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Research article

Assessment of correlated measurement errors in presence of missing data using ranked set sampling

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Abstract: A miniscule amount of work was done for the assessment of measurement errors in the existence of missing data using a few sampling techniques, while no work was available for the assessment of correlated measurement errors in the existence of missing data. This study aimed to propose some general imputation methods and the corresponding resultant estimators in the existence of missing data under ranked set sampling, provided the data was contaminated with the correlated measurement errors. The mean square error of the developed resultant estimators was established to the first order approximation. The potency of the developed imputation methods and corresponding resultant estimators was assessed by a comprehensive simulation experiment relying on a hypothetically created population. The findings indicated that the proposed imputation methods and the resultant estimators surpassed the traditional imputation methods and the resultant estimators. In addition, a real data application of the proposed imputation methods was also provided.

Keywords: correlated measurement errors; searls type power ratio imputation methods; searls type power ratio estimators; simulation
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1. Introduction

Survey sampling is used in a variety of disciplines like epidemiology, ecology, medicine, and agriculture, among others, with the underlying premise that all variables are accurately assessed. However, as recorded correct values in exercise are nearly impossible, this criterion is usually ignored. Consequently, errors are always present in the data recorded. The discrepancy between the recorded value and the actual value is known as measurement error (ME) and arises due to the faulty instrument, faulty experimental techniques, interviewers, participants, and data interpreters used. Several authors have assessed the effect of MEs, including [1–10], among others.

The aforementioned authors assumed that the MEs in study and auxiliary variables are independent, but presumably it may not be adequate to assume that the MEs are independent in both study and auxiliary variables, as the same investigator frequently gathers data on both variables. Instead, they will be correlated and dependent, and this dependency in the MEs may be brought on by the hidden underlying deportment of the data. This assumption suggests that MEs arise due to inherent characteristics of the data-generating process rather than external factors like instrument flaws or human mistakes. However, it does not explicitly define the specific nature of this underlying behavior. Some important sources of correlated MEs include instrumental bias, calibration issues, observer bias, environmental factors, sampling errors, processing errors, time-dependent changes, interference effects, memory effects, methodological constraints, etc. The issue of correlated ME was primarily introduced by [11], who measured the effect of the correlated MEs on the efficacy of the usual ratio and product estimators of the population mean. Recently, the effect of correlated MEs on the efficacy of the various estimators was assessed by [12, 13].

The ranked set sampling (RSS) is the efficient alternative to simple random sampling (SRS). It was originated by [14] to estimate the production of the mean pasture, but it may be efficiently utilized in the situations where ranking of the observations is much easier than obtaining their exact values. These situations frequently occur in the fields of medicine [15, 16], forestry [17], environmental monitoring [18], and reliability [19]. Essentially, the RSS was developed by [14] for the population mean estimation. However, to date, it has been applied to almost all statistical problems, including statistical inference [20], estimation of the population mean [21–28], the population variance [29], the population proportion [30], and the cumulative distribution function [31].

In survey sampling, situations may arise when the data is contaminated with the missing values. In such situations, it becomes tedious for the survey professionals to draw a valid conclusion. To solve the issue of missing values in the data, the imputation is used. There are several imputation techniques that have been proposed by different authors, including [32–38], but literature contains no work to tackle the issue of missing data when the data is contaminated with the correlated MEs. This work has the following objectives:

(i). To develop some fundamental theory under missing completely at random for RSS in the case of missing data when the data is contaminated with correlated MEs.

(ii). To adapt imputation methods like mean, ratio, product, and power ratio imputation methods and to propose Searls type power ratio imputation methods for estimating the population mean without modifying the initial responses while imputing the missing values.

(iii). To assess the effect of correlated MEs on the efficacy of the adapted and suggested imputation methods.

In Section 2, we define the theory and notations used throughout this study. In Section 3, we adapt some fundamental imputation methods such as mean, ratio, and product imputation methods, while in Section 4, we suggest Searls type power ratio imputation methods along with their properties for estimating the population mean to evaluate the impact of correlated MEs under RSS. In Section 5, we present a mathematical comparison to evaluate the performance of the imputation methods. Section 6 offers an extensive simulation study and the interpretation of simulation results. Application of the adapted and proposed methods is discussed in Section 7. The conclusion is provided in Section 8.

2. Methodology and notations

In the fundamental RSS process, one first determines a set size H. Then, H^2 sample units are randomly selected from the population and divided into H sets, each of size H. The units in each set are given judgment ranking without obtaining actual measurements. The i^{th} judgement order statistic is quantified from the i^{th} $(i = 1, 2, ..., H)$ set, and the remaining unquantified units are returned to the population. This completes a cycle of RSS. The complete cycle may be replicated K times to obtain a ranked set sample of size $n = HK$. The i^{th} judgement order statistic for the l^{th} cycle is $X_{[i]l}$. The ranking procedure may contain some errors, which is shown by the usage of square brackets, whereas the square brackets are changed to round brackets if the rankings are perfect. As compared to SRS of equivalent size, the RSS frequently results in more effective inference. This is due to the fact that a ranked set sample includes information from both the preliminary rankings and the quantified observations in addition to the information provided by the observations themselves.

Suppose that the true measurements on the j^{th} unit of the study and auxiliary variables are X_j and Y_j, respectively. Although true measurements on these units cannot be obtained due to some reasons, they may still be measured as x_j and y_j using the ME v_j and u_j for the j^{th} unit of the corresponding variables. Let $x_j = X_j + v_j$ and $y_j = Y_j + u_j$, where $j = 1, 2, ..., n$. Let $(\bar{y}, \bar{x})$ be the sample means, (μ_y, μ_x) be the population means, (σ_y^2, σ_x^2) be the population variances, (C_y, C_x) be the population coefficients of variation of variables (y, x), respectively, and ρ_{xy} be the coefficient of correlation between variables x and y. The MEs (u_j, v_j) are also unobservable, having means of (0, 0), variances (σ_u^2, σ_v^2), population coefficients of variation (C_u, C_v), and correlation coefficient of ρ_{uv}.

Let $H_1 = HP$ be the number of responding units out of sampled H units, where P is the probability that the i^{th} respondent belongs to the responding group r_u and $(1 - P)$ is the probability that the i^{th} respondent belongs to the non-responding group $\bar{r}_u$ such that $s = r_u \cup \bar{r}_u$. Let $r = HKP$ be the responding units out of sampled n units such that $n > r$. The value Y_i, $i \in r_u$ is observable for each unit, except for the units $i \in \bar{r}_u$ where the values are missing and require imputation to construct the complete structure of data to make a valid conclusion. The known auxiliary variable population data assists to execute the imputation of missing Y values. Suppose $\bar{x}_{r,rss} = \sum_{i=1}^{H_1} \sum_{l=1}^{K} x_{(i:i)l}/HKP$ and $\bar{y}_{r,rss} = \sum_{i=1}^{H_1} \sum_{l=1}^{K} y_{[i:i]l}/HKP$ are the traditional estimators of μ_x and μ_y, respectively, such that $x_{(i:i)l}$ and $y_{[i:i]l}$ are the i^{th} order statistics and i^{th} judgement order in the i^{th} sample of size H in cycle l for variables X and Y, respectively. For convenience, we have denoted $x_{(i:i)l}$ and $y_{[i:i]l}$ by $x_{(i)}$ and $y_{[i]}$, respectively. To determine the mean square error (MSE) of the proposed estimators under MEs, we use the following notations: $\bar{y}_{r,rss} = \mu_y(1 + \delta_0)$, $\bar{x}_{r,rss} = \mu_x(1 + \delta_1)$, and $\bar{x}_{n,rss} = \mu_x(1 + \delta_2)$, where δ_i, $i = 0, 1, 2$ are the error terms, provided that

$$
\begin{aligned}
E(\delta_0) &= E(\delta_1) = E(\delta_2) = 0,\\
E(\delta_0^2) &= (\varnothing^* C_y^2 - W_y^{2*} + \varnothing^* C_u^2 - W_u^{2*}) = \Delta_{02}^*,\\
E(\delta_1^2) &= (\varnothing^* C_x^2 - W_x^{2*} + \varnothing^* C_v^2 - W_v^{2*}) = \Delta_{20}^*,\\
E(\delta_2^2) &= (\varnothing C_x^2 - W_x^2 + \varnothing C_v^2 - W_v^2) = \Delta_{20},\\
E(\delta_0\delta_1) &= (\varnothing^* \rho_{xy} C_x C_y - W_{xy}^* + \varnothing^* \rho_{uv} C_u C_v - W_{uv}^*) = \Delta_{11}^*,\\
E(\delta_0\delta_2) &= (\varnothing \rho_{xy} C_x C_y - W_{xy} + \varnothing \rho_{uv} C_u C_v - W_{uv}) = \Delta_{11},\\
E(\delta_1\delta_2) &= E(\delta_2^2) = (\varnothing C_x^2 - W_x^2 + \varnothing C_v^2 - W_v^2) = \Delta_{20},
\end{aligned}
$$

where

$$\varnothing = \frac{1}{HK};\ \varnothing^* = \frac{1}{HKP};\ C_x = \frac{S_x}{\mu_x};\ C_v = \frac{S_v}{\mu_x};\ C_y = \frac{S_y}{\mu_y};\ C_u = \frac{S_u}{\mu_y};\ W_y^{2*} = \frac{1}{H^2KP}\sum_{i=1}^{H}\frac{(\mu_{y_{[i]}} - \mu_y)^2}{\mu_y^2};$$

$$W_x^2 = \frac{1}{H^2K}\sum_{i=1}^{H}\frac{(\mu_{x_{(i)}} - \mu_x)^2}{\mu_x^2};\ W_x^{2*} = \frac{1}{H^2KP}\sum_{i=1}^{H}\frac{(\mu_{x_{(i)}} - \mu_x)^2}{\mu_x^2};\ W_{xy} = \frac{1}{H^2K}\sum_{i=1}^{H}\frac{(\mu_{x_{(i)}} - \mu_x)(\mu_{y_{[i]}} - \mu_y)}{\mu_x\mu_y};$$

$$W_{xy}^* = \frac{1}{H^2KP}\sum_{i=1}^{H}\frac{(\mu_{x_{(i)}} - \mu_x)(\mu_{y_{[i]}} - \mu_y)}{\mu_x\mu_y};\ W_u^2 = \frac{1}{H^2K}\sum_{i=1}^{H}\frac{(\mu_{u_{[i]}} - \mu_u)^2}{\mu_y^2};\ W_u^{2*} = \frac{1}{H^2KP}\sum_{i=1}^{H}\frac{(\mu_{u_{[i]}} - \mu_u)^2}{\mu_y^2};$$

$$W_v^2 = \frac{1}{H^2K}\sum_{i=1}^{H}\frac{(\mu_{v_{(i)}} - \mu_v)^2}{\mu_x^2};\ W_v^{2*} = \frac{1}{H^2KP}\sum_{i=1}^{H}\frac{(\mu_{v_{(i)}} - \mu_v)^2}{\mu_x^2};\ W_{uv_{[i]}} = \frac{1}{H^2K}\sum_{i=1}^{H}\frac{(\mu_{u_{[i]}} - \mu_y)(\mu_{x_{(i)}} - \mu_v)}{\mu_u\mu_v};$$

$$W_{uv}^* = \frac{1}{H^2KP}\sum_{i=1}^{H}\frac{(\mu_{u_{[i]}} - \mu_y)(\mu_{x_{(i)}} - \mu_v)}{\mu_u\mu_v};\mu_{x_{(i)}} = E(x_{(i)});\ \text{and}\ \mu_{y_{[i]}} = E(y_{[i]}).$$

The above results can be easily extended from Al-Omari and Bouza (2014).

