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Research article

Analysis and design for a class of fuzzy control nonlinear systems with state observer under disturbances

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Abstract: The stabilization problem of a class of fuzzy systems with particular uncertainty restrictions is addressed in this paper using an observer design. We construct a fuzzy controller that ensures the Takagi-Sugeno fuzzy systems' uncertain solutions will converge. The ability to examine the convergence of trajectories using an estimated state controller towards a certain region of the origin that defines the system's asymptotic behavior is one benefit of the methodology employed in this work. Additionally, we provide an example to demonstrate the primary result's validity.

Keywords: fuzzy control nonlinear system; non-observer; disturbances; state estimation; convergence and stability
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1. Introduction

The idea of observability was initially presented in relation to linear systems theory. In this context, the Kalman filter and the Luenberger observer were introduced in the stochastic and deterministic contexts, respectively (see [1, 2] and references therein). The observability concept is independent of the control function for linear systems; for nonlinear systems, this is no longer the case. The output-feedback problem is typically tackled for more generic systems by expanding the findings of observer synthesis, which is a difficult challenge in and of itself. Many techniques have been developed during the last few decades to build nonlinear observers for nonlinear systems. A general sufficient Lyapunov condition for the observer design of a general class of nonlinear systems is provided by Lyapunov-based approaches [3–6], and the proposed observer is a direct extension of the Luenberger observer in the linear case. Research on the control of linear systems with unmeasurable states and bounded time variant shocks has been ongoing (see [7–14]). Many classes of nonlinear systems can therefore be considered; for example, the authors in [15] design an interval observer for switched nonlinear partial differential equation systems. Asymptotic stability is conservative in many real-world applications due to unmodeled dynamics, measurement noise, and other perturbations. Consequently, an intriguing characteristic that is frequently demonstrated for these systems in the presence of disturbances is that the solutions persist in a neighborhood of the origin for a long enough amount of time. Inspired by the idea of practical stability [16–26], one can explore the idea of a practical observer; in this instance, the error equation can be estimated. The origin was not intended to be the system's equilibrium point in these investigations. Therefore, designing a controller that ensures the stability of the origin as an equilibrium point is no longer possible in the presence of uncertainties. In [22,27], some controllers are constructed to guarantee exponential stability of a ball containing the origin of the state space where the radius of this ball can be made arbitrarily small. An intriguing class for explaining dynamic processes is the bilinear models, which are positioned somewhere in the middle of the linear and nonlinear systems [3,28]. Because of the existence of products between the state variables and inputs, bilinear models maintain the nonlinearity of the system while recalling the linear form in their structural design. In [29], the problem of prescribed-time optimal control using reinforcement learning technology is studied, where the authors proposed a prescribed-time adaptive dynamic programming control approach that ensures both optimality and prescribed-time stability. Inspired by Zadeh's approach [30], Takagi and Sugeno's (see [31–35]) T-S fuzzy models are nonlinear systems that are given local linear approximations of an underlying system by a series of if-then rules. The original nonlinear system is then accurately approximated by the T-S fuzzy system. Using this approach, several works and applications are given for various classes of control systems [5,28,36–44]. The observer-controller method works well for control problems involving unquantifiable system states. The control gain matrices can be derived by solving the stability criteria of T-S control systems using a convex optimization process. These conditions can be characterized as a sequence of linear matrix inequalities (LMIs). The authors in [14,45–48], proposed some fuzzy models for certains classes of nonlinear systems and fuzzy controllers for stabilization. New sufficient conditions with and without uncertainties have been given [36,49]. We focus on the study of observers for certain dynamic fuzzy systems of the Takagi-Sugeno (T-S) type. Many authors are interested in these classes of systems which are important in several applications (see [7,45,50–54]).

An observer is built to measure the states precisely where an estimation of the disturbance term in the observer configuration is considered. The system is composed of several affine local systems that are interpolated by weighting functions resulting from a fuzzy partitioning of the state space subject to the observation condition, which ensures the existence of the estimator that gives an approximation of the state. Numerous techniques, including sliding mode, backstepping, adaptive fuzzy control, and others, are offered to obtain a satisfactory control performance for complex nonlinear system control issues (see, for example, [41–43]). The fuzzy sliding mode observers integrate fuzzy logic with sliding mode observers to create robust observers capable of handling disturbances while ensuring stability. These are especially useful when dealing with highly uncertain or noisy environments. T-S fuzzy observers offer smoother state estimation compared to sliding mode observers because fuzzy logic enables smoother transitions between the fuzzy rules. This makes the T-S fuzzy observer less prone to the chattering phenomenon inherent in sliding mode observers. Some other studies propose adaptive observers where fuzzy rules are tuned in real time based on the estimated disturbances, improving system performance in dynamic environments with unpredictable changes. The fuzzy-controlled nonlinear systems use fuzzy logic to model and control nonlinear dynamics, where traditional control methods might fail due to the system's complexity or uncertainty. In our work, we use Lyapunov-like functions with state-dependent gains, refining the analysis of global exponential stability. We consider more sophisticated nonlinear Lyapunov functions that can provide guarantees even in the presence of highly nonlinear disturbances. We introduce an approach to solve the observer-based controller problem. The system consists of multiple affine local systems, interpolated using weighting functions resulting from a fuzzy partitioning of the state space subject to the observation condition which ensures the existence of the estimator that gives an approximation of the state.

In this paper, we consider the following fuzzy system model:

$$\dot{\xi} = \sum_{\kappa=1}^{r} \mu_\kappa(z)(A_\kappa \xi + B_\kappa u + d_\kappa(t)),$$

where ξ is the state, u is the control and y is the output of the system, $d_\kappa(t), \kappa = 1,\ldots,r$, represent the uncertainties. We prove that, the state can be estimated by a Kalman-type observer based on the observability of the system. Additionally, we examine the situation of these systems when there are disturbances. As an application by supposing the existence of a stabilizing controller, we show that the system in the presence of disturbances can be stabilized by an estimated feedback coming from an observer. Moreover, we provide an example to verify the validity of the main result.

2. Fuzzy observer design

The Takagi-Sugeno model has been shown to be useful for researching nonlinear systems. In fact, it provides a more straightforward mathematical formulation for describing the behavior of nonlinear systems. Because of the convex inherent property of the weighting functions, several methods created in the linear domain can be applied to nonlinear systems in a broad way. This representation is quite intriguing as it streamlines the observer design challenge. Let $M_{\kappa p}$ x is x the x fuzzy x set x (k x = x 1, x 2, ..., p), x z(t) x =[z1(t), ..., zp(t) $]^T$ x is x the x premise x variable vector associated x with x the x system x states x and x inputs x with r x is x the x number x of x fuzzy x rules.

