Mathematics

https://www.aimspress.com/journal/Math

AIMS Mathematics, 10(5): 11435–11443.
DOI: 10.3934/math.2025520
Received: 25 February 2025
Revised: 08 May 2025
Accepted: 14 May 2025
Published: 20 May 2025

Research article

Weight distributions of a class of skew cyclic codes over $M_2(\mathbb{F}_2)$

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Abstract: Let $M_2(\mathbb{F}_2)$ be the ring of matrices of order 2×2 over finite field $\mathbb{F}_2$ and $\omega \in M_2(\mathbb{F}_2)$ be a cubic primitive root of unity. For any even positive integer t, the weight distributions of the skew cyclic codes of length $3t$ with parity check polynomials $x^t - \omega^i, i = 0,\ 1,\ 2$ and $(x^t - \omega^j)(x^t - \omega^k)$, $0 \leq j < k \leq 2$ were determined.

Keywords: skew cyclic code; matrix ring; weight distribution; module homomorphism
Mathematics Subject Classification: 11T71, 94B05, 94B15

1. Introduction

Let $M_2(\mathbb{F}_2)$ be the full matrix ring over finite field $\mathbb{F}_2$ of order 2×2 and C be a linear code over $M_2(F_2)$ with parameters $[n, k, d]$, where n is the length, k is the dimension, and d is the minimum distance. For an automorphism θ of $M_2(\mathbb{F}_2)$ and a unit λ in $M_2(\mathbb{F}_2)$, the linear code C is said to be a skew constacyclic code or a (θ, λ)- constacyclic code if it is closed under the (θ, λ) shift $\kappa_{\theta,\lambda}$ defined by

$$\kappa_{\theta,\lambda}((c_0, c_1, \ldots, c_{n-1})) = (\theta(\lambda c_{n-1}), \theta(c_0), \ldots, \theta(c_{n-2})).$$

In particular, such codes are called skew cyclic or skew negacyclic codes when λ equals 1 or -1, respectively. Specifically, when θ is the identity automorphism, then skew constacyclic code C becomes a classical constacyclic, cyclic, and negacyclic code.

The weight distribution of C is defined by the sequence $(1 = A_0, A_1, A_2, \ldots, A_n)$, where A_i is the number of codewords with Hamming weight i of C. The weight enumerator of C is defined by

$$1 + A_1 z + A_2 z^2 + \cdots + A_n z^n.$$

The set $\Omega = \{i : A_i \neq 0\}$ is called its weight set. In coding theory, the weight distribution of codes plays an important role, as it can help us analyze the minimum distance between codewords and to understand the error correction performance. By analyzing the weight distribution of the code, the error correction ability and decoding performance of the code can be improved.

The weight distributions of cyclic codes have been studied for many years and are well-known in some cases. There are extensive studies on the weight distributions of reducible and irreducible cyclic codes. The weight distribution of irreducible cyclic codes over finite fields has been thoroughly investigated in [9, 21, 25], and the weight distribution of reducible cyclic codes over finite fields can be found in [5, 22, 24]. The weight distribution of cyclic codes with few weights is studied in [12, 13, 20]. The weight distribution of fixed-length cyclic codes was studied in [10, 15, 26]. The complete weight distribution of two classes of cyclic codes was studied in [7]. Furthermore, there are significant findings and further researches on weight distributions of cyclic codes contained in [2, 8, 11].

In the history of error-correcting code theory, skew cyclic codes have also attracted significant attention. D. Boucher conducted initial studies of skew cyclic codes in [3], and D. Boucher also studied skew cyclic codes over Galois rings in [4]. S.Jitman studied skew constacyclic codes over finite chain rings in [6]. M. Shi studied skew cyclic codes over non-chain ring in [17], and I. Siap studied skew cyclic codes of arbitrary lengths in [19]. In addition, there are also studies on skew cyclic codes over chain rings (see [1, 14, 16]).

