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Research article

Intelligent optimization algorithm for strategic planning in economics with multi-factors assessment: A MEREC-driven Heronian mean framework

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Abstract: Strategic planning in economics solves interconnected challenges such as dynamic interdependencies, conflicting objectives, and uncertain outcomes. Many such complexities present challenges for traditional approaches in terms of the evaluation and outcomes, requiring an innovative decision approach. An intelligent optimization algorithm is introduced in this paper, which assists policymakers in strategic planning through a multi-factor comprehensive assessment. A robust mechanism for analyzing, prioritizing, and resolving conflicting objectives is proposed based on integrating economic, social, and risk-related parameters within a highly sophisticated circular interval-valued framework. Unlike the traditional model, the proposed methodology is applied to reveal more adaptive, dynamic, and interdependent factors that provide actionable strategies that are consistent with policy goals. The comparison analysis demonstrates the proposed algorithm's validation and efficacy, highlighting the reliability and adaptability and depicting that the algorithm can transform the process of making economic strategy. The proposed work provides theoretical innovation and practical applications such as macroeconomic policy, financial risk management, and sustainable infrastructure development. It is a compelling tool for policymakers and strategists seeking to maximize economic outcomes in uncertain and dynamic environments.

Keywords: fuzzy logic-based optimization; multi-factor assessment algorithms; strategic economic planning; Heronian mean; MEREC approach
Mathematics Subject Classification: 03E72, 62C86, 68U35, 90B50

1. Introduction

In economics, strategic planning is creating long-term policies and activities to reach specific objectives soon, such as the allocation of resources, risk management, and sustainable development [1]. In a rapidly evolving economic landscape, the decisionmakers face challenges in handling multiple competing aspects, which consist of economic performance together with social fundamentals, environmental sustainability and policy workability. The complexity that exists between multiple system factors makes traditional methods ineffective, so the application of intelligent optimization emerges as a modern solution. The method establishes extended-term policies and frameworks designed for particular economic targets while considering distribution complexities, stakeholder preferences, and market elements. Through this framework, policymakers, together with organizations and governments, obtain the capability to make strategic choices that secure both sustainable economic growth and social equality with sustainability. Economic systems maintain a dynamic nature because they depend on multiple interdependent factors consisting of technological innovation together with population growth and global business activities and environmental transformations as well as geopolitical risks. The interrelated nature of economic systems needs evaluation during the planning of strategic measures and the building of adaptive policies with resilience characteristics. Addressing the challenges of unemployment, income disparity, inflation, climate change, and resource scarcity demands a comprehensive assessment approach.

Strategic planning is a cornerstone of effective decision-making and long-term goal achievement across various applied disciplines, from tourism to housing, governance, agriculture, foreign policy, and urban development. The adaptability of strategic frameworks means they can be used in any sector to bring organizations and governments to align resources, anticipate challenges, and achieve sustainable growth. The Jordan Tourism Board's 2023 Strategic Planning [2] is an example of how modern strategies and digital engagement can be merged to increase a nation's tourism capabilities using social media to extend their reach to a worldwide audience. Similarly, housing planning in the context of addressing affordability is embedded in administrative studies for affordable housing [3], on the allocation of finances and affordability. Strategic planning is equally important in local government. The case studies [4] highlight how municipalities utilize specific frameworks tailored to the success of community development, economic development, and infrastructure expansion. Further, integrating technology and market strategy [5] is discussed in agricultural economics to improve productivity and sustainability and adapt to the increasingly competitive market in a global setting. The foreign policy analysis [6] stands as a groundbreaking resource at the nexus of governance and policy that can inform diplomatic or business decisions, covering risks while enhancing international cooperation. Another set of works that also demonstrates the effectiveness of strategic plans at the regional level is, for example, The Evaluation of the Regional Long-Time Development Plan [7], which assesses the effects of these plans on socioeconomic development and governance. Moreover, the use of economic indicators [8] as instruments for determining market trends and forecasting future performance means data merits use as a strategic decision platform.

The innovation districts have become strategic elements of spatial planning on an urban scale [9]. Urban projects are the pathways to foster urban innovation, economic growth, and societal advancement, which provide pathways for the latter. This comprehensive exploration of strategic planning shows how it can solve complex problems, foster innovation, and underpin long-term sustainability. This analysis examines its use in tourism, housing, governance, agriculture, and urban planning to illuminate the principles, methodologies, and outcomes of strategic planning as a transformative tool for contemporary development.

The comprehensive exploration of strategic planning highlights its versatility and necessity in addressing complex challenges, fostering innovation, and achieving long-term sustainability. By examining its application across tourism, housing, governance, agriculture, and urban planning, this analysis aims to illuminate the principles, methodologies, and outcomes of strategic planning as a transformative tool for modern development.

1.1. Key elements of strategic planning in economics

The key elements of strategic planning are displayed in Figure 1, and their description is given below:

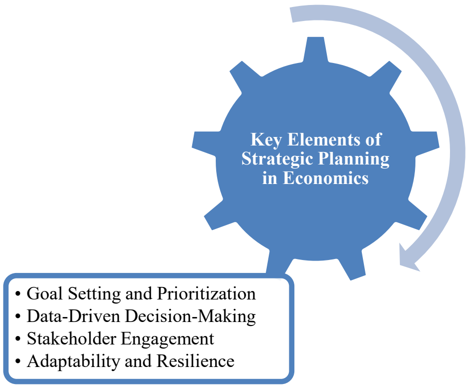

Figure 1. Key elements of strategic planning.

a) **Goal setting and prioritization.** The first part of strategic planning defines clear objectives connected to national or organizational visions. Some goals are increasing GDP growth, reducing poverty, promoting green technologies, and conducting balanced regional development. Prioritization uses limited resources in the areas with the most significant impact potential.

b) **Data-driven decision-making.** Economic planning necessarily requires analysis of large datasets; macroeconomic indicators (GDP, inflation rates, unemployment); social factors (educational levels, healthcare access); and environmental metrics (carbon emissions, biodiversity indices). Data, though, is only valuable when it is transformed into actionable insights, and that's where advanced analytics and intelligent algorithms play a critical role in providing valuable insights.

c) **Stakeholder engagement.** The stakeholders are essential, and governments, businesses, communities, and international organizations must be considered. Equitable, socially acceptable, and politically 'feasible' inclusive policymaking is defined.

d) **Adaptability and resilience.** In an uncertain environment, strategies must be flexible enough to deal with unexpected disruptions (pandemics, economic recessions, or technological milestones) that affect the lives of customers and employees, respectively. Moreover, building flexibility relies on scenario planning and real-time monitoring.

1.2. Background and motivation

Strategic planning is a mechanism to navigate the complex, dynamic challenges in global tourism, housing, governance, agriculture, urban innovation, and public policy. It helps us provide structure alignment so that the objectives are ready and resource allocation can be optimized to help them achieve sustainable growth in increasingly competitive and resource-constrained environments. Tourism planning can also do this, as the Jordan tourism board's strategies [2] show how tourism planning can preserve culture, balance it with economic expansion, and bridge tradition and modernization. The affordable housing initiative project [3] necessitates a creative, cost-efficient approach toward tackling critical social concerns, maintaining and ensuring financial sustainability, and inclusivity over the long term. Strategic planning in the agricultural sector [10] Plugs in technological innovations and market-driven convergences to revolutionize productivity and food security through a blueprint for urban strategic development to boost creativity and economic resilience. Innovation districts, too, are a burgeoning passion for strategic urban planning rooted in collaboration, entrepreneurship, the development of technological solutions, and transforming cities into engines of economic and creative growth. From these diverse examples, it becomes clear how strategic planning could be used to solve real-world problems. Despite its inherent strengths, it also allows for common and glaring weaknesses, including the incomplete use of imprecise data, balancing multiple priorities, and quantifying the less tangible results, such as social impact, cultural value, and long-term adaptability. Increasingly, advanced decision-making frameworks are needed to address the requirements of strategic plans, ensuring diverse stakeholder views, anticipation of evolving market trends, and RIM to unfavorable changes. A strategic planning system that leverages the interdependencies and complexity of the present globalized and resource-scarce environment is emerging, one that embraces uncertainty, fosters innovation, and delivers solutions that solve in a dynamic, adaptable way when designed to the realities of each sector's specific challenges.

The motivation for this research is to address the above complexities by considering strategic planning to be a decision-making problem and providing a comprehensive decision-making tool to systematically evaluate the optimal strategy by considering the imprecision inherent in the environment. Using such an approach, the strategic planning processes enable robust data-driven models to produce optimized answers that address sector requirements. For instance, decision-making tools can enhance the viability of affordable housing initiatives with criteria like economic feasibility, social impact, and environmental sustainability. Likewise, it can be valuable in urban innovation projects to prioritize actions deriving from interlocking issues such as infrastructure, stakeholder collaboration, and scalability over time. By reframing strategic planning as a decision-making problem, stakeholders can make informed, adaptive, and context-sensitive choices; ensure the alignment of resources and goals and promote innovation and resilience in a complex and interconnected world.

1.3. Problem statement

Strategic planning is essential for tackling complex, joint challenges in various sectors, such as tourism, housing, governance, agriculture, and urban development. By drawing on prior research on the transformative potential of strategic frameworks, this paper calls into question the competence of studies in that regard, highlighting significant gaps in dimensions like uncertainty, interdependencies, and flexibility. Tourism strategic planning is a good example of how digital platforms can be used.

Still, it fails to provide the methodology for tackling conflicting priorities and evolving dynamics. Like works on affordable housing investment analysis, the financial viability of a project is emphasized without considering the integrated frameworks for socio-economic and environmental factors. Studies of technology integration with market strategic management in the agricultural sector frequently neglect the complexities of reconciling short-term gains with long-term sustainability. Moreover, studies focusing on urban creativity and economic growth tend to leave out contextual factors such as environmental sustainability, infrastructure, and community equity.

Further, the traditional decision-making models in these domains heavily rely on deterministic methods that do not account for imprecision, ambiguity, and complex relationships between criteria. Although the existing fuzzy [11] frameworks such as interval valued intuitionistic fuzzy sets (IVIFS) [12], pythagorean fuzzy sets (PyFS) [13], and q-rung fuzzy sets (q-RFS) [14] are innovative and used by various scholars [15–17], they fail to tackle real-world decision-making problems with a circular framework [18] that depicts the interrelationships and radius between membership and non-membership values. We show that these methods typically produce inconsistent rankings, are limited in their ability to consider various stakeholder preferences, and cannot opportunistically adapt to changing contexts over time.

To address these gaps, strategic planning has to shift to a multi-criteria decision-making (MCDM) problem capable of dealing with uncertainty, flexibility, and interdependencies. In this study, a new robust framework scheme using the IVIF Heronian mean within the circular framework (CIVIFHM) and its extension, including the weighted Heronian mean (CIVIFWHM), generalized Heronian mean (CIVIFGHM), and weighted generalized Heronian mean (CIVIFWGHM), was presented. The proposed operators are ideally suited to evaluate such alternatives as industrial modernization, renewable energy development, education and workforce training, and infrastructure expansion. They also offer flexibility to address changing priorities and circular interrelationships that consider the complexity of the environment. The study employs this innovative approach to not only bridge the existing gaps in previous methodologies but also to create the most complete tool for a systematic evaluation and ranking of alternatives that the field of public policy has to offer policymakers. The proposed approach overcomes the weaknesses of existing methods in providing robust, flexible, and contextually sensitive solutions to strategic planning problems in a world of increasing interconnectedness and uncertainty.

1.4. Objective and contribution

This research aims to develop a robust and vigorous decision-making approach for assessing economic strategies, including stagnation, unemployment, and resource limitation, while promoting equity, growth, and sustainability. This research specifically aims to integrate the MEREC approach with a CIVIF framework to identify the preference of various criteria such as job creation, growth of the economy, social equity, and environmental sustainability. By exploiting advanced methodologies, the study offers policymakers a systematic and balanced tool to balance competing priorities, navigate uncertainties, and find optimal solutions for strategic economic planning.

In the field of economic strategy, this study makes several significant contributions. First, it allows for a comprehensive assessment of multi-criteria problems while resolving uncertainty and imprecision. It introduces a novel combination of the interval-valued circular intuitionistic framework and the MEREC technique. Second, it provides a structured methodology to assess trade-offs among critical factors like creating jobs, equity, economic growth, and sustainability, and offers actionable solutions for decision-makers. Third, the research proves its practical application by applying this approach to a real-world case study, highlighting that it has the potential to assist policymakers in crafting sustainable and balanced economic growth. Last, the research opens the way for future study by proposing a versatile approach that could be adapted by other complex decision-making fields, enhancing strategic planning and intelligent optimization.

1.5. Layout

This paper is organized as follows: in the introductory Section 1, the introduction, background history, motivation, problem statement, objectives, and contribution are defined. Section 2 presents the literature on the existing model and discusses the research gap in economic strategy planning. In Section 3, some basic concepts that form the foundation of the proposed methodology are presented in Section 4. The proposed method is utilized to evaluate the economic strategy planning problem, and its numerical evaluation is given in Section 5. The last section, Section 6, concludes the findings by discussing their limitations and future direction.

2. Literature review

This section gives an overview of the existing models in economic strategy by highlighting their methodologies, their areas of application, and their drawbacks. Further, it points out the research gap in existing models and does not fully address the problem of dealing with uncertainties.