The input-output T-S x fuzzy x model x in presence of uncertainties will be as follows:

Rule x κ x : x If x $z_1(t)$ x xis $M_{\kappa 1}$ x and x $z_2(t)$ is $M_{\kappa 2}$ x and x $z_p(t)$ x is $M_{\kappa p}$, then

$$\dot{\xi} = A_\kappa \xi + B_\kappa u + d_\kappa(t), \quad y = C_\kappa \xi, \text{for all } \kappa = 1, \ldots, r.$$

We first consider the following fuzzy system model:

$$\dot{\xi} = \sum_{\kappa=1}^{r} \mu_\kappa(z)(A_\kappa \xi + B_\kappa u), \quad y = \sum_{\kappa=1}^{r} \mu_\kappa(z) C_\kappa \xi, \tag{2.1}$$

where $\xi \in \mathbb{R}^n$ is the state, u is the control input, and $y \in \mathbb{R}^q$ is the output. The known matrices A_κ, B_κ and C_κ are of appropriate dimension, $r \geq 2$ is the number of rules, and z is the premise vector, which may include unmeasurable variables, and is assumed to be $\mu_\kappa(z) \geq 0$, for all $\kappa = 1, \ldots, r$ and $\sum_{\kappa=1}^{r} \mu_\kappa(z) = 1$, for all $t \geq 0$.

In many practical control problems, the physical state variables of systems are partially or entirely unavailable for measurement, as they cannot be accessed by sensing devices that are either unavailable or prohibitively expensive. In such contexts, observer-based control schemes can be configured to estimate the state for (2.1). Taking a new output $\hat{y}$ defined by: $\hat{y} = \sum_{\kappa=1}^{r} \mu_\kappa(z) C_\kappa \hat{\xi}$.
In such a case, an observer can take the form:

$$\dot{\hat{\xi}} = \sum_{\kappa=1}^{r} \mu_\kappa(z)(A_\kappa \hat{\xi} + B_\kappa u) - \sum_{\kappa=1}^{r} \mu_\kappa(z)\, L_\kappa(\hat{y} - y). \tag{2.2}$$

The gain matrices L_κ are chosen in such a way that the error $\bar{\xi} = \hat{\xi} - \xi$ will approaches to the origin or a small neighborhood of the origin, which characterizes the asymptotic behavior of the solutions, when t goes to infinity. Now, in the presence of uncertainties, by considering the perturbed fuzzy systems associated be (2.1) as follows:

$$\dot{\xi} = \sum_{\kappa=1}^{r} \mu_\kappa(z)(A_\kappa \xi + B_\kappa u + d_\kappa(t)),$$

we would like to design an estimator, part of which is a copy of the system that has the same form as (2.2):

$$\dot{\hat{\xi}} = \sum_{\kappa=1}^{r} \mu_\kappa(z) \left(A_\kappa \hat{\xi} + B_\kappa u + \hat{d}_\kappa(t) \right) - \sum_{\kappa=1}^{r} \mu_\kappa(z)\, L_\kappa(\hat{y} - y),$$

where $\hat{d}_\kappa(t)$ are some known continuous functions that will be chosen so that the observer converges. In this situation, the error equation will take the following form:

$$\dot{\bar{\xi}} = \sum_{\kappa=1}^{r} \mu_\kappa(z) \left(A_\kappa \bar{\xi} + \hat{d}_\kappa(t) - d_\kappa(t) \right) - \sum_{\kappa=1}^{r} \mu_\kappa(z)\, L_\kappa(\hat{y} - y).$$

Our goal is to use the T-S approach to show that, under certain conditions, the state of the last differential equation converges to a small neighborhood of the origin in the presence of uncertainties. Recursively, the majority of observers for a dynamical system Σ: $\dot{\xi} = \Upsilon(\xi, u)$, $y = h(\xi)$ are defined as a dynamical system with the measured variables $(u,\ y)$ as its input and the state estimate $\hat{\xi}$ as its output, like $\hat{\Sigma}$: $\dot{\hat{\xi}} = \hat{\Upsilon}(\xi, u, y)$, $y = h(\xi)$. Note that, if the system Σ and the asymptotic observer $\hat{\Sigma}$ start from the same initial condition, then the state trajectories of these systems should always remain the same. In actuality, this criterion is the foundation for the convergence and the observer concept. Therefore, the convergence criterion $\lim_{t\to\infty}(\hat{\xi}(t) - \xi(t))$ when t goes to infinity, is the primary characteristic of the study of the convergence of an observer $\hat{\Sigma}$ and the entire basis for its existence. Consequently, the following difference $(\hat{\xi} - \xi)$ is of particular relevance for this study. Next, we combine the system equation Σ and observer equation $\hat{\Sigma}$ to create the differential equation, which is called error equation. In fact, it frequently happens that the state of the error equation that is asymptotically or exponentially stable actually presents a steady-state inaccuracy in the presence of uncertainties. When this is the case, to adjust certain gains, the error can be reduced at will by considering another mode of convergence of solutions. We can see this strategy through a very basic introductory example: Let consider the scalar system $\dot{\xi}(t) = -\alpha\xi(t) + d(t)$, where ξ is the state, $\alpha > 0$, and $d(t)$ is an external disturbance supposed bounded by a nonnegative constant δ. One has the following estimation on the solution: $|\xi(t)| \leq (|\xi(0)| - \delta\alpha)e^{-\alpha t} + \delta\alpha.$ So, for initial condition taken outside the ball centered at the origin and of the radius $\delta\alpha$, the solution approaches this ball when t goes to infinity. It is in this sense that we will study the behavior of the state of the error equation in the presence of uncertainties for the convergence of the observer for a class of fuzzy systems.

