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Research article

Decision analysis with known and unknown weights in the complex N-cubic fuzzy environment: An application of accident prediction models

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Abstract: The component of Pakistan's road safety management (RSM) systems that appears to be the least reliable is the evaluation of road safety measures. Road safety initiatives' daily operations, such as allocating specific financial resources and incorporating measures for road safety into the fabric of culture, are only sometimes observed by governments. When this happens, the analysis usually concentrates on issues related to the infrastructure and the enforcement of laws; thorough evaluations of road safety initiatives are incredibly uncommon. Road authorities, practitioners, and architects of road safety depend on prediction tools, often known as accident prediction models (APMs). These instruments are employed to assess safety concerns, pinpoint areas for improvement, and calculate the expected safety consequences of these modifications. The goal of this research is to use the complex N-cubic fuzzy set (CNCFS), an innovative and practical tool for decision making that excels at handling imprecise or ambiguous data in real-world decision-making processes, in the context. This study also proposes a novel entropy approach to multi-attribute group decision-making issues in RSM. We also investigate the assessment of accident forecasting models in RSM to demonstrate the feasibility and efficacy of the suggested strategy. Further, the advantages and superiority of the proposed strategy are explained using the experimental data and comparisons with known and unknown weights obtained by the entropy method. The study's conclusions demonstrate that the suggested approach is more workable and compatible with other current strategies.

Keywords: inter-valued fuzzy set; fuzzy set; N-cubic fuzzy set; entropy technique; decision-making
Mathematics Subject Classification: 03E72, 94D05

1. Introduction

The most lacking component of road safety management road safety management (RSM) systems in Pakistan is usually thought to be the evaluation of measures to improve road safety. Highway authorities and safety experts need to know how to prevent accidents. Models like these make it easier to look at safety problems, find ways to make things safer, and estimate how these changes would affect the number of collisions that occur. The purpose of this study is to apply the complex N-cubic fuzzy aggregation operators in conjunction with multi-attribute decision making (MADM) techniques to provide a novel solution for the problems of road safety management. To further illustrate the usefulness and superiority of the recommended approach, we execute a case study to evaluate RSM accident prediction methods. Ultimately, the advantages and superiority of the proposed technique are explained through an analysis of the experiment data and comparisons with current approaches. The study's conclusions demonstrate that the suggested strategy is more workable and consistent with other strategies already in use. Their goal is to improve transportation infrastructure, safety, and management. It has been demonstrated that accident prediction models (APMs) are crucial tools in the RSM sector. These complex mathematical frameworks forecast the possibility of accidents happening in specific locations or under circumstances by utilizing historical data, weather, patterns of traffic, and other relevant variables. Road safety officials can strategically manage resources, conduct targeted operations, and establish comprehensive programs to lower the incidence of accidents by recognizing dangerous places and possible areas of risk. In addition to improving overall road safety and allocating resources as efficiently as possible, these models lower the monetary and human consequences of collisions. These models for forecasting will become increasingly effective and precise as technology develops, contributing significantly to everyone's ability to live in safer and more secure neighborhoods. Table 1 provides a review of the RSM literature.

Table 1. Research that has already been done on RSM.

Reference	Approach	Application
Deveci et al. [1]	Logarithmic approach of additive weights based on fuzzy Einstein and TOPSIS technique	Assessment of applications of metaverse traffic safety
Xie et al. [2]	Fuzzy logic	Thorough assessment of the risks associated with driving on motorways
Cubranic-Dobrodolac et al. [3]	Application of fuzzy inference systems of interval type-2	Examine how perceptions of speed and space affect the frequency of traffic accidents.
Al-Omari et al. [4]	Fuzzy logic and geographic information system	Prediction the locations of high-traffic accidents
Zaranezhad et al. [5]	Ant colony optimisation algorithm, fuzzy systems, genetic algorithm, and artificial neural network	Creation of prediction models for incidents involving maintenance and repairs at oil refineries
Mitrovic Simic et al. [6]	A new fuzzy MARCOS model, fuzzy complete consistency approach, and data envelope analysis	Road segment safety is assessed using the road's geometric parameters.
Koçar et al. [7]	Fuzzy logic	A methodology for evaluating the risk of road accidents
Stevic et al. [8]	Multi-criteria decision-making model with integration	Traffic risk assessment of two-lane road segments
Ahammad et al. [9]	Technique for crash prediction models	A new strategy to prevent traffic accidents and create traffic safety regulations
Gecchele et al. [10]	The fuzzy delphi procedure of analytic hierarchy	A versatile method for choosing areas for vehicle traffic counting
Jafarzadeh Ghoushchi et al. [11]	Combined MARCOS and SWARA techniques in a spherically fuzzy environment	Risk prioritisation and road safety evaluation
Mohammadi et al. [12]	Fuzzy-analytical hierarchy process	Road traffic accidents involving pedestrians in urban areas
Garnaik et al. [13]	Fuzzy inference system	Highway design's effect on traffic safety
Gaber et al. [14]	Fuzzy logic algorithm	Data on traffic safety on rural roads: analysis and modelling
Benallou et al. [15]	Fuzzy Bayesian	An assessment of the probability of accidents brought on by truck drivers
Gou et al. [16]	linguistic preference orderings	Medical health resources allocation evaluation in public health emergencies
Hussain, A., & Ullah, K. [17]	spherical fuzzy	An intelligent decision support system and real-life applications
Wang et al. [18]	complex intuitionistic fuzzy	Application for resilient green supplier selection
Xiao [19]	Quantum X-entropy	Quantum X-entropy in generalized quantum evidence theory
Xiao [20]	Intuitionistic fuzzy set	A distance measure and its application to pattern classification problems

The fuzzy set (FS) notion was first presented by Zadeh [21] as a way to deal with uncertainty in real-world decision-making circumstances. FS-based MAGDM methods and FS extensions were then presented. Just the degree of membership (MD) is employed in the context of FS to describe ambiguous facts; there is no discussion of the non-membership degree (NMD). Thus, the concept of intuitionistic FS (IFS) was created by Atanassov [22], where two parts define every component: first is MD, indicated by the symbol u, and the NMD, denoted by the symbol v. Yager [23] developed the Pythagorean FS (PyFS), which is restricted as $u + v \geq 1$ but $u^2 + v^2 \geq 1$ in order to get around the IFS's shortcoming. MDs and NMDs are utilized to generate PyFS, just like IFS. Since PyFS has less restrictive limitations than IFS, MD or NMD can't have squares larger than one. People are unable to function in circumstances when the total squared MD and NMD levels are more than 1. Yager [24] extended the traditional FS theory with the idea of q-rung orthopair FS (q-ROFS). To solve the previously mentioned PyFS constraint, they enforced the condition that $\mu^q + \nu^q \geq 1$. As a result, the development of q-ROFS-based MAGDM techniques has drawn considerable interest as a novel field of study, resulting in the publication of numerous creative approaches to decision-making. Consequently, q-ROFS is better equipped to deal with unclear data flexibly and suitably. Numerous researchers have examined the q-ROFS, and their findings have led to a number of developments [25,26]. In addition to being valuable tools for handling MADM or MAGDM issues, aggregation operators (AOs) also offer advantages. Deveci et al. [27] looked at three alternative implementation options for driverless autos in the virtual world. The suggested MADM technique was used for those alternatives in order to evaluate them based on twelve distinct attributes. The characteristics were divided into four groups: technology, ethical and legal transportation, and sociological. Demir Uslu et al. [28] emphasized the main barriers to a sustainable healthcare plan during COVID-19. The overall compromise solution is a hybrid decision-making approach that was proposed by Deveci et al. [29]. Included were the weighted q-ROF Hamacher average and weighted q-ROF Hamacher geometric mean AOs. Fetanat and Tayebi created a novel decision support system known as q-ROFS-based multi-attributive ideal-real comparative analysis [30]. Pakistani national road governments, designers, and road safety engineers must make greater use of the APM despite its strong scientific foundation, which makes it easier to assess and choose road safety measures and permits effective decision-making under financial constraints. That goal can be achieved by applying APM research, which shows a strong demand for adoption but is not readily available at this time. FS theory can handle the problem of ambiguous, interpretative, and uncertain judgments. One kind of FS used to deal with ambiguity and imprecision in decision-making is called q-ROFS. It is a PyFS generalization that enables the analysis and representation of complicated data.

Moreover, Soujanya and Reddy [31] developed the concept of N-cubic picture fuzzy linear spaces. Madasi et al. [32] presented the idea of N-cubic structure to q-ROFSs in decision-making problems. Kavyasree and Reddy [33] presented $P(R)$-union of internal, $P(R)$-intersection and external cubic picture hesitant linear spaces and their properties. Tanoli et al. [34] expressed some algebraic operations for complex cubic fuzzy sets and their structural properties. Karazma et al. [35] expressed some new results in $\mathcal{N}$-cubic sets. Many of the scholars worked on cubic and N-cubic fuzzy sets and their extended versions in decision-making problems; for this, see the papers [36–42]. Furthermore, Khoshaim et al. [43] developed the picture CFSs theory and presented some operations for picture CFS. Muhaya and Alsager [44] introduced a new concept about cubic bipolar neutrosophic sets. Sajid et al. [45] modified some aggregations for the cubic intuitionistic fuzzy hypersoft set. Recently, in 2024, many researchers worked on cubic sets, picture cubic sets, picture fuzzy cubic graphs and N-cubic Sets; for this literature, see [46–50].

Adjusting this value to fit distinct decision-making scenarios can help us handle instances where varying degrees of uncertainty exist. CNCFS is an excellent decision-making tool that helps us deal with complicated and unpredictable situations more efficiently. It enables DMs to consider both the imprecision and the uncertainty of the data when representing and analyzing complicated linguistic information in decision-making situations. Our motivation for developing the CNCFSs method is as follows:

- The CNCF approach offers a legitimate and compatible framework for characterizing the NCF information, extending the structure of earlier models. The method is predicated on NCF assessments, which are subsequently transformed into fuzzy data. This provides decision-making information and significantly increases the accuracy of decision making.
- A wide range of applications for the suggested approach is made possible by the flexibility of the NCFS representation model, which also improves comprehension of how to apply quantitative CNCF information in decision-making situations.
- Since the approach analyses the superiority, equality, and inferiority relations of the best option to other alternatives as well as the visual representation, it is more wonderful and suitable for producing interesting ranking outcomes.