Based on the algebraic structures of skew cyclic codes over $M_2(\mathbb{F}_2)$ obtained by [18], we study the weight distributions of a class of skew cyclic codes over $M_2(\mathbb{F}_2)$. Let $\mathbb{F}_4$ be the splitting field of $f(x) = x^2+x+1$ and ω be a root of $f(x)$ in $\mathbb{F}_4$. For any even positive integer t, the weight distributions of the skew cyclic codes of length $3t$ with parity check polynomials $x^t-\omega^i$, $i = 0,\ 1,\ 2$ and $(x^t-\omega^j)(x^t-\omega^k)$, $0 \le j < k \le 2$ are determined. The major results are presented in the following theorems.

Theorem 1.1. *Let C be the skew cyclic code with parity check polynomials $h(x) = x^t - \omega^i,\ i = 0,\ 1,\ 2.$ The weight distribution of C is the sequence $(A_0, A_1, A_2, \ldots, A_n)$, where*

$$A_i = \begin{cases} 0, & 3 \nmid i; \\ \binom{t}{i/3} 15^{i/3}, & 3 \mid i. \end{cases}$$

Theorem 1.2. *Let C be the skew cyclic code with parity check polynomials $(x^t - \omega^j)(x^t - \omega^k)$, $0 \le j < k \le 2$. Then, the weight set of C is $\Omega = \{3i + 2j \mid 0 \le i \le t,\ 0 \le j \le t - i\}$, and the weight distribution is*

$$A_k = \sum_{\substack{i=0 \\ 3i+2j=k}}^{t} \sum_{j=0}^{t-i} \binom{t}{i}\binom{t-i}{j} 210^i 45^j,\ k = 0, 1, \cdots, n.$$

This paper is organized as follows. We review the algebraic structure of skew cyclic codes over $M_2(\mathbb{F}_2)$ briefly in Section 2. In Section 3, we prove our major results and give some examples. In Section 4, we concludes this paper.

2. Preliminaries

We review some algebraic properties of skew cyclic codes over $M_2(\mathbb{F}_2)$ briefly in this section.

We denote the 2×2 matrix ring over finite field $\mathbb{F}_2$ by $M_2(\mathbb{F}_2)$. By [23, p. 111], the matrix ring $M_2(\mathbb{F}_2)$ is isomorphic to the $\mathbb{F}_4$-cyclic algebra $\mathcal{R} = \mathbb{F}_4 \oplus e\mathbb{F}_4$ with $e^2 = 1$ under map δ,

$$\delta : \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \mapsto e,\ \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \mapsto \omega.$$

Denote σ the Frobenius map on $\mathbb{F}_4$. The Frobenius σ can be extended to a homomorphism θ on $\mathcal{R}$ that is defined by $\theta(a+eb) = \sigma(a) + e\sigma(b)$, $a, b \in \mathbb{F}_4$. The multiplication in $\mathcal{R}$ is given by $re = e\theta(r)$ for any $r \in \mathcal{R}$, and the addition is usual.

A linear code C on $\mathcal{R}$ is a left submodule of $\mathcal{R}^n$. The linear code C is said to be l-quasi cyclic if it is closed under $\mathcal{T}^l$, where $\mathcal{T}$ is the linear shift mapping defined by

$$\mathcal{T}(c_0, c_1, \cdots, c_{n-1}) = (c_{n-1}, c_0, \cdots, c_{n-2}).$$

In particular, the linear code C is a cyclic code if $l = 1$. The linear code C is called a skew cyclic code if it is closed under the map $\mathcal{T}_\theta$, defined by

$$\mathcal{T}_\theta(c_0, c_1, \cdots, c_{n-1}) = (\theta(c_{n-1}), \theta(c_0), \cdots, \theta(c_{n-2})).$$