3. Adapted imputation methods

In this section, we adapted some fundamental imputations to sort out the missing data problems when the data is contaminated with correlated MEs.

3.1. Mean imputation method

We propose mean imputation of the population mean by extending the results of [33] for single value imputation when y values of the i^{th} sample unit under RSS are missing and require imputation. The techniques of imputation for population mean are given as

$$y_{.i_m} = \begin{cases} y_{[i]} & \text{for } i \in r_u, \\ \bar{y}_{r,rss} & \text{for } i \in \bar{r}_u. \end{cases}$$

The resultant estimator is given by

$$t_m = \bar{y}_{r,rss}.$$

The variance of the estimator t_m is given by

$$V(t_m) = \mu_y^2\Delta_{02}^*. \tag{3.1}$$

Considering the additional auxiliary information into account, the imputation methods are categorized as

Scheme I: When μ_x is known and $\bar{x}_{n,rss}$ is used.

Scheme II: When μ_x is known and $\bar{x}_{r,rss}$ is used.

Scheme III: When μ_x is not known and $\bar{x}_{n,rss}$, $\bar{x}_{r,rss}$ are used.

3.2. Conventional ratio imputation methods

The classical ratio imputation methods in the presence of ME are given as
Scheme I

$$y_{.i_{r_1}} = \begin{cases} y_{[i]} & \text{for } i \in r_u, \\ \frac{1}{n-r}\left[n\bar{y}_{r,rss}\frac{\mu_x}{\bar{x}_{n,rss}} - r\bar{y}_{r,rss}\right] & \text{for } i \in \bar{r}_u. \end{cases}$$

Scheme II

$$y_{.i_{r_2}} = \begin{cases} y_{[i]} & \text{for } i \in r_u, \\ \frac{1}{n-r}\left[n\bar{y}_{r,rss}\frac{\mu_x}{\bar{x}_{r,rss}} - r\bar{y}_{r,rss}\right] & \text{for } i \in \bar{r}_u. \end{cases}$$

Scheme III

$$y_{.i_{r_3}} = \begin{cases} y_{[i]} & \text{for } i \in r_u, \\ \frac{1}{n-r}\left[n\bar{y}_{r,rss}\frac{\bar{x}_{n,rss}}{\bar{x}_{r,rss}} - r\bar{y}_{r,rss}\right] & \text{for } i \in \bar{r}_u. \end{cases}$$

Under the above schemes, the resultant estimators are

$$\begin{aligned} t_{r_1} &= \bar{y}_{r,rss}\frac{\mu_x}{\bar{x}_{n,rss}}, \\ t_{r_2} &= \bar{y}_{r,rss}\frac{\mu_x}{\bar{x}_{r,rss}}, \\ \text{and} \quad t_{r_3} &= \bar{y}_{r,rss}\frac{\bar{x}_{n,rss}}{\bar{x}_{r,rss}}. \end{aligned}$$

The MSE equations of the above resultant estimators are given by

$$MSE(t_{r_1}) = \mu_y^2(\Delta_{02}^* + \Delta_{20} - 2\Delta_{11}), \tag{3.2}$$

$$MSE(t_{r_2}) = \mu_y^2(\Delta_{02}^* + \Delta_{20}^* - 2\Delta_{11}^*), \tag{3.3}$$

$$\text{and} \quad MSE(t_{r_3}) = \mu_y^2[\Delta_{02}^* + (\Delta_{20}^* - \Delta_{20}) - 2(\Delta_{11}^* - \Delta_{11})]. \tag{3.4}$$

3.3. Conventional product imputation methods

The conventional product imputation methods in the presence of ME are given as
Scheme I

$$y_{.i_{p_1}} = \begin{cases} y_{[i]} & \text{for } i \in r_u, \\ \frac{1}{n-r}\left[n\bar{y}_{r,rss}\frac{\bar{x}_{n,rss}}{\mu_x} - r\bar{y}_{r,rss}\right] & \text{for } i \in \bar{r}_u. \end{cases}$$

Scheme II

$$y_{.i_{p_2}} = \begin{cases} y_{[i]} & \text{for } i \in r_u, \\ \frac{1}{n-r}\left[n\bar{y}_{r,rss}\frac{\bar{x}_{r,rss}}{\mu_x} - r\bar{y}_{r,rss}\right] & \text{for } i \in \bar{r}_u. \end{cases}$$

Scheme III

$$y_{.i_{p_3}} = \begin{cases} y_{[i]} & \text{for } i \in r_u, \\ \frac{1}{n-r}\left[n\bar{y}_{r,rss}\frac{\bar{x}_{r,rss}}{\bar{x}_{n,rss}} - r\bar{y}_{r,rss}\right] & \text{for } i \in \bar{r_u}. \end{cases}$$

Under the above schemes, the resultant estimators are

$$\begin{aligned} t_{p_1} &= \bar{y}_{r,rss}\frac{\bar{x}_{n,rss}}{\mu_x}, \\ t_{p_2} &= \bar{y}_{r,rss}\frac{\bar{x}_{r,rss}}{\mu_x}, \\ \text{and} \quad t_{p_3} &= \bar{y}_{r,rss}\frac{\bar{x}_{r,rss}}{\bar{x}_{n,rss}}. \end{aligned}$$

The MSE equations of the above resultant estimators are given by

$$MSE(t_{p_1}) = \mu_y^2(\Delta_{02}^* + \Delta_{20} + 2\Delta_{11}), \tag{3.5}$$

$$MSE(t_{p_2}) = \mu_y^2(\Delta_{02}^* + \Delta_{20}^* + 2\Delta_{11}^*), \tag{3.6}$$

$$\text{and} \quad MSE(t_{p_3}) = \mu_y^2[\Delta_{02}^* + (\Delta_{20}^* - \Delta_{20}) + 2(\Delta_{11}^* - \Delta_{11})]. \tag{3.7}$$

3.4. Power ratio imputation methods

The power ratio imputation methods in the case of correlated MEs are as follows:
Scheme I

$$y_{.i_{g_1}} = \begin{cases} y_{[i]} & \text{for } i \in r_u, \\ \frac{1}{n-r}\left[n\bar{y}_{r,rss}\left(\frac{\mu_x}{\bar{x}_{n,rss}}\right)^{\Theta_1} - r\bar{y}_{r,rss}\right] & \text{for } i \in \bar{r_u}. \end{cases}$$

Scheme II

$$y_{.i_{g_2}} = \begin{cases} y_{[i]} & \text{for } i \in r_u, \\ \frac{1}{n-r}\left[n\bar{y}_{r,rss}\left(\frac{\mu_x}{\bar{x}_{r,rss}}\right)^{\Theta_2} - r\bar{y}_{r,rss}\right] & \text{for } i \in \bar{r_u}. \end{cases}$$

Scheme III

$$y_{.i_{g_3}} = \begin{cases} y_{[i]} & \text{for } i \in r_u, \\ \frac{1}{n-r}\left[n\bar{y}_{r}\left(\frac{\bar{x}_{n,rss}}{\bar{x}_{r,rss}}\right)^{\Theta_3} - r\bar{y}_{r,rss}\right] & \text{for } i \in \bar{r_u}. \end{cases}$$

Under the aforementioned schemes, the resultant estimators will be given by

$$\begin{aligned} t_{g_1} &= \bar{y}_{r,rss}\left(\frac{\mu_x}{\bar{x}_{n,rss}}\right)^{\Theta_1}, \\ t_{g_2} &= \bar{y}_{r,rss}\left(\frac{\mu_x}{\bar{x}_{r,rss}}\right)^{\Theta_2}, \\ \text{and} \quad t_{g_3} &= \bar{y}_{r,rss}\left(\frac{\bar{x}_{n,rss}}{\bar{x}_{r,rss}}\right)^{\Theta_3}, \end{aligned}$$

where Θ_1, Θ_2, and Θ_3 are duly selected scalars.

The minimum MSE of the resultant power ratio estimators t_{g_1}, t_{g_2}, and t_{g_3} at the optimum values of $\Theta_{1(opt)} = \Delta_{11}/\Delta_{20}$, $\Theta_{2(opt)} = \Delta^*_{11}/\Delta^*_{20}$, and $\Theta_{3(opt)} = (\Delta^*_{11} - \Delta_{11})/(\Delta^*_{20} - \Delta_{20})$, respectively, is expressed by

$$min.MSE(t_{g_1}) = \mu_y^2\left(\Delta^*_{02} - \frac{\Delta_{11}^2}{\Delta_{20}}\right), \tag{3.8}$$

$$min.MSE(t_{g_2}) = \mu_y^2\left(\Delta^*_{02} - \frac{\Delta_{11}^{*^2}}{\Delta^*_{20}}\right), \tag{3.9}$$

$$\text{and} \quad min.MSE(t_{g_3}) = \mu_y^2\left[\Delta^*_{02} - \frac{(\Delta^*_{11} - \Delta_{11})^2}{(\Delta^*_{20} - \Delta_{20})}\right]. \tag{3.10}$$

4. Suggested imputation methods

In the sampling theory, one of the objectives of the researchers is to improve the efficiency of their estimators. In order to improve the efficiency of the estimators, [39] suggested a procedure which is based on pre-multiplying a tuning parameter in the estimators. Employing Searls philosophy, we multiplied a tuning parameter in the power ratio imputation methods and suggested Searls type power ratio imputation methods for the estimation of population mean in the presence of missing data when the data is contaminated with the correlated MEs. The proposed imputation methods are given by
Scheme I

$$y_{.i_{s_1}} = \begin{cases} y_{[i]} & \text{for } i \in r_u, \\ \frac{1}{n-r}\left[n\Lambda_1\bar{y}_{r,rss}\left(\frac{\mu_x}{\bar{x}_{n,rss}}\right)^{\Theta_1} - r\bar{y}_{r,rss}\right] & \text{for } i \in \bar{r}_u. \end{cases}$$

Scheme II

$$y_{.i_{s_2}} = \begin{cases} y_{[i]} & \text{for } i \in r_u, \\ \frac{1}{n-r}\left[n\Lambda_2\bar{y}_{r,rss}\left(\frac{\mu_x}{\bar{x}_{r,rss}}\right)^{\Theta_2} - r\bar{y}_{r,rss}\right] & \text{for } i \in \bar{r}_u. \end{cases}$$

Scheme III

$$y_{.i_{s_3}} = \begin{cases} y_{[i]} & \text{for } i \in r_u, \\ \frac{1}{n-r}\left[n\Lambda_3\bar{y}_r\left(\frac{\bar{x}_{n,rss}}{\bar{x}_{r,rss}}\right)^{\Theta_3} - r\bar{y}_{r,rss}\right] & \text{for } i \in \bar{r}_u. \end{cases}$$

Under the aforementioned schemes, the resultant estimators are given by

$$T_{s_1} = \Lambda_1\bar{y}_{r,rss}\left(\frac{\mu_x}{\bar{x}_{n,rss}}\right)^{\Theta_1},$$

$$T_{s_2} = \Lambda_2\bar{y}_{r,rss}\left(\frac{\mu_x}{\bar{x}_{r,rss}}\right)^{\Theta_2},$$

$$\text{and} \quad T_{s_3} = \Lambda_3\bar{y}_{r,rss}\left(\frac{\bar{x}_{n,rss}}{\bar{x}_{r,rss}}\right)^{\Theta_3},$$

where $\Lambda_j,\ j = 1, 2, 3$ and Θ_j are appropriately selected scalars.

