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Theory article

Disturbance decoupling of large-scale Boolean networks based on network aggregation and state-flipping control

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Abstract: This paper addresses the disturbance decoupling problem for large-scale Boolean networks. First, the original network is decomposed into several smaller subnetworks via a network aggregation approach, which significantly reduces computational complexity. Then, a state-flipping control strategy is applied to achieve disturbance decoupling within these subnetworks. Necessary and sufficient conditions are established under both uncontrolled and controlled scenarios, leading to the overall disturbance decoupling of the original large-scale Boolean network. Furthermore, this paper proposes two algorithms: One for verifying the feasibility of disturbance decoupling in the original network, and the other for finding the minimum set of flipped nodes required to achieve disturbance decoupling in each subnetwork. Finally, a numerical example illustrates the effectiveness of the proposed methodology.

Keywords: large-scale Boolean networks; disturbance decoupling; network aggregation; state-flipping control; semi-tensor product
Mathematics Subject Classification: 93D09, 94C11

1. Introduction

Since its introduction by Kauffman, Boolean networks have become a classical discrete dynamic model for studying gene regulatory networks [1, 2]. In this model, the state of a gene is simplified as a binary variable: 1 denotes expression, while 0 denotes non-expression. The introduction of the semi-tensor product as a mathematical tool has brought significant transformations to the analytical methods of Boolean networks and game theory [3, 4]. Building on this, the algebraic state-space representation (ASSR) method further achieved linear modeling of Boolean networks, facilitating a series of studies including robust model construction [5], stability analysis [6, 7], observability [8, 9], controllability [10, 11], synchronization [12–14], and disturbance decoupling problems [15].

Large-scale Boolean networks (LBNs) represent an extremely important direction in Boolean network research. Gene regulation is a typical large-scale system, with tens of thousands of genes in a single cell interacting and regulating each other. Studying LBNs is a critical step from theoretical simplified models toward biologically realistic systems. Current research on LBNs has formed several key areas, including systemic stability analysis [16–20], synchronization issues [21], and so on.

The core challenge in analyzing LBNs lies in the large network scale, which leads to extremely high computational complexity. Reducing this complexity is therefore critical. Among various solutions, network aggregation and state-flipping control offer effective approaches from the dimensions of model simplification and precise intervention, respectively. Network aggregation decomposes the original LBN into several subnetworks, that are much smaller than the original network, thereby making previously infeasible tasks computationally tractable. On the other hand, state-flipping control also exhibits unique advantages in handling large-scale Boolean networks. Traditional matrix methods face the bottleneck of exponentially increasing computational complexity as the number of nodes grows, while pinning control requires modifying the logical rules of nodes to exert influence, which may alter the original state transition structure of the network. In contrast, state-flipping control achieves intervention simply by directly flipping the state values of specific nodes without modifying any logical rules, thereby minimizing the impact on the network. Based on this advantage, this research direction has received widespread attention in recent years. Currently, there is a wealth of research on network aggregation methods and state-flipping control, such as [22,23], where the authors introduced the network aggregation method, but its application is limited to attractor identification and controllability analysis, without involving controller design. Building on this, [24] designed a state feedback controller; this paper adopts state-flipping control, which can achieve the target state more quickly. [25], on the other hand, focuses on robust set stability but still does not address the disturbance decoupling problem. Furthermore, the network aggregation method exhibits strong potential in practical applications, as demonstrated by its effective use in the 31-node Pseudomonas aeruginosa quorum sensing system. Research on state-flipping control methods primarily covers areas such as weak stability [26], complete synchronization [27], and so on. However, up to now, the issue of disturbance decoupling in LBNs has not been fully explored, and research on this issue is of great significance. To address this challenge, we plan to conduct in-depth research using a combination of network aggregation and state-flipping control methods.

In gene regulatory networks, external disturbances may disrupt the stability of critical functions, leading to abnormal cell cycles and even diseases. Therefore, how to design control strategies to protect key system outputs from these disturbances is one of the core issues in systems biology. Disturbance decoupling is an important control objective proposed to address this need. By constructing appropriate controllers, the system output can be made completely independent of external disturbances, thereby ensuring that core functions operate stably under disturbance environments. In practical applications, research on this issue can significantly enhance network robustness, safeguarding key processes such as the cell cycle from signal fluctuations [28]. For example, in the cell cycle network, ensuring that regulatory factors are not affected by drug interference helps maintain normal division. Over the past few decades, research on disturbance decoupling has evolved significantly, progressing from its origins in Boolean networks [29] to applications in switched [30] and time-delay Boolean networks [31]. Unlike these traditional methods, [15] proposed a novel perspective to address the disturbance decoupling problem based on a profound understanding of its essence. Building on the research, [32] further explored the issue of finite-time disturbance decoupling, contributing to the advancement of the field. It should be noted that the most existing research on LBNs has primarily concentrated on issues of stability and synchronization, with studies on disturbance decoupling in such networks still being quite limited. Therefore, this paper explores the challenging issues of disturbance decoupling in LBNs.

The main contributions of this paper are summarized as follows:

(1) Unlike [19], this paper introduces a network aggregation method. This approach decomposes the original large-scale Boolean network into multiple smaller and more manageable subnetworks, significantly reducing computational complexity without destroying the overall structure of the original network.

(2) In contrast to the research of [33] on small-scale Boolean networks, this paper focuses on the disturbance decoupling problem of large-scale Boolean networks, which better aligns with practical networks. From the perspective of redundant variable separation, necessary and sufficient conditions for disturbance decoupling in subnetworks under both uncontrolled and controlled scenarios are proposed, thereby enabling the solution of the decoupling problem for the original network.

(3) Compared to [25], this paper further designs subnetwork state-flipping controllers after network decomposition. Based on this, two algorithms are developed: Algorithm 1 verifies the decouple ability of the entire network, and Algorithm 2 identifies the minimum set of flipping nodes, effectively reducing control costs while ensuring decoupling performance.

The structure of this article is as follows: Section 2 introduces relevant theoretical concepts and necessary preliminary knowledge; Section 3 is the main body of the article, which provides a detailed introduction to the entire process of implementing disturbance decoupling in large-scale Boolean networks; Section 4 demonstrates the effectiveness of the results through a specific example; and Section 5 concludes the article.

