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Research article

A novel numerical method for stochastic conformable fractional differential systems

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Abstract: This study introduced the conformable fractional discrete Temimi–Ansari method (CFDTAM), a novel numerical framework designed to solve fractional stochastic nonlinear differential equations with enhanced efficiency and accuracy. By leveraging the conformable fractional derivative (CFD), the CFDTAM unifies classical and fractional-order systems while maintaining computational simplicity. The method's efficacy was demonstrated through applications to a stochastic population model and the Brusselator system, showcasing its ability to handle nonlinear dynamics with high precision. A comprehensive convergence analysis was also conducted to validate the reliability and stability of the proposed method. All computations were performed using Mathematica 12 software, ensuring accuracy and consistency in numerical simulations. CFDTAM sets a new benchmark in fractional stochastic modeling, paving the way for advancements in partial differential equations, delay systems, and hybrid models.

Keywords: stochastic differential equations; white noise; fractional-order systems; population model; conformable derivative
Mathematics Subject Classification: 34A08, 65L05, 60H10

1. Introduction

Fractional calculus dates back to 1695, when L'Hôpital and Leibniz explored the concept of extending the differential operator to fractional powers, such as 1/2 [1,2]. Fractional calculus involves the study of derivatives and integrals of non-integer orders, which can be real, rational, or complex [1]. The field is gaining increasing attention due to its versatility and ability to model systems with memory-dependent behaviors. Fractional-order systems (FOS) are particularly valued for providing more accurate representations of physical phenomena compared to their integer-order counterparts, while utilizing simpler mathematical models [3]. This advantage is mostly due to the tunability provided by fractional orders, which act as extra parameters in the modeling process.

Fractional-order calculus has several technical applications, including bioengineering [4,5], control systems [6,7], signal filtering [8], oscillatory systems [9], energy storage and conversion [10], encryption techniques [11], and chaos theory [12]. These examples demonstrate the use of fractional calculus in both theoretical and practical breakthroughs.

Stochastic differential equations (SDEs) improve the modeling accuracy of dynamic systems by including random fluctuations, making them essential for capturing real-world uncertainty. SDEs have been successfully used in a variety of disciplines, including stochastic control, neural networks, financial economics, electrical engineering, and population dynamics [13]. However, the nonlinear and complex character of SDEs frequently makes exact solutions impractical, necessitating the development of efficient numerical and analytical approaches to approximating their solutions.

Fractional calculus has been recently developed, offering a powerful framework for generalizing classical calculus. At its core, fractional calculus extends the concept of differentiation and integration to non-integer orders, with applications across mathematics, physics, and engineering. Various fractional derivatives and integrals definitions have emerged, including the Riemann–Louville, Grunwald–Letnikov, conformable, and Caputo formulations [14–20]. More recent advancements have introduced the Caputo–Fabrizio [21] and Atangana–Baleanu [22] derivatives, which incorporate non-singular kernels to address the limitations of earlier approaches. Despite advancements in the field, choosing the most suitable fractional derivative for a given problem remains a major challenge. Among the most commonly used definitions, the Riemann–Liouville and Caputo derivatives stand out for their traditional applications across various domains, including models based on memory mechanisms [23], fractional diffusion equations [24], and quantum dynamics, such as Brownian motion and anomalous diffusion [25].

However, traditional fractional derivatives often lack essential properties of classical calculus, such as the product, quotient, and chain rules. To address these challenges, Khalil et al. [26] introduced the conformable fractional derivative (CFD), a novel definition that extends the ordinary limit definition of derivatives while preserving many classical properties. The CFD simplifies computations and provides a consistent framework for extending classical theorems to fractional calculus [20]. Motivated by the advantages of the CFD, this work leverages its capabilities to address complex differential models. By incorporating the CFD into the proposed numerical framework, we aim to develop a robust and efficient approach for solving fractional differential equations, expanding the applicability of fractional calculus to real-world scenarios.

Fractional calculus has advanced significantly in recent years with the introduction of new definitions and applications for fractional derivatives. One important contribution is the CFD, which was presented by Khalil et al. in 2014 [26]. This concept offers a novel viewpoint on fractional differentiation by generalizing the conventional derivative while retaining obvious ties to classical calculus.