The suggested work's flexibility helps to clarify the problems and addresses the shortcomings of the previous approaches. Utilizing various competing qualities, a case study is conducted to assess the superiority, equity, and inadequacy of APMs for RSM utilizing the proposed CNCF averaging aggregation operators' technique. A thorough and sorted list of possibilities is then produced using the CNCF approach. We use the existing models to choose the APMs for RSM to evaluate the validity of the new method. The comparison findings provide a decreasing ranking of the answers and demonstrate that the proposed method may be applied successfully to MAGDM situations. Using a novel methodology, this work advances the development and evolution of the decision-making scenario. The contributions made to the paper are as follows:

- Using the proposed techniques, we offer a unique methodology in a CNCF context.
- We established the notion of a WPA operator.
- We conduct a case study to assess APMs for RSM using the MAGDM approach.
- A comparative analysis is conducted using known and unknown weights obtained by the entropy method.

In most practical problems, particularly in MAGDM problems, the determination of attribute weights plays a crucial role in the aggregation process. The importance of each attribute directly influences the final decision outcome, as different attributes contribute unequally to the overall evaluation of alternatives. Proper weight allocation ensures that the decision-making process accurately reflects the relative significance of each criterion, leading to a more rational and reliable assessment. The entropy method is a widely used objective weighting technique in decision making, especially under uncertainty. This method ensures unbiased weight distribution by relying purely on the intrinsic characteristics of the data, eliminating subjective bias. The entropy method determines the weight of each criterion based on the amount of information it provides. Many researchers [51–53] work in decision making with the help of the entropy technique.

The rest of this manuscript is structured as follows: A few basic ideas regarding the NCFS and CNCFS are covered in Section 2. Section 3 provides a full explanation of the CNCFA and CNCFG decision analysis approach used to solve the MAGDM problem. Section 4 presents the details of CNCFOWA and CNCFOWG operators. In Section 5, we discuss about CNCF hybrid weighted averaging and geometric operators. Section 6 presents an example of APMs for RSM in Pakistan, which is solved using the methods outlined in Sections 3–5. Section 7 presents the research's conclusions, limitations, and recommendations for further work. The article's primary job is displayed in Figure 1.

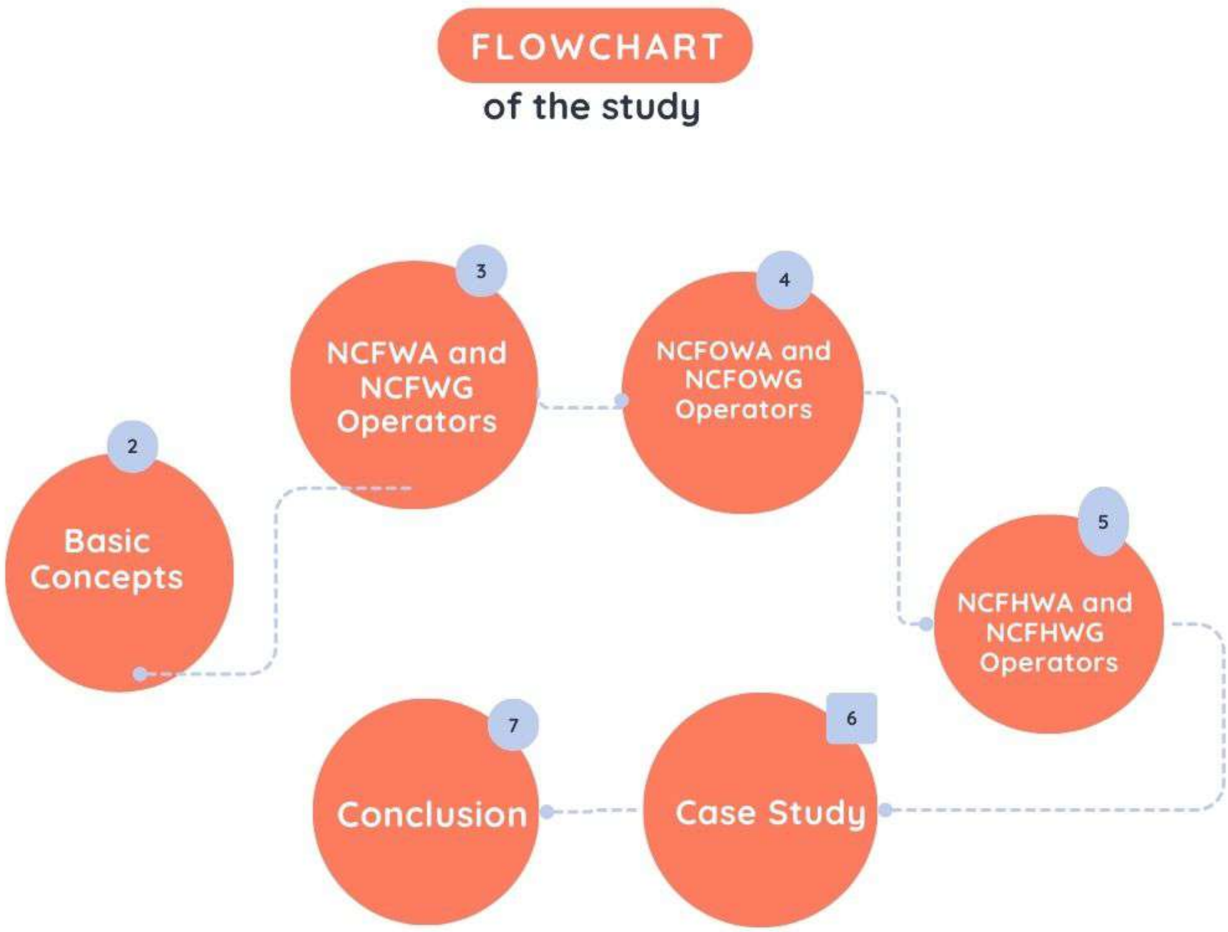

Figure 1. The framework of the study.

2. Basic concepts

In the following section, we use the universal set X to describe a few fundamental concepts regarding IFSs and CNCFSs.

Definition 2.1. An IFS z is defined on X is given by

$$z = \{(u, e_z(u), c_z(u))|u \in X\}, \tag{2.1}$$

where e_z is MD and c_z is NMD, such that $0 \leq (u, e_z(u), c_z(u)) \leq 1$ for each $u \in X$.

Definition 2.2. A CIFS [54] z is defined on X is given by

$$z = \left\{\left(u, \left(e_z(u), \omega_{e_z(u)}\right), \left(c_z(u), \omega_{c_z(u)}\right)\right) |u \in X\right\}, \tag{2.2}$$

with $\left(e_z(u), \omega_{e_z(u)}\right)$ and $\left(c_z(u), \omega_{c_z(u)}\right)$ representing the MD and NMD of the element, respectively.

3. Construction of aggregation operators of CNCF

This section employs the new definition of the complex N-cubic fuzzy set, score function and introduces some operational laws of CNCFSs. Later, we will modify the CNCFWA and CNCFWG aggregation operators.

Definition 3.1. A complex N-cubic fuzzy set z defined on a universe X is given by:

$$z = \left\{\left(u, \left[\left(e_z^-(u), \omega_{e_z^-(u)}\right), \left(e_z^+(u), \omega_{e_z^+(u)}\right)\right], \left(c_z(u), \omega_{c_z(u)}\right)\right) \mid u \in X\right\}, \tag{3.1}$$

where $e_z^-(u)$ and $e_z^+(u)$ denote the lower and upper bounds of the membership degree, respectively, and $\omega_{e_z^-(u)}$, $\omega_{e_z^+(u)}$ represent their associated complex lower and upper bounds.

This structure extends the traditional N-cubic fuzzy set, providing a more refined representation of uncertainty in decision-making problems.

Definition 3.2. Let $z_j = ((e_j, \omega_{e_j}), (c_j, \omega_{c_j}))(j = 1, 2, ...m)$ be a complex N-cubic value (CNCV). Then, the score function is

$$s(z) = \frac{1}{6}\left(e_1^-(z) + e_1^+(z) + c_1(z) + \frac{\omega_{e_1^-}(z) + \omega_{e_1^+}(z) + \omega_{c_1}(z)}{2\pi}\right), \tag{3.2}$$

where $s(z) \in [-1, 0]$. Then,

(1) $s(z_2) \prec s(z_1)$, its mean $z_2 \prec z_1$.
(2) $s(z_1) \prec s(z_2)$, its mean $z_1 \prec z_2$.
(3) $s(z_2) = s(z_1)$, its mean $z_1 = z_2$.

Definition 3.3. For two CNCFNs $z_1 = ((e_1, \omega_{e_1}), (c_1, \omega_{c_1}))$, $z_2 = ((e_2, \omega_{e_2}), (c_2, \omega_{c_2}))$ and λ is real number,

(1) $z_1 \oplus z_2 = \Big(\Big[\Big(-1+\prod_{j=1}^{2}\Big(1+e_j^-\Big), -1+\prod_{j=1}^{2}\Big(1+\omega_{e_j^-}\Big)\Big),$
$\Big(-1+\prod_{j=1}^{2}\Big(1+e_j^+\Big), -1+\prod_{j=1}^{2}\Big(1+\omega_{e_j^+}\Big)\Big)\Big], \Big(\prod_{j=1}^{2} c_j), (-1)^{n+1}\prod_{j=1}^{2}\omega_{c_j})\Big)\Big).$

(2) $z_1 \oplus z_2 = \Big(\Big((-1)^{n+1}\prod_{j=1}^{2} e_j), (-1)^{n+1}\prod_{j=1}^{2}\omega_{e_j})\Big), \Big[\Big(-1+\prod_{j=1}^{2}\Big(1+c_j^-\Big), -1+\prod_{j=1}^{2}\Big(1+\omega_{c_j^-}\Big)\Big),$
$\Big(-1+\prod_{j=1}^{2}\Big(1+c_j^+\Big), -1+\prod_{j=1}^{2}\Big(1+\omega_{c_j^+}\Big)\Big)\Big]\Big).$

(3) $\lambda z_1 = \Big(\Big[\Big(-1+(1+e_1^-)^\lambda, -1+(1+\omega_{e_1^-})^\lambda\Big), \Big(-1+(1+e_1^+)^\lambda, 1-(1+\omega_{e_1^+})^\lambda\Big)\Big], \Big((-1)^{n+1}(c_1)^\lambda, ((-1)^{n+1}\omega_{c_1})^\lambda\Big)\Big).$

(4) $z_1^\lambda = \Big(\Big((-1)^{n+1}(e_1)^\lambda, (-1)^{n+1}(\omega_{e_1})^\lambda\Big), \Big[\Big(-1+(1+c_1^-)^\lambda, -1+(1+\omega_{c_1^-})^\lambda\Big), \Big(-1+(1+c_1^+)^\lambda, -1+(1+\omega_{c_1^+})^\lambda\Big)\Big]\Big).$

3.1. CNCFWA and CNCFWG operators

Suppose that $F(X)$ be a set of all $CNCFNs$. This section express some weighted and geometric AOs.