Notation:

- $\mathcal{N}_+$ is the set of positive integers.
- The binary set $\mathcal{D} = \{0, 1\}$.
- δ_n^i is the i-th column of the identity matrix I_n.
- $\Delta_n = \left\{\delta_n^1, \ldots, \delta_n^n\right\}$ denotes the set of columns of I_n.
- $L = \left[\delta_m^{i_1}, \ldots, \delta_m^{i_s}\right]$ is a logical matrix, simplified by $\delta_m [i_1, \ldots, i_s]$.
- $\mathcal{L}_{p\times q}$, $\mathcal{H}_{m\times n}$, $\mathfrak{B}_{m\times n}$ are the sets of $m \times n$ logical matrices, real matrices, and Boolean matrices, respectively.
- $[m, n] = \{m, m + 1, \ldots, n\}$, where $m < n \in \mathcal{N}_+$.
- $\mathcal{P}_U = \{A \mid A \subseteq U\}$ is the power set of U.
- $\mathrm{Col}_i(A)$ ($\mathrm{Row}_i(A)$) is the i-th column (row) of matrix A, and $\mathrm{Col}(A)$ ($\mathrm{Row}(A)$) is the set of columns (rows) of A.
- " $\vee$ ", " $\wedge$ ", " $\oplus$ ", "$\neg$", " $\rightarrow$ " represent the logical operators disjunction, conjunction, exclusive or, negation, conditional, respectively.
- $\mathbf{0}_n = [\underbrace{0 \quad 0 \quad \cdots \quad 0}_{n}]^\top$.

- For two matrices X and Y, the intersection of matrices is defined as $Z = X \bigwedge Y = (z_{ij})_{m\times n}$, where $z_{ij} = x_{ij} \wedge y_{ij}$.

2. Preliminaries

2.1. Problem formulation

Consider the following large-scale Boolean network with n nodes, q disturbance inputs, and p outputs:

$$\begin{cases} X_1(t+1) = f_1(X(t), \Xi(t)), \\ \vdots \\ X_n(t+1) = f_n(X(t), \Xi(t)), \\ y_j(t) = g_j(X(t)), j = 1\cdots p, \end{cases} \tag{2.1}$$

where $X(t) = (X_1(t), \dots, X_n(t)) \in \mathcal{D}^n$, $\Xi(t) = \left(\gamma_1(t), \dots, \gamma_q(t)\right) \in \mathcal{D}^q$, and $Y(t) = (y_1(t), \dots, y_p(t)) \in \mathcal{D}^p$ are the state, disturbance, and output of system (2.1), respectively; $f_i : \mathcal{D}^{n+q} \to \mathcal{D}$, $i \in [1, n]$ and $g_j : \mathcal{D}^n \to \mathcal{D}$, $j \in [1, p]$ are Boolean functions.

Disturbance decoupling describes a system with outputs that are completely immune to external disturbances. Motivated by [30], the disturbance decoupling problem is solvable, if the closed-loop system becomes

$$\begin{cases} X_i(t+1) = \widehat{f_i}(\widehat{X}(t)), i = 1, \dots, \varepsilon, \\ X_k(t+1) = \widehat{f_k}(X(t), \Xi(t)), k = \varepsilon+1, \dots, n, \\ y_j(t) = g_j(\widehat{X}(t)), j = 1, \dots, p, \end{cases} \tag{2.2}$$

where $\widehat{X}(t) = (X_1(t), \dots, X_\varepsilon(t)) \in \mathcal{D}^\varepsilon$, $\widehat{f_i} : \mathcal{D}^\varepsilon \to \mathcal{D}$, $i \in [1, \varepsilon]$ and $\widehat{f_k} : \mathcal{D}^{n+q} \to \mathcal{D}$, $k \in [\varepsilon+1, n]$ are Boolean functions.

This paper aims to accomplish disturbance decoupling for system (2.1) based on network aggregation and state-flipping control.

2.2. Network aggregation

The network aggregation method is an effective approach for handling LBNs. According to [24], network aggregation must not yield isolated blocks. Thus, first we assume that the network diagram of system (2.1) is weakly connected. It is worth noting that many real-world networks can be regarded as weakly connected structures. For example, in the T-cell receptor signaling pathway, the overall network remains weakly connected. The network graph of a system is said to be weakly connected if there always exists a path between any two nodes after ignoring the direction of edges [22]. This property enables the system to derive global behavior step by step from local characteristics. When the network does not satisfy weak connectivity and contains multiple independent subsystems, the proposed method can be applied to each subsystem separately.

Then, the state node $N = \{X_1, X_2, \dots, X_n\}$ in system (2.1) is partitioned into m blocks:

$$N = N_1 \cup N_2 \cup \ldots \cup N_m, \tag{2.3}$$

where N_ω is a proper subset of N, $N_{\omega_1} \cap N_{\omega_2} = \emptyset$, $\omega_1 \neq \omega_2$ and $N_\omega = \left\{X_1^\omega, X_2^\omega, \dots, X_{m_\omega}^\omega\right\}$, each block constituting a subnetwork denoted as Σ_ω, $\omega \in [1, m]$. It should be pointed out that these subnetworks are not mutually independent and may exhibit interconnecting edges. The source nodes of incoming edges are defined as input nodes, denoted by $Z_\omega = \left\{z_1^\omega, z_2^\omega, \ldots, z_{\nu_\omega}^\omega\right\}$.

Remark 2.1. Regarding the partitioning of subnetworks, this paper follows the principles established in the literature [23]: A) Each subnetwork should be sufficiently small in scale to reduce computational burden; and B) The number of interaction nodes between subnetworks should be minimized. For large-scale networks and clustering algorithms based on node degree or connectivity, such as the Louvain community detection algorithm and agglomerative clustering, can be employed for partitioning to maximize intra-subnetwork coupling and minimize inter-subnetwork coupling [34].

For subnetwork Σ_ω, without loss of generality, assume that only the first ε_ω ($\varepsilon_\omega < m_\omega$) state variables affect the output. Therefore, the disturbance decoupling problem is solvable for subnetwork Σ_ω if the following condition holds:

$$\begin{cases} X_i^\omega(t+1) = \widehat{f_i^\omega}(Z_\omega(t), \widehat{X}_\omega(t)), i = 1, \ldots, \varepsilon_\omega, \\ X_k^\omega(t+1) = \widehat{f_k^\omega}(Z_\omega(t), X_\omega(t), \Xi_\omega(t)), k = \varepsilon_\omega + 1, \ldots, m_\omega, \\ y_j(t) = g_j(\widehat{X}_1(t), \widehat{X}_2(t), \ldots, \widehat{X}_m(t)), j = 1, \ldots, p, \end{cases} \tag{2.4}$$

