AIMS Mathematics

AIMS Mathematics, 11(3): 7015–7046.
DOI: 10.3934/math.2026288
Received: 19 January 2026
Revised: 19 February 2026
Accepted: 03 March 2026
Published: 18 March 2026

https://www.aimspress.com/journal/Math

Research article

A Spherical fuzzy entropy–driven TOPSIS framework with hybrid weighting for group decision-making

Ebru Aydoğdu*

Department of Basic Sciences, Naval Academy, Turkish National Defence University, İstanbul 34940, Türkiye

* **Correspondence:** Email: ebru.aydogdu@msu.edu.tr, ebrudiyarbakirli@gmail.com.

Abstract: Decision-making problems involving multiple conflicting criteria and hesitant evaluations require weighting mechanisms that are both theoretically consistent and practically reliable. In spherical fuzzy group decision-making environments, expert importance is often assumed or externally assigned, while objective and subjective criterion weights are not systematically integrated. To address this limitation, this study proposes a new spherical fuzzy entropy measure to objectively derive expert weights directly from the decision matrix. In addition, objective entropy-based criterion weights and SWARA-based subjective assessments are coherently combined through a multiplicative aggregation structure within a TOPSIS framework. The proposed model is applied to a medical waste treatment technology selection problem. Comparative analyses and sensitivity experiments demonstrate stable rankings under different entropy formulations and aggregation strategies, while the proposed entropy exhibits stronger discriminatory capacity. These findings indicate that the framework provides a consistent and informative weighting structure for group decision-making under uncertainty.

Keywords: spherical fuzzy sets; entropy measure; TOPSIS; SWARA; combined criteria weighting; medical waste management; multi-criteria decision making
Mathematics Subject Classification: 94D05, 90B50, 28E10, 03E72

Appendix A. List of abbreviations

Abbreviation	Full term
ARAS	Additive ratio assessment
ASFDM	Aggregated spherical fuzzy decision matrix
AWSFDM	Aggregated weighted spherical fuzzy decision matrix
CCW	Combined criteria weight
CRITIC	Criteria importance through intercriteria correlation
CoCoSo	Combined compromise solution
CODAS	Combinative distance-based assessment
COPRAS	Complex proportional assessment
CPT	Cumulative prospect theory
DMs	Decision makers
EW	Entropy weight
EW-CCW-TOPSIS	Entropy weighted combined criterion weight technique for order preference by similarity to ideal solution
MABAC	Multi-attributive border approximation area comparison
MARCOS	Measurement of alternatives and ranking according to compromise solution
MCDM	Multi-criteria decision making
MEREC	Method based on the removal effects of criteria
NIS	Negative ideal solution
NSFDM	Normalized spherical fuzzy decision matrix
PIS	Positive ideal solution
PROMETHEE	Preference ranking organization method for enrichment evaluation
SFS	Spherical fuzzy set
SFN	Spherical fuzzy number
SWARA	Stepwise weight assessment ratio analysis
TODIM	An acronym in Portuguese for interactive and multi-criteria decision making
TOPSIS	Technique for order preference by similarity to ideal solution
VIKOR	VlseKriterijumska Optimizacija i Kompromisno Resenje
WASPAS	Weighted aggregated sum product assessment

1. Introduction

Decision-making problems involving multiple conflicting criteria and uncertain information are inherently complex, particularly in group decision-making environments where expert judgments may be imprecise, hesitant, or partially inconsistent. In such contexts, the accurate modeling of uncertainty and the systematic aggregation of expert opinions play a central role in ensuring reliable and robust decision outcomes. Medical waste management represents a typical decision context of this nature, where economic, environmental, technical, and safety-related criteria must be evaluated simultaneously under significant ambiguity.

Multi-criteria decision-making (MCDM) methods offer a structured mathematical framework for addressing these challenges. However, classical MCDM models based on crisp information are insufficient to capture the vagueness and hesitation inherent in human evaluations. To overcome this limitation, fuzzy set theory and its extensions have been extensively incorporated into decision-making models. Following the introduction of fuzzy sets by Zadeh [46], several generalized fuzzy structures have been proposed, including intuitionistic fuzzy sets [4], Pythagorean fuzzy sets [45], picture fuzzy sets [13], and, more recently, spherical fuzzy sets (SFSs) [28].

Spherical fuzzy sets provide a flexible and expressive uncertainty modeling framework by allowing independent degrees of membership, non-membership, and hesitancy subject to a spherical constraint. This structure enables a richer representation of expert uncertainty and makes SFSs particularly suitable for group decision-making problems characterized by incomplete or inconsistent information. As a result, SFS-based MCDM models have been increasingly adopted in the literature and integrated with various decision-making techniques such as ARAS [7], COPRAS [9], CPT-based models [48], entropy-driven approaches [31,32], entropy-weighted TOPSIS frameworks applied under uncertainty conditions [38], and recently, with advanced aggregation-based hybrid frameworks such as the Hamacher AHP–COPRAS integration under spherical fuzzy environments [22]. Recent contributions have further expanded the application of spherical fuzzy theory in decision-making environments. For instance, prioritized aggregation operators based on spherical fuzzy sets have been developed to enhance information fusion in data analytical enterprises [24]. Similarly, spherical fuzzy soft geometric aggregation operators have been introduced to model complex AI-driven decision scenarios in engineering applications [14]. In transportation safety research, spherical fuzzy AHP-based models have also been proposed to quantify behavioral factors under uncertainty [35]. While these studies demonstrate the flexibility of spherical fuzzy structures in modeling and aggregation, they primarily focus on operator development or preference modeling. The systematic integration of entropy-based expert weighting with a coherent hybrid criterion weighting mechanism within a unified TOPSIS framework remains relatively underexplored.

Among classical MCDM methods, the technique for order preference by similarity to ideal solution (TOPSIS) is widely used due to its clear mathematical logic, computational efficiency, and ability to simultaneously identify the most and least desirable alternatives. Numerous extensions of TOPSIS within the spherical and complex spherical fuzzy environment have been proposed, addressing diverse decision contexts including supplier selection [28], robot selection [10], and sustainability-oriented problems [43], and entropy-based complex spherical fuzzy TOPSIS models [8]. However, existing entropy-based spherical or complex spherical fuzzy TOPSIS models primarily focus on objective criterion weighting and generally assume predefined or equal importance of experts. Nevertheless, a careful examination of existing SFS and complex SFS-TOPSIS-based studies reveals substantial methodological differences in terms of expert weighting assumptions and criterion weight determination strategies. In particular, most existing approaches, including entropy-based models, either assume equal importance among experts, rely on subjectively assigned expert weights, or employ exclusively objective or exclusively subjective criterion weighting schemes.

Fully integrated frameworks that objectively determine expert importance while coherently combining objective and subjective criterion weights within a unified TOPSIS structure remain limited. Motivated by this gap, this study proposes an entropy-based expert weighting and combined criterion weighting TOPSIS (EW–CCW–TOPSIS) framework under a spherical fuzzy environment. By deriving expert weights endogenously from entropy measures and integrating objective and subjective criterion information in a systematic manner, the proposed approach enhances methodological consistency and robustness in group decision-making problems under uncertainty.

Table 1 presents a structured comparison of representative spherical fuzzy set (SFS)-based TOPSIS models with respect to group decision-making assumptions, expert weighting strategies, criterion weighting mechanisms, and application domains. The comparison reveals that most existing

SFS-TOPSIS studies either assume expert importance a priori, assign equal weights to decision-makers, or rely solely on objective or subjective criterion weighting.

Table 1. Methodological characteristics of existing SFS-based TOPSIS models.

Ref.	Ranking method	Group MCDM	DM weights			Criteria weights			Application area
			Subj.	Obj.	Apr.	Subj.	Obj.	Apr.	
[28]	TOPSIS	yes	yes	-	given	yes	-	Given	Supplier selection problem
[25]	TOPSIS	yes	yes	-	given	yes	-	Given	Hospital location selection problem
[10]	TOPSIS	yes	-	yes	with distance	-	yes	Entropy weight	Robot selecting problem
[34]	TOPSIS	no	-	-	-	-	yes	AHP	Advanced manufacturing system selection
[30]	TOPSIS	no	-	-	-	yes	-	Given	Optimal site selection of electrical vehicle charging station
[15]	TOPSIS	yes	yes	-	assign equal weights	-	yes	Maximizing deviation	Choosing the best advertising strategy for food industry companies
[43]	TOPSIS	yes	yes		given	yes	-	Given	Evaluation of circular economy business models for SMEs

In particular, entropy-based approaches predominantly focus on criterion weighting, while expert weighting is often neglected or requires additional external information. Conversely, subjective methods such as the stepwise weight assessment ratio analysis (SWARA) method, originally proposed by Kersuliene et al. [26], effectively capture expert preferences by allowing decision-makers to determine the relative importance of criteria through a structured sequential evaluation process; however, when used independently, they fail to reflect the intrinsic information dispersion of the decision matrix.

Although recent studies have attempted to combine objective and subjective weighting schemes within spherical fuzzy MCDM frameworks, such as the BWM–entropy–COPRAS model proposed by Gao et al. [16], these approaches commonly exhibit at least one of the following limitations: (i) expert weights are externally assigned or assumed, (ii) objective and subjective criterion weights are combined without a systematic integration mechanism, or (iii) additional behavioral or psychological parameters are required, which may not always be available or reliable.

Existing spherical fuzzy MCDM studies have predominantly focused either on objective entropy-based weighting or on subjective preference-driven methods. However, the efficiency and discriminatory performance of entropy measures in determining expert reliability have not been sufficiently examined, particularly in group decision-making contexts. Moreover, although hybrid weighting strategies have been introduced in the literature, the systematic integration of entropy-based expert weighting with combined objective–subjective criterion weighting remains limited. This methodological gap becomes particularly critical in medical waste treatment selection, where both data dispersion and expert domain knowledge must be simultaneously reflected to ensure reliable and risk-aware decisions.

Motivated by these methodological gaps, this study proposes a novel spherical fuzzy group decision-making framework that integrates entropy-based expert weighting with combined criterion weighting within a TOPSIS structure. The main contributions of this study can be summarized as follows:

• A new spherical fuzzy entropy measure is introduced and employed to objectively derive expert weights directly from the decision matrix, without requiring auxiliary information.

• A combined criterion weighting scheme is developed by systematically integrating entropy-based objective weights and SWARA-based subjective weights using a multiplicative aggregation strategy.

• The proposed EW–CCW–TOPSIS framework ensures a coherent balance between data-driven information and expert judgment, enhancing robustness and discrimination capability in group decision-making problems.

• The effectiveness, stability, and consistency of the proposed approach are demonstrated through a real-world medical waste treatment technology selection problem, supported by comparative and sensitivity analyses.

The remainder of the paper is organized as follows: Section 2 introduces the fundamental concepts of spherical fuzzy sets. Section 3 presents the proposed entropy measure and the integrated EW–CCW–TOPSIS methodology. Section 4 illustrates the applicability of the proposed framework through a case study. Section 5 provides comparative and robustness analyses, and Section 6 concludes the paper with final remarks and future research directions.

2. Preliminaries

This section retrospectively covers some notions of SFSs and entropy measures on SFS.

2.1. Spherical fuzzy set

Definition 2.1. [28] *Let X be the universe of discourse. The spherical fuzzy set (SFS) is defined as*

$$A = \{\langle x, (\mu_A(x), \iota_A(x), \nu_A(x)) \mid x \in X\rangle\}$$

where $\mu_A(x) : X \to [0,1]$, $\iota_A(x) : X \to [0,1]$, $\nu_A(x) : X \to [0,1]$, and $(\mu_A(x))^2 + (\iota_A(x))^2 + (\nu_A(x))^2 \leq 1$, $\forall x \in X$. In the meantime, for each x, the numbers $\mu_A(x)$, $\iota_A(x)$, and $\nu_A(x)$ are the degree of positive-membership, neutral membership, and negative-membership of x to A, respectively. In addition, $\pi_A(x) = \sqrt{1 - (\mu_A(x))^2 - (\nu_A(x))^2 - (\iota_A(x))^2}$ is called the refusal degree of x in X.

For simplicity, $\mathrm{A} = (\mu_\mathrm{A}, \iota_\mathrm{A}, \nu_\mathrm{A})$ is said to be a spherical fuzzy number (SFN). In the following, some basic operators on SFNs are given.