2.1. Existing model in economics strategic planning

Strategic planning models have evolved in economics to solve challenges: resource optimization, forecasting, performance analysis, etc. Such models are usually theoretical but include practical tools to evaluate alternatives, predict trends, and inform policymaking. Thus, they are effective only to the extent that they can embed uncertainty, interdependencies, and stakeholder preferences. Several existing models assist in illustrating the use of strategic planning in several economic contexts, including key performance indicators (KPIs) and data envelopment analysis (DEA) [19] which are applied to Greek tourism and integrated with DEA to assess the efficiency of destinations. It helps to make strategic plans by identifying the factors of strengths, weaknesses, and areas of improvement, but it only conducts linear efficiency evaluation and lacks multi-faceted interdependence. Artificial neural networks (ANN) [20], applies machine learning (ML) techniques to predict the trend and is used for predicting air traffic demand based on socio-economic parameters. Despite its predictive capability, this approach is untransparent when it makes decisions and cannot incorporate qualitative arguments. In strategic planning and performance perceptions [21], the mediation analysis examines relationships between strategic planning and citizen and manager perceptions. Although it provides stakeholder views, it is hard to use in quantitative, data-driven cases where alternatives must be judged. The digital transformation in the supply chain optimization model [22] emphasizes the economic cost of supply chain transformation with economic theory and practical tools and focuses on the economic impacts of digital transformation. It applies to technological applications but does not address broader strategic issues such as social equity or environmental sustainability. The foresight research and economic complexity [23] examine synergies between economic complexity and foresight research within the framework of small countries in technological revolutions. However, regarding innovation and adaptability, there is little framework to decide when, where, and how to allocate resources in detail. The existing studies based on economic strategic planning are shown in Table 1.

Table 1. Existing model in economics strategic planning.

Model	Application	Strengths	Limitations
Key Performance Indicators and Data Envelopment Analysis (KPIs and DEA)	Evaluation in tourism	Identifies strengths and weaknesses in performance	Limited to linear efficiency evaluation; does not address multi-criteria dependencies
Artificial Neural Networks (ANN)	Predicting air traffic demand	Accurate forecasting of socio-economic trends	Lacks decision transparency; does not incorporate qualitative factors
Strategic Planning and Performance Perceptions	Managerial and citizen perspectives in planning	Captures stakeholder relationships effectively	Ineffective for quantitative, data-driven evaluations
Digital Transformation in Supply Chain Optimization	Supply chain efficiency and digital impact	Technological integration and economic analysis	Ignores broader strategic goals like equity and sustainability
Foresight Research and Economic Complexity	Innovation for small economies	Encourages adaptability and fosters innovation	Does not provide detailed frameworks for decisionmaking.

To address the limitations of existing economic strategic planning frameworks, the proposed model, based upon the CIVIF framework, provides robust, flexible, and efficient solutions to strategic planning problems in a world of increasing complexities and uncertainties. In contrast to the literature, the proposed model conditions DEA models to handle nonlinear interdependencies and a complete set of criteria like economic growth, social equity, and environmental impact. Moreover, it is also superior to machine learning models (MLM), like artificial neural networks (ANN), which use a process that can be seen by humans but is not transparent and interpretable in terms of both qualitative and quantitative. The model bridges the stakeholder perception gap and quantitative analysis by providing inclusion and a high degree of data. It also tackles the shortcomings of existing supply chain optimization and foresight research models by embedding sustainability, equity, and adaptability into the decision-making framework. The proposed model featuring advanced aggregation operators (AOs) not only performs well in dealing with imprecision but also considers conflicting objectives, providing an attractive, robust, and flexible solution to complex economic strategic planning problems.

Moreover, the proposed approach extends the traditional decision-making models by integrating a CIVIF framework that allows a more nuanced representation of uncertainty and interdependence between economic factors. Unlike traditional decision-making approaches that use subjective weight values [24], this framework uses the MEREC-driven Heronian Mean approach to dynamically evaluate the objective weighting values, resulting in higher adaptability. Mathematically, this improves the aggregation process by maintaining interrelationships, reducing bias, and increasing decision consistency. The approach is reliable with current developments in computational economics and fuzzy decision theory, bridging the gap between qualitative expert opinion and quantitative optimization. By addressing critical drawbacks of existing models, such as rigid preference structures and sensitivity to input variations, this framework offers a more flexible solution for complicated economic strategy planning.

2.2. Research gap

Despite a vast generalization of economic strategic planning research, many problems remain to be resolved in a complex setting, compared to real-world decision-making scenarios. Models that are KPIs plus DEA or artificial neural networks and supply chain optimization frameworks are perfect when applied to particular circumstances. Yet, they do not integrate the various interdependencies and uncertainties in a multi-criteria evaluation. However, complex economic environments often result in imprecision in data and non-linear relationships between criteria and conflicting objectives, rendering these models inapplicable. For example, whereas DEA models can only perform linear efficiency analysis, ML models lack transparency and do not account for qualitative preferences. As stakeholder-focused approaches can tell you who cares and what they care about, they can also capture perceptions, but not robust quantitative methods for selecting one option over another. In parallel with such models, foresight and economic complexity models also focus on adaptability but do not include structured resource allocation and prioritization frameworks.

Unlike traditional models such as ANN and DEA, which struggle with interpretability and fail to capture the complex interdependencies of economic factors, our proposed framework introduces a circular interval-valued intuitionistic fuzzy (CIVIF) approach. This innovative methodology seamlessly integrates quantitative economic indicators and qualitative risk assessments, providing policymakers with a robust, data-driven decision-making tool that adapts to uncertainty and conflicting objectives in strategic economic planning. To address the gaps with a decision-making model that can seamlessly handle imprecision, manage interdependencies between criteria, and enable transparency and inclusivity when faced with diverse priority specifications. This research aims to develop the IVIF Heronian mean within the circular framework (CIVIFHM) framework and its extension, which offers an alternative to existing models using an advanced decision-making model that considers the weighted relationships and dynamic AOs to develop a comprehensive, flexible, and robust tool for addressing various multi-dimensional strategic planning challenges.

3. Preliminaries

This section recalls some basic concepts of CIFS, CIVIFS, its operational laws, HM and GHM AO, and some of its properties.

Definition 3.1. [18] A circular intuitionistic fuzzy set (CIFS) C is define by;

$$\mathrm{C} = \left\{\left(\mathrm{C}, \left(m_{\mathrm{C}}(\mathrm{c}), n_{\mathrm{C}}(\mathrm{c}), r(\mathrm{c})\right)\right); \mathrm{c} \in \mathrm{C}\right\}. \tag{1}$$

The CIFS consists of a triplet that depicts a circle's membership, non-membership, and radius. Moreover, each term of the triplet lies within the zero-one interval, and the sum of $\left(m_{\mathrm{C}}(\mathrm{c}), n_{\mathrm{C}}(\mathrm{c})\right)$ also lies within the zero-one interval. For convenience, we will write it as (m, n, r).

Definition 3.2. [25] A circular interval-valued intuitionistic fuzzy set (CIVIFS) C is defined by;

$$\mathrm{C} = \left\{\left(\mathrm{c}, \left(m_{\mathrm{C}}(\mathrm{c}), n_{\mathrm{C}}(\mathrm{c}), r(\mathrm{c})\right)\right); \mathrm{c} \in \mathrm{C}\right\}. \tag{2}$$

Where $m_{\mathrm{C}}(\mathrm{c}) = \left[m^{\ell}, m^{u}\right]; n_{\mathrm{C}}(\mathrm{c}) = \left[n^{\ell}, n^{u}\right]; r(\mathrm{c}) = \left[r^{\ell}, r^{u}\right]$. Each term is bounded between the lower and upper bounds such that each term lies within the zero-one interval, and a condition that the sum of its upper bounds, i.e., $(m^{u} + n^{u})$, lies within the zero-one interval.

Definition 3.3. [25] Let $\mathrm{C} = \left(\left[m^{\ell}, m^{u}\right], \left[n^{\ell}, n^{u}\right], \left[r^{\ell}, r^{u}\right]\right)$ depict the CIVIFS. Then, the score value is determined by

$$S_{\mathrm{C}} = \frac{S_{\mathrm{IFS}}\left(m^{\ell}, n^{\ell}\right) + S_{\mathrm{IFS}}(m^{u}, n^{u}) \times \left(r^{\ell} + r^{u}\right)}{2},$$
$$S_{\mathrm{C}} = \frac{\left(\left(m^{\ell} + m^{u} - n^{\ell} - n^{u}\right) \times \left(r^{\ell} + r^{u}\right)\right)}{2}. \tag{3}$$

Definition 3.4. [25] Let $\mathrm{C}_{i} = \left(\left[m^{\ell}, m^{u}\right], \left[n^{\ell}, n^{u}\right], \left[r^{\ell}, r^{u}\right]\right); (i = 1,2)$ represent a CIVIFSs. Then;

a) $$\mathrm{C}_1 \oplus \mathrm{C}_2 = \left(\begin{array}{c} \left[\begin{array}{c}\left({m^{\ell}}_1 + {m^{\ell}}_2 - {m^{\ell}}_1 {m^{\ell}}_2\right), \\ ({m^{u}}_1 + {m^{u}}_2 - {m^{u}}_1 {m^{u}}_2)\end{array}\right], \left[\left({n^{\ell}}_1 {n^{\ell}}_2\right), ({n^{u}}_1 {n^{u}}_2)\right], \\ \left[\begin{array}{c}\left({r^{\ell}}_1 + {r^{\ell}}_2 - {r^{\ell}}_1 {r^{\ell}}_2\right), \\ ({r^{u}}_1 + {r^{u}}_2 - {r^{u}}_1 {r^{u}}_2)\end{array}\right]\end{array}\right), \tag{4}$$

b) $$\mathrm{C}_1 \otimes \mathrm{C}_2 = \left(\begin{array}{c} \left[\left({m^{\ell}}_1 {m^{\ell}}_2\right), ({m^{u}}_1 {m^{u}}_2)\right], \left[\begin{array}{c}\left({n^{\ell}}_1 + {n^{\ell}}_2 - {n^{\ell}}_1 . {n^{\ell}}_2\right), \\ ({n^{u}}_1 + {n^{u}}_2 - {n^{u}}_1 . {n^{u}}_2)\end{array}\right], \\ \left[\left({r^{\ell}}_1 {r^{\ell}}_2\right), ({r^{u}}_1 {r^{u}}_2)\right]\end{array}\right), \tag{5}$$

c) $$\hbar \mathrm{C}_1 = \left(\begin{array}{c} \left[1 - \left(1 - {m^{\ell}}_1\right)^{\hbar}, 1 - (1 - {m^{u}}_1)^{\hbar}\right], \left[\left({n^{\ell}}_1\right)^{\hbar}, ({n^{u}}_1)^{\hbar}\right], \\ \left[1 - \left(1 - {r^{\ell}}_1\right)^{\hbar}, 1 - (1 - {r^{u}}_1)^{\hbar}\right]\end{array}\right), \tag{6}$$

d) $$(\mathrm{C}_1)^{\hbar} = \left(\begin{array}{c} \left[\left({m^{\ell}}_1\right)^{\hbar}, ({m^{u}}_1)^{\hbar}\right], \left[1 - \left(1 - {n^{\ell}}_1\right)^{\hbar}, 1 - (1 - {n^{u}}_1)^{\hbar}\right], \\ \left[\left({r^{\ell}}_1\right)^{\hbar}, ({r^{u}}_1)^{\hbar}\right]\end{array}\right). \tag{7}$$

3.1. Heronian mean (HM) AO and its properties

To depict the interrelationship between the multiple parameters, HM AO [26] plays a significant role by accumulating and aggregating the provided data and assisting the experts in making decisions. So, the HM is defined as follows:

Definition 3.5. [26] Let $C_i(i = 1,2,\dots,n) \in N^+$. Then,

$$HM^{i,j}(C_1, C_2, \dots, C_n) = \left(\left(2/_{n^2 + n}\right) \sum_{p=1}^{n} \sum_{q=1}^{n} \sqrt{C_p^i C_q^j}\right). \quad (8)$$

Here $i, j > 0$.

The HM AO satisfies the following properties;

a) $HM^{i,j}(0,0,\dots,0) = 0$; If $\forall\, C_i = 0, \forall i$;

b) $HM^{i,j}(C_1, C_2, \dots, C_n) = C$; If $\forall\, C_i = C, \forall i$;

c) $HM^{i,j}(C_1, C_2, \dots, C_n) \geq HM^{i,j}(C_1', C_2', \dots, C_n')$; If $C_i \geq C_i', \ \forall i$;

d) $\min(C_i) \leq HM^{i,j}(C_1, C_2, \dots, C_n) \leq \max(C_i)$;

Definition 3.6. [26] Let $C_i(i = 1,2,\dots,n) \in N^+$. Then,

$$GHM^{i,j}(C_1, C_2, \dots, C_n) = \left(\left(2/_{n^2 + n}\right) \sum_{p=1}^{n} \sum_{q=1}^{n} C_p^i C_q^j\right)^{1/_{i+j}}. \quad (9)$$

Here $i, j > 0$. If $i = j = 1/2$ Eq (9) is reduced to Eq (8).

