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Research article

Exponential input-to-state stability of nonlinear systems under impulsive disturbance via aperiodic intermittent control

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Abstract: In this paper, the exponential input-to-state stabilization (EISS) problem for nonlinear systems subject to impulsive disturbance and continuous external inputs is addressed by an aperiodic intermittent control (APIC), which is further classified as either time-triggered APIC (TAPIC) or event-triggered APIC (EAPIC). To establish sufficient conditions for the realization of EISS, the Lyapunov approach is used. It is shown that the suggested APIC can successfully reduce the negative consequences of continuous external inputs and impulsive disturbance. Limiting the percentage of the active interval in the control procedure yields a range of impulse moments under TAPIC. The relationship among impulse disturbance, intermittent control parameters, the event-triggered mechanism (ETM), and the threshold is established under EAPIC to guarantee EISS. The predesigned ETM is used to generate a series of impulse disturbance moments. Furthermore, the Zeno phenomenon is excluded. Finally, an example of Chua's oscillator is presented to show how effective the system is under TAPIC and EAPIC.

Keywords: impulse disturbance; aperiodic intermittent control; time-triggered and event-triggered; input-to-state stabilization
Mathematics Subject Classification: 93C10, 93D40

1. Introduction

In real life, impulse phenomena are used in a variety of fields, including aerospace [1], digital signal processing [2], ecological domain [3], and orbital adjustment of satellite [4], typically representing a sudden change in the system's state at a specific moment in time. In mathematics, impulse phenomena are usually described using impulse differential equations, which not only describe the dynamic behavior between successive impulses but also define an instantaneous update or reset of the system's state at discrete time points. Impulses can be classified into stable and unstable types on the basis of the effect on the system. Stable impulses help the system achieve the desired performance and positively contribute to its stability. For example, Zhang et al. [5] and He et al. [6] investigated the finite-time stability of general nonlinear system and positive switched linear delayed system under impulse control, respectively. In [7], the authors revealed that stable impulses accelerate the convergence of the system state based on the Lyapunov method. In [8], impulse control is then applied to multi-agent systems. In contrast, unstable impulses or impulse disturbances occur when the system encounters unavoidable transient disturbances. These disturbances can adversely affect the stability of the system and even disrupt its dynamic behavior. Recently, impulse disturbance has received increasing attention in areas such as synchronization of the network system and network security. Zhang et al. [9] established a Zeno-free event-triggered impulsive control framework to achieve input-to-state stability (ISS) for nonlinear systems under arbitrarily timed impulsive disturbance. In [10], the results indicated that impulsive disturbance had a negative effect on system stability and needed to be suppressed by increasing the feedback gain in order to achieve stability. Yang et al. [11] investigated the synchronization problem of Boolean control networks with a drive-response structure under impulsive disturbance. The paper [12] was the first to address the synchronization problem under impulsive disturbance without imposing a lower bound on the impulse intervals.

When discussing how the disturbance impacts the performance of a system, we also need to consider ISS, as it mathematically characterizes how exogenous signals relate to state variable evolution. Sontag was the first to introduce this concept in [13]. Regardless of where the system begins in the state space, ISS implies that the state will eventually converge near the origin. External inputs exist in dynamic systems because of unavoidable negative consequences such as component friction and environmental noise in real-world situations. Therefore, studying the ISS of systems has garnered increasing interest from researchers, and there are many articles on the ISS properties of systems (see e.g., [14–16]). In [17], the ISS of a nonlinear system was analyzed using an event-triggered impulse control method. Sufficient conditions were proposed to establish the connection between impulse intensity and the event-triggered mechanism (ETM). The problems of ISS and integral ISS for nonlinear systems with hybrid inputs and delayed impulses were examined in [18], together with the impact of hybrid delayed impulses on both stable and unstable impulses. In [19], ISS was extended to infinite-dimensional stochastic impulsive systems, and the results are applicable to both stable and unstable impulses of the system.

Intermittent control (IC) has attracted much attention as an efficient control mechanism and has been extensively used in the control of dynamic systems. Compared with continuous control, IC does not require the constant application of control inputs to the system and can effectively reduce resource consumption. In some practical applications, such as aircraft control, robot control, and smart grids, these controls do not require continuous control signals as in conventional control methods. In these cases, intermittent control proves highly effective because it inputs control signals to the system only at specific time intervals, which is consistent with the dynamics of the system. IC is easily separated into periodic intermittent control (PIC) (see e.g., [20–22]) and aperiodic intermittent control (APIC) (see e.g., [23–27]), which differ in terms of when and how the control signals are applied. While PIC updates the control signals at fixed intervals, ensuring regularity, it lacks flexibility and may lead to unnecessary control actions when the system's state is already stable. In contrast, APIC triggers control signals on the basis of specific conditions or events, offering greater adaptability and improved efficiency. This flexibility enables APIC to dynamically respond to the system's requirements, reduce resource consumption, and enhance overall performance. Furthermore, APIC can adjust the control intervals in real time, which improves its ability to manage external disturbances and uncertainties. This is especially useful for complicated nonlinear systems. Zhang et al. [25] investigated the exponential input-to-state stabilization (EISS) of semi-linear system via APIC and aperiodic intermittent sampled-data control, respectively. In [26], the authors proposed an APIC strategy that flexibly regulated the control period and width and combined it with a Lyapunov approach to suppress system's uncertainties and impulsive disturbance, achieving finite-time stabilization of nonlinear systems. In [28], the global asymptotic stability of nonlinear time-delay systems under PIC was investigated, while [29] demonstrates that APIC achieves EISS through constraining the control width ratio, thereby enhancing the robustness against impulsive disturbance. These advantages highlight APIC as a superior choice for modern control applications that demand high levels of efficiency and adaptability.

In the engineering field, time-triggered control (TC) and event-triggered control (EC) are additional options to the IC mentioned above, which utilize different control methods. Although TC is simpler to design and implement than EC, it frequently updates the control signals due to having fixed time intervals, leading to the waste of computational and communication resources. However, EC can reduce unnecessary waste by designing a suitable ETM to determine when system control is applied. So far, there are numerous research results on EC due to its effective performance in achieving dynamic system performance (see e.g., [30–34]). For example, Peng et al. [30] discussed the stability of the pth moment of the impulsive stochastic delay system under the event-triggered control strategy framework. However, EC implies that the event generator and the controller operate continuously even if the system is not really needed. This issue has been addressed by the combination of IC and EC, known as event-triggered intermittent control (EIC). In [34], the authors achieved the stabilization of nonlinear delayed systems under impulsive disturbance for a finite period of time by designing an APIC strategy and verified, through numerical simulations, that event-triggered APIC (EAPIC) outperforms time-triggered APIC (TAPIC).

According to the available results, it is clear that there has been limited research on the ISS of nonlinear dynamical systems under APIC, especially considering the effects of impulsive disturbances. Therefore, it is important and challenging to study the ISS of nonlinear systems with impulsive disturbance and external disturbance by designing appropriate APIC strategies. The present study examines the EISS for such systems employing both TAPIC and EAPIC, with impulsive effects manifesting at each instant of control initiation. The main contributions of this paper include the following.

(1) We propose a novel APIC strategy designed for nonlinear systems subject to both continuous and impulsive disturbances. Furthermore, numerical simulations based on Chua's oscillator demonstrate that, under identical disturbance and initial conditions, EAPIC significantly reduces control effort compared with TAPIC.

(2) Under TAPIC, the active interval width and impulse moments are determined by the link between the control parameters and the impulse disturbance, providing a lower bound on range of the control width. This not only preserves the advantages of intermittent control but also ensures that the beneficial system properties resulting from the strategy are maintained over a sufficiently long period. Under EAPIC, the links among impulse disturbance, control parameters, ETM, and the threshold are established, and the series of impulse disturbance moments is produced using the predesigned ETM while excluding Zeno behavior.