(1) When $\Lambda_j \neq \Theta_j = 1$, the Searls type power ratio imputation methods $y_{.i_{s_j}}$ and the resultant Searls type power ratio estimators T_{s_j} deform into the usual ratio imputation methods $y_{.i_{r_j}}$ and the resultant ratio estimators t_{r_j}.
(2) When $\Lambda_j = 1$ and $\Theta_j = -1$, the Searls type power ratio imputation methods $y_{.i_{s_j}}$ and the resultant Searls type power ratio estimators T_{s_j} deform into the usual product imputation methods $y_{.i_{p_j}}$ and the resultant product estimators t_{p_j}.
(3) When $\Lambda_j = 1$, the Searls type power ratio imputation methods $y_{.i_{s_j}}$ and the resultant Searls type power ratio estimators T_{s_j} deform into the power ratio imputation methods $y_{.i_{g_j}}$ and the resultant power ratio estimators t_{g_j}.

Theorem 4.1. *The minimum MSE of the resultant Searls type power ratio estimators* $T_{s_j},\ j = 1, 2, 3$ *is expressed up to first-order approximation as*

$$min.MSE(T_{s_j}) = \mu_y^2\left(1 - \frac{M_j^2}{L_j}\right). \tag{4.1}$$

Proof. Expressing the estimator T_{s_1} in terms of errors as

$$\begin{aligned} T_{s_1} &= \Lambda_1\mu_y(1+\delta_0)\left[\frac{\mu_x}{\mu_x(1+\delta_2)}\right]^{\Theta_1}, \\ &= \Lambda_1\mu_y(1+\delta_0)(1+\delta_2)^{-\Theta_1}, \\ &= \mu_y\Lambda_1\left[1+\delta_0-\Theta_1\delta_2-\Theta_1\delta_0\delta_2+\frac{\Theta_1(\Theta_1+1)}{2}\delta_2^2\right]. \end{aligned} \tag{4.2}$$

Subtracting μ_y from both sides of (4.2) provides (4.3):

$$T_{s_1} - \mu_y = \mu_y\left[\Lambda_1\left(1+\delta_0-\Theta_1\delta_2-\Theta_1\delta_0\delta_2+\frac{\Theta_1(\Theta_1+1)}{2}\delta_2^2\right)-1\right]. \tag{4.3}$$

Taking the expectation from each side of (4.3), we obtain

$$Bias(T_{s_1}) = \mu_y\left[\Lambda_1\left(1+\frac{\Theta_1(\Theta_1+1)}{2}\Delta_{20}-\Theta_1\Delta_{11}\right)-1\right].$$

Do square and take the expectation from each side of (4.3), and we obtain

$$\begin{aligned} MSE(T_{s_1}) &= \mu_y^2\left[\begin{array}{l} 1+\Lambda_1^2(1+\Delta_{02}^*+(2\Theta_1^2+\Theta_1)\Delta_{20}-4\Theta_1\Delta_{11}) \\ -2\Lambda_1\left(1+\frac{\Theta_1(\Theta_1+1)}{2}\Delta_{20}-\Theta_1\Delta_{11}\right) \end{array}\right], \\ &= \mu_y^2(1+\Lambda_1^2L_1-2\Lambda_1M_1), \end{aligned} \tag{4.4}$$

where $L_1 = 1+\Delta_{02}^*+(2\Theta_1^2+\Theta_1)\Delta_{20}-4\Theta_1\Delta_{11}$ and $M_1 = 1+\frac{\Theta_1(\Theta_1+1)}{2}\Delta_{20}-\Theta_1\Delta_{11}$.

Minimization of the (4.4) w.r.t. Λ_1 provides:

$$\Lambda_{1(opt)} = \frac{M_1}{L_1}.$$

Use of $\Lambda_{1(opt)}$ in (4.4) provides:

$$minMSE(T_{s_1}) = \mu_y^2\left(1 - \frac{M_1^2}{L_1}\right).$$

In the same way, we can obtain the minimum MSE of the rest of the resultant proposed estimators $T_{s_j},\ j = 2, 3$. We may usually write

$$MSE(T_{s_j}) = \mu_y^2(1 + \Lambda_j^2 L_j - 2\Lambda_j M_j), \tag{4.5}$$

where $L_2 = 1 + \Delta_{02}^* + (2\Theta_2^2 + \Theta_2)\Delta_{20}^* - 4\Theta_2\Delta_{11}^*$, $M_2 = 1 + \frac{\Theta_2(\Theta_2+1)}{2}\Delta_{20}^* - \Theta_2\Delta_{11}^*$, $L_3 = 1 + \Delta_{02}^* + (2\Theta_3^2 + \Theta_3)(\Delta_{20}^* - \Delta_{20}) - 4\Theta_3(\Delta_{11}^* - \Delta_{11})$, and $M_3 = 1 + \frac{\Theta_3(\Theta_3+1)}{2}(\Delta_{20}^* - \Delta_{20}) - \Theta_3(\Delta_{11}^* - \Delta_{11})$.

Minimization of (4.5) w.r.t. Λ_j provides:

$$\Lambda_{j(opt)} = \frac{M_j}{L_j}.$$

Use of $\Lambda_{j(opt)}$ in (4.5) provides:

$$minMSE(T_{s_j}) = \mu_y^2\left(1 - \frac{M_j^2}{L_j}\right).$$

Note that minimizing Λ_j and Θ_j simultaneously is a typical task. Thus, putting $\Lambda_j = 1$ in the respective estimators and minimizing the MSE w.r.t. Θ_j provides the optimum values of Θ_j as

$$\Theta_{1(opt)} = \frac{\Delta_{11}}{\Delta_{20}},$$

$$\Theta_{2(opt)} = \frac{\Delta_{11}^*}{\Delta_{20}^*},$$

$$\text{and} \quad \Theta_{3(opt)} = \frac{(\Delta_{11}^* - \Delta_{11})}{(\Delta_{20}^* - \Delta_{20})}.$$

□

Remark 4.1. *By setting $\rho_{uv} = 0$ in the above results, the case of uncorrelated MEs can be obtained. These results are more extensive and all-encompassing, and they specifically contain the results of the uncorrelated MEs.*

5. Mathematical comparison

The mathematical comparison of the proposed Searls type power ratio imputation methods is done with the adapted imputation methods in this section, and the following efficiency conditions are developed.

Lemma 5.1. *The suggested estimators T_{s_j} repress the usual mean estimator t_m, if*

$$V(t_m) > MSE(T_{s_j}) \implies \mu_y^2\Delta_{02}^* > \mu_y^2\left(1 - \frac{M_j^2}{L_j}\right) \implies \frac{M_j^2}{L_j} > 1 - \Delta_{02}^*.$$

Lemma 5.2. *(i). The suggested estimator T_{s_1} represses the ratio estimator t_{r_1} under scheme I, if*

$$MSE(t_{r_1}) > MSE(T_{s_1})$$
$$\Longrightarrow \mu_y^2(\Delta_{02}^* + \Delta_{20} - 2\Delta_{11}) > \mu_y^2\left(1 - \frac{M_1^2}{L_1}\right)$$
$$\Longrightarrow \frac{M_1^2}{L_1} > 1 - \Delta_{02}^* - \Delta_{20} + 2\Delta_{11}.$$

(ii). The suggested estimator T_{s_2} represses the ratio estimator t_{r_2} under scheme II, if

$$MSE(t_{r_2}) > MSE(T_{s_2})$$
$$\Longrightarrow \mu_y^2(\Delta_{02}^* + \Delta_{20}^* - 2\Delta_{11}^*) > \mu_y^2\left(1 - \frac{M_2^2}{L_2}\right)$$
$$\Longrightarrow \frac{M_2^2}{L_2} > 1 - \Delta_{02}^* - \Delta_{20}^* + 2\Delta_{11}^*.$$

(iii). The suggested estimator T_{s_3} represses the ratio estimator t_{r_3} under scheme III, if

$$MSE(t_{r_3}) > MSE(T_{s_3})$$
$$\Longrightarrow \mu_y^2[\Delta_{02}^* + (\Delta_{20}^* - \Delta_{20}) - 2(\Delta_{11}^* - \Delta_{11})] > \mu_y^2\left(1 - \frac{M_3^2}{L_3}\right)$$
$$\frac{M_3^2}{L_3} > 1 - \Delta_{02}^* - (\Delta_{20}^* - \Delta_{20}) + 2(\Delta_{11}^* - \Delta_{11}).$$

Lemma 5.3. *(i). The suggested estimator T_{s_1} represses the product estimator t_{p_1} under scheme I, if*

$$MSE(t_{p_1}) > MSE(T_{s_1})$$
$$\Longrightarrow \mu_y^2(\Delta_{02}^* + \Delta_{20} + 2\Delta_{11}) > \mu_y^2\left(1 - \frac{M_1^2}{L_1}\right)$$
$$\Longrightarrow \frac{M_1^2}{L_1} > 1 - \Delta_{02}^* - \Delta_{20} - 2\Delta_{11}.$$

(ii). The suggested estimator T_{s_2} represses the product estimator t_{p_2} under scheme II, if

$$MSE(t_{p_2}) > MSE(T_{s_2})$$
$$\Longrightarrow \mu_y^2(\Delta_{02}^* + \Delta_{20}^* + 2\Delta_{11}^*) > \mu_y^2\left(1 - \frac{M_2^2}{L_2}\right)$$
$$\Longrightarrow \frac{M_2^2}{L_2} > 1 - \Delta_{02}^* - \Delta_{20}^* - 2\Delta_{11}^*.$$

(iii). The suggested estimator T_{s_3} represses the product estimator t_{p_3} under scheme III, if

$$MSE(t_{p_3}) > MSE(T_{s_3})$$
$$\Longrightarrow \mu_y^2[\Delta_{02}^* + (\Delta_{20}^* - \Delta_{20}) + 2(\Delta_{11}^* - \Delta_{11})] > \mu_y^2\left(1 - \frac{M_3^2}{L_3}\right)$$
$$\frac{M_3^2}{L_3} > 1 - \Delta_{02}^* - (\Delta_{20}^* - \Delta_{20}) - 2(\Delta_{11}^* - \Delta_{11}).$$