The GHM AO satisfies the following properties;

a) $GHM^{i,j}(0,0,\dots,0) = 0$; If $\forall\, C_i = 0, \forall i$.

b) $GHM^{i,j}(C_1, C_2, \dots, C_n) = C$; If $\forall\, C_i = C, \forall i$.

c) $GHM^{i,j}(C_1, C_2, \dots, C_n) \geq GHM^{i,j}(C_1', C_2', \dots, C_n')$.

d) If $C_i \geq C_i', \forall i; \min(C_i) \leq GHM^{i,j}(C_1, C_2, \dots, C_n) \leq \max(C_i)$.

4. Methodology

This section delves into the framework, which is characterized by circular features, and facilitates a more precise approach to addressing uncertainty and imprecision over the interval. This section introduces the CIVIFS by utilizing the HM AO, which is pivotal in decision analysis. Moreover, it also defines the theorems, properties, and axioms of CIVIFS within the HM, GHM, WHM, and WGHM AOs. Further, the weight values are evaluated using the MEREC (method based on the removal effect of criteria) approach. The proposed framework enhances traditional strategic planning by incorporating a structured fuzzy decision-making process that accounts for uncertainty and conflicting objectives. By integrating the MEREC method, our approach systematically removes less significant criteria and prioritizes impactful economic factors, resulting in an adaptive and resilient economic planning tool.

4.1. MEREC approach

This approach is proposed by Ghorabaee et al. [27], which is also considered a weight-determining approach that computes the impact of each criterion by removing each criterion one by one. The step of this approach is defined as follows:

a) Formation of a decision matrix such that each element reflects the performance value of an alternative concerning a specific criterion.

$$(\mathcal{D})_{\mathbb{m}\times \mathrm{n}} = \begin{matrix} \\ s_1 \\ s_2 \\ \vdots \\ s_{\mathbb{m}} \end{matrix} \begin{bmatrix} Ł_1 & Ł_2 & \dots & Ł_{\mathrm{n}} \\ \mathrm{C}_{11} & \mathrm{C}_{12} & \dots & \mathrm{C}_{1\mathrm{n}} \\ \mathrm{C}_{21} & \mathrm{C}_{22} & \dots & \mathrm{C}_{2\mathrm{n}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{C}_{\mathbb{m}1} & \mathrm{C}_{\mathbb{m}2} & \dots & \mathrm{C}_{\mathbb{m}\mathrm{n}} \end{bmatrix}.$$

b) Each term in the decision matrix is normalized to ensure consistency and compatibility across each attribute value. The normalized matrix is obtained by

$$(N)_{\mathbb{m}\times \mathrm{n}} = \begin{cases} \dfrac{\mathrm{C}_{\mathbb{m}\mathrm{n}}}{\max(\mathrm{C}_{\mathbb{m}\mathrm{n}})}; cost \\ \dfrac{\min(\mathrm{C}_{\mathbb{m}\mathrm{n}})}{\mathrm{C}_{\mathbb{m}\mathrm{n}}}; benefit \end{cases}$$

c) Calculate the alternative's overall performance, representing the effectiveness across all criteria.

$$\mathcal{O}_i = \frac{1}{\mathrm{n}} \sum_j \ln\left(1 - N_{ij}\right).$$

d) To evaluate the impact of each attributive value, the performance of each alternative is recalculated by excluding the contribution of each criterion one by one.

$$\mathcal{O}'_{ij} = \frac{1}{\mathrm{n}} \sum_{k\neq j} \ln(1 - N_{ik}).$$

e) Now calculate the deviation degree between the overall performance and the recalculated performance to measure the influence of each criterion.

$$\delta_j = \sum_i \left|\mathcal{O}'_{ij} - \mathcal{O}_i\right|.$$

f) By summing up the deviation degree, the weight values w_j are determined, ensuring that each criterion is prioritized appropriately according to relative importance.

$$w_j = \frac{\delta_j}{\sum_k \delta_k}.$$

4.2. IVIF heronian mean within the circular framework (CIVIFHM) AO

Definition 4.1. Let $\mathrm{C}_i(i = 1,2,\dots,\mathrm{n})$ be circular interval-valued intuitionistic fuzzy values (CIVIFVs). Then,

$$CIVIFWHM^{i,j}(C_1, C_2, \dots, C_n) = \left(\left(2/_{n^2+n}\right) \sum_{p=1}^{n} \sum_{q=1}^{n} \sqrt{\begin{matrix}(w_p C_p)^{i} \times \\ (w_q C_q)^{j}\end{matrix}} \right). \tag{10}$$

$$CIVIFHM^{i,j}(C_1, C_2, \dots, C_n) = \left(\left(2/_{n^2+n}\right) \sum_{p=1}^{n} \sum_{q=1}^{n} \sqrt{C_p^{i} C_q^{j}} \right). \tag{11}$$

Here $i, j > 0$.

Definition 4.2. Let $C_i (i = 1,2, \dots, n)$ be a CIVIFV and $w_i (i = 1,2, \dots, n)$ be the weight value that defines the significance of each C_i with a condition $\sum w_i = 1$ Then,

Theorem 4.1. Let $C_i (i = 1,2, \dots, n)$ be a CIVIFV. Then:

$$\begin{aligned} &CIVIFHM^{i,j}(C_1, C_2, \dots, C_n) \\ &= \left(\begin{matrix} \left[1 - \prod_{q=1,p=1}^{n} \left(1 - \sqrt{\left((m_p{}^{\ell})^{i} (m_q{}^{\ell})^{j}\right)}\right)^{\left(2/_{n^2+n}\right)}, 1 - \prod_{q=1,p=1}^{n} \left(1 - \sqrt{\left((m_p{}^{u})^{i} (m_q{}^{u})^{j}\right)}\right)^{\left(2/_{n^2+n}\right)} \right], \\ \left[\prod_{q=1,p=1}^{n} \left(1 - \sqrt{\begin{matrix}(1-n_p{}^{\ell})^{i} \times \\ (1-n_q{}^{\ell})^{j}\end{matrix}}\right)^{\left(2/_{n^2+n}\right)}, \prod_{q=1,p=1}^{n} \left(1 - \sqrt{\begin{matrix}(1-n_p{}^{u})^{i} \times \\ (1-n_q{}^{u})^{j}\end{matrix}}\right)^{\left(2/_{n^2+n}\right)} \right], \\ \left[1 - \prod_{q=1,p=1}^{n} \left(1 - \sqrt{(r_p{}^{\ell})^{i} (r_q{}^{\ell})^{j}}\right)^{\left(2/_{n^2+n}\right)}, 1 - \prod_{q=1,p=1}^{n} \left(1 - \sqrt{(r_p{}^{u})^{i} (r_q{}^{u})^{j}}\right)^{\left(2/_{n^2+n}\right)} \right] \end{matrix} \right). \end{aligned} \tag{12}$$

Proof. See Appendix A.

Theorem 4.2. Let $C_i (i = 1,2, \dots, n)$ be a CIVIFV and $w_i (i = 1,2, \dots, n)$ be the weight value that defines the significance of each C_i with a condition $\sum w_i = 1$. Then:

$$\begin{aligned} &CIVIFWHM^{i,j}(C_1, C_2, \dots, C_n) \\ &= \left(\begin{matrix} \left[1 - \prod_{q=1,p=1}^{n} \left(1 - \sqrt{\left(\begin{matrix}\left(1-(1-m_p{}^{\ell})^{w_i}\right)^{i} \times \\ \left(1-(1-m_q{}^{\ell})^{w_i}\right)^{j}\end{matrix}\right)}\right)^{\left(2/_{n^2+n}\right)}, 1 - \prod_{q=1,p=1}^{n} \left(1 - \sqrt{\left(\begin{matrix}\left(1-(1-m_p{}^{u})^{w_i}\right)^{i} \times \\ \left(1-(1-m_q{}^{u})^{w_i}\right)^{j}\end{matrix}\right)}\right)^{\left(2/_{n^2+n}\right)} \right], \\ \left[\prod_{q=1,p=1}^{n} \left(1 - \left(\sqrt{\begin{matrix}\left(1-(n_p{}^{\ell})^{w_i}\right)^{i} \times \\ \left(1-(n_q{}^{\ell})^{w_i}\right)^{j}\end{matrix}}\right)\right)^{\left(2/_{n^2+n}\right)}, \prod_{q=1,p=1}^{n} \left(1 - \left(\sqrt{\begin{matrix}\left(1-(n_p{}^{u})^{w_i}\right)^{i} \times \\ \left(1-(n_q{}^{u})^{w_i}\right)^{j}\end{matrix}}\right)\right)^{\left(2/_{n^2+n}\right)} \right], \\ \left[1 - \prod_{q=1,p=1}^{n} \left(1 - \sqrt{\left(\begin{matrix}\left(1-(1-r_p{}^{\ell})^{w_i}\right)^{i} \times \\ \left(1-(1-r_q{}^{\ell})^{w_i}\right)^{j}\end{matrix}\right)}\right)^{\left(2/_{n^2+n}\right)}, 1 - \prod_{q=1,p=1}^{n} \left(1 - \sqrt{\left(\begin{matrix}\left(1-(1-r_p{}^{u})^{w_i}\right)^{i} \times \\ \left(1-(1-r_q{}^{u})^{w_i}\right)^{j}\end{matrix}\right)}\right)^{\left(2/_{n^2+n}\right)} \right] \end{matrix} \right). \end{aligned} \tag{13}$$

Proof. Follow the steps of Theorem 4.1.

4.2.1. Properties of CIVIFHM AO

The CIVIFHM AO satisfies the following properties:

a) **Idempotency.** Let $CC_i = (m_{C_i}, n_{C_i}, r_{C_i}); (i = 1,2, \dots, n)$ and $C_i = (m_C, n_C, r_C) = C; \forall i$ represents a CIVIFV. Then;

$$CIVIFHM^{i,j}(C_1, C_2, \dots, C_n) = C.$$

Here, $(m_{C_i} = [m^\ell, m^u], n_{C_i} = [n^\ell, n^u], r_{C_i} = [r^\ell, r^u])$.

b) **Boundedness.** Let a CIVIFV $C_i = (m_{C_i}, n_{C_i}, r_{C_i}); (i = 1,2, \dots, n)$ and

$$C^- = \begin{pmatrix} \min_i({m_{C_i}}^\ell), \min_i({m_{C_i}}^u) \\ \max_i({n_{C_i}}^\ell), \max_i({n_{C_i}}^u) \\ \min_i({r_{C_i}}^\ell), \min_i({r_{C_i}}^u) \end{pmatrix}; C^+ = \begin{pmatrix} \max_i({m_{C_i}}^\ell), \max_i({m_{C_i}}^u) \\ \min_i({n_{C_i}}^\ell), \min_i({n_{C_i}}^u) \\ \max_i({r_{C_i}}^\ell), \max_i({r_{C_i}}^u) \end{pmatrix}; \forall i.$$

Then,

$$C^- \leq CIVIFHM^{i,j}(C_1, C_2, \dots, C_n) \leq C^+.$$

c) **Monotonicity.** Let $C_i = (m_{C_i}, n_{C_i}, r_{C_i})$ and $C_i' = ({m_{C_i}}', {n_{C_i}}', {r_{C_i}}'); (i = 1,2, \dots, n)$ be a CIVIFV.

If $C_i \leq C_i'; \forall i$ means that; ${m_{C_i}}^\ell < {m_{C_i}}'^\ell$, ${m_{C_i}}^u < {m_{C_i}}'^u$; ${n_{C_i}}^\ell > {n_{C_i}}'^\ell, {n_{C_i}}^u > {n_{C_i}}'^u$; and ${r_{C_i}}^\ell < {r_{C_i}}'^\ell, {r_{C_i}}^u < {r_{C_i}}'^u$. Then,

$$CIVIFHM^{i,j}(C_1, C_2, \dots, C_n) \leq CIVIFHM^{i,j}(C_1', C_2', \dots, C_n').$$

4.3. IVIF generalized heronian mean within circular framework (CIVIFGHM) AO

Definition 4.3. Let $C_i (i = 1,2, \dots, n)$ be a CIVIFN and $i, j > 0$. Then,

$$CIVIFGHM^{i,j}(C_1, C_2, \dots, C_n) = \left(\left({}^{2}\!/_{n^2 + n} \right) \sum_{p=1}^{n} \sum_{q=1}^{n} C_p^i C_q^j \right)^{1/_{i+j}}. \tag{14}$$

Definition 4.4. Let $C_i (i = 1,2, \dots, n)$ be a CIVIFV and $w_i (i = 1,2, \dots, n)$ s.t $\sum w_i = 1$ that displays the weight vector which represents the importance of each C_i. Then,

$$CIVIFWGHM^{i,j}(C_1, C_2, \dots, C_n) = \left(\left({}^{2}\!/_{n^2 + n} \right) \sum_{p=1}^{n} \sum_{q=1}^{n} (w_p C_p)^i \times (w_q C_q)^j \right)^{1/_{i+j}}. \tag{15}$$

Theorem 4.3. Let $C_i(i = 1,2,\ldots,n)$ be a CIVIFV. Then, Eq (15) becomes

$$CIVIFGHM^{i,j}(C_1, C_2, \ldots, C_n) = \left(\begin{array}{c} \left[\left(1 - \prod_{q=1,p=1}^{n} \left(1 - \left((m_p{}^{\ell})^{i}(m_q{}^{\ell})^{j}\right)\right)^{\left(2/n^2+n\right)}\right)^{1/i+j}, \left(1 - \prod_{q=1,p=1}^{n} \left(1 - \left((m_p{}^{u})^{i}(m_q{}^{u})^{j}\right)\right)^{\left(2/n^2+n\right)}\right)^{1/i+j} \right], \\ \left[\left(\prod_{q=1,p=1}^{n} \left(1 - \begin{pmatrix} (1-n_p{}^{\ell})^{i} \\ (1-n_q{}^{\ell})^{j} \end{pmatrix}\right)^{\left(2/n^2+n\right)}\right)^{1/i+j}, \left(\prod_{q=1,p=1}^{n} \left(1 - \begin{pmatrix} (1-n_p{}^{u})^{i} \\ (1-n_q{}^{u})^{j} \end{pmatrix}\right)^{\left(2/n^2+n\right)}\right)^{1/i+j} \right], \\ \left[\left(1 - \prod_{q=1,p=1}^{n} \left(1 - \left((r_p{}^{\ell})^{i}(r_q{}^{\ell})^{j}\right)\right)^{\left(2/n^2+n\right)}\right)^{1/i+j}, \left(1 - \prod_{q=1,p=1}^{n} \left(1 - \left((r_p{}^{u})^{i}(r_q{}^{u})^{j}\right)\right)^{\left(2/n^2+n\right)}\right)^{1/i+j} \right] \end{array} \right). \quad (16)$$

Proof. See Appendix B.