(3) Liu et al. [23] achieved EISS for continuous nonlinear systems with continuous external inputs by employing APIC. However, their work solely addressed continuous system dynamics, whereas the present study incorporates transient external disturbance.

(4) In contrast to prior studies, particularly [28], which solely focuses on fixed control widths, the proposed method accommodates flexible control widths and event-triggering conditions while rigorously guaranteeing the exclusion of Zeno behavior.

Notations: Let $\mathbb{R}$ signify all real numbers, $\mathbb{R}_+ = [0, +\infty)$. $\mathbb{R}^n$ is the n-dimensional real vector space, $\mathbb{R}^{n\times m}$ is the $n \times m$-dimensional real space, $\mathbb{Z}_+$ indicates the set of positive integers, and $\mathbb{Z}_0^+ = \{0\} \cup \mathbb{Z}_+$. The identity matrix of compatible dimensions is denoted by I. $X > 0$ or $X < 0$ signify that the matrix X is positive or negative definite. For any matrix X, the operations X^T and X^{-1} yield its transpose and inverse, respectively. $\lambda_M(X)$ denotes the maximum eigenvalue of the matrix X. Respectively, $z(t_r^+)$ and $z(t_r^-)$ stand for the right and left limits. $C(E, F) = \{\eta : E \to F \textit{ is continuous}\}$. $\star$ indicates the symmetric block in a symmetric matrix. $\mathbb{L}$ indicates the set of Lebesgue measurable functions $\omega(t) \in C[\mathbb{R}_+, \mathbb{R}_+)$. For $\omega(t) \in \mathbb{L}$, define $|\omega(t)|_G$ as the essential supremum of $|\omega(t)|$ over G, and $|\omega(t)|_\infty$ as the essential supremum of $|\omega(t)|$ with $G = \mathbb{R}_+$. A function $\vartheta(d) \in C(\mathbb{R}_+, \mathbb{R}_+)$ belongs to the class K_∞ if it is strictly increasing in d and $\vartheta(d) \to +\infty$ as $d \to +\infty$.

2. Preliminaries

We study the following system in this paper:

$$\begin{cases} \dot{z}(t) = \mathscr{A} z(t) + g(z(t)) + u(t) + \omega(t), \quad t \geq t_0, \\ z(t_0) = z_0, \end{cases} \tag{2.1}$$

where $z(t) \in \mathbb{R}^n$ represents the state vector, $\dot{z}(t)$ signifies the right-hand derivative of $z(t)$, $\mathscr{A} \in \mathbb{R}^{n\times n}$, $g(z) = (g_1(z_1), \ldots, g_n(z_n))^T \in C(\mathbb{R}^n, \mathbb{R}^n)$, $u(t)$ indicates the control input, and $\omega(t) \in \mathbb{L}$ denotes external disturbance.

Design the rth aperiodic intermittent control (APIC) $(\mathscr{H}, \{t_r\}, \{\tau_r\})$ as

$$u(t) = \begin{cases} \mathscr{H} z(t), & t \in [t_r, t_r + \tau_r), \\ 0, & t \in [t_r + \tau_r, t_{r+1}), \end{cases} \tag{2.2}$$

where $\mathscr{H} \in \mathbb{R}^{n\times n}$ is the control strength, and t_r is the impulse instant, which is also the starting instant of the rth APIC. Assume that the sequence $\{t_r\}_{r=1}^{\infty}$ satisfies $0 = t_0 < t_1 < \cdots < t_r < \ldots$ and $\lim_{r\to\infty} t_r = \infty$, $\mathscr{F}_c$ indicates such kinds of sequences. Similarly, $\mathscr{F}_\omega$ indicates the control width sequences $\{\tau_r\}$ satisfying $0 < \tau_r < \Theta_r = t_{r+1} - t_r, r \geq 0$. Here, $[t_r, t_r + \tau_r)$ represents the active interval and $[t_r + \tau_r, t_{r+1})$ represents the inactive interval. For the given $\{t_r\}$ and $\{\tau_r\}$, let $\mathscr{V}(\mathscr{H}, \{t_r\}, \{\tau_r\})$ be the set of APIC composed of the control gain $\mathscr{H}$. Moreover, we define $\tau_{max} = max_{r\in\mathbb{Z}_0^+}\{\tau_r\}$, $\tau_{min} = min_{r\in\mathbb{Z}_0^+}\{\tau_r\}$, and $\Theta_{max} = max_{r\in\mathbb{Z}_0^+}\{\Theta_r\}$.

Then the system (2.1) can be changed to the form below under APIC (2.2):

$$\begin{cases} \dot{z}(t) = (\mathscr{A} + \mathscr{H}) z(t) + g(z(t)) + \omega(t), & t \in [t_r, t_r + \tau_r), \\ \dot{z}(t) = \mathscr{A} z(t) + g(z(t)) + \omega(t), & t \in [t_r + \tau_r, t_{r+1}). \end{cases} \tag{2.3}$$

As can be observed, the controller works only during the active interval, and input data are not transferred during the other interval. In addition, considering that the state of (2.3) may suddenly jump at a particular moment due to input noise and uncertainty, thus generating the impulsive phenomenon, which we regard as impulsive disturbance. Then the system (2.3) is disturbed by an impulse and can be rewritten as

$$\begin{cases} \dot{z}(t) = (\mathscr{A} + \mathscr{H})z(t) + g(z(t)) + \omega(t), & t \in [t_r, t_r + \tau_r),\ \ t \neq t_r, \\ \dot{z}(t) = \mathscr{A}z(t) + g(z(t)) + \omega(t), & t \in [t_r + \tau_r, t_{r+1}), \\ z(t_r) = (I + \mathscr{D})z(t_r^-), & t = t_r,\ \ r \in \mathbb{Z}_+, \end{cases} \tag{2.4}$$

where $\mathscr{D} \in \mathbb{R}^{n\times n}$ is a constant matrix with respect to the impulse gain, and we assume that the state variable $z(t)$ of the system (2.4) is right continuous. Let $z(t) = z(t, t_0, z_0)$ denote the solution of the system (2.4) with the initial condition $z(t_0) = z_0$. Moreover, assume that (2.4) with $u = 0$ is non-ISS. Designing an appropriate APIC to achieve the EISS of system (2.4) is the aim of this paper. Some definitions, lemmas, and assumptions to be used in the following are given below.

Definition 2.1. *The right upper Dini derivative of the local Lipschitz function V:* $\mathbb{R}^n \to \mathbb{R}_+$ *is defined as*

$$D^+V(z) = \lim_{\Delta t\to 0^+} sup\frac{V(z+\Delta t) - V(z)}{\Delta t}.$$

Definition 2.2. *[23] For the given constants* $\rho > 0, \varpi > 0$*, and* $\Gamma \in \mathrm{K}_\infty$*, if an APIC that belongs to* $\mathscr{V}(\mathscr{H}, \{t_r\}, \{\tau_r\})$ *exists such that the solution* $z(t)$ *of the system* (2.4) *satisfies*

$$|z(t)| \leq \varpi|z_0|exp(-\rho(t-t_0)) + \Gamma(|\omega(t)|_{[t_0,t]}),\ \forall t \geq t_0.$$

Then the system (2.4) *is said to be exponentially input-to-state stabilized (EISS) with respect to* $\omega(t)$*.*

Definition 2.3. *[35] For an impulse time sequence* $\{t_r\}, N_0 > 0$ *denotes the chatter bound, where* T^* *is called the average dwell time, if*

$$N(t, t^*) \leq N_0 + \frac{t - t^*}{T^*}, \tag{2.5}$$

where $N(t, t^*)$ *indicates the number of impulses within the interval* $[t^*, t)$*.*

Lemma 2.1. *[36] For a positive definite matrix M and a constant k, where* $k > 0$*, there is*

$$2\mu^T\nu \leq k\mu^T M\mu + k^{-1}\nu^T M^{-1}\nu,\ \mu \in \mathbb{R}^n,\ \nu \in \mathbb{R}^n.$$

Assumption 2.1. *Consider a positive constant* l_m *satisfying*

$$|g(z_1) - g(z_2)| \leq l_m|z_1 - z_2|,\ z_1, z_2 \in \mathbb{R}^n,\ m = 1, 2, \ldots, r.$$

Then the function $g(\cdot)$ *satisfies the Lipschitz condition.*

3. Main results

The EISS of the nonlinear system (2.4) over $\mathscr{V}(\mathscr{H}, \{t_r\}, \{\tau_r\})$ is guaranteed by some criteria given in this section. We concentrate on TAPIC and EAPIC.