2.1. Observer configuration

The class of linear systems has the following form: $\dot{\xi}(t) = A\xi(t) + Bu,\ \ \xi(0) = \xi_0,\ y(t) = C\xi(t),$ where ξ is the state, u is the control, and y is the output of the system. In general, the states are not available for measurement. In such a case, an observer can be employed to estimate the states, and its structure is as follows: $\dot{\hat{\xi}}(t) = A\hat{\xi}(t) + Bu - L(C\hat{\xi}(t) - y(t)),$ where $\hat{\xi}(t)$ is the state of the observer. It is required that the estimation error, $\bar{\xi}(t) = \hat{\xi}(t) - \xi(t)$ converges to zero for an appropriate choice of the gain matrix L. The majority of existing techniques result in the creation of an exponential observer, with exponential stability being the most desired. We shall assume the observability of the pair (A, C); in this case, there exists a gain matrix $L(n \times p)$ such that the error equation: $\dot{\bar{\xi}}(t) = (A - LC)\bar{\xi}(t)$ will be exponentially stable. Accordingly, a Kalman-like observer is employed, where the gain matrix is defined as $L = S_\Theta^{-1}C^T$. We can design a state observer as follows (see [11,12,16,20]):

$$\dot{\hat{\xi}}(t) = A\hat{\xi}(t) + Bu - S_\Theta^{-1}C^T(C\hat{\xi}(t) - y(t)),$$

where S_Θ satisfies the following stationary equation: $0 = -\Theta S_\Theta - A^T S_\Theta - S_\Theta A + C^T C,\ \Theta > 0,$ with $S_\Theta = lim_{t\to+\infty} S_t$ with $S_t \in S^+$ the cone of symmetric positive definite matrices on $\mathbb{R}^n$ which satisfies: $S_t = -\Theta S_t - A(u)^T S_t - S_t A(u) + C^T C.$ Since the pair (A, C) is observable, a gain matrix L can be determined such that $Reel\lambda(A(u) - LC) < 0$ for any control u.

In the sequel, we will examine the asymptotic behaviors of the solutions of the fuzzy system in the presence of uncertainties, in the sense that all state trajectories remain bounded and converge to a sufficiently small region around the origin. It is also desirable that the state move quickly to get to the origin, or at least to a suitably small ball.

Let consider the following perturbed fuzzy system:

$$\dot{\xi} = \sum_{\kappa=1}^{r} \mu_\kappa(z)(A_\kappa\xi + B_\kappa u + d_\kappa(t)), \tag{2.3}$$

and the associated approximate system:

$$\dot{\hat{\xi}} = \sum_{\kappa=1}^{r} \mu_\kappa(z)(A_\kappa\hat{\xi} + B_\kappa u + \hat{d}_\kappa(t)) - \sum_{\kappa=1}^{r} \mu_\kappa(z)\, L_\kappa(\hat{y} - y), \tag{2.4}$$

where $\hat{d}_\kappa(t) \in \mathbb{R}^n$ is a known function such that $\parallel \hat{d}_\kappa(t) - d_\kappa(t) \parallel \leq \delta_\kappa(t),\ \forall t \geq 0,$ with $\delta_\kappa(t)$ is a continuous nonnegative known function, for all κ. Let $\bar{d}_\kappa(t) = \hat{d}_\kappa(t) - d_\kappa(t)$. Therefore, the error equation will be:

$$\dot{\bar{\xi}} = \sum_{\kappa=1}^{r} \mu_\kappa(z)(A_\kappa \bar{\xi} + \bar{d}_\kappa(t)) - \sum_{\kappa=1}^{r} \mu_\kappa(z)\, L_\kappa(\hat{y} - y). \tag{2.5}$$

We are in a position to show, under certain restrictions on uncertain terms, the existence of an observer in the presence of perturbations. We intend to study the convergence of the error to arrive at an exponential estimate. We suppose that, the pairs (A_κ, C_κ), $\kappa = 1, \ldots, r$, are observable. In these conditions, for all $\kappa = 1, \ldots, r$, there exists $\Theta_\kappa > 0$ such that for all $\Theta > \Theta_\kappa$, $\kappa = 1, \ldots, r$, there exists gain matrices L_κ such that $Reel\lambda(A_\kappa - S_\Theta^{-1} C_\kappa^T C_\kappa) < 0$. Remark that, it suffices to take $\widetilde{\Theta} = \max\Theta_\kappa$, $\kappa = 1, \ldots, r$, and $\Theta > \widetilde{\Theta}$. Furthermore, for each κ, the matrix S_{Θ_κ} satisfies the following stationary equations:

$$0 = -\Theta S_{\Theta_\kappa} - A_\kappa^T S_{\Theta_\kappa} - S_{\Theta_\kappa} A_\kappa + C_\kappa^T C_\kappa, \qquad \kappa = 1, \ldots, r.$$

Note that, in absence of uncertainties, the state of the error equation satisfies:

$$\dot{\bar{\xi}} = \sum_{\kappa=1}^{r} \sum_{\tau=1}^{r} \mu_\kappa(z)\mu_\tau(z)(A_\kappa - L_\tau C_\kappa)\bar{\xi}.$$

So, it suffices to take as gain matrices: $L_\kappa = S_\Theta^{-1} C_\kappa^T$. This implies that, under the observability of the pairs (A_κ, C_κ), $\kappa = 1, \ldots, r$, there exists $L_\kappa > 0$ such that the error satisfies an estimation of the form:

$$\| \bar{\xi}(t) \| \leq \ell_{L_\kappa} \| \bar{\xi}(0) \| \, e^{-\gamma_{L_\kappa} t}, \ \forall t \geq 0, \ \ell_{L_\kappa} \geq 1, \ \gamma_{L_\kappa} > 0. \tag{2.6}$$

The requirement that the constants are independent of the beginning conditions gives rise to the term uniform in the estimation above. The homogeneity attribute is important for time-varying systems because it offers some resistance against external disturbances. Now, with $\bar{d}_\kappa(t) = \hat{d}_\kappa(t) - d_\kappa(t)$, we can define the error between the estimated states $\hat{\xi}$ and the real states ξ. Also we assume that μ solely contains measurable parameters, that is, does not depend on the estimated states; then, for $u < \min_\kappa u_\kappa$, we obtain the following error differential equation:

$$\dot{\bar{\xi}} = \sum_{\kappa=1}^{r} \sum_{\tau=1}^{r} \mu_\kappa(z)\mu_\tau(z)(A_\kappa - L_\tau C_\kappa)\bar{\xi} + \sum_{\kappa=1}^{r} \mu_\kappa \bar{d}_\kappa(t),$$

and so

$$\dot{\bar{\xi}} = \sum_{\kappa=1}^{r} \mu_\kappa^2 G_{\kappa\kappa}\bar{\xi}(t) + 2\sum_{\kappa<\tau}^{r} \mu_\kappa\mu_\tau G_{\kappa\tau}\bar{\xi}(t) + \sum_{\kappa=1}^{r} \mu_\kappa \bar{d}_\kappa(t),$$

where

$$G_{\kappa\kappa} = A_\kappa - L_\kappa C_\kappa,$$

and

$$G_{\kappa\tau} = \frac{1}{2}(A_\kappa - L_\tau C_\kappa + A_\tau - L_\kappa C_\tau).$$