Theorem 4.4. Let $C_i(i = 1,2,\ldots,n)$ be CIVIFV and $w_i(i = 1,2,\ldots,n)$ such that $\sum w_i = 1$ displays a weight vector that represents the importance of C_i. Then,

$$CIVIFWGHM^{i,j}(C_1, C_2, \ldots, C_n) = \left(\begin{array}{c} \left[\begin{array}{c} \left(1 - \prod_{q=1,p=1}^{n} \left(1 - \left(1 - (1 - m_p{}^{\ell})^{w_i}\right)^{i}\left(1 - (1 - m_q{}^{\ell})^{w_i}\right)^{j}\right)^{\left(2/n^2+n\right)}\right)^{1/i+j}, \\ \left(1 - \prod_{q=1,p=1}^{n} \left(1 - \left(1 - (1 - m_p{}^{u})^{w_i}\right)^{i}\left(1 - (1 - m_q{}^{u})^{w_i}\right)^{j}\right)^{\left(2/n^2+n\right)}\right)^{1/i+j} \end{array} \right], \\ \left[\begin{array}{c} \left(\prod_{q=1,p=1}^{n} \left(1 - \left(1 - (n_p{}^{\ell})^{w_i}\right)^{i}\left(1 - (n_q{}^{\ell})^{w_i}\right)^{j}\right)^{\left(2/n^2+n\right)}\right)^{1/i+j}, \\ \left(\prod_{q=1,p=1}^{n} \left(1 - \left(1 - (n_p{}^{u})^{w_i}\right)^{i}\left(1 - (n_q{}^{u})^{w_i}\right)^{j}\right)^{\left(2/n^2+n\right)}\right)^{1/i+j} \end{array} \right], \\ \left[\begin{array}{c} \left(1 - \prod_{q=1,p=1}^{n} \left(1 - \left(1 - (1 - r_p{}^{\ell})^{w_i}\right)^{i}\left(1 - (1 - r_q{}^{\ell})^{w_i}\right)^{j}\right)^{\left(2/n^2+n\right)}\right)^{1/i+j}, \\ \left(1 - \prod_{q=1,p=1}^{n} \left(1 - \left(1 - (1 - r_p{}^{u})^{w_i}\right)^{i}\left(1 - (1 - r_q{}^{u})^{w_i}\right)^{j}\right)^{\left(2/n^2+n\right)}\right)^{1/i+j} \end{array} \right] \end{array} \right). \quad (17)$$

Proof. Follow the steps of Theorem 4.3.

4.3.1. Properties of CIVIFG AO

The CIVIFGHM AO satisfies the following properties:

a) **Idempotency.** Let $\mathrm{C}_i = \left(m_{\mathrm{C}_i}, n_{\mathrm{C}_i}, r_{\mathrm{C}_i}\right); (i = 1,2, \dots, \mathrm{n})$ and $\mathrm{C}_i = (m_{\mathrm{C}}, n_{\mathrm{C}}, r_{\mathrm{C}}) = \mathrm{C}; \forall i$ represents a CIVIFV. Then,

$$CIVIFHM^{i,j}(\mathrm{C}_1, \mathrm{C}_2, \dots, \mathrm{C}_{\mathrm{n}}) = \mathrm{C}.$$

Here, $\left(m_{\mathrm{C}_i} = \left[m^{\ell}, m^{u}\right], n_{\mathrm{C}_i} = \left[n^{\ell}, n^{u}\right], r_{\mathrm{C}_i} = \left[r^{\ell}, r^{u}\right]\right)$.

b) **Boundedness.** Let $\mathrm{C}_i = \left(m_{\mathrm{C}_i}, n_{\mathrm{C}_i}, r_{\mathrm{C}_i}\right); (i = 1,2, \dots, \mathrm{n})$ be a CIVIFV and

$$\mathrm{C}^{-} = \begin{pmatrix} \min\limits_i\left(m_{\mathrm{C}_i}{}^{\ell}\right), \min\limits_i\left(m_{\mathrm{C}_i}{}^{u}\right) \\ \max\limits_i\left(n_{\mathrm{C}_i}{}^{\ell}\right), \max\limits_i\left(n_{\mathrm{C}_i}{}^{u}\right), \\ \min\limits_i\left(r_{\mathrm{C}_i}{}^{\ell}\right), \min\limits_i\left(r_{\mathrm{C}_i}{}^{u}\right) \end{pmatrix}; \ \mathrm{C}^{+} = \begin{pmatrix} \max\limits_i\left(m_{\mathrm{C}_i}{}^{\ell}\right), \max\limits_i\left(m_{\mathrm{C}_i}{}^{u}\right), \\ \min\limits_i\left(n_{\mathrm{C}_i}{}^{\ell}\right), \min\limits_i\left(n_{\mathrm{C}_i}{}^{u}\right), \\ \max\limits_i\left(r_{\mathrm{C}_i}{}^{\ell}\right), \max\limits_i\left(r_{\mathrm{C}_i}{}^{u}\right) \end{pmatrix}; \forall i.$$

Then,

$$\mathrm{C}^{-} \leq CIVIFHM^{i,j}(\mathrm{C}_1, \mathrm{C}_2, \dots, \mathrm{C}_{\mathrm{n}}) \leq \mathrm{C}^{+}.$$

c) **Monotonicity.** Let $\mathrm{C}_i = \left(m_{\mathrm{C}_i}, n_{\mathrm{C}_i}, r_{\mathrm{C}_i}\right)$ and $\mathrm{C}_i{}' = \left(m_{\mathrm{C}_i}{}', n_{\mathrm{C}_i}{}', r_{\mathrm{C}_i}{}'\right); (i = 1,2, \dots, \mathrm{n})$ represent a CIVIFV. If $\mathrm{C}_i \leq \mathrm{C}_i{}'; \forall i$ means that $m_{\mathrm{C}_i}{}^{\ell} < m_{\mathrm{C}_i}{}'^{\ell}$, $m_{\mathrm{C}_i}{}^{u} < m_{\mathrm{C}_i}{}'^{u}, ; n_{\mathrm{C}_i}{}^{\ell} > n_{\mathrm{C}_i}{}'^{\ell}, n_{\mathrm{C}_i}{}^{u} > n_{\mathrm{C}_i}{}'^{u}$ and $r_{\mathrm{C}_i}{}^{\ell} < r_{\mathrm{C}_i}{}'^{\ell}, r_{\mathrm{C}_i}{}^{u} < r_{\mathrm{C}_i}{}'^{u}$.

Then,

$$CIVIFHM^{i,j}(\mathrm{C}_1, \mathrm{C}_2, \dots, \mathrm{C}_{\mathrm{n}}) \leq CIVIFHM^{i,j}(\mathrm{C}_1{}', \mathrm{C}_2{}', \dots, \mathrm{C}_{\mathrm{n}}{}').$$

5. Strategic planning in economics

Planning systematically organizes strategies, policies, and resources in economics to achieve goals and envision future economic conditions. It is an effort in a structured way to address societal needs, manage resources effectively, and promote equity, growth, and sustainability. Planning is not only an estimation or a set of calculations regarding the future. It is an active decision-making process that is based on actions with demanded outcomes in a revolutionary economic landscape. The fundamentals of economic planning focus on answering three primary questions: (a) What are the goals? (b) What policies and resources are needed? (c) How to achieve these goals? A combination of data-driven research, strategic vision, and policymaking is required to answer these questions. From the national government crafting fiscal policies to corporate design strategies for market expansion, several levels of economic planning can be carried out. Various strategies, such as projecting future trends by evaluating current conditions, identifying the objectives, and organizing actionable strategies to meet goals, are involved. The inherent complexities of modern economic systems are responsible for the need for planning in economics. The effective allocation of resources (being finite) is required to achieve economic growth while maintaining environmental and social balance.

The planning gives policymakers an advantage in optimizing the resources used, prioritizing initiatives, and mitigating risks. It ensures that informed and deliberate choices guide development so it is not left to chance. For instance, a country may use strategic planning to gain long-term stability, diversify its economy, and lower dependency on volatile sectors. A range of challenges must be overcome in economic planning. The most painful issues are inequality, uncertainty in the world market, degradation in the environment, and limitation of resources. Planners must manage balancing conflicting priorities, such as encouraging rapid industrial growth while reducing unemployment (so inflation does not trigger) and maintaining environmental resources. These challenges are made worse by external factors such as technological disruptions, geopolitical instability, and world economic trends, which can have unpredictable influences on local economies.

Strategic planning in economics takes this process to the next step by introducing a long-term vision, defined objectives, and applicable strategies to achieve economic goals. Traditional planning focuses on instant needs, but unlike this, strategic planning focuses on broad perspectives and future scenarios. It is a decision-making process shaping tomorrow, such as investing in renewable energy resources to concentrate on existing monetary needs and future sustainability challenges. In several regions, poverty and energy availability are inseparable, as a lack of cheap electricity limits economic prospects, healthcare, and education. Strategic planning in renewable energy is critical for tackling environmental issues and strengthening low-income areas. Governments that invest in clean energy infrastructure may create jobs, lower energy costs, and provide long-term solutions that fuel economic growth. This long-term approach guarantees that renewable energy serves as an accelerator for poverty alleviation, providing communities with an opportunity for financial security and improved living conditions. Strategic planning offers a systematic framework for making decisions, enabling industries and policymakers to align with the resources according to priorities while maintaining the adaptation to change. Multiple factors are vital for effective economic planning. These factors include economic indicators such as employment rates, GDP growth, trade balance, and inflation; environmental factors such as conservation of resources and sustainability; and social factors such as healthcare, education, and equity. The absence of one factor can significantly influence the others, as they are deeply interconnected. For example, if a plan oversees environmental sustainability only, it might result in short-term gains in return for long-term damage, undermining economic stability and social well-being. Similarly, disparities could occur due to a lack of focus on social equity, eventually weakening the economic framework and causing unrest in the social factors. Hence, systemic planning is mandatory to ensure sustainable, resilient, and inclusive growth. It assists in reducing the risks associated with inequality, resource scarcity, and external shocks. Combining multiple factors and balancing the objectives and planning could create a blueprint for long-term success.

5.1. Case study: Evaluating a strategic planning problem with multi-factors assessment

A government task is to address a rising unemployment rate in an area suffering from economic stagnation. Although the region has substantial economic potential, it still encounters challenges like insufficient training and education programs, unequal allocation of resources, outdated organization, and environmental concerns. Many regional organizations are working on old traditional methods, making them incompetent in the modern world. Meanwhile, workers do not have the skills required for the new job criteria. Thus, it is harder for people to find work due to a mismatch between skills and available jobs. Another vital issue caused by inefficient organizations is environmental damage. The pollution created by these industries harms nature and the area's health, making it difficult for investors to begin new businesses. In the future, this could worsen the economic problems, limiting opportunities. In addition, basic needs and resources such as healthcare, education, and infrastructure are unequally distributed. For instance, rural areas are often less developed, leaving them behind while urban areas get more funding. This unequal distribution makes it difficult for everyone to grow, leading to frustration. As a result, the gap between rich and poor gets wider and wider. To handle all these challenges, the government must have a systematic and solid plan to create jobs and grow the economy while ensuring everyone gets an equal chance to participate.

So, policymakers are hired by the government, whose tasks are to identify outdated organizations, conduct training programs, build education sectors, and encourage greener technologies to create jobs so that they stimulate the growth of the economy while ensuring societal equity and environmental sustainability. The following criteria [28] are considered to evaluate this problem: ($Ł_1$) growth of the economy, ($Ł_2$) creation of new jobs, ($Ł_3$) sustainability of the environment, and ($Ł_4$) ensuring a balanced assessment that addresses long-term goals by viewing challenges. Four strategies s_i are identified by policymakers based on these criteria: modernization of industries (s_1), renewable energy development (s_2), workforce training and education (s_3), and expansion of infrastructure (s_4). Each strategy is tested for its capacity to meet the goals and objectives of the criteria and ensure that the selected solution aligns with the area's priorities. With a systematic and balanced approach, the region can convert its challenges into opportunities and build its prosperous and sustainable future.