Definition 2.2. [28] *Assume that $A = (\mu_A, \iota_A, \nu_A)$ and $B = (\mu_B, \iota_B, \nu_B)$ are two SFNs, then:*

(1) The sum of A and B:

$$A \oplus B = \left(\sqrt{\mu_A^2 + \mu_B^2 - \mu_A^2\mu_B^2}, \quad \sqrt{(1-\mu_B^2)\iota_A^2 + (1-\mu_A^2)\iota_B^2 - \iota_A^2\iota_B^2}, \nu_A . \nu_B\right).$$

(2) The product of A and B:

$$A\otimes B=\left(\mu_A.\mu_B\,,\quad \sqrt{(1-\nu_B^2)\iota_B^2+(1-\nu_A^2)\iota_B^2-\iota_A^2\iota_B^2}\,,\quad \sqrt{\nu_A^2+\nu_B^2-\nu_A^2\nu_B^2}\right).$$

(3) The scalar multiplication of A:

$$\lambda.A=\left(\sqrt{1-(1-\mu_A^2)^\lambda}\,,\sqrt{(1-\mu_A^2)^\lambda-(1-\mu_A^2-\iota_A^2)^\lambda}\,,\nu_A^\lambda\right),\qquad \lambda>0.$$

(4) The power of A:

$$A^\lambda=\left(\mu_A^\lambda\,,\sqrt{(1-\nu_A^2)^\lambda-(1-\nu_A^2-\iota_A^2)^\lambda}\,,\sqrt{1-(1-\nu_A^2)^\lambda}\right),\qquad \lambda>0.$$

Definition 2.3. [3] *It is assumed that $A=(\mu_A,\iota_A,\nu_A)$ and $B=(\mu_B,\iota_B,\nu_B)$. Then, one has the following:*

(1) $A=B$ *iff* $\mu_A=\mu_B,\iota_A=\iota_B,\nu_A=\nu_B$,

(2) $A^c=(\nu_A,\iota_A,\mu_A)$,

(3) $A\cup B=(max\{\mu_A,\mu_B\},min\{\iota_A,\iota_B\},min\,\{\nu_A,\nu_B\})$,

(4) $A\cap B=(min\{\mu_A,\mu_B\},min\{\iota_A,\iota_B\},max\{\nu_A,\nu_B\})$.

Definition 2.4. [28] *Let $A_1,A_2,\dots,A_n$ be a set of SFNs and $w=(w_1,w_2,\dots,w_n)^T$ the corresponding weight vector, with the conditions $w_i\in[0,1]$, $\sum_{i=1}^n w_i=1$. Then, the spherical weighted arithmetic mean (SWAM) operator is defined as:*

$$SWAM_w(A_1,A_2,\dots,A_n)=w_1.A_1\oplus w_2.A_2\oplus\dots\oplus w_n.A_n$$
$$=\left(\sqrt{1-\prod_{i=1}^n\left(1-\mu_{A_i}^2\right)^{w_i}}\,,\sqrt{\prod_{i=1}^n\left(1-\mu_{A_i}^2\right)^{w_i}-\prod_{i=1}^n\left(1-\mu_{A_i}^2-\iota_{A_i}^2\right)^{w_i}}\,,\quad \prod_{i=1}^n\left(\nu_{A_i}\right)^{w_i}\right).$$

Definition 2.5. [2] *For two SFNs $A=(\mu_A,\iota_A,\nu_A)$ and $B=(\mu_B,\iota_B,\nu_B)$, the comparative rules, established upon the score function S and the accuracy function Ac, are articulated as follows:*

(1) If $S(A)>S(B)$, then $A>B$.

(2) If $S(A)=S(B)$ and $Ac(A)<Ac(B)$, then $A<B$.

(3) If $S(A)=S(B)$ and $Ac(A)=Ac(B)$, then $A=B$.

Definition 2.6. [17] *It is assumed that $A=(\mu_A,\iota_A,\nu_A)$. The score function of A is defined as follows:*

$$S(A)=\frac{1}{2}\left(1+\mu_A^2-\nu_A^2-(\iota_A^2+\pi_A^2).\frac{log_2(1+\iota_A^2+\pi_A^2)}{100}\right).$$

This score function contains the term $(1+\mu^2+\pi^2)$, which is always greater than or equal to 1 under spherical fuzzy constraints. Consequently, the logarithmic operations involved in the proposed method are always well-defined, and no smoothing or numerical adjustment is required.

2.2. Existing spherical fuzzy entropy measures

Entropy measure is an effective mathematical tool for measuring fuzziness degree and one of the most cited objective weighting approaches in multi-criteria decision-making domain. In this subsection, we first review prior entropy measures for SFS sets. Assume that $A=\{\langle x_i,\mu_A(x_i),\iota_A(x_i),\nu_A(x_i)\rangle\}$, $B=\{\langle x_i,\mu_B(x_i),\iota_B(x_i),\nu_B(x_i)\rangle\}$ are two SFS on X and $x_i\in X$ $(i=1,2,3,\dots,n)$. The existing entropy measures are summarized below:

(1) Barukab et al. [10] gave the entropy measure induced by generalized distance measures on spherical fuzzy set as follows, for all $x_i \in X$:

$$E_{BAAAK}(A) = \frac{1}{n}\sum_{i=1}^{n}\left[\left(1 - d(A, A^c)\right).\frac{1 + \left(\pi_A(x_i)\right)^2}{2}\right]$$

where the generalized distance measure is

$$d(A,B) = \left(\frac{1}{2n}\sum_{i=1}^{n}\begin{pmatrix} \left|\left(\mu_A(x_i)\right)^2 - \left(\mu_B(x_i)\right)^2\right|^{\phi} \\ +\left|\left(\iota_A(x_i)\right)^2 - \left(\iota_B(x_i)\right)^2\right|^{\phi} \\ +\left|\left(\nu_A(x_i)\right)^2 - \left(\nu_B(x_i)\right)^2\right|^{\phi} \end{pmatrix}\right)^{\frac{1}{\phi}}, \qquad \phi > 0\ (\in \mathbb{R}).$$

Therefore, the SFS entropy of Barukab et al. [10] is as follows for $\phi = 1$:

$$E_{BAAAK}(A) = \frac{1}{n}\sum_{i=1}^{n}\left[\left(1 - \left|\left(\mu_A(x_i)\right)^2 - \left(\nu_A(x_i)\right)^2\right|\right).\frac{2 - \mu_A^2(x_i) - \nu_A^2(x_i) - \iota_A^2(x_i)}{2}\right].$$

(2) Aydoğdu and Gül [6] proposed a new entropy measure for SFS, as follows:

$$E_{AG}(A) = \frac{1}{n}\sum_{i=1}^{n} 1 - \frac{4}{5}\left[\left|\mu_A^2(x_i) - \nu_A^2(x_i)\right| + \left|\iota_A^2(x_i) - 0.25\right|\right].$$

(3) Jin et al. [24] presented the spherical fuzzy entropy as follows for all $x_i \in X$:

$$E_{JAA}(A) = -\frac{1}{n}\sum_{i=1}^{n}\left(\mu_A(x_i)\, log\left(\mu_A(x_i)\right) + \iota_A(x_i)\, log\left(\iota_A(x_i)\right) + \nu_A(x_i)\, log\left(\nu_A(x_i)\right)\right).$$

(4) Aydın and Kahraman [5] gave a new entropy on SFS for all $x_i \in X$:

$$E_{AK}(A) = \frac{1}{n}\sum_{i=1}^{n}\frac{\iota_A^2(x_i) + 1 - \left|\mu_A^2(x_i) - \nu_A^2(x_i)\right|}{\iota_A^2(x_i) + 1 + \left|\mu_A^2(x_i) - \nu_A^2(x_i)\right|}.$$

(5) Li et al. [29] introduced a new entropy measure for all $x_i \in X$:

$$E_{LLMY}(A) = 1 - \frac{1}{n\sqrt{2}}\sum_{i=1}^{n}\sqrt{\mu_A^4(x_i) + \nu_A^4(x_i) + \iota_A^4(x_i) + \left(\pi_A^2(x_i) - 1\right)^2}.$$

(6) Kumar et al. [27] gave the entropy measure by expanding to Shannon entropy for *all* $x_i \in X$:

$$E_{KYH}(A) = -\frac{1}{2n}\sum_{i=1}^{n}\begin{pmatrix} \mu_A^2(x_i).\, log_2\left(\mu_A^2(x_i)\right) + \nu_A^2(x_i).\, log_2\left(\nu_A^2(x_i)\right) \\ +\iota_A^2(x_i).\, log_2\left(\iota_A^2(x_i)\right) + \pi_A^2(x_i).\, log_2\left(\pi_A^2(x_i)\right) \end{pmatrix}.$$

3. New spherical fuzzy entropy measure

In this section, we propose a novel entropy measure on SFS. Then, we conduct a comparative study of the proposed entropy with the existing entropy measure in SFS to show the superiority of the proposed entropy measure.

3.1. New entropy measure

Definition 3.1. *Let A and B be two SFSs on X. A real-valued function $E: SFS \to [0,1]$ is an entropy measure for SFSs if it has the following properties:*

i. *Sharpness: $E(A) = 0$ if A is crisp set.*

ii. *Maximality: $E(A) = 1$ if and only if $\mu_A^2(x_i) = \nu_A^2(x_i)$ and $\iota_A^2(x_i) + \mu_A^2(x_i) = 0.5$ for all $x_i \in X$.*

iii. *Symmetry: $E(A) = E(A^c)$.*

iv. *Resolution: $E(A) \le E(B)$ if $\mu_A^2(x_i) \ge \mu_B^2(x_i)$,$\nu_A^2(x_i) = \nu_B^2(x_i)$, $\iota_A^2(x_i) \ge \iota_B^2(x_i)$ for all $x_i \in X$.*

Theorem 3.2. *Let $A = (\mu_A, \iota_A, \nu_A)$ be an SFN. The function $E_A: SFS \to [0,1]$*

$$E_A(A) = \frac{1}{n}\sum_{i=1}^{n} \frac{1 - |\mu_A^2(x_i) - \nu_A^2(x_i)| - |\iota_A^2(x_i) - \pi_A^2(x_i)|}{1 + |\mu_A^2(x_i) - \nu_A^2(x_i)| + |\iota_A^2(x_i) - \pi_A^2(x_i)|} \tag{1}$$

is an entropy measure.

Proof:

i. Let A be a crisp set. Then, $\mu_A(x) = 1, \nu_A(x) = 0, \iota_A(x) = 0$ or $\mu_A(x) = 0, \nu_A(x) = 1, \iota_A(x) = 0$ for all $x \in X$. If $\mu_A(x) = 0, \nu_A(x) = 1, \iota_A(x) = 0$, we get

$$\begin{aligned} E_A(A) &= \frac{1}{n}\sum_{i=1}^{n} \frac{1 - |\mu_A^2(x_i) - \nu_A^2(x_i)| - |\iota_A^2(x_i) - \pi_A^2(x_i)|}{1 + |\mu_A^2(x_i) - \nu_A^2(x_i)| + |\iota_A^2(x_i) - \pi_A^2(x_i)|} \\ &= \frac{1}{n}\sum_{i=1}^{n} \frac{1 - |0 - 1| - |0 - 0|}{1 + |0 - 1| + |0 - 0|} \\ &= 0. \end{aligned}$$

Similarly, if $\mu_A(x) = 1, \nu_A(x) = 0, \iota_A(x) = 0$ then $E_A(A) = 0$.

ii. Let

$$\frac{1}{n}\sum_{i=1}^{n} \frac{1 - |\mu_A^2(x_i) - \nu_A^2(x_i)| - |\iota_A^2(x_i) - \pi_A^2(x_i)|}{1 + |\mu_A^2(x_i) - \nu_A^2(x_i)| + |\iota_A^2(x_i) - \pi_A^2(x_i)|} = 1.$$

Then

$$\frac{1 - |\mu_A^2(x_i) - \nu_A^2(x_i)| - |\iota_A^2(x_i) - \pi_A^2(x_i)|}{1 + |\mu_A^2(x_i) - \nu_A^2(x_i)| + |\iota_A^2(x_i) - \pi_A^2(x_i)|} = 1 \text{ for all } x_i \in X.$$

Thus, $|\mu_A^2(x_i) - \nu_A^2(x_i)| + |\iota_A^2(x_i) - \pi_A^2(x_i)| = 0$ for all $x_i \in X$. Hence, $\mu_A^2(x_i) = \nu_A^2(x_i)$ and $\iota_A^2(x_i) = \pi_A^2(x_i)$. From the equality of $\pi_A^2(x_i) = 1 - \mu_A^2(x_i) - \iota_A^2(x_i) - \nu_A^2(x_i)$, we derive

$$\iota_A^2 + \mu_A^2 = 0.5.$$

The converse is clear.

iii. Since $A^c = \{\nu_A(x), \iota_A(x), \mu_A(x) \mid x \in X\}$, we can get

$$E_A(A) = \frac{1}{n}\sum_{i=1}^{n} \frac{1 - |\mu_A^2(x_i) - \nu_A^2(x_i)| - |\iota_A^2(x_i) - \pi_A^2(x_i)|}{1 + |\mu_A^2(x_i) - \nu_A^2(x_i)| + |\iota_A^2(x_i) - \pi_A^2(x_i)|}$$

$$= \frac{1}{n}\sum_{i=1}^{n}\frac{1-|\nu_A^2(x_i)-\mu_A^2(x_i)|-|\iota_A^2(x_i)-\pi_A^2(x_i)|}{1+|\nu_A^2(x_i)-\mu_A^2(x_i)|+|\iota_A^2(x_i)-\pi_A^2(x_i)|}$$
$$= E_A(A^c).$$

iv. Since $\mu_A^2(x_i) \geq \mu_B^2(x_i)$, $\nu_A^2(x_i) = \nu_B^2(x_i)$, $\iota_A^2(x_i) \geq \iota_B^2(x_i)$, we get

$$\pi_B^2 = 1-\mu_B^2-\nu_B^2-\iota_B^2 \geq 1-\mu_A^2-\nu_A^2-\iota_A^2 = \pi_A^2.$$

Since $\pi_B^2 \geq \pi_A^2$ *and* $\iota_B^2 \leq \iota_A^2$, we obtain

$$|\iota_B^2-\pi_B^2| \leq |\iota_A^2-\pi_A^2|. \tag{2}$$

For $\mu_B^2(x_i) \leq \mu_A^2(x_i)$ and $\nu_A^2(x_i) = \nu_B^2(x_i)$, we have

$$|\mu_B^2(x_i)-\nu_B^2(x_i)| \leq |\mu_A^2(x_i)-\nu_A^2(x_i)|. \tag{3}$$

From inequalities (2) and (3), we derive

$$|\mu_B^2(x_i)-\nu_B^2(x_i)|+|\iota_B^2-\pi_B^2| \leq |\mu_A^2(x_i)-\nu_A^2(x_i)|+|\iota_A^2-\pi_A^2|.$$

Thus

$$\frac{1-|\mu_A^2(x_i)-\nu_A^2(x_i)|-|\iota_A^2(x_i)-\pi_A^2(x_i)|}{1+|\mu_A^2(x_i)-\nu_A^2(x_i)|+|\iota_A^2(x_i)-\pi_A^2(x_i)|} \leq \frac{1-|\mu_B^2(x_i)-\nu_B^2(x_i)|-|\iota_B^2(x_i)-\pi_B^2(x_i)|}{1+|\mu_B^2(x_i)-\nu_B^2(x_i)|+|\iota_B^2(x_i)-\pi_B^2(x_i)|}.$$

Consequently, $E_A(A) \leq E_A(B)$.