Now, for $\Theta > \widetilde{\Theta}$, where $\widetilde{\Theta} = \max\Theta_\kappa$, there exists gain matrices L_κ such that

$$Reel\lambda(A_\kappa - S_\Theta^{-1} C_\kappa^T C_\kappa) < 0.$$

In order to construct an observer design, we should verify that assumption (A_κ, C_κ) is observable for $\kappa = 1, \ldots, r,$ and then for all $\kappa = 1, \ldots, r,$ (A_κ, C_κ) is observable, we can consider S_Θ as a common symmetric positive definite matrix that is a solution of:

$$A_\kappa^T S_\Theta + S_\Theta A_\kappa = -\Theta S_\Theta + C_\kappa^T C_\kappa, \ \kappa = 1, \ldots, r.$$

So, by using the gain matrices $L_\kappa = S_\theta^{-1} C_\kappa^T,$ $\kappa = 1, \ldots, r,$ one can find some matrices $\bar{Q}_\kappa$ and $\bar{Q}_{\kappa\tau}$ that are positive definite symmetric satisfying:

$$(G_{\kappa\kappa}^T S_\Theta + S_\Theta G_{\kappa\kappa}) = -Q_\kappa, \ \kappa = 1, \ldots, r,$$

and

$$(G_{\kappa\tau}^T S_\Theta + S_\Theta G_{\kappa\tau}) = -Q_{\kappa\tau}, \ \kappa = 1, \ldots, r, \ \tau = 1, \ldots, r.$$

Note that, the fuzzy system is asymptotically stable when these requirements are met. By converting the design work into a convex problem, linear matrix inequalities optimization can effectively solve it. Local state feedback gains are achieved if the solution is feasible, which means that the stability constraints are satisfied. By considering a fuzzy model system with known modeling errors and additional uncertainty with known upper bounds, we suppose the following condition is required for the convergence of the observer under the disturbances terms. The system (2.3) is said to be uniformly globally practically exponentially stable, if there exists a ball

$$B_\eta = \{ \xi \in \mathbb{R}^n \ / \ \| \xi \| \leq \eta \}$$

such that B_η is uniformly globally practically exponentially stable, it means that, one has an estimation on the solutions as follows:

$$\| \xi(t) \| \leq \ell \ \| \xi(0) \| \ e^{-\gamma t} + \eta\ , \ \forall t \geq 0, \ \ell \geq 1, \ \gamma > 0.$$

Note that, if the bound of the perturbation term depends on a small parameter $\mathcal{E} > 0$ that can be made as small as we want, it means that $\eta = \eta\ (\mathcal{E})$ goes to zero as $\mathcal{E}$ tends to zero; then the estimated state with the error converges to the origin exponentially when t tends to infinity. The solutions would converge under these circumstances to a tiny ball whose resulting radius is very small. In particular this can be viewed as a robustness result with respect to a small parameter. It turns out that one can consider more general class of systems, and by taking the radius of the obtained ball, which attracts the solutions, small enough to obtain the asymptotic behavior of the solutions near the origin, which is not necessarily an equilibrium point.

(A) There exist some nonnegative continuous functions $\delta_\kappa(t)$, such that

$$\| \bar{d}_\kappa(t) \| \leq \delta_\kappa(t), \ \forall \kappa = 1, \ldots, r, \ t \geq 0, \ \text{with} \ \left(\sum_{\kappa=1}^{r} \delta_\kappa(t)^2\right) < +\infty.$$

Remark that, one can write the error fuzzy system:

$$\dot{\bar{\xi}} = \sum_{\kappa=1}^{r} \sum_{\tau=1}^{r} \mu_\kappa(z)\mu_\tau(z)(A_\kappa - L_\tau C_\kappa)\bar{\xi} + \sum_{\kappa=1}^{r} \mu_\kappa \bar{d}_\kappa(t),$$

as

$$\dot{\bar{\xi}} = \Sigma(\bar{\xi}) + \Delta(t),$$

where

$$\Sigma(\bar{\xi}) = \sum_{\kappa=1}^{r} \sum_{\tau=1}^{r} \mu_\kappa(z)\mu_\tau(z)(A_\kappa - L_\tau C_\kappa)\bar{\xi},$$

and

$$\Delta(t) = \sum_{\kappa=1}^{r} \mu_\kappa \bar{d}_\kappa(t).$$

Therefore, taking into account the assumption (A), one can construct an observer that converges to a small ball.

Theorem 2.1. *Suppose that, the assumption* (A) *is satisfied and for all* $\kappa = 1,\dots,r$, (A_κ, C_κ) *are observable; then the error state converges to a small ball* B_ρ, $\rho > 0$, *uniformly and exponentially.*

Proof. Consider the Lyapunov function candidate $\upsilon(t,\bar{\xi}) = \bar{\xi}^T S_\Theta \bar{\xi}$. Its derivative with respect to time is given by:

$$\dot{\upsilon}(t,\bar{\xi}) = 2\ \bar{\xi}^T S_\Theta \left(\Sigma(\bar{\xi}) + \Delta(t)\right),$$

which implies that

$$\dot{\upsilon}(t,\bar{\xi}) = \sum_{\kappa=1}^{r} \mu_\kappa^2 x^T (G_{\kappa\kappa}^T S_\Theta + S_\Theta G_{\kappa\kappa})x + 2\sum_{\kappa<\tau}^{r} \mu_\kappa\mu_\tau \bar{\xi}^T (G_{\kappa\tau}^T S_\Theta + S_\Theta G_{\kappa\tau})\bar{\xi} + 2\bar{\xi}^T S_\Theta \sum_{\kappa=1}^{r} \mu_\kappa \bar{d}_\kappa(t).$$

The first two terms on the right-hand side represent the derivative of the Lyapunov function υ concerning the nominal system, while the third term accounts for the perturbation's effect. On the one hand, we observe that:

$$\bar{\xi}^T (G_{\kappa\kappa}^T S_\Theta + S_\Theta G_{\kappa\kappa})\bar{\xi} \le -\lambda_{min}(Q_\kappa) \parallel \bar{\xi} \parallel^2,\ \kappa = 1,2,\dots,r,$$

and

$$\bar{\xi}^T (G_{\kappa\tau}^T S_\Theta + S_\Theta G_{\kappa\tau})\bar{\xi} \le -\lambda_{min}(Q_{\kappa\tau}) \parallel \bar{\xi} \parallel^2,\ 1 \le \kappa < \tau \le r.$$