Lemma 5.4. *(i). The suggested estimator T_{s_1} represses the power ratio estimator t_{g_1} under scheme I, if*

$$\begin{aligned} MSE(t_{g_1}) &> MSE(T_{s_1}) \\ \implies \mu_y^2\left(\Delta_{02}^* - \frac{\Delta_{11}^2}{\Delta_{20}}\right) &> \mu_y^2\left(1 - \frac{M_1^2}{L_1}\right) \\ \frac{M_1^2}{L_1} &> 1 - \Delta_{02}^* + \frac{\Delta_{11}^2}{\Delta_{20}}. \end{aligned}$$

(ii). The suggested estimator T_{s_2} represses the power ratio estimator t_{g_2} under scheme II, if

$$\begin{aligned} MSE(t_{g_2}) &> MSE(T_{s_2}) \\ \implies \mu_y^2\left(\Delta_{02}^* - \frac{\Delta_{11}^{*2}}{\Delta_{20}^*}\right) &> \mu_y^2\left(1 - \frac{M_2^2}{L_2}\right) \\ \frac{M_2^2}{L_2} &> 1 - \Delta_{02}^* + \frac{\Delta_{11}^{*2}}{\Delta_{20}^*}. \end{aligned}$$

(iii). The suggested estimator T_{s_3} represses the power ratio estimator t_{g_3} under scheme III, if

$$\begin{aligned} MSE(t_{g_3}) &> MSE(T_{s_3}) \\ \implies \mu_y^2\left[\Delta_{02}^* - \frac{(\Delta_{11}^* - \Delta_{11})^2}{(\Delta_{20}^* - \Delta_{20})}\right] &> \mu_y^2\left(1 - \frac{M_2^2}{L_2}\right) \\ \frac{M_3^2}{L_3} &> 1 - \Delta_{02}^* + \frac{(\Delta_{11}^* - \Delta_{11})^2}{(\Delta_{20}^* - \Delta_{20})}. \end{aligned}$$

The suggested imputation methods repress the adapted imputation methods if the mathematical conditions determined under the above lemmas are satisfied.

6. Simulation

To strengthen the mathematical results as well as to see the effect of the correlated MEs on the adapted and suggested imputation methods and the resultant estimators, a comprehensive simulation is conducted on a hypothetically created population. The simulation algorithm is delineated under the following points.

(i). A population of size $N = 1000$ is created utilizing a four-variate multivariate normal distribution through R software as $W = (X, Y, u, v)'$ with the mean vector $\mu_w = (\mu_x, \mu_y, 0, 0)'$ and covariance matrix:

$$\begin{pmatrix} \sigma_x^2 & \rho_{xy}\sigma_x\sigma_y & 0 & 0 \\ \rho_{xy}\sigma_x\sigma_y & \sigma_y^2 & 0 & 0 \\ 0 & 0 & \sigma_u^2 & \rho_{uv}\sigma_u\sigma_v \\ 0 & 0 & \rho_{uv}\sigma_u\sigma_v & \sigma_v^2 \end{pmatrix}.$$

The descriptives used to create the population are as follows: $\mu_y = 12$, $\mu_x = 18$, $\sigma_y^2 = 125$, $\sigma_x^2 = 136$, ρ_{xy}=-0.9, -0.5, -0.1, 0, 0.5, 0.9, $\sigma_u^2 = 12$, $\sigma_v^2 = 14$, and ρ_{uv}=-0.9, -0.5, 0, 0.5, 0.9.

(ii). Take 12000 ranked set samples of size $n = 15$ from the above population using RSS.

(iii). Consider the samples taken in step (*ii*), and the 12000 values of each resultant estimator are obtained.

(iv). Utilizing the descriptives described in (*i*), the percent relative efficiency (PRE) of several estimators is obtained for the responding probability $P = 0.67$ as well as for different combinations of MEs such as 5%, 10%, and 15%. The PRE is calculated employing the following expression:

$$PRE = \frac{\sum_{i=1}^{12000}(t_{mi} - \mu_y)^2}{\sum_{i=1}^{12000}(T_i^* - \mu_y)^2} \times 100$$

where $T_i^* = t_m,\ t_{r_1},\ t_{r_2},\ t_{r_3},\ t_{p_1},\ t_{p_2},\ t_{p_3},\ t_{g_1},\ t_{g_2},\ t_{g_3},\ T_{s_1},\ T_{s_2},\ T_{s_3}$.

The findings of the simulation are displayed in Tables 1–4.

Tables 1–4 contains the simulation results (PRE) of the resultant adapted and proposed estimators. The important results are construed in the following points:

(1). The findings of Table 1 are given for positive correlation coefficients $\rho_{xy} = 0,\ 0.5,\ 0.9$ from where it is noticed that:

- The percent relative efficiency of the ratio estimator t_{r_1} under scheme I grows with the consecutive growth in the valuations of ρ_{xy} from 0 to 0.9. The percent relative efficiency also grows with the consecutive growth in the magnitude and direction of ρ_{uv} from −0.9 to 0.9. These observations are clearly visible from Figure 1. The percent relative efficiency of the ratio estimators t_{r_2} and t_{r_3} under schemes II and III, respectively, follow the same demeanor for which the figures can be provided, if needed.
- The percent relative efficiency of the product estimator t_{p_1} under scheme I diminishes with the consecutive diminishment in the valuations of ρ_{xy} from −0.1 to −0.9. The percent relative efficiency also diminishes with the consecutive diminishment in the magnitude and direction of ρ_{uv} from −0.9 to 0.9. These observations are clearly visible from Figure 2. The percent relative efficiency of the product estimators t_{p_2} and t_{p_3} under schemes II and III, respectively, follow the same demeanor for which the figures can be provided, if needed.
- The percent relative efficiency of the power ratio estimator t_{g_1} under scheme I grows with the consecutive growth in the valuations of ρ_{xy} from 0 to 0.9. The percent relative efficiency also grows with the consecutive growth in the magnitude and direction of ρ_{uv} from −0.9 to 0.9. These observations are clearly visible from Figure 3. The percent relative efficiency of the power ratio estimators t_{g_2} and t_{g_3} under schemes II and III, respectively, follow the same demeanor for which the figures can be provided, if needed.
- The percent relative efficiency of the Searls type power ratio estimator T_{s_1} under scheme I grows with the consecutive growth in the valuations of ρ_{xy} from 0 to 0.9. The percent relative efficiency also grows with the consecutive growth in the magnitude and direction of ρ_{uv} from −0.9 to 0.9. These observations are clearly visible from Figure 4. The percent relative efficiency of the Searls type power ratio estimators T_{s_2} and T_{s_3} under schemes II and III, respectively, follow the same demeanor for which the figures can be provided, if needed.
- The proposed Searls type power ratio estimators repress the ratio, product and power ratio estimators under the respective schemes for different valuations of correlation coefficients.

(2). The findings of Table 2 are given for negative correlation coefficient $\rho_{xy} = -0.1, \ -0.5, \ -0.9$ from where it is noticed that:

- The percent relative efficiency of the ratio estimator t_{r_1} under scheme I diminishes with the consecutive diminish in the valuations of ρ_{xy} from -0.1 to -0.9, but the percent relative efficiency grows with the consecutive growth in the magnitude and direction of ρ_{uv} from -0.9 to 0.9. These observations are clearly visible from Figure 5. The percent relative efficiency of the ratio estimators t_{r_2} and t_{r_3} under schemes II and III, respectively, follow the same demeanor for which the figures can be provided, if needed.
- The percent relative efficiency of the product estimator t_{p_1} under scheme I grows with the consecutive diminishment in the valuations of ρ_{xy} from -0.1 to -0.9. The percent relative efficiency also diminishes with the consecutive growth in the magnitude and direction of ρ_{uv} from -0.9 to 0.9. These observations are clearly visible from Figure 6. The percent relative efficiency of the product estimators t_{p_2} and t_{p_3} under schemes II and III, respectively, follow the same demeanor for which the figures can be provided, if needed.
- The percent relative efficiency of the power ratio estimator t_{g_1} under scheme I grows with the consecutive diminishment in the valuations of ρ_{xy} from -0.1 to -0.9, but the percent relative efficiency diminishes with the consecutive growth in the magnitude and direction of ρ_{uv} from -0.9 to 0.9. These observations are clearly visible from Figure 7. The percent relative efficiency of the power ratio estimators t_{g_2} and t_{g_3} under schemes II and III, respectively, follow the same demeanor for which the figures can be provided, if needed.
- The percent relative efficiency of the Searls type power ratio estimator T_{s_1} under scheme I grows with the consecutive diminishment in the valuations of ρ_{xy} from -0.1 to -0.9. The percent relative efficiency also grows with the consecutive growth in the magnitude and direction of ρ_{uv} from -0.9 to 0.9. These observations are clearly visible from Figure 8. The percent relative efficiency of the Searls type power ratio estimators T_{s_2} and T_{s_3} under schemes II and III, respectively, follow the same demeanor for which the figures can be provided, if needed.
- The proposed Searls type power ratio estimators repress the ratio, product and power ratio estimators under the respective schemes for different valuations of correlation coefficients.

(3). The findings of Table 3 are given for positive correlation coefficient $\rho_{xy} = 0, \ 0.5, \ 0.9$ with different percentages of ME. When ME=5%, then it is noticed that:

- The percent relative efficiency of the ratio estimator t_{r_1} under scheme I grows with the consecutive growth in the valuations of ρ_{xy} from 0 to 0.9. The percent relative efficiency also grows with the consecutive growth in the magnitude and direction of ρ_{uv} from -0.9 to 0.9. These observations are clearly visible from Figure 9. The percent relative efficiency of the ratio estimators t_{r_2} and t_{r_3} under schemes II and III, respectively, follow the same demeanor for which the figures can be provided, if needed.
- The percent relative efficiency of the product estimator t_{p_1} under scheme I diminishes with the consecutive diminish in the valuations of ρ_{xy} from -0.1 to -0.9. The percent relative efficiency also diminishes with the consecutive diminish in the magnitude and direction of ρ_{uv} from -0.9 to 0.9. These observations are clearly visible from Figure 10. The percent relative efficiency of the product estimators t_{p_2} and t_{p_3} under schemes II and III, respectively,

follow the same demeanor for which the figures can be provided, if needed.

- The percent relative efficiency of the power ratio estimator t_{g_1} under scheme I grows with the consecutive growth in the valuations of ρ_{xy} from 0 to 0.9. The percent relative efficiency also grows with the consecutive growth in the magnitude and direction of ρ_{uv} from -0.9 to 0.9. These observations are clearly visible from Figure 11. The percent relative efficiency of the power ratio estimators t_{g_2} and t_{g_3} under schemes II and III, respectively, follow the same demeanor for which the figures can be provided, if needed.
- The percent relative efficiency of the Searls type power ratio estimator T_{s_1} under scheme I grows with the consecutive growth in the valuations of ρ_{xy} from 0 to 0.9. The percent relative efficiency also grows with the consecutive growth in the magnitude and direction of ρ_{uv} from -0.9 to 0.9. These observations are clearly visible from Figure 12. The percent relative efficiency of the Searls type power ratio estimators T_{s_2} and T_{s_3} under schemes II and III, respectively, follow the same demeanor for which the figures can be provided, if needed.
- The proposed Searls type power ratio estimators repress the adapted ratio, product and power ratio estimators under the respective schemes for different valuations of correlation coefficients.
- Moreover, the above observations are also true for the other percentages of ME, like 10% and 15%.