3.2. Comparison of the proposed entropy with existing entropy measures

In this subsection, we give an example of a proposed entropy measure. Then, we compare the proposed entropy E_A with prior entropies E_{BAAAK}, E_{AG}, E_{JAA}, E_{AK}, E_{LLMY}, E_{ZWCW}, E_{KYH} to show that the proposed entropy measure is accurate and reliable.

Example 3.3. *Let* $X = \{x_1, x_2, x_3\}$ *be a universal set and* $A = \{\mu_A(x_i), \iota_A(x_i), \nu_A(x_i)\}$ *be SFS as follows:*

$$A = \{(x_1, 0.78, 0.25, 0.11), (x_2, 0.52, 0.59, 0.55), (x_3, 0.51, 0.45, 0.45)\}.$$

To describe linguistic variables, we define A as "Large". Based on the iterative application of the power operator to A, A^2 *can be interpreted as "Very Large",* A^3 *as "Quite Very Large", and* A^4 *as "Very Very Large". In this way,*

$$A^2 = \{(x_1, 0.6084, 0.3458, 0.1550), (x_2, 0.2704, 0.6036, 0.7165), (x_3, 0.2601, 0.5310, 0.6033)\},$$
$$A^3 = \{(x_1, 0.4745, 0.4143, 0.1893), (x_2, 0.1406, 0.5446, 0.8128), (x_3, 0.1326, 0.5445, 0.7019)\},$$
$$A^4 = \{(x_1, 0.3701, 0.4680, 0.2180), (x_2, 0.0731, 0.4709, 0.8736), (x_3, 0.0676, 0.5283, 0.7716)\}.$$

The proposed entropy and prior entropy measures of A, A^2, A^3, A^4 are calculated and shown in Table 2. From a mathematical perspective and considering human intuition, the entropy measure should adhere to the following inequality:

$$E(A) > E(A^2) > E(A^3) > E(A^4) \tag{4}$$

Table 2. Entropy values and standard deviation.

	E_{BAAAK}	E_{AG}	E_{JAA}	E_{AK}	E_{LLMY}	E_{KYH}	E_A
A	0.4723	0.7282	$\mathbf{0.4090}$	0.7143	$\mathbf{0.3515}$	0.8511	0.4327
A^2	0.4106	0.6374	$\mathbf{0.4121}$	0.5547	$\mathbf{0.3777}$	0.8118	0.2814
A^3	0.3773	0.6061	$\mathbf{0.3845}$	0.5079	$\mathbf{0.3575}$	0.7154	0.1910
A^4	0.3593	0.5928	$\mathbf{0.3529}$	0.4885	$\mathbf{0.3283}$	0.6288	0.1740
Std d.	0.043	0.0528	0.0237	0.0887	0.0176	0.0864	$\mathbf{0.1025}$

The empirical findings presented in Table 2 underscore that the proposed E_A measure strictly adheres to the axiomatic requirements of Eq (4), whereas several existing measures (specifically E_{JAA} and E_{LLMY}) fail to maintain this essential hierarchical order. A pivotal motivation for introducing this novel entropy formulation is to augment the discriminatory capacity of objective weights, thereby minimizing potential information loss during the synthesis of expert intuition and empirical observations. While measures like E_{BAAAK}, E_{AG}, E_{AK}, and E_{KYH} satisfy the requisite inequality, they exhibit significantly lower standard deviations compared to E_A. The robust distributional spread of E_A (0.1025) ensures that even subtle nuances within the spherical fuzzy decision matrices are effectively captured, facilitating a more rigorous and reliable differentiation between competing medical waste treatment alternatives in complex decision environments.

As illustrated in Figure 1, the proposed entropy measure (E_A) exhibits a more dynamic response to changes in fuzzy states compared to existing measures. While conventional functions tend to produce flatter curves—indicating a loss of discriminatory power—the higher slope of the E_A curve demonstrates its superior sensitivity in distinguishing between subtle degrees of uncertainty. This characteristic is crucial for multi-criteria decision-making, as it ensures that the resulting objective weights are more representative of the actual information load within the decision matrix.

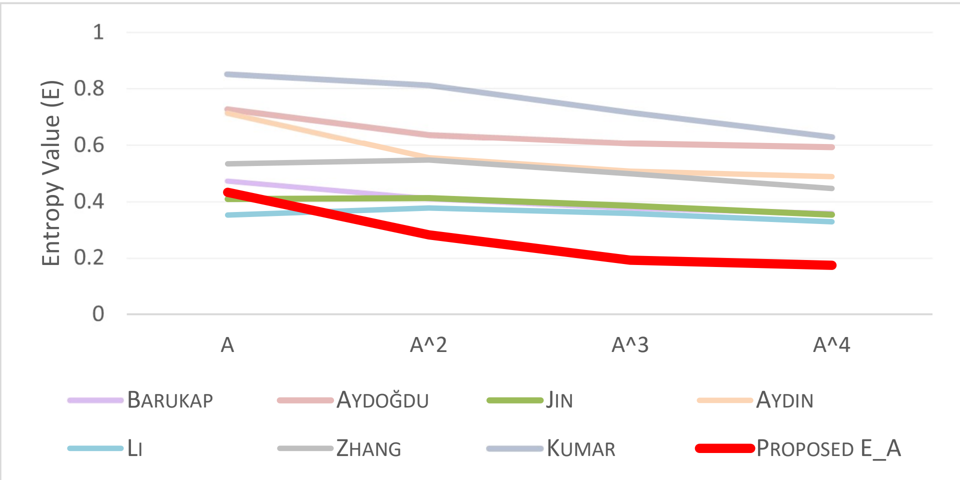

Figure 1. Comparison of spherical fuzzy entropy measures.

The larger standard deviation obtained by the proposed entropy indicates a wider distribution of entropy values, which enhances the discriminatory power of the objective weighting process. This wider distinction prevents weight homogenization and allows the decision model to better capture subtle informational differences among criteria. Such a property is particularly important in the medical waste treatment selection problem, where alternative technologies often exhibit comparable performance levels across multiple technical, environmental, and economic criteria. In such high-risk and regulation-sensitive decisions, small variations in criterion importance may significantly influence the overall evaluation. Therefore, the higher sensitivity and wider differentiation provided by the proposed entropy enable a more informative and reliable weighting structure for complex decision environments.

4. Proposed method

In this section, a TOPSIS method is proposed for SF-based MCGDM, with the utilization of an entropy measure for objective decision-maker weighting and a combined method for criteria weighting that incorporates objective and subjective inputs. The proposed TOPSIS methodology (named EW-CCW-TOPSIS) includes two techniques for determining weights. The first, determination of the entropy-based weight (EW), is a technique that uses the proposed entropy to objectively determine the weights of DMs. The second is the combining criteria weights (CCW) technique, which integrates objective and subjective criterion weights. In the combining weights technique, objective criteria weights are determined by the proposed entropy, and subjective criteria weights are calculated with the SWARA method.

In the context of decision-making, we define $A=\{A_1,A_2,\dots,A_k\}$ as the collection of k alternatives, $C=\{C_1,C_2,\dots,C_m\}$ as the set of m criteria, and $H=\{H_1,H_2,\dots,H_n\}$ as the set of n decision makers (DMs) involved in process. By considering how criterion C_q affects alternatives A_p, each expert H_r evaluates alternatives A_p relative to C_q and generates a spherical fuzzy decision matrix (SFDM) $D^{(r)}=\left(d_{pq}^{(r)}\right)_{k\times m}$, where $d_{pq}^{(r)}=\left(\mu_{D_{pq}^{(r)}},\iota_{D_{pq}^{(r)}},\nu_{D_{pq}^{(r)}}\right)$ for all $r=1,2,\dots,n$, $p=1,2,\dots,k$ and $q=1,2,\dots,m$. The SFDM built by expert H_r is presented in the following way:

$$\mathbf{D}^{(r)}=\begin{pmatrix} d_{11}^{(r)} & d_{12}^{(r)} & \cdots & d_{1m}^{(r)} \\ d_{21}^{(r)} & d_{22}^{(r)} & \cdots & d_{2m}^{(r)} \\ \vdots & \cdots & \vdots & \\ d_{k1}^{(r)} & d_{k2}^{(r)} & \cdots & d_{km}^{(r)} \end{pmatrix}.$$

The procedure of the new EW-CCW-TOPSIS method consists of the following steps:

Step 1. (Normalized decision matrix) As a first step, expert-provided information is normalized, as the SFDM likely involves various benefit and cost criteria, as follows:

$$s_{pq}^{(r)}=\begin{cases} d_{pq}^{(r)}, & for\ benefit\ criteria\ C_q \\ \left(d_{pq}^{(r)}\right)^c, & for\ cost\ criteria\ C_q \end{cases} \tag{5}$$

where $\left(d_{pq}^{(r)}\right)^c$ is a complement of $d_{pq}^{(r)}$. Thus, the normalized spherical fuzzy decision matrix (NSFDM) $D_N^{(r)}=\left(s_{pq}^{(r)}\right)_{k\times m}$, where $s_{pq}^{(r)}=\left(\mu_{pq}^{(r)},\iota_{pq}^{(r)},\nu_{pq}^{(r)}\right)$ for all $r=1,2,\dots,n$, is written as follows:

$$\boldsymbol{D}_{\boldsymbol{N}}^{(\boldsymbol{r})}=\begin{pmatrix} s_{11}^{(r)} & s_{12}^{(r)} & \cdots & s_{1m}^{(r)} \\ s_{21}^{(r)} & s_{22}^{(r)} & \cdots & s_{2m}^{(r)} \\ \vdots & \cdots & \vdots & \\ s_{k1}^{(r)} & s_{k2}^{(r)} & \cdots & s_{km}^{(r)} \end{pmatrix}.$$

Step 2. (Computation of objective weights of expert) In this step, for each expert, the entropy of the recommended values against the alternative value of each criterion is calculated via the proposed entropy. Then, the $EW^{(r)}$ value for each expert is found using the calculated entropy values $E_A\left(s_{pq}^{(r)}\right)$, as given by Li et al. [29]. Afterward, the expert weights are obtained as $\omega^{(r)}$.

Let $\Omega = \left(\omega^{(1)}, \omega^{(2)}, \ldots, \omega^{(n)}\right)$ be the SF weight matrix, where $\omega^{(r)}$ is assigned to expert E_r.

$$\omega^{(r)} = \frac{1 - EW^{(r)}}{\sum_{r=1}^{n} 1 - EW^{(r)}}, \qquad \forall r = 1,2,\ldots,n. \tag{6}$$

In this formula, for all $r = 1,2,\ldots,n$

$$EW^{(r)} = -\frac{1}{\ln(k \times m)} \sum_{p=1}^{k} \sum_{q=1}^{m} \frac{E_A\left(s_{pq}^{(r)}\right)}{\sum_{p=1}^{k} \sum_{q=1}^{m} E_A\left(s_{pq}^{(r)}\right)} . \ln\left(\frac{E_A\left(s_{pq}^{(r)}\right)}{\sum_{p=1}^{k} \sum_{q=1}^{m} E_A\left(s_{pq}^{(r)}\right)}\right), \tag{7}$$

where the proposed entropy

$$E_A\left(s_{pq}^{(r)}\right) = \frac{1}{n} \sum_{i=1}^{n} \frac{1 - |\mu^2(x_i) - \nu^2(x_i)| - |\iota^2(x_i) - \pi^2(x_i)|}{1 + |\mu^2(x_i) - \nu^2(x_i)| + |\iota^2(x_i) - \pi^2(x_i)|}. \tag{8}$$

The entropy-based expert weight formulation in Eq (7) follows the structure introduced by Li et al. [29]. In this formulation, the normalization term $\sum_{p=1}^{k} \sum_{q=1}^{m} E_A\left(s_{pq}^{(r)}\right)$ is strictly positive since entropy values lie in $[0,1]$, and the decision matrix does not contain fully crisp spherical fuzzy evaluations. Therefore, the logarithmic term remains well-defined and no smoothing procedure is required.