Here is a description of the criteria and alternatives.

Criteria for Evaluation: To assess potential strategies, four key criteria are identified:

Criteria	Interpretation
Economic Growth potential	The capacity of a strategy to attract investments and increase GDP.
Creation of Job	The ability of the strategy to create sustainable opportunities for employment.
Sustainability of Environment	The extent to which the strategy enhances green practices and reduces environmental damage.
Social Equity	The ability of a strategy to encourage equitable resource distribution and reduce disparity.

Alternatives: Four potential strategies are established:

Alternative (strategies)	Interpretation
Modernization of Industries	Updating the existing organization with advanced technologies to enhance competitiveness and productivity.
Renewable Energy Development	Investing in wind, solar, and other green energy projects to promote sustainability and create jobs.
Education and Workforce Training	Escalating educational enterprises and skill development programs to meet the market demands.
Development of infrastructure	Constructing the networks of transportation and public utilities to enhance connectivity and attract investments.

5.2. Systematic problem-solving strategy

This section evaluates the decision-making problem using the CIVIF framework by considering criteria and alternatives from policymakers' opinions. The weight values of the criteria are determined by using the MEREC approach. So, to address this problem systematically, the problem-solving process involved a series of steps, which have been structured as below:

a) Formation of a decision matrix such that each element reflects the performance value of the alternative concerning a specific criterion.

$$(\mathcal{D})_{\mathrm{m}\times\mathrm{n}} = \begin{matrix} s_1 \\ s_2 \\ \vdots \\ s_{\mathrm{m}} \end{matrix} \begin{bmatrix} Ł_1 & Ł_2 & \cdots & Ł_{\mathrm{n}} \\ \mathrm{C}_{11} & \mathrm{C}_{12} & \cdots & \mathrm{C}_{1\mathrm{n}} \\ \mathrm{C}_{21} & \mathrm{C}_{22} & \cdots & \mathrm{C}_{2\mathrm{n}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{C}_{\mathrm{m}1} & \mathrm{C}_{\mathrm{m}2} & \cdots & \mathrm{C}_{\mathrm{mn}} \end{bmatrix}.$$

b) Normalization of each criterion to ensure consistency and compatibility across each attributive value. The normalized matrix is obtained by

$$(N_{ij})_{\mathrm{m}\times\mathrm{n}} = \begin{cases} \dfrac{\mathrm{C}_{\mathrm{mn}}}{\max(\mathrm{C}_{\mathrm{mn}})}; cost \\ \dfrac{\min(\mathrm{C}_{\mathrm{mn}})}{\mathrm{C}_{\mathrm{mn}}}; benefit \end{cases}.$$

c) Defuzzifed the information defined in the CIVIF framework by utilizing the score function, which gives the information of each criterion.

$$S_{\mathrm{C}} = \frac{\left(\left(m^{\ell} + m^{u} - n^{\ell} - n^{u}\right) \times \left(r^{\ell} + r^{u}\right)\right)}{2}. \tag{18}$$

d) Calculate the alternative's overall performance $\mathcal{O}_i$, representing its effectiveness across all criteria, using the MEREC approach.

$$\mathcal{O}_i = \frac{1}{\mathrm{n}} \sum_{j} \ln\left(1 - N_{ij}\right). \tag{19}$$

e) To evaluate the impact of each attributive value, the performance of each alternative $\mathcal{O}'_{ij}$ is recalculated by excluding the contribution of each criterion one by one.

$$\mathcal{O}'_{ij} = \frac{1}{\mathrm{n}} \sum_{k \neq j} \ln(1 - N_{ik}). \tag{20}$$

f) Now calculate the deviation degree between the overall performance and the recalculated performance to measure the influence of each criterion.

$$\delta_j = \sum_{i} \left|\mathcal{O}'_{ij} - \mathcal{O}_i\right|.$$

g) The weight values are determined by summarizing the deviation degree, which ensures that each attributive value is prioritized appropriately according to its relative importance.

$$w_j = \frac{\delta_j}{\sum_k \delta_k}.$$

h) Now, apply the proposed operators on the decision matrix to aggregate each alternative across the attributive values. The proposed operators are defined in (Eqs 12, 13, 16, and 17).

i) Evaluate the performance of each alternative by computing the scored value (Eq 18) and rank them accordingly.

$$S_{\mathrm{C}} = \frac{\left(\left(m^{\ell} + m^{u} - n^{\ell} - n^{u}\right) \times \left(r^{\ell} + r^{u}\right)\right)}{2}. \tag{21}$$

By leveraging a structured decision-making approach, the proposed framework systematically balances economic growth, sustainability, and risk mitigation. Unlike conventional models, it dynamically integrates economic indicators such as inflation rates, market trends, and investment risks by adaptive weighting mechanisms and data-driven evaluation. This ensures that strategic planning remains resilient, responsive to shifting economic conditions, and aligned with long-term policy goals. The algorithm of the proposed methodology is as follows:

	Algorithm
Input	Decision matrix (comprises of the criteria and the alternatives).
Required Output	Rank of alternatives and optimal selection of alternatives.
Requirement	The preference value (weight) of each criterion, Normalization Approach, and Aggregation Operators that aggregate the given input.
Problem-Solving Steps	
	Formation of decision matrix which reflects the performance value.
	Normalize the decision matrix (by the defined formula).
	Defuzzifed the given information, which resulted in the collective information of each criterion.
	Utilized the MEREC approach to compute the weight values.
	Apply the proposed operators on the decision matrix to aggregate each alternative.
	Rank alternatives based on aggregated scores.
	Select the optimal alternative.

The defined algorithm employs a systematic decision-making procedure to deal with multidimensional economic planning challenges efficiently. To ensure that each criterion receives the proper priority, input parameters such as economic, social, and risk-related parameters are first processed by evaluating the weights using the MEREC technique. The model uses the CIVIFHM and MEREC approach, which assesses the trade-offs between competing criteria and offers a balance between conflicting objectives. The algorithm facilitates adaptive optimization under uncertainty by continuously refining solutions dynamically adjusting weight distributions through sensitivity analysis. This iterative process enhances decision-making resilience, allowing the framework to adapt to real-time economic fluctuations and policy shifts. It ensures that the final strategy proposals are both practically and mathematically in line with actual policymaking restrictions.

5.3. Numerical evaluation

To validate the proposed framework, here we utilize a hypothetical dataset that closely mirrors real-world economic conditions, including key factors such as unemployment rates, resource distribution, industrial modernization, social equity, and environmental sustainability. The use of simulated data allows for a controlled assessment of the model's decision-making capabilities, ensuring that its adaptability and robustness can be tested across diverse economic scenarios. While the dataset is not derived from real-world case studies, it is designed to reflect realistic policy challenges and trade-offs faced by decision-makers. This approach enables a systematic evaluation of how the algorithm processes multi-factor dependencies and resolves conflicting objectives. Moreover, using a hypothetical dataset eliminates biases and external inconsistencies, allowing for a more focused assessment of the model's effectiveness.

For the numerical evaluation of the problem, the above steps defined in the algorithm are followed as:

a) To access the performance value of alternatives concerning a specific criterion, the policy-makers define the linguistic assessment scale (Table 2), which is then utilized to form a decision matrix.

Table 2. Assessment scale.

Assessment scale	CIVIFVs					
	m^{ℓ}	m^{u}	n^{ℓ}	n^{u}	r^{ℓ}	r^{u}
Exceptional Contribution (EC)	0.220	0.270	0.150	0.220	0.186	0.245
Significant Potential (SP)	0.200	0.250	0.170	0.235	0.187	0.243
Satisfactory Impact (SI)	0.180	0.230	0.190	0.236	0.188	0.233
Limited Impact (LI)	0.160	0.210	0.210	0.237	0.189	0.224
Balanced Impact (BI)	0.140	0.190	0.230	0.238	0.189	0.214

Table 3 shows the decision matrix formed from policymakers' opinions based on the assessment scale.

Table 3. Opinion of policymakers based on assessment scale.

	Ł$_1$	Ł$_2$	Ł$_3$	Ł$_4$
s_1	EC	BI	LI	SI
s_2	SP	LI	EC	SP
s_3	LI	SP	SI	BI
s_4	BI	SI	BI	EC

Table 4 displays the values corresponding to the assessment scale.

Table 4. Opinion of policymakers based on assessment scale values.

	Ł$_1$						Ł$_2$					
	m^ℓ	m^u	n^ℓ	n^u	r^ℓ	r^u	m^ℓ	m^u	n^ℓ	n^u	r^ℓ	r^u
S_1	0.220	0.270	0.150	0.220	0.186	0.245	0.140	0.190	0.230	0.238	0.189	0.214
S_2	0.200	0.250	0.170	0.235	0.187	0.243	0.160	0.210	0.210	0.237	0.189	0.224
S_3	0.160	0.210	0.210	0.237	0.189	0.224	0.200	0.250	0.170	0.235	0.187	0.243
S_4	0.140	0.190	0.230	0.238	0.189	0.214	0.180	0.230	0.190	0.236	0.188	0.233
	Ł$_3$						Ł$_4$					
	m^ℓ	m^u	n^ℓ	n^u	r^ℓ	r^u	m^ℓ	m^u	n^ℓ	n^u	r^ℓ	r^u
S_1	0.160	0.210	0.210	0.237	0.189	0.224	0.180	0.230	0.190	0.236	0.188	0.233
S_2	0.220	0.270	0.150	0.220	0.186	0.245	0.200	0.250	0.170	0.235	0.187	0.243
S_3	0.180	0.230	0.190	0.236	0.188	0.233	0.140	0.190	0.230	0.238	0.189	0.214
S_4	0.140	0.190	0.230	0.238	0.189	0.214	0.220	0.270	0.150	0.220	0.186	0.245

b) To ensure consistency and compatibility across each attributive value, each criterion is normalized (by the above step b) and displayed in Table 5.

Table 5. Normalization of decision matrix.

	Ł$_1$						Ł$_2$					
	m^ℓ	m^u	n^ℓ	n^u	r^ℓ	r^u	m^ℓ	m^u	n^ℓ	n^u	r^ℓ	r^u
S_1	1.000	1.00	0.652	0.924	0.984	1.000	0.70	0.760	1.000	1.000	1.000	0.882
S_2	0.909	0.925	0.739	0.987	0.989	0.989	0.800	0.840	0.913	0.995	1.000	0.921
S_3	0.727	0.777	0.913	0.995	1.000	0.912	1.000	1.000	0.739	0.987	0.989	1.000
S_4	0.636	0.7037	1.000	1.000	1.000	0.873	0.900	0.920	0.826	0.991	0.994	0.960
	Ł$_3$						Ł$_4$					
	m^ℓ	m^u	n^ℓ	n^u	r^ℓ	r^u	m^ℓ	m^u	n^ℓ	n^u	r^ℓ	r^u
S_1	0.727	0.777	0.913	0.995	1.000	0.912	0.818	0.851	0.826	0.991	0.994	0.951
S_2	1.000	1.000	0.652	0.924	0.984	1.000	0.909	0.925	0.739	0.987	0.989	0.989
S_3	0.818	0.851	0.826	0.991	0.994	0.951	0.636	0.703	1.000	1.000	1.000	0.873
S_4	0.636	0.703	1.000	1.000	1.000	0.873	1.000	1.000	0.652	0.924	0.984	1.000

c) Defuzzifed the information defined in the CIVIF framework by utilizing the score function (Eq 18), which gives the information of each criterion, as displayed in Table 6.

Table 6. Defuzzification of CIVIF information.

	Ł$_1$	Ł$_2$	Ł$_3$	Ł$_4$
s_1	0.42010	-0.50827	-0.38607	-0.14364
s_2	0.10736	-0.25831	0.42010	0.10736
s_3	-0.38607	0.27203	-0.14364	-0.61818
s_4	-0.61818	0.00226	-0.61818	0.42010

d) Table 7 displays the alternative's overall performance (Eq 19), representing its effectiveness across all criteria.

Table 7. Overall performance of alternative.

	s_1	s_2	s_3	s_4
$\mathcal{O}_i$	0.0816	−0.1356	0.15613	0.10386

e) To evaluate the impact of each attributive value, the performance of each alternative is recalculated by excluding the contribution of each criterion one by one (Eq 20), as shown in Table 8.

Table 8. Impact of each attributive value.

$\mathcal{O}'_{ij}$	Ł$_1$	Ł$_2$	Ł$_3$	Ł$_4$
s_1	0.217914	−0.021049	0.000072	0.048136
s_2	−0.107175	−0.193011	0.000654	−0.107175
s_3	0.074508	0.235499	0.122572	0.035801
s_4	−0.016464	0.104429	−0.016464	0.240085

f) To measure the influence of each criterion, the deviation degree (by following the above defined steps) between the overall performance and the recalculated performance has been checked and displayed in Table 9.

Table 9. Deviation degree.

	Ł$_1$	Ł$_2$	Ł$_3$	Ł$_4$
δ_j	0.3665	0.2401	0.3717	0.3185

g) The weight values are determined, ensuring that each criterion is prioritized appropriately according to its relative importance by summing up the deviation degree in Table 10.

Table 10. Weight values of criterion.

	Ł$_1$	Ł$_2$	Ł$_3$	Ł$_4$
w_j	0.2826	0.1851	0.2866	0.2455

Figure 2 displays the pictorial representation of each strategy's deviation degree and weight values.

