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Research article

Hamming distance-based knowledge measure and entropy for interval-valued Pythagorean fuzzy sets

Li Li* **and Xin Wang**

College of Mathematical Sciences, Bohai University, Jinzhou 121000, China

* **Correspondence:** Email: lll_dz@163.com.

Abstract: The development of knowledge measures and uncertainty measures for constructing interval-valued Pythagorean fuzzy sets (IVPFS) have garnered significant attention in recent years. Nevertheless, existing uncertainty measures predominantly depend on entropy-based approaches, which exhibit limitations in effectively characterizing the knowledge inherent in interval intuitionistic fuzzy sets. This study extends the axiomatic framework of knowledge measures for fuzzy sets by introducing a novel distance-based knowledge measure function. The proposed measure is rigorously validated through comprehensive mathematical analysis and supported by extensive numerical examples. Furthermore, this research extends the entropy properties from interval-valued intuitionistic fuzzy sets to their Pythagorean counterparts while providing rigorous proofs of their compliance with axiomatic definitions. To demonstrate practical applicability, the proposed entropy measure is implemented in multi-attribute group decision-making scenarios involving unknown interval-valued Pythagorean fuzzy information. Experimental results substantiate both the validity and practical utility of the proposed measures.

Keywords: interval-valued Pythagorean fuzzy set; knowledge measure; entropy; distance; multiple attribute decision problem
Mathematics Subject Classification: 03E72

1. Introduction

Since Zadeh proposed the fuzzy set theory [1] in 1965, scholars have conducted extensive and in-depth research on it. In 1983, the Bulgarian scholar Atanassov proposed the concept [2] of the intuitionistic fuzzy set. The intuitionistic fuzzy set considers membership, non-membership, and hesitation, leading to extensive research and fruitful results [3–9]. However, the intuitionistic fuzzy set can only describe the situation where the sum of membership and non-membership is less than or equal to 1, which makes the decision-making process greatly limited and affects its scope of application. To this end, Yager, by defining the Pythagorean fuzzy complement operation, proposes that the Pythagorean fuzzy set [10, 11] where the sum of membership and non-membership exceeds 1 and the sum of squares is no more than 1, forcing the decision maker to modify the intuitionistic fuzzy decision value and make a decision [12, 13]. Mishra and Rani [14] employed an intuitionistic fuzzy multi-criteria decision-making (MCDM) approach to evaluate and prioritize blockchain networks, demonstrating the application of fuzzy theory in emerging technology decision-making. The assessment of blockchain networks involves numerous uncertain factors, such as the difficulty in precisely quantifying performances across security, scalability, and efficiency dimensions among different blockchains. Intuitionistic fuzzy sets (IFS) effectively address this uncertainty by incorporating membership degrees, non-membership degrees, and hesitation degrees, thereby providing a more comprehensive reflection of experts' evaluations across various aspects of blockchain networks and generating more accurate decision-making bases.

In the field of multi-attribute decision making (MADM), the generalization of fuzzy theory has rendered decision-making processes more aligned with real-world scenarios. Traditional MADM methods exhibit limitations when handling vague and uncertain information, whereas fuzzy sets and their extended theories effectively compensate for these shortcomings. For instance, the application of IFS in MADM allows decision-makers to simultaneously express support, opposition, and hesitation toward attribute values of alternatives, thereby enriching decision-making information. Interval-valued intuitionistic fuzzy sets (IVIFS) further expand this expressiveness by representing membership and non-membership degrees in interval forms, enabling more nuanced characterization of uncertainties. Building upon IFS and IVIFS, interval-valued Pythagorean fuzzy sets relax the constraint on the squared sum of membership and non-membership degrees, offering greater flexibility in addressing complex decision problems. With continuous advancements of fuzzy theory in MADM, novel decision-making methods and models have emerged. These approaches, rooted in diverse fuzzy set theories and integrated with various information processing techniques such as entropy and distance measures, enhance the accuracy and reliability of decisions. In practical applications, the promotion of fuzzy theory in MADM has provided robust support for decision-making across fields like engineering design, economic management, and environmental assessment, assisting decision-makers in navigating complex uncertain environments to formulate more rational choices. Currently, Pythagorean fuzzy sets are widely used in decision making, medical diagnosis, and pattern recognition. Promotion work has always been an important part of fuzzy mathematics research. Literature [10] generalizes the range of membership and non-membership from the number between pure [0,1] to [0,1] sub intervals, proposing the concept of interval-valued Pythagorean fuzzy set.The generalization based on the original theory is the hot topic of fuzzy mathematical theory research. At present, many achievements have been made in the fuzzy set. For example, literature [22] presents the operator of interval-valued Pythagorean fuzzy set and its basic properties; literature [22] defines two precision functions of interval-valued Pythagorean fuzzy set and their application in the decision process; literature [24] proposes a multi-attribute decision method based on interval-valued Pythagorean fuzzy set language information.

Entropy in fuzzy set theory can effectively measure fuzzy information; entropy is an important information measurement tool in fuzzy set theory. Zadeh in 1965 first proposed the entropy of fuzzy set [1]; literature [15] proposed the definition of interval direct fuzzy set entropy; literature [17] proposed several kinds of Pythagorean fuzzy set entropy definitions and studied the relationship between entropy, distance, and similarity. Thus, the entropy and knowledge measures of the interval-valued Pythagorean fuzzy set have great promise.