Step 3. (Construction of aggregated spherical fuzzy decision matrix) The combined decision of all experts is achieved by integrating the independent decisions of each expert with their weights using the SWAM operator, and an aggregated spherical fuzzy decision matrix (ASFDM) $\mathbf{D} = \left(s_{pq}\right)_{k \times m}$ is then constructed as follows:

$$s_{pq} = SWAM_{\omega^{(r)}}\left(s_{pq}^{(1)}, s_{pq}^{(2)}, \ldots, s_{pq}^{(n)}\right) = \omega^{(1)}. s_{pq}^{(1)} \oplus \omega^{(2)}. s_{pq}^{(2)} \oplus \ldots \oplus \omega^{(n)}. s_{pq}^{(n)}$$
$$= \left(\sqrt{1 - \prod_{i=1}^{n}\left(1 - \mu_{s_{pq}^{(i)}}^2\right)^{\omega^{(i)}}}, \sqrt{\prod_{i=1}^{n}\left(1 - \mu_{s_{pq}^{(i)}}^2\right)^{\omega^{(i)}} - \prod_{i=1}^{n}\left(1 - \mu_{s_{pq}^{(i)}}^2 - \iota_{s_{pq}^{(i)}}^2\right)^{\omega^{(i)}}}, \prod_{i=1}^{n}\left(\nu_{s_{pq}^{(i)}}\right)^{\omega^{(i)}}\right).$$

If we denote $s_{pq} = \left(\mu_{X_p}(c_q), \iota_{X_p}(c_q), \nu_{X_p}(c_q)\right)$ for all $p = 1,2,\ldots,k$ and $q = 1,2,\ldots,m$, then $\mathrm{D} = \left(s_{pq}\right)_{k \times m}$ is represented as follows:

$$\mathbf{D} = \begin{pmatrix} s_{11} & s_{12} & \cdots & s_{1m} \\ s_{21} & s_{22} & \cdots & s_{2m} \\ \vdots & \cdots & \vdots & \\ s_{k1} & s_{k2} & \cdots & s_{km} \end{pmatrix}.$$

Step 4. (Determining the combined weights of criteria) In MCDM problems, the criteria weights, which demonstrate the significance of each criterion, are essential parameters. We have determined the combined criteria weights by employing both objective and subjective weights. The subjective weights were derived using the SWARA method, while the objective weights were computed with our proposed entropy measure.

a) *Objective weights.* The method mainly computes criteria weights according to the information of the original data. Let $\overline{w_q} = (\overline{w_1}, \overline{w_2}, \ldots, \overline{w_m})$ be the objective weight of the criteria. First, calculate the entropy of each attribute by the proposed entropy measure:

$$E_A(c_q) = \frac{1}{k}\sum_{p=1}^{k} \frac{1 - \left|\mu_{X_p}^2(c_q) - \nu_{X_p}^2(c_q)\right| - \left|\iota_{X_p}^2(c_q) - \pi_{X_p}^2(c_q)\right|}{1 + \left|\mu_{X_p}^2(c_q) - \nu_{X_p}^2(c_q)\right| + \left|\iota_{X_p}^2(c_q) - \pi_{X_p}^2(c_q)\right|}, \forall q = 1,2, \ldots, m.$$

In accordance with information entropy theory, a lesser degree of entropy signifies a greater degree of criterion importance; conversely, a greater degree of entropy signifies a lesser degree of criterion importance. The objective weights are then determined through the following calculation:

$$\overline{w_q} = \frac{1 - E_A(c_q)}{\sum_{q=1}^{m}\left(1 - E_A(c_q)\right)}, \forall q = 1,2, \ldots, m. \tag{9}$$

b) *Subjective weights.* The SWARA method is used to calculate subjective weights. This method compares all criteria and determines the subjective weights for MCDM problems. The subjective weights of the criteria are calculated by the SWARA method in the following way:

Substep 1: Using linguistic variables, each expert's subjective preferences for the criteria are obtained. Subsequently, these preferences are integrated via the SWAM operator, by utilizing the objective expert weights ($\omega^{(r)}$) derived in Step 2.

Substep 2: Calculate the score function of criteria (given by [17]) as follows:

$$s(c_q) = \frac{1}{2}\left(\begin{array}{c} 1 + \left(\mu(c_q)\right)^2 - \left(\nu(c_q)\right)^2 \\ -\left(\left(\iota(c_q)\right)^2 + \left(\pi(c_q)\right)^2\right). \dfrac{\log_2\left(1 + \left(\iota(c_q)\right)^2 + \left(\pi(c_q)\right)^2\right)}{100} \end{array}\right) \tag{10}$$

for all $p = 1,2, \ldots, k$ and $q = 1,2, \ldots, m$.

Substep 3: Order the criteria by score, from largest to smallest.

Substep 4: (Determine the comparative significance value of the resulting score.) The comparative significance is determined based on the second top criteria, and the following comparative significance s_q^* is obtained by taking the difference between score values $s(c_{q-1})$ and $s(c_q)$.

Substep 5: Determine the comparative coefficient k_q^* as below:

$$k_q^* = \begin{cases} 1, & q = 1 \\ s_q^* + 1 & q > 1. \end{cases} \tag{11}$$

Substep 6: Compute the relative weight Ω_q^* as follows:

$$\Omega_q^* = \begin{cases} 1, & q = 1 \\ \dfrac{\Omega_q^*}{k_q^*}, & q > 1. \end{cases} \tag{12}$$

Substep 7: Calculate the subjective weights of each criteria as follows:

$$\widetilde{w_q} = \frac{\Omega_q^*}{\sum_{q=1}^{m}\Omega_q^*}. \tag{13}$$

c) *Combined weights.* The integrated weight matrix $w_q = (w_1, w_2, \ldots, w_m)$ is determined by utilizing the following equation:

$$w_q = \frac{\overline{w_q} \times \widetilde{w_q}}{\sum_{q=1}^{m} \overline{w_q} \times \widetilde{w_q}}, \forall q = 1,2, \ldots, m. \tag{14}$$

A multiplicative aggregation is preferred in this step to generate a synergistic weight vector, ensuring that a criterion attains high global significance only when supported by both data-driven entropy and expert-led SWARA assessments, thereby avoiding the information dilution common in additive models.

Step 5. Find the aggregated weighted spherical fuzzy decision matrix (AWSFDM) $\mathbf{D}' = \left(s'_{pq}\right)_{k\times m}$ by considering the ASFDM matrix $\mathbf{D}$ and the combined weight matrix w_q. The calculation of the s'_{pq} element of $\mathbf{D}'$ is as follows:

$$s'_{pq} = \left(\begin{array}{c} \sqrt{1-\left(1-\mu_{X_p}^2(c_q)\right)^{w_q}}, \sqrt{\left(1-\mu_{X_p}^2(c_q)\right)^{w_q} - \left(1-\mu_{X_p}^2(c_q) - \iota_{X_p}^2(c_q)\right)^{w_q}}, \\ \left(\nu_{X_p}(c_q)\right)^{w_q} \end{array} \right) \tag{15}$$

for all $p = 1,2, \ldots k$ and $q = 1,2, \ldots, m$. If we denote $s'_{pq} = \left(\mu'_{X_p}(c_q), \iota'_{X_p}(c_q), \nu'_{X_p}(c_q)\right)$, then the AWSFDM is as follows:

$$\mathbf{D}' = \begin{pmatrix} s'_{11} & s'_{12} & \cdots & s'_{1m} \\ s'_{21} & s'_{22} & \cdots & s'_{2m} \\ \vdots & \cdots & \vdots & \\ s'_{k1} & s'_{k2} & \cdots & s'_{km} \end{pmatrix}.$$

Step 6. Because the components of the AWSFDM $\mathbf{D}'$ are spherical fuzzy number, to ascertain the positive and negative ideal solutions, the score matrix $\mathbf{D}^* = \left(s^*_{pq}\right)_{k\times m}$ is formed by applying the score function given by Gao [13].

Step 7. (Determine the positive ideal solution (PIS) A^+ and negative ideal solution (NIS) A^-) Let benefit type and cost type criteria be denoted by c_B and c_C, respectively. PIS A^+ and NIS A^- are determined as follows:

$$A^+ = \{A^+(c_1), A^+(c_2), \ldots, A^+(c_m)\},$$

$$A^- = \{A^-(c_1), A^-(c_2), \ldots, A^-(c_m)\}$$

where $A^+(c_q)$ and $A^-(c_q)$ are evaluated for all $q = 1,2, \ldots, m$ as:

$$A^+(c_q) = \left\{\left(\max_p s^*_{pq} \,\middle|\, c_q \in c_B\right), \left(\min_p s^*_{pq} \,\middle|\, c_q \in c_C\right) \mid 1 \le p \le k\right\}, \tag{16}$$

$$A^-(c_q) = \left\{\left(\min_p s^*_{pq} \,\middle|\, c_q \in c_B\right), \left(\max_p s^*_{pq} \,\middle|\, c_q \in c_C\right) \mid 1 \le p \le k\right\}. \tag{17}$$

Step 8. (Determine the distance between each alternative with A^+ and A^-) To calculate the distance, we use the Euclidean distance as below:

$$d(A_p, A^+) = \sqrt{\sum_{q=1}^{m} \left(X_p(c_q) - A^+(c_q)\right)^2},$$

$$d(A_p, A^-) = \sqrt{\sum_{q=1}^{m} \left(X_p(c_q) - A^-(c_q)\right)^2}.$$

Step 9. (Computation of closeness coefficient) The closeness coefficient of an alternative A_p is calculated with the equation below, in order to quantify its closeness to the PIS and its remoteness from the NIS:

$$CC(A_p) = \frac{d(A_p, A^-)}{d(A_p, A^-) + d(A_p, A^+)}$$

for all $p = 1,2, \dots, k$.

Step 10. (Selection of the best alternative.) Rank the alternatives in descending order based on their closeness coefficient $CC(A_p)$, thereby comparing them.

5. Numerical illustration and discussion results

5.1. Case application: Strategic selection of medical waste treatment technologies

Medical waste poses a significant risk due to toxic wastes, which include synthetic compound materials like used syringes, soiled dressings, and discarded pharmaceuticals. Moreover, medical waste is a major driver of disease transmission and death worldwide, causing approximately 5.2 million deaths each year, of which a disproportionately large portion, approximately four million, are children [12]. Singh et al. (2020) [41] reported that, in India, 70% of biomedical waste undergoes suitable treatment, whereas the remaining 30% is either inadequately discarded or observed as roadside debris. The COVID-19 pandemic has exacerbated this issue, resulting in a swift escalation of medical waste volume [39]. Before the pandemic, daily medical waste generation was recorded at 619 tons, which surged to 774 tons during the pandemic period [12].

Determining the optimal approach for handling this waste stream is vital due to the increasing risks and limitations of the current system. Therefore, this study aims to examine the problem of medical waste treatment (data taken from [37]) to validate the applicability of the proposed EW-CCW-TOPSIS methodology. Pattel et al. ([37]) addressed this problem in the intuitionistic fuzzy environment. In this problem, medical waste treatment alternatives were determined as autoclaving (X1), microwave (X2), plasma pyrolysis (X3), chemical disinfection (X4), and incineration (X5). These alternatives are assessed based on 10 criteria belonging to four different dimensions given in Table 3. The set of criteria is $C = \{c_1, c_2, c_3, \dots, c_{10}\}$, where $c_1 =$ cost, $c_2 =$ land requirement, $c_3 =$ waste residual, $c_4 =$ energy consumption, $c_5 =$ microbial inactivation, $c_6 =$ treatment effectiveness, $c_7 =$ reliability, $c_8 =$level of automation, $c_9 =$ need for skilled operator, and $c_{10} =$ public acceptance.

Table 3. Different criteria for medical waste treatment alternatives [37].

Dimension	Criteria	Type of criteria
Economic	Cost (c_1)	Cost
	Land requirement (c_2)	Cost
Environmental	Waste residual (c_3)	Cost
	Energy consumption (c_4)	Cost
	Microbial inactivation (c_5)	Benefit
Technical	Treatment effectiveness (c_6)	Benefit
	Reliability (c_7)	Benefit
	Level of automation (c_8)	Benefit
	Need for skilled operator (c_9)	Benefit
Social	Public acceptance (c_{10})	Benefit

There are four experts H_1, H_2, H_3, and H_4 to decide the solution for this problem. Each expert H_r (r = 1, 2, 3,4) evaluates the alternatives X_p (p = 1, 2, …, 5) with respect to C_q (q = 1, 2,…, 10) with the help of linguistic measures (taken from [37]), shown in Table 4. Linguistic measures of importance with the corresponding SFSs are given in Table 5 [29].

Table 4. Spherical fuzzy decision matrix in terms of linguistic measures.

Exper	Alternativ	Attributes									
		c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
H_1	X_1	G	P	P	VG	F	F	F	VG	G	VG
	X_2	VG	VP	F	G	F	MP	VG	G	G	VG
	X_3	G	VP	P	G	G	MP	P	G	G	MG
	X_4	VG	MP	MG	G	MG	MG	P	VG	MG	F
	X_5	VG	G	F	VG	F	MG	F	VG	MG	G
H_2	X_1	F	MG	F	G	MG	P	G	P	VP	G
	X_2	P	G	F	G	MG	P	G	F	MP	MG
	X_3	P	G	G	G	P	F	F	F	P	VG
	X_4	P	MG	F	MG	MG	G	MG	G	P	MG
	X_5	VG	VG	MG	F	MG	G	MG	MG	G	G
H_3	X_1	MP	G	P	P	G	P	VG	F	MG	VG
	X_2	F	MG	P	MP	G	MG	G	MG	F	G
	X_3	F	G	P	P	G	MP	F	F	F	G
	X_4	F	F	P	P	VG	MP	F	G	F	MG
	X_5	P	P	G	P	G	MP	F	G	P	MP
H_4	X_1	F	F	P	P	P	VG	MG	F	G	MG
	X_2	F	MG	VP	P	MP	G	VG	F	VG	P
	X_3	P	VP	F	VP	MG	G	G	G	MG	MP
	X_4	F	P	VP	P	MP	G	G	VG	F	MP
	X_5	P	P	MG	MG	MP	F	F	G	F	P

Table 5. Linguistic terms and their corresponding SFNs.