3.1. Stabilization to EISS via TAPIC

Theorem 3.1. *Under the conditions of Assumption 2.1, presume that β_1, β_2, b_1, and b_2 are positive constants and $\iota \geq 1$, where $\beta_2 \geq \beta_1$, $\beta^* = \sqrt{\frac{\beta_2}{\beta_1}}$ and the matrix $P > 0$ satisfies*

(i) $\beta_1 I \leq P \leq \beta_2 I$,

(ii) $(\mathscr{A} + \mathscr{H})^T P + P(\mathscr{A} + \mathscr{H}) + (2l^*\beta^* + b_1)P \leq 0$,

(iii) $\mathscr{A}^T P + P\mathscr{A} + (2l^*\beta^* - b_2)P \leq 0$,

(iv) $\begin{bmatrix} -\iota\mathscr{K} & \mathscr{K}(I + \mathscr{D})^T \\ \star & -\mathscr{K} \end{bmatrix} \leq 0$,

(v) $\frac{ln\iota}{T^*} - \tilde{\xi} < 0$,

where $\mathscr{K} = P^{-1}$. Then for $\{t_r\} \in \mathscr{F}_c$ and $\{\tau_r\} \in \mathscr{F}_w$, the system (2.4) *over $\mathscr{V}(\mathscr{H}, \{t_r\}, \{\tau_r\})$ is EISS, which satisfies*

$$\frac{b_2}{b_1 + b_2} < \inf_{r\in\mathbb{Z}_0^+} \frac{\{\tau_r\}}{\{\Theta_r\}} < 1. \tag{3.1}$$

Proof. Assume the Lyapunov function $V(z(t)) = z^T(t)Pz(t)$. For ease of understanding, let $V(z(t))$ be $V(t)$. When $t \neq t_r$, for $t \in [t_r, t_r + \tau_r)$, we get

$$\begin{aligned} D^+V(t) &= 2z^T(t)P\dot{z}(t) \\ &= z^T(t)((\mathscr{A} + \mathscr{H})^T P + P(\mathscr{A} + \mathscr{H}))z(t) + 2l^* z^T(t)Pz(t) + 2z^T(t)P\omega(t), \end{aligned} \tag{3.2}$$

where $l^* = diag\left\{l_1^2, l_2^2, \ldots, l_r^2\right\}$.

From Lemma 2.1, $a_1 > 0$ exists satisfying

$$2z^T(t)P\omega(z) \leq a_1 z^T(t)Pz(t) + a_1^{-1}\lambda_M(P)|\omega(t)|^2. \tag{3.3}$$

Combining (3.2), (3.3), and (*ii*), one has

$$\begin{aligned} D^+V(t) &\leq z^T(t)((\mathscr{A} + \mathscr{H})^T P + P(\mathscr{A} + \mathscr{H}) + 2l^*\beta^* P + a_1 P)z(t) + a_1^{-1}\lambda_M(P)|\omega(t)|^2 \\ &\leq -\tilde{b}_1 V(t) + |\omega(t)|^2 a_1^{-1}\lambda_M(P), \end{aligned}$$

where $\tilde{b}_1 = b_1 - a_1$. Integrating the aforementioned inequality over the interval $[t_r, t]$, let $\gamma_1 = \tilde{b}_1^{-1}a_1^{-1}\lambda_M(P)$, one gets

$$V(t) \leq exp(-\tilde{b}_1(t - t_r))V(t_r) + |\omega(t)|^2_{[t_r,t]}\gamma_1. \tag{3.4}$$

Similarly, for $t \in [t_r + \tau_r, t_{r+1})$, from (*iii*), $a_2 > 0$ exists, which yields

$$D^+V(t) \leq z^T(t)(\mathscr{A}^T P + P\mathscr{A} + 2l^*\beta^* P + a_2 P)z(t) + a_2^{-1}\lambda_M(P)|\omega(t)|^2 \leq \tilde{b}_2 V(t) + |\omega(t)|^2 a_2^{-1}\lambda_M(P),$$

where $\tilde{b}_2 = b_2 + a_2$. Let $\gamma_2 = \tilde{b}_2^{-1}a_2^{-1}\lambda_M(P)$. Then, by integrating the inequality above, one obtains

$$V(t) \leq exp(\tilde{b}_2(t - t_r - \tau_r))V(t_r + \tau_r) + |\omega(t)|^2_{[t_r+\tau_r,t]}\gamma_2. \tag{3.5}$$

When $t = t_r$, from (*iv*), we have

$$V(t) = z^T(t^-)(I + \mathscr{D})^T P(I + \mathscr{D})Z(t^-) \leq \iota V(t^-). \tag{3.6}$$

From (3.4), we derive

$$V(t_r+\tau_r) \le exp(-\tilde{b}_1\tau_r)V(t_r) + |\omega(t)|^2_{[t_r,t]}\gamma_1. \tag{3.7}$$

Let $\gamma = \gamma_1+\gamma_2$, $\Delta_{r,i} = -(\tilde{b}_1+\tilde{b}_2)\sum_{j=i}^{r-1}\tau_j + \tilde{b}_2(t-t_i)$ and $P_r(t) = \sum_{i=0}^{r-1}\iota^{N(t,t_i+\tau_i)}exp(\tilde{b}_1\tau_i + \Delta_{r,i}(t))$, where $r\in\mathbb{Z}_+$, $i\in\mathbb{Z}_0^+$ with $r>i$.

Then for $t\in[t_0,t_0+\tau_0)$ and $N(t,t_0)=0$, by (3.4), one gets

$$V(t) \le exp(-\tilde{b}_1(t-t_0))V(t_0) + |\omega(t)|^2_{[t_0,t]}\gamma_1 = \iota^{N(t,t_0)}exp(-\tilde{b}_1(t-t_0))V(t_0) + |\omega(t)|^2_{[t_0,t]}\gamma_1. \tag{3.8}$$

For $t\in[t_0+\tau_0,t_1)$ and $N(t,t_0)=N(t,t_0+\tau_0)=0$, from (3.5) and (3.7), we have

$$\begin{aligned}V(t) &\le exp(\tilde{b}_2(t-t_0-\tau_0))V(t_0+\tau_0) + |\omega(t)|^2_{[t_0+\tau_0,t]}\gamma_2\\ &\le exp(-(\tilde{b}_1+\tilde{b}_2)\tau_0 + \tilde{b}_2(t-t_0))V(t_0) + |\omega(t)|^2_{[t_0,t]}(\gamma_1 exp(\tilde{b}_2(t-t_0-\tau_0)) + \gamma_2)\\ &\le exp(\Delta_{1,0}(t))V(t_0) + |\omega(t)|^2_{[t_0,t]}\gamma\iota^{N(t,t_0+\tau_0)}exp(\tilde{b}_1\tau_0 + \Delta_{1,0}(t))\\ &= \iota^{N(t,t_0)}exp(\Delta_{1,0}(t))V(t_0) + |\omega(t)|^2_{[t_0,t]}\gamma P_1(t).\end{aligned} \tag{3.9}$$