In the intuitionistic fuzzy set of interval values proposed by our predecessors, we discuss the entropy and knowledge measure based on the distance of interval values. Since there are few examples of interval-valued Pythagorean fuzzy sets using entropy to solve practical problems. When hesitation degree is not taken into account, interval-valued intuitionistic fuzzy numbers are the same as interval-valued Pythagorean fuzzy numbers. Therefore, this paper mainly uses the distance-based knowledge measure and entropy of the interval-valued intuitionistic fuzzy set to extend to the interval-valued Pythagorean fuzzy set. Show that it conforms to the axiomatic definition of entropy and knowledge measures; the entropy proposed in this paper can get different entropy forms according to the different functions. Second, taking the knowledge measure presented in this paper, explain its rationality. Finally, entropy is applied to multi-attribute decision problems with unknown weights. Explains the rationality and practicability of distance-based knowledge measures and the entropy of interval-valued Pythagorean fuzzy sets. The entropy based on distance proposed in this paper can derive different forms of entropy according to different functions and can address multi-attribute decision-making problems with unknown weights, which is the most significant distinction from previous entropy concepts. It also fills the gap in interval-valued Pythagorean fuzzy entropy.

Section 2 of this section briefly reviews the definitions of the intuitionistic fuzzy set and the interval-valued Pythagorean fuzzy set, including the entropy properties in research use. In Section 3 we study and prove the entropy and knowledge measure of the interval-valued Pythagorean fuzzy set; in addition, we propose the entropy and knowledge measure based on Hemming distance. Section 4 applies the entropy proposed in this paper to the multi-attribute decision problem with unknown weights, and Section 5 summarizes the study.

2. Preliminary data

This section reviews the fundamental theoretical knowledge of fuzzy sets. We begin by introducing the definitions of interval-valued intuitionistic fuzzy sets and fuzzy sets, followed by the axiomatic definitions of entropy and knowledge measures, and existing examples of entropy and knowledge measures for intuitionistic fuzzy sets.

2.1. Interval-valued Pythagorean fuzzy set

Definition 1. (Intuitionistic fuzzy set) An intuitionistic fuzzy set (IFS) A on X can be defined as: $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$ where $\mu_A(x)$ is the membership function of A, $\nu_A(x)$ is the non-membership function of A, and for all $\forall x \in X, \mu_A(x) \in [0,1], \nu_A(x) \in [0.1]$, both have $0 \le \mu_A(x) + \nu_A(x) \le 1$.

Definition 2. (Interval-valued intuitionistic fuzzy set) An interval-valued intuitionistic fuzzy set (IVIFS) B on X can be defined as: $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle\ x \in X\}$, where is $\mu_B(x) = [\mu_B{}^-(x), \mu_B{}^+(x)] \subseteq [0,1]$ the membership function, $\nu_B(x) = [\nu_B{}^-(x), \nu_B{}^-(x)] \subseteq [0,1]$ is the non-membership function, both have $0 \le \mu_B(x)^2 + \nu_B(x)^2 \le 1$. For the interval hesitation margin $\pi_B(x) = [\pi_B^-(x), \pi_B^+(x)]$, we have $\pi_B^-(x) = [1 - \mu_B^+(x) - \nu_B^+(x)]$ and $\pi_B^+(x) = [1 - \mu_B^-(x) - \nu_B^-(x)]$. Specifically, if $\mu_B^-(x) = \mu_B^+(x)$ and $\nu_B^-(x) = \nu_B^+(x)$, then the IVIFS B is reduced to an IFS. Furthermore, reference [7] pointed out that the $[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]_x = \left\{\left[\frac{1}{3}, \frac{1}{3}\right], \left[\frac{1}{3}, \frac{1}{3}\right], \left[\frac{1}{3}, \frac{1}{3}\right]\right\}$.

Definition 3. (Interval-valued Pythagorean fuzzy set) An interval-valued Pythagorean fuzzy set (IVPFS) P on X can be defined as: $P = \{\langle x, \mu_P(x), \nu_P(x)\rangle | x \in X\}$, where $\mu_P(x) = [\mu_P{}^-(x), \mu_P{}^+(x)] \subseteq$

$[0,1]$. The membership function, $v_P(x) = [v_P{}^-(x), v_P{}^+(x)] \subseteq [0,1]$ is the non-membership function; both have $0 \leq \mu_P x^2 + v_P x^2 \leq 1. \pi_P(x) = [\pi_P{}^-(x), \pi_P{}^+(x)]$ is a hesitant degree and $\pi_P{}^-(x) = \sqrt{1-\mu_P{}^+(x)-v_P{}^+(x)}, \pi_P{}^+(x) = \sqrt{1-\mu_P{}^-(x)-v_P{}^-(x)}$.

Definition 4. For the arbitrary: $P1 \leq P2 \in IvPFS(U)$

(1) $P1 \leq P2 \Leftrightarrow \forall x \in U$, $\mu_{P1}{}^+(x) \leq \mu_{P2}{}^+(x), \mu_{P1}{}^-(x) \leq \mu_{P2}{}^-(x)$, $v_{P1}{}^+(x) \geq v_{P2}{}^+(x)$, and $v_{P1}{}^-(x) \geq v_{P2}{}^-(x)$.

(2) $P1 = P2 \Leftrightarrow \forall x \in U, \mu_{P1}{}^+(x) = \mu_{P2}{}^+(x), \mu_{P1}{}^-(x) = \mu_{P2}{}^-(x), v_{P1}{}^+(x) = v_{P2}{}^+(x) and v_{P1}{}^-(x) = v_{P2}{}^-(x)$.

(3) $P1 \cup P2 = \left\{ \left\langle \begin{array}{l} x, [\max(\mu_{P1}{}^-(x), \mu_{P2}{}^-(x)), \max(\mu_{P1}{}^+(x), \mu_{P2}{}^+(x))], \\ {}[\min(v_{P1}{}^-(x), v_{P2}{}^-(x)), \min(v_{P1}{}^+(x), v_{P2}{}^+(x))] \end{array} \right\rangle | x \in U \right\}$.