Let f : [0,∞) → R be a function. Then the CFD of the order α is defined by

$$T_\alpha f(t) = \frac{d^\alpha f(t)}{dt^\alpha} = f^\alpha(t) = \lim_{\varepsilon\to 0}\frac{f(t+\varepsilon t^{1-\alpha})-f(t)}{\varepsilon}, \quad (1)$$

where $t > 0$ and $0 < \alpha \leq 1$. This formulation serves as a potential generalization of the classical derivative. When $\alpha = 1$, the CFD reduces to the standard derivative:

$$f'(t) = \lim_{\varepsilon\to 0}\frac{f(t+\varepsilon)-f(t)}{\varepsilon}, \quad (2)$$

for functions that are α-differentiable in some $(0, b), b > 0$, and if the limit $\lim_{t\to 0^+} f^\alpha(t)$ exists, the value of the derivative at time 0 is:

$$f^\alpha(0) = \lim_{t\to 0^+} f^\alpha(t). \quad (3)$$

Khalil et al. [26] also discovered some essential features of the CFD, which they formulated as theorems in their work. These qualities demonstrate the definition's utility and consistency with classical calculus.

Theorem 1. (Fundamental properties of the conformable fractional derivative) Let $\alpha \in (0,1]$ and suppose f and g are α-differentiable at $t > 0$. Then, the conformable fractional derivative satisfies the following properties:

1) Linearity: $T_\alpha(af + bg) = aT_\alpha(f) + bT_\alpha(g)$ for all $a, b \in \Re$.
2) Power rule: $T_\alpha(t^p) = pt^{p-\alpha}$, for all $p \in \Re$.
3) Constant rule: $T_\alpha(\mu) = 0$, for all constant functions $f(t) = \mu$.
4) Product rule: $T_\alpha(fg) = fT_\alpha(g) + gT_\alpha(f)$.
5) Quotient rule: $T_\alpha(f\backslash g) = \frac{gT_\alpha(f)-fT_\alpha(g)}{g^2}$.
6) Higher-order representation: $T_\alpha f(t) = t^{n+1-\alpha}\frac{d^{n+1}}{dt^{n+1}}f(t), \alpha \in [n, n+1]$.

If $f(t)$ is $(n + 1)$ differentiable at $t > 0$ for $n = 0$, this simplifies to:

$$T_\alpha f(t) = t^{1-\alpha}\frac{df(t)}{dt}.$$

Proof. To validate the power rule, consider $f(t) = t^p$. By applying the CFD definition, we derive: $T_\alpha f(t) = \lim_{\epsilon\to 0}\frac{(t+\epsilon\, t^{1-\alpha})^p - t^p}{\epsilon}$.

Using binomial expansion and simplifying, we confirm: $T_\alpha f(t) = pt^{p-\alpha}$, establishing the power rule. Similar procedures apply to prove the remaining properties.

The advent of the CFD opened the door for new approaches to fractional calculus, providing an elegant balance of simplicity and generality. Its compatibility with classical calculus procedures makes it an effective tool for modeling complicated systems in a variety of scientific and engineering disciplines.

The fundamental goal and contribution of this paper is to introduce a unique numerical framework, known as the conformable fractional discrete Temimi–Ansari approach (CFDTAM), to deal with Itô stochastic fractional differential equations with nonlinear terms, which include both single equations and systems. This study expands on previous work [27] and the application of DTAM to stochastic nonlinear differential equations [28] by incorporating the CFD into the DTAM framework.

This is the first use of the CFD for stochastic nonlinear differential equations. The suggested CFDTAM is a novel, efficient, and computationally simple technique that is fundamentally comparable to the integer-order example but retains accuracy and processing speed. The CFDTAM framework is designed to find the solution of fractional stochastic nonlinear differential equations in a straightforward, rapid, and efficient manner, making it appropriate for use in real applications.

$$D_t^{\alpha}\mho(t) = F(t,\mho) + G(t) + f(t)\,\eta(t),\ \mho(0) = a, \tag{4}$$

D_t^{α} represents the fractional derivative with conformable sense CFD. The order of the fractional derivative is $\alpha \in [0,1]$ and $t > 0$. $\mho(t)$ is the unidentified operator, the autonomous variable is t, $F(t,\mho)$, and $G(t)$ are either nonlinear or linear operations, and $\eta(t)$ is white noise with Gaussian shape that may be generated from the Wiener process $\varpi(t)$ by:

$$\eta(t) = \delta\frac{d\varpi(t)}{dt}, \tag{5}$$

$E[\eta(t)] = 0$ has a mean of 0 and a variance of δ^2 with $\delta = 1$.

In this study, we propose the CFDTAM, a novel numerical framework for solving stochastic fractional differential equations. By leveraging the advantages of CFD, CFDTAM offers improved numerical stability, enhanced accuracy, and computational efficiency. The effectiveness of the proposed method is demonstrated through applications to a stochastic population model and the Brusselator system, both of which illustrate its capability in handling nonlinear and stochastic dynamics with high precision.

The remainder of this paper is organized as follows: Section 2 provides an overview of the numerical methodology and the development of CFDTAM. Section 3 presents numerical experiments to validate the method's effectiveness, followed by a discussion of the results. Finally, Section 4 concludes the study and outlines potential future research directions.