For $t\in[t_1,t_1+\tau_1)$ and $N(t,t_0)=N(t,t_0+\tau_0)=1$, together with (3.4), (3.6), and (3.9), it holds that

$$\begin{aligned}V(t) &\le exp(-\tilde{b}_1(t-t_1))V(t_1) + |\omega(t)|^2_{[t_1,t]}\gamma_1\\ &\le \iota exp(-\tilde{b}_1(t-t_1))V(t_1^-) + |\omega(t)|^2_{[t_1,t]}\gamma_1\\ &\le \iota exp(-\tilde{b}_1(t-t_1) + \Delta_{1,0}(t_1))V(t_0) + |\omega(t)|^2_{[t_1,t]}\gamma_1 + |\omega(t)|^2_{[t_0,t]}\gamma exp(-\tilde{b}_1(t-t_1) + \tilde{b}_1\tau_0 + \Delta_{1,0}(t_1))\\ &\le \iota^{N(t,t_0)}exp(\Delta_{1,0}(t_1))V(t_0) + |\omega(t)|^2_{[t_0,t]}(\gamma_1 + \gamma\iota^{N(t,t_0+\tau_0)}exp(\tilde{b}_1\tau_0 + \Delta_{1,0}(t_1)))\\ &= \iota^{N(t,t_0)}exp(\Delta_{1,0}(t_1))V(t_0) + |\omega(t)|^2_{[t_0,t]}(\gamma_1 + \gamma P_1(t_1)).\end{aligned}$$

For $t\in[t_1+\tau_1,t_2)$, $N(t,t_0)=1, N(t,t_i+\tau_i)=1-i$, and $i=0,1$, from (3.5)–(3.7), we have

$$\begin{aligned}V(t) &\le exp(\tilde{b}_2(t-t_1-\tau_1))V(t_1+\tau_1) + |\omega(t)|^2_{[t_1+\tau_1,t]}\gamma_2\\ &\le \iota exp(\tilde{b}_2(t-t_1-\tau_1) - \tilde{b}_1\tau_1 + \Delta_{1,0}(t_1))V(t_0) + |\omega(t)|^2_{[t_1,t]}(\gamma_1 exp(\tilde{b}_2(t-t_1-\tau_1) + \gamma_2)\\ &\quad + |\omega(t)|^2_{[t_0,t]}\gamma\iota exp(-\tilde{b}_1\tau_1 + \tilde{b}_1\tau_0 + \Delta_{1,0}(t_1) + \tilde{b}_2(t-t_1-\tau_1))\\ &= \iota exp(\Delta_{2,0}(t))V(t_0) + |\omega(t)|^2_{[t_1,t]}(\gamma_1 exp(\tilde{b}_1\tau_1 + \Delta_{2,1}(t)) + \gamma_2) + |\omega(t)|^2_{[t_0,t]}\gamma\iota exp(\tilde{b}_1\tau_0 + \Delta_{2,0}(t))\\ &\le \iota^{N(t,t_0)}exp(\Delta_{2,0}(t))V(t_0) + |\omega(t)|^2_{[t_0,t]}(\gamma\iota^{N(t,t_1+\tau_1)}exp(\tilde{b}_1\tau_1 + \Delta_{2,1}(t))\\ &\quad + |\omega(t)|^2_{[t_0,t]}\iota^{N(t,t_0+\tau_0)}exp(\tilde{b}_1\tau_0 + \Delta_{2,0}(t))\\ &= \iota^{N(t,t_0)}exp(\Delta_{2,0}(t))V(t_0) + |\omega(t)|^2_{[t_0,t]}\gamma P_2(t).\end{aligned}$$

Similarly, for $t\in[t_r,t_r+\tau_r)$, $N(t,t_0)=r$, $N(t,t_i+\tau_i)=r-i$, $r\in\mathbb{Z}_+$, $i\in\mathbb{Z}_0^+$ and $r>i$, we have

$$V(t) \le \iota^{N(t,t_0)}exp(\Delta_{r,0}(t))V(t_0) + |\omega(t)|^2_{[t_0,t]}(\gamma_1 + \gamma P_r(t)). \tag{3.10}$$

For $t\in[t_r+\tau_r,t_{r+1})$, $N(t,t_0)=r$, and $N(t,t_i+\tau_i)=r-i$, we have

$$V(t) \le \iota^{N(t,t_0)}exp(\Delta_{r+1,0}(t))V(t_0) + |\omega(t)|^2_{[t_0,t]}\gamma P_{r+1}(t). \tag{3.11}$$

Then according to (3.8)–(3.11), for all $t\ge t_0$, $r\in\mathbb{Z}_+$ yields the following:

$$V(t) = \begin{cases}\iota^{N(t,t_0)}exp(\Delta_{1,0}(t))V(t_0) + |\omega(t)|^2_{[t_0,t]}\gamma_1 P_1(t), & t\in[t_0,t_1),\\ \iota^{N(t,t_0)}exp(\Delta_{r,0}(t))V(t_0) + |\omega(t)|^2_{[t_0,t]}(\gamma_1 + \gamma P_r(t)), & t\in[t_r,t_r+\tau_r),\\ \iota^{N(t,t_0)}exp(\Delta_{r+1,0}(t))V(t_0) + |\omega(t)|^2_{[t_0,t]}\gamma P_{r+1}(t), & t\in[t_r+\tau_r,t_{r+1}).\end{cases} \tag{3.12}$$

According to (3.1), it follows directly that for some $\xi \in (0, b_1)$, the following holds:

$$\frac{\xi + b_2}{b_1 + b_2} = \frac{\tilde{\xi} + \tilde{b}_2}{\tilde{b}_1 + \tilde{b}_2} \leq \frac{\tau_r}{\Theta_r} < 1, \quad \forall r \in \mathbb{Z}_0^+,$$

where $a_1 = a_2 = a < \xi < b_1$ and $\tilde{\xi} = \xi - a$.

From the definition of $\Delta_{r,i}$, it can be asserted that for all $t \in [t_r, t_r + \tau_r)$,

$$\Delta_{r,i} \leq (\tilde{\xi} + \tilde{b}_2)\Big(t - t_i - \sum_{j=i}^{r-1} \Theta_j\Big) - \tilde{\xi}(t - t_i) \leq (\tilde{\xi} + \tilde{b}_2)(t - t_r) - \tilde{\xi}(t - t_i) \leq (\tilde{\xi} + \tilde{b}_2)\tau_r - \tilde{\xi}(t - t_i).$$

Then from (2.5), it is possible to obtain the following for all $t \in [t_r, t_r + \tau_r)$:

$$\iota^{N(t,t_0)} exp(\Delta_{r,0}(t)) \leq \iota^{N_0} exp((\tilde{\xi} + \tilde{b}_2)\tau_{max}) exp\Big((\frac{ln\iota}{T^*} - \tilde{\xi})(t - t_0)\Big),$$

and

$$\begin{aligned} P_r(t) &\leq \iota^{N_0} \sum_{i=0}^{r-1} exp\Big(\frac{ln\iota}{T^*}(t - t_i - \tau_i)\Big) exp\Big(\tilde{b}_1\tau_i - (\tilde{\xi} + \tilde{b}_2)\sum_{j=i}^{r-1}\Theta_j + \tilde{b}_2(t - t_i)\Big) \\ &\leq \iota^{N_0} \sum_{i=0}^{r-1} exp\Big((\tilde{b}_1 + \tilde{b}_2)\tau_{max} + (\frac{ln\iota}{T^*} - \tilde{\xi})(r - i)\tau_{min}\Big) \\ &\leq \iota^{N_0} exp((\tilde{b}_1 + \tilde{b}_2)\tau_{max})\Big(1 - exp((\frac{ln\iota}{T^*} - \tilde{\xi})\tau_{min})\Big)^{-1}. \end{aligned}$$

Let $\varphi = \iota^{N_0} exp((\tilde{b}_1 + \tilde{b}_2)\tau_{max})(1 - exp((\frac{ln\iota}{T^*} - \tilde{\xi})\tau_{min}))^{-1}$, then $max_{r\in\mathbb{Z}_0^+}(P_r(t)) \leq \varphi$.