(4). The findings of Table 4 are given for negative correlation coefficients $\rho_{xy} = -0.1, \ -0.5, \ -0.9$ with different percentages of ME. When ME=5%, then it is noticed that:

- The percent relative efficiency of the ratio estimator t_{r_1} under scheme I diminishes with the consecutive diminishment in the valuations of ρ_{xy} from -0.1 to -0.9, but the percent relative efficiency grows with the consecutive growth in the magnitude and direction of ρ_{uv} from -0.9 to 0.9. These observations are clearly visible from Figure 13. The percent relative efficiency of the ratio estimators t_{r_2} and t_{r_3} under schemes II and III, respectively, follow the same demeanor for which the figures can be provided, if needed.
- The percent relative efficiency of the product estimator t_{p_1} under scheme I grows with the consecutive diminishment in the valuations of ρ_{xy} from -0.1 to -0.9. The percent relative efficiency also diminishes with the consecutive growth in the magnitude and direction of ρ_{uv} from -0.9 to 0.9. These observations are clearly visible from Figure 14. The percent relative efficiency of the product estimators t_{p_2} and t_{p_3} under schemes II and III, respectively, follow the same demeanor for which the figures can be provided, if needed.
- The percent relative efficiency of the power ratio estimator t_{g_1} under scheme I grows with the consecutive diminishment in the valuations of ρ_{xy} from -0.1 to -0.9, but the percent relative efficiency diminishes with the consecutive growth in the magnitude and direction of ρ_{uv} from -0.9 to 0.9. These observations are clearly visible from Figure 15. The percent relative efficiency of the power ratio estimators t_{g_2} and t_{g_3} under schemes II and III, respectively, follow the same demeanor for which the figures can be provided, if needed.
- The percent relative efficiency of the Searls type power ratio estimator T_{s_1} under scheme I grows with the consecutive diminishment in the valuations of ρ_{xy} from -0.1 to -0.9. The percent relative efficiency also grows with the consecutive growth in the magnitude and direction of ρ_{uv} from -0.9 to 0.9. These observations are clearly visible from Figure 16. The percent relative efficiency of the Searls type power ratio estimators T_{s_2} and T_{s_3} under

schemes II and III, respectively, follow the same demeanor for which the figures can be provided, if needed.

- The proposed Searls type power ratio estimators repress the ratio, product and power ratio estimators under the respective schemes for different valuations of correlation coefficients.
- Moreover, the above observations are also true for the other percentages of ME, like 10% and 15%.

Table 1. PRE of different estimators when $\rho_{xy} = 0,\ 0.5,\ 0.9$.

		Scheme I				Scheme II				Scheme III			
ρ_{xy}	ρ_{uv}	t_{r_1}	t_{p_1}	t_{g_1}	T_{s_1}	t_{r_2}	t_{p_2}	t_{g_2}	T_{s_2}	t_{r_3}	t_{p_3}	t_{g_3}	T_{s_3}
0	-0.9	84.80	88.87	102.19	126.76	64.79	72.50	106.96	130.45	73.31	79.74	104.56	123.86
	-0.5	85.92	87.83	102.07	126.84	66.82	70.43	106.54	130.47	75.03	78.05	104.29	125.04
	0	87.15	86.46	102.04	126.93	69.13	67.82	106.44	130.78	76.97	75.88	104.23	126.46
	0.5	88.60	85.00	102.14	127.18	71.95	65.15	106.79	131.60	79.29	73.62	104.45	128.19
	0.9	89.79	83.98	102.33	127.50	74.38	63.36	107.40	132.60	81.25	72.08	104.84	129.64
0.5	-0.9	85.96	81.59	102.27	121.91	66.90	59.38	107.21	126.90	75.10	68.58	104.72	123.49
	-0.5	87.26	80.59	102.42	122.24	69.33	57.81	107.71	127.77	77.14	67.16	105.04	125.00
	0	88.75	79.26	102.73	122.61	72.24	55.77	108.75	129.11	79.53	65.30	105.70	126.85
	0.5	90.46	77.87	103.22	123.16	75.78	53.72	110.44	131.13	82.36	63.41	106.76	129.06
	0.9	91.86	76.90	103.71	123.74	78.84	52.35	112.16	133.15	84.76	62.11	107.83	130.93
0.9	-0.9	91.92	80.23	103.46	124.86	78.97	57.25	111.27	133.22	84.86	66.65	107.28	129.88
	-0.5	93.32	79.16	103.90	125.29	82.19	55.63	112.84	134.97	87.32	65.17	108.25	131.58
	0	95.28	77.64	104.72	126.08	86.96	53.40	115.81	138.19	90.87	63.10	110.07	134.22
	0.5	97.44	76.44	105.69	127.26	92.62	51.70	119.49	142.38	94.93	61.51	112.27	137.21
	0.9	98.99	75.52	106.52	128.17	97.00	50.45	122.78	145.99	97.97	60.31	114.20	139.49

Table 2. *PRE* of different estimators when $\rho_{xy} = -0.1,\ -0.5,\ -0.9$.

		Scheme I				Scheme II				Scheme III			
ρ_{xy}	ρ_{uv}	t_{r_1}	t_{p_1}	t_{g_1}	T_{s_1}	t_{r_2}	t_{p_2}	t_{g_2}	T_{s_2}	t_{r_3}	t_{p_3}	t_{g_3}	T_{s_3}
-0.1	-0.9	82.77	88.95	102.34	125.95	61.32	72.65	107.44	129.66	70.29	79.86	104.86	121.99
	-0.5	83.89	87.86	102.13	125.97	63.21	70.50	106.76	129.48	71.94	78.10	104.43	123.16
	0	85.10	86.44	102.02	125.99	65.34	67.79	106.37	129.55	73.78	75.85	104.18	124.54
	0.5	86.53	84.92	102.03	126.19	67.95	65.02	106.40	130.15	75.99	73.51	104.20	126.27
	0.9	87.72	83.87	102.14	126.47	70.21	63.18	106.78	130.97	77.86	71.91	104.44	127.71
-0.5	-0.9	79.10	93.18	103.71	126.63	55.53	81.85	112.16	132.31	65.08	87.06	107.83	118.47
	-0.5	80.13	91.97	103.23	126.41	57.10	79.08	110.47	131.18	66.52	84.94	106.78	119.44
	0	81.24	90.42	102.78	126.17	58.83	75.69	108.92	130.22	68.08	82.30	105.80	120.58
	0.5	82.53	88.76	102.43	126.08	60.93	72.27	107.73	129.73	69.95	79.55	105.05	122.04
	0.9	83.62	87.60	102.27	126.14	62.75	69.97	107.21	129.76	71.55	77.67	104.72	123.27
-0.9	-0.9	76.65	99.03	106.55	129.13	52.00	97.13	122.88	141.43	61.79	98.06	114.25	116.47
	-0.5	77.54	97.55	105.70	128.52	53.25	92.92	119.53	138.47	62.96	95.15	112.29	117.14
	0	78.74	95.48	104.70	127.81	55.00	87.44	115.76	135.34	64.59	91.22	110.03	118.16
	0.5	80.23	93.63	103.91	127.52	57.25	82.90	112.85	133.46	66.66	87.86	108.26	119.60
	0.9	81.25	92.28	103.46	127.36	58.86	79.78	111.27	132.50	68.10	85.48	107.28	120.68

Table 3. *PRE* of the estimators for different levels of MEs with $\rho_{xy} = 0,\ 0.5,\ 0.9$.

			Scheme I				Scheme II				Scheme III			
ME %	ρ_{xy}	ρ_{uv}	t_{r_1}	t_{p_1}	t_{g_1}	T_{s_1}	t_{r_2}	t_{p_2}	t_{g_2}	T_{s_2}	t_{r_3}	t_{p_3}	t_{g_3}	T_{s_3}
5%	0	-0.9	86.06	87.75	102.24	126.27	67.07	70.27	107.11	130.34	75.25	77.91	104.65	124.56
		-0.5	86.61	87.15	102.21	126.30	68.10	69.11	107.01	130.42	76.11	76.96	104.59	125.17
		0	87.31	86.50	102.21	126.39	69.43	67.89	107.02	130.66	77.22	75.94	104.60	125.94
		0.5	87.31	86.50	102.21	126.39	69.43	67.89	107.02	130.66	77.22	75.94	104.60	125.94
		0.9	88.71	85.21	102.31	126.66	72.16	65.53	107.35	131.44	79.46	73.94	104.81	127.57
	0.5	-0.9	87.58	80.29	102.66	121.80	69.95	57.35	108.53	127.93	77.65	66.74	105.56	124.83
		-0.5	88.24	79.72	102.79	121.96	71.24	56.47	108.98	128.50	78.71	65.95	105.84	125.63
		0	89.07	79.10	102.98	122.22	72.90	55.53	109.62	129.34	80.06	65.08	106.25	126.66
		0.5	89.97	78.36	103.24	122.51	74.75	54.45	110.50	130.39	81.55	64.08	106.80	127.81
		0.9	90.72	77.88	103.46	122.81	76.33	53.74	111.28	131.36	82.80	63.42	107.29	128.77
	0.9	-0.9	94.72	79.27	104.43	125.92	85.55	55.80	114.76	137.00	89.83	65.33	109.43	132.70
		-0.5	95.43	78.76	104.72	126.23	87.33	55.03	115.83	138.19	91.14	64.62	110.08	133.65
		0	96.31	78.21	105.11	126.66	89.61	54.22	117.26	139.80	92.79	63.87	110.94	134.83
		0.5	97.28	77.54	105.58	127.15	92.20	53.26	119.09	141.79	94.64	62.97	112.03	136.20
		0.9	98.07	77.11	105.98	127.60	94.39	52.64	120.63	143.51	96.17	62.39	112.94	137.31
10%	0	-0.9	84.90	89.01	102.20	126.86	64.98	72.77	106.97	130.55	73.47	79.96	104.57	123.94
		-0.5	85.89	87.78	102.06	126.82	66.77	70.34	106.52	130.45	74.99	77.97	104.28	125.04
		0	87.22	86.53	102.04	126.98	69.25	67.95	106.44	130.83	77.07	75.99	104.23	126.51
		0.5	88.68	85.05	102.14	127.24	72.10	65.25	106.79	131.66	79.41	73.70	104.45	128.26
		0.9	89.93	84.09	102.33	127.61	74.66	63.55	107.42	132.71	81.47	72.24	104.85	129.75
	0.5	-0.9	86.07	81.74	102.27	122.01	67.10	59.64	107.20	126.98	75.27	68.80	104.71	123.55
		-0.5	87.25	80.56	102.42	122.19	69.30	57.76	107.72	127.72	77.12	67.12	105.04	124.96
		0	88.82	79.35	102.73	122.66	72.39	55.90	108.75	129.15	79.64	65.42	105.70	126.88
		0.5	90.54	77.94	103.22	123.21	75.96	53.83	110.45	131.19	82.51	63.51	106.77	129.11
		0.9	92.00	77.03	103.72	123.85	79.14	52.53	112.18	133.28	84.99	62.29	107.84	131.02
	0.9	-0.9	91.97	80.33	103.45	124.89	79.07	57.40	111.23	133.21	84.94	66.79	107.26	129.86
		-0.5	93.38	79.28	103.89	125.40	82.32	55.80	112.81	135.05	87.42	65.33	108.23	131.64
		0	95.34	77.73	104.72	126.13	87.10	53.52	115.80	138.22	90.98	63.22	110.06	134.24
		0.5	97.51	76.53	105.69	127.33	92.81	51.83	119.50	142.44	95.07	61.63	112.27	137.24
		0.9	99.06	75.60	106.53	128.21	97.20	50.56	122.80	146.04	98.11	60.41	114.21	139.50
15%	0	-0.9	83.77	90.10	102.26	127.48	63.01	75.02	107.18	131.01	71.77	81.76	104.70	123.41
		-0.5	85.38	88.55	101.98	127.53	65.83	71.84	106.25	130.78	74.20	79.20	104.10	125.10
		0	87.14	86.56	101.89	127.62	69.09	68.00	105.97	131.14	76.94	76.03	103.92	127.12
		0.5	89.41	84.58	102.09	128.15	73.60	64.42	106.60	132.61	80.62	72.99	104.33	129.83
		0.9	91.00	83.00	102.45	128.59	76.94	61.71	107.80	134.25	83.28	70.63	105.10	131.90
	0.5	-0.9	84.64	83.00	102.04	122.29	64.52	61.71	106.46	126.50	73.07	70.63	104.24	122.47
		-0.5	86.48	81.52	102.14	122.68	67.85	59.28	106.78	127.42	75.91	68.48	104.44	124.58
		0	88.58	79.57	102.51	123.14	71.92	56.25	108.03	129.13	79.26	65.74	105.24	127.18
		0.5	91.22	77.71	103.25	124.12	77.42	53.50	110.53	132.29	83.66	63.20	106.82	130.55
		0.9	93.14	76.19	104.03	124.88	81.74	51.36	113.29	135.34	86.98	61.18	108.53	133.21
	0.9	-0.9	90.13	81.78	102.83	125.01	75.09	59.69	109.11	131.61	81.81	68.85	105.93	128.37
		-0.5	92.08	80.21	103.30	125.49	79.32	57.21	110.72	133.51	85.13	66.62	106.93	130.65
		0	94.45	77.41	104.31	125.93	84.88	53.07	114.33	137.00	89.34	62.79	109.17	133.93
		0.5	97.94	76.23	105.73	128.13	94.01	51.42	119.63	143.46	95.90	61.24	112.35	138.51
		0.9	100.21	74.94	107.01	129.57	100.63	49.67	124.78	149.13	100.42	59.56	115.35	141.97