2. Development and convergence analysis of the CFDTAM

This section will revisit the classical discrete Temimi–Ansari method (DTAM) as the underlying framework. We use the CFD to apply the approach to stochastic nonlinear differential equations with fractional-order coefficients (α). We integrate these parts to create the final numerical scheme and determine the necessary time step ($\hbar$) for convergence.

Analyze the differential equation below, which has the following form:

$$L[\mho(t)] + \mathcal{N}[\mho(t)] + \mathrm{g}(t) = 0, \tag{6a}$$

Subject to initial conditions

$$\mathcal{I}\left(\mho,\frac{d^j\mho}{dt^j}\right) = 0. \tag{6b}$$

The operators L and $\mathcal{N}$ denote the linear and nonlinear components, respectively, while $\mathrm{g}(t)$ stands for the nonhomogeneous term. Equation (1) is solved using the Temimi–Ansari method, as demonstrated below: To obtain the initial approximate function $\mho_0$, resolve the following starting value issue:

$$L[\mho_0(t)] + \mathrm{g}(t) = 0, \mathcal{I}\left(\mho_0,\frac{d^j\mho_0}{dt^j}\right) = 0. \tag{7}$$

The following issue needs to be resolved in order to obtain the subsequent sacrificial function $\mho_1(t)$

$$L[\mho_1(t)] + \mathcal{N}[\mho_1(t)] + g(t) = 0,\ \mathcal{J}\left(\mho_1, \frac{d^j \mho_1}{dt^j}\right) = 0. \tag{8}$$

The nth approximate function, $\mho_n(t)$, can be computed similarly. Then

$$L[\mho_n(t)] + \mathcal{N}[\mho_n(t)] + g(t) = 0, n = 2,3, \dots,\ \mathcal{J}\left(\mho_n, \frac{d^j \mho_n}{dt^j}\right) = 0. \tag{9}$$

As the number of iterations increases, the resulting iterative solution approaches the precise solution.

$$\mho(t) = \lim_{n\to\infty} \mho_n(t). \tag{10}$$

Several research has extensively examined TAM's error behavior and convergence qualities when applied to ordinary differential equations and then extended to differential equation systems. Temimi and Ansari (2015) proposed a computational iterative framework for solving nonlinear ordinary differential equations, along with a fundamental study of the method's performance and application [29]. Expanding on this idea, a new method was introduced for handling differential algebraic equations, highlighting the flexibility and effectiveness of TAM in handling complex systems, as detailed in [30]. Further applications and tests have proved TAM's ability to solve numerous types of equations. For example, Ebrahimi et al. (2013) used an iterative method to solve partial differential equations, including the Korteweg-de Vries equations, demonstrating the flexibility of iterative approaches such as TAM in solving nonlinear and fractional problems [31]. Arafa et al. (2021) extended the method to fractional differential equations by developing the fractional Temimi–Ansari method (FTAM) and performed a comprehensive convergence analysis for solving physical equations, stressing the method's robustness in fractional-order contexts [32]. Similarly, Odibat and Momani (2008) developed a generalized differential transform approach for fractional-order linear partial differential equations, demonstrating the promise of iterative systems in fractional calculus [33].

These works together give a solid foundation for understanding TAM's convergence criteria, error analysis, and broad applicability to ordinary, partial, and fractional differential equations. These contributions serve as the foundation for future research and development of TAM, as proven in this study.

To offer a study on convergence, we shall begin with:

$$\begin{cases} \varsigma_0 = U_0(t), \\ \varsigma_1 = \Theta[\varsigma_0], \\ \varsigma_2 = \Theta[\varsigma_0 + \varsigma_1], \\ \vdots \\ \varsigma_n = \Theta[\varsigma_0 + \varsigma_1 + \dots + \varsigma_{n-1}]. \end{cases} \tag{11}$$

Define the component $\Theta[\mho(t)]$ as

$$\Theta[\varsigma_n] = U_n(t) - \sum_{i=0}^{n-1} U_i(t), i = 1,2,3, \dots\dots. \tag{12}$$

Considering the response for TAM is $\mho_n(t)$.

The theorems that follow use these criteria to explore convenient provisions for TAM convergence.