Definition 3.4. Let $z_j = ((e_j, \omega_{e_j}), (c_j, \omega_{c_j}))(j = 1, 2, ...m)$ be the collection of CNCV. Then, the definition of $CNCFWA$ operator is

$$CNCWA_w(z_1, z_2, \ldots, z_m) = (\zeta_1 z_1 \oplus \zeta_2 z_2 \oplus \cdots \oplus \zeta_m z_m), \tag{3.3}$$

where $\zeta = (\zeta_1, \zeta_2, ..., \zeta_m)^T$ is weight vector (WV) and $\sum_{j=1}^{m} \zeta_j = 1$.

Theorem 3.1. *Assuming $z_j = ((e_j, \omega_{e_j}), (c_j, \omega_{c_j}))(j = 1, 2, ...m)$ is a set of CNCFNs, then the obtained result by CNCWA operator is still a CNCFN, and*

$$CNCFWA(\zeta_1, \zeta_2, ..., \zeta_m) = \left\{\left[\left(-1+\prod_{j=1}^{m}\left(1+e_j^-\right)^{\zeta_j}, -1+\prod_{j=1}^{m}\left(1+\omega_{e_j^-}\right)^{\zeta_j}\right),\right.\right. \tag{3.4}$$

$$\left.\left.\left(-1+\prod_{j=1}^{m}\left(1+e_j^+\right)^{\zeta_j}, -1+\prod_{j=1}^{m}\left(1+\omega_{e_j^+}\right)^{\zeta_j}\right)\right], \prod_{j=1}^{m}(-1)^{n+1}(c_j)^{\zeta_j}, \prod_{j=1}^{m}(-1)^{n+1}(\omega_{c_j})^{\zeta_j}\right\}. \tag{3.5}$$

Proof. See the proof in Appendix A.

□

Definition 3.5. Let $z_j = ((e_j, \omega_{e_j}), (c_j, \omega_{c_j}))(j = 1, 2, ...m)$ be the collection of CNCV. Then, the definition of complex N-cubic weighted geometric average operator is

$$CNCWG_w(z_1, z_2, \ldots, z_m) = ((z_1)^{\zeta_1} \otimes (z_2)^{\zeta_2} \otimes \cdots \otimes (z_m)^{\zeta_m}), \tag{3.6}$$

where $\zeta = (\zeta_1, \zeta_2, ..., \zeta_m)^T$ is WV and $\sum_{j=1}^{m} \zeta_j = 1$.

Theorem 3.2. *Let $z_j = ((e_j, \omega_{e_j}), (c_j, \omega_{c_j}))(j = 1, 2, ...m)$ be a collection of CNCFNs. Then, the aggregated value by the CNCWG operator is still a CNCFN and*

$$CNCFWG(\zeta_1, \zeta_2, ..., \zeta_m) = \left\{\left(\prod_{j=1}^{m}(-1)^{n+1}(e_j)^{\zeta_j}, \prod_{j=1}^{m}(-1)^{n+1}(\omega_{e_j})^{\zeta_j}\right),\right. \tag{3.7}$$

$$\left[\left(-1+\prod_{j=1}^{m}\left(1+c_j^-\right)^{\zeta_j},-1+\prod_{j=1}^{m}\left(1+\omega_{c_j^-}\right)^{\zeta_j}\right),\right. \tag{3.8}$$

$$\left.\left.\left(-1+\prod_{j=1}^{m}\left(1+c_j^+\right)^{\zeta_j},-1+\prod_{j=1}^{m}\left(1+\omega_{c_j^+}\right)^{\zeta_j}\right)\right]\right\}. \tag{3.9}$$

Proof. See proof in Appendix B. □

4. CNCFOWA and CNCFOWG operators

The CNCFSs can effectively represent the fuzzy information, and the traditional NCFSs operator can only process the real numbers. It is crucially essential to generalize aggregation operators to process CNCFNs. In this section, some aggregation operators are introduced to fuse CNCFNs. Firstly, the order weighted aggregation operator is generalized under CNCF information, and some CNCFOWA and CNCFOWG aggregation operators are presented.

Definition 4.1. Let $z_j = ((e_j, \omega_{e_j}), (c_j, \omega_{c_j}))(j = 1, 2, ...m)$ be the collection of CNCV. Then, the definition of CNCFOWA operator is

$$CNCOWA_w(z_1, z_2, \ldots, z_m) = (\zeta_1 z_{\sigma(1)} \oplus \zeta_2 z_{\sigma(2)} \oplus \cdots \oplus \zeta_m z_{\sigma(m)}), \tag{4.1}$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(m))$ is a permutation of $(1, 2, \ldots, n)$, such that $z_{\sigma(j-1)} \geq z_{\sigma(j)}$ and $\zeta = (\zeta_1, \zeta_2, ..., \zeta_m)^T$ is WV and $\sum_{j=1}^{m} \zeta_j = 1$.

Theorem 4.1. *Assuming $z_j = ((e_j, \omega_{e_j}), (c_j, \omega_{c_j}))(j = 1, 2, ...m)$ is a set of CNCFNs. Then, the obtained result by CNCOWA operator is still a CNCFN, and*

$$CNCFOWA(\zeta_1, \zeta_2, ..., \zeta_m) = \left\{\left[\left(-1+\prod_{j=1}^{m}\left(1+e_{\sigma(j)}^-\right)^{\zeta_j},-1+\prod_{j=1}^{m}\left(1+\omega_{e_{\sigma(j)}^-}\right)^{\zeta_j}\right),\right.\right. \tag{4.2}$$

$$\left.\left(-1+\prod_{j=1}^{m}\left(1+e_{\sigma(j)}^+\right)^{\zeta_j},-1+\prod_{j=1}^{m}\left(1+\omega_{e_{\sigma(j)}^+}\right)^{\zeta_j}\right)\right], \tag{4.3}$$

$$\left.\left(\prod_{j=1}^{m}(-1)^{n+1}(c_{\sigma(j)})^{\zeta_j},\prod_{j=1}^{m}(-1)^{n+1}(\omega_{c_{\sigma(j)}})^{\zeta_j}\right)\right\}. \tag{4.4}$$

Proof. Theorem 3.1 can be used to illustrate this theorem. □

Definition 4.2. Let $z_j = ((e_j, \omega_{e_j}), (c_j, \omega_{c_j}))(j = 1, 2, ...m)$ be the collection of CNCV. Then, the definition of CNCFOWG operator is

$$CNCOWG_w(z_1, z_2, \ldots, z_m) = ((z_{\sigma(1)})^{\zeta_1} \otimes (z_{\sigma(2)})^{\zeta_2} \otimes \cdots \otimes (z_{\sigma(m)})^{\zeta_m})), \tag{4.5}$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(m))$ is a permutation of $(1, 2, \ldots, n)$, such that $z_{\sigma(j-1)} \geq z_{\sigma(j)}$ and $\zeta = (\zeta_1, \zeta_2, ..., \zeta_m)^T$ is WV and $\sum_{j=1}^{m} \zeta_j = 1$.

Theorem 4.2. *Assuming $z_j = ((e_j, \omega_{e_j}), (c_j, \omega_{c_j}))(j = 1, 2, ...m)$ is a set of CNCFNs, then the obtained result by CNCOWG operator is still a CNCFN. and*

$$CNCFOWG(\zeta_1, \zeta_2, ..., \zeta_m) = \left\{ \left(\left(\prod_{j=1}^{m} (-1)^{n+1} (e_{\sigma(j)})^{\zeta_j}, \prod_{j=1}^{m} (-1)^{n+1} (\omega_{e_{\sigma(j)}})^{\zeta_j} \right), \right. \right. \tag{4.6}$$

$$\left[\left(-1 + \prod_{j=1}^{m} \left(1 + c^{-}_{\sigma(j)} \right)^{\zeta_j}, -1 + \prod_{j=1}^{m} \left(1 + \omega_{c^{-}_{\sigma(j)}} \right)^{\zeta_j} \right), \right. \tag{4.7}$$

$$\left. \left. \left(-1 + \prod_{j=1}^{m} \left(1 + c^{+}_{\sigma(j)} \right)^{\zeta_j}, -1 + \prod_{j=1}^{m} \left(1 + \omega_{c^{+}_{\sigma(j)}} \right)^{\zeta_j} \right) \right] \right\}. \tag{4.8}$$

Proof. Theorem 3.2 can be used to illustrate this theorem. □

5. CNCF hybrid weighted averaging and geometric operators

CNCFSs can effectively represent fuzzy information, while the traditional NCFS operator can only process real numbers. It is crucial to generalize aggregation operators to process CNCFNs. In this section, some aggregation operators are introduced to fuse CNCFNs. To start, the order-weighted aggregation operator is generalized under CNCF information, and some CNCFHWA and CNCFHWG aggregation operators are presented.

Definition 5.1. Let $z_j = ((e_j, \omega_{e_j}), (c_j, \omega_{c_j}))(j = 1, 2, ...m)$ be the collection of CNCV. Then, the definition of CNCFHWA operator is

$$CNCHWA_w(z_1, z_2, \ldots, z_m) = (\zeta_1 \dot{z}_{\sigma(1)} \oplus \dot{\zeta}_2 z_{\sigma(2)} \oplus \cdots \oplus \zeta_m \dot{z}_{\sigma(m)}), \tag{5.1}$$

where $z_{\sigma(j)}$ is the j^{th} largest element of the CNCF arguments $z_j(\dot{z}_j = (n\zeta j)z_j)$, $j = (1, 2, \ldots, m)$.