Table 4. *PRE* of the estimators for different levels of MEs with $\rho_{xy} = -0.1, \ -0.5, \ -0.9$.

			Scheme I				Scheme II				Scheme III			
ME %	ρ_{xy}	ρ_{uv}	t_{r_1}	t_{p_1}	t_{g_1}	T_{s_1}	t_{r_2}	t_{p_2}	t_{g_2}	T_{s_2}	t_{r_3}	t_{p_3}	t_{g_3}	T_{s_3}
5%	-0.1	-0.9	83.93	87.78	102.30	125.36	63.29	70.32	107.33	129.31	72.01	77.96	104.79	122.60
		-0.5	84.48	87.15	102.23	125.36	64.24	69.12	107.09	129.29	72.83	76.97	104.64	123.20
		0	85.17	86.48	102.19	125.41	65.47	67.85	106.94	129.39	73.89	75.90	104.55	123.96
		0.5	85.92	85.67	102.18	125.50	66.82	66.37	106.90	129.63	75.03	74.65	104.52	124.84
		0.9	86.56	85.14	102.20	125.63	68.00	65.40	106.97	129.94	76.02	73.83	104.56	125.57
	-0.5	-0.9	80.02	92.07	103.46	125.86	56.92	79.30	111.28	131.30	66.36	85.12	107.29	118.80
		-0.5	80.52	91.39	103.24	125.73	57.69	77.78	110.50	130.76	67.06	83.94	106.80	119.29
		0	81.15	90.64	103.01	125.64	58.69	76.16	109.71	130.28	67.96	82.66	106.30	119.91
		0.5	81.83	89.75	102.79	125.56	59.77	74.30	108.98	129.89	68.92	81.18	105.84	120.64
		0.9	82.41	89.15	102.66	125.56	60.73	73.07	108.53	129.73	69.77	80.19	105.56	121.26
	-0.9	-0.9	77.05	97.20	105.95	127.51	52.57	91.98	120.51	138.58	62.32	94.48	112.87	116.18
		-0.5	77.51	96.46	105.56	127.24	53.21	89.98	118.98	137.24	62.93	93.06	111.97	116.54
		0	78.09	95.63	105.13	126.98	54.05	87.84	117.37	135.94	63.71	91.51	111.01	117.09
		0.5	78.70	94.67	104.70	126.72	54.95	85.41	115.75	134.65	64.54	89.73	110.03	117.54
		0.9	79.24	94.00	104.41	126.59	55.75	83.79	114.68	133.87	65.28	88.53	109.38	118.03
10%	-0.1	-0.9	82.89	89.09	102.34	126.05	61.51	72.94	107.45	129.76	70.46	80.09	104.87	122.08
		-0.5	83.86	87.82	102.13	125.97	63.16	70.40	106.75	129.47	71.90	78.02	104.43	123.16
		0	85.18	86.51	102.01	126.05	65.47	67.92	106.36	129.61	73.89	75.96	104.18	124.61
		0.5	86.62	84.98	102.03	126.26	68.11	65.13	106.4	130.21	76.12	73.60	104.20	126.35
		0.9	87.86	83.98	102.14	126.58	70.48	63.37	106.79	131.08	78.09	72.08	104.45	127.83
	-0.5	-0.9	79.22	93.31	103.72	126.73	55.72	82.15	112.18	132.41	65.25	87.29	107.84	118.57
		-0.5	80.10	91.92	103.22	126.41	57.05	78.97	110.45	131.17	66.47	84.86	106.77	119.44
		0	81.32	90.48	102.77	126.23	58.95	75.83	108.91	130.27	68.19	82.40	105.80	120.65
		0.5	82.62	88.81	102.42	126.15	61.08	72.37	107.72	129.79	70.08	79.63	105.04	122.13
		0.9	83.77	87.70	102.27	126.24	63.01	70.17	107.20	129.85	71.77	77.83	104.71	123.40
	-0.9	-0.9	76.73	99.09	106.55	129.18	52.10	97.31	122.90	141.51	61.88	98.18	114.26	116.53
		-0.5	77.64	97.63	105.71	128.62	53.40	93.16	119.57	138.57	63.10	95.31	112.31	117.24
		0	78.82	95.53	104.70	127.87	55.12	87.58	115.75	135.39	64.71	91.33	110.03	118.24
		0.5	80.33	93.68	103.90	127.58	57.41	83.03	112.83	133.51	66.80	87.96	108.25	119.70
		0.9	81.35	92.32	103.45	127.41	59.00	79.87	111.23	132.53	68.23	85.55	107.26	120.76
15%	-0.1	-0.9	81.84	90.22	102.47	126.77	59.80	75.28	107.89	130.43	68.95	81.96	105.15	121.64
		-0.5	83.44	88.61	102.08	126.74	62.45	71.96	106.59	129.92	71.28	79.30	104.32	123.30
		0	85.18	86.55	101.87	126.74	65.48	67.98	105.90	129.96	73.90	76.01	103.88	125.30
		0.5	87.44	84.50	101.93	127.20	69.67	64.28	106.09	131.12	77.42	72.87	104.00	127.99
		0.9	89.01	82.87	102.20	127.58	72.77	61.49	106.98	132.53	79.95	70.44	104.57	130.06
	-0.5	-0.9	78.40	94.37	104.03	127.58	54.49	84.70	113.29	133.68	64.12	89.20	108.53	118.36
		-0.5	79.88	92.58	103.25	127.23	56.72	80.46	110.53	131.80	66.17	86.01	106.82	119.76
		0	81.47	90.34	102.58	126.88	59.19	75.53	108.25	130.44	68.40	82.17	105.38	121.43
		0.5	83.55	88.10	102.14	126.93	62.64	70.95	106.78	130.14	71.45	78.47	104.44	123.77
		0.9	84.97	86.33	102.04	127.04	65.11	67.58	106.46	130.49	73.58	75.67	104.24	125.54
	-0.9	-0.9	76.06	100.19	107.04	130.27	51.18	100.57	124.89	143.47	61.01	100.38	115.41	116.56
		-0.5	77.32	98.03	105.74	129.36	52.94	94.25	119.71	139.07	62.67	96.07	112.40	117.51
		0	79.08	95.10	104.33	128.00	55.51	86.50	114.39	134.60	65.06	90.53	109.20	118.72
		0.5	81.21	92.41	103.30	128.06	58.79	80.07	110.72	132.71	68.04	85.71	106.94	121.26
		0.9	82.74	90.56	102.83	128.04	61.27	75.98	109.10	132.08	70.25	82.52	105.92	123.01

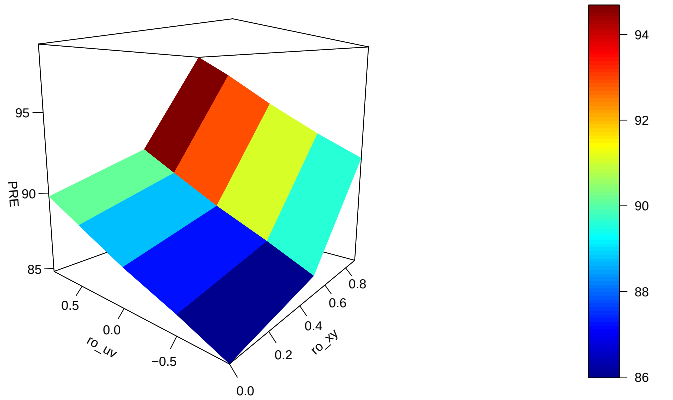

Figure 1. PRE of the estimator t_{r_1} reported in Table 1 for $\rho_{xy} = (0,\ 0.5,\ 0.9)$, and $\rho_{uv} = (-0.9,\ -0.5,\ 0,\ 0.5,\ 0.9)$.

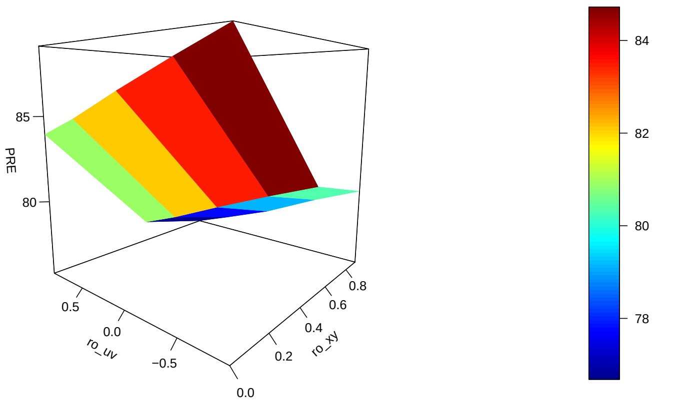

Figure 2. PRE of the estimator t_{p_1} reported in Table 1 for $\rho_{xy} = (0,\ 0.5,\ 0.9)$, and $\rho_{uv} = (-0.9,\ -0.5,\ 0,\ 0.5,\ 0.9)$.