Lemma 1. (Convergence of the Temimi–Ansari iteration) Given a sequence defined by the recurrence:

$$\mho_{n+1}(t) = \mho_n(t) + ht^{\alpha-1}F(t,\mho_n) + \delta t^{\alpha-1}(G(t,\mho_n) + f(t))(\varpi_{i+1} - \varpi_i),$$

where F and G satisfy:

$$\|F(t,\text{U}) - F(t,y)\| \leq L\|\text{U} - y\|, \|G(t,\text{U}) - G(t,y)\| \leq L\|\text{U} - y\|.$$

For some $L > 0$, the sequence $\{\text{U}_n(t)\}$ converges to the exact solution $\text{U}(t)$.
Proof. Define the error term $e_n(t) = \text{U}_n(t) - \text{U}(t)$. The recurrence yields:

$$e_{n+1}(t) = e_n(t) + ht^{\alpha-1}[F(t,\text{U}_n) - F(t,\text{U})] + \delta t^{\alpha-1}[G(t,\text{U}_n) - G(t,\text{U})](\varpi_{i+1} - \varpi_i).$$

Applying the Lipschitz condition and iterating, we obtain:

$$\|e_{n+1}(t)\| \leq (1 + L\,h\,t^{\alpha-1})\|e_n(t)\|.$$

Choosing sufficiently small ensures convergence, proving the result.
Theorem 2. Assume Θ in Eq (12) is a variable from $\mathbb{K}$ to $\mathbb{K}$, where $\mathbb{K}$ is a Hilbert space. The chain solution $\text{U}(t) = \lim_{n\to\infty} \text{U}_n(t)$ comes together if $\exists$ 0<μ<1 such that

$$H\|\Theta[\varsigma_0 + \varsigma_1 + \ldots + \varsigma_n]\| \leq \mu\|\Theta[\varsigma_0 + \varsigma_1 + \ldots + \varsigma_{n-1}]\| \forall\mu \in \mathbb{N} \cup \{0\}. \tag{13}$$

This idea represents a specific instance of the fixed-point concept and is adequate for establishing the convergence of TAM.
Proof in [32,33].
Corollary 1. (Error bound for CFDTAM approximation) If the chain solution $\sum_{i=0}^{\infty} \text{U}_i(t)$ converges to $\text{U}(t)$, the greatest error will be

$$E_n(t) \leq \frac{1}{1-\wp}\wp^n\|\text{U}_0\|, \tag{14}$$

where the chain $\sum_{i=0}^{n-1} \text{U}_i(t)$ is used to tackle a broad range of nonlinear problems and $\wp = (1 - L\,h\,t^{\alpha-1})$ ensures contraction.
Proof in [32,33].
The TAM's acquired solution meets the accurate solution, so $\exists$ 0<μ<1

$$B_{n-1} = \begin{cases} \frac{\|\varsigma_n\|}{\|\varsigma_{n-1}\|}, & \|\varsigma_{n-1}\| \neq 0, \\ 0, & \|\varsigma_{n-1}\| = 0. \end{cases} \tag{15}$$

The power chain solution $\sum_{i=0}^{\infty} \text{U}_i(t)$ meets the exact answer $\text{U}(t)$ under the condition $0 \leq B_{n-1} < 1, \ \forall n = 0,1,2,\ldots$

The fractional time derivative D_t^{α} is the conformable fractional derivative (CFD). For a fractional operator, we suggest using CFDTAM to solve the stochastic nonlinear differential Eq (4).

The FDTAM approach for the Caputo fractional operator for calculating the problem's solution (4) consists of the following:
Consider an n-point regular grid on $[0, \text{Ţ}]$ as $\{i: i = 1, \ldots, n\}$, with $0 < t_1 < t_2 \ldots < t_n = \text{Ţ}$ and $t_i - t_{i-1} = q$. Assume a fixed constant, $\text{h} \in (0, q]$ for a given h.

To approximate $\frac{d\text{U}_0(t_{j+1})}{dt}$ and $\frac{d\varpi(t_{j+1})}{dt}$, use property 6 of the CFD first, followed by the finite difference form.

$$\frac{d\text{U}_0(t_{i+1})}{dt} = \frac{\text{U}_0^{i+1} - \text{U}_0^i}{\text{h}} + O(\text{h}), \quad \frac{d\varpi(t_{i+1})}{dt} = \frac{\varpi_{i+1} - \varpi_i}{\text{h}}. \tag{16}$$

The first iterative equation for the conformable scheme to approximate $D_t^{\alpha}Ʋ_0(t_{j+1})$ is provided by

$$Ʋ_0(t_{i+1}) = Ʋ_0(t_i) + \delta\, t^{\alpha-1} f(t_i)\,[\varpi(t_{i+1}) - \varpi(t_i)], \tag{17}$$

where $Ʋ_0(t_{i+1}) = Ʋ_0^{i+1}, Ʋ_0(t_i) = Ʋ_0^{i}, \varpi(t_{i+1}) = \varpi_{i+1}\ and\ \varpi(t_i) = \varpi_i$. The initial improved equation to estimate the initial approximated function, $Ʋ_0(t_{i+1})$, is

$$Ʋ_0^{i+1} = Ʋ_0^{i} + \delta {t_i}^{\alpha-1} f(t_i)(\varpi_{i+1} - \varpi_i). \tag{18a}$$