It follows that,

$$\dot{\upsilon}(t,\bar{\xi}) \le -\sum_{\kappa=1}^{r} \mu_\kappa^2 \lambda_{min}(Q_\kappa) \parallel \bar{\xi} \parallel^2 - 2\sum_{\kappa<\tau}^{r} \mu_\kappa\mu_\tau \lambda_{min}(Q_{\kappa\tau}) \parallel \bar{\xi} \parallel^2 + 2\bar{\xi}^T S_\Theta \sum_{\kappa=1}^{r} \mu_\kappa \bar{d}_\kappa(t).$$

Thus,

$$\dot{\upsilon}(t,\bar{\xi}) \le -\left(\sum_{\kappa=1}^{r} \mu_\kappa^2 \lambda_{min}(Q_\kappa) + 2\sum_{\kappa<\tau}^{r} \mu_\kappa\mu_\tau \lambda_{min}(Q_{\kappa\tau})\right) \parallel \bar{\xi} \parallel^2 + \ 2\bar{\xi}^T S_\Theta \sum_{\kappa=1}^{r} \mu_\kappa \bar{d}_\kappa(t).$$

Then, one obtains

$$\dot{\upsilon}(t,\bar{\xi}) \leq -\Lambda_0 \parallel \bar{\xi} \parallel^2 \sum_{\kappa=1}^{r} \sum_{\kappa=1}^{r} \mu_\kappa \mu_\tau + 2\bar{\xi}^T S_\Theta \sum_{\kappa=1}^{r} \mu_\kappa \bar{d}_\kappa(t),$$

where

$$\Lambda_0 = \inf\{(\lambda_{\min}(Q_\kappa); \kappa = 1, \dots, r); (\lambda_{\min}(Q_{\kappa\tau}); 1 \leq \kappa < \tau \leq r)\},$$

$\lambda_{\min(\max)}$ denotes the smallest (largest) eigenvalue of the matrix.

Since,

$$\sum_{\kappa=1}^{r} \sum_{\tau=1}^{r} \mu_\kappa \mu_\tau = 1,$$

then, we have

$$\dot{\upsilon}(t,\bar{\xi}) \leq -\Lambda_0 \parallel \bar{\xi} \parallel^2 + 2\bar{\xi}^T S_\Theta \sum_{\kappa=1}^{r} \mu_\kappa \bar{d}_\kappa(t).$$

On the other hand, we have

$$\parallel \sum_{\kappa=1}^{r} \mu_\kappa \bar{d}_\kappa(t) \parallel \leq \sum_{\kappa=1}^{r} \mu_i(\delta_\kappa(t)).$$

Taking into account the above expressions, it follows that

$$\dot{\upsilon}(t,\bar{\xi}) \leq -\Lambda_0 \parallel \bar{\xi} \parallel^2 + 2 \parallel \bar{\xi} \parallel \parallel S_\Theta \parallel \sum_{\kappa=1}^{r} \mu_\kappa(\delta_\kappa(t)).$$

On the other hand, by using the Cauchy-Schwartz inequality, one has

$$\dot{\upsilon}(t,\bar{\xi}) \leq -\Lambda_0 \parallel \bar{\xi} \parallel^2 + 2 \parallel \bar{\xi} \parallel \parallel S_\Theta \parallel (\sum_{\kappa=1}^{r} \mu_\kappa^2)^{\frac{1}{2}} (\sum_{\kappa=1}^{r} \delta_\kappa(t)^2)^{\frac{1}{2}}.$$

It follows that,

$$\dot{\upsilon}(t,\bar{\xi}) \leq -\Lambda_0 \parallel \bar{\xi} \parallel^2 + 2 \parallel S_\Theta \parallel (\sum_{\kappa=1}^{r} \delta_\kappa(t)^2)^{\frac{1}{2}} \parallel \bar{\xi} \parallel.$$

Let $\eta_\delta = (\sum_{\kappa=1}^{r} \delta_\kappa(t)^2)^{\frac{1}{2}} < +\infty$, for all $t \geq 0$.

One obtains,

$$\dot{\upsilon}(t,\bar{\xi}) \leq -\Lambda_0 \parallel \bar{\xi} \parallel^2 + 2 \parallel S_\Theta \parallel \eta_\delta \parallel \bar{\xi} \parallel.$$

Using the fact that,

$$\lambda_{min}(S_\Theta) \parallel \bar{\xi} \parallel^2 \leq \upsilon(t,\bar{\xi}) = \bar{\xi}^T S_\Theta \bar{\xi} \leq \lambda_{max}(S_\Theta) \parallel \bar{\xi} \parallel^2,$$

and by taking $\parallel S_\Theta \parallel = \lambda_{max}(S_\Theta)$, yields

$$\dot{\upsilon}(t,\bar{\xi}) \leq -\Lambda_0 \lambda_{max}^{-1}(S_\Theta) \upsilon(t,\bar{\xi}) + 2\lambda_{max}(S_\Theta) \lambda_{min}^{\frac{-1}{2}}(S_\Theta) \eta_\delta \upsilon(t,\bar{\xi})^{\frac{1}{2}}.$$

Denoting,

$$\rho = \Lambda_0 \lambda_{max}^{-1}(S_\Theta) \quad \text{and} \quad \theta = 2\lambda_{max}(S_\Theta) \lambda_{min}^{\frac{-1}{2}}(S_\Theta) \eta_\delta.$$

It follows that,

$$\dot{v}(t,\bar{\xi}) \leq -\rho v(t,\bar{\xi}) + \theta v(t,\bar{\xi})^{\frac{1}{2}}.$$

Using the above expression, and by considering

$$w(t) = v(t,\bar{\xi})^{\frac{1}{2}},$$

the derivative with respect to time is given by:

$$\dot{w}(t) = \dot{v}(t,\bar{\xi})/(2v(t,\bar{\xi})^{\frac{1}{2}}).$$

This implies that,

$$\dot{w}(t) \leq -\frac{1}{2}\rho w(t) + \frac{1}{2}\theta.$$

Therefore, a simple computation gives:

$$\dot{w}(t) \leq (w(0) - \theta\rho)e^{-\frac{1}{2}\rho\, t} + \frac{\theta}{\rho}.$$

Thus,

$$\parallel \bar{\xi}(t) \parallel \leq \lambda_{min}^{\frac{-1}{2}}(S_\Theta)(\lambda_{max}^{\frac{1}{2}}(S_\Theta) \parallel \bar{\xi}(0) \parallel -\frac{\theta}{\rho})e^{-\frac{1}{2}\rho t} + \frac{\theta}{\rho}.$$

Hence, with $\eta = \frac{\theta}{\rho}$, the ball B_η is globally uniformly practically exponentially stable and so the error state converges to B_η.