For all $t \geq t_0$, review (3.12)

$$\begin{aligned} V(t) &\leq \iota^{N(t,t_0)} exp(\Delta_{r,0}(t))V(z_0) + |\omega(t)|^2_{[t_0,t]}\gamma(1 + P_r(t)) \\ &\leq \alpha_1 exp\Big((\frac{ln\iota}{T^*} - \tilde{\xi})(t - t_0)\Big)V(z_0) + |\omega(t)|^2_{[t_0,t]}\alpha_2, \end{aligned}$$

where $\alpha_1 = \iota^{N_0} exp((\tilde{\xi} + \tilde{b}_2)\tau_{max})$, $\alpha_2 = \gamma(1 + \varphi)$. By ($i$), let $\beta^* = \sqrt{\frac{\beta_2}{\beta_1}}$, then we can also obtain

$$|z(t)| \leq \alpha_1^{\frac{1}{2}}\beta^* exp\Big(\frac{1}{2}(\frac{ln\iota}{T^*} - \tilde{\xi})(t - t_0)\Big)|z_0| + |\omega(t)|^2_{[t_0,t]}\alpha_2^{\frac{1}{2}}. \tag{3.13}$$

By combining (3.13) with (v), it can be obtained that the system (2.4) is EISS, that is, the system (2.1) can be stabilized to EISS over $\mathscr{V}(\mathscr{H}, \{t_r\}, \{\tau_r\})$ under TAPIC and impulse disturbance. □

Remark 3.1. *Condition (ii) indicates that when the control input $u(t) = \mathscr{H}z(t)$, the system* (2.1) *can be stabilized to EISS. In other words, the state does not grow indefinitely due to external disturbance $\omega(t)$. Condition (iii) shows that when the control input $u(t) = 0$, the system* (2.1) *may be non-EISS; that is, in the absence of a control input, the system state $z(t)$ may diverge due to the external disturbance $\omega(t)$.*

Remark 3.2. *If we rewrite* (2.2) *as*

$$u(t) = \begin{cases} \mathscr{H}z(t), & t \in [rJ, rJ + \tau), \\ 0, & t \in [rJ + \tau, (r+1)J), \end{cases} \tag{3.14}$$

then, APIC becomes PIC so that (2.1) *stabilizes with* $\omega = 0$*. This is a special instance of* (2.2)*, where the active interval is implied by* $0 < \tau < J$ *and the control period is represented by* $J > 0$*. As can be seen,* J *and* τ *are fixed and* (3.14) *is time-triggered. Moreover, the stabilization can be proved under PIC* (3.14) *in an approach analogous to that of Theorem 3.1.*

Remark 3.3. *Notably, inequalities* (3.4) *and* (3.5) *are guaranteed to hold by conditions* (*ii*) *and* (*iii*)*, which govern continuous dynamics. Here,* $\tilde{b}_1$*and* $\tilde{b}_2$ *are introduced to describe the decay and growth rates during the active and inactive control intervals, respectively. A larger* b_1 *accelerates stabilization during active intervals, while a smaller* b_2 *mitigates divergence during inactive intervals. Their ratio* $\frac{b_2}{b_1+b_2}$ *directly influences the lower bound of the control width. The impulse coefficient* ι *quantifies the impact of the impulse disturbance at* $\{t_r\}$*, where a smaller* ι *indicates a weaker disturbance effect. According to condition* (*v*)*,* $\tilde{\xi}$ *is proportional to* $\ln\iota$*. Moreover, the reduction in the control interval* $\Theta_r = t_{r+1} - t_r$ *leads to the rise in the control frequency. If we assume that* $\tilde{b}_2$*,* $\tilde{\xi}$ *are fixed, two scenarios should be considered when choosing a smaller* τ_r *in light of the inequality* (3.1) *as follows.*
(1) *If* Θ_r *remains constant, then* $\tilde{b}_1$ *grows. This implies that for the given impulse intensity, achieving EISS requires a stronger control intensity when the control frequency is fixed.*
(2) *If* $\tilde{b}_1$ *remains constant,* Θ_r *decreases. In this case, when the control intensity is fixed, a greater control frequency is required to provide EISS for the given impulse intensity.*

It should also be emphasized that the specified series of starting moments $\{t_r\}$ serves as the basis for the creation of the control width sequence $\{\tau_r\}$ in Theorem 3.1. Both sequences are designed in accordance with the APIC strategy, and thus Theorem 3.2 provides a means of determining the activation of the control action for a given sequence of control width. The proof of Theorem 3.2 is not included here, since it is equivalent to that of Theorem 3.1.

Theorem 3.2. *Let conditions (i)–(v) of Theorem 3.1 hold. Then for a given* $\{\tau_r\} \in \mathscr{F}_w$*, the system* (2.4) *is EISS over* $\mathscr{V}(\mathscr{H}, \{t_r\}, \{\tau_r\})$*, where* $\{t_r\} \in \mathscr{F}_c$ *satisfies* (3.1).

Theorems 3.1 and 3.2 have established the validity of the general APIC mechanism, and two corollaries are presented below to address specific situations of this mechanism. Specifying (3.1) for these situations is essentially adequate, and the corresponding stabilization results can be obtained using a similar approach. For the sake of conciseness, the following proofs are left out.

Corollary 3.1. *When the hypotheses of Theorem 3.1 are satisfied, if* $t_r = rJ$*,* $\tau_r = \tau$ *and*

$$\frac{b_2}{b_1 + b_2} J < \tau < J,$$

then the system (2.4) *is EISS over* $\mathscr{V}(\mathscr{H}, \{rJ\}, \tau)$.

Corollary 3.2. *When the hypotheses of Theorem 3.2 are satisfied, if* $\tau_r = \tau$ *and*

$$\frac{b_2}{b_1 + b_2}\Theta_r < \tau < \Theta_r, \forall r \in \mathbb{Z}_0^+,$$

then the system (2.4) *is EISS over* $\mathscr{V}(\mathscr{H}, \{t_r\}, \tau)$.

Remark 3.4. *Under the same presumptions, Theorem* 3.2 *can be regarded as a complementary expression for Theorem 3.1. In the APIC mechanism* $\mathscr{V}(\mathscr{H}, \{t_r\}, \{\tau_r\})$, *the control gain* $\mathscr{H}$, *the control start time* $\{t_r\}$, *and the control width* $\{\tau_r\}$ *are considered as the design parameters, where* $\mathscr{H}$ *is determined by the sufficient condition based on the linear matrix inequalities (LMI). The link between the control width* τ_r *and the control period* Θ_r *should be taken into consideration when designing APIC strategy, since* Θ_r *depends on two consecutive control start times. Consequently, to handle these two parameters, this paper adopts the approach of setting one parameter and constructing another. Specifically, in Theorem 3.1,* $\{t_r\}$ *is given in advance and* $\{\tau_r\}$ *is treated as the design parameter. Conversely, in Theorem* 3.2, $\{\tau_r\}$ *is given and* $\{t_r\}$ *is treated as a design parameter.*

3.2. Stabilization to EISS via EAPIC

In this section, we propose EAPIC for stabilization to EISS. Unlike TAPIC, EAPIC is state-dependent and can lessen the conservatism of TAPIC. The following ETM is defined:

$$t_{r+1} = \begin{cases} inf\{t : t \in \Phi_r(t_r + \tau_r, t_r + \epsilon]\}, & if\ \Phi_r(t_r + \tau_r, t_r + \epsilon] \neq \emptyset, \\ t_r + \epsilon, & if\ \Phi_r(t_r + \tau_r, t_r + \epsilon] = \emptyset, \end{cases} \tag{3.15}$$

where $\Phi_r(\eta, t] = \{t : t > \eta + \tau_r, V(z(t)) \geq e^{\sigma} V(z(\eta + \tau_r)) + |\omega(t)|^2_{[\eta+\tau_r,t]}\gamma^*\}, \tau_r > 0$, $\sigma > 0$, e^{σ} is threshold-value, ϵ is a relatively large positive constant that represents the check-period, and $\gamma^* \geq max(\gamma_1, \gamma_2)$. A simple structure of EAPIC is shown in Figure 1. The EAPIC framework containing an impulse disturbance is shown in Figure 2.