To calculate the next discrete approximation function $Ʋ_1(t_{i+1})$ and the nth discrete approximate functions $Ʋ_n(t_{i+1})$, follow these steps:

$$Ʋ_1^{i+1} = Ʋ_1^{i} + \mathrm{h}{t_i}^{\alpha-1} F\left(t_i, Ʋ_0^{i}\right) + \delta t^{\alpha-1}\left(G\left(t_i, Ʋ_0^{i}\right) + f(t_i)\right)\left(\varpi_{i+1} - \varpi_i\right), \tag{18b}$$

$$Ʋ_n^{i+1} = Ʋ_n^{i} + \mathrm{h}{t_i}^{\alpha-1} F\left(t_i, Ʋ_{n-1}^{i}\right)$$

$$+\delta {t_i}^{\alpha-1}\left(G\left(t_i, Ʋ_{n-1}^{i}\right) + f(t_i)\right)\left(\varpi_{i+1} - \varpi_i\right), n{:}\, 2,3,\ldots \tag{18c}$$

The answer will be determined using ķ iterations of different trends in the Wiener process $\varpi(t)$. The improved scheme (18) can be expressed in the following format:

$$Ʋ_{0,ķ}^{i+1} = Ʋ_{0,ķ}^{i} + \delta {t_i}^{\alpha-1} f(t_i)\,[\varpi_{i+1,ķ} - \varpi_{i,ķ}], \tag{19a}$$

$$Ʋ_{1,ķ}^{i+1} = Ʋ_{1`,ķ}^{i} + \mathrm{h}{t_i}^{\alpha-1}\left[F\left(t_i, Ʋ_{0,ķ}^{i}\right) + \left(G\left(t_i, Ʋ_{0,ķ}^{i}\right) + f(t_i)\,\frac{\varpi_{i+1,ķ} - \varpi_{i,ķ}}{\mathrm{h}}\right)\right], \tag{19b}$$

$$Ʋ_{n,ķ}^{i+1} = Ʋ_{n-1,ķ}^{i} + \mathrm{h}\, t_i^{\alpha-1}\left[F\left(t_i, Ʋ_{n-1,ķ}^{i}\right) + \left(G\left(t_i, Ʋ_{n-1,ķ}^{i}\right) + f(t_i)\,\frac{\varpi_{i+1,ķ} - \varpi_{i,ķ}}{\mathrm{h}}\right)\right]. \tag{19c}$$

To ensure system convergence (19a)–(19c), the time step should be selected. To calculate the convergence scale of the fixed-point repeat, differentiate the rightmost portion of Eq (19b) about $Ʋ_{0,ķ}$, as follows:

$$\mathrm{h}t_i^{\alpha-1}\frac{\partial F(t_i,Ʋ_{0,ķ}^{i})}{\partial Ʋ_{0,ķ}^{i}} + t_i^{\alpha-1}\frac{\partial G(t_i,Ʋ_{0,ķ}^{i})}{\partial Ʋ_{0,ķ}^{i}}(\varpi_{i+1,ķ} - \varpi_{i,ķ}) < 0, \tag{20}$$

$$\mathrm{h} < -\frac{\frac{\partial G(t_i,Ʋ_{0,ķ}^{i})}{\partial Ʋ_{0,ķ}^{i}}}{\frac{\partial F\left(t_i,Ʋ_{0,ķ}^{i}\right)}{\partial Ʋ_{0,ķ}^{i}}}\left(\varpi_{i+1,ķ} - \varpi_{i,ķ}\right), \tag{21}$$

$$\text{Let } \mathrm{f}_1 = \frac{\partial G(t_i,Ʋ_{0,ķ}^{i})}{\partial Ʋ_{0,ķ}^{i}}, \mathrm{f}_2 = \frac{\partial F(t_i,Ʋ_{0,ķ}^{i})}{\partial Ʋ_{0,ķ}^{i}}, \tag{22}$$

then we have

$$\mathrm{h} < -\frac{\mathrm{f}_1}{\mathrm{f}_2}\left(\varpi_{i+1,ķ} - \varpi_{i,ķ}\right). \tag{23}$$

The condition $\mathrm{h} < -\frac{\mathrm{f}_1}{\mathrm{f}_2}(\varpi_{i+1,\mathrm{k}} - \varpi_{i,\mathrm{k}})$ is enough to ensure the convergence of the time step in the finite difference method (FDM). Given the structural similarities of the other two equations, (19a) and (19c), this requirement can also be applied to numerical approximations made with FDM in those circumstances.

In conclusion, the overall expectation and variance of the answer can be obtained by calculating the expectation and variance of the sequence $\mho_{n,1}, \mho_{n,2}, \mho_{n,3}, \ldots, \mho_{n,k}$.