Denote

$$\mathcal{R}[x,\theta] = \left\{r_n x^n + r_{n-1}x^{n-1} + \cdots + r_1 x + r_0 \mid r_i \in \mathcal{R},\ 0 \le i \le n,\ n \in \mathbb{N}\right\}.$$

For any $r \in \mathcal{R}$ and any natural number i, define the multiplication by $x^i \cdot r = \theta^i(r)x^i$. Then, for any $a + eb \in \mathcal{R}$, we have

$$x^i(a+eb) = \begin{cases} (a+eb)x^i, & \text{if } i \text{ is even,} \\ (a^2+eb^2)x^i, & \text{if } i \text{ is odd.} \end{cases}$$

We can see that $\mathcal{R}[x,\theta]$ forms a skew polynomial ring under this multiplication and the usual polynomial addition. We also denote $\mathcal{R}[x,\theta]_t$ as the subset of $\mathcal{R}[x,\theta]$ of polynomial of degree less than t.

Lemma 2.1. [18, Lemma 2.1] Let $f(x)$, $g(x) \in \mathcal{R}[x,\theta]$ with the leading coefficient of $g(x)$ as invertible. Then, there exist two unique polynomials $q(x)$, $r(x) \in \mathcal{R}[x,\theta]$, such that

$$f(x) = q(x)g(x) + r(x),$$

with $r(x) = 0$ or $\deg(r(x)) < \deg(g(x))$. The polynomials $q(x)$ and $r(x)$ are called the right quotient and right remainder, respectively. The polynomial $g(x)$ is called a right divisor of $f(x)$ if $r(x) = 0$.

Lemma 2.2. The center $Z(\mathcal{R}[x,\theta])$ of a polynomial ring $\mathcal{R}[x,\theta]$ is $\mathbb{F}_2[x^2]$.

Proof. For any $r \in \mathcal{R}$, we have $x^{2i}r = (\theta^2)^i(r)x^{2i} = rx^{2i}$. Thus, $x^{2i} \in Z(\mathcal{R}[x,\theta])$. This implies that if $r_j \in \mathbb{F}_2$, then $\sum_{j=0}^{s} r_j x^{2j}$ lies in $Z(\mathcal{R}[x,\theta])$. Conversely, for any $f_z \in Z(\mathcal{R}[x,\theta])$ and $r \in \mathcal{R}$, if $rf_z = f_z r$ and $xf_z = f_z x$, then the coefficients of f_z are in $\mathbb{F}_2$ and $f_z \in \mathcal{R}[x^2,\theta]$. Therefore, $f_z \in \mathbb{F}_2[x^2]$. □

Lemma 2.3. [18, Proposition 2.6.] Let n be a positive integer and C be a skew cyclic code of length n over $\mathcal{R}$ with a polynomial of minimum degree $d(x)$, where the leading coefficient of $d(x)$ is a unit. Then, C is a free $\mathcal{R}[x,\theta]$-submodule. $\mathcal{R}[x]$-submodule of $\mathcal{R}_n$, such that $C =< d(x) >$, where $d(x)$ is a right divisor of $x^n - 1$. Moreover, the code C has a basis $\mathcal{B} = \{d(x), xd(x), ..., x^{n-deg(d(x))-1}d(x)\}$, and the number of codewords in C is $|\mathcal{R}|^{n-deg(d(x))}$.

Let t be an even integer and $n = 3t$. Denote $\mathcal{R}_n = \mathcal{R}[x,\theta]/\langle x^n - 1\rangle$. By Lemma 2.2, the sets $\mathcal{R}_n = \mathcal{R}[x,\theta]/\langle x^n - 1\rangle$ and $\mathcal{R}[x,\theta]/\langle x^t - 1\rangle$ are two rings. It is easy to check $x^t - 1$, $x^t - \omega$ and $x^t - \omega^2$ commute with each other, so in $\mathcal{R}[x,\theta]$, we have

$$x^n - 1 = x^{3t} - 1 = (x^t - 1)(x^t - \omega)(x^t - \omega^2).$$

Under the canonical projection ψ, it is known by Chinese remainder theorem, as left $\mathcal{R}[x,\theta]$-modules,

$$\mathcal{R}[x,\theta]/\langle x^n-1\rangle \cong \mathcal{R}[x,\theta]/\langle x^t-1\rangle \oplus \mathcal{R}[x,\theta]/\langle x^t-\omega\rangle \oplus \mathcal{R}[x,\theta]/\langle x^t-\omega^2\rangle. \tag{2.1}$$