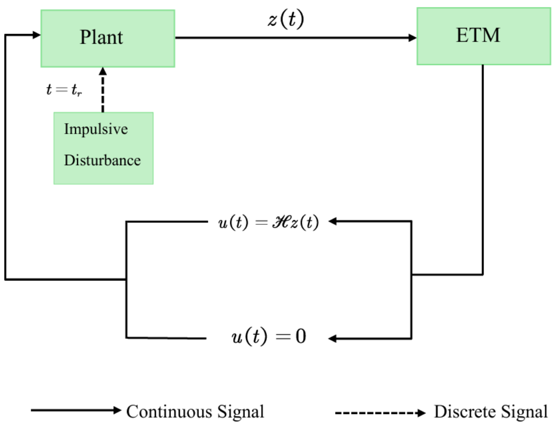

Figure 1. Structure of EAPIC.

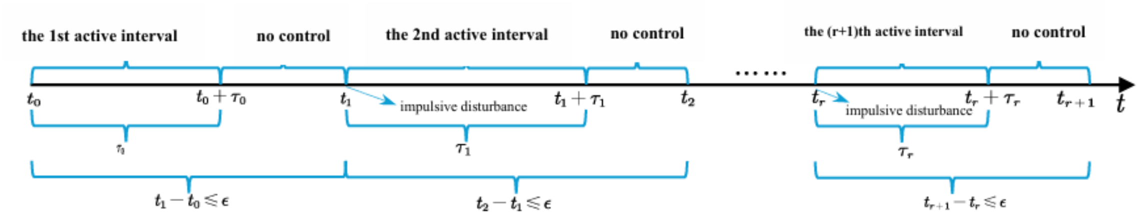

Figure 2. Framework of EAPIC.

Theorem 3.3. *Under assumptions (i)–(iv) specified in Theorem 3.1, positive constants* $\tilde{b}_1$*,* $\tilde{b}_2$ *exist that satisfy*

$$\frac{\sigma}{\tilde{b}_1} < \tau_{min} \leq \tau_r \leq \epsilon - \frac{\sigma}{\tilde{b}_2}, \tag{3.16}$$

as in Theorem 3.1, $0 < a = a_1 = a_2 < b_1$*,* $\tilde{b}_1 = b_1 - a$*, and* $\tilde{b}_2 = b_2 + a$*. Consequently, the Zeno phenomenon is excluded and EISS is achieved for the system* (2.4) *via the ETM* (3.15).

Proof. For ease of understanding, let $V(z(t))$ be $V(t)$.

As in Theorem 3.1, $t = t_r$, we get

$$V(z(t)) = z^T(t^-)(I + \mathscr{D}^T)P(I + \mathscr{D})Z(t^-) \leq \iota V(z(t^-)). \tag{3.17}$$

For $t \neq t_r$, one obtains

$$\begin{cases} V(t) \leq exp(-\tilde{b}_1(t - t_r))V(t_r) + |\omega(t)|^2_{[t_r,t]}\gamma_1, & t \in [t_r, t_r + \tau_r), \\ V(t) \leq exp(\tilde{b}_2(t - t_r - \tau_r))V(t_r + \tau_r) + |\omega(t)|^2_{[t_r+\tau_r,t]}\gamma_2, & t \in [t_r + \tau_r, t_{r+1}), \end{cases} \tag{3.18}$$

and

$$V(t_{r+1}) \leq exp(\tilde{b}_2(t_{r+1} - t_r - \tau_r))V(t_r + \tau_r) + |\omega(t)|^2_{[t_r+\tau_r,t_{r+1}]}\gamma_2. \tag{3.19}$$

If $\Phi_r(t_r + \tau_r, t_r + \epsilon] \neq \emptyset$, then $t_{r+1} - t_r \leq \epsilon$, until the event generator function is greater than zero, the next event will not be triggered, that is

$$V(t_{r+1}) = e^{\sigma}V(t_r + \tau_r) + |\omega(t)|^2_{[t_r+\tau_r,t_{r+1}]}\gamma^*. \tag{3.20}$$

In accordance with (3.19) and (3.20), we get

$$\frac{\sigma}{\tilde{b}_2} \leq t_{r+1} - t_r - \tau_r \leq \epsilon - \tau_{min}, \forall r \in \mathbb{Z}_0^+. \tag{3.21}$$

If $\Phi_r(t_r + \tau_r, t_r + \epsilon] = \emptyset$, then $t_{r+1} - t_r = \epsilon$, and (3.21) still holds.

Since $\frac{\sigma}{\tilde{b}_2} > 0$, according to (3.16) and (3.21),

$$0 < (\frac{1}{\tilde{b}_1} + \frac{1}{\tilde{b}_2})\sigma \leq t_{r+1} - t_r \leq \epsilon. \tag{3.22}$$

By combining (3.21) and (3.22), it is known that Zeno behavior is avoided.

For $\forall t \in [t_r, t_r + \tau_r)$, by (3.18), we get

$$V(t_r + \tau_r) \leq exp(-\tilde{b}_1\tau_r)V(t_r) + |\omega(t)|^2_{[t_r,t_r+\tau_r]}\gamma_1. \tag{3.23}$$

From (3.15), (3.18), and (3.23), we have, for $\forall r \in \mathbb{Z}_0^+$,

$$V(t) \leq e^{\sigma}V(t_r + \tau_r) + |\omega(t)|^2_{[t_r+\tau_r,t]}\gamma^*, t \in [t_r + \tau_r, t_{r+1}). \tag{3.24}$$

$$V(t) \leq e^{\sigma}V(t_r) + |\omega(t)|^2_{[t_r,t]}\gamma^*, t \in [t_r, t_{r+1}). \tag{3.25}$$

From (3.23)–(3.25), we get

$$V(t_{r+1}) \leq exp(\sigma - \tilde{b}_1\tau_r)V(t_r) + e^{\sigma}|\omega(t)|^2_{[t_r,t_r+\tau_r]}\gamma_1 + |\omega(t)|^2_{[t_r+\tau_r,t_{r+1}]}\gamma^*$$

$$\leq exp(\sigma - \tilde{b}_1\tau_r)V(t_r) + (e^{\sigma}\gamma_1 + \gamma^*)|\omega(t)|^2_{[t_r,t_{r+1}]}, \ \forall r \in \mathbb{Z}_0^+. \tag{3.26}$$

For $t \in [t_0, t_1)$, by (3.25), we have

$$V(t) \leq e^{\sigma}V(t_0) + |\omega(t)|^2_{[t_0,t]}\gamma^*. \tag{3.27}$$

For $t \in [t_r, t_{r+1})$ and $r \in \mathbb{Z}_+$, let $q_r = \sigma - \tilde{b}_1\tau_r$, $\bar{\gamma} = e^{\sigma}\gamma_1 + \gamma^*$; then, by combining (3.16), (3.17), and (3.26), one has

$$\begin{aligned}
V(t_r) &\leq \iota V(t_r^-) \\
&\leq \iota exp(q_{r-1})V(t_{r-1}) + \iota|\omega(t)|^2_{[t_{r-1},t_r]}\bar{\gamma} \\
&\leq \iota^2 exp(q_{r-1} + q_{r-2})V(t_{r-2}) + \iota^2 exp(q_{r-1})|\omega(t)|^2_{[t_{r-2},t_{r-1}]}\bar{\gamma} + \iota|\omega(t)|^2_{[t_{r-1},t_r]}\bar{\gamma} \\
&\leq \iota^3 exp(q_{r-1} + q_{r-2} + q_{r-3})V(t_{r-3}) + \iota^3 exp(q_{r-1} + q_{r-2})|\omega(t)|^2_{[t_{r-3},t_{r-2}]}\bar{\gamma} \\
&\quad + \iota^2 exp(q_{r-1})|\omega(t)|^2_{[t_{r-2},t_{r-1}]}\bar{\gamma} + \iota|\omega(t)|^2_{[t_{r-1},t_r]}\bar{\gamma} \\
&\quad \vdots \\
&\leq \iota^r exp(\sum_{i=0}^{r-1} q_i)V(t_0) + \iota^r\Big(\sum_{i=1}^{r-1} exp(\sum_{j=i}^{r-1} q_j) + 1\Big)|\omega(t)|^2_{[t_0,t_r]}\bar{\gamma},
\end{aligned}$$

where $q = \tilde{b}_1\tau_{min} - \sigma > 0$ and $\tilde{\gamma} = \bar{\gamma}(1 - e^{-q})^{-1}$, and thus one gets

$$V(t_r) \leq \iota^r exp(-rq)V(z_0) + \iota^r|\omega(t)|^2_{[t_0,t_r]}\tilde{\gamma}. \tag{3.28}$$