This novel combination of the discrete Temimi–Ansari method (DTAM), the CFD, and the proposed numerical scheme creates a highly efficient and effective tool dealing with fractional stochastic nonlinear differential equations. This method's simplicity and speed, which enable the rapid execution of numerous iterations without incurring significant processing costs, are among its main advantages. Traditional methods, on the other hand, frequently struggle with the intricacy of calculating the mean and variance, rendering them unsuitable for large-scale calculations. Our results align well with the stochastic Runge-Kutta method for integer-order cases $(\alpha = 1)$, as implemented in Mathematica 12. Furthermore, the CFD significantly benefits the old Caputo definition of fractional derivatives. Unlike Caputo-based alternatives, which frequently involve cumbersome implementations, the suggested system is strongly related to the structure of integer-order methods, considerably simplifying its application and decreasing computational complexity. Furthermore, the numerical results show that the variance amplitude achieved with our technique is significantly lower than prior findings described in [34]. This reduction shows the framework's improved accuracy and efficiency, underlining its superiority in dealing with fractional stochastic systems.

3. Numerical assessments

This section presents two models to demonstrate the variety and effectiveness of the suggested strategy. The first example considers a stochastic population model, while the second considers the stochastic form of the Brusselator system. These examples demonstrate that the proposed method can handle a variety of complex equations and systems. All computations were done with Mathematica software, which ensured the numerical simulations were accurate and efficient.

3.1. Stochastic population model

The stochastic population model is a key framework used in a variety of biological and financial applications. It roughly resembles the Black–Scholes model, except it incorporates an additional nonlinear component to account for system losses. These nonlinear losses pose major obstacles to analytical solutions, making numerical analysis a viable and effective alternative. Although there are various proven numerical techniques for studying nonlinear systems (e.g., [34,35]), extending these approaches to fractional nonlinear systems is still an open area of research.

In this example, we offer a numerical technique for assessing the nonlinear fractional system generated by the CFDTAM. This technique bridges the gap between classical and fractional nonlinear systems, allowing for the effective treatment of complicated stochastic models.

Although the computational method is explained for a first-order approximation, it is easy to extend to achieve greater precision. Examine the population model that has quadratic losses:

$$D_t^{\alpha}\mho(t) = (a\mho(t) - \epsilon\mho(t)^2) + \lambda\mho(t)\eta(t) \quad ; \mho(0) = \mho_0, \quad t > 0, \tag{24}$$

$a, \lambda,$ and ϵ are deterministic variables. The term $\epsilon\mho(t)^2$ denotes nonlinear quadratic losses in the

system owing to internal or external causes.
For a deterministic system $(\lambda = 0)$, we get:

$$D_t^{\alpha}\text{U}(t) = \text{U}(t)\big(a - \epsilon\text{U}(t)\big);\ \text{U}(0) = \text{U}_0. \tag{25}$$

This represents a Bernoulli differential equation, and its solution is given as follows:

$$\text{U}(t) = \frac{a}{\epsilon + \left(\frac{a}{\text{U}(t)} - \epsilon\right)e^{-at}}. \tag{26}$$

For $a > 0$, the average solution $\text{U}(t)^{(0)}$ reaches a capacity for transport $K = a/\epsilon$ as t approaches infinity. For a value less than 0, the solution decays to zero. The model (24) has a unique positive solution $\text{U}(t)$ [36] at $\text{U}_0 > 0$:

$$\text{U}(t) = \frac{\text{U}_0\, exp^0(at + \lambda\omega(t))}{1 + \text{U}_0\epsilon\int_0^t exp^0(at + \ \omega(t))dt}. \tag{27}$$

In conventional calculus, the comparable answer would be:

$$\text{U}(t) = \frac{\text{U}_0\, exp(\left(a - \frac{\lambda^2}{2}\right)t + \lambda\omega(t))}{1 + \text{U}_0\epsilon\int_0^t exp(\left(a - \frac{\lambda^2}{2}\right)t + \lambda\omega(t)dt}, \tag{28}$$

the solution (28) converges to a stable Gamma distribution g when $\gamma(\frac{2a}{\lambda^2 - 1}, \frac{\lambda^2}{2\epsilon})$ is less than $2a$. For $\lambda^2 > 2a$, the solution converges virtually certainly to zero. If $\lambda^2 = 2a$ [33].