Theorem 5.1. *Assuming $z_j = ((e_j, \omega_{e_j}), (c_j, \omega_{c_j}))(j = 1, 2, ...m)$ is a set of CNCFNs, then the obtained result by CNCHWA operator is still a CNCFN, and*

$$CNCFHWA(\zeta_1, \zeta_2, ..., \zeta_m) = \left\{ \left[\left(-1 + \prod_{j=1}^{m} \left(1 + \dot{e}^{-}_{\sigma(j)} \right)^{\zeta_j}, -1 + \prod_{j=1}^{m} \left(1 + \omega_{\dot{e}^{-}_{\sigma(j)}} \right)^{\zeta_j} \right), \right. \right. \tag{5.2}$$

$$\left. \left(-1 + \prod_{j=1}^{m} \left(1 + \dot{e}^{+}_{\sigma(j)} \right)^{\zeta_j}, -1 + \prod_{j=1}^{m} \left(1 + \omega_{\dot{e}^{+}_{\sigma(j)}} \right)^{\zeta_j} \right) \right], \tag{5.3}$$

$$\left. \left(\prod_{j=1}^{m} (-1)^{n+1} (\dot{c}_{\sigma(j)})^{\zeta_j}, \prod_{j=1}^{m} (-1)^{n+1} (\omega_{\dot{c}_{\sigma(j)}})^{\zeta_j} \right) \right\}. \tag{5.4}$$

Proof. Theorem 3.1 can be used to illustrate this theorem. □

Definition 5.2. Let $z_j = ((e_j, \omega_{e_j}), (c_j, \omega_{c_j}))(j = 1, 2, ...m)$ be the collection of CNCV. Then, the definition of CNCFHWG operator is

$$CNCHWG_w(z_1, z_2, \ldots, z_m) = ((\dot{z}_{\sigma(1)})^{\zeta_1} \otimes (\dot{z}_{\sigma(2)})^{\zeta_2} \otimes \cdots \otimes (\dot{z}_{\sigma(m)})^{\zeta_m}), \tag{5.5}$$

where $z_{\sigma(j)}$ is the j^{th} largest element of the CNCF arguments $z_j(\dot{z}_j = (n\zeta j)z_j), j = (1, 2, \ldots, m)$.

Theorem 5.2. *Let $z_j = ((e_j, \omega_{e_j}), (c_j, \omega_{c_j}))(j = 1, 2, ...m)$ is a set of CNCFNs. Then, the obtained result by CNCHWG operator is still a CNCFN, and*

$$CNCFHWG(\zeta_1, \zeta_2, ..., \zeta_m) = \left\{ \left(\prod_{j=1}^{m} (-1)^{n+1} (\dot{e}_{\sigma(j)})^{\zeta_j}, \prod_{j=1}^{m} (-1)^{n+1} (\omega_{\dot{e}_{\sigma(j)}})^{\zeta_j} \right), \right. \tag{5.6}$$

$$\left[\left(-1 + \prod_{j=1}^{m} \left(1 + \dot{c}^{-}_{\sigma(j)}\right)^{\zeta_j}, -1 + \prod_{j=1}^{m} \left(1 + \omega_{\dot{c}^{-}_{\sigma(j)}}\right)^{\zeta_j} \right), \right. \tag{5.7}$$

$$\left. \left. \left(-1 + \prod_{j=1}^{m} \left(1 + \dot{c}^{+}_{\sigma(j)}\right)^{\zeta_j}, -1 + \prod_{j=1}^{m} \left(1 + \omega_{\dot{c}^{+}_{\sigma(j)}}\right)^{\zeta_j} \right) \right] \right\}. \tag{5.8}$$

Proof. Theorem 3.2 can be used to illustrate this theorem. □

6. Case study

In this part, a case study is used to demonstrate the proposed method's flexibility and efficacy. To validate our efforts, we engage in the challenging task of identifying which APMs are most beneficial to Pakistani RSMs. APMs are instruments that can help manage traffic safety by forecasting the expected frequency and impact of crashes at a particular road intersection. They can also be applied to choose investments in road safety and assess the effectiveness of existing measures. APMs are derived from statistical evaluation of accident problem data, which includes the road network's structure, traffic flow, speed limitation, and other criteria. Over 30,000 people are killed and 500,000 are injured in Pakistani road accidents each year, making them serious problems for the country's economy and public health. However, Pakistan does not regularly deploy APMs to control traffic safety. Some of the reasons why the APMs are not used in Pakistan are the dearth of collision data and the inadequate quality of the road infrastructures. Another obstacle is the lack of awareness and experience among road authorities, road planners, and road safety experts about the benefits and uses of APMs. Some research has attempted to create and implement APMs for various road types in Pakistan, such as motorways, rural roads, and urban highways. The current research employs a variety of methodologies and data sources, such as cluster analysis, machine learning techniques and regression models. However, the validity and scope of these studies are limited, and little attention has been paid to the results or how they were used. As a result, more accurate and reliable APMs are needed to capture the unique characteristics and difficulties of road safety in Pakistan.

6.1. Ethical considerations related to RSM decisions

RSM ethical considerations are essential to guaranteeing everyone's safety when driving. One ethical consideration is the duty of drivers to follow traffic rules and laws. This entails observing posted speed limits, ceding to pedestrians, and stopping at stop signs and red lights. Drivers who disregard these restrictions put themselves in danger and other road users in danger. Another ethical factor is the duty of transportation organizations and the government to maintain secure and well-maintained roads. This entails ensuring that roads are appropriately planned, built, inspected, and repaired regularly, as well as installing the right signage and traffic controls. To pinpoint areas that require enhancement, it is equally critical that these organizations gather and examine data on traffic safety. To encourage safe driving habits and guarantee the security of every person on the road, RSM ethical concerns are crucial. When making judgments on road safety, moral values, justice, and accountability are all part of the ethical considerations surrounding RSM decisions. The following are some moral considerations that affect RSM decisions:

- Life for people and welfare: The main ethical issue is putting the lives of people and welfare first. Considering the effects on people, families, and communities, decisions should be made with the goal of reducing damage, injuries, and fatalities on roadways.
- Fairness and equity: It's critical to make sure that decisions on road safety are equitable. This entails resolving differences in safety protocols among various populations or geographical areas, striving for a fair allocation of resources, and steering clear of choices that unjustly impact groups.
- Accountability and transparency: Ethical RSM calls for open communication and public participation regarding rules, policies, and their implementation, as well as transparency in decision-making procedures. It is essential to hold individuals in charge of putting road safety measures into place and enforcing them accountable.
- Priority balancing: Ethical issues entail striking a balance between different demands and primacy. For instance, there may be times when the need for safety conflicts with other goals, such as personal freedom or financial considerations. The trick is striking a reasonable balance without sacrificing safety.
- Risk assessment and mitigation: Making ethical decisions requires precisely identifying risks and implementing countermeasures. This entails considering the advantages and disadvantages of putting in place suitable safety regulations, keeping up with infrastructure, and using efficient enforcement techniques.
- Respect for laws, rules, and industry standards: Adherence to these legal and regulatory requirements is a crucial ethical factor. When feasible, aiming to go above and above the bare minimum required by law to improve safety shows a dedication to moral RSM.
- Constant improvement and adaptability: Ethical RSM necessitates a dedication to constant improvement and adaptation, which is founded on data, developing technology and guidelines. It is crucial to be receptive to fresh concepts and developments for safer road infrastructure.
- Public education and engagement: Ethical concerns stress the significance of educating people about road safety and involving the public. It is crucial to arm communities with information and promote a driving culture of responsibility.

By considering these ethical factors, RSM decisions can be more thorough, equitable, and focused on protecting lives and well-being while tackling the intricacies and difficulties of a contemporary transport system.

6.2. *Standards for the case study*

This section explains the criteria for choosing APMs for RSM in Pakistan and how the structure we suggest could be applied to reduce traffic accidents in other nations. One could classify Pakistan's APM selection for RSM as a classic MAGDM dilemma. This research presents the CNCFS approach to evaluating APM efficacy in an RSM environment in Pakistan. Artificial intelligence algorithms ($\tilde{N}_1$), text mining analysis ($\tilde{N}_2$), road design features and infrastructure ($\tilde{N}_3$), vehicle-related factors ($\tilde{N}_4$), and factors connected to the environment ($\tilde{N}_5$) are five potential APMs. The following criteria can be used to choose APMs real-time updates ($\tilde{M}_1$), analysis of feature importance ($\tilde{M}_2$), methods for machine learning ($\tilde{M}_3$), and visualization and reporting ($\tilde{M}_4$). Thus, DMs might provide CNCFS according to their preferences to evaluate the APMs for RSM in Pakistan. This case study offers a methodical approach to assessing various APMs by leveraging the various perspectives of DMs. To make data aggregation easier, four DMs use the CNCFNs to offer their CNCFS based on four qualities. The assessment values provided by four DMs for every attribute of each choice are shown in the decision matrix Tables 2–5.

6.3. *Algorithm for the case study*

Step 1. We introduce the CNCF assessment matrix. These matrices express the assessments of four DMs.
Step 2. Aggregate the collective decision matrix by applying CNCFWA operator.
Step 3. To find the unknown weight, we utilise the entropy formula as given in Eq 6.4.
Step 4. Find the aggregate values $\tilde{N}_i$ by using the unknown weights.
Step 5. Use the Definition 3.2 to find the score functions of $\tilde{T}_i$.
Step 6. Rank the various alternatives by using the CNCFWA, CNCFOWA, and CNCFHWA operators.

6.4. *Assessment of the case study*

The evaluation method used to choose APMs in RSM is described in this subsection. For this, the CNCFS is employed.
Step 1. We introduce the CNCF assessment matrix. These matrices express the assessments of four DMs as given in Tables 2–5.

Table 2. CNCF information table given by S_1.