For $t \geq t_0$, together with (3.22), (3.24), (3.25), (3.27), and (3.28), we have

$$\begin{aligned}
V(t) &\leq e^{\sigma}V(t_r) + (\gamma^* + \tilde{\gamma})|\omega(t)|^2_{[t_0,t]} \\
&\leq \iota^r exp(\sigma + q)exp\Big(\frac{-q}{\epsilon}(t - t_0)\Big)V(z_0) + (\gamma^* + (\iota^r e^{\sigma} + 1)\tilde{\gamma})|\omega(t)|^2_{[t_0,t]}.
\end{aligned} \tag{3.29}$$

By (3.29), as in Theorem 3.1, let $\beta^* = \sqrt{\frac{\beta_2}{\beta_1}}$, which yields

$$|z(t)| \leq \iota^{\frac{r}{2}}\beta^* exp\Big(\frac{\sigma + q}{2}\Big)exp\Big(\frac{-q(t - t_0)}{2\epsilon}\Big)|z_0| + \big(\gamma^* + (\iota^r e^{\sigma} + 1)\tilde{\gamma}\big)^{\frac{1}{2}}|\omega(t)|_{[t_0,t]}.$$

Therefore, it can be seen that the system (2.4) is EISS; that is, system (2.1) can be stabilized to EISS over $\mathscr{V}(\mathscr{H}, \{t_r\}, \{\tau_r\})$ under ETM (3.15) and impulse disturbance. □

Remark 3.5. *In ETM* (3.15)*, when the event condition is not satisfied, meaning the event generator function does not exceed the preset threshold, the controller continues to operate for a certain duration, known as the check-period ϵ. Without this check-period, the controller would stop controlling the system once the event-triggering condition is not met, which could result in a loss of control. By introducing the check-period ϵ, the controller ensures that control inputs are applied at the end of the check-period. This mechanism ensures that the system achieves EISS, even when the system's state does not reach the predefined threshold throughout the entire check-period.*

Remark 3.6. *The performance of the EAPIC strategy is highly sensitive to several key parameters, such as the triggering threshold e^{σ}, the control width τ_r, the check-period ϵ, and the disturbance-related gain γ^*. A larger σ leads to fewer control updates, but may slow down the system's convergence. Conversely, a smaller σ results in more frequent control actions, enhancing the response speed at the cost of higher resource consumption. The parameter ϵ serves as an upper bound on the inter-event time. Choosing a larger ϵ helps reduce the triggering rate but requires careful balancing with σ to avoid performance degradation. The lower bound on τ_r, as given in Theorem 3.3, ensures the exclusion of Zeno behavior while directly affecting how quickly the system reacts to impulses. The choice of $\gamma^* \geq max(\gamma_1, \gamma_2)$ provides robustness against external disturbances. While increasing γ^* improves disturbance tolerance, it may make the triggering condition more conservative.*

4. Example

In this section, Chua's oscillator is used as an example to examine the validity of the conclusions produced, taking both TAPIC and EAPIC into account.

Example 4.1. Consider the following underlying system:

$$\dot{z} = \mathscr{A}z + g(z) + u + \omega, \tag{4.1}$$

where

$$\mathscr{A} = \begin{pmatrix} -\alpha(1+b) & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix}, \quad g(z) = \begin{pmatrix} g(z) \\ 0 \\ 0 \end{pmatrix},$$

where $g(z) = -\frac{\alpha}{2}(a-b)(|z_1+1| - |z_1-1|)$, and a, b, α, β are parameters. As in [37], we choose $\alpha = 9.215, \beta = 15.9946$, $a = -1.2495$, and $b = -0.75735$. When $z(0) = (4, 1, 3)^T$, $u = \omega = 0$. Figure 3 shows the trajectory of z_1, z_2, z_3. It is evident that (4.1) is an unstable system.

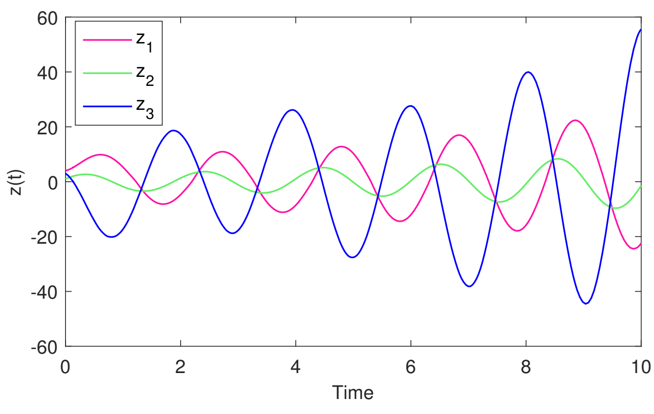

Figure 3. State trajectories of the system (4.1).

If we assume $z(t) = (z_1, z_2, z_3)^T$, then by considering APIC, impulse disturbance, and continuous external inputs, then we can rewrite the system (4.1) as

$$\begin{cases} \dot{z} = \mathscr{A}z + g(z) + u + \omega, & t \geq 0, t \neq t_r, r \in \mathbb{Z}_+, \\ z(t_r) = (I + \mathscr{D})z(t_r^-), & t = t_r, \end{cases} \tag{4.2}$$

where

$$u(t) = \begin{cases} \mathscr{H} z(t), & t \in [t_r, t_r + \tau_r), \\ 0, & t \in [t_r + \tau_r, t_{r+1}). \end{cases}$$

Now, we design TAPIC and EAPIC. Firstly, think about the system (4.2) with $P = I$ and $\mathscr{D} = diag\{0.3, 0.1, 0.2\}$. By combining the conditions (i)–(v) and using the LMI toolbox, one gets $b_1 = 6.2490$, $b_2 = 9.07097$, and $\mathscr{H} = -7.66I$. In the following, in order to facilitate the later comparison between TAPIC and EAPIC, the same data are used for the simulation. We use the same control width $\tau_r = 1.5$ and the same continuous external disturbance $\omega(t) = (3sin(t), 3cos(t)$, and $3rand(1))^T$.

Case 1. Stabilization to EISS via TAPIC: According to Theorem 3.2, for TAPIC with ($\mathscr{H} = -7.66I, \{t_r\}$, and $\{\tau_r = 1.5\}$), if $\{t_r\}$ is chosen to meet $t_{r+1} \in (t_r + 1.5, t_r + 1.5 + 1.0334)$, then (4.2) is stabilized to EISS by the TAPIC. In the simulation, $t_{r+1} = t_r + 2 + 0.5333rand(1)$. Figure 4 shows the trajectories of $z(t)$, and Figure 5 illustrates the intermittent control $u(t)$.