Obtaining the mean and variance of $\text{U}(t)$ using the exact solution (28) might be challenging. In the present study, CFDTAM and mathematical methods are used to derive the kernels and statistics, as shown below. The constructed iterative scheme (19a)–(19c) for the stochastic fractional population model with quadratic losses is

$$\text{U}_{0,\text{k}}^{i+1} = \text{U}_{0,\text{k}}^{i} + \lambda {t_i}^{\alpha-1}\,\text{U}_{0,\text{k}}^{i}\,[\omega_{i+1,\text{k}} - \omega_{i,\text{k}}], \tag{29a}$$

$$\text{U}_{1,\text{k}}^{i+1} = \text{U}_{1`,\text{k}}^{i} + \text{h}\,{t_i}^{\alpha-1}\left(a\text{U}_{0,\text{k}}^{i} - \epsilon\text{U}_{0,\text{k}}^{2}\right) + \lambda {t_i}^{\alpha-1}\,\text{U}_{0,\text{k}}^{i}\,[\omega_{i+1,\text{k}} - \omega_{i,\text{k}}], \tag{29b}$$

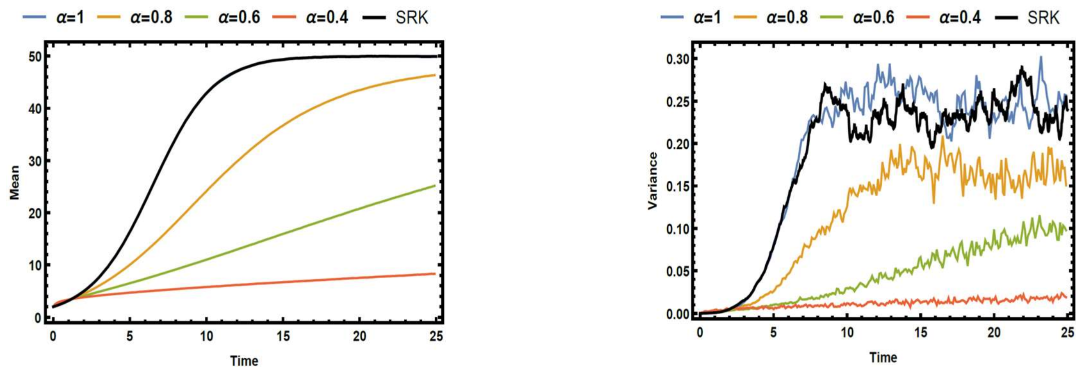
$$\text{U}_{n,\text{k}}^{i+1} = \text{U}_{n-1,\text{k}}^{i} + \text{h}\,{t_i}^{\alpha-1}\left(a\text{U}_{n-1,\text{k}}^{i} - \epsilon\text{U}_{n-1,\text{k}}^{2}\right) + \lambda {t_i}^{\alpha-1}\,\text{U}_{n-1,\text{k}}^{i}\,[\omega_{i+1,\text{k}} - \omega_{i,\text{k}}], \tag{29c}$$

Simulations with $u(0) = 2$ and $k = 5000$, for $\text{U}_0 = 0.5$, $\text{h} = 0.05$, $a = 0.5$, $\epsilon = 0.01$, and $\lambda = 0.02$ parameters were performed on Wiener process samples. Figures 1(a),(b) show the expected and variance values obtained using the suggested and stochastic Runge–Kutta methods, respectively. These figures show that the suggested approach's solutions are consistent and compatible with those given by the stochastic Runge–Kutta method. Increasing the Hurst parameter results in a smoother solution with fewer oscillations but at the expense of increased variance. Furthermore, when λ is small as in the current example, the mean solution is relatively insensitive to Hurst parameter changes. In such instances, the stochastic (diffusion) factor has just a tiny influence on the mean solution.

Figure 2 illustrates the comparison of the mean (a) and variance (b) for the population model, evaluated using the CFDTAM method at α = 1 and the stochastic Runge–Kutta (SRK) method. The results indicate that both methods effectively capture the statistical characteristics of the system. Examining the mean values, the CFDTAM method produces results that closely correspond to those obtained using the SRK method, suggesting its accuracy in approximating population dynamics. Minor discrepancies may be observed due to differences in numerical schemes and stochastic integration approaches.

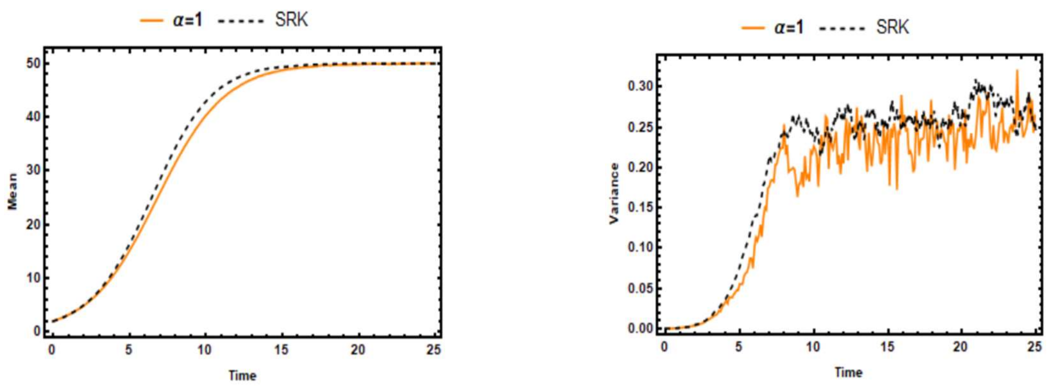
Regarding variance, both methods display similar patterns, demonstrating their ability to represent stochastic fluctuations within the model. The consistency between the two techniques indicates that CFDTAM can serve as a reliable alternative to SRK for stochastic population modeling.