where $X_\omega(t) = (X_1^\omega(t), X_2^\omega(t), \ldots, X_{m_\omega}^\omega(t)) \in \mathcal{D}^{m_\omega}$ denotes the state variables of subnetwork, Σ_ω, $Z_\omega(t) = (z_1^\omega(t), z_2^\omega(t), \ldots, z_{\nu_\omega}^\omega(t)) \in \mathcal{D}^{\nu_\omega}$ denotes the input variables, $\Xi_\omega(t) = (\gamma_1^\omega(t), \gamma_2^\omega(t), \ldots, \gamma_{q_\omega}^\omega(t)) \in \mathcal{D}^{q_\omega}$ denotes the disturbance variables, $\widehat{X}_\omega(t) = (X_1^\omega(t), X_2^\omega(t), \ldots, X_{\varepsilon_\omega}^\omega(t)) \in \mathcal{D}^{\varepsilon_\omega}$ denotes the first ε_ω state variables of subnetwork Σ_ω, $\widehat{f_i^\omega} : \mathcal{D}^{\varepsilon_\omega+\nu_\omega} \to \mathcal{D}$, $i \in [1, \varepsilon_\omega]$, and $\widehat{f_k^\omega} : \mathcal{D}^{\nu_\omega+m_\omega+q_\omega} \to \mathcal{D}$, $k \in [\varepsilon_\omega + 1, m_\omega]$ denotes the Boolean functions.

Remark 2.2. When the original system lacks the structure in which the first ε_ω variables directly govern the output, the coordinate transformation technique from [29] can be applied to achieve the required form.

In this paper, after network aggregation, by uniformly treating inputs from other subnetworks as switching signals, each subnetwork can be processed as an independent switched subsystem. For this reason, we first address the disturbance decoupling problem for each subnetwork with switching signals, and then analyze the disturbance decoupling problem of the entire large network.

2.3. State-flipping control

State-flipped control enables the attainment of target states without disrupting the network structure.

Definition 2.1. (Flip function): [35] Set $V = \{i_1, i_2, \ldots, i_v\} \subseteq [1, n]$; the flip function $f_V^\updownarrow(X)$ of V is defined as

$$\overline{X}_V = f_V^\updownarrow(X) = \left(X_1, \ldots, \overline{X}_{i_1}, \ldots, \overline{X}_{i_v}, \ldots, X_n\right), \tag{2.5}$$

where $X_i \in \mathcal{D}$.

State-flipped control operates by selecting a set of nodes $X_{i_1}, X_{i_2}, \ldots, X_{i_v}$ and flipping their values. If the initial state of node X_{i_t} is 1 (or 0) for $t \in [1, v]$, its state becomes $\overline{X}_{i_t} = 0$ (or 1) after flipping, where $\overline{X}_{i_t}$ denotes the flipped state value of node X_{i_t}.

Definition 2.2. (Flip matrix): [35] Set $V = \{i_1, i_2, \ldots, i_v\}$. $\mathbf{f}_V$ represents the flipping matrix relative to V, which can be expressed as

$$\mathrm{Col}_j(\mathbf{f}_V) = \delta_{2^n}^i,\ j \in [1, 2^n],\ \text{if } X \sim \delta_{2^n}^j \xrightarrow{f_V^{\updownarrow}} \overline{X}_V \sim \delta_{2^n}^i. \tag{2.6}$$

Given the equation $\overline{X}_V = \mathbf{f}_V X$, where $(\mathbf{f}_V)_{ij} = 1$ means that $\overline{X}_V \sim \delta_{2^n}^i$ can be achieved from $X \sim \delta_{2^n}^j$ by changing the logical vector $X_{i_1}, X_{i_2}, \ldots, X_{i_v}$, we refer to $X \longrightarrow \overline{X}_V$ as a flipping transformation.

Definition 2.3. (Combinatorial flip matrix): [35] Denote $U = \{i_1, i_2, \ldots, i_u\} \subseteq [1, n]$; the combination flip matrix relative to U is given as

$$(\mathcal{F}_U)_{ij} = \begin{cases} 1, & \text{if } \exists\, V \in \mathcal{P}_U \text{ such that } \delta_{2^n}^j \xrightarrow{f_V^{\updownarrow}} \delta_{2^n}^i, \\ 0, & \text{otherwise.} \end{cases} \tag{2.7}$$

When element $(\mathcal{F}_U)_{ij} = 1$, it indicates that there exists a flipping subset $V \in \mathcal{P}_U$ such that $(\mathbf{f}_V)_{ij} = 1$. The following conclusion can be derived:

$$\mathcal{F}_U = \sum_{V \in \mathcal{P}_U} \mathbf{f}_V. \tag{2.8}$$

3. Main results

In this section, necessary and sufficient conditions for the solvability of disturbance decoupling for each subnetwork under both controlled and uncontrolled conditions are established. Based on these conditions, two algorithms are proposed: A algorithm is proposed to verify whether LBNs can achieve disturbance decoupling, and the other is proposed for finding the minimum set of flipped nodes required for each subnetwork to achieve disturbance decoupling.

First, using the algebraic state space representation [3], the subnetwork Σ_ω can be represented as follows:

$$\mathcal{X}_\omega(t+1) = L_\omega Z_\omega(t) \mathcal{X}_\omega(t) \gamma_\omega(t), \tag{3.1}$$

where $\mathcal{X}_\omega(t) = \ltimes_{j=1}^{m_\omega} \mathcal{X}_j^\omega(t) \in \Delta_{2^{m_\omega}}$, $Z_\omega(t) = \ltimes_{j=1}^{v_\omega} z_j^\omega(t) \in \Delta_{2^{v_\omega}}$, $\gamma_\omega(t) = \ltimes_{j=1}^{q_\omega} \gamma_j^\omega(t) \in \Delta_{2^{q_\omega}}$, $\omega \in [1, m]$, and $L_\omega \in \mathcal{L}_{2^{m_\omega} \times 2^{m_\omega + v_\omega + q_\omega}}$.

In subnetwork Σ_ω, since the output is directly influenced only by the first ε_ω state variables, we will disregard the dynamic evolution of X_k^ω in the following analysis, $k \in [\varepsilon_\omega + 1, m_\omega]$. Then, we have

$$\widehat{\mathcal{X}}_\omega(t+1) = F_\omega Z_\omega(t) \widehat{\mathcal{X}}_\omega(t) \widetilde{\mathcal{X}}_\omega(t) \gamma_\omega(t), \tag{3.2}$$

where $F_\omega = (I_{2^{\varepsilon_\omega}} \otimes \mathbf{1}_{2^{m_\omega - \varepsilon_\omega}}^\top) L_\omega$, $\widehat{\mathcal{X}}_\omega(t) = \ltimes_{j=1}^{\varepsilon_\omega} \mathcal{X}_j^\omega(t) \in \Delta_{2^{\varepsilon_\omega}}$, and $\widetilde{\mathcal{X}}_\omega(t) = \ltimes_{j=\varepsilon_\omega+1}^{m_\omega} \mathcal{X}_j^\omega(t) \in \Delta_{2^{m_\omega - \varepsilon_\omega}}$.