Linguistic terms	(μ, ι, ν)
Extremely good (EG)	(0.9,0.0,0.1)
Very good (VG)	(0.8,0.1,0.2)
Good (G)	(0.7,0.2,0.3)
Medium good (MG)	(0.6,0.3,0.4)
Fair (F)	(0.5,0.4,0.4)
Medium poor (MP)	(0.4,0.3,0.6)
Poor (P)	(0.3,0.2,0.7)
Very poor (VP)	(0.2,0.1,0.8)
Extremely poor (EP)	(0.1,0.0,0.9)

Step 1. In this problem, c_1, c_2, c_3, and c_4 are cost type criteria. The normalized spherical fuzzy decision matrix (NSFDM) $D_N^{(r)}$ is constructed using Eq (5), and NSFDMs $\left(D_N^{(1)}, D_N^{(2)}, D_N^{(3)}, D_N^{(4)}\right)$ are given in Tables 6–9.

Step 2. In this step, the objective weights of experts are calculated by using the proposed entropy measure. First, for each expert, the entropy of the recommended values against the alternative value of each criterion is calculated using Eq (8). Then, the $EW^{(r)}$ value for each expert is calculated using calculated entropy using Eq (7). Hence, the weights of each expert are calculated by using Eq (6). $EW^{(r)}$ values and the experts' weights $\omega^{(r)}$ are shown in Table 10.

Table 10 indicates that expert H_1 receives the highest weight (0.332), as this expert has the lowest entropy value among the group. Since entropy reflects the degree of uncertainty or dispersion in the evaluations, a lower entropy implies more consistent and less ambiguous judgments across criteria and alternatives. In contrast, experts H_2 and H_3 exhibit higher entropy values, indicating relatively more dispersed assessments and consequently lower importance weights. This result demonstrates that the proposed entropy-based weighting mechanism assigns greater influence to experts who provide more stable and informative evaluations, thereby improving the reliability of the group decision-making process.

Table 6. NSFDM of expert H_1.

$D_N^{(1)}$	c_1	c_2	c_3	c_4	c_5
X_1	(0.3, 0.2, 0.7)	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.3)	(0.2, 0.1, 0.8)	(0.5, 0.4, 0.4)
X_2	(0.2, 0.1, 0.8)	(0.8, 0.1, 0.2)	(0.4, 0.4, 0.5)	(0.3, 0.2, 0.7)	(0.5, 0.4, 0.4)
X_3	(0.3, 0.2, 0.7)	(0.8, 0.1, 0.2)	(0.7, 0.2, 0.3)	(0.3, 0.2, 0.7)	(0.7, 0.3, 0.3)
X_4	(0.2, 0.1, 0.8)	(0.4, 0.3, 0.6)	(0.4, 0.3, 0.6)	(0.3, 0.2, 0.7)	(0.6, 0.3, 0.4)
X_5	(0.2, 0.1, 0.8)	(0.3, 0.2, 0.7)	(0.4, 0.4, 0.5)	(0.2, 0.1, 0.8)	(0.5, 0.4, 0.4)
	c_6	c_7	c_8	c_9	c_{10}
X_1	(0.5, 0.4, 0.4)	(0.5, 0.4, 0.4)	(0.8, 0.1, 0.2)	(0.7, 0.2, 0.3)	(0.8, 0.1, 0.2)
X_2	(0.4, 0.3, 0.6)	(0.8, 0.1, 0.2)	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.3)	(0.8, 0.1, 0.2)
X_3	(0.4, 0.3, 0.6)	(0.3, 0.2, 0.7)	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.3)	(0.6, 0.3, 0.4)
X_4	(0.6, 0.3, 0.4)	(0.3, 0.2, 0.7)	(0.8, 0.1, 0.2)	(0.6, 0.3, 0.4)	(0.5, 0.4, 0.4)
X_5	(0.6, 0.3, 0.4)	(0.5, 0.4, 0.4)	(0.8, 0.1, 0.2)	(0.6, 0.3, 0.4)	(0.7, 0.2, 0.3)

Table 7. NSFDM of expert H_2.

$D_N^{(2)}$	c_1	c_2	c_3	c_4	c_5
X_1	(0.4, 0.4, 0.5)	(0.4, 0.3, 0.6)	(0.4, 0.4, 0.5)	(0.3, 0.2, 0.7)	(0.6, 0.3, 0.4)
X_2	(0.7, 0.2, 0.3)	(0.3, 0.2, 0.7)	(0.4, 0.4, 0.5)	(0.3, 0.2, 0.7)	(0.6, 0.3, 0.4)
X_3	(0.7, 0.2, 0.3)	(0.3, 0.2, 0.7)	(0.3, 0.2, 0.7)	(0.3, 0.2, 0.7)	(0.3, 0.2, 0.7)
X_4	(0.7, 0.2, 0.3)	(0.4, 0.3, 0.6)	(0.4, 0.4, 0.5)	(0.4, 0.3, 0.6)	(0.6, 0.3, 0.4)
X_5	(0.2, 0.1, 0.8)	(0.2, 0.1, 0.8)	(0.4, 0.3, 0.6)	(0.4, 0.4, 0.5)	(0.6, 0.3, 0.4)
	c_6	c_7	c_8	c_9	c_{10}
X_1	(0.3, 0.2, 0.7)	(0.7, 0.2, 0.3)	(0.3, 0.2, 0.7)	(0.2, 0.1, 0.8)	(0.7, 0.2, 0.3)
X_2	(0.3, 0.2, 0.7)	(0.7, 0.2, 0.3)	(0.5, 0.4, 0.4)	(0.4, 0.3, 0.6)	(0.6, 0.3, 0.4)
X_3	(0.5, 0.4, 0.4)	(0.5, 0.4, 0.4)	(0.5, 0.4, 0.4)	(0.3, 0.2, 0.7)	(0.8, 0.1, 0.2)
X_4	(0.7, 0.2, 0.3)	(0.6, 0.3, 0.4)	(0.7, 0.2, 0.3)	(0.3, 0.2, 0.7)	(0.6, 0.3, 0.4)
X_5	(0.7, 0.2, 0.3)	(0.6, 0.3, 0.4)	(0.6, 0.3, 0.4)	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.3)

Table 8. NSFDM of expert H_3.

$D_N^{(3)}$	c_1	c_2	c_3	c_4	c_5
X_1	(0.6, 0.3, 0.4)	(0.3, 0.2, 0.7)	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.3)
X_2	(0.4, 0.4, 0.5)	(0.4, 0.3, 0.6)	(0.7, 0.2, 0.3)	(0.6, 0.3, 0.4)	(0.7, 0.2, 0.3)
X_3	(0.4, 0.4, 0.5)	(0.3, 0.2, 0.7)	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.3)
X_4	(0.4, 0.4, 0.5)	(0.4, 0.4, 0.5)	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.3)	(0.8, 0.1, 0.2)
X_5	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.3)	(0.3, 0.2, 0.7)	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.3)
	c_6	c_7	c_8	c_9	c_{10}
X_1	(0.3, 0.2, 0.7)	(0.8, 0.1, 0.2)	(0.5, 0.4, 0.4)	(0.6, 0.3, 0.4)	(0.8, 0.1, 0.2)
X_2	(0.6, 0.3, 0.4)	(0.7, 0.2, 0.3)	(0.6, 0.3, 0.4)	(0.5, 0.4, 0.4)	(0.7, 0.2, 0.3)
X_3	(0.4, 0.3, 0.6)	(0.5, 0.4, 0.4)	(0.5, 0.4, 0.4)	(0.5, 0.4, 0.4)	(0.7, 0.2, 0.3)
X_4	(0.4, 0.3, 0.6)	(0.5, 0.4, 0.4)	(0.7, 0.2, 0.3)	(0.5, 0.4, 0.4)	(0.6, 0.3, 0.4)
X_5	(0.4, 0.3, 0.6)	(0.5, 0.4, 0.4)	(0.7, 0.2, 0.3)	(0.3, 0.2, 0.7)	(0.4, 0.3, 0.6)

Table 9. NSFDM of expert H_4.

$D_N^{(4)}$	c_1	c_2	c_3	c_4	c_5
X_1	(0.4, 0.4, 0.5)	(0.4, 0.4, 0.5)	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.3)	(0.3, 0.2, 0.7)
X_2	(0.4, 0.4, 0.5)	(0.4, 0.3, 0.6)	(0.8, 0.1, 0.2)	(0.7, 0.2, 0.3)	(0.4, 0.3, 0.6)
X_3	(0.7, 0.2, 0.3)	(0.8, 0.1, 0.2)	(0.4, 0.4, 0.5)	(0.8, 0.1, 0.2)	(0.6, 0.3, 0.4)
X_4	(0.4, 0.4, 0.5)	(0.7, 0.2, 0.3)	(0.8, 0.1, 0.2)	(0.7, 0.2, 0.3)	(0.4, 0.3, 0.6)
X_5	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.3)	(0.4, 0.3, 0.6)	(0.4, 0.3, 0.6)	(0.4, 0.3, 0.6)
	c_6	c_7	c_8	c_9	c_{10}
X_1	(0.8, 0.1, 0.2)	(0.6, 0.3, 0.4)	(0.5, 0.4, 0.4)	(0.7, 0.2, 0.3)	(0.6, 0.3, 0.4)
X_2	(0.7, 0.2, 0.3)	(0.8, 0.1, 0.2)	(0.5, 0.4, 0.4)	(0.8, 0.1, 0.2)	(0.3, 0.2, 0.7)
X_3	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.3)	(0.6, 0.3, 0.4)	(0.4, 0.3, 0.6)
X_4	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.3)	(0.8, 0.1, 0.2)	(0.5, 0.4, 0.4)	(0.4, 0.3, 0.6)
X_5	(0.5, 0.4, 0.4)	(0.5, 0.4, 0.4)	(0.7, 0.2, 0.3)	(0.5, 0.4, 0.4)	(0.3, 0.2, 0.7)

Table 10. The weights of experts.

	$EW^{(r)}$	DM weights
H_1	0.93494	0.332
H_2	0.96035	0.202
H_3	0.96032	0.203
H_4	0.94894	0.261

Step 3. The ASFDM is calculated by considering NSFDM and experts' weights, as shown in Table 11.

Table 11. ASFDM.

D	c_1	c_2	c_3	c_4	c_5
X_1	(0.1836, 0.3306, 0.5344)	(0.2768, 0.2797, 0.4686)	(0.4356, 0.2403, 0.3327)	(0.2920, 0.1867, 0.4938)	(0.2936, 0.3027, 0.4367)
X_2	(0.2063, 0.2974, 0.5271)	(0.3560, 0.2102, 0.4294)	(0.3916, 0.2792, 0.3547)	(0.2716, 0.2311, 0.5007)	(0.3082, 0.3168, 0.4194)
X_3	(0.3156, 0.2499, 0.4412)	(0.4754, 0.1342, 0.3325)	(0.3465, 0.2587, 0.4071)	(0.3650, 0.1725, 0.4248)	(0.3913, 0.2723, 0.3840)
X_4	(0.2063, 0.2974, 0.5271)	(0.2626, 0.2942, 0.4824)	(0.3916, 0.2457, 0.3769)	(0.3157, 0.2236, 0.4577)	(0.3886, 0.2593, 0.3863)
X_5	(0.2842, 0.1746, 0.5073)	(0.2969, 0.1939, 0.4853)	(0.1462, 0.3249, 0.5826)	(0.2064, 0.2648, 0.5527)	(0.3082, 0.3168, 0.4194)
	c_6	c_7	c_8	c_9	c_{10}
X_1	(0.3303, 0.2589, 0.4188)	(0.4267, 0.2752, 0.3277)	(0.3890, 0.2728, 0.3557)	(0.3928, 0.2178, 0.3880)	(0.5509, 0.1757, 0.2602)
X_2	(0.2908, 0.2584, 0.4757)	(0.5853, 0.1420, 0.2357)	(0.3612, 0.3154, 0.3634)	(0.4427, 0.2370, 0.3292)	(0.4467, 0.1913, 0.3467)
X_3	(0.2794, 0.2971, 0.4610)	(0.2768, 0.3008, 0.4470)	(0.4035, 0.2845, 0.3371)	(0.3418, 0.2776, 0.4071)	(0.4161, 0.2383, 0.3644)
X_4	(0.3913, 0.2532, 0.3800)	(0.2997, 0.2766, 0.4470)	(0.5853, 0.1420, 0.2357)	(0.2601, 0.3409, 0.4480)	(0.2756, 0.3374, 0.4446)
X_5	(0.3267, 0.3087, 0.4096)	(0.2737, 0.3802, 0.4)	(0.5244, 0.1889, 0.2778)	(0.3157, 0.2978, 0.4227)	(0.3434, 0.2226, 0.4389)

Step 4. To obtain the combined criterion weight, the criterion weights are calculated both objectively and subjectively:

a) Objective weights (entropy): The objective weights of the criteria are calculated using the proposed entropy-based approach. First, the entropy value of each criterium is calculated by applying Eq (1). Then, the calculated entropy is used in Eq (9). for obtaining objective criteria weights, as given in Table 12.

b) Subjective weights (SWARA): The subjective weights of the criteria are calculated with the SWARA method as follows:

Substep 1: The subjective preferences of criteria via each expert using the linguistic variables are given in Table 13. Then, aggregated subjective preferences of criteria are calculated by incorporating the objectively derived expert weights as obtained in Step 3, given in Table 13.