3. Stabilization via an estimated small feedback law

In general, the class of controllers that can be derived from the observer-controller configuration is obviously limited: these controllers can only be formed by combining static state feedback with an asymptotic observer. Consequently, in comparison to a more general dynamic control scheme, the observer-controller arrangement imposes stricter limitations on the behavior that can be assigned to a closed-loop system. That being said, the observer-controller arrangement is significant. It is important because stabilization approaches for nonlinear systems are not always easy to come by. First, it offers a very generic technique for the stabilization of nonlinear systems. Second, a fractional representation of the system can be derived using a stabilizing controller that was acquired through the observer-controller configuration. First, we shall estimate the state ξ from the available signals u and y.

The closed-loop system under state feedback (observer-based controller)

$$\Rightarrow \begin{cases} \begin{bmatrix} \dot{\hat{\xi}} \\ \dot{\bar{\xi}} \end{bmatrix} & = \begin{bmatrix} A+BK & BK \\ 0 & A-LC \end{bmatrix} \begin{bmatrix} \hat{\xi} \\ \bar{\xi} \end{bmatrix} + \Omega(t) \\ y & = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} \hat{\xi} \\ \bar{\xi} \end{bmatrix} \end{cases} \tag{3.1}$$

with $\Omega(t) = \begin{bmatrix} \hat{d}(t) \\ \bar{d}(t) \end{bmatrix}$.

Since the eigenvalues of

$$\begin{bmatrix} T & Y \\ 0 & R \end{bmatrix} = \{\, Eigenvalues \;\; of \quad T\,\} \cup \{\, Eigenvalues \;\; of \quad R\,\},$$

so closed-loop poles are at the eigenvalues of $(A + BK)$ and those of $(A - LC)$.

If $\Omega(t)$ is bounded, then one can obtain an estimation as:

$$\| (\hat{\xi}, \bar{\xi}) \| \leq le^{-\gamma} \| (\hat{\xi}(0), \bar{\xi}(0)) \| + \rho, \qquad l > 0, \qquad \rho > 0, \qquad \forall \;\, t \geq 0.$$

We suppose that we know the three matrices $(A; B; C)$. Suppose that the pair $(C; A)$ is observable with the hypothesis (A). Let us suppose also the existence of a stabilizing feedback $u = u(\xi) = K\xi$ that stabilizes the linear system in the presence of perturbations. The "observer design based-controller configuration" has the following schema, as shown in the following Figure 1:

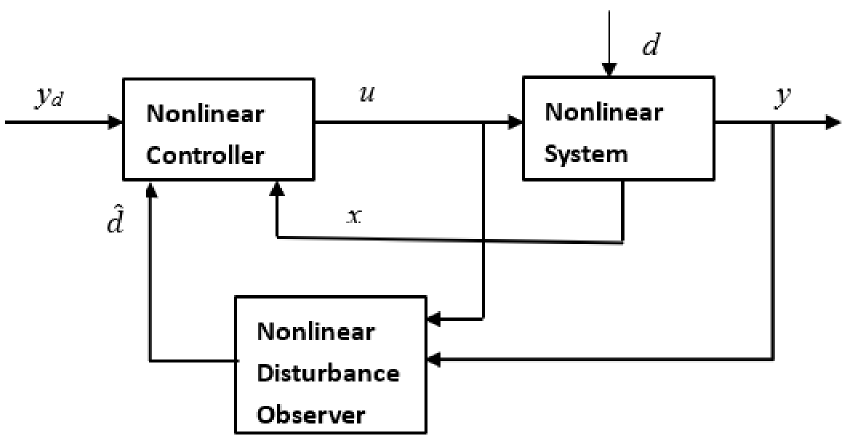

Figure 1. Observer based controller configuration.

We will adapt this procedure for the class of fuzzy systems given at the beginning. Hence, by considering the estimator (2.4), the closed-loop system via the state estimated feedback gives a composite system formed by the closed-loop state equation with the estimated fuzzy controller and the error equation. The system under consideration is:

$$\dot{\xi} = \sum_{\kappa=1}^{r} \mu_\kappa(z)(A_\kappa \xi + B_\kappa u + d_\kappa(t)).$$

The observer considered is of the form:

$$\dot{\hat{\xi}} = \sum_{\kappa=1}^{r} \mu_\kappa(z)(A_\kappa\hat{\xi} + B_\kappa u + \hat{d}_\kappa(t))\ ) - \sum_{\kappa=1}^{r} \mu_\kappa(z)\, L_\kappa(\hat{y} - y).$$

The two last equations provide the error equation:

$$\dot{\bar{\xi}} = \sum_{\kappa=1}^{r} \sum_{\tau=1}^{r} \mu_\kappa(z)\mu_\tau(z)(A_\kappa - L_\tau C_\kappa)\bar{\xi} + \sum_{\kappa=1}^{r} \mu_\kappa \bar{d}_\kappa(t).$$

Taking into account the state estimation error with the following estimated fuzzy controller:

$$u(\hat{\xi}) = \sum_{j=1}^{r} \mu_j(z)K_j\hat{\xi}(t),$$

and the fact that

$$y = \sum_{\kappa=1}^{r} \mu_\kappa(z)C_\kappa\xi,$$

we obtain by augmenting the states of the system, the following $2n$ dimensional state equations for the observer based controller closed-loop system:

$$\begin{cases} \dot{\hat{\xi}} &= \displaystyle\sum_{\kappa=1}^{r} \mu_\kappa(z)(A_\kappa\hat{\xi} + B_\kappa u(\hat{\xi}) + \hat{d}_\kappa(t))\ ) - \sum_{\kappa=1}^{r} \mu_\kappa(z)\, L_\kappa(\hat{y} - y) \\ \dot{\bar{\xi}} &= \displaystyle\sum_{\kappa=1}^{r}\sum_{\tau=1}^{r} \mu_\kappa(z)\mu_\tau(z)(A_\kappa - L_\tau C_\kappa)\bar{\xi} + \sum_{\kappa=1}^{r} \mu_\kappa \bar{d}_\kappa(t) \end{cases}$$