(4) $P1 \cap P2 = \left\{ \left\langle \begin{array}{l} x, [\min(\mu_{P1}{}^-(x), \mu_{P2}{}^-(x)), \min(\mu_{P1}{}^+(x), \mu_{P2}{}^+(x))], \\ {}[\max(v_{P1}{}^-(x), v_{P2}{}^-(x)), \max(v_{P1}{}^+(x), v_{P2}{}^+(x))] \end{array} \right\rangle | x \in U \right\}$.

(5) $P^c = \{\langle x, [v_P{}^-(x), v_P{}^+(x)], [\mu_P{}^-(x), \mu_P{}^+(x)] \rangle | x \in X\}$.

Definition 5. For any $M, N \in IVPFS(x)$, a mapping F: $IVPFS(x) \rightarrow [0,1]$: is called the entropy of IVPFS if F satisfies the following conditions:

(1) $F(M) = 0$ iff M a crisp set;

(2) $F(M) = 1$,iff all three descriptions of the IVPFS interval satisfy $F(M) = 1$;

(3) $F(M) = F(M^C)$;

(4) if $Z(M, \left[\frac{1}{3}, \frac{1}{3}\right]_x) \geq Z(N, \left[\frac{1}{3}, \frac{1}{3}\right]_x))$, then $F(M) \leq F(N)$, where Z is a distance measure.

Definition 6. Z is a distance on $IVPFS(x) \times IVPFS(x)$then A, B, C belong to IVPFS (X); the following properties hold:

(1) $Z(A,B) = 0$, iff $A = B$;

(2) $Z(A,B) = Z(B,A)$;

(3) $Z(A,C) \leq Z(A,B) + Z(B,C)$;

(4) If $A \prec B \prec C$, then $Max\{Z(A.B), Z(B,C)\} \leq Z(A,C)$.

2.2. *Some existing entropy and knowledge measures for IVIFS or IVPFS*

Early research focused on the entropy of membership and non-membership of intuitionistic fuzzy sets, expressed in the following forms:

$$F_A = \frac{1}{n}\sum_{i=1}^{n}(1-\mu_A(x)+v_A(x))e^{1-\mu_A(x)+v_A(x)}, \tag{1}$$

where $\mu_A(x) = [\mu_A{}^-(x), \mu_A{}^+(x)] \subseteq [0,1]$ and $v_A(x) = [v_A{}^-(x), v_A{}^+(x)] \subseteq [0,1]$.

$$E(\tilde{A}) = \frac{1}{n}\sum_{i=1}^{n}\frac{(\mu_{\tilde{A}}^-(x_i))^2 \wedge (v_{\tilde{A}}^-(x_i))^2 + (\mu_{\tilde{A}}^+(x_i))^2 \wedge (v_{\tilde{A}}^+(x_i))^2}{(\mu_{\tilde{A}}^-(x_i))^2 \vee (v_{\tilde{A}}^-(x_i))^2 + (\mu_{\tilde{A}}^+(x_i))^2 \vee (v_{\tilde{A}}^+(x_i))^2}, \tag{2}$$

$$F_B = 1 - \frac{1}{2}\sum_{i=1}^{n}\left[\left|\mu_B{}^-(x)-0.5\right| + \left|\mu_B{}^+(x)-0.5\right| + \left|v_B{}^-(x)-0.5\right| + \left|v_B{}^+(x)-0.5\right|\right]. \tag{3}$$

Moreover, some measures link membership and non-membership to trigonometric functions:

$$F_C = \frac{1}{n}\sum_{i=1}^{n}\left\{\frac{1}{\sqrt{2}-1}\left[\sqrt{2}\cos\frac{\mu_C{}^-(x)+\mu_C{}^+(x)-v_C{}^-(x)-v_C{}^+(x)}{8}\pi - 1\right]\right\}, \tag{4}$$

$$F_D = \frac{1}{n}\sum_{i=1}^{n}\cot\left\{\frac{\pi}{4} + \frac{\mu_D^{-}(x) + \mu_D^{+}(x) - \nu_D^{-}(x) - \nu_D^{+}(x)}{4(4 - \mu_D^{-}(x) - \mu_D^{+}(x) - \nu_D^{-}(x) - \nu_D^{+}(x))}\right\}. \tag{5}$$

Additionally, there is a knowledge measure describing interval-valued intuitionistic fuzzy sets:

$$H_1 = 1 - \frac{1}{2n}\sum_{i=1}^{n}\left(1 - \frac{1}{2}\left(\left|\mu_H^{-}(x) - \nu_H^{-}(x)\right| + \left|\mu_H^{+}(x) - \nu_H^{+}(x)\right|\right)\right) \times \left[1 + \frac{1}{2}(\pi_H^{-}(x) + \pi_H^{+}(x))\right]. \tag{6}$$

However, research on the entropy and knowledge measures of interval-valued Pythagorean fuzzy sets remains limited. The following sections establish the knowledge measure and entropy of Hamming distance-based interval-valued Pythagorean fuzzy sets to verify their rationality and applicability in real-world scenarios.

3. Construct knowledge measure and entropy with Hemming distance for IVPFS

3.1. Knowledge measure based on Hamming distance

Definition 7. $A \in IVPFS(x)$, $H : IvPFS(x) \to [0,1]$ is a distance-based knowledge measure of interval-valued Pythagorean fuzzy set if it satisfies:
(1) $H(A) = 1 \Leftrightarrow A$ is a crisp set;
(2) $H(A) = 0 \Leftrightarrow \pi_H^{-}(x) = \pi_H^{+}(x) = 1$;
(3) $H(A) \geq H(B) \Leftrightarrow$, A contains more knowledge measures than does B, that is: $Z(A,[0,0,1]_x) \geq Z(B,[0,0,1]_x)$, $[0,0,1]_x = \langle [0,0], [0,0], [1,1] \rangle$;
(4) $H(A) = H(A^c)$.