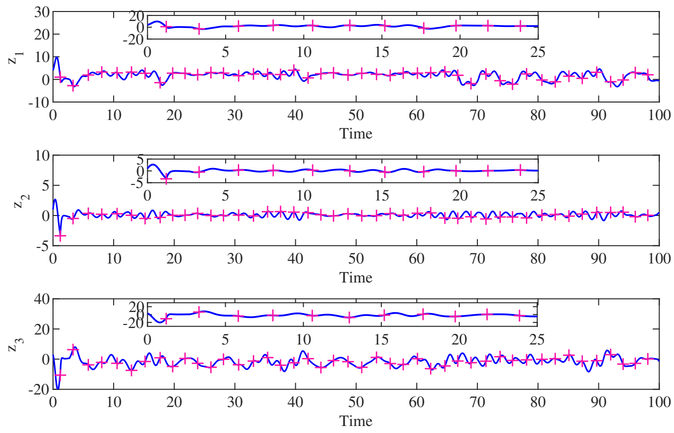

Figure 4. Stabilization to EISS of (4.2) by TAPIC, where + stands for the moment of impulse.

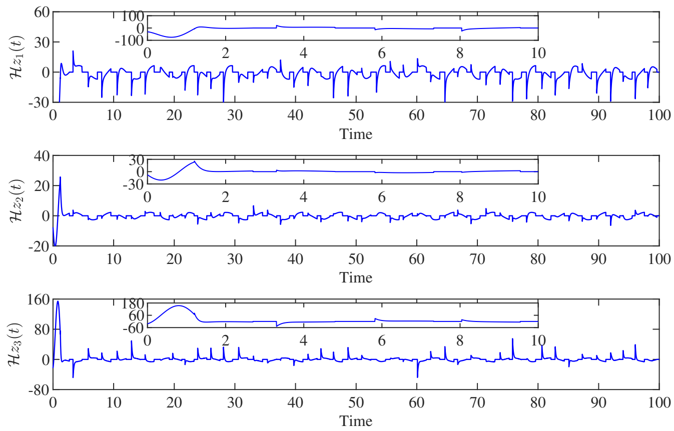

Figure 5. Intermittent control input of (4.2) under TAPIC.

Case 2. Stabilization to EISS via EAPIC: For EAPIC (3.15) and (3.16), we set $\gamma^* = 5 \geq max(\gamma_1, \gamma_2) = max(0.1905, 0.0993) = 0.1905$, $e^\sigma = 100$, $\epsilon = 10$, and $\{t_r\}$ is determined by

$$t_{r+1} = \begin{cases} inf\{t : t \in \Phi_r(t_r + 1.5, t_r + 10]\}, & if\ \Phi_r(t_r + 1.5, t_r + 10] \neq \emptyset, \\ t_r + 10, & if\ \Phi_r(t_r + 1.5, t_r + 10] = \emptyset, \end{cases}$$

where $\Phi_r(\eta, t] = \{t : t > \eta + 1.5, V(z(t)) \geq 100V(z(\eta + 1.5)) + 5|\omega(t)|^2_{[\eta+1.5,t]}\}$, and $V(z(t)) = z^T(t)Pz(t)$. From Theorem 3.3, (4.2) is stabilized to EISS by the EAPIC. Figures 6 and 7 show the state trajectories of $z(t)$ and the intermittent control inputs of $u(t)$, respectively. Figure 8 shows the event-triggered intervals of the system (4.2).

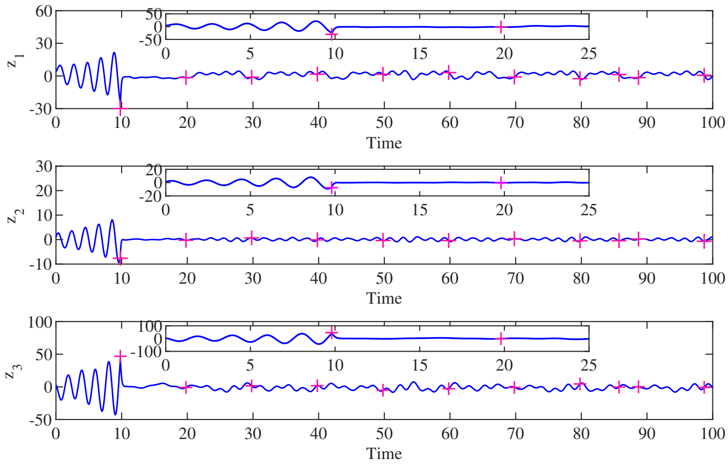

Figure 6. Stabilization to EISS of (4.2) by EAPIC, where + stands for the moment of impulse.

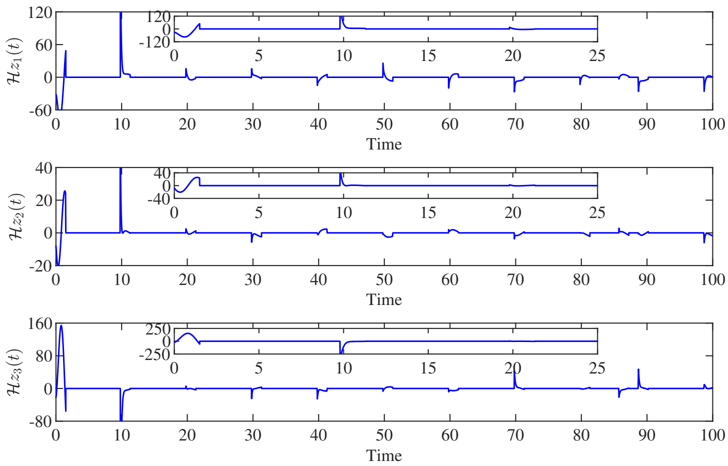

Figure 7. Intermittent control input of (4.2) under EAPIC.

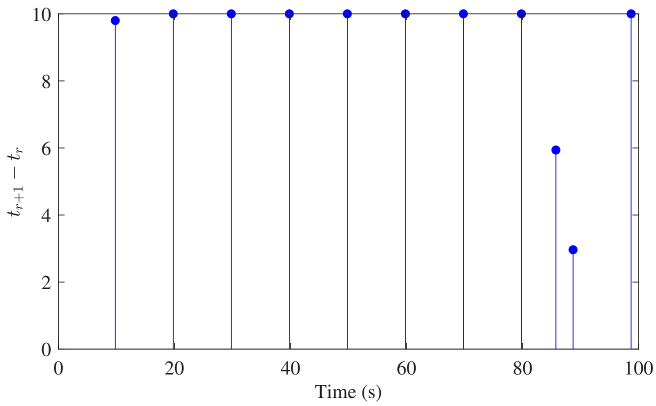

Figure 8. Event-triggered intervals of (4.2).

Remark 4.1. *In the example of Chua's oscillator, we have used the same control gain matrix $\mathscr{H}$, the same parameters, the same length of the active interval, and the same initial value, except for the different impulse instant t_r. With both control schemes, TAPIC and EAPIC, it is clear that at the same time, under EAPIC, the trigger frequency is lower, which is more resource-saving and less costly to control. This indicates that EAPIC outperforms TAPIC in terms of efficiency.*

5. Conclusions

This study investigates the EISS of nonlinear systems under impulsive disturbance and external inputs by means of APIC strategies, including TAPIC and EAPIC strategies. Compared with PIC [20–22], which requires fixed control intervals, the proposed APIC framework provides enhanced flexibility through its adaptive control mechanism, while EAPIC further reduces resource consumption compared with conventional EC [30–33] by synergistically combining intermittent execution with state-dependent triggering. Unlike the existing impulsive stabilization methods [17], our approach explicitly addresses hybrid disturbances (both impulsive and continuous inputs) and establishes tractable LMI-based conditions for EISS. Numerical simulations of Chua's oscillator demonstrate the superiority of EAPIC over TAPIC. Future work could extend this framework to stochastic systems. In addition, EISS for nonlinear impulsive systems can be studied via the EAPIC framework.

Author contributions

Siyue Yao: Writing–original draft; Jin-E Zhang: Supervision, Writing–review and editing. Both authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflicts of interest

The authors declare no conflicts of interest.

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