Therefore, the closed-loop fuzzy system under estimated feedback (fuzzy-observer based fuzzy controller) will be as follows:

$$\begin{cases} \begin{bmatrix} \dot{\hat{\xi}} \\ \dot{\bar{\xi}} \end{bmatrix} &= \sum_{\kappa=1}^{r} \sum_{\tau=1}^{r} \sum_{s=1}^{r} \mu_\kappa\mu_\tau\mu_s \begin{bmatrix} A_\kappa + B_\kappa K_s & B_\tau K_s \\ 0 & A_\tau - L_\tau C_s \end{bmatrix} \begin{bmatrix} \hat{\xi} \\ \bar{\xi} \end{bmatrix} + \sum_{\kappa=1}^{r} \sum_{\tau=1}^{r} \Omega_\kappa(t) \\ y &= \sum_{\kappa=1}^{r} \mu_\kappa(z)C_\kappa\xi \end{cases} \tag{3.2}$$

with $\Omega_\kappa(t) = \begin{bmatrix} \hat{d}_\kappa(t) \\ \bar{d}_\kappa(t) \end{bmatrix}$.

Since $\hat{d}_\kappa(t)$ and $\hat{d}_\kappa(t)$ are bounded continuous functions for all $\kappa = 1, \ldots, r,$ and $t \geq 0$, then one gets an estimation as in assumption (A); there exists some nonnegative continuous functions $\tilde{\delta}_\kappa(t)$ such that $\| \Omega_\kappa(t) \| \leq \tilde{\delta}_\kappa(t)$, $\forall \kappa = 1, \ldots, r,$ and $t \geq 0$, with $(\sum_{\kappa=1}^{r} \tilde{\delta}_\kappa(t)^2) < +\infty$. It follows that, the solutions converge to a small ball centered at the origin of $\mathbb{R}^n \mathrm{x} \mathbb{R}^n$, it means that there exists $\tilde{\rho} > 0$, such that,

$$\| \left(\hat{\xi}(\mathrm{t}), \bar{\xi}(\mathrm{t})\right) \| \leq \ell e^{-\gamma} \| \left(\hat{\xi}(0), \bar{\xi}(0)\right) \| + \tilde{\rho},\ \ell > 0, \tilde{\rho} > 0\ ,\ \forall\ t \geq 0.$$

In summary, by using fuzzy rules to approximate the system dynamics, the Takagi-Sugeno fuzzy technique offers a framework for creating observers and controllers for systems with disturbances.

The fuzzy observer is used to estimate the state of a mass-spring-damper system with bounded perturbations. A state feedback control law is then applied to stabilize the system while taking perturbations into consideration. Robust stabilization can be achieved by carefully adjusting the control law, observer gains, and fuzzy model. Fuzzy control has proven to be an effective control approach for many complex nonlinear or even nonanalytical systems. It has been suggested as an alternative approach to conventional control techniques in many situations. These features make T-S fuzzy observers highly attractive for applications involving complex nonlinear systems with disturbances, providing a more practical and efficient solution compared to existing methods. The Lyapunov-based approach to proving global exponential stability is still the most common and rigorous method. However, recent works focus on refining these Lyapunov functions to address more complex system behaviors, such as nonlinearities and external disturbances. For instance, nonlinear disturbance estimation, or nonlinear disturbance observers, has been developed, allowing for better disturbance compensation and smoother convergence of the state estimation error.

Based on the analysis presented above, the T-S fuzzy system design process can be summed up as follows.

Step 1. Verify that the pairs (A_κ, C_κ) are observable for $l = \kappa, \ldots, r$.

Step 2. For all $\kappa = 1, \ldots, r$, there exists $\Theta_\kappa > 0$ such that the matrix S_{Θ_κ} satisfies the following stationary equations: $0 = -\Theta S_{\Theta_\kappa} - A_\kappa^T S_{\Theta_\kappa} - S_{\Theta_\kappa} A_\kappa + C_\kappa^T C_\kappa$, $\kappa = 1, \ldots, r$. Then take $\widetilde{\Theta} = \max \Theta_\kappa$, $\kappa = 1, \ldots, r$, and $\Theta > \widetilde{\Theta}$.

Step 3. Verify that assumption (A) is satisfied.

Step 4. Construct the fuzzy controller, which is expected to provide an estimation of the state.

Step 6. Verify that the pairs (A_κ, B_κ) are stabilizable for $\kappa = 1, \ldots, r$.

Step 7. Set up stabilization via an estimated controller via the $2n$-cascade system.

Step 8. Characterization of the convergence of the looped system via an estimated controller through a small ball centered at the origin.

Example. Let us consider a simple second-order linear system, which can represent a mass-spring-damper: $\dot{\xi}(t) = A\xi(t) + Bu(t) + d(t)$, where: $\xi(t) = [\xi_1(t)\xi_2(t)]^T \in \mathbb{R}^2$ is the state vector, with $\xi_1(t)$ as position and $\xi_2(t)$ as velocity. u(t) is the control input. *d*(t) is a perturbation (bounded disturbance).

The system matrices are:

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{\mathrm{k}}{\mathrm{m}} & -\frac{\mathrm{b}}{\mathrm{m}} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{1}{\mathrm{m}} \end{bmatrix},$$

where: k is the spring constant, m is the mass of the object, and b is the damping coefficient. The perturbation d(t) is assumed to be bounded. The position variable captures the potential energy stored in the spring, while the velocity variable captures the kinetic energy stored by the mass. The damper only dissipates energy; it does not store energy. Often when choosing state variables, it is helpful to consider what variables capture the energy stored in the system. The Takagi-Sugeno fuzzy model represents the system as a set of fuzzy rules.

For this example, we will use two fuzzy rules to approximate the system dynamics.

Therefore, we can define the fuzzy rules as follows:

Rule 1. If ξ_1 is M_{11} then $\dot{\xi}(t) = A_1\xi(t) + B_1 u(t)$

Rule 2. If ξ_1 is M_{21} then $\dot{\xi}(t) = A_2\xi(t) + B_2u(t)$.