In this section, we use the TOPSIS distance method [26], first defining two positive ideal solutions: $K_1 = \langle x, [1,1], [0,0] \rangle$, $K_2 = \langle x, [0,0], [1,1] \rangle$ and a negative ideal solution: $Q = \langle x, [0,0], [0,0] \rangle$ element $A = \left\{\left\langle x, \left[\nu_P^{-}(x), \nu_P^{+}(x)\right], [\mu_P^{-}(x), \mu_P^{+}(x)]\right\rangle | x \in X\right\} \in IvPFS(x)$, α represents the influence of the positive ideal solution, negative ideal solution, and human factors.

Definition 8. A mapping $H : IvPFS(x) \to L[0,1]$ is a measure of interval-valued Pythagorean fuzzy set based on knowledge of Hamming distance, defined as:

$$H(A) = \frac{\alpha D(A,Q)}{(1-\alpha)D(A,K_1)D(A,K_2) + \alpha D(A,Q)}, \tag{7}$$

$L[0,1] = \{[a,b] : 0 \leq a \leq b \leq 1\}$,$\alpha \in [0,1]$ is the influence of subjective attitude, $D(A,Q)$ is the distance between A and the negative ideal solution, $D(A,K_1)D(A,K_2)$ is the distance between A and the positive ideal solutions K_1 and K_2.

Theorem 1. $H(A)$ as defined in (8) is a fuzzy knowledge measure of the interval-valued Pythagorean set. The rationality and validity of the proposed interval-valued Pythagorean fuzzy set knowledge measure are proved below.

Proof. Theorem 1 follows from (1)–(4) of Definition 7.
(1) $H(A) = 1 \Leftrightarrow A$ is a crisp set: According to the definition of distance, if $H(A) = 1$, then $(1-\alpha)Z(A,K_1)Z(A,K_2) = 0$, that is, $Z(A,K_1) = 0$ or $Z(A,K_2) = 0$. This means $A = K_1$ or $A = K_2$, so A is a crisp set.
(2) $H(A) = 0 \Leftrightarrow \pi_H^{-}(x) = \pi_H^{+}(x) = 1$: From the definition of IVPFS, if $P(A) = 0$, then $Z(A,N) = 0$, and

obviously $\pi_H^-(x) = \pi_H^+(x), x \in X$.
(3) $H(A) = H(A^c)$: According to the definition of IVPFS, $Z(A,Q) = Z(A^C,Q)$, $Z(A,K_1) = Z(A^C,K_1)$, and $Z(A,K_2) = Z(A^C,K_2)$, so $H(A) = H(A^C)$.
(4) $H(A) \geq H(B)$ contains more knowledge measures: Calculate the partial derivatives of $H(A)$ with respect to $Z(A,Q)$, $Z(A,K_1)$, and $Z(A,K_2)$. $\frac{\partial H(A)}{\partial Z(A,Q)} = \frac{\alpha[(1-\alpha)Z(A,K_1)Z(A,K_2)]}{[(1-\alpha)Z(A,K_2)Z(A,K_2)+\alpha Z(A,Q)]^2} > 0$, $\frac{\partial H(A)}{\partial Z(A,Q)} = \frac{-(1-\alpha)Z(A,K_2)}{[(1-\alpha)Z(A,K_2)Z(A,K_2)+\alpha Z(A,Q)]^2} < 0$, $\frac{\partial H(A)}{\partial Z(A,K_2)} = \frac{-(1-\alpha)Z(A,K_1)}{[(1-\alpha)Z(A,K_2)Z(A,K_2)+\alpha Z(A,Q)]^2} < 0$. This shows that $H(A)$ increases monotonically with $Z(A,Q)$ and decreases monotonically with $Z(A,K_1)$ and $Z(A,K_2)$. If A contains more knowledge measures, that is, $Z(A,Q) \geq Z(B,Q)$, $Z(A,K_1) \leq Z(B,K_1)$, and $Z(A,K_2) \leq Z(B,K_2)$, then $H(A) \geq H(B)$.
In the interval-valued Pythagorean fuzzy set, A is the interval-valued Pythagorean fuzzy set, so the normalized mine distance between A and the positive ideal solution and the negative ideal solution is as follows:

$$Z(A,Q) = \frac{1}{4n}\sum_{i=1}^{n}[\mu_H{}^+(x)^2 + \mu_H{}^-(x)^2 + \nu_H{}^+(x)^2 + \nu_H{}^-(x)^2],$$

$$Z(A,K_1) = \frac{1}{4n}\sum_{i=1}^{n}[2 - \mu_H{}^+(x)^2 - \mu_H{}^-(x)^2 + \nu_H{}^+(x)^2 + \nu_H{}^-(x)^2],$$

$$Z(A,K_2) = \frac{1}{4n}\sum_{i=1}^{n}[2 + \mu_H{}^+(x)^2 + \mu_H{}^-(x)^2 - \nu_H{}^+(x)^2 - \nu_H{}^-(x)^2].$$

Here we use $Z(A,Q) = F$, $Z(A,K_1) = F_1$, $Z(A,K_2) = F_2$, the knowledge measure based on the Hemming distance is:

$$H(A) = \frac{\alpha Z(A,Q)}{(1-\alpha)Z(A,K_1)Z(A,K_2) + \alpha Z(A,Q)} = \frac{\alpha F}{(1-\alpha)F_1F_2 + \alpha F}. \tag{8}$$

3.2. Construct entropy measure with distance for IVPFS

Entropy is a tool to measure set ambiguity, widely used in medical diagnosis, attribute decision making, and pattern recognition. The larger the entropy, the more ambiguous the information. This section defines and studies the entropy on the interval-valued Pythagorean fuzzy set according to literature [26].