	$\tilde{M}_1$	$\tilde{M}_2$
$\tilde{N}_1$	$\langle[(-0.41,-0.90),(-0.17,-0.70)],(-0.32,-0.52)\rangle$	$\langle[(-0.72,-0.59),(-0.18,-0.30)],(-0.33,-0.40)\rangle$
$\tilde{N}_2$	$\langle[(-0.30,-0.69),(-0.06,-0.29)],(-0.21,-0.42)\rangle$	$\langle[(-0.20,-0.32),(-0.06,-0.15)],(-0.11,-0.42)\rangle$
$\tilde{N}_3$	$\langle[(-0.70,-0.52),(-0.46,-0.29)],(-0.71,-0.72)\rangle$	$\langle[(-0.60,-0.49),(-0.36,-0.22)],(-0.61,-0.38)\rangle$
$\tilde{N}_4$	$\langle[(-0.52,-0.51),(-0.12,-0.30)],(-0.43,-0.22)\rangle$	$\langle[(-0.53,-0.36),(-0.23,-0.10)],(-0.52,-0.52)\rangle$
$\tilde{N}_5$	$\langle[(-0.80,-0.67),(-0.07,-0.45)],(-0.51,-0.60)\rangle$	$\langle[(-0.42,-0.83),(-0.11,-0.42)],(-0.11,-0.31)\rangle$
	$\tilde{M}_3$	$\tilde{M}_4$
$\tilde{N}_1$	$\langle[(-0.43,-0.91),(-0.09,-0.32)],(-0.34,-0.38)\rangle$	$\langle[(-0.44,-0.72),(-0.10,-0.42)],(-0.35,-0.32)\rangle$
$\tilde{N}_2$	$\langle[(-0.73,-0.72),(-0.36,-0.42)],(-0.11,-0.19)\rangle$	$\langle[(-0.10,-0.65),(-0.07,-0.42)],(-0.61,-0.39)\rangle$
$\tilde{N}_3$	$\langle[(-0.50,-0.55),(-0.26,-0.22)],(-0.61,-0.65)\rangle$	$\langle[(-0.39,-0.52),(-0.15,-0.19)],(-0.30,-0.39)\rangle$
$\tilde{N}_4$	$\langle[(-0.23,-0.91),(-0.10,-0.50)],(-0.31,-0.42)\rangle$	$\langle[(-0.41,-0.85),(-0.16,-0.32)],(-0.21,-0.60)\rangle$
$\tilde{N}_5$	$\langle[(-0.60,-0.62),(-0.13,-0.32)],(-0.34,-0.31)\rangle$	$\langle[(-0.31,-0.51),(-0.16,-0.32)],(-0.11,-0.41)\rangle$

Table 3. CNCF information table given by S_2.

	$\tilde{M}_1$	$\tilde{M}_2$
$\tilde{N}_1$	$\langle[(-0.80,-0.52),(-0.16,-0.29)],(-0.61,-0.81)\rangle$	$\langle[(-0.50,-0.60),(-0.13,-0.13)],(-0.66,-0.40)\rangle$
$\tilde{N}_2$	$\langle[(-0.70,-0.69),(-0.15,-0.29)],(-0.31,-0.51)\rangle$	$\langle[(-0.50,-0.54),(-0.06,-0.29)],(-0.21,-0.29)\rangle$
$\tilde{N}_3$	$\langle[(-0.43,-0.55),(-0.35,-0.22)],(-0.60,-0.32)\rangle$	$\langle[(-0.44,-0.23),(-0.14,-0.10)],(-0.35,-0.59)\rangle$
$\tilde{N}_4$	$\langle[(-0.50,-0.85),(-0.08,-0.32)],(-0.39,-0.41)\rangle$	$\langle[(-0.49,-0.41),(-0.09,-0.16)],(-0.30,-0.61)\rangle$
$\tilde{N}_5$	$\langle[(-0.30,-0.90),(-0.21,-0.70)],(-0.11,-0.20)\rangle$	$\langle[(-0.34,-0.80),(-0.14,-0.17)],(-0.25,-0.21)\rangle$
	$\tilde{M}_3$	$\tilde{M}_4$
$\tilde{N}_1$	$\langle[(-0.60,-0.38),(-0.36,-0.11)],(-0.31,-0.52)\rangle$	$\langle[(-0.40,-0.90),(-0.26,-0.42)],(-0.70,-0.32)\rangle$
$\tilde{N}_2$	$\langle[(-0.80,-0.62),(-0.16,-0.21)],(-0.21,-0.50)\rangle$	$\langle[(-0.60,-0.52),(-0.23,-0.22)],(-0.11,-0.59)\rangle$
$\tilde{N}_3$	$\langle[(-0.53,-0.63),(-0.12,-0.41)],(-0.37,-0.61)\rangle$	$\langle[(-0.74,-0.85),(-0.36,-0.50)],(-0.73,-0.90)\rangle$
$\tilde{N}_4$	$\langle[(-0.72,-0.32),(-0.42,-0.11)],(-0.23,-0.51)\rangle$	$\langle[(-0.55,-0.61),(-0.15,-0.41)],(-0.26,-0.42)\rangle$
$\tilde{N}_5$	$\langle[(-0.79,-0.59),(-0.56,-0.22)],(-0.71,-0.31)\rangle$	$\langle[(-0.65,-0.72),(-0.54,-0.42)],(-0.31,-0.52)\rangle$

Table 4. CNCF information table given by S_3.

	$\tilde{M}_1$	$\tilde{M}_2$
$\tilde{N}_1$	$\langle[(-0.71,-0.72),(-0.27,-0.60)],(-0.32,-0.50)\rangle$	$\langle[(-0.62,-0.72)(-0.28,-0.51)],(-0.33,-0.31)\rangle$
$\tilde{N}_2$	$\langle[(-0.38,-0.52),(-0.16,-0.22)],(-0.11,-0.32)\rangle$	$\langle[(-0.43,-0.51)(-0.16,-0.31)],(-0.21,-0.54)\rangle$
$\tilde{N}_3$	$\langle[(-0.30,-0.82),(-0.16,-0.29)],(-0.72,-0.72)\rangle$	$\langle[(-0.64,-0.70)(-0.37,-0.31)],(-0.60,-0.50)\rangle$
$\tilde{N}_4$	$\langle[(-0.73,-0.93),(-0.22,-0.22)],(-0.35,-0.58)\rangle$	$\langle[(-0.43,-0.62)(-0.23,-0.40)],(-0.42,-0.30)\rangle$
$\tilde{N}_5$	$\langle[(-0.63,-0.81),(-0.41,-0.42)],(-0.52,-0.29)\rangle$	$\langle[(-0.32,-0.42)(-0.12,-0.20)],(-0.31,-0.22)\rangle$
	$\tilde{M}_3$	$\tilde{M}_4$
$\tilde{N}_1$	$\langle[(-0.53,-0.55),(-0.29,-0.22)],(-0.34,-0.52)\rangle$	$\langle[(-0.74,-0.50)(-0.40,-0.20)],(-0.45,-0.30)\rangle$
$\tilde{N}_2$	$\langle[(-0.70,-0.83),(-0.46,-0.42)],(-0.19,-0.49)\rangle$	$\langle[(-0.50,-0.50)(-0.26,-0.30)],(-0.41,-0.50)\rangle$
$\tilde{N}_3$	$\langle[(-0.59,-0.52),(-0.26,-0.29)],(-0.65,-0.23)\rangle$	$\langle[(-0.41,-0.70)(-0.25,-0.50)],(-0.29,-0.70)\rangle$
$\tilde{N}_4$	$\langle[(-0.43,-0.67),(-0.18,-0.45)],(-0.31,-0.72)\rangle$	$\langle[(-0.71,-0.80)(-0.36,-0.30)],(-0.50,-0.60)\rangle$
$\tilde{N}_5$	$\langle[(-0.62,-0.90),(-0.33,-0.70)],(-0.14,-0.82)\rangle$	$\langle[(-0.32,-0.60)(-0.16,-0.50)],(-0.42,-0.30)\rangle$

Table 5. CNCF information table given by S_4.

	$\tilde{M}_1$	$\tilde{M}_2$
$\tilde{N}_1$	$\langle[(-0.60,-0.89),(-0.16,-0.49)],(-0.41,-0.50)\rangle$	$\langle[(-0.60,-0.90),(-0.13,-0.49)],(-0.16,-0.20)\rangle$
$\tilde{N}_2$	$\langle[(-0.50,-0.57),(-0.39,-0.32)],(-0.11,-0.30)\rangle$	$\langle[(-0.58,-0.72),(-0.16,-0.52)],(-0.21,-0.68)\rangle$
$\tilde{N}_3$	$\langle[(-0.63,-0.50),(-0.35,-0.20)],(-0.21,-0.80)\rangle$	$\langle[(-0.39,-0.73),(-0.24,-0.43)],(-0.30,-0.58)\rangle$
$\tilde{N}_4$	$\langle[(-0.50,-0.41),(-0.28,-0.21)],(-0.19,-0.30)\rangle$	$\langle[(-0.69,-0.78),(-0.39,-0.42)],(-0.20,-0.34)\rangle$
$\tilde{N}_5$	$\langle[(-0.60,-0.22),(-0.21,-0.10)],(-0.41,-0.50)\rangle$	$\langle[(-0.54,-0.67),(-0.24,-0.37)],(-0.29,-0.80)\rangle$
	$\tilde{M}_3$	$\tilde{M}_4$
$\tilde{N}_1$	$\langle[(-0.40,-0.72),(-0.16,-0.42)],(-0.31,-0.90)\rangle$	$\langle[(-0.48,-0.55),(-0.26,-0.22)],(-0.51,-0.32)\rangle$
$\tilde{N}_2$	$\langle[(-0.80,-0.63),(-0.46,-0.44)],(-0.29,-0.59)\rangle$	$\langle[(-0.60,-0.52),(-0.28,-0.29)],(-0.19,-0.61)\rangle$
$\tilde{N}_3$	$\langle[(-0.53,-0.59),(-0.13,-0.22)],(-0.27,-0.52)\rangle$	$\langle[(-0.54,-0.82),(-0.16,-0.29)],(-0.43,-0.52)\rangle$
$\tilde{N}_4$	$\langle[(-0.62,-0.23),(-0.12,-0.10)],(-0.53,-0.49)\rangle$	$\langle[(-0.55,-0.50),(-0.25,-0.20)],(-0.56,-0.80)\rangle$
$\tilde{N}_5$	$\langle[(-0.59,-0.36),(-0.36,-0.10)],(-0.30,-0.42)\rangle$	$\langle[(-0.65,-0.52),(-0.34,-0.20)],(-0.21,-0.50)\rangle$

Step 2. Aggregate the collective decision matrix by applying CNCFWA operator as given in Table 6.

Table 6. CNCF information table given by S_5.