The results shown above demonstrate the CFDTAM's efficiency in producing accurate solutions with a small number of terms. Extending this strategy to models with fractional-order derivatives is a natural step. When assessed with CFDTAM, the outcome of the consistent system will contain both integer-order and fractional-order derivatives, in addition to the method's inherent fractional-order integrals. To address such cases, the computational algorithm must be changed to accurately approximate the mixed derivatives and integrals, ensuring that the approach is still accurate and relevant.



(a) (b)

Figure 1. Mean (a) and variance (b) for the population model at $\alpha = 0.4, 0.6, 0.8, 1$.



(a) (b)

Figure 2. Comparison of the mean (a) and variance (b) for the population model, applying the CFDTAM method at $\alpha = 1$ alongside the stochastic Runge–Kutta (SRK) method.

3.2. Stochastic Brusselator system

Consider the stochastic version of the Brusselator system [28]:

$$\begin{cases} {}_{0}^{C}D_{t_0}^{\alpha}\mho(t) = (\eta-1)\mho(t) + \eta\,\mho^2(t) + \big(1+\mho(t)\big)^2 y(t) + \gamma\,\mho(t)\big(1+\mho(t)\big)\frac{d\omega(t)}{dt}, \\ {}_{0}^{C}D_{t_0}^{\alpha}y(t) = -\eta\mho(t) - \eta\mho^2(t) - \big(1+\mho(t)\big)^2 y(t) - \gamma\,\mho(t)\big(1+\mho(t)\big)\frac{d\omega(t)}{dt}. \end{cases} \tag{30}$$

With $y(t_0) = 0, \mho(t_0) = -0.1$. This nonlinear system exhibits unforced periodic oscillations observed in specific chemical reactions. The simulation is conducted for $\eta = 1.9$ and $\gamma = 0.1$, and its iterative scheme is presented as follows.

$$\mho_{0,k}^{i+1} = \mho_{0,k}^{i} \tag{31a}$$

$$\mho_{1,k}^{i+1} = \mho_{1,k}^{i} + 0.9\text{h}\,{t_i}^{\alpha-1}\mho_{0,k}^{i} + 1.9\text{h}\,{t_i}^{\alpha-1}\left(\mho_{0,k}^{i}\right)^2 + \text{h}\,{t_i}^{\alpha-1}\left(1+\left(\mho_{0,k}^{i}\right)\right)^2 y_{0,k}^{i}$$
$$+0.1{t_i}^{\alpha-1}\mho_{0,k}^{i}\left(1+\left(\mho_{0,k}^{i}\right)\right)\left(\omega_{i+1,k}-\omega_{i,k}\right), \tag{31b}$$

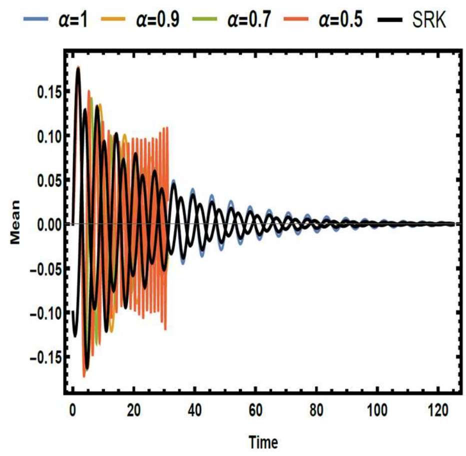
$$\mho_{n,k}^{i+1} = \mho_{n,k}^{i} + 0.9\text{h}\,{t_i}^{\alpha-1}\mho_{n-1,k}^{i} + 1.9\text{h}\,{t_i}^{\alpha-1}\left(\mho_{n-1,k}^{i}\right)^2 + \text{h}\,{t_i}^{\alpha-1}\left(1+\left(\mho_{n-1,k}^{i}\right)\right)^2 y_{0,k}^{i}$$
$$+0.1{t_i}^{\alpha-1}\mho_{n-1,k}^{i}\left(1+\left(\mho_{n-1,k}^{i}\right)\right), n{:}\ 2,3,\ldots\ldots, \tag{31c}$$

$$y_{0,k}^{i+1} = y_{0,k}^{i}, \tag{31d}$$