We consider the membership functions as:

$$\mu_1(\xi_1(t)) = \frac{1-\sin(\xi_1(t))}{2}; \mu_2(\xi_1(t)) = \frac{\sin(\xi_1(t))+1}{2}. \quad (4.4)$$

Here we can suppose the fuzzy sets for the states: ξ_1 (Position): Small, Large; ξ_2 (Velocity): Small, Large. Therefore, we have the following fuzzy system model:

$$\dot{\xi} = \sum_{\kappa=1}^{2} \mu_\kappa(z)\big(A_\kappa\xi + B_\kappa u + d(t)\big), \quad y = \sum_{\kappa=1}^{2} \mu_\kappa(z)C_\kappa\xi,$$

where $y \in \mathbb{R}^2$ is the output, $\mu_\kappa(z)$ are the fuzzy weights (membership functions), and the approximate system with $\hat{\xi}$ denotes the state estimate:

$$\dot{\hat{\xi}} = \sum_{\kappa=1}^{2} \mu_\kappa(z)(A_\kappa\hat{\xi} + B_\kappa u + \hat{d}_\kappa(t)) - \sum_{\kappa=1}^{2} \mu_\kappa(z)\, L_\kappa(\hat{y} - y),$$

where $\hat{d}_\kappa(t) \in \mathbb{R}^n$ is a known function such that

$$\| \hat{d}_\kappa(t) - d_\kappa(t) \| \leq \delta(t), \ \forall t \geq 0,$$

with $\delta(t)$ are some continuous nonnegative known functions satisfying:

$$\eta_\delta = (\sum_{\kappa=1}^{2} \delta_\kappa(t)^2)^{\frac{1}{2}} < +\infty, \qquad \text{for all } t \geq 0.$$

L_κ are suitable gains matrices with y as the measured output and $\hat{y}$ the estimated output.

Let $\bar{d}_\kappa(t) = \hat{d}_\kappa(t) - d_\kappa(t)$. It follows that the error equation is given by:

$$\dot{\bar{\xi}} = \sum_{\kappa=1}^{2} \mu_\kappa(z)(A_\kappa\bar{\xi} + \bar{d}_\kappa(t)) - \sum_{\kappa=1}^{2} \mu_\kappa(z)\, L_\kappa(\hat{y} - y).$$

By guaranteeing convergence and stability of the solutions, we make sure that the observer and controller are built to manage constrained perturbations. This can be achieved by minimizing the impact of the perturbation terms by modifying the fuzzy rule-based observer gains. By choosing suitable fuzzy membership functions and adjusting the observer gains, the stability of the system in the presence of disturbances can be guaranteed under any circumstances.

Let us choose the following values for the system parameters and apply them to the observer-based controller.

Let k= 1 N/m (spring constant), m = 1 kg (mass), b= 0.2 N·s/m (damping coefficient), and η_δ=0.1 N (perturbation bound). The T-S fuzzy model consists of the following matrices:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & -0.2 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ -2 & -0.4 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Based on this choice, each nominal local model is controllable. Moreover, for the observer design, we choose L_1=[10 10] and L_2=[10 10] as the observer gains, which need to be designed for stability under the following estimated fuzzy controller:

$$u(\hat{\xi}) = \sum_{j=1}^{2} \mu_j(z) K_j \hat{\xi}(t),$$

with K_1=[-5 -5] and K_2=[-5 -5]. This allows us to consider the fuzzy controller estimated by the fuzzy observer, which stabilizes the system in a closed-loop, where the time evolution of the solutions is shown in Figure 2.

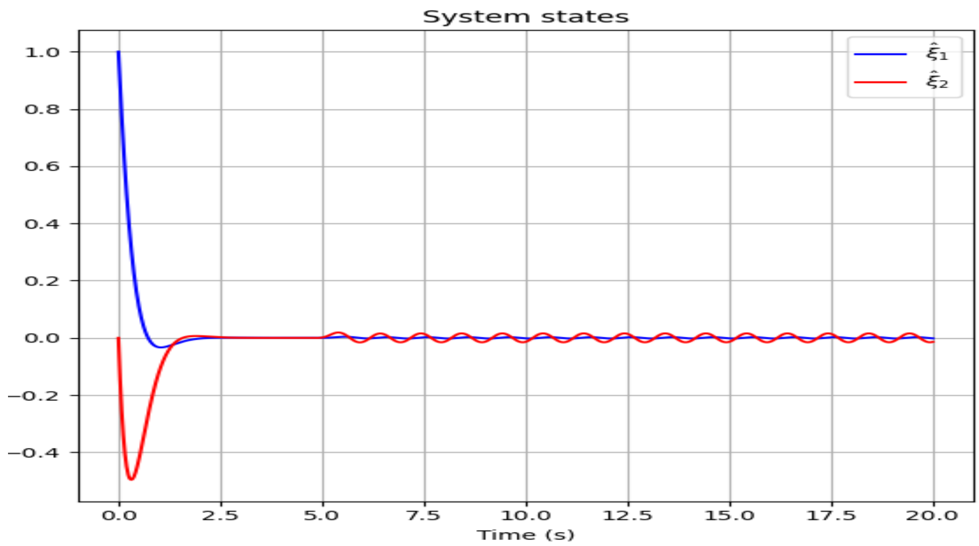

Figure 2. Time evolution of the states.

4. Conclusions

It is shown in this paper that the state of an uncertain fuzzy system can be estimated via an observer. It is proven that the solutions of the error equation converge to a certain ball, where the nominal system is linear and the uncertainties are uniformly bounded. We employ the Takagi-Sugeno fuzzy technique to address the inherent nonlinearities. Local nonlinear models are used to handle nonlinearities relying on unmeasured states and to minimize the amount of fuzzy rules. It is possible to answer the design conditions efficiently since they are expressed as linear matrix inequalities. The synthesis conditions lead to the resolution of some constraints that can be solved with numerical tools that are related with the spectrum of the nominal system. In the presence of the fuzzy observer, it is demonstrated that the state estimate fuzzy controller can stabilize the composite system. When compared to the linearization approach, the proposed approach does not require knowing the uncertainties of the system but just information on the term that increases them. A numerical example is used to simulate and test the obtained conditions. Using the proposed approach, some interesting future works can be done for Takagi-Sugeno fuzzy Cohen-Grossberg neural networks with uncertainties.

Author contributions

Omar Kahouli: Writing-original draft preparation, Methodology; Tarak Maatoug: Validation, Formal analysis; François Delmotte: Investigation, Visualization; Mohamed Ali Hammami: Writing review and Editing; Naim Ben Ali: Software, Validation; Mohammad Alshammari: Resources, Data curation. All authors have read and approved the final version of the manuscript for publication.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflict of interest.

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