In fact, it is easy to prove the injection and the module homomorphism, whereas the surjection follows since we can define

$$f(x) = f_1(x)(x^{2t}+x^t+1) + f_2(x)(\omega x^{2t}+\omega^2 x^t+1) + f_3(x)(\omega^2 x^{2t}+\omega x^t+1)$$

for any $(\overline{f_1(x)},\ \overline{f_2(x)},\ \overline{f_3(x)}) \in \mathcal{R}[x,\theta]/\langle x^t-1\rangle \oplus \mathcal{R}[x,\theta]/\langle x^t-\omega\rangle \oplus \mathcal{R}[x,\theta]/\langle x^t-\omega^2\rangle$.

3. Proof of the major theorems

In this section, let t be an even integer and $n = 3t$. The code C is the skew cyclic code with parity check polynomial $h(x)$ of the form $x^t - \omega^i,\ i = 0,\ 1,\ 2$ or $(x^t-\omega^j)(x^t-\omega^k),\ 0 \le j < k \le 2$. We determine the weight distributions of C and prove the major theorems.

For any $\phi(x) \in \mathcal{R}[x,\theta]/\langle x^n-1\rangle$, there exists unique $(r_0(x), r_1(x), r_2(x)) \in (\mathcal{R}[x,\theta]_t)^3$ such that $\phi(x) = \sum_{i=0}^{2} r_i(x)x^{ti}$, therefore,

$$\psi(\phi(x)) = (b_0(x), b_1(x), b_2(x)), \tag{3.1}$$

where $b_j(x) = \sum_{i=0}^{2} r_i(x)(\omega^i)^j$, $j = 0,1,2$. Denote

$$M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & 1+\omega \\ 1 & 1+\omega & \omega \end{bmatrix},$$

then

$$\psi(\phi(x)) = (b_0(x), b_1(x), b_2(x)) = (r_0(x), r_1(x), r_2(x))M. \tag{3.2}$$

Therefore, we have

$$(r_0(x), r_1(x), r_2(x)) = (b_0(x), b_1(x), b_2(x))M^{-1}, \tag{3.3}$$

where

$$M^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1+\omega & \omega \\ 1 & \omega & 1+\omega \end{bmatrix}.$$

3.1. Case $h(x) = x^t - \omega^i,\ 0 \le i \le 2$

As the proofs are similar, we assume $h(x) = x^t - 1$. Then under the canonical projection ψ defined by (2.1),

$$C \cong \mathcal{R}[x,\theta]/\langle x^t-1\rangle,$$

and for any $\sum_{i=0}^{2} r_i x^{ti} \in C$, we have $\psi(\sum_{i=0}^{2} r_i x^{ti}) = (b_0(x), 0, 0)$. Hence, by (3.3), we have

$$b_0(x) = r_0(x) = r_1(x) = r_2(x). \tag{3.4}$$

Theorem 3.1. *The weight distribution of C is the sequence $(A_0, A_1, A_2, \ldots, A_n)$, where*

$$A_i = \begin{cases} 0, & 3 \nmid i; \\ \binom{t}{i/3} 15^{i/3}, & 3 \mid i. \end{cases}$$