According to (2.4), the disturbance decoupling of the subsystem Σ_ω is achievable when the dynamics of the first ε_ω state variables remain unaffected by $\left\{X_{\varepsilon_\omega+1}^w, \ldots, X_{m_\omega}^w\right\}$ and $\left\{\gamma_1^w, \ldots, \gamma_{q_w}^w\right\}$. This implies that these sets of variables are regarded as redundant variables.

Considering system (3.2), divide F_ω into 2^{v_ω} equal blocks, denoted as $(F_\omega)_i$, where $\omega \in \{1, \ldots, m\}$ and $i \in \{1, \ldots, 2^{v_\omega}\}$. Next, partition each $(F_\omega)_i$ into 2^{ε_ω} equal blocks, represented as $(F_\omega)_{ij}$, where $\omega \in \{1, \ldots, m\}$, $i \in \{1, \ldots, 2^{v_\omega}\}$, and $j \in \{1, \ldots, 2^{\varepsilon_\omega}\}$. The following results can be obtained.

Theorem 3.1. Considering the subnetwork Σ_ω, the disturbance decoupling problem is solvable if and only if, for any $i \in [1, 2^{\nu_\omega}]$ and any $j \in [1, 2^{\varepsilon_\omega}]$, the following equation holds:

$$(Q_\omega)_{ij} = \bigwedge_{l=1}^{2^{m_\omega-\varepsilon_\omega+q_\omega}} \mathrm{Col}_l[(F_\omega)_{ij}] \neq \mathbf{0}_{2^{\varepsilon_\omega}}. \tag{3.3}$$

Proof. (Sufficiency) Assume that (3.3) holds. From (3.2), it can be seen that after partitioning $F_\omega = [(F_\omega)_1, (F_\omega)_2, \ldots, (F_\omega)_{2^{\nu_\omega}}]$, each block corresponds to a fixed $Z_\omega = \delta^i_{2^{\nu_\omega}}$ for $i \in [1, 2^{\nu_\omega}]$. $(F_\omega)_i = [(F_\omega)_{i1}, (F_\omega)_{i2}, \ldots, (F_\omega)_{i2^{\varepsilon_\omega}}]$, where each block corresponds to a fixed $Z_\omega = \delta^i_{2^{\nu_\omega}}$ and $\widehat{X}_\omega(t) = \delta^j_{2^{\varepsilon_\omega}}$ with $i \in [1, 2^{\nu_\omega}]$, $j \in [1, 2^{\varepsilon_\omega}]$.

Specifically, let $\widetilde{X}_\omega(t) = \delta^\alpha_{2^{m_\omega-\varepsilon_\omega}}$, $\gamma_\omega(t) = \delta^\beta_{2^{q_\omega}}$, and

$$\begin{aligned}\widehat{X}_\omega(t+1) &= F_\omega \delta^i_{2^{\nu_\omega}} \delta^j_{2^{\varepsilon_\omega}} \delta^\alpha_{2^{m_\omega-\varepsilon_\omega}} \delta^\beta_{2^{q_\omega}} = F_\omega \delta^i_{2^{\nu_\omega}} \delta^j_{2^{\varepsilon_\omega}} \delta^{(\alpha-1)2^{q_\omega}+\beta}_{2^{m_\omega-\varepsilon_\omega+q_\omega}} \\ &= (F_\omega)_{ij} \delta^l_{2^{m_\omega-\varepsilon_\omega+q_\omega}} = \mathrm{Col}_l[(F_\omega)_{ij}],\end{aligned}$$

where $l = (\alpha-1)2^{q_\omega} + \beta$. The validity of condition (3.3) implies that

$$\mathrm{Col}_1[(F_\omega)_{ij}] = \mathrm{Col}_2[(F_\omega)_{ij}] = \cdots = \mathrm{Col}_{2^{m_\omega-\varepsilon_\omega+q_\omega}}[(F_\omega)_{ij}];$$

this indicates that for fixed i and j, all columns of matrix $(F_\omega)_{ij}$ be equal, i.e., $\widehat{X}_\omega(t+1)$ is independent of the redundant states $\widetilde{X}_\omega(t)$ and the disturbance $\gamma_\omega(t)$. Therefore, this guarantees the solvability of the disturbance decoupling problem for subnetwork Σ_ω.

(Necessity) Assuming the disturbance decoupling problem for the subnetwork Σ_ω is solvable, the output $y(t)$ depends solely on the first ε_ω relevant state variables $\widehat{X}_\omega(t)$. This implies that $\widehat{X}_\omega(t+1)$ must be uniquely determined only by the current relevant state $\widehat{X}_\omega(t)$ and the switching signal $Z_\omega(t)$.

If $\exists\ i_0 \in [1, 2^{\nu_\omega}]$, $j_0 \in [1, 2^{\varepsilon_\omega}]$ such that

$$(Q_\omega)_{i_0j_0} = \bigwedge_{l=1}^{2^{m_\omega-\varepsilon_\omega+q_\omega}} \mathrm{Col}_l[(F_\omega)_{i_0j_0}] = \mathbf{0}_{2^{\varepsilon_\omega}};$$

this means that there exists at least $l_1 \neq l_2$ such that $\mathrm{Col}_{l_1}[(F_\omega)_{i_0j_0}] \neq \mathrm{Col}_{l_2}[(F_\omega)_{i_0j_0}]$.

The column index l corresponds to different combinations of redundant states and disturbances. This means that under identical conditions $Z_\omega(t)$ and $\widehat{X}_\omega(t)$, the system produces different subsequent states, with the variation originating from different values of redundant states and disturbances. In other words, the system fails to achieve disturbance decoupling. This contradicts the initial assumption. Therefore, condition (3.3) must hold. ◊

If a subnetwork Σ_ω is not inherently disturbance decouplable, external control must be applied. In such cases, we focus on the design of state-flipping controllers to achieve disturbance decoupling.

For subnetwork Σ_ω, the state-flipping matrix is

$$(\overline{Q}_\omega)_i = (Q_\omega)_i \mathcal{F}_{U_\omega}, \tag{3.4}$$

where $(Q_\omega)_i = [(Q_\omega)_{i1}\ (Q_\omega)_{i2} \cdots (Q_\omega)_{i2^{\varepsilon_\omega}}]$, $i \in [1, 2^{\nu_\omega}]$, $(Q_\omega)_i \in \mathcal{L}_{2^{\varepsilon_\omega}\times 2^{\varepsilon_\omega}}$, $\mathcal{F}_{U_\omega} \in \mathfrak{B}_{2^{\varepsilon_\omega}\times 2^{\varepsilon_\omega}}$, and $(\overline{Q}_\omega)_i \in \mathcal{H}_{2^{\varepsilon_\omega}\times 2^{\varepsilon_\omega}}$.