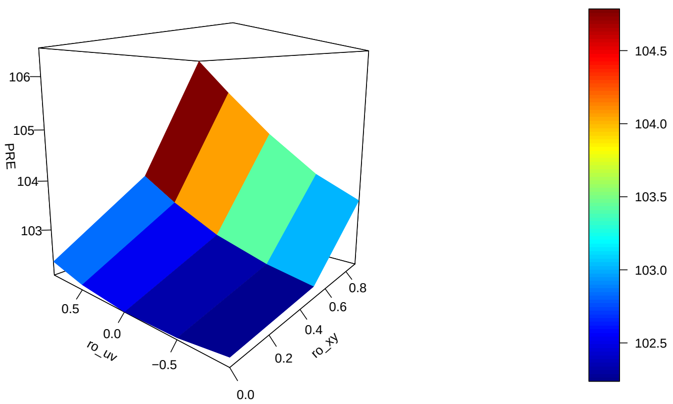

Figure 3. PRE of the estimator t_{g_1} reported in Table 1 for $\rho_{xy} = (0,\ 0.5,\ 0.9)$, and $\rho_{uv} = (-0.9,\ -0.5,\ 0,\ 0.5,\ 0.9)$.

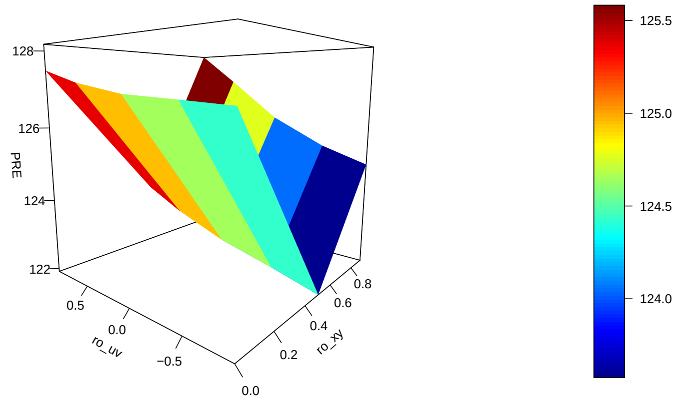

Figure 4. PRE of the estimator T_{s_1} reported in Table 1 for $\rho_{xy} = (0,\ 0.5,\ 0.9)$, and $\rho_{uv} = (-0.9,\ -0.5,\ 0,\ 0.5,\ 0.9)$.

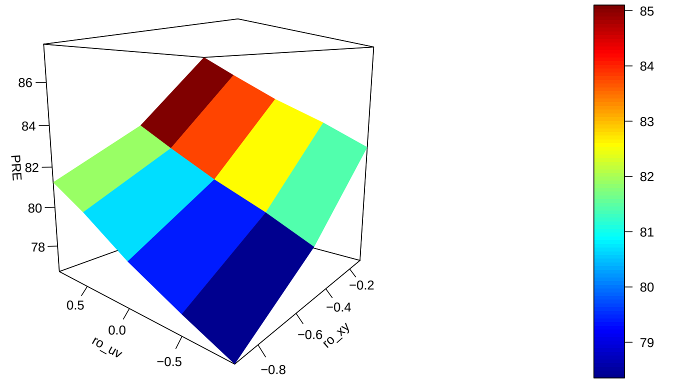

Figure 5. PRE of the estimator t_{r_1} reported in Table 2 for $\rho_{xy} = (-0.1,\ -0.5,\ -0.9)$, and $\rho_{uv} = (-0.9,\ -0.5,\ 0,\ 0.5,\ 0.9)$.

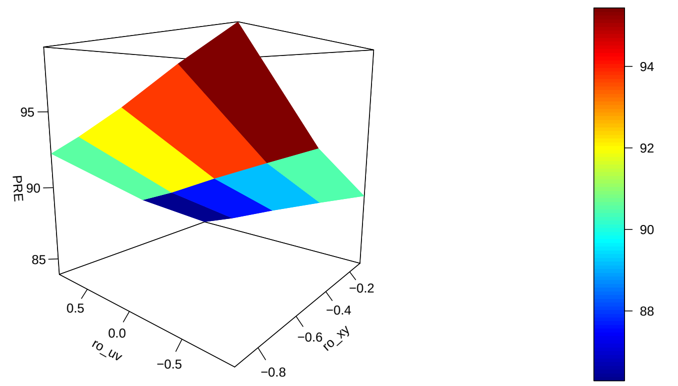

Figure 6. PRE of the estimator t_{p_1} reported in Table 2 for $\rho_{xy} = (-0.1,\ -0.5,\ -0.9)$, and $\rho_{uv} = (-0.9,\ -0.5,\ 0,\ 0.5,\ 0.9)$.

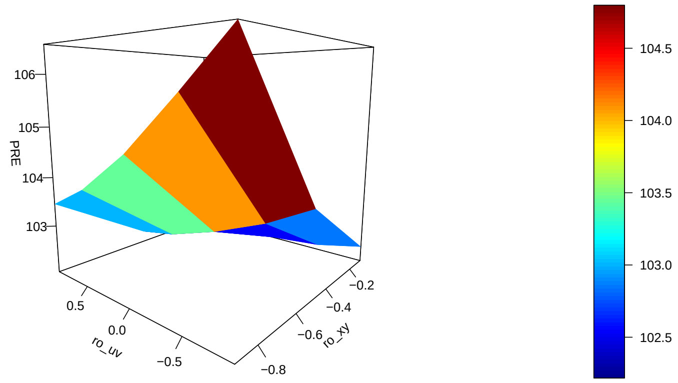

Figure 7. PRE of the estimator t_{g_1} reported in Table 2 for $\rho_{xy} = (-0.1, \ -0.5, \ -0.9)$, and $\rho_{uv} = (-0.9, \ -0.5, \ 0, \ 0.5, \ 0.9)$.

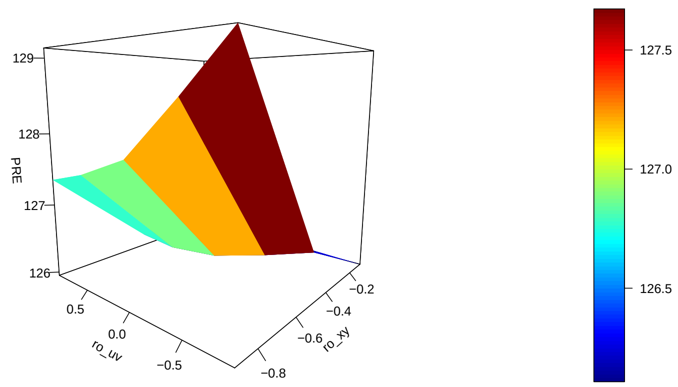

Figure 8. PRE of the estimator T_{s_1} reported in Table 2 for $\rho_{xy} = (-0.1, \ -0.5, \ -0.9)$, and $\rho_{uv} = (-0.9, \ -0.5, \ 0, \ 0.5, \ 0.9)$.

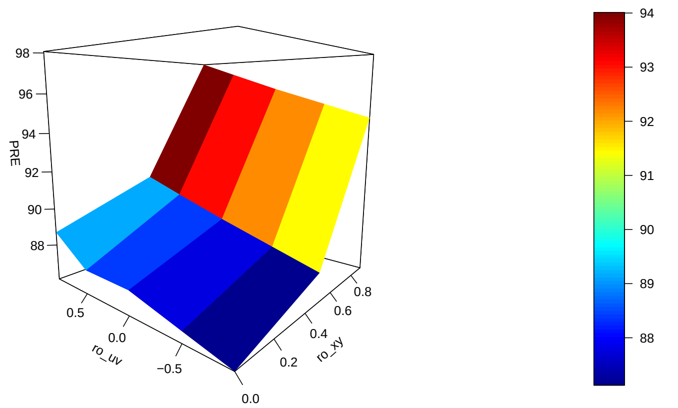

Figure 9. PRE of the estimator t_{r_1} reported in Table 3 for ME=5%, $\rho_{xy} = (0, \ 0.5, \ 0.9)$, and $\rho_{uv} = (-0.9, \ -0.5, \ 0, \ 0.5, \ 0.9)$.

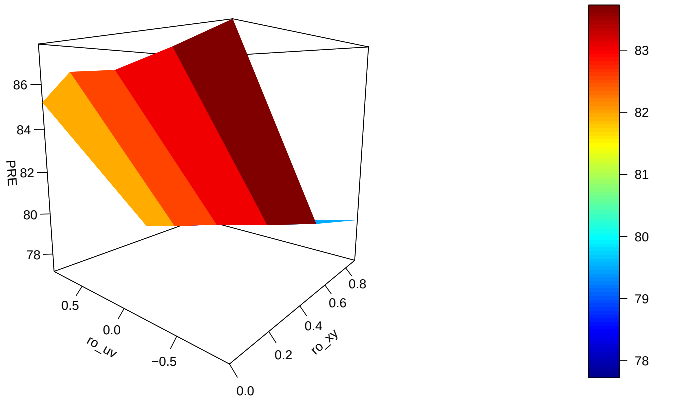

Figure 10. PRE of the estimator t_{p_1} reported in Table 3 for ME=5%, $\rho_{xy} = (0,\ 0.5,\ 0.9)$, and $\rho_{uv} = (-0.9,\ -0.5,\ 0,\ 0.5,\ 0.9)$.

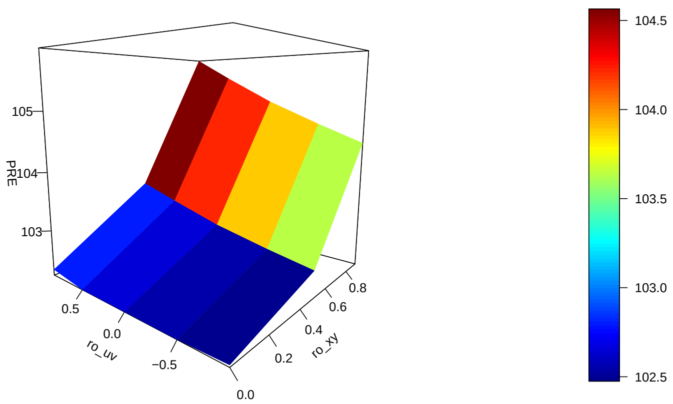

Figure 11. PRE of the estimator t_{g_1} reported in Table 3 for ME=5%, $\rho_{xy} = (0,\ 0.5,\ 0.9)$, and $\rho_{uv} = (-0.9,\ -0.5,\ 0,\ 0.5,\ 0.9)$.