Definition 9. Let J be a strictly monotonically decreasing function from [0,1] to [0,1] E is the entropy on the interval-valued Pythagorean fuzzy set, defined as:

$$F(A) = \frac{J(\frac{9}{4}Z(A,[\frac{1}{3},\frac{1}{3},\frac{\sqrt{7}}{3}]_x)) - J(1)}{J(0) - J(1)}. \tag{9}$$

Theorem 2. To prove that the Definition 9 is correct according to the known axiomatic definition of entropy and to prove it as follows:
(1) From the definition of entropy and distance, it is crisp that: $F(A) = 0 \Leftrightarrow \frac{9}{4}Z(A,[\frac{1}{3},\frac{1}{3},\frac{\sqrt{7}}{3}]_x) = 1 \Leftrightarrow Z(A,[\frac{1}{3},\frac{1}{3},\frac{\sqrt{7}}{3}]_x) = \frac{4}{9}$.
(2) From the definition of distance, it is clear that $F(A) = 1 \Leftrightarrow \frac{9}{4}Z(A,[\frac{1}{3},\frac{1}{3},\frac{\sqrt{7}}{3}]_x) = 0 \Leftrightarrow Z(A,[\frac{1}{3},\frac{1}{3},\frac{\sqrt{7}}{3}]_x) = 0$, then $A = [\frac{1}{3},\frac{1}{3},\frac{\sqrt{7}}{3}]_x$, $\mu_A(x) = \nu_A(x) = [\frac{1}{3},\frac{1}{3}]_x$.

(3) Because J is strictly monotonically decreasing, if $Z(A, [\frac{1}{3}, \frac{1}{3}, \frac{\sqrt{7}}{3}]_x) \geq Z(B, [\frac{1}{3}, \frac{1}{3}, \frac{\sqrt{7}}{3}]_x)$, then $J(\frac{9}{4}Z(A, [\frac{1}{3}, \frac{1}{3}, \frac{\sqrt{7}}{3}]_x)) \leq J(\frac{9}{4}Z(B, [\frac{1}{3}, \frac{1}{3}, \frac{\sqrt{7}}{3}]_x))$, then $F(A) \leq F(B)$.
(4) According to the definition of the complement of the interval-valued Pythagorean fuzzy set, we can know that $Z(A, [\frac{1}{3}, \frac{1}{3}, \frac{\sqrt{7}}{3}]_x) = Z(A^C, [\frac{1}{3}, \frac{1}{3}, \frac{\sqrt{7}}{3}]_x)$ and then $F(A) = F(A^C)$.
We can get different entropy according to different functions, such as: Make $J(x) = 1 - x$, the distance is the Hamming distance, and then it can be obtained:

$$F(A) = 1 - \frac{9}{4} \times \frac{1}{4n} \sum_{i=1}^{n} \left[\left| \mu_A{}^-(x)^2 - \frac{1}{9} \right| + \left| \mu_A{}^+(x)^2 - \frac{1}{9} \right| + \left| v_A{}^-(x)^2 - \frac{1}{9} \right| + \left| v_A{}^+(x)^2 - \frac{1}{9} \right| \right]. \tag{10}$$

At $J(x) = 2 - x$, the distance is the marine distance, which can be obtained by:

$$F(A) = 2 - \frac{9}{4} \times \frac{1}{4n} \sum_{i=1}^{n} \left[\left| \mu_A{}^-(x)^2 - \frac{1}{9} \right| + \left| \mu_A{}^+(x)^2 - \frac{1}{9} \right| + \left| v_A{}^-(x)^2 - \frac{1}{9} \right| + \left| v_A{}^+(x)^2 - \frac{1}{9} \right| \right]. \tag{11}$$

Make $J(x) = 1 - x$, the distance is the marine distance, we weight, and then we can get:

$$\begin{aligned} F^{\omega}(A) = 1 - \tfrac{9}{4} \times \tfrac{1}{4n} \sum_{i=1}^{n} \omega_i \left[\left| \mu_A{}^-(x)^2 - \tfrac{1}{9} \right| + \left| \mu_A{}^+(x)^2 - \tfrac{1}{9} \right| + \left| v_A{}^-(x)^2 - \tfrac{1}{9} \right| + \left| v_A{}^+(x)^2 - \tfrac{1}{9} \right| \right], \\ \omega_i = \{\omega_1, \omega_2, \omega_3 ... \omega_n\}, \sum_{i=1}^{n} \omega_i = 1. \end{aligned} \tag{12}$$

We use the distance-based knowledge measure and entropy of the interval-valued intuitionistic fuzzy set to extend to the interval-valued Pythagorean fuzzy set. Show that it conforms to the axiomatic definition of entropy and knowledge measures the entropy proposed in this paper can get different entropy forms according to the different functions.