Algorithm 1 Verification of solvability for disturbance decoupling in LBNs

1: **Input**: $G = (V,\ E)$, F_ω.
2: **Output**: φ.
3: Initialize $\varphi = \emptyset$, $\lambda = \emptyset$
4: **for** $\omega = 1$ to m **do**
5:     Calculate Z_ω
6:     φ_ω = True
7:       **for** $i = 1$ to 2^{m_ω} **do**
8:         **for** $j = 1$ to 2^{ε_ω} **do**
9:             **if** $(Q_\omega)_{ij} = \bigwedge_{l=1}^{2^{m_\omega-\varepsilon_\omega+q_\omega}} \mathrm{Col}_l[(F_\omega)_{ij}] = \mathbf{0}_{2^{\varepsilon_\omega}}$ **then**
10:                 φ_ω = False
11:                 add ω to λ
12:                 **break**
13:             **end if**
14:         **end for**
15:        **if** φ_ω = False **then**
16:           **break**
17:        **end if**
18:       **end for**
19:     **if** φ_ω = True **then**
20:        Append φ_ω to φ
21:     **end if**
22: **end for**
23: **for** $\omega \in \lambda$ **do**
24:     Find $\mathcal{F}_{U_\omega}$ and calculate $(\overline{Q}_\omega)_k$
25:     φ_ω = True
26:       **for** $\kappa = 1$ to 2^{ν_ω} **do**
27:         **for** $j = 1$ to 2^{ε_ω} **do**
28:             **if** $\mathbf{1}^\top_{2^{\varepsilon_\omega}} \cdot \mathrm{Col}_j[(\overline{Q}_\omega)_k] = 0$ **then**
29:                 φ_ω = False
30:                 **break**
31:             **end if**
32:         **end for**
33:        **if** φ_ω = False **then**
34:           go to step 24
35:        **end if**
36:       **end for**
37:     Append φ_ω to φ
38: **end for**
39: **if** φ = {True, True, . . . , True} **then**
40:     **Return** φ
41:        **break**
42: **end if**

Theorem 3.2. For the controlled subnetwork Σ_ω, the disturbance decoupling problem is solvable if and only if there exists a flipping control such that

$$\mathbf{1}_{2^{\varepsilon_\omega}}^{\top} \cdot \mathrm{Col}_j[(\overline{Q}_\omega)_i] > 0 \tag{3.5}$$

holds for all $i \in [1, 2^{\nu_\omega}]$ and $j \in [1, 2^{\varepsilon_\omega}]$.

Proof. For Eq (3.4), $(Q_\omega)_i$ is the state transition matrix of the uncontrolled subnetwork under a fixed switching signal $Z_\omega = \delta_{2^{\nu_\omega}}^i$; $\mathcal{F}_{U_\omega}$ is the combinatorial flip matrix, satisfying

$$(\mathcal{F}_{U_\omega})_{\beta j} = 1 \Longleftrightarrow \exists\, V \subseteq \mathcal{P}_{U_\omega}, \delta_{2^{\varepsilon_\omega}}^j \xrightarrow{V} \delta_{2^{\varepsilon_\omega}}^\beta.$$

Therefore, under the switching signal $Z_\omega = \delta_{2^{\nu_\omega}}^i$ and starting from the state $\delta_{2^{\varepsilon_\omega}}^j$,

$$[(\overline{Q}_\omega)_i]_{\alpha j} \geq 1 \Longleftrightarrow \exists\, V \subseteq \mathcal{P}_{U_\omega}, \forall \widetilde{X}_\omega(t), \gamma_\omega(t), \widehat{X}_\omega(t+1) = \delta_{2^{\varepsilon_\omega}}^\alpha,$$

where $[(\overline{Q}_\omega)_i]_{\alpha j}$ represents the element in the α-th row and j-th column of matrix $(\overline{Q}_\omega)_i$.

(Necessity) Assume that under state-flipping control, the subnetwork Σ_ω achieves disturbance decoupling. The matrix $(\overline{Q}_\omega)_i$ aggregates all such intersection cases for each relevant state under switching signal $Z_\omega = \delta_{2^{\nu_\omega}}^i$ after applying state-flipping control.

If $\exists\, i_0 \in [1, 2^{\nu_\omega}]$, $j_0 \in [1, 2^{\varepsilon_\omega}]$ such that

$$\mathbf{1}_{2^{\varepsilon_\omega}}^{\top} \cdot \mathrm{Col}_{j_0}[(\overline{Q}_\omega)_{i_0}] = 0,$$

i.e.,

$$(\overline{Q}_\omega)_{i_0 j_0} = \mathbf{0}_{2^{\varepsilon_\omega}} \Longleftrightarrow [(\overline{Q}_\omega)_{i_0}]_{\alpha j_0} = 0, \forall\, \alpha \in [1, 2^{\varepsilon_\omega}].$$

From the above analysis, it follows that

$$\forall\, V \subseteq \mathcal{P}_{U_\omega}, \exists\, \widetilde{X}_\omega(t), \gamma_\omega(t), \widehat{X}_\omega(t+1) \neq \delta_{2^{\varepsilon_\omega}}^\alpha.$$

Consequently, this state becomes influenced by redundant states and disturbances. Since this state affects the output, the system cannot achieve disturbance decoupling, contradicting the initial assumption. Therefore, condition (3.5) must hold.

(Sufficiency) Assume that condition (3.5) holds. Under switching signal $Z_\omega = \delta_{2^{\nu_\omega}}^i$, for each column j of the matrix $(\overline{Q}_\omega)_i$, there exists at least one row α^*, $\alpha^* \in [1, 2^{\varepsilon_\omega}]$, such that

$$\mathrm{Row}_{\alpha^*}[\mathrm{Col}_j[(\overline{Q}_\omega)_i]] \geq 1,$$

i.e., there exists a flipping subset $V \subseteq \mathcal{P}_{U_\omega}$ such that, starting from the current state $\delta_{2^{\varepsilon_\omega}}^j$, the next relevant state is uniquely determined as $\delta_{2^{\varepsilon_\omega}}^{\alpha^*}$ under all redundant states and disturbances. This implies that disturbances and redundant states cannot influence the evolution of the relevant states. Furthermore, condition (3.5) holds for all switching signals $i \in [1, 2^{\nu_\omega}]$. Therefore, the system can achieve solvability of disturbance decoupling. ◊

In this paper, the input Z_ω is regarded as an arbitrary switching signal to cover all possible scenarios. Condition (3.5) in Theorem 3.2 holds for all Z_ω, so regardless of whether Z_ω is affected by disturbances or originates from inter-subnetwork coupling, the disturbance decoupling property of each subnetwork remains unaffected. When all subnetworks satisfy Theorem 3.2, the global output $y_j(t)$ is determined solely by the relevant states that are independent of disturbances. Consequently, even if a flipping operation induces state fluctuations that propagate through cycles, disturbances cannot re-enter the relevant states, and the global disturbance decoupling property under the closed-loop setting is preserved.