	$\tilde{M}_1$	$\tilde{M}_2$
$\tilde{N}_1$	$\langle[(-0.6678, -0.6612), (-0.1943, -0.5346)], (-0.4020, -0.5685)\rangle$	$\langle[(-0.6118, -0.7498), (-0.1846, -0.3860)], (-0.3228, -0.3089)\rangle$
$\tilde{N}_2$	$\langle[(-0.5001, -0.6173), (-0.1595, -0.2795)], (-0.1622, -0.3731)\rangle$	$\langle[(-0.4564, -0.5572), (-0.1645, -0.3430)], (-0.1845, -0.4678)\rangle$
$\tilde{N}_3$	$\langle[(-0.5275, -0.6371), (-0.3270, -0.2493)], (-0.4919, -0.6048)\rangle$	$\langle[(-0.5266, -0.5896), (-0.3352, -0.2823)], (-0.4363, -0.5135)\rangle$
$\tilde{N}_4$	$\langle[(-0.5826, -0.7778), (-0.1851, -0.2599)], (-0.4462, -0.3667)\rangle$	$\langle[(-0.5475, -0.5938), (-0.2461, -0.2986)], (-0.3298, -0.4136)\rangle$
$\tilde{N}_5$	$\langle[(-0.6081, -0.7354), (-0.2479, -0.4520)], (-0.2611, -0.3541)\rangle$	$\langle[(-0.4117, -0.7014), (-0.1571, -0.2902)], (-0.2236, -0.3300)\rangle$
	$\tilde{M}_3$	$\tilde{M}_4$
$\tilde{N}_1$	$\langle[(-0.4988, -0.6891), (-0.2392, -0.2760)], (-0.3241, -0.5663)\rangle$	$\langle[(-0.5495, -0.7106), (-0.2743, -0.3125)], (-0.4961, -0.3143)\rangle$
$\tilde{N}_2$	$\langle[(-0.7621, -0.7167), (-0.1595, -0.3793)], (-0.3475, -0.4284)\rangle$	$\langle[(-0.5442, -0.5442), (-0.3293, -0.3047)], (-0.4918, -0.5232)\rangle$
$\tilde{N}_3$	$\langle[(-0.5765, -0.5745), (-0.2773, -0.2915)], (-0.4397, -0.4503)\rangle$	$\langle[(-0.5642, -0.7586), (-0.2379, -0.3947)], (-0.4090, -0.6120)\rangle$
$\tilde{N}_4$	$\langle[(-0.5458, -0.6167), (-0.2191, -0.3048)], (-0.3325, -0.5345)\rangle$	$\langle[(-0.5799, -0.7142), (-0.2428, -0.3086)], (-0.3592, -0.5931)\rangle$
$\tilde{N}_5$	$\langle[(-0.6621, -0.6932), (-0.3723, -0.3964)], (-0.4451, -0.4418)\rangle$	$\langle[(-0.5172, -0.5997), (-0.3230, -0.3735)], (-0.2470, -0.4206)\rangle$

Step 3. To find the unknown weight, we apply the entropy approach. In order to obtain the weights for each property, we utilise the entropy formula or technique found in Table 6. The formula is

$$\frac{1}{2n}\left(2-\left[\left(1-\left(\frac{1}{n}\sum_{i=1}^{n}e_i^{+}+\frac{1}{n}\sum_{i=1}^{n}e_i^{-}\right)\right)+\left(1-\left(\frac{1}{n}\sum_{i=1}^{n}\omega_{e_i^{+}}+\frac{1}{n}\sum_{i=1}^{n}\omega_{e_i^{-}}\right)\right)+\left(1-\left(\frac{1}{n}\sum_{i=1}^{n}c_i+\frac{1}{n}\sum_{i=1}^{n}\omega_{c_i}\right)\right)\right]\right).$$

Then the obtained weights are: $w_1 = 0.2613$, $w_2 = 0.2431$, $w_3 = 0.2630$, $w_4 = 0.2399$.

Step 4. To determine the aggregate values $\tilde{N}_i$, use the purposed approaches and its associated unknown weights (that is obtained by entropy formula) ($w_1 = 0.2613$, $w_2 = 0.2431$, $w_3 = 0.2630$, $w_4 = 0.2399$) and known weights (that are the supposed weights given to the attributes by the experts) ($w_1 = 0.21$, $w_2 = 0.25$, $w_3 = 0.28$, $w_4 = 0.26$).

Step 5. Use the Definition 3.2. to find the score functions of $\tilde{T}_i$.

Step 6. Rank the various alternatives by using the CNCFWA, CNCFOWA, and CNCFHWA operators.

In Table 7, we have computed values of the CNCFWA for each alternatives by using the unknown and known weights.

Table 7. Computed values of the CNCFWA for each alternatives by using the unknown and known weights.

Weights	$\tilde{N}_1$	$\tilde{N}_2$	$\tilde{N}_3$	$\tilde{N}_4$	$\tilde{N}_5$
By unknown	-0.0898	-0.1008	-0.0771	-0.0843	-0.1104
By known	-0.0890	-0.1011	-0.0775	-0.0722	-0.1108

From the Figure 2 and Table 8, the best alternative is $\gimel_3$.

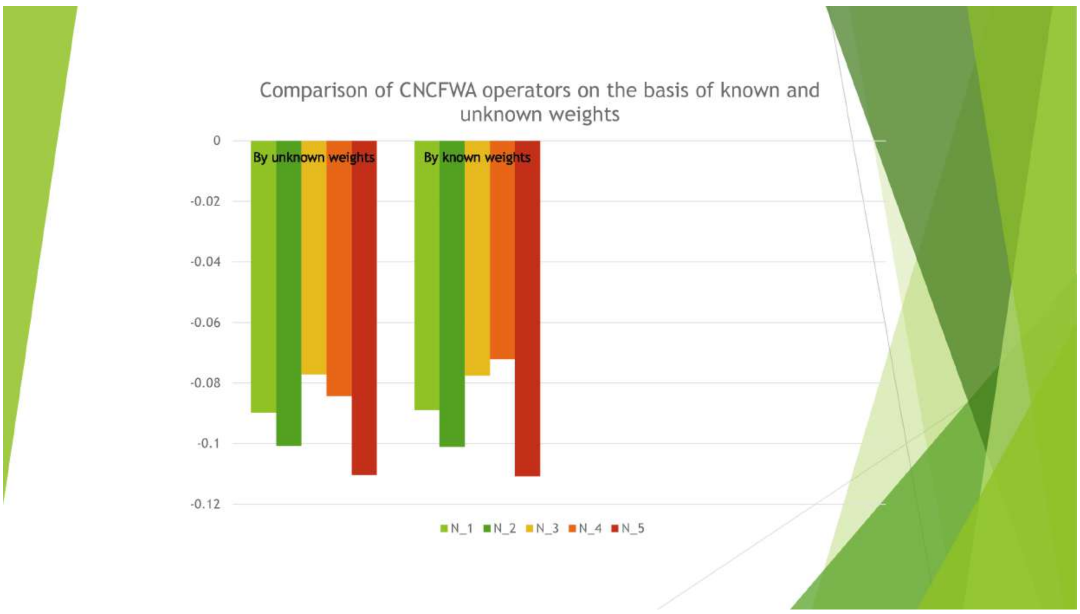

Figure 2. The comparison of CNCFWA for each alternatives by using the unknown and known weights.

Table 8. The ranking of alternatives using CNCFWA operator.

Weights	Ranking
By unknown	$\gimel_3 \succ \gimel_4 \succ \gimel_1 \succ \gimel_2 \succ \gimel_5$
By known	$\gimel_3 \succ \gimel_4 \succ \gimel_1 \succ \gimel_2 \succ \gimel_5$

In Table 9, we have computed values of the CNCFOWA for each alternatives by using the unknown and known weights.

Table 9. Computed values of the CNCFOWA for each alternatives by using the unknown and known weights.

Weights	$\tilde{N}_1$	$\tilde{N}_2$	$\tilde{N}_3$	$\tilde{N}_4$	$\tilde{N}_5$
By unknown	-0.0894	-0.1005	-0.0795	-0.0834	-0.1109
By known	-0.0904	-0.1016	-0.0775	-0.0848	-0.1125

From the Figure 3 and Table 10, the best alternative is $\gimel_3$.

Table 10. The ranking of alternatives using CNCOWA operator.

Weights	Ranking
By unknown	$\gimel_3 \succ \gimel_4 \succ \gimel_1 \succ \gimel_2 \succ \gimel_5$
By known	$\gimel_3 \succ \gimel_4 \succ \gimel_1 \succ \gimel_2 \succ \gimel_5$

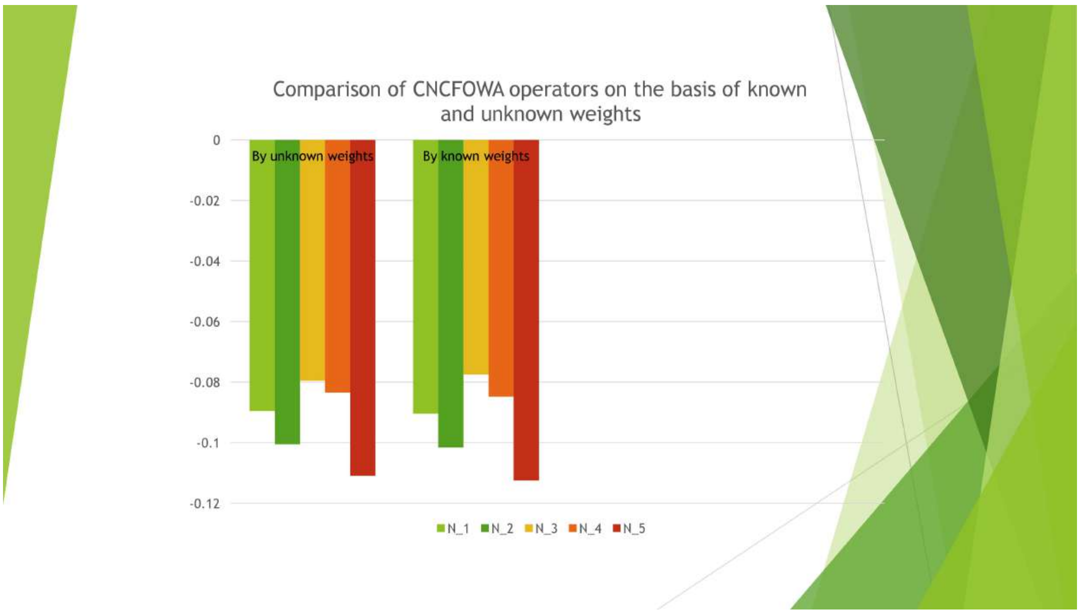

Figure 3. The comparison of CNCFOWA for each alternatives by using the unknown and known weights.