4. Application of entropy and knowledge measure

Taking interval-valued Pythagorean fuzzy entropy as an example, we compare it with known interval-valued intuitionistic fuzzy entropy and interval-valued Pythagorean fuzzy entropy, without considering hesitation. Subsequently, for instance, we compare the known entropy and knowledge measurement with those presented in this paper. Table 1 shows the results obtained by comparing the known entropy using six intervals of Pythagorean fuzzy numbers. To facilitate this comparison, we refer to the example in reference [25] and calculate as follows:
Example 1.

$$A_1 = \langle x, [0.7, 0.7], [0.2, 0.2] \rangle, A_2 = \langle x, [0.5, 0.5], [0.3, 0.3] \rangle, A_3 = \langle x, [0.5, 0.5], [0.5, 0.5] \rangle,$$
$$A_4 = \langle x, [0.5, 0.5], [0.4, 0.4] \rangle, A_5 = \langle x, [0.6, 0.6], [0.2, 0.2] \rangle, A_6 = \langle x, [0.4, 0.4], [0.4, 0.4] \rangle.$$

Table 1. A comparison of the entropy.

	F_A	E_A	F_B	F_C	F_D	F	P_1	P
A_1	0.74	0.08	0.38	0.11	0.62	0.55	0.31	0.363
A_2	0.96	0.36	0.71	0.24	0.87	0.82	0.34	0.236
A_3	1.00	1	1.00	0.00	1.00	0.44	0.50	0.300
A_4	0.83	0.64	0.50	0.24	0.71	0.76	0.28	0.276
A_5	0.83	0.11	0.50	0.24	0.71	0.76	0.28	0.276
A_6	1.00	1	1.00	0.24	1.00	0.89	0.4	0.215

Note that for two different sets, A_3 and A_6, the results of F_A, F_B, F_C and F_D, E_A are phase-ton identical; there are counterintuitive cases. In addition, the calculation results of F_C for setsA_1 and A_4 and A_2, A_5 and A_6 are still the same, so the entropy fails in this case, but F, P_1, P and the entropy and knowledge measure are consistent with the ranking of entropy and knowledge measures in [26], which also indicates that the entropy can be applied to the interval-valued Pythagorean fuzzy set.

Example 2. Consider a scenario involving the selection of a cloud service from four available options: SAP Sales on Demand (X_1), Salesforce Sales Cloud (X_2), Microsoft Dynamic CRM (X_3) and Oracle Cloud CRM (X_3). When assessing these options, experts consider five critical attributes: Performance, cost (or pay), reputation, scalability, and safety. The evaluations of these attributes by experts are provided in Table 2, with scores ranging from 1 to 5: $aj = (1, 2, 3, 4, 5)$, where a higher score indicates a better evaluation. Additionally, each attribute has a specific weight in the decision-making process, captured by a weight vector. The ultimate objective is to identify the best cloud service option based on the combined evaluations and weights.

Table 2. A comparison of the entropy.

	X_1	X_2	X_3	X_4
$a1$	([0.5,0.7],[0.2,0.3])	([0.4,0.5],[0.3,0.4])	([0.4,0.6],[0.3,0.4])	([0.3,0.5],[0.3,0.5])
$a2$	([0.3,0.5],[0.3,0.4])	([0.2,0.3],[0.2,0.4])	([0.7,0.8],[0.1,0.2])	([0.1,0.3],[0.6,0.8])
$a3$	([0.7,0.8],[0.1,0.2])	([0.3,0.4],[0.4,0.5])	([0.6,0.8],[0.1,0.1])	([0.1,0.2],[0.6,0.7])
$a4$	([0.5,0.7],[0.1,0.2])	([0.2,0.3],[0.5,0.6])	([0.5,0.7],[0.2,0.3])	([0.3,0.4],[0.4,0.6])
$a5$	([0.2,0.4],[0.3,0.5])	([0.7,0.8],[0.2,0.2])	([0.6,0.7],[0.1,0.3])	([0.2,0.4],[0.5,0.6])

Step 1. Since all five attributes are advantageous, normalization of the attributes in R is not required.

Step 2. Using the interval-valued Pythagorean fuzzy weighted average (IVPFWA) operator mentioned in literature [22], the individual evaluation value Z_{ij} and attribute weight vector $\omega = (0.2214, 0.090, 0.5121, 0.1126, 0.1010)^T$, $Y_i = (1, 2, 3, 4)$,

$$\begin{aligned} Y_1 &= \langle [0.5724, 0.7284], [0.1637, 0.2773] \rangle, \\ Y_2 &= \langle [0.3527, 0.4563], [0.3555, 0.4681 \rangle, \\ Y_3 &= \langle [0.5751, 0.7633], [0.1591, 0.2215] \rangle, \\ Y_4 &= \langle [0.1804, 0.3251], [0.5227, 0.6687] \rangle. \end{aligned}$$

Step 3. Based on the overall collective value of based on the alternative, when $\alpha = 0.3$, we use Eq (7) to calculate the knowledge measure of IvPFS. Therefore, we get: $K(Y_1) = 0.7062, K(Y_2) = 0.5924, K(Y_3) = 0.7198, K(Y_4) = 0.6678$, IVPFS The order of these knowledge measures is as follows:

$K(Y_3) \succ K(Y_1) \succ K(Y_4) \succ K(Y_2)$, Therefore, given the knowledge measure, we can derive the alternative ordering $X_3 \succ X_1 \succ X_4 \succ X_2$; thus, X_3 is the best cloud service. This demonstrates the practicality and rationality of the proposed knowledge measurement.