Substep 2: Defuzzification of aggregated criteria preferences is made by using the score function in Eq (10), which is shown in Table 13.

Substep 3: The score value of criteria arranged in descending order, which is given in Table 14.

Substep 4: Comparative significance of each criteria, as determined in Table 14.

Substeps 5–6: Comparative coefficient k_q^* and relative weight Ω_q^* are calculated by using Eqs (11) and (12), respectively, which are given in Table 14.

Substep 7: Subjective weights of each criteria $\tilde{w}$ are calculated using Eq (13), as shown in Table 14.

c) Combined weights (Entropy-SWARA): The integrated weights of the criteria w_q are determined with Eq (14) and are shown in Table 12. A closer examination of Table 12 reveals that for criterion c_4, the synthesis of an objective weight (0.1054) and a subjective weight (0.1223) culminates in a fortified combined significance of 0.1463. This outcome explicitly validates the proposed framework's proficiency in importance accentuation, effectively ensuring that pivotal decision factors are strategically prioritized rather than being neutralized during the weight aggregation phase.

Table 12. Weights of criteria.

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
Objective weights $\bar{w}$	0.1052	0.1033	0.0977	0.1054	0.0933	0.0972	0.1002	0.0999	0.0964	0.1008
Subjective weights $\tilde{w}$	0.0900	0.0825	0.0838	0.1223	0.1239	0.1021	0.1177	0.1186	0.0800	0.0785
Combined weights w	0.1075	0.0967	0.0930	0.1463	0.1313	0.1126	0.1340	0.1346	0.0875	0.0899

Table 13. Importance of criteria by experts, aggregated, and score value.

	H_1	H_2	H_3	H_4	Aggregated SFNs	Score values (s)
c_1	MP	F	P	MP	(0.4069,0.3029,0.5701)	0.4187
c_2	P	VP	F	VP	(0.3210,0.2869,0.6647)	0.3293
c_3	P	P	F	VP	(0.3357,0.2950,0.6470)	0.3457
c_4	VG	MG	VG	G	(0.7462,0.1893,0.2559)	0.7448
c_5	VG	G	VG	G	(0.7594,0.1693,0.2414)	0.7584
c_6	MP	G	G	MP	(0.5603,0.2867,0.4528)	0.5531
c_7	VG	MG	MG	G	(0.7085,0.1992,0.2945)	0.7065
c_8	G	G	MG	VG	(0.7158,0.1623,0.2860)	0.7148
c_9	P	MP	P	VP	(0.3034,0.2203,0.7025)	0.2982
c_{10}	VP	P	VP	MP	(0.2879,0.1661,0.7222)	0.2796

Table 14. Computation of subjective criteria weights.

Criteria	Score value	Comparative significance	Comparative coefficient	Related weight	Subjective weight
c_5	0.7584	-	1	1	0.1239
c_4	0.7448	0.0136	1.0136	0.9865	0.1223
c_8	0.7142	0.0305	1.0305	0.9573	0.1186
c_7	0.7065	0.0077	1.0077	0.9499	0.1177
c_6	0.5531	0.1534	1.1534	0.8235	0.1021
c_1	0.4187	0.1343	1.1343	0.7260	0.0900
c_3	0.3457	0.0730	1.0730	0.6766	0.0838
c_2	0.3293	0.0164	1.0164	0.6657	0.0825
c_9	0.2982	0.0311	1.0311	0.6456	0.0800
c_{10}	0.2796	0.0185	1.0185	0.6338	0.0785

As depicted in Figure 2, the integration of SWARA and the proposed entropy measure provides a balanced yet highly sensitive criteria weighting framework. The objective weights (blue line) ensure that the intrinsic informational characteristics of the data are preserved, while the subjective weights (green line) incorporate professional expertise, particularly emphasizing critical criteria like c_4, c_5, and c_8. The resulting combined weights (red line) do not merely average these two sources; instead, through the multiplicative aggregation mechanism, they amplify the distinction between criteria. This amplification is vital for medical waste treatment technology selection, as it prevents “rank reversal” and provides a more robust foundation for the final TOPSIS evaluation.

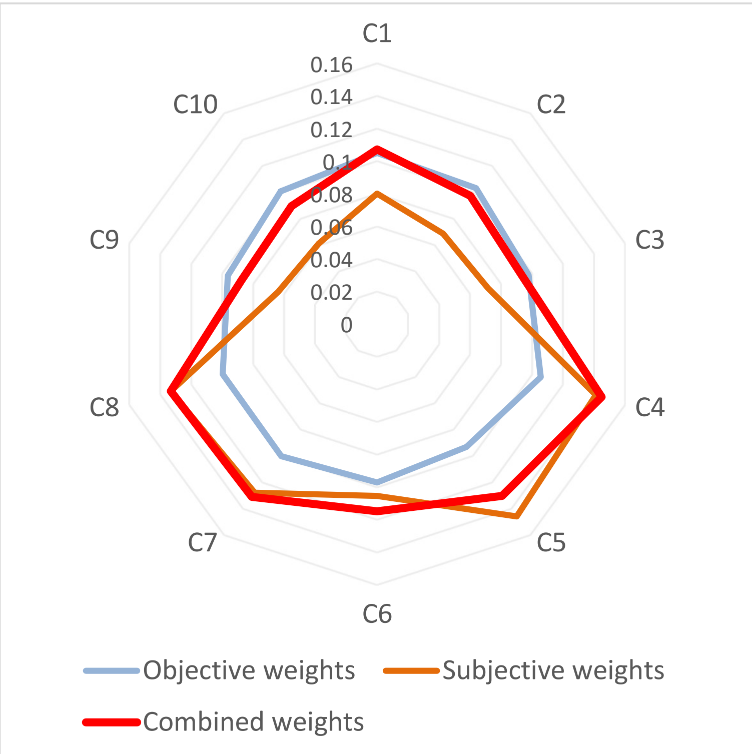

Figure 2. Radar chart comparison of objective, subjective, and integrated combined weights.

Step 5. After determining the weights of the criteria by using Eq (15), the AWSFDM $\mathbf{D}' = (s'_{pq})_{k\times m}$ matrix is obtained, as shown in Table 15.

Table 15. AWSFDM.

$\boldsymbol{D}'$	c_1	c_2	c_3	c_4	c_5
X_1	(0.0606, 01130, 0.9348)	(0.0876, 0.0920, 0.9292)	(0.1392, 0.0820, 0.9026)	(0.1138, 0.0748, 0.9018)	(0.1085, 0.1167, 0.8968)
X_2	(0.0683, 0.1015, 0.9334)	(0.1141, 0.0703, 0.9214)	(0.1240, 0.0938, 0.9080)	(0.1056, 0.0925, 0.9037)	(0.1141, 0.1230, 0.8921)
X_3	(0.1059, 0.0872, 0.9157)	(0.1565, 0.0471, 0.8989)	(0.1088, 0.0851, 0.9197)	(0.1439, 0.0706, 0.8822)	(0.1470, 0.1082, 0.8818)
X_4	(0.0683, 0.1015, 0.9334)	(0.0830, 0.0966, 0.9318)	(0.1240, 0.0822, 0.9132)	(0.1235, 0.0905, 0.8919)	(0.1458, 0.1027, 0.8825)
X_5	(0.0949, 0.0599, 0.9296)	(0.0943, 0.0635, 0.9324)	(0.0448, 0.1027, 0.9509)	(0.0797, 0.1049, 0.9168)	(0.1141, 0.1230, 0.8921)
	c_6	c_7	c_8	c_9	c_{10}
X_1	(0.1137,0.0930, 0.9065)	(0.1630, 0.1122, 0.8611)	(0.1478, 0.1095, 0.8701)	(0.1207, 0.0705, 0.9204)	(0.1789, 0.0627, 0.8859)
X_2	(0.0995,0.0917, 0.9196)	(0.2338, 0.0627, 0.8239)	(0.1365, 0.1261, 0.8726)	(0.1375, 0.0787, 0.9072)	(0.1407, 0.0641, 0.9091)
X_3	(0.0954,0.1057, 0.9164)	(0.1030, 0.1165, 0.8977)	(0.1537, 0.1152, 0.8638)	(0.1040, 0.0887, 0.9243)	(0.1302, 0.0791, 0.9132)
X_4	(0.1362,0.0931, 0.8967)	(0.1119, 0.1075, 0.8977)	(0.2343, 0.0629, 0.8232)	(0.0782, 0.1073, 0.9321)	(0.0841, 0.1080, 0.9297)
X_5	(0.1124,0.1117, 0.9043)	(0.1019, 0.1492, 0.8844)	(0.2058, 0.0805, 0.8416)	(0.0956, 0.0946, 0.9273)	(0.1059, 0.0716, 0.9270)

Step 6. The aggregated values for each alternative, presented as spherical fuzzy (SF) values in the column, are provided in Table 15. By employing the score function, these numbers are defuzzified, and the subsequent score matrix D^* is computed and illustrated in Table 16.

Table 16. Score matrix.

$\boldsymbol{D}^*$	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
X_1	0.0647	0.0719	0.1021	0.0995	0.1034	0.0953	0.1421	0.1320	0.0835	0.1233
X_2	0.0665	0.0818	0.0952	0.0970	0.1083	0.0819	0.1874	0.1282	0.0977	0.0964
X_3	0.0861	0.180	0.0828	0.1209	0.1217	0.0844	0.1021	0.1383	0.0781	0.0913
X_4	0.0665	0.0691	0.0905	0.1096	0.1209	0.1070	0.1031	0.1881	0.0685	0.0712
X_5	0.0723	0.0696	0.0487	0.0826	0.1083	0.0972	0.1137	0.1665	0.0744	0.0757

Step 7. PIS and NIS are calculated using Eqs (16) and (17), which given in Table 17.

Steps 8–10. The distance between each alternative with positive and negative ideal solutions are calculated using Euclidean distance. Then, relative closeness and ranking are determined as in Table 18.

As a result, the alternatives are ordered as

$$X_2 > X_5 > X_1 > X_4 > X_3$$

where “>” signifies “preferred over”. Accordingly, X_2 is the preferred alternative.

Table 17. PIS and NIS value.

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
PIS A^+	0.0677	0.0691	0.0487	0.0826	0.1217	0.1070	0.1874	0.1881	0.0977	0.1233
NIS A^-	0.0861	0.1080	0.1021	0.1209	0.1034	0.0819	0.1021	0.1282	0.0685	0.0712

Table 18. Distance of alternative from PIS and NIS and ranking.

Alternatives	$d(X_p, A^+)$	$d(X_p, A^-)$	Relative Closeness Index	Ranking
X_1	0.0948	0.0833	0.4675	3
X_2	0.0874	0.1023	0.5393	1
X_3	0.1274	0.0361	0.2210	5
X_4	0.1146	0.0817	0.4161	4
X_5	0.0949	0.0888	0.4833	2

5.2. Comparative and robustness analysis

This section validates the proposed framework through a structured methodological comparison, benchmarking analysis, and robustness assessment. The evaluation is designed to examine how different assumptions regarding expert importance, criterion weighting, and aggregation mechanisms influence decision outcomes in spherical fuzzy group decision-making environments.

5.2.1. Method analysis

Existing spherical fuzzy MCDM approaches can be broadly categorized into three main groups: entropy-based objective weighting methods, SWARA-based subjective weighting methods, and hybrid weighting frameworks. Entropy-driven models provide a fully data-oriented weighting structure and ensure objectivity; however, they are generally limited to criterion weighting and fail to adequately capture differences in expert reliability and influence. Conversely, SWARA-based approaches are effective in reflecting expert preferences in criterion weighting, yet their exclusive reliance on subjective judgments neglects the intrinsic information structure of the decision matrix.

Hybrid weighting frameworks have been proposed to address these limitations by combining objective and subjective weighting schemes. Nevertheless, as summarized in Tables 19–21, most existing hybrid models assume expert weights a priori or require additional behavioral parameters, reference points, or externally assigned priorities. Consequently, expert importance cannot be inferred directly from the decision data, which restricts their applicability in environments characterized by information scarcity or uncertain expert reliability.

In contrast, the proposed method introduces a fully endogenous weighting architecture in which expert weights are objectively derived using entropy, while criterion weights are obtained through a systematic integration of entropy-based objective weights and SWARA-based subjective assessments. This unified structure eliminates the need for auxiliary information beyond the spherical fuzzy decision matrix and ensures that both data-driven variability and expert judgment are coherently incorporated. As a result, the proposed approach enhances methodological consistency, reduces subjective bias, and improves robustness in group decision-making contexts.

Table 19 illustrates that entropy-based SF-MCDM studies predominantly focus on objective criterion weighting, while expert importance is either ignored or externally specified. CPT-based models, in particular, require additional behavioral inputs, limiting their effectiveness when expert reliability information is incomplete.

Table 19. Analysis of existing MCDM methods in SF using the entropy weighting method.