Proof. Let s be the number of nonzero coefficients of $b_0(x)$, then by (3.4), the number of nonzero coefficients of $r_0(x)$, $r_1(x)$ and $r_2(x)$ is $3s$. Hence, the weight set of C is $\Omega = \{3s \mid 0 \le s \le t\}$. Furthermore, since there are $\binom{t}{i}15^i$ polynomials $b_0(x)$ in $\mathcal{R}[x,\theta]_t$ of weight i, Theorem 3.1 is proved. □

Example 1. *Here are two examples:*

(1) When $t = 8$*, then* C *is a* $[24, 8, 3]$ *skew cyclic code over* $M_2(\mathbb{F}_2)$ *with weight enumerator*

$$1 + 120z^3 + 6300z^6 + 18900z^9 + 3543750z^{12} + 42525000z^{15} + 318937500z^{18} + 1366875000z^{21} + 2562890625z^{24}.$$

(2) When $t = 10$*, then* C *is a* $[30, 10, 3]$ *skew cyclic code over* $M_2(\mathbb{F}_2)$ *with weight enumerator*

$$1 + 150z^3 + 10125z^6 + 405000z^9 + 10631250z^{12} + 191362500z^{15} + 2392031250z^{18} + 20503125000z^{21} + 115330078125z^{24} + 384433593750z^{27} + 576650390625z^{30}.$$

3.2. Case $h(x) = (x^t - \omega^j)(x^t - \omega^k),\ 0 \le j < k \le 2$

We assume $h(x) = (x^t - 1)(x^t - \omega)$ as the proofs of other cases are similar. Then, then under the canonical projection ψ defined by (2.1),

$$C \cong \mathcal{R}[x,\theta]/\langle x^t - 1\rangle \oplus \mathcal{R}[x,\theta]/\langle x^t - \omega\rangle$$

and for any $\sum_{i=0}^{2} r_i(x)x^{ti} \in C$, we have $\psi(\sum_{i=0}^{2} r_i(x)x^{ti}) = (b_0(x), b_1(x), 0)$. By (3.3), we get

$$\begin{cases} b_0(x) + b_1(x) = r_0(x), \\ b_0(x) + (1+\omega)b_1(x) = r_1(x), \\ b_0(x) + \omega b_1(x) = r_2(x). \end{cases} \tag{3.5}$$

For $i = 0, 1, 2$, write $b_i(x) = \sum_{s=0}^{t-1} b_{i,s}x^s$ and $r_i(x) = \sum_{s=0}^{t-1} r_{i,s}x^s$. By comparing the coefficients on both sides of (3.4), we obtain t system of equations as follows:

$$\begin{cases} b_{0s} + b_{1s} = r_{0s}, \\ b_{0s} + (1+\omega)b_{1s} = r_{1s},\ 0 \le s \le t-1. \\ b_{0s} + \omega b_{1s} = r_{2s}, \end{cases} \tag{3.6}$$

Since any two vectors of $(1,1)$, $(1,\omega)$ and $(1, 1+\omega)$ are linearly independent, then for any $(b_{0s}, b_{1s}) \in \mathcal{R}^2$, (3.6) implies the number of nonzero entries of $(r_{0s}, r_{1s}, r_{2s}) \in \mathcal{R}^3$ cannot be 1; therefore, the Hamming weight $w_H(r_{0s}, r_{1s}, r_{2s})$ lies in $S = \{0, 2, 3\}$. Hence, the Hamming weight set of C is

$$\Omega = \{k \mid k = k_0 + k_1 + \cdots + k_{t-1},\ k_i \in S,\ 0 \le i \le t-1\}.$$

By counting the number of those k_i's that equal 2 and 3, we have

$$\Omega = \{3i + 2j \mid 0 \le i \le t,\ 0 \le j \le t - i\}. \tag{3.7}$$

From the Hamming weight set Ω obtained in (3.7), it can be seen that C is a $[n = 3t, 2t, 2]$ code and Theorem 1.2 follows from Theorem 3.2.