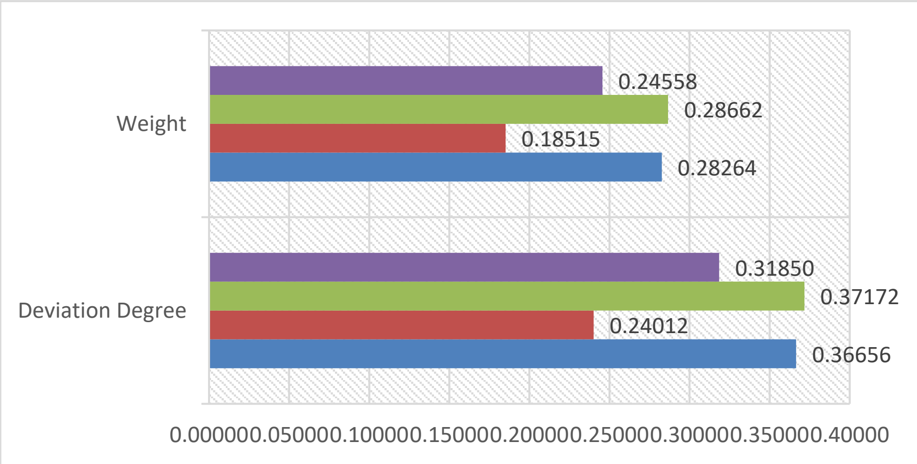

Figure 2. Deviation degree and weight values.

h) To aggregate each alternative across the attributive values, the proposed operators (Eqs 12, 13, 16, and 17) are applied to the decision matrix, and their results values are shown in Table 11.

Table 11. Aggregation of each alternative.

	CIVIFHM						CIVIFWHM					
	m^ℓ	m^u	n^ℓ	n^u	r^ℓ	r^u	m^ℓ	m^u	n^ℓ	n^u	r^ℓ	r^u
S_1	0.2636	0.3340	0.0726	0.0970	0.2834	0.3279	0.0714	0.0933	0.5326	0.5547	0.0758	0.0944
S_2	0.2926	0.3617	0.0612	0.0964	0.2823	0.3324	0.0799	0.1021	0.5119	0.5351	0.0754	0.0986
S_3	0.2569	0.3275	0.0759	0.0996	0.2837	0.3234	0.0665	0.0883	0.5442	0.5643	0.0760	0.0928
S_4	0.2560	0.3268	0.0755	0.0972	0.2834	0.3279	0.0676	0.0894	0.5404	0.5514	0.0758	0.0925

	CIVIFGHM						CIVIFWGHM					
	m^ℓ	m^u	n^ℓ	n^u	r^ℓ	r^u	m^ℓ	m^u	n^ℓ	n^u	r^ℓ	r^u
S_1	0.2204	0.2826	0.4321	0.4912	0.2365	0.2872	0.000001	0.000008	0.677853	0.731684	0.000169	0.000001
S_2	0.2453	0.3072	0.4003	0.4898	0.2356	0.2989	0.000208	0.000001	0.650538	0.730913	0.000166	0.000208
S_3	0.2142	0.2763	0.4407	0.4968	0.2368	0.2865	0.000102	0.000001	0.692677	0.736702	0.000170	0.000102
S_4	0.2142	0.2764	0.4396	0.4916	0.2365	0.2843	0.000114	0.000010	0.686617	0.731867	0.000169	0.000114

i) The performance of each alternative is computed by the scored value (Eq 18) and their ranking is obtained accordingly, as shown in Table 12.

Table 12. Score and ranking of each alternative.

S_C	CIVIFHM		CIVIFWHM		CIVIFGHM		CIVIFWGHM	
	S_C	Rank	S_C	Rank	S_C	Rank	S_C	Rank
S_1	0.1308	2	−0.0785	2	−0.1101	2	−0.000041	3
S_2	0.1527	1	−0.0753	1	−0.0902	1	−0.000045	4
S_3	0.1242	4	−0.0805	4	−0.1170	4	−0.000040	2
S_4	0.1254	3	−0.0787	3	−0.1147	3	−0.000039	1

The graphical representation of score values of alternatives by utilizing the proposed AOs is shown in Figure 3.

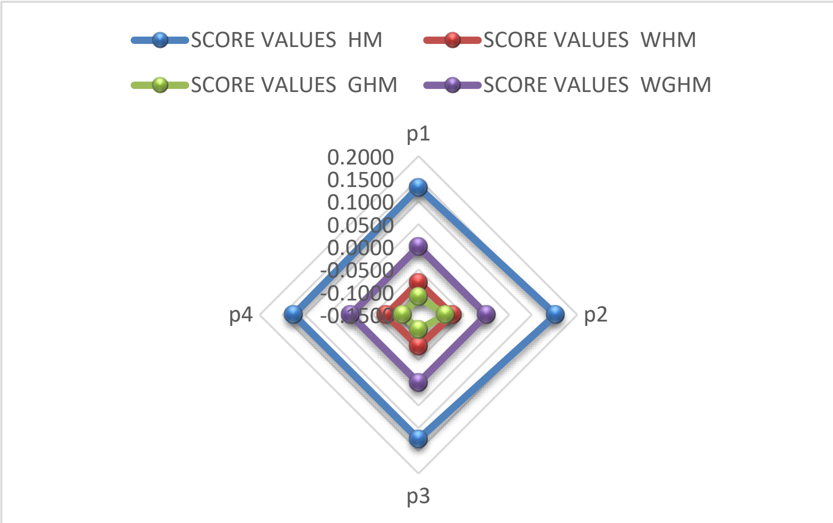

Figure 3. Score values by Aos.

5.4. Result discussion and comparison analysis

In Figure 3, evaluating strategic planning in economics by utilizing the different AOs including the HM, WHM, GHM, and WGHM, within the circular interval-valued framework provides insights into each strategy. The strategies (alternatives) were assessed based on their score values and ranking outcomes. The above figure highlights the score and ranking outcomes of defined AOs. By CIVIFHM, the S_2 strategy is the most optimal for strategic planning, then $S_1, S_4,$and S_3 renewable energy emerged as the most impactful alternative due to its significant contribution to multiple criteria, such as environmental impact and job creation, which aligns with the previous studies in sustainable economic planning [29,30]. However, unlike traditional models focusing solely on environmental and financial benefits, the proposed approach integrates a circular interval framework to capture dynamic interdependencies, making the decision-making process more adaptable. Similarly, by CIVIFWHM, the S_2 strategy is the most optimal for strategic planning followed by $S_1, S_4,$and S_3. The findings highlighted the dominance of renewable energy in strategic economic goals, despite the weighted modification that, as a result, slightly varied performance value. By generalized AO, the alternatives displayed distinct impacts, particularly by favoring the renewable energy, i.e., S_2, and highlighting the challenges associated with S_3, i.e., education and training in this context. The emphasis on education and training S_3 in the CIVIFWGHM outcome supports the findings [31], which highlight that skill development is crucial in maximizing the economic benefits of renewable energy projects. By the CIVIFWGHM, the most refined evaluation has been displayed with the ranking values of $S_3 > S_4 > S_1 > S_2$, highlighting the impacts of education and training by considering a balanced weight structure. The analysis suggests that a hybrid approach combining education initiatives with investments in renewable energy would yield the most balanced and sustainable outcomes. So, the above discussion and result outcome highlighted that different operators yield varying optimal alternatives. This demonstrates that change in weight values or operator selection can affect the decisions and allow the experts to customize the approach based on their preference, making it more flexible and versatile.

To validate the superiority of the proposed framework, a comparative analysis (Table 13) has been conducted between the CIVIF-based models and existing methods which show a detailed comparison of prior defined approaches, including interval-valued intuitionistic fuzzy sets (IVIFS) [12], Heronian mean-based models such as interval-valued intuitionistic fuzzy Heronian mean (IVIFHM) [32], interval-valued intuitionistic fuzzy generalized Heronian mean (IVIFGHM) [32], and circular intuitionistic fuzzy sets (CIFS) [18] framework, which reveals that these approaches address basic imprecision and are unable to incorporate the radius between membership and non-membership functions or account for interrelationships between criteria except the CIFS. However, in this scenario, the policymakers' values are a circular interval framework to make it more comprehensive and accommodate the imprecisions and uncertainty.

Table 13. Comparison analysis.

Framework	Score value	Ranking outcomes	Optimal strategy
IVIFS [12]	-	*failed*	−
IVIFHM [32]	-	*failed*	−
IVIFGHM [32]	-	*failed*	−
CIFS [18]	-	*failed*	−
CIVIFS [25]	$S_1 = -0.0094, S_2 = -0.007, S_3 = -0.0019, S_4 = 0.0140$	$S_C(S_3) > S_C(S_1) > S_C(S_2) > S_C(S_4)$	S_3
CIVIFHM	$S_1 = 0.13078, S_2 = 0.15266, S_3 = 0.12416, S_4 = 0.12535$	$S_C(S_2) > S_C(S_1) > S_C(S_4) > S_C(S_3)$	S_2
CIVIFWHM	$S_1 = -0.07839, S_2 = -0.07514, S_3 = -0.08013, S_4 = -0.07857$	$S_C(S_2) > S_C(S_1) > S_C(S_4) > S_C(S_3)$	S_2
CIVIFGHM	$S_1 = -0.11007, S_2 = -0.09023, S_3 = -0.11697, S_4 = -0.11473$	$S_C(S_2) > S_C(S_1) > S_C(S_4) > S_C(S_3)$	S_2
CIVIFWGHM	$S_1 = -0.0000402, S_2 = -0.0000446, S_3 = -0.0000403, S_4 = -0.00003921$	$S_C(S_3) > S_C(S_4) > S_C(S_2) > S_C(S_1)$	S_3

The comparison demonstrates that variation in the outcomes of the results occurs due to the inherent mathematical properties of each operator. The variations in score values between the proposed models and existing methods such as IVIFS and IVIFHM depict how each model handles uncertainty and interdependencies. Traditional approaches tend to treat criteria as independent variables, whereas the defined framework accounts for circular interval-valued relationships, leading to more stable and realistic decision outcomes. The proposed approach considered the relationships between each criterion and emphasized proportionality and interdependence more effectively. Moreover, the weight values affect the score values and then the ranking outcomes, which shows that their sensitivity corresponds to each criterion. This adaptability to varying the opinion of expertise, weight values, and preference models underscores the reliability and flexibility of the proposed framework in decision-making scenarios. So, this framework integrates the policymaking processes and provides a decision-support tool that helps policymakers navigate complex economic decisions. By offering clear, data-driven insights, it cuts through complexity. It offers precision, reduces bias, and ensures that decisions are reactive and strategically aligned with changing economic and social realities.

The algorithm can be embedded into government decision-making systems, assisting policymakers in evaluating economic policies, allocating resources efficiently, and forecasting socio-economic impacts. In politically stable environments, it enhances long-term strategic planning, while in volatile economies, it aids in rapid response strategies by adjusting the real-time data within the specific framework. By providing structured, objective insights, the framework helps governments balance economic growth, social equity, and environmental sustainability.

6. Conclusions

This study utilized an IVIF within the circular framework and the MEREC approach to evaluate the weighting criteria values and report on the crucial problem of creating an effective economic strategy. The framework allowed a detailed assessment of various strategies by capturing the interdependencies inherent in economic planning, imprecision, and uncertainties. The MEREC application certified that the weighting of criteria, including creating jobs, social equity, economic growth, and environmental sustainability, was vigorous and highlighted the area's policy goals and priorities. These outcomes confirm that the method is highly suitable for handling complex, multi-criteria decision-making scenarios, particularly in economic and policy-driven applications. The analysis provides a holistic assessment of alternatives by combining these weighted criteria within the CIVIF framework and balancing long-term objectives with short-term needs. The result shows that the framework effectively finds strategies to address economic stagnation, unemployment, equity, and sustainability. Moreover, the findings highlight the dominance of renewable energy strategies in economic planning, as they consistently ranked highest across multiple AOs. However, combining workforce training and education with renewable energy development, the hybrid approach is recommended, demonstrating how decision-making tools can assist policymakers toward actionable and balanced solutions. This work highlights the importance of advanced evaluation frameworks in economic strategy planning to prioritize strategic objectives, navigate multifactorial challenges, and offer policymakers actionable insights by underscoring a flexible methodology that can be addressed to multiple economic contexts. Therefore, this study contributes to economic strategy planning by viewing how the defined framework is successfully applied to real-world problems. The proposed framework is highly suitable for strategic economic planning as it effectively captures interdependencies, handles uncertainty, and provides a flexible, data-driven approach to decision-making. By offering an adaptable, comprehensive, and structured evaluation process, policymakers are equipped with a potent tool for crafting strategies that highlight ongoing economic needs while safeguarding future sustainability and inclusivity.

6.1. Challenges in economic strategic planning

The challenges faced during the economic strategic planning are as follows:

- ***a)*** **Complexity and interdependencies.** Multiple interconnected factors significantly influence the economic system, making it difficult to predict future outcomes correctly. For example, promoting industrial growth may boost GDP but worsen environmental degradation.
- ***b)*** **Conflicting objectives.** Policymakers often struggle with trade-offs among competing priorities, such as ensuring environmental sustainability versus achieving fast development or fostering economic growth versus minimizing inequality.
- ***c)*** **Limited resources.** Systematic and strategic planning required a balanced, limited financial, human, and natural resources while maximizing impact across diverse sectors.
- ***d)*** **Uncertainty and risk.** Uncertainty in external shocks such as natural disasters, trade wars, market trends, and political stability complicates long-term planning.