Theorem 3.3. The large-scale Boolean network can achieve disturbance decoupling, if all its subnetworks Σ_ω can achieve disturbance decoupling under arbitrary switching signals and arbitrary disturbances, $\omega \in [1, m]$.

Proof. Assume that for each $\omega \in \{1, \ldots, m\}$, the subnetwork Σ_ω satisfies the conditions of Theorem 3.2. That is, there exists a flipping control such that under its switching signal Z_ω and all disturbances, the evolution of its relevant states $\widehat{X}_\omega$ is independent of redundant variables. Formally, for any two distinct disturbances $\gamma_\omega^{(1)}$ and $\gamma_\omega^{(2)}$, we have

$$\widehat{X}_\omega(t; Z_\omega, \gamma_\omega^{(1)}) = \widehat{X}_\omega(t; Z_\omega, \gamma_\omega^{(2)}), \forall\, t \geq 0, \forall\, Z_\omega \in \Delta_{2^{v_\omega}},$$

where $\widehat{X}_\omega(t; Z_\omega, \gamma_\omega^{(1)})$ represents the state at time t under the switching signal Z_ω and disturbance $\gamma_\omega^{(1)}$. From the output equation in (2.4), the global output y_j is a Boolean function of the relevant states of each subnetwork:

$$y_j(t) = g_j(\widehat{X}_1(t), \widehat{X}_2(t), \ldots, \widehat{X}_m(t)), j = 1, \ldots, p.$$

Since each $\widehat{X}_\omega$ is unaffected by disturbances, the output of the entire original system is also unaffected by disturbances. Therefore, the large-scale Boolean network achieves solvability of disturbance decoupling. ◊

Remark 3.1. Each subnetwork independently designs its controller, treating inputs from other subnetworks as arbitrary switching signals. According to Theorem 3.2, as long as each subnetwork achieves disturbance decoupling under all switching signals, its decoupling property remains unaffected regardless of whether the control across subnetworks is executed synchronously or asynchronously. Since the global output is determined solely by the relevant states of each subnetwork, once all subnetworks are decoupled, the entire network can achieve global disturbance decoupling, independent of differences in control timing.

To analyze disturbance decoupling solvability in large-scale Boolean networks, Algorithm 1 is developed for verification. Key notations include: $G = (V, E)$ representing the network structure, and $\varphi = \{\varphi_1, \varphi_2, \ldots, \varphi_m\}$ denoting the solvability set where $\varphi_\omega =$ True indicates solvable subnetworks and $\varphi_\omega =$ False unsolvable ones. The algorithm's implementation details are presented as follows.

The state-flipping control strategy adopted in this paper takes flip cost as the optimization objective. Although flipping all nodes is guaranteed to achieve the desired control effect, the associated cost is prohibitively high. Therefore, Algorithm 2 is designed to solve for the minimal set of nodes to flip that can still achieve the control objective, thereby significantly reducing the control cost while ensuring effectiveness. Before presenting the algorithm, the following key notations are introduced: τ denotes the number of flipped nodes, μ denotes the index of a specific arrangement under the same number of flipped nodes, and ρ_τ denotes the total number of combinations when the number of flipped nodes is τ. All candidate sets are lexicographically ordered based on node indices in $1, 2, \ldots, m_\omega$.

For example, considering $m_\omega = 3$, when $\tau = 1$, there exist three cases: $\mu = 1$ corresponds to the set $\{1\}$, $\mu = 2$ corresponds to the set $\{2\}$, and $\mu = 3$ corresponds to the set $\{3\}$. In this scenario, $\rho_1 = 3$. When $\tau = 2$, there exist three cases: $\mu = 1$ corresponds to the set $\{1, 2\}$, $\mu = 2$ corresponds to the set $\{1, 3\}$, and $\mu = 3$ corresponds to the set $\{2, 3\}$. In this scenario, $\rho_2 = 3$. When $\tau = 3$, there exist one case: $\mu = 1$ corresponds to the set $\{1, 2, 3\}$. In this scenario, $\rho_3 = 1$.

In Algorithm 2, U_ω is defined as a candidate node set rather than a fixed flipping instruction. Since the optimal flipping subset required to achieve disturbance decoupling may vary across different states, this paper predetermines U_ω so that each state can select a feasible subset V according to its own needs, rather than applying a uniform operation. To identify the flipping subset with the minimum cardinality while ensuring feasibility, it is necessary to enumerate all candidate subsets in increasing order of cardinality for verification.

Algorithm 2 Find the minimum flip set

Input: F_ω.
Output: U_ω.
Initialize $\tau = 1$, $\mu = 1$, $U_\omega = \emptyset$
for $\omega \in \lambda$ **do**
    for $\kappa = 1$ to 2^{ν_ω} **do**
        for $j = 1$ to 2^{ε_ω} **do**
            if $\mathbf{1}^{\top}_{2^{\varepsilon_\omega}} \cdot \mathrm{Col}_j[(\overline{Q}_\omega)_k] = 0$ **then**
                if $\mu < \rho_\tau$ **then**
                    $\mu + +$
                else
                    $\tau + +$
                    $\mu \longleftarrow 1$
                end if
            else
                Return U_ω
            end if
        end for
    end for
end for

Remark 3.2. According to the disjoint partition principle in (2.3), the global flipping set can be obtained directly by taking the union of the minimum flipping sets of each subnetwork, i.e., $U = \bigcup_{\omega=1}^{m} U_\omega$. However, this union only ensures local minimality at the subnetwork level and is not necessarily globally optimal, since achieving global optimality requires considering the coupling relationships between subnetworks. This paper aims to reduce computational complexity by decomposing the original problem through network aggregation, rather than pursuing global optimality. Nevertheless, the proposed method guarantees a minimum number of flipped nodes within each subnetwork. Future work will focus on optimizing the global flipping set while preserving decoupling performance.