5. A multi-attribute decision problem approach based on the Interval-valued Pythagorean fuzzy set entropy of Hemming distance

In previous studies, it is not hard to see that when dealing with a multiple attribute decision problem (MADM), entropy can be used as a criterion decision information content attribute weight analysis tool, namely the attribute entropy weight. The more about a property, the greater the related weight is. In addition to entropy, in the background of IVPFS, knowledge measure also has this attribute. Then, the more you know about an attribute, the greater the importance weight associated with it. We call this the knowledge weight of the attributes. Next, a practical example adapted from [17] is provided to illustrate the application of the entropy formula proposed herein to the MAGDM of IVPFS. A city is planning to build a city library. The Urban Development Commissioner now faces the question of what kind of air conditioning system libraries should install. The contractor provided four feasible solutions, $A_i = (1, 2, 3, 4)$. At the same time, there are five attributes $C_j = (1, 2, 3, 4, 5)$ that need to be considered comprehensively. They are C_1-performance; C_2-maintainability; C_3-flexibility; C_4-$\cos t$; C_5-security. These options were evaluated by a committee of four experts, $D_K = (K = 1, 2, 3, 4)$, assuming that the weight vector of the experts was $\lambda = (0.3, 0.2, 0.3, 0.2)^T$and that the importance weight of the attributes was completely unknown. The order attribute weight vector is $\omega = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5)^T$, where $\sum_{i=1}^{5} \omega_i = 1$, $D_K$personal opinion on $A_i$relative to $C_j$is provided in the form of an interval-valued intuitionistic fuzzy decision matrix, expressed as $R_K = (r_{ij}^k)_{4\times5}, r_{ij}^k = [\mu_{ij}^k, v_{ij}^k], \mu_{ij}^k = [\mu_{ij}^{-k}, \mu_{ij}^{+k}], v_{ij}^k = [v_{ij}^{-k}, v_{ij}^{+k}]$ as the interval-valued Pythagorean fuzzy number. The decision matrix is drawn from four experts (Tables 3–6).

Table 3. Decision matrix given by the first decision maker (R_1).

	A_1	A_2	A_3	A_4
C_1	([0.5,0.7],[0.2,0.3])	([0.4,0.5],[0.3,0.4])	([0.4,0.6],[0.3,0.4])	([0.3,0.5],[0.3,0.5])
C_2	([0.3,0.5],[0.3,0.4])	([0.2,0.3],[0.2,0.4])	([0.7,0.8],[0.1,0.2])	([0.1,0.3],[0.6,0.8])
C_3	([0.7,0.8],[0.1,0.2])	([0.3,0.4],[0.4,0.5])	([0.6,0.8],[0.1,0.1])	([0.1,0.2],[0.6,0.7])
C_4	([0.5,0.7],[0.1,0.2])	([0.2,0.3],[0.5,0.6])	([0.5,0.7],[0.2,0.3])	([0.3,0.4],[0.4,0.6])
C_5	([0.2,0.4],[0.3,0.5])	([0.7,0.8],[0.2,0.2])	([0.6,0.7],[0.1,0.3])	([0.2,0.4],[0.5,0.6])

Table 4. Decision matrix given by the first decision maker (R_2).

	A_1	A_2	A_3	A_4
C_1	([0.4,0.5],[0.2,0.4])	([0.3,0.4],[0.4,0.5])	([0.4,0.6],[0.3,0.4])	([0.3,0.4],[0.4,0.6])
C_2	([0.3,0.4],[0.4,0.6])	([0.1,0.3],[0.3,0.7])	([0.6,0.8],[0.1,0.2])	([0.1,0.2],[0.6,0.8])
C_3	([0.6,0.7],[0.1,0.2])	([0.3,0.4],[0.4,0.5])	([0.7,0.8],[0.1,0.2])	([0.1,0.2],[0.7,0.8])
C_4	([0.5,0.6],[0.1,0.3])	([0.2,0.3],[0.6,0.7])	([0.4,0.6],[0.3,0.4])	([0.3,0.4],[0.4,0.6])
C_5	([0.1,0.3],[0.3,0.5])	([0.6,0.8],[0.1,0.2])	([0.5,0.6],[0.2,0.4])	([0.2,0.4],[0.5,0.6])

Table 5. Decision matrix given by the first decision maker (R_3).

	A_1	A_2	A_3	A_4
C_1	([0.4,0.7],[0.1,0.2])	([0.4,0.5],[0.2,0.4])	([0.2,0.4],[0.3,0.4])	([0.3,0.4],[0.2,0.4])
C_2	([0.3,0.5],[0.3,0.4])	([0.2,0.4],[0.4,0.5])	([0.6,0.8],[0.1,0.2])	([0.1,0.2],[0.6,0.8])
C_3	([0.6,0.7],[0.1,0.2])	([0.4,0.5],[0.3,0.4])	([0.5,0.7],[0.1,0.3])	([0.1,0.3],[0.5,0.7])
C_4	([0.5,0.6],[0.1,0.3])	([0.1,0.2],[0.7,0.8])	([0.5,0.7],[0.2,0.3])	([0.2,0.3],[0.5,0.7])
C_5	([0.3,0.5],[0.4,0.5])	([0.6,0.7],[0.2,0.3])	([0.6,0.8],[0.1,0.2])	([0.1,0.2],[0.6,0.8])

Table 6. Decision matrix given by the first decision maker (R_4).

	A_1	A_2	A_3	A_4
C_1	([0.6,0.7],[0.2,0.3])	([0.4,0.5],[0.4,0.5])	([0.4,0.5],[0.3,0.4])	([0.3,0.4],[0.4,0.5])
C_2	([0.3,0.4],[0.3,0.4])	([0.1,0.3],[0.3,0.7])	([0.6,0.7],[0.1,0.2])	([0.1,0.3],[0.6,0.7])
C_3	([0.7,0.8],[0.1,0.2])	([0.3,0.4],[0.5,0.6])	([0.5,0.8],[0.1,0.2])	([0.1,0.2],[0.5,0.8])
C_4	([0.5,0.6],[0.1,0.3])	([0.2,0.3],[0.4,0.6])	([0.4,0.5],[0.2,0.3])	([0.2,0.3],[0.4,0.5])
C_5	([0.1,0.2],[0.5,0.7])	([0.6,0.7],[0.1,0.2])	([0.5,0.6],[0.3,0.4])	([0.3,0.4],[0.5,0.6])

Table 7. Group opinion matrix (R).