6.2. Advantages

- ***a)*** The model provides more precise decision-making outcomes by effectively handling uncertainty and interdependencies between criteria and reducing inconsistencies and biases in rankings.
- ***b)*** The proposed framework dynamically adjusts to data inputs, allowing policymakers to make informed decisions even in rapidly evolving economic environments.
- ***c)*** By integrating expert-driven weight adjustments and multi-factor analysis, the model reduces subjective biases and ensures a more interpretable and structured decision-making process.

6.3. Limitations and future direction

While the proposed integration of the MEREC approach with the CIVIF framework highlights the significant potential in tailoring complex economic strategy planning, certain limits should be acknowledged. First, the framework dramatically depends on the completeness and accuracy of input data, for instance, social parameters, economic indicators, and environmental matrices. Outdated or inaccurate data may include the reliability of the results. Moreover, there is a chance of subjectivity, as the framework relies on expert judgment for assigning CIF values, which could influence the consistency of the assessment process across diverse contexts. Second, the static nature of analysis is another limitation. The framework demonstrates a snapshot of the decision-making landscape. Still, it does not account for the dynamic changes in geopolitical changes, economic conditions, and technological advancements that could significantly impact the effectiveness and feasibility of strategies over time. In real-world applications, the algorithm may struggle with scalability when handling extremely large and complex datasets, potentially increasing computational demands and processing time.

In the future, the study should address these limitations by integrating machine learning (ML) techniques and real-time data analytics to update the inputs dynamically and exploring alternative weight-determination methods in response to rapidly changing conditions. Adding a participatory approach (like incorporating a wider range of stakeholders) can assist in reducing subjectivity and make the evaluation process more precise, inclusive, and reflective. Furthermore, adding scenario analysis or probabilistic modeling to the framework can improve its capacity to consider uncertainty and evaluate the long-term effects of selected tactics. Future research could also aim to apply the framework in other regions and sectors to assess its generalization and adaptability. These advancements would strengthen the robustness and utilization of this approach, making it a strong and valuable tool for multifaceted and complex decision-making in economic planning and many more.

Moreover, the proposed model can be extended into the neutrosophic framework [24], soft set framework [33], hyper soft framework [34], complex framework [35] and can be analyzed by the WASPAS method [36], hybrid method [37], DEMATEL method [38], ORESTE method [39], fuzzy neural network [40], EDAS method [41], and metric spaces [42].

Use of Generative-AI tools declaration

The authors acknowledge that no generative AI tools were used in preparing this manuscript. However, the grammarly tool was utilized primarily for language refinement, grammar correction, and improving the clarity of explanations in the literature and case study sections.

Conflict of interest

The author declares no conflicts of interest.

References

1. G. A. Steiner, *Strategic planning*, New York: Free Press, 1979.
2. A. R. Arabeyyat, R. Aldweik, A comprehensive exploration of the jordan tourism board's 2023 strategic planning, news, and social media, *Emir. J. Bus. Econ. Soc. Stud.*, **3** (2024), 32–41. https://doi.org/10.54878/79sfh467
3. A. A. Akinsulire, C. Idemudia, A. C. Okwandu, O. Iwuanyanwu, Strategic planning and investment analysis for affordable housing: Enhancing viability and growth, *Magna Sci. Adv. Res. Rev.*, **11** (2024), 119–131. https://doi.org/10.30574/msarr.2024.11.2.0114
4. R. Kemp, *Strategic planning in local government: A casebook*, New York: Routledge, 1992. https://doi.org/10.4324/9781003571957
5. E. Raji, T. I. Ijomah, O. G. Eyieyien, Integrating technology, market strategies, and strategic management in agricultural economics for enhanced productivity, *Int. J. Manag. Entrep. Res.*, **6** (2024), 2112–2124. https://doi.org/10.51594/ijmer.v6i7.1260
6. D. Frei, D. Ruloff, *Handbook of foreign policy analysis: Methods for practical application in foreign policy planning, strategic planning and business risk assessment*, Martinus Nijhoff, 1989.
7. M. J. Rahayu, A. H. Juwita, S. Bintariningtyas, E. F. Rini, T. Wahyuni., The effectiveness of the regional long-term development plan of Purworejo Regency: The evaluation of strategic planning, *Plan. Malays.*, **22** (2024). https://doi.org/10.21837/pm.v22i30.1436
8. G. Zakhidov, Economic indicators: Tools for analyzing market trends and predicting future performance, *Int. Multidiscip. J. Univ. Sci. Prospect.*, **2** (2024), 23–29.
9. H. Kalliomäki, P. Oinas, T. Salo, Innovation districts as strategic urban projects: The emergence of strategic spatial planning for urban innovation, *Eur. Plan. Stud.*, **32** (2024), 78–96. https://doi.org/10.1080/09654313.2023.2216727
10. A. Bazina, Strategic planning and its impact on persistent profitability and productivity in African agriculture sector, *Multi-Knowl. Electron. Compr. J. Educ. Sci. Publ.*, 2021.
11. L. A. Zadeh, Fuzzy sets, *Inf. Control*, **8** (1965), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X
12. K. T. Atanassov, *Interval-valued intuitionistic fuzzy sets*, Cham: Springer, 2020. https://doi.org/10.1007/978-3-030-32090-4

13. R. R. Yager, Pythagorean fuzzy subsets, In: *2013 Joint IFSA world congress and NAFIPS annual meeting (IFSA/NAFIPS)*, 2013, 57–61. https://doi.org/10.1109/IFSA-NAFIPS.2013.6608375
14. R. R. Yager, Generalized orthopair fuzzy sets, *IEEE Trans. Fuzzy Syst.*, **25** (2017), 1222–1230. https://doi.org/10.1109/TFUZZ.2016.2604005
15. R. Imran, K. Ullah, Z. Ali, M. Akram, A multi-criteria group decision-making approach for robot selection using interval-valued intuitionistic fuzzy information and Aczel-Alsina Bonferroni means, *Spectr. Decis. Mak. Appl.*, **1** (2024), 1–32. https://doi.org/10.31181/sdmap1120241
16. S. Biswas, A. Sanyal, D. Pamucar, Students' perceptions about the webinars: An intuitionistic fuzzy force field analysis, *Spec. Oper. Res.*, **2** (2025), 113–133.
17. S. Kousar, A. Ansar, N. Kausar, G. Freen, Multi-criteria decision-making for smog mitigation: A comprehensive analysis of health, economic, and ecological impacts, *Spectr. Decis. Mak. Appl.*, **2** (2025), 53–67. https://doi.org/10.31181/sdmap2120258
18. K. T. Atanassov, Circular intuitionistic fuzzy sets, *J. Intell. Fuzzy Syst.*, **39** (2020), 5981–5986. https://doi.org/10.3233/JIFS-189072
19. S. Varelas, G. Tsoupros, Key performance indicators and data envelopment analysis in Greek tourism: A strategic planning tool for destinations and DMMOs, *Sustainability*, **16** (2024), 3453. https://doi.org/10.3390/su16083453
20. M. S. Alam, J. B. Deb, A. A. Amin, S. Chowdhury, An artificial neural network for predicting air traffic demand based on socio-economic parameters, *Decis. Anal. J.*, **10** (2024), 100382. https://doi.org/10.1016/j.dajour.2023.100382
21. L. Vandersmissen, B. George, J. Voets, Strategic planning and performance perceptions of managers and citizens: Analysing multiple mediations, *Public Manag. Rev.*, **26** (2024), 514–538. https://doi.org/10.1080/14719037.2022.2103172
22. H. E. Adama, O. A. Popoola, C. D. Okeke, A. E. Akinoso, Economic theory and practical impacts of digital transformation in supply chain optimization, *Int. J. Adv. Econ.*, **6** (2024), 95–107. https://doi.org/10.51594/ijae.v6i4.1072
23. M. Tiits, E. Karo, T. Kalvet, Small countries facing the technological revolution: Fostering synergies between economic complexity and foresight research, *Compet. Rev.*, **35** (2025), 53–75. https://doi.org/10.1108/CR-03-2023-0051
24. R. Imran, K. Ullah, Z. Ali, M. Akram, An approach to multi-attribute decision-making based on single-valued neutrosophic hesitant fuzzy aczel-alsina aggregation operator, *Neutrosophic Syst. Appl.*, **22** (2024), 7. https://doi.org/10.61356/j.nswa.2024.22387
25. E. Boltürk, C. Kahraman, Interval-valued and circular intuitionistic fuzzy present worth analyses, *Informatica*, **33** (2022), 693–711. https://doi.org/10.15388/22-INFOR478
26. A. Hussain, K. Ullah, H. Garg, T. Mahmood, A novel multi-attribute decision-making approach based on T-spherical fuzzy Aczel Alsina Heronian mean operators, *Granul. Comput.*, **9** (2024), 21. https://doi.org/10.1007/s41066-023-00442-6
27. M. Keshavarz-Ghorabaee, M. Amiri, E. K. Zavadskas, Z. Turskis, J. Antucheviciene, Determination of objective weights using a new method based on the removal effects of criteria (MEREC), *Symmetry*, **13** (2021), 525. https://doi.org/10.3390/sym13040525
28. M. A. Ali, M. Kamraju, D. B. Sonaji, Economic policies for sustainable development: Balancing growth, social equity, and environmental protection, *ASEAN J. Econ. Econ. Educ.*, **3** (2024), 23–28.

29. B. Ma, A. Wang, Exploring the role of renewable energy in green job creation and sustainable economic development: An empirical approach, *Energy Strategy Rev.*, **58** (2025), 101642. https://doi.org/10.1016/j.esr.2025.101642
30. S. A. Khan, A. Q. Patoli, H. Ahmed, I. Ahmed, Innovations in green technologies: analyzing their contribution to job creation and sustainable economic transitions, *Rev. Appl. Manag. Soc. Sci.*, **8** (2025), 263–277. https://doi.org/10.47067/ramss.v8i1.456
31. R. Wang, M. Qamruzzaman, S. Karim, Unveiling the power of education, political stability and ICT in shaping technological innovation in BRI nations, *Heliyon*, **10** (2024), e30142. https://doi.org/10.1016/j.heliyon.2024.e30142
32. W. Wang, Y. Feng, Group decision making based on generalized intuitionistic fuzzy yager weighted heronian mean aggregation operator, *Int. J. Fuzzy Syst.*, **26** (2024), 1364–1382. https://doi.org/10.1007/s40815-023-01672-1
33. M. Saqlain, R. Imran, S. Hassan, TOPSIS technique of MCDM under cubic intuitionistic fuzzy soft set environment, *Sci. Inquiry Rev.*, **7** (2023), 33–52. https://doi.org/10.32350/sir.71.03
34. M. Saqlain, M. Riaz, R. Imran, F. Jarad, Distance and similarity measures of intuitionistic fuzzy hypersoft sets with application: Evaluation of air pollution in cities based on air quality index, *AIMS Mathematics*, **8** (2023), 6880–6899. https://doi.org/10.3934/math.2023348
35. R. Imran, K. Ullah, Z. Ali, M. Akram, T. Senapati, The theory of prioritized Muirhead mean operators under the presence of complex single-valued neutrosophic values, *Decis. Anal. J.*, **7** (2023), 100214. https://doi.org/10.1016/j.dajour.2023.100214
36. H. Wang, W. Zhao, J. Zheng, Improved q-rung orthopair fuzzy WASPAS method based on softmax function and frank operations for investment decision of community group-buying platform, *J. Soft Comput. Decis. Anal.*, **2** (2024), 188–212. https://doi.org/10.31181/jscda21202442
37. K. Kara, H. Özyürek, G. C. Yalçın, N. Burgaz, Enhancing financial performance evaluation: The MEREC-RBNAR hybrid method for sustainability-indexed companies, *J. Soft Comput. Decis. Anal.*, **2** (2024), 236–257. https://doi.org/10.31181/jscda21202444
38. T. Xu, H. Wang, L. Feng, Y. Zhu, Risk factors assessment of smart supply chain in intelligent manufacturing services using DEMATEL method with linguistic q-ROF information, *J. Oper. Intell.*, **2** (2024), 129–152. https://doi.org/10.31181/jopi21202417
39. Y. Wang, X. Han, W. Wang, A fermatean fuzzy ORESTE method for evaluating the resilience of the food supply chain, *J. Oper. Intell.*, **2** (2024), 78–94. https://doi.org/10.31181/jopi2120249
40. J. Ge, S. Zhang, Adaptive inventory control based on fuzzy neural network under uncertain environment, *Complexity*, **2020** (2020), 6190936. https://doi.org/10.1155/2020/6190936
41. R. Imran, K. Ullah, Circular intuitionistic fuzzy edas approach: A new paradigm for decision-making in the automotive industry sector, *Spectr. Eng. Manag. Sci.*, **3** (2025), 76–92. https://doi.org/10.31181/sems31202537i
42. M. Sarwar, H. Humaira T. Li, Fuzzy fixed point results and applications to ordinary fuzzy differential equations in complex valued metric spaces, *Hacet. J. Math. Stat.*, **49** (2018), 1712–1728. https://doi.org/10.15672/HJMS.2018.633