In Table 11 we have computed values of the CNCFHWA for each alternatives by using the unknown and known weights.

Table 11. Computed values of the CNCFHWA of each alternatives by using the unknown and known weights, here transform known weights (0.21, 0.25, 0.28, 0.26).

Weights	$\tilde{N}_1$	$\tilde{N}_2$	$\tilde{N}_3$	$\tilde{N}_4$	$\tilde{N}_5$
By unknown	-0.0896	-0.1003	-0.0771	-0.0844	-0.1105
By known	-0.0886	-0.0995	-0.0768	-0.0844	-0.1091

From the Figure 4 and Table 12, the best alternative is $\gimel_3$.

Table 12. The ranking of alternatives using CNCFHWA operator.

Weights	Ranking
By unknown	$\gimel_3 \succ \gimel_4 \succ \gimel_1 \succ \gimel_2 \succ \gimel_5$
By known	$\gimel_3 \succ \gimel_4 \succ \gimel_1 \succ \gimel_2 \succ \gimel_5$

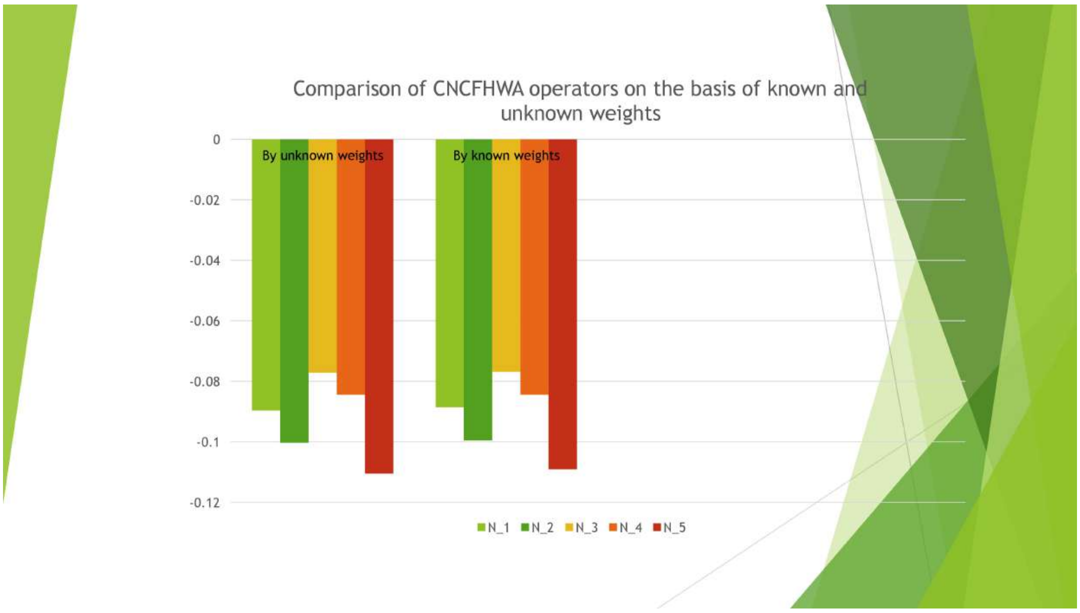

Figure 4. The comparison of CNCFHWA for each alternatives by using the unknown and known weights.

6.5. Discussion

The results presented in the study highlight the effectiveness and reliability of the proposed CNCF aggregation operators in addressing MAGDM problems. Each operator consistently identified $\gimel_3$ as the best alternative, irrespective of whether the weights were known or unknown. This uniform ranking across all methods underscores the robustness of the CNCF environment and its ability to handle varying levels of information. The graphical representations corroborate the tabular results, visually emphasizing the dominance of $\gimel_3$ in all scenarios. These findings affirm that the proposed methodology is not only reliable but also versatile and capable. It also emphasizes our framework's natural ability to support clear and logical decision-making procedures, which is a critical component of RSM in Pakistan. By employing several MAGDM approaches, the study seeks to provide a comprehensive assessment of decision-making techniques associated with the RSM setting in Pakistan.

6.6. Advantages

The proposed approach has the following advantages over the existing ones: (i) The approach created in this piece of work satisfies the requirements that the squared MD and NMD sum is not equal to one and is commonly used for imprecise data. The DMs struggle to manage the CNCFS efficiently when they are given such information. This specific situation can be successfully handled using the CNCFS technique. (ii) By including the experts' levels of confidence in their knowledge and comprehension of the assessed choices, the CNCF framework can improve the accuracy, dependability, and thoroughness of the decision-making process. (iii) The ability to work seamlessly with both known and unknown weights ensures their adaptability to real-world problems, where exact weights are often unavailable or difficult to determine. (iv) To sum up, our approach is adaptable and ideal for solving issues in CNCF-MAGDM and is more capable of managing fuzzy data.

7. Conclusions

APMs are essential parts of contemporary RSM methods. By anticipating potential accident dangers using data and predictive analytics, these models enable authorities to take preventative action to reduce crashes and enhance road safety in general. By evaluating historical accident data, traffic patterns, meteorological conditions, and other relevant factors, these models pinpoint high-risk areas and times, enabling resources to be directed where they are most needed. To reduce the dangers that have been identified, road safety officials can next implement focused interventions like better signs, altered traffic patterns, or more police. Finally, APMs offer a data-driven framework for informed decision making, accident reduction, and road safety for both cars and pedestrians. The concept of CNCFS was used in this work, and its fundamental functions and related characteristics were investigated. To incorporate personal preferences for decisions into a collective one, a WPA operator with CNCFNs was also proposed. A novel approach known as the entropy method was proposed to handle the MAGDM problem in a CNCF environment. A real-world scenario was used to illustrate this approach, which made use of the CNCFNs. A comparison analysis was carried out to determine the superiority and efficacy of the suggested approach. The current study and comparative analysis with known and unknown weights show that the strategy used in this work is more flexible and has a broader range of applications for successfully conveying unclear information. The proposed method is believed to be appropriate for integrating ambiguous and unclear data in the context of decision making since it permits a more precise and conclusive representation of the information supplied. Road safety management relies heavily on APMs' assistance in identifying high-risk areas and facilitating preventative measures. These models, however, mainly depend on historical data on accidents, which might not fully capture contemporary patterns or changes in driving behavior. Furthermore, the intricate interactions between a wide range of factors that impact road safety, including weather, road maintenance, and real-time traffic dynamics, are sometimes difficult for such models to account for. The change in traffic conditions makes it significantly harder for these models to offer accurate and reliable estimates. For preventative actions to be beneficial, they must be effective in APMs, but for financial, policy, or human reasons, they may not always be practical. Finding the correct variables for a situation might be tricky, even if CNCFS is an effective tool for processing complex information.

Limitations and future work: The approach may not work effectively with precise data or clearly defined criteria for concluding because it is meant to handle imprecise and ambiguous information. To overcome these limitations and create RSM strategies, future research should employ a thorough approach that considers both qualitative insights and quantitative data. Future research can discuss several AOs for linguistic CNCFS, including distance measurements, similarity measures, OWG AOs, and OWA AOs in a specific context. Combining linguistic CNCFNs with MAGDM challenges, such as information fusion, pattern recognition, green supplier selection, and material selection, may help us tackle a range of real-world issues in diverse industries.

Author contributions

All authors of this article have been contributed equally. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors state that they have no conflicts of interest.

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Appendix

A. Appendix A

We apply mathematical induction (MI) on n to validate Eq (3.4).

For $n = 2$, we have $z_1 = (e_1, c_1)$ and $z_2 = (e_2, c_2)$. Thus, by the operation of CNCFNs, we get

$$\zeta_1 z_1 = \left\{\left[\left(-1 + (1 + e_1^-)^{\zeta_1}, -1 + (1 + \omega_{e_1^-})^{\zeta_1}\right), \left(-1 + (1 + e_1^+)^{\zeta_1}, -1 + (1 + \omega_{e_1^+})^{\zeta_1}\right)\right], (-1)^{n+1}(c_1)^{\zeta_1}, (-1)^{n+1}(\omega_{c_1})^{\zeta_1}\right\}, \tag{A.1}$$

$$\zeta_2 z_2 = \left\{\left[\left(-1 + (1 + e_2^-)^{\zeta_2}, -1 + (1 + \omega_{e_2^-})^{\zeta_2}\right), \left(-1 + (1 + e_2^+)^{\zeta_2}, -1 + (1 + \omega_{e_2^+})^{\zeta_2}\right)\right], (-1)^{n+1}(c_2)^{\zeta_2}, (-1)^{n+1}(\omega_{c_2})^{\zeta_2}\right\}, \tag{A.2}$$

and

$$CNCFWA(z_1, z_2) = \zeta_1 z_1 \otimes \zeta_2 z_2$$
$$= \left\{\left[\left(-1 + (1 + e_1^-)^{\zeta_1}, -1 + (1 + \omega_{e_1^-})^{\zeta_1}\right), \left(-1 + (1 + e_1^+)^{\zeta_1}, -1 + (1 + \omega_{e_1^+})^{\zeta_1}\right)\right], \left((-1)^{n+1}(c_1)^{\zeta_1}, (-1)^{n+1}(\omega_{c_1})^{\zeta_1}\right)\right\}$$
$$\otimes \left\{\left[\left(-1 + (1 + e_2^-)^{\zeta_2}, -1 + (1 + \omega_{e_2^-})^{\zeta_2}\right), \left(-1 + (1 + e_2^+)^{\zeta_2}, -1 + (1 + \omega_{e_2^+})^{\zeta_2}\right)\right], \left((-1)^{n+1}(c_2)^{\zeta_2}, (-1)^{n+1}(\omega_{c_2})^{\zeta_2}\right)\right\}, \tag{A.3}$$

$$= \left\{\left[\left(-1 + \prod_{j=1}^{2}\left(1 + e_j^-\right)^{\zeta_j}, -1 + \prod_{j=1}^{2}\left(1 + \omega_{e_j^-}\right)^{\zeta_j}\right),\right.\right. \tag{A.4}$$

$$\left.\left(-1 + \prod_{j=1}^{2}\left(1 + e_j^+\right)^{\zeta_j}, -1 + \prod_{j=1}^{2}\left(1 + \omega_{e_j^+}\right)^{\zeta_j}\right)\right], \tag{A.5}$$

$$\left.\left(\prod_{j-1}^{2}(-1)^{n+1}(c_j)^{\zeta_j}, \prod_{j-1}^{2}(-1)^{n+1}(\omega_{c_j})^{\zeta_j}\right)\right\}. \tag{A.6}$$