	A_1	A_2	A_3	A_4
C_1	[0.47, 0.63] [0.17, 0.29]	[0.35, 0.45] [0.34, 0.50]	[0.34, 0.49] [0.30, 0.43]	[0.30, 0.43] [0.34, 0.49]
C_2	[0.30, 0.46] [0.35, 0.47]	[0.13, 0.31] [0.28, 0.47]	[0.63, 0.78] [0.10, 0.22]	[0.10, 0.22] [0.63, 0.78]
C_3	[0.56, 0.72] [0.13, 0.43]	[0.33, 0.43] [0.56, 0.68]	[0.54, 0.77] [0.10, 0.23]	[0.10, 0.23] [0.43, 0.61]
C_4	[0.50, 0.63] [0.10, 0.27]	[0.17, 0.30] [0.56, 0.68]	[0.43, 0.61] [0.22, 0.32]	[0.22, 0.32] [0.43, 0.61]
C_5	[0.16, 0.37] [0.37, 0.54]	[0.63, 0.75] [0.13, 0.23]	[0.53, 0.66] [0.19, 0.31]	[0.31, 0.91] [0.53, 0.66]

Here, we use the method of calculating the entropy given by Eq (12) to derive the importance weight of the properties. All of the above comments were summarized in order to further evaluate the alternatives. Table 7 summarizes all opinions

Step 1. Summarize the personal opinions of all experts and form the comprehensive evaluation decision matrix $R_k = (r_{ij}^k)_{4\times5}$. We use the interval-valued Pythagoras weighted average operator by referring to [21],

$$\begin{aligned} r_{ij} &= IVPFWA(r_{ij}^1, r_{ij}^2, r_{ij}^3, r_{ij}^4) \\ &= ([\sum_{i=1}^{n} \omega_i \mu_{ij}^-, \sum_{i=1}^{n} \omega_i \mu_{ij}^+], [\sum_{i=1}^{n} \omega_i v_{ij}^-, \sum_{i=1}^{n} \omega_i v_{ij}^+]). \end{aligned}$$

Step 2. Use Eq (11) to calculate the entropy F_j of each $C_j = (1, 2, 3, 4, 5)$ of the interval-valued Pythagorean fuzzy set. When the criterion weights are completely unknown, we use the following equation $\omega_j = \frac{1-E_j}{5-\sum_{i=1}^{5} F_j}(j = 1, 2, 3, 4, 5)$, then each C_j is $\omega = (0.0407, 0.0958, 0.1129, 0.1335, 0.2203)^T$.

Step 3. Use formula (13) and its weight vector $\omega = (0.0407, 0.0958, 0.1129, 0.1335, 0.2203)^T$ to calculate the entropy of each alternative scheme. Meanwhile, the smaller the entropy value, the better the scheme is. The calculated results are shown in Table 8.
Step 4. Rank the calculation results. According to Table 8, we find that it is the best choice, which is the same as the results of Park, which show the rationality and practicability of entropy for the multi-attribute decision problem (MADM).

Table 8. Entropy values of the best air conditioning selection.

	E_j^{ω}	Park
A_1	0.9656	3
A_2	0.9733	4
A_3	0.9567	1
A_4	0.9592	2

The decision-making approach based on entropy calculations aligns with results from other methods, validating its reasonableness. A smaller entropy value indicates a superior alternative because entropy quantifies informational uncertainty. Lower entropy signifies more deterministic and concentrated performance across attributes, better aligning with decision-making objectives. In uncertain decision-making environments, knowledge measurement reflects the effective information contained in an alternative. A higher value indicates proximity to the ideal state and reduced uncertainty, while entropy gauges attribute-level uncertainty. Low entropy implies stable and predictable attribute performance. Thus, selecting alternatives with high knowledge measurement values and low entropy ensures optimal outcomes while accounting for uncertainty.

6. Conclusions

We place a new axiomatic definition of the IVPFS knowledge measure in the present work. Also, the knowledge measure and entropy of IVPFS based on the axiomatic definition of distance are set up. The two formulations recommended in this paper can be a range of knowledge measures and entropy expansion. To analyze the performance of our proposed entropy model, an example is provided. Finally, through an interval-based solution to the multi-attribute decision problem with unknown weights, we demonstrate the potential and efficacy of this method. I deem that distance-based entropy can encompass different types of entropy via various functions in accordance with different forms and can also resolve multi-attribute decision-making having an unknown weight, which is the most conclusive, rather than previous entropy specifications. Moreover, it may fill the present voids in Interval-Valued Pythagorean fuzzy entropy.

In the future, the research will be more focused on the realization of the relationship between the knowledge measure and entropy applied to multi-attribute decision problems, data analysis, and risk assessment. Exploring Hybrid Measures: Integrate other uncertainty measurement methods (e.g., fuzzy measures, Dempster-Shafer theory of evidence) to construct hybrid knowledge measurement models. This could combine the strengths of diverse approaches, enhancing the ability to process complex uncertain information. For instance, combining fuzzy measures with the distance-based knowledge measurement proposed here could better account for interactions between attributes.

Expanding Application Domains: Beyond multi-attribute decision-making problems, apply the knowledge measurement framework to fields such as feature selection in machine learning or cluster analysis in data mining. In feature selection, knowledge measurement could evaluate feature importance, selecting those contributing most to model performance to improve accuracy and efficiency.

Authors contributions

Xin Wang: Writing-review and editing, software, validation; Li Li: Writing-review and editing, software, validation. All authors have read and approved the final version for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare no conflict of interest.

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