability to forecast how market participants process economic policy signals from policymakers [101]. In turn, this yields valuable information about how they will likely react to potential policy changes, which may improve the efficiency and effectiveness of the monetary policy framework aimed at achieving and presuming both price and macro-financial stability. In this context, policymakers should adopt the ASV-GHskw model rather than existing alternatives, such as the widely accepted ASV-st model, to better assess the potential causes and consequences of significant price fluctuations occurring in the examined stock and crude oil markets.

Turning to financial risk analysis, market risk levels calculated using the ASV-GHskw model under Basel IV are significantly higher than those under Basel III in all cases. This suggests that under the latest Basel capital regulations, banks and other institutions will hold more capital to meet market risk capital requirements arising from their positions in these financial markets. This, in turn, underscores the crucial role of using models that more appropriately account for skewness and heavy-tailedness. As Assaf [102] and Lin et al. [103], among others, point out, adequately capturing the accurate tail characteristics of the financial variables returns distributions, in particular when they exhibit highly left skewed distributions, as in the case of this study, is essential for banks and other financial institutions to obtain more accurate market risk estimates since the choice of the accurate market risk measurement model has remarkable impacts on regulatory capital requirements and, consequently, on the development process of the financial system [102–104].

In investment analysis, ES estimates based on the ASV-GHskw model can also provide valuable information for risk-averse investors, who expect a higher average return by holding less risky financial assets. This requires more accurate estimates of the positions' actual market risk in the related stock and crude oil markets. Thus, risk-averse investors in stock and crude oil markets should consider the aforementioned outcomes when formulating optimal investment strategies.

As a suggestion for future studies, the performance of the ASV-GHskw model in other financial risk management applications, such as asset allocation, option pricing, and hedging, can be examined.

Use of Generative-AI tools declaration

The author declares that he has not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The author declares that he has no conflict of interest.

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