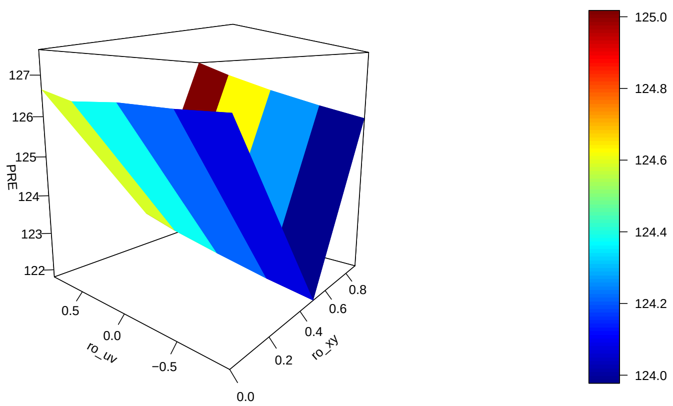

Figure 12. PRE of the estimator T_{s_1} reported in Table 3 for ME=5%, $\rho_{xy} = (0,\ 0.5,\ 0.9)$, and $\rho_{uv} = (-0.9,\ -0.5,\ 0,\ 0.5,\ 0.9)$.

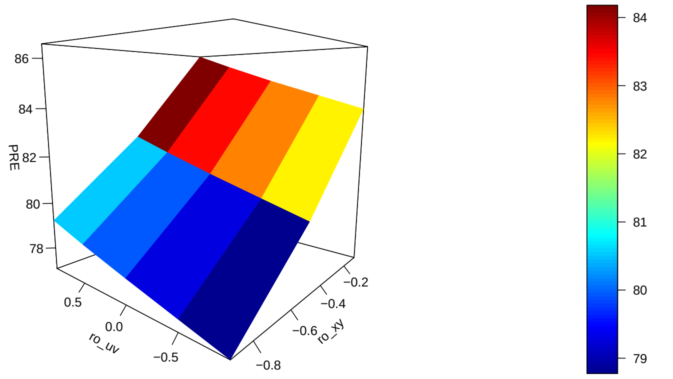

Figure 13. PRE of the estimator t_{r_1} reported in Table 4 for ME=5%, $\rho_{xy} = (-0.1, \ -0.5, \ -0.9)$, and $\rho_{uv} = (-0.9, \ -0.5, \ 0, \ 0.5, \ 0.9)$.

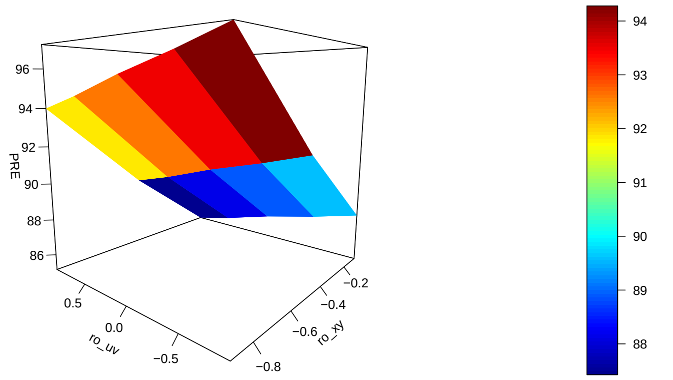

Figure 14. PRE of the estimator t_{p_1} reported in Table 4 for ME=5%, $\rho_{xy} = (-0.1, \ -0.5, \ -0.9)$, and $\rho_{uv} = (-0.9, \ -0.5, \ 0, \ 0.5, \ 0.9)$.

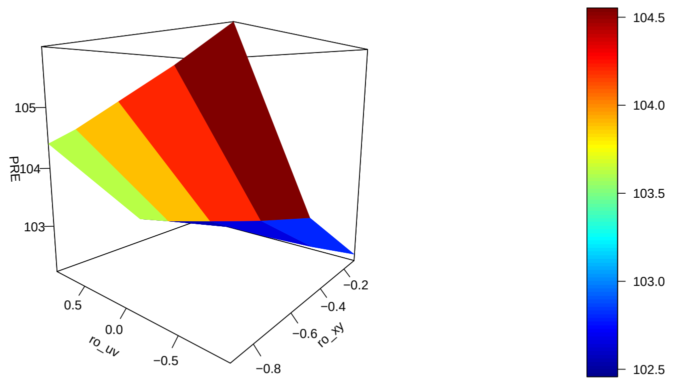

Figure 15. PRE of the estimator t_{g_1} reported in Table 4 for ME=5%, $\rho_{xy} = (-0.1, \ -0.5, \ -0.9)$, and $\rho_{uv} = (-0.9, \ -0.5, \ 0, \ 0.5, \ 0.9)$.

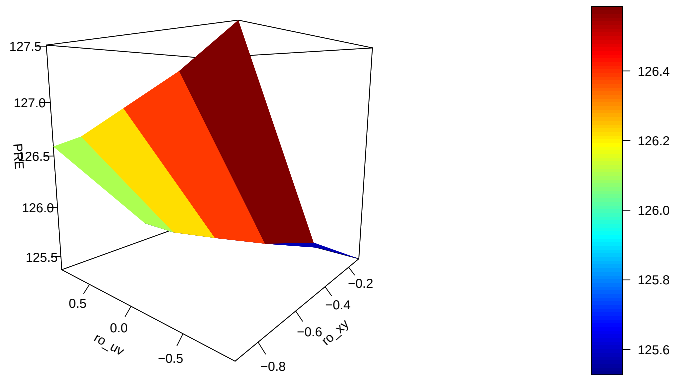

Figure 16. PRE of the estimator T_{s_1} reported in Table 4 for ME=5%, $\rho_{xy} = (-0.1, \ -0.5, \ -0.9)$, and $\rho_{uv} = (-0.9, \ -0.5, \ 0, \ 0.5, \ 0.9)$.

7. Application

This section provides the application of the adapted and suggested imputation methods. The imputation methods are applied over two different real datasets which are described below.

Dataset 1 is taken from [40], pp. 652–659, which is based on 284 municipalities of Sweden. A municipality typically consists of a town and its neighboring areas. The sizes and other characteristics of the municipalities vary greatly. The population consists of 284 municipalities. In the present study, the total number of seats in 1982 (S82) in municipal council is chosen as study variable y and the number of conservative seats in 1982 (CS82) in the municipal council is chosen as the auxiliary variable x. Following are the necessary descriptive statistics: N=284, $n = 15$, $H = 3$, $K = 5$, $P = 0.67$, $\bar{Y}$=46.0704, $\bar{X}$=9.095, σ_y^2=158.5533, σ_x^2=24.3690, $\sigma_u^2 = 15.8553$, $\sigma_v^2 = 2.4369$, ρ_{xy}=0.6878, and $\rho_{uv} = -0.0155$.

Dataset 2 is based on the humidity of Karachi, Pakistan, which was recently used by [41]. The humidity levels of Karachi vary throughout the year. The highest levels occur in August, reaching 82% (very high), while the lowest is recorded in January at 54%. Throughout the year, the average humidity in Karachi is 70%. Humidity plays an important role in how temperatures are felt. During the warmest month, May, the maximum average temperature is around 36^0C. Combined with high humidity during this period, the temperature can feel even warmer than the thermometer shows. During the coldest month, January, the maximum average temperature is around 26^0C. This period has moderate humidity. Here, we considered the daily basis maximum percentage of humidity. The humidity (%) in the year 2022 is taken as the study variable y, while the year 2021 is taken as the auxiliary variable x. The imputation methods are implemented over the humidity data of Karachi. Following are the required descriptive statistics: N=365, $n = 50$, $H = 3$, $K = 5$, $P = 0.67$, $\bar{Y}$= 90.09, $\bar{X}$=90.82, σ_y^2=55.08, σ_x^2=52.98, $\sigma_u^2 = 7.84$, $\sigma_v^2 = 6.90$, ρ_{xy}=0.76, and $\rho_{uv} = 0.04$.

For the above datasets, the PRE of the adapted and proposed estimators is calculated by employing the following expression:

$$PRE = \frac{V(t_m)}{MSE(T^*)} \times 100 \tag{7.1}$$

where $T^* = t_m,\ t_{r_1},\ t_{r_2},\ t_{r_3},\ t_{p_1},\ t_{p_2},\ t_{p_3},\ t_{g_1},\ t_{g_2},\ t_{g_3}, T_{s_1},\ T_{s_2},\ T_{s_3}$. The outcomes displayed in Table 5 show the outperformance of the suggested estimators over the adapted estimators for both datasets.

Table 5. *PRE* of the adapted and suggested estimators for real datasets

Estimators	Scheme I				Scheme II				Scheme III			
	t_{r_1}	t_{p_1}	t_{g_1}	T_{s_1}	t_{r_2}	t_{p_2}	t_{g_2}	T_{s_2}	t_{r_3}	t_{p_3}	t_{g_3}	T_{s_3}
Dataset 1	59.35	24.57	123.36	124.81	49.45	17.92	139.39	141.28	74.78	39.81	110.28	125.30
Dataset 2	137.41	46.17	140.70	141.10	168.44	36.49	175.99	176.08	115.49	63.52	116.62	119.09

8. Conclusions

In sampling theory, very few researchers have studied the problem of mean estimation in the presence of missing data provided the data is contaminated with MEs under a few sampling designs. However, the problem of mean estimation in the presence of missing data provided the data is contaminated with the correlated MEs has not yet been studied by any researcher. This paper has provided a fundamental effort to adapt the classical mean, ratio, product, and power ratio imputation methods and propose Searls type power ratio imputation methods along with their resultant estimators. The MSE of the adapted and proposed resultant estimators is obtained by employing the first-order approximation. The mathematical comparison is performed to derive the efficiency conditions under which the proposed estimators would repress the adapted estimators. Theoretical insights are enriched by a comprehensive simulation that additionally assesses the effect of the correlated MEs on the efficacy of the resultant estimators. The simulation findings are displayed in Tables 1–4 by PRE. The findings of Tables 1 and 2 demonstrate that the PRE of the proposed Searls type power ratio estimators $T_{s_j},\ j = 1, 2, 3$ grows as ρ_{xy} varies from 0 to 0.9 and reduces as the value of ρ_{xy} grows from -0.9 to -0.1, which also relies on the direction and magnitude of ρ_{uv}. Further, the same pattern is also observed in the PREs of the adapted and suggested resultant estimators reported in Tables 3 and 4 for various percentages of MEs. It is also noticed that the PRE of the suggested estimators is significantly different in the case of uncorrelated and correlated MEs. In addition, the suggested Searls type power ratio estimators repress the adapted estimators for duly opted values of σ_y^2, σ_u^2, σ_x^2, σ_v^2, ρ_{xy}, ρ_{uv}, and percentages of MEs. The suggested estimators were also applied to two different real datasets. The results of both real datasets were reported in Table 5 and demonstrated the dominance of the suggested estimators over the adapted ones under each scheme. Henceforth, the adapted and suggested imputation methods and the resultant estimators are strongly recommended to the surveyors to solve the real-life challenges of uncorrelated and correlated MEs.

Author contributions

Anoop Kumar: Methodology, Conceptualization, Writing-original draft; Shashi Bhushan: Writing-review & editing, Validation; Abdullah Mohammed Alomair: Project administration, Funding acquisition. All authors have read and agreed to the published version of the manuscript.

Use of generative AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Competing interests

The authors declare no competing interests.

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