Appendix A

Proof. (Theorem 4.1) By utilizing the basic operational laws defined for CIVIFS;

$$\text{ꝑ}_{p}^{\zeta}=\left(\left[\left(\vartheta_{p}{}^{\mathrm{r}}\right)^{\zeta},\left(\vartheta_{p}{}^{\text{ɨ}}\right)^{\zeta}\right],\left[1-\left(1-\text{ß}_{p}{}^{\mathrm{r}}\right)^{\zeta},1-\left(1-\text{ß}_{p}{}^{\text{ɨ}}\right)^{\zeta}\right],\left[\left(\varrho_{p}{}^{\mathrm{r}}\right)^{\zeta},\left(\varrho_{p}{}^{\text{ɨ}}\right)^{\zeta}\right]\right),$$

$$\text{ꝑ}_{q}^{\eta}=\left(\left[\left(\vartheta_{q}{}^{\mathrm{r}}\right)^{\eta},\left(\vartheta_{q}{}^{\text{ɨ}}\right)^{\eta}\right],\left[1-\left(1-\text{ß}_{q}{}^{\mathrm{r}}\right)^{\eta},1-\left(1-\text{ß}_{q}{}^{\text{ɨ}}\right)^{\eta}\right],\left[\left(\varrho_{q}{}^{\mathrm{r}}\right)^{\eta},\left(\varrho_{q}{}^{\text{ɨ}}\right)^{\eta}\right]\right).$$

So,

$$\text{ꝑ}_{p}^{\zeta}\times\text{ꝑ}_{q}^{\eta}=\begin{pmatrix}\left[\left(\vartheta_{p}{}^{\mathrm{r}}\right)^{\zeta}\left(\vartheta_{q}{}^{\mathrm{r}}\right)^{\eta},\left(\vartheta_{p}{}^{\text{ɨ}}\right)^{\zeta}\left(\vartheta_{q}{}^{\text{ɨ}}\right)^{\eta}\right],\left[1-\left(1-\text{ß}_{p}{}^{\mathrm{r}}\right)^{\zeta}\left(1-\text{ß}_{q}{}^{\mathrm{r}}\right)^{\eta}\right],\\ \left[\left(\varrho_{p}{}^{\mathrm{r}}\right)^{\zeta}\left(\varrho_{q}{}^{\mathrm{r}}\right)^{\eta},\left(\varrho_{p}{}^{\text{ɨ}}\right)^{\zeta}\left(\varrho_{q}{}^{\text{ɨ}}\right)^{\eta}\right]\end{pmatrix}.$$

Then,

$$\sum_{q=1}^{\mathfrak{y}}\sum_{p=1}^{\mathfrak{y}}\sqrt{\text{ꝑ}_{p}^{\zeta}\text{ꝑ}_{q}^{\eta}}$$

$$=\begin{pmatrix}\left[1-\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\sqrt{\left(\left(\vartheta_{p}{}^{\mathrm{r}}\right)^{\zeta}\left(\vartheta_{q}{}^{\mathrm{r}}\right)^{\eta}\right)}\right),1-\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\sqrt{\left(\left(\vartheta_{p}{}^{\text{ɨ}}\right)^{\zeta}\left(\vartheta_{q}{}^{\text{ɨ}}\right)^{\eta}\right)}\right)\right],\\ \left[\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\left(\sqrt{\begin{matrix}\left(1-\text{ß}_{p}{}^{\mathrm{r}}\right)^{\zeta}\\ \left(1-\text{ß}_{q}{}^{\mathrm{r}}\right)^{\eta}\end{matrix}}\right)\right),\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\left(\sqrt{\begin{matrix}\left(1-\text{ß}_{p}{}^{\text{ɨ}}\right)^{\zeta}\\ \left(1-\text{ß}_{q}{}^{\text{ɨ}}\right)^{\eta}\end{matrix}}\right)\right)\right],\\ \left[1-\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\sqrt{\left(\varrho_{p}{}^{\mathrm{r}}\right)^{\zeta}\left(\varrho_{q}{}^{\mathrm{r}}\right)^{\eta}}\right),1-\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\sqrt{\left(\varrho_{p}{}^{\text{ɨ}}\right)^{\zeta}\left(\varrho_{q}{}^{\text{ɨ}}\right)^{\eta}}\right)\right]\end{pmatrix}.$$

So,

$$\left(\left({}^{2}/_{\mathfrak{y}^{2}+\mathfrak{y}}\right)\sum_{p=1}^{\mathfrak{y}}\sum_{q=1}^{\mathfrak{y}}\sqrt{\text{ꝑ}_{p}^{\zeta}\text{ꝑ}_{q}^{\eta}}\right)$$

$$=\begin{pmatrix}\left[1-\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\sqrt{\left(\left(\vartheta_{p}{}^{\mathrm{r}}\right)^{\zeta}\left(\vartheta_{q}{}^{\mathrm{r}}\right)^{\eta}\right)}\right)^{\left({}^{2}/_{\mathfrak{y}^{2}+\mathfrak{y}}\right)},1-\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\sqrt{\left(\left(\vartheta_{p}{}^{\text{ɨ}}\right)^{\zeta}\left(\vartheta_{q}{}^{\text{ɨ}}\right)^{\eta}\right)}\right)^{\left({}^{2}/_{\mathfrak{y}^{2}+\mathfrak{y}}\right)}\right],\\ \left[\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\left(\sqrt{\begin{matrix}\left(1-\text{ß}_{p}{}^{\mathrm{r}}\right)^{\zeta}\\ \left(1-\text{ß}_{q}{}^{\mathrm{r}}\right)^{\eta}\end{matrix}}\right)\right)^{\left({}^{2}/_{\mathfrak{y}^{2}+\mathfrak{y}}\right)},\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\left(\sqrt{\begin{matrix}\left(1-\text{ß}_{p}{}^{\text{ɨ}}\right)^{\zeta}\\ \left(1-\text{ß}_{q}{}^{\text{ɨ}}\right)^{\eta}\end{matrix}}\right)\right)^{\left({}^{2}/_{\mathfrak{y}^{2}+\mathfrak{y}}\right)}\right],\\ \left[1-\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\sqrt{\left(\varrho_{p}{}^{\mathrm{r}}\right)^{\zeta}\left(\varrho_{q}{}^{\mathrm{r}}\right)^{\eta}}\right)^{\left({}^{2}/_{\mathfrak{y}^{2}+\mathfrak{y}}\right)},1-\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\sqrt{\left(\varrho_{p}{}^{\text{ɨ}}\right)^{\zeta}\left(\varrho_{q}{}^{\text{ɨ}}\right)^{\eta}}\right)^{\left({}^{2}/_{\mathfrak{y}^{2}+\mathfrak{y}}\right)}\right]\end{pmatrix}$$

$$=CIVIFHM^{\zeta,\eta}\left(\text{ꝑ}_{1},\text{ꝑ}_{2},\ldots,\text{ꝑ}_{\mathfrak{y}}\right).$$

Appendix B

Proof. (Theorem 4.3) By using the operations defined for CIVIFS;

$$\wp_{p}^{\zeta} = \left(\left[\left(\vartheta_{p}{}^{\mathrm{r}}\right)^{\zeta}, \left(\vartheta_{p}{}^{\mathrm{i}}\right)^{\zeta}\right], \left[1-\left(1-ß_{p}{}^{\mathrm{r}}\right)^{\zeta}, 1-\left(1-ß_{p}{}^{\mathrm{i}}\right)^{\zeta}\right], \left[\left(\varrho_{p}{}^{\mathrm{r}}\right)^{\zeta}, \left(\varrho_{p}{}^{\mathrm{i}}\right)^{\zeta}\right]\right),$$

$$\wp_{q}^{\eta} = \left(\left[\left(\vartheta_{q}{}^{\mathrm{r}}\right)^{\eta}, \left(\vartheta_{q}{}^{\mathrm{i}}\right)^{\eta}\right], \left[1-\left(1-ß_{q}{}^{\mathrm{r}}\right)^{\eta}, 1-\left(1-ß_{q}{}^{\mathrm{i}}\right)^{\eta}\right], \left[\left(\varrho_{q}{}^{\mathrm{r}}\right)^{\eta}, \left(\varrho_{q}{}^{\mathrm{i}}\right)^{\eta}\right]\right),$$

$$\wp_{p}^{\zeta} \times \wp_{q}^{\eta} = \begin{pmatrix} \left[\left(\vartheta_{p}{}^{\mathrm{r}}\right)^{\zeta}\left(\vartheta_{q}{}^{\mathrm{r}}\right)^{\eta}, \left(\vartheta_{p}{}^{\mathrm{i}}\right)^{\zeta}\left(\vartheta_{q}{}^{\mathrm{i}}\right)^{\eta}\right], \\ \left[1-\left(1-ß_{p}{}^{\mathrm{r}}\right)^{\zeta}\left(1-ß_{q}{}^{\mathrm{r}}\right)^{\eta}, 1-\left(1-ß_{p}{}^{\mathrm{i}}\right)^{\zeta}\left(1-ß_{q}{}^{\mathrm{i}}\right)^{\eta}\right], \\ \left[\left(\varrho_{p}{}^{\mathrm{r}}\right)^{\zeta}\left(\varrho_{q}{}^{\mathrm{r}}\right)^{\eta}, \left(\varrho_{p}{}^{\mathrm{i}}\right)^{\zeta}\left(\varrho_{q}{}^{\mathrm{i}}\right)^{\eta}\right] \end{pmatrix}.$$

Then,

$$\sum_{q=1}^{\mathfrak{y}}\sum_{p=1}^{\mathfrak{y}} \wp_{p}^{\zeta}\wp_{q}^{\eta} = \begin{pmatrix} \left[1-\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\left(\left(\vartheta_{p}{}^{\mathrm{r}}\right)^{\zeta}\left(\vartheta_{q}{}^{\mathrm{r}}\right)^{\eta}\right)\right), 1-\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\left(\left(\vartheta_{p}{}^{\mathrm{i}}\right)^{\zeta}\left(\vartheta_{q}{}^{\mathrm{i}}\right)^{\eta}\right)\right)\right], \\ \left[\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\left(1-ß_{p}{}^{\mathrm{r}}\right)^{\zeta}\left(1-ß_{q}{}^{\mathrm{r}}\right)^{\eta}\right), \prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\left(1-ß_{p}{}^{\mathrm{i}}\right)^{\zeta}\left(1-ß_{q}{}^{\mathrm{i}}\right)^{\eta}\right)\right], \\ \left[1-\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\left(\left(\varrho_{p}{}^{\mathrm{r}}\right)^{\zeta}\left(\varrho_{q}{}^{\mathrm{r}}\right)^{\eta}\right)\right), 1-\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\left(\left(\varrho_{p}{}^{\mathrm{i}}\right)^{\zeta}\left(\varrho_{q}{}^{\mathrm{i}}\right)^{\eta}\right)\right)\right] \end{pmatrix}.$$

So,

$$\left(\left({}^{2}/_{\mathfrak{y}^{2}+\mathfrak{y}}\right)\sum_{p=1}^{\mathfrak{y}}\sum_{q=1}^{\mathfrak{y}} \wp_{p}^{\zeta}\wp_{q}^{\eta}\right)$$

$$= \begin{pmatrix} \left[\left(1-\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\left(\left(\vartheta_{p}{}^{\mathrm{r}}\right)^{\zeta}\left(\vartheta_{q}{}^{\mathrm{r}}\right)^{\eta}\right)\right)^{\left({}^{2}/_{\mathfrak{y}^{2}+\mathfrak{y}}\right)}\right), \left(1-\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\left(\left(\vartheta_{p}{}^{\mathrm{i}}\right)^{\zeta}\left(\vartheta_{q}{}^{\mathrm{i}}\right)^{\eta}\right)\right)^{\left({}^{2}/_{\mathfrak{y}^{2}+\mathfrak{y}}\right)}\right)\right], \\ \left[\left(\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\left(1-ß_{p}{}^{\mathrm{r}}\right)^{\zeta}\left(1-ß_{q}{}^{\mathrm{r}}\right)^{\eta}\right)^{\left({}^{2}/_{\mathfrak{y}^{2}+\mathfrak{y}}\right)}\right), \left(\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\left(1-ß_{p}{}^{\mathrm{i}}\right)^{\zeta}\left(1-ß_{q}{}^{\mathrm{i}}\right)^{\eta}\right)^{\left({}^{2}/_{\mathfrak{y}^{2}+\mathfrak{y}}\right)}\right)\right], \\ \left[\left(1-\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\left(\left(\varrho_{p}{}^{\mathrm{r}}\right)^{\zeta}\left(\varrho_{q}{}^{\mathrm{r}}\right)^{\eta}\right)\right)^{\left({}^{2}/_{\mathfrak{y}^{2}+\mathfrak{y}}\right)}\right), \left(1-\prod_{q=1,p=1}^{\mathfrak{y}}\left(1-\left(\left(\varrho_{p}{}^{\mathrm{i}}\right)^{\zeta}\left(\varrho_{q}{}^{\mathrm{i}}\right)^{\eta}\right)\right)^{\left({}^{2}/_{\mathfrak{y}^{2}+\mathfrak{y}}\right)}\right)\right] \end{pmatrix}$$

$$= CIVIFGHM^{\zeta,\eta}\left(\wp_{1}, \wp_{2}, \dots, \wp_{\mathfrak{y}}\right).$$