Remark 3.3. If directly processing the original large-scale Boolean network, the dimension of the state transition matrix is $O(2^n \times 2^{n+q})$, leading to excessively high computational complexity. This paper decomposes the original network into multiple small-scale subnetworks through network aggregation, where the number of nodes m_ω, inputs ν_ω, and disturbances q_ω in each subnetwork are all much smaller than n. This reduces the matrix size to $O(2^{m_\omega} \times 2^{m_\omega+\nu_\omega+q_\omega})$, significantly lowering the computational complexity. Meanwhile, the enumeration search is performed at the subnetwork level. Algorithm 2 adopts a search strategy that proceeds in increasing order of cardinality and terminates once a feasible solution is found, so the actual search space is much smaller than the worst-case 2^{m_ω}, further improving computational efficiency and making the method applicable to large-scale networks.

4. An illustrative example

Example 4.1. Consider the following large-scale Boolean network:

$$\begin{cases} X_1(t+1) = X_2(t) \wedge X_3(t) \wedge X_4(t) \\ X_2(t+1) = X_1(t) \vee X_2(t) \vee \gamma_1(t) \\ X_3(t+1) = X_2(t) \vee X_3(t) \\ X_4(t+1) = X_4(t) \vee X_5(t) \vee (X_5(t) \rightarrow X_6(t)) \vee \gamma_2(t) \\ X_5(t+1) = X_5(t) \vee X_6(t) \\ X_6(t+1) = X_3(t) \vee (\neg(X_4(t) \vee X_5(t))) \\ X_7(t+1) = X_7(t) \vee X_8(t) \\ X_8(t+1) = X_8(t) \\ X_9(t+1) = X_6(t) \vee X_7(t) \vee \gamma_3(t) \\ X_{10}(t+1) = X_{10}(t) \wedge X_{11}(t) \\ X_{11}(t+1) = (X_{10}(t) \rightarrow X_{11}(t)) \wedge X_{12}(t) \\ X_{12}(t+1) = X_{10}(t) \vee X_{11}(t) \vee \gamma_4(t) \\ X_{13}(t+1) = X_9(t) \vee X_{11}(t) \vee X_{12}(t) \\ X_{14}(t+1) = X_{14}(t) \wedge X_{16}(t) \\ X_{15}(t+1) = X_{15}(t) \rightarrow X_{16}(t) \\ X_{16}(t+1) = X_{14}(t) \vee X_{15}(t) \vee X_{16}(t) \\ X_{17}(t+1) = (X_{10}(t) \rightarrow X_{15}(t)) \wedge X_{16}(t) \wedge \gamma_5(t), \end{cases} \tag{4.1}$$

where $X_i(t) \in \mathcal{D}$ represents state nodes with $i \in [1,17]$, and $\gamma_j \in \mathcal{D}$ represents disturbance with $j \in [1,5]$. Assume output equations are

$$\begin{cases} y_1(t) = X_1(t) \vee X_4(t) \\ y_2(t) = X_1(t) \wedge X_5(t) \wedge X_7(t) \\ y_3(t) = X_2(t) \vee X_8(t) \vee X_{12}(t) \\ y_4(t) = X_{10}(t) \rightarrow X_{11}(t) \\ y_5(t) = X_{14}(t) \wedge X_{15}(t) \wedge X_{16}(t), \end{cases} \tag{4.2}$$

where $y_\epsilon \in \mathcal{D}$, $\epsilon \in [1,5]$.

Firstly, by using network aggregation, the state nodes of LBN are divided into 5 subnetworks, and the state nodes and input nodes of each subnetwork are as follows: $\Sigma_1 = \{X_1, X_2, X_3\}$, $Z_1 = \left\{z_1^1 = X_4\right\}$; $\Sigma_2 = \{X_4, X_5, X_6\}$, $Z_2 = \left\{z_1^2 = X_3\right\}$; $\Sigma_3 = \{X_7, X_8, X_9\}$, $Z_3 = \left\{z_1^3 = X_6\right\}$; $\Sigma_4 = \{X_{10}, X_{11}, X_{12}, X_{13}\}$, $Z_4 = \left\{z_1^4 = X_9\right\}$; $\Sigma_5 = \{X_{14}, X_{15}, X_{16}, X_{17}\}$, and $Z_5 = \left\{z_1^5 = X_{10}\right\}$. The specific network structure diagram is depicted in Figure 1.

According to (3.2), the ASSR of each subnetwork Σ_ω can be established:

$$\widehat{X}_\omega(t+1) = F_\omega Z_\omega(t)\widehat{X}_\omega(t)\widetilde{X}_\omega(t)\gamma_\omega(t), \omega \in [1, 5]. \tag{4.3}$$

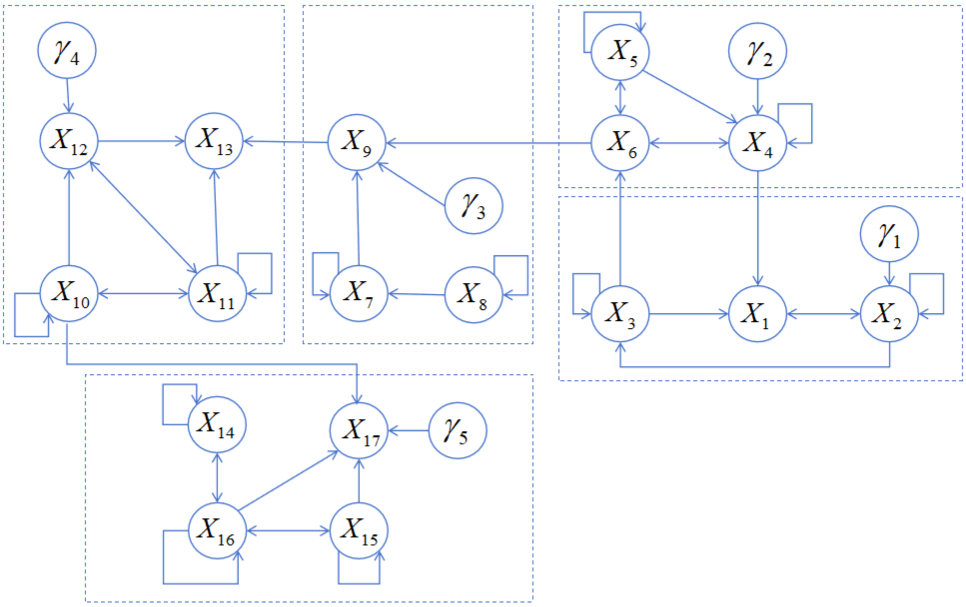

Figure 1. Network structure diagram.

Second, we will investigate the disturbance decoupling solvability of each subnetwork, thereby achieving disturbance decoupling solvability of the LBN.