Now, for $n = m > 2$ then

$$= \left\{\left[\left(-1 + \prod_{j=1}^{m}\left(1 + e_j^-\right)^{\zeta_j}, -1 + \prod_{j=1}^{m}\left(1 + \omega_{e_j^-}\right)^{\zeta_j}\right),\right.\right. \tag{A.7}$$

$$\left(-1+\prod_{j=1}^{m}\left(1+e_j^{+}\right)^{\zeta_j},-1+\prod_{j=1}^{m}\left(1+\omega_{e_j^{+}}\right)^{\zeta_j}\right)\Bigg], \tag{A.8}$$

$$\left(\prod_{j-1}^{m}(-1)^{n+1}(c_j)^{\zeta_j},\prod_{j-1}^{m}(-1)^{n+1}(\omega_{c_j})^{\zeta_j}\right)\Bigg\}. \tag{A.9}$$

For $n = m + 1$, we have

$$CNCFWA(z_1, z_2, \ldots, z_{m+1}) = CNCFWG(z_1, z_2, \ldots, z_m \otimes z_{m+1}^{\zeta_{m+1}})$$

$$= \Bigg\{\Bigg[\left(-1+\prod_{j=1}^{m}\left(1+e_j^{-}\right)^{\zeta_j},-1+\prod_{j=1}^{m}\left(1+\omega_{e_j^{-}}\right)^{\zeta_j}\right), \tag{A.10}$$

$$\left(-1+\prod_{j=1}^{m}\left(1+e_j^{+}\right)^{\zeta_j},-1+\prod_{j=1}^{m}\left(1+\omega_{e_j^{+}}\right)^{\zeta_j}\right)\Bigg],\left(\prod_{j-1}^{m}(-1)^{n+1}(c_j)^{\zeta_j},\prod_{j-1}^{m}(-1)^{n+1}(\omega_{c_j})^{\zeta_j}\right)\Bigg\}\otimes \tag{A.11}$$

$$\Big\{\Big[\Big(-1+(1+e_{m+1}^{-})^{\zeta_{m+1}},-1+(1+\omega_{e_{m+1}^{-}})^{\zeta_{m+1}}\Big),$$

$$\Big(-1+(1+e_{m+1}^{+})^{\zeta_{m+1}},-1+(1+\omega_{e_{m+1}^{+}})^{\zeta_{m+1}}\Big)\Big],\Big((c_{m+1})^{\zeta_{m+1}},(\omega_{c_{m+1}})^{\zeta_{m+1}}\Big)\Big\}, \tag{A.12}$$

$$= \Bigg\{\Bigg[\left(-1+\prod_{j=1}^{m+1}\left(1+e_j^{-}\right)^{\zeta_j},-1+\prod_{j=1}^{m+1}\left(1+\omega_{e_j^{-}}\right)^{\zeta_j}\right), \tag{A.13}$$

$$\left(-1+\prod_{j=1}^{m+1}\left(1+e_j^{+}\right)^{\zeta_j},-1+\prod_{j=1}^{m+1}\left(1+\omega_{e_j^{+}}\right)^{\zeta_j}\right)\Bigg], \tag{A.14}$$

$$\left(\prod_{j-1}^{m+1}(-1)^{n+1}(c_j)^{\zeta_j},\prod_{j-1}^{m+1}(-1)^{n+1}(\omega_{c_j})^{\zeta_j}\right)\Bigg\}, \tag{A.15}$$

Thus, it is hold for $n = m + 1$.

Consequently, the result holds $\forall \in \mathbb{Z}^{+}$.

B. Appendix B

We apply mathematical induction (MI) on n to validate Eq (3.7).

For $n = 2$, we have $z_1 = (e_1, c_1)$ and $z_2 = (e_2, c_2)$. Thus, by the operation of CNCFNs, we get

$$z_1^{\zeta_1} = \Big(\Big((-1)^{n+1}(e_1)^{\zeta_1},(-1)^{n+1}(\omega_{e_1})^{\zeta_1}\Big),\Big[\Big(-1+(1+c_1^{-})^{\zeta_1},-1+(1+\omega_{c_1^{-}})^{\zeta_1}\Big),\Big(-1+(1+c_1^{+})^{\zeta_1},-1+(1+\omega_{c_1^{+}})^{\zeta_1}\Big)\Big]\Big), \tag{B.1}$$

$$z_2^{\zeta_2} = \Big\{\Big((-1)^{n+1}(e_2)^{\zeta_j},(-1)^{n+1}(\omega_{e_2})^{\zeta_j},\Big)\Big[\Big(-1+(1+c_2^{-})^{\zeta_2},-1+(1+\omega_{c_2^{-}})^{\zeta_2}\Big),\Big(-1+(1+c_2^{+})^{\zeta_2},-1+(1+\omega_{c_2^{+}})^{\zeta_2}\Big)\Big]\Big\}, \tag{B.2}$$

and

$$CNCFWG(z_1,z_2)=z_1^{\zeta_1}\otimes z_2^{\zeta_2}$$

$$=\Big\{\Big((-1)^{n+1}(e_1)^{\zeta_1},(-1)^{n+1}(\omega_{e_1})^{\zeta_1}\Big),\Big[\Big(-1+(1+c_1^-)^{\zeta_1},-1+(1+\omega_{c_1^-})^{\zeta_1}\Big),$$

$$\Big(-1+(1+c_1^+)^{\zeta_1},-1+(1+\omega_{c_1^+})^{\zeta_1}\Big)\Big]\Big\}\otimes \tag{B.3}$$

$$\Big\{\Big((-1)^{n+1}(e_2)^{\zeta_2},(-1)^{n+1}(\omega_{e_2})^{\zeta_2}\Big),\Big[\Big(-1+(1+c_2^-)^{\zeta_2},-1+(1+\omega_{c_2^-})^{\zeta_2}\Big), \tag{B.4}$$

$$\Big(-1+(1+c_2^+)^{\zeta_2},-1+(1+\omega_{c_2^+})^{\zeta_2}\Big)\Big]\Big\}, \tag{B.5}$$

$$=\left\{\left(\prod_{j-1}^{2}(-1)^{n+1}(e_j)^{\zeta_j},\prod_{j-1}^{2}(-1)^{n+1}(\omega_{e_j})^{\zeta_j}\right), \tag{B.6}$$

$$\left[\left(-1+\prod_{j=1}^{2}\left(1+c_j^-\right)^{\zeta_j},-1+\prod_{j=1}^{2}\left(1+\omega_{c_j^-}\right)^{\zeta_j}\right), \tag{B.7}$$

$$\left(-1+\prod_{j=1}^{2}\left(1+c_j^+\right)^{\zeta_j},-1+\prod_{j=1}^{2}\left(1+\omega_{c_j^+}\right)^{\zeta_j}\right)\right]\right\}. \tag{B.8}$$

Now, for $n=m>2$ then

$$=\left\{\left(\prod_{j-1}^{m}(-1)^{n+1}(e_j)^{\zeta_j},\prod_{j-1}^{m}(-1)^{n+1}(\omega_{e_j})^{\zeta_j}\right), \tag{B.9}$$

$$\left[\left(-1+\prod_{j=1}^{m}\left(1+c_j^-\right)^{\zeta_j},-1+\prod_{j=1}^{m}\left(1+\omega_{c_j^-}\right)^{\zeta_j}\right), \tag{B.10}$$

$$\left(-1+\prod_{j=1}^{m}\left(1+c_j^+\right)^{\zeta_j},-1+\prod_{j=1}^{m}\left(1+\omega_{c_j^+}\right)^{\zeta_j}\right)\right]\right\}. \tag{B.11}$$

For $n=m+1$, we have

$$CNCFWG(z_1,z_2,\ldots,z_{m+1})=CNCFWG(z_1,z_2,\ldots,z_m\otimes z_{m+1}^{\zeta_{m+1}})$$

$$=\left\{\left(\prod_{j-1}^{m}(-1)^{n+1}(e_j)^{\zeta_j},\prod_{j-1}^{m}(-1)^{n+1}(\omega_{e_j})^{\zeta_j}\right),\left[\left(-1+\prod_{j=1}^{m}\left(1+c_j^-\right)^{\zeta_j},-1+\prod_{j=1}^{m}\left(1+\omega_{c_j^-}\right)^{\zeta_j}\right), \tag{B.12}$$

$$\left(-1+\prod_{j=1}^{m}\left(1+c_j^+\right)^{\zeta_j},-1+\prod_{j=1}^{m}\left(1+\omega_{c_j^+}\right)^{\zeta_j}\right)\right]\right\}\otimes \tag{B.13}$$

$$\Big\{\Big((-1)^{n+1}(e_{m+1})^{\zeta_{m+1}},(-1)^{n+1}(\omega_{e_{m+1}})^{\zeta_{m+1}}\Big),\Big[\Big(-1+(1+c_{m+1}^-)^{\zeta_{m+1}},-1+(1+\omega_{c_{m+1}^-})^{\zeta_{m+1}}\Big),$$

$$\left(-1+(1+c_{m+1}^{+})^{\zeta_{m+1}},-1+(1+\omega_{c_{m+1}^{+}})^{\zeta_{m+1}}\right)\Big]\Big\}, \quad \text{(B.14)}$$

$$=\left\{\left(\left(\prod_{j-1}^{m+1}(-1)^{n+1}(e_j)^{\zeta_j},\prod_{j-1}^{m+1}(-1)^{n+1}(\omega_{e_j})^{\zeta_j}\right),\right.\right. \quad \text{(B.15)}$$

$$\left[\left(-1+\prod_{j=1}^{m+1}\left(1+c_j^{-}\right)^{\zeta_j},-1+\prod_{j=1}^{m+1}\left(1+\omega_{c_j^{-}}\right)^{\zeta_j}\right),\right. \quad \text{(B.16)}$$

$$\left.\left.\left(-1+\prod_{j=1}^{m+1}\left(1+c_j^{+}\right)^{\zeta_j},-1+\prod_{j=1}^{m+1}\left(1+\omega_{c_j^{+}}\right)^{\zeta_j}\right)\right]\right\}. \quad \text{(B.17)}$$

Thus, it is hold for $n = m + 1$.

Consequently, the result holds $\forall \in \mathbb{Z}^+$.