Theorem 3.2. *The weight distribution of C is*

$$A_k = \sum_{\substack{i=0 \\ 3i+2j=k}}^{t} \sum_{j=0}^{t-i} \binom{t}{i}\binom{t-i}{j} 210^i 45^j,\ k = 0, 1, \cdots, n.$$

Proof. Considering the following system of equations

$$\begin{cases} x_1 + x_2 = 0, \\ x_1 + (1+\omega)x_2 = 0, \\ x_1 + \omega x_2 = 0. \end{cases} \tag{3.8}$$

For $0 \leq k \leq 3$, let $M(k)$ be the number of the points $(x_1, x_2) \in \mathcal{R}^2$ that satisfies some k equations of (3.8), but does not satisfy the other, and let $N(k)$ be the number of $(b_{0s}, b_{1s}) \in \mathcal{R}^2$ such that the corresponding Hamming weight of $(r_{0s}, r_{1s}, r_{2s}) \in \mathcal{R}^3$ equals k, then $N(k) = M(3-k)$.

Since any two equations of (3.8) have only common zero solution, then $M(2) = 0$ and $M(3) = 1$. For each equation, there are 15 nonzero solutions, and therefore $M(1) = 3 \times 15 = 45$, which implies that $M(0) = 16^2 - 1 - 45 = 210$. In conclusion, values of $M(k)s$ and $N(k)s$ are listed in Tables 1 and 2.

Table 1. Values of $M(k)s$.

k	0	1	2	3
$M(k)$	210	45	0	1

Table 2. Values of $N(k)s$.

k	0	1	2	3
$N(k)$	1	0	45	210

For any $0 \leq k \leq n = 3t$, the number A_k of codewords $\sum_{i=0}^{2} r_i(x)x^{ti}$ of weight k in C can be obtained by collecting the number of all those $(b_{0s}, b_{1s}) \in \mathcal{R}^2$, such that $w_H(r_{0s}, r_{1s}, r_{2s}) = k_s$ with $k = k_0 + k_1 + \cdots + k_{t-1}$, then

$$\begin{aligned} A_k &= \sum_{\substack{(k_0,\ldots,k_{t-1}) \in S^t \\ k = k_0 + \cdots + k_{t-1}}} N(k_0)N(k_1)\cdots N(k_{t-1}) \\ &= \sum_{\substack{i=0 \\ 3i+2j=k}}^{t} \sum_{j=0}^{t-i} \binom{t}{i}\binom{t-i}{j} 210^i 45^j. \end{aligned}$$

Theorem 3.2 can be proved. □

Example 2. *We list two examples:*

(1) Let $t = 2$, then C is a $[6, 4, 2]$ skew cyclic code over $M_2(\mathbb{F}_2)$ with weight enumerator

$$1 + 90z^2 + 420z^3 + 2025z^4 + 18900z^5 + 44100z^6.$$

(2) Let $t = 4$*, then* C *is a* $[12, 8, 2]$ *skew cyclic code over* $M_2(\mathbb{F}_2)$ *with weight enumerator*

$$1 + 180z^2 + 840z^3 + 12150z^4 + 113400z^5 + 629100z^6 + 5103000z^7 + 27914625z^8 + 113589000z^9 + 535815000z^{10} + 1666980000z^{11} + 1944810000z^{12}.$$

4. Conclusions

In this paper, for any even positive integer t, we present two major theorems for calculating the weight distribution of a class of skew cyclic codes of length $3t$. It is interesting to study case t, as it is odd.

Author contributions

Zhen Du: Conceptualization, Writing-original draft; Chuanze Niu: Conceptualization, Writing-review and editing. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

We thank the editors and the reviewers for their useful feedback that improved this paper. This work is supported by the Natural Science Foundation of Shandong Province, China (Grant number ZR2019BA011) and by the National Natural Science Foundation of China (Grant number 11401285).

Conflict of interest

The authors declare no conflicts of interest.

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