No	Ref.	Group DM	Weights of experts	Weights of criteria	Used entropy	Methodology
1	[27]	No	-	**Entropy (objective)**	E_{KYH}	TODIM-VIKOR
2	[24]	Yes	Assuming (subjective)	**Entropy (objective)**	E_{JAA}	With logarithmic aggregation operator
3	[6]	Yes	Assuming (subjective)	**Entropy (objective)**	E_{AG}	WASPAS
4	[47]	Yes	Assuming (subjective)	**Entropy (objective)**	E_{ZWCW}	With Hamacher power aggregation operator
5	[1]	Yes	Assuming (subjective)	**Entropy (objective)**	ShannonEntropy[29]	PROMETHEE-1 PROMETHEE-2
6	[10]	Yes	Computing (objective)	**Entropy (objective)**	E_{BAAAK}	TOPSIS
7	[36]	Yes	Computing (objective)	**Entropy (objective)**	E_{AG}	WASPAS
8	[33]	Yes	Computing (objective)	**Entropy (objective)**	E_{M}	Failure mode and effect analysis
9	[29]	Yes	**Entropy (objective)**	**Entropy (objective)**	E_{LLMY}	VIKOR
10	[48]	Yes	CPT (objective)	Combined (obj.+subj.) (**entropy** + given weights)	ShannonEntropy[40]	MABAC
11	Proposed method	Yes	**Entropy (objective)**	Combined (obj.+subj.) (**entropy** + SWARA)	E_{A}	TOPSIS

Table 20 shows that SWARA-based SF-MCDM approaches largely rely on subjectively assumed expert weights, even in group decision-making environments. Although SWARA effectively captures preference structures, this dependency increases sensitivity to subjective bias.

Table 20. Analysis of existing MCDM methods in SF using the SWARA weighting method.

No	Ref.	Group DM	Weights of experts	Weights of criteria	Methodology
1	[42]	No	-	SWARA (subjective)	MARCOS
2	[18]	Yes	Assuming (subjective)	SWARA (subjective)	WASPAS
3	[19]	Yes	Assuming (subjective)	SWARA (subjective)	CoCoSo
4	[20]	Yes	Assuming (subjective)	SWARA (subjective)	MARCOS
5	[21]	Yes	Assuming (subjective)	SWARA (subjective)	CODAS
6	[44]	Yes	Assuming (subjective)	SWARA (subjective)	WASPAS
7	[12]	Yes	Assuming (subjective)	SWARA (subjective)	WASPAS
8	[17]	Yes	Assuming (subjective)	Combined (obj.+subj.) (MEREC + SWARA)	ARAS
9	Proposed Method	Yes	Entropy (objective)	Combined (obj.+subj.) (Entropy + SWARA)	TOPSIS

Table 21 confirms that existing combined weighting frameworks differ mainly in their treatment of expert importance. The proposed method advances this line of research by deriving expert weights purely from entropy while simultaneously integrating objective and subjective criterion weights within a single decision framework.

Table 21. Analysis of existing MCDM methods in SF using the combining weighting criteria method.

No	Ref.	Group DM	Weights of experts	Weights of criteria	Methodology
1	[17]	Yes	Assuming (subjective)	Combined (obj.+subj.) (MEREC + SWARA)	ARAS
2	[48]	Yes	CPT (objective)	Combined (obj.+subj.) (Entropy + given weights)	MABAC
3	[49]	Yes	CPT (objective)	Combined (obj.+subj.) (CRITIC + SWARA)	TODIM
4	[30]	Yes	AHP	SWARA	CoCoSo
5	Proposed method	Yes	Entropy (objective)	Combined (obj.+subj.) (Entropy + SWARA)	TOPSIS

5.2.2. Benchmarking with existing MCGDM frameworks

In this subsection, the proposed method is benchmarked against three representative spherical fuzzy MCDM approaches selected to reflect different expert–criterion weighting configurations. SF-TOPSIS [28] represents a fully subjective structure, SF-MEREC-SWARA-ARAS [17] employs subjective expert weights with hybrid criterion weighting, and SF-ENT-TOPSIS [10] relies exclusively on objective weighting.

As shown in Table 22, the proposed approach produces consistent results with methods that incorporate subjective information in the weighting process. In both the supplier selection and digital marketing technology evaluation problems, the best alternative coincides with those obtained by the corresponding reference models.

Table 22. Analysis of existing MCDM methods.

	Result of the relevant method	Result of the proposed method (EW-CCW-TOPSIS)	DM weights		Criteria weights		Best alternative
			Obj	Subj.	Subj.	Obj.	
[28]	$X_1 > X_4 > X_2 > X_3$	$X_1 > X_4 > X_2 > X_3$	-	yes	yes	no	Same
[17]	$X_4 > X_5 > X_1 > X_2 > X_3$	$X_4 > X_1 > X_5 > X_3 > X_2$	-	yes	yes	yes	Same
[10]	$X_1 > X_2 > X_4 > X_5 > X_3$	$X_2 > X_5 > X_1 > X_4 > X_3$	yes	-	no	yes	Not same
	$X_2 > X_1 > X_4 > X_5 > X_3$		yes	-	yes	yes	Same

Differences are observed when compared with SF-ENT-TOPSIS, where both expert and criterion weights are determined purely by entropy. In this case, variations in the ranking appear. However, when subjective criterion assessments are incorporated into the entropy-based structure, the resulting best alternative becomes identical. This indicates that integrating subjective evaluations contributes to improved ranking stability.

Overall, the benchmarking results demonstrate that the proposed method provides a flexible weighting structure capable of reproducing consistent decisions across different weighting assumptions, thereby enhancing decision reliability in group decision-making problems.

5.2.3. Robustness with respect to entropy variations

Robustness with respect to entropy formulation is examined by replacing the proposed entropy with alternative spherical fuzzy entropy measures while preserving the remaining methodological structure. As shown in Table 23, the best and worst alternatives remain invariant across all entropy configurations. This indicates that the proposed framework maintains ranking stability and does not rely on a specific entropy structure to generate consistent decisions.

Although the ranking remains stable across entropy formulations, this does not imply that the proposed entropy is redundant. The results indicate that the proposed framework is structurally robust, while the higher dispersion and sensitivity of the proposed entropy (Section 3.2) provide stronger discriminatory power in the objective weighting process. Such enhanced sensitivity becomes particularly important in decision environments where alternatives exhibit close performance levels, as it allows subtle informational differences to be more effectively reflected in the weight structure.

Table 23. Sensitivity analysis results: Proposed method with different entropy.

Entropy given by	X_1	X_2	X_3	X_4	X_5	Ranking
[10]	0.4767	0.5637	0.1995	0.4124	0.4815	$X_2 > X_5 > X_1 > X_4 > X_3$
[24]	0.4611	0.5437	0.2231	0.4217	0.4924	$X_2 > X_5 > X_1 > X_4 > X_3$
[6]	0.4750	0.5453	0.2083	0.4194	0.4873	$X_2 > X_5 > X_1 > X_4 > X_3$
[5]	0.4922	0.5789	0.1822	0.4092	0.4735	$X_2 > X_1 > X_5 > X_4 > X_3$
[29]	0.4595	0.5438	0.2223	0.4188	0.4879	$X_2 > X_5 > X_1 > X_4 > X_3$
[47]	0.4613	0.5324	0.2265	0.4220	0.4944	$X_2 > X_5 > X_1 > X_4 > X_3$
[27]	0.4647	0.5395	0.2185	0.4240	0.4933	$X_2 > X_5 > X_1 > X_4 > X_3$
Proposed entropy E_A	0.4675	0.5393	0.2210	0.4161	0.4833	$X_2 > X_5 > X_1 > X_4 > X_3$

5.2.4. Sensitivity analysis of weight aggregation strategies

Sensitivity analysis is conducted by comparing multiplicative and linear aggregation strategies for combining objective and subjective criterion weights. As reported in Table 24 and in Figure 3, the linear aggregation produces a more uniform weight distribution, while the multiplicative approach amplifies inter-criterion contrast. Despite these numerical differences, the final ranking of alternatives remains unchanged across both aggregation mechanisms. This invariance confirms the structural stability of the proposed framework and its resilience to variations in weight integration strategies.

Table 24. Comparative results of different weight aggregation strategies and ranking consistency check.

	Objective weight ($\overline{w}$)	Subjective weight ($\widetilde{w}$)	Multiplicative combined weight (w_{mult})	Linear combined weight (w_{lin})
c_1	0.1052	0.0900	0.1075	0.0976
c_2	0.1033	0.0825	0.0967	0.0929
c_3	0.0977	0.0838	0.0930	0.0908
c_4	0.1054	0.1223	**0.1463**	0.1138
c_5	0.0933	0.1239	0.1313	0.1086
c_6	0.0972	0.1021	0.1126	0.0996
c_7	0.1002	0.1177	0.1340	0.1090
c_8	0.0999	0.1186	0.1346	0.1093
c_9	0.0964	0.0800	0.0875	0.0882
c_{10}	0.1008	0.0785	0.0899	0.0897
Standard deviation	**0.0037**	**0.0181**	**0.0206**	**0.0091**
Rank (X1-X5)	-	-	2-5-1-4-3	2-5-1-4-3

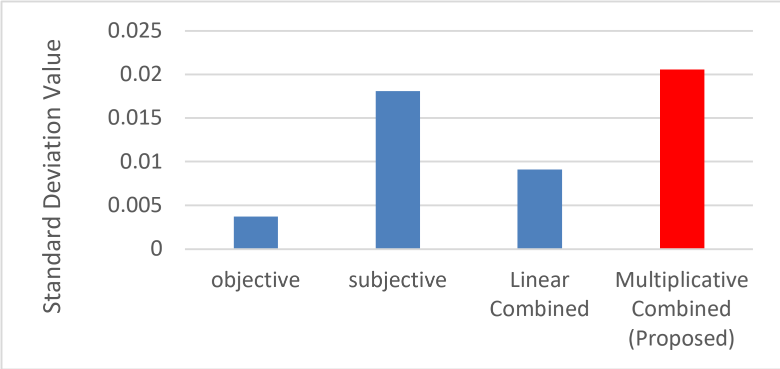

Figure 3. Standard deviation.

Although both aggregation strategies produce the same ranking of alternatives, this result should not be interpreted as redundancy of the multiplicative approach. The invariance of the ranking indicates the structural robustness of the decision model, whereas the multiplicative integration provides a higher level of weight dispersion and inter-criterion discrimination. In high-risk decision environments such as medical waste treatment selection, enhanced discrimination among critical criteria improves the sensitivity and interpretability of the decision process without compromising stability.

Overall, the analyses in Sections 5.2.1–5.2.4 demonstrate that the proposed framework provides a coherent and reliable solution for spherical fuzzy group decision-making problems. By objectively deriving expert importance, systematically integrating objective and subjective criterion weights, and maintaining ranking stability under varying entropy formulations and aggregation mechanisms, the proposed approach overcomes structural limitations observed in existing SF-based MCDM methods and ensures robust decision outcomes under uncertainty.

6. Conclusions

This study developed a spherical fuzzy group decision-making framework that integrates entropy-based expert weighting with a hybrid criterion weighting mechanism within a TOPSIS structure. The main motivation was to address a methodological gap in existing SFS-based models, where expert importance is frequently predefined or detached from criterion weighting procedures. In group decision environments characterized by uncertainty and heterogeneous expert judgments, this separation may lead to structural inconsistency in weight determination. By introducing a new spherical fuzzy entropy measure and employing it to derive expert weights directly from the decision matrix, the proposed framework establishes a fully endogenous and structurally consistent weighting architecture.

The proposed entropy measure was analytically validated and comparatively examined against several existing spherical fuzzy entropy formulations. The results reveal that it generates a wider dispersion of entropy values and exhibits higher sensitivity to variations in fuzzy evaluations. This enhanced responsiveness strengthens the discriminatory power of the objective weighting process, allowing subtle informational differences among criteria and experts to be more explicitly captured. In the medical waste treatment case study, the entropy-based expert weighting reflected differences in evaluation ambiguity across decision-makers, resulting in a transparent and interpretable distribution of influence levels.

The hybrid weighting structure, constructed through multiplicative integration of entropy-based objective weights and SWARA-based subjective weights, ensures that a criterion attains global significance only when supported by both data-driven evidence and expert judgment. Although multiplicative aggregation increases weight contrast (as reflected in higher standard deviation values), sensitivity analysis confirmed that the final ranking remains invariant under alternative aggregation mechanisms. Moreover, robustness analysis using different entropy formulations demonstrated that the decision outcomes are structurally stable and not dependent on a specific entropy specification.

From a practical standpoint, the proposed framework offers three main advantages: (i)elimination of arbitrary expert weight assignment through objective entropy-based derivation, (ii)coherent integration of objective and subjective criterion information within a unified structure, and (iii)preservation of informational contrast without compromising ranking stability. These properties are particularly valuable in complex and regulation-sensitive problems such as medical waste treatment selection, where alternatives often display closely comparable performance across technical, environmental, and economic criteria.

Nevertheless, the model assumes complete spherical fuzzy evaluations and may require preprocessing when assessments are incomplete or highly sparse. In addition, while multiplicative integration enhances discrimination, it may amplify minor fluctuations under highly noisy decision environments. Future research may extend the framework to interval-valued spherical fuzzy structures, large-scale group decision contexts, or alternative nonlinear aggregation mechanisms.

Use of Generative-AI tools declaration

The author declares that no Artificial Intelligence (AI) tools were used in the preparation of this article.

Conflict of interest

The author declares no conflict of interest.

References

1. M. Akram, K. Zahid, C. Kahraman, A PROMETHEE based outranking approach for the construction of Fangcang shelter hospital using spherical fuzzy sets, *Artif. Intell. Med.*, **135** (2023), 102456. https://doi.org/10.1016/j.artmed.2022.102456
2. S. Ashraf, T. Mahmood, S. Abdullah, Q. Khan, Different approaches to multi-criteria group decision making problems for picture fuzzy environment, *B. Braz. Math. Soc.*, New Series, **50** (2019), 373–397. https://doi.org/10.1007/s00574-018-0103-y
3. S. Ashraf, S. Abdullah, Spherical aggregation operators and their application in multiattribute group decision-making, *Int. J. Intell. Syst.*, **34** (2019), 493–523. https://doi.org/10.1002/int.22062
4. K. T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Set. Syst.*, **20** (1986), 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3
5. S. Aydın, C. Kahraman, *A spherical fuzzy multi-expert MCDM method based on entropy and cosine similarity*, in Developments of Artificial Intelligence Technologies in Computation and Robotics, Proceedings of the 14th International FLINS Conference, 2020. https://doi.org/10.1142/9789811223334_0020

6. A. Aydoğdu, S. Gül, A novel entropy proposition for spherical fuzzy sets and its application in multiple attribute decision-making, *Int. J. Intell. Syst.*, **35** (2020), 1354–1374. https://doi.org/10.1002/int.22256
7. A. Aydoğdu, S. Gül, New entropy propositions for interval-valued spherical fuzzy sets and their usage in an extension of ARAS (ARAS-IVSFS), *Expert Syst.*, **39** (2022), e12898. https://doi.org/10.1111/exsy.12898
8. E. Aydoğdu, E. Güner, B. Aldemir, H. Aygün, Complex spherical fuzzy TOPSIS based on entropy, *Expert Syst. Appl.*, **215** (2023), 119331. https://doi.org/10.1016/j.eswa.2022.119331
9. E. Aydoğdu, B. Aldemir, E. Güner, H. Aygün, A novel entropy measure with its application to the COPRAS method in complex spherical fuzzy environment, *Informatica*, **34** (2023), 679–711. https://doi.org/10.15388/23-INFOR539
10. O. Barukab, S. Abdullah, S. Ashraf, M. Arif, S. A. Khan, A new approach to fuzzy TOPSIS method based on entropy measure under spherical fuzzy information, *Entropy*, **21** (2019), 1231. https://doi.org/10.3390/e21121231
11. Central pollution control board, annual report on biomedical waste management as per biomedical waste management rules, 2016 for the year 2020, 2020.
12. M. B. Bouraima, Y. Qiu, E. Ayyildiz, A. Yildiz, Prioritization of strategies for a sustainable regional transportation infrastructure by hybrid spherical fuzzy group decision-making approach, *Neural Comput. Appl.*, **35** (2023), 17967–17988. https://doi.org/10.1007/s00521-023-08660-4
13. B. C. Cuong, Picture fuzzy sets, *J. Comput. Sci. Cybern.*, **30** (2014), 409–420.
14. W. Emam, J. Ahmmad, T. Mahmood, T. Senapati, Impact of AI techniques in electrical engineering with spherical fuzzy soft geometric aggregation operators, *IEEE Access*, **12** (2024), 119803–119828. https://doi.org/10.1109/ACCESS.2024.3447061
15. E. Farrokhizadeh, S. A. Seyfi-Shishavan, F. K. Gündoğdu, Y. Donyatalab, C. Kahraman, S. H. Seifi, A spherical fuzzy methodology integrating maximizing deviation and TOPSIS methods, *Eng. Appl. Artif. Intell.*, **101** (2021). https://doi.org/10.1016/j.engappai.2021.104212
16. K. Gao, T. Liu, Y. Rong, V. Simic, H. Garg, T. Senapati, A novel BWM-entropy-COPRAS group decision framework with spherical fuzzy information for digital supply chain partner selection, *Complex Intell. Syst.,* **10** (2024), 6983–7008. https://doi.org/10.1007/s40747-024-01500-5
17. K. Gao, T. Liu, D. Yue, V. Simic, Y. Rong, H. Garg, An integrated spherical fuzzy multi-criterion group decision-making approach and its application in digital marketing technology assessment, *Int. J. Comput. Intell. Syst.*, 2023. https://doi.org/10.1007/s44196-023-00298-3
18. S. J. Ghoushchi, S. R. Bonab, A. M. Ghiaci, G. Haseli, H. Tomaskova, M. Hajiaghaei-Keshteli, Landfill site selection for medical waste using an integrated SWARA-WASPAS framework based on spherical fuzzy set, *Sustainability*, **13** (2021), 13950. https://doi.org/10.3390/su132413950
19. S. J. Ghoushchi, S. M. Jalalat, S. R. Bonab, A. M. Ghiaci, G. Haseli, H. Tomaskova, Evaluation of wind turbine failure modes using the developed SWARA-CoCoSo methods based on the spherical fuzzy environment, *IEEE Access*, **10** (2022), 86750–86764. https://doi.org/10.1109/ACCESS.2022.3199359
20. S. J. Ghoushchi, S. S. Haghshenas, A. M. Ghiaci, G. Guido, A. Vitale, Road safety assessment and risks prioritization using an integrated SWARA and MARCOS approach under spherical fuzzy environment, *Neural Comput. Appl.*, **35** (2023), 4549–4567. https://doi.org/10.1007/s00521-022-07929-4
21. S. J. Ghoushchi, H. Garg, S. R. Bonab, A. Rahimi, An integrated SWARA-CODAS decision-making algorithm with spherical fuzzy information for clean energy barriers evaluation, *Expert Syst. Appl.*, **223** (2023), 119884. https://doi.org/10.1016/j.eswa.2023.119884

22. E. Güner, B. Aldemir, E. Aydoğdu, H. Aygün, Novel Hamacher aggregation operators with applications to the AHP-COPRAS method for spherical fuzzy environment, *Neural Comput. Appl.*, **37** (2025), 12941–1299. https://doi.org/10.1007/s00521-025-11167-9
23. A. Hussain, S. Yin, K. Ullah, M. Waqas, S. Moslem, T. Senapati, Decision analysis for data analytical enterprises with an innovative approach of spherical fuzzy prioritized aggregation operators, *J. Open Innov.-Techn., Market Comp.*, **11** (2025), 100642. https://doi.org/10.1016/j.joitmc.2025.100642
24. Y. Jin, S. Ashraf, S. Abdullah, Spherical fuzzy logarithmic aggregation operators based on entropy and their application in decision support systems, *Entropy*, **21** (2019), 628. https://doi.org/10.3390/e21070628
25. C. Kahraman, F. K. Gundogdu, S. Ç. Onar, B. Oztaysi, *Hospital location selection using spherical fuzzy TOPSIS*, in 11th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT 2019), 2019, 77–82. https://doi.org/10.2991/eusflat-19.2019.12
26. V. Keršuliene, E. K. Zavadskas, Z. Turskis, Selection of rational dispute resolution method by applying new step-wise weight assessment ratio analysis (SWARA), *J. Bus. Econ. Manag.*, **11** (2010), 243–258. https://doi.org/10.3846/jbem.2010.12
27. S. Kumar, J. Younis, A. Hussain, A novel spherical fuzzy VIKOR approach based on TODIM for evaluating and ranking the opinion polls with Shannon entropy and Jensen-Shannon divergence measure, *IEEE Access*, 2023.
28. F. K. Gündoğdu, C. Kahraman, Spherical fuzzy sets and spherical fuzzy TOPSIS method, *J. Intell. Fuzzy Syst.*, **36** (2019), 337–352. https://doi.org/10.3233/JIFS-181401
29. F. K. Gündoğdu, C. Kahraman, A novel spherical fuzzy analytic hierarchy process and its renewable energy application, *Soft Comput.*, **24** (2020), 4607–4621. https://doi.org/10.1007/s00500-019-04222-w
30. F. K. Gündoğdu, C. Kahraman, *Optimal site selection of electric vehicle charging station by using spherical fuzzy TOPSIS method,* in Decision Making with Spherical Fuzzy Sets: Theory and Applications, Springer, Cham, 2020, 201–216. https://doi.org/10.1007/978-3-030-45461-6_8
31. Z. Li, A. Liu, J. Miao, Y. Yang, A three-phase method for spherical fuzzy environment and application to community epidemic prevention management, *Expert Syst. Appl.*, **211** (2023), 118601. https://doi.org/10.1016/j.eswa.2022.118601
32. R. Mahajan, A. Baidya, P. Majumder, U. K. Bera, A novel hybrid spherical fuzzy AHP-SWARA-CoCoSo based group decision-making approach and their application in evaluating women empowerment policies, *Sādhanā*, **50** (2025), 118. https://doi.org/10.1007/s12046-025-02770-6
33. Q. X. Ma, X. M. Zhu, K. Y. Bai, R. T. Zhang, D. W. Liu, A novel failure mode and effect analysis method with spherical fuzzy entropy and spherical fuzzy weight correlation coefficient, *Eng. Appl. Artif. Intell.*, **122** (2023). https://doi.org/10.1016/j.engappai.2023.106163
34. M. Mathew, R. K. Chakrabortty, M. J. Ryan, A novel approach integrating AHP and TOPSIS under spherical fuzzy sets for advanced manufacturing system selection, *Eng. Appl. Artif. Intell.*, **96** (2020), 103988. https://doi.org/10.1016/j.engappai.2020.103988
35. S. Moslem, D. Farooq, D. Esztergár-Kiss, G. Yaseen, T. Senapati, M. Deveci, A novel spherical decision-making model for measuring the separateness of preferences for drivers' behavior factors associated with road traffic accidents, *Expert Syst. Appl.*, **238** (2024), 122318. https://doi.org/10.1016/j.eswa.2023.122318

36. P. H. Nguyen, T. T. Dang, K. A. Nguyen, H. A. Pham, Spherical fuzzy WASPAS-based entropy objective weighting for international payment method selection, *CMC-Comput. Mater. Con.,* **72** (2022). https://doi.org/10.32604/cmc.2022.025532
37. A. Patel, S. Jana, J. Mahanta, Intuitionistic fuzzy EM-SWARA-TOPSIS approach based on new distance measure to assess the medical waste treatment techniques, *Appl. Soft Comput.*, **144** (2023), 110521. https://doi.org/10.1016/j.asoc.2023.110521
38. S. Sen, L. Sahoo, K. Tiwary, T. Senapati, Entropy weighted TOPSIS based cluster head selection in wireless sensor networks under uncertainty, *Telecom*, **4** (2023), 678–692. https://doi.org/10.3390/telecom4040030
39. M. M. Rahman, M. Bodrud-Doza, M. D. Griffiths, M. A. Mamun, Biomedical waste amid COVID-19: perspectives from Bangladesh, *Lancet Glob. Health*, **8** (2020), e1262. https://doi.org/10.1016/S2214-109X(20)30349-1
40. C. E. Shannon, A mathematical theory of communication, *Bell Syst. Techn. J.*, **27** (1948), 379–423. https://doi.org/10.1002/j.1538-7305.1948.tb01338.x
41. A. Singh, K. Saha, COVID-19 and biomedical waste management, 2020. Available from: https://sprf.in/covid-19-and-biomedical-waste-management/.
42. M. A. Taş, E. Çakır, Z. Ulukan, Spherical fuzzy SWARA-MARCOS approach for green supplier selection, *3C Tecnología*, 2021, 115–133. https://doi.org/10.17993/3ctecno.2021.specialissue7.115-133
43. K. Toker, A. Görener, Evaluation of circular economy business models for SMEs using spherical fuzzy TOPSIS: An application from a developing countries' perspective, *Environ. Dev. Sustain.,* **25** (2023), 1700–1741. https://doi.org/10.1007/s10668-022-02119-7
44. Y. J. Qiu, M. B. Bouraima, C. K. Kiptum, E. Ayyildiz, Strategies for enhancing industry 4.0 adoption in East Africa: An integrated spherical fuzzy SWARA-WASPAS approach, *J. Ind. Intell.,* **1** (2023), 87–100. https://doi.org/10.56578/jii010202
45. R. R. Yager, *Pythagorean fuzzy subsets*, in 2013 Joint IFSA World Congress and NAFIPS Annual Meeting, Canada, 2013, 57–61. https://doi.org/10.1109/IFSA-NAFIPS.2013.6608375
46. L. A. Zadeh, Fuzzy sets, *Inform. Control*, **8** (1965), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X
47. H. Zhang, H. Wang, Q. Cai, G. Wei, Spherical fuzzy Hamacher power aggregation operators based on entropy for multiple attribute group decision making, *J. Intell. Fuzzy Syst.,* **44** (2023), 8743–8771. https://doi.org/10.3233/JIFS-224468
48. H. Zhang, G. Wei, X. Chen, CPT-MABAC method for spherical fuzzy multiple attribute group decision making and its application to green supplier selection, *J. Intell. Fuzzy Syst.*, **41** (2021), 1009–1019. https://doi.org/10.3233/JIFS-202954
49. H. Zhao, H. Zang, Investment decision framework for hybrid wind-solar-hydroelectric-hydrogen storage power generation projects using spherical fuzzy SWARA-CRITIC and CPT-TODIM, *Int. J. Hydrogen Energ.*, **177** (2025), 151565. https://doi.org/10.1016/j.ijhydene.2025.151565