In subnetwork Σ_1, we consider X_4 as an arbitrary switching signal, and it can be calculated that

$$(F_1)_1 = \delta_4[1,\ 1,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 1,\ 1,\ 3,\ 3,\ 3,\ 4,\ 3,\ 4],$$

$$(F_1)_2 = \delta_4[3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 4,\ 3,\ 4].$$

Then, we have $(Q_1)_{11} = \mathbf{0}_4$, $(Q_1)_{13} = \mathbf{0}_4$, $(Q_1)_{14} = \mathbf{0}_4$, and $(Q_1)_{24} = \mathbf{0}_4$. This indicates that the conditions of Theorem 3.1 are not met, and Σ_1 cannot achieve the solvability of disturbance decoupling without control. To achieve the goal, state-flipping control is applied. Considering $U_1 = \{1, 2\}$ and $\mathcal{P}_{U_1} = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$, it can be derived that

$$(\overline{Q}_1)_1 = (Q_1)_1\mathcal{F}_{U_1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{and } (\overline{Q}_1)_2 = (Q_1)_2\mathcal{F}_{U_1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 3 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

This implies that

$$\mathbf{1}_4^{\top} \cdot \mathrm{Col}_j[(\overline{Q}_1)_i] > 0$$

holds for any $i \in [1, 2]$, $j \in [1, 4]$; then, the conditions of Theorem 3.2 are satisfied. It should be noted that different states within the same subnetwork have completely different flipping requirements. Specifically, when the switching signal is δ_2^1, state δ_4^1 requires flipping node $\{2\}$; state δ_4^2 requires no flipping; state δ_4^3 requires flipping $\{1, 2\}$; and state δ_4^4 requires flipping $\{1\}$. When the switching signal is δ_2^2, state δ_4^1, δ_4^2, and δ_4^3 require no flipping; state δ_4^4 can achieve the target by flipping $\{1\}$ or $\{2\}$ or $\{1, 2\}$. Therefore, the controlled Σ_1 can achieve the solvability of disturbance decoupling.

Similarly, it has been calculated that subnetworks Σ_2 and Σ_4 cannot achieve the goal without control. By applying state-flipping control sets $U_2 = \{5\}$ and $U_4 = \{10\}$, respectively,

$$(\overline{Q}_2)_1 = (Q_2)_1 \mathcal{F}_{U_2} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$(\overline{Q}_2)_2 = (Q_2)_2 \mathcal{F}_{U_2} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$(\overline{Q}_4)_1 = (Q_4)_1 \mathcal{F}_{U_4} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$(\overline{Q}_4)_2 = (Q_4)_2 \mathcal{F}_{U_4} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

the conditions of Theorem 3.2 can be met. Therefore, they can achieve disturbance decoupling.

For subnetworks Σ_3 and Σ_5, it can be calculated that

$$(Q_3)_{ij} = \bigwedge_{l=1}^{4} \mathrm{Col}_l[(F_3)_{ij}] \neq \mathbf{0}_4, \forall\, i \in [1, 2], j \in [1, 4],$$

$$(Q_5)_{ij} = \bigwedge_{l=1}^{4} \mathrm{Col}_l[(F_5)_{ij}] \neq \mathbf{0}_8, \forall\, i \in [1,2], j \in [1,8].$$

This inherently satisfies the conditions of Theorem 3.1 without requiring additional control, and can thus achieve disturbance decoupling without control.

According to Theorem 3.3, when each subnetwork achieves solvability of disturbance decoupling, meaning that the outputs are unaffected by disturbances, then the entire large-scale Boolean network can achieve disturbance decoupling.

Finally, taking Σ_1 and Σ_3 as examples, the state transition diagrams with and without control are presented to clearly demonstrate the effectiveness of the results. Figures 2 and 3 show the state transitions of Σ_1 without control and with control, respectively. Without control, states δ_4^1, δ_4^3, and δ_4^4 do not reach a unique output under switching signal δ_2^1, and state δ_4^4 does not reach a unique output under switching signal δ_2^2. However, with control, all states reach a unique output under any switching signal and any disturbance. Figure 4 shows the state transitions of Σ_3, where all states reach a unique output under any switching signal and disturbance even without control. In these diagrams, blue arrows indicate state transitions under switching signal δ_2^1, while orange arrows indicate state transitions under switching signal δ_2^2.

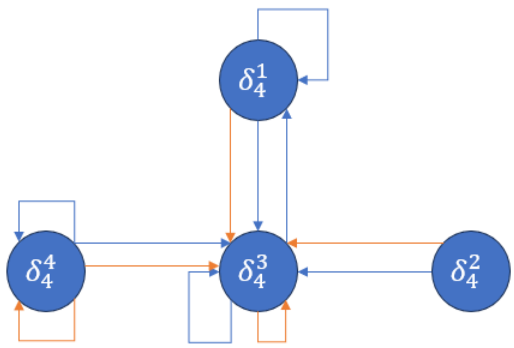

Figure 2. The state transition diagram of Σ_1 without control.

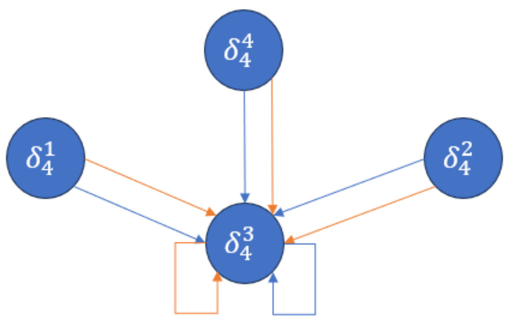

Figure 3. The state transition diagram of Σ_1 with control.

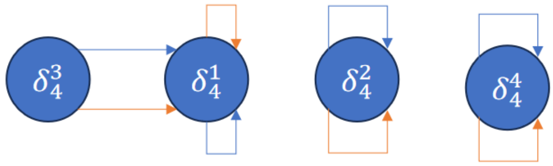

Figure 4. The state transition diagram of Σ_3.

5. Conclusions

This paper has conducted a study on disturbance decoupling in large-scale Boolean networks. By employing network aggregation, the original network has been decomposed into smaller subnetworks, significantly reducing computational complexity. A state-flipping control strategy has been adopted to achieve disturbance decoupling for these subnetworks, with necessary and sufficient conditions derived under both uncontrolled and controlled scenarios. Two algorithms are proposed, including a verification algorithm and an algorithm for identifying the minimum set of flipping nodes. A numerical example has demonstrated the effectiveness of the methods. Future research may explore other complexity reduction approaches to enable more efficient disturbance decoupling, offering new insights for controlling complex networks in practical applications.

Author contributions

Peilian Guo: Conceptualization, validation, writing–review, editing; Juhan Li: Investigation, writing-original draft; Yuanhua Wang: Conceptualization, writing–review, editing, formal analysis; Ben Niu: Writing–review, editing. All authors have read and agreed to the published version of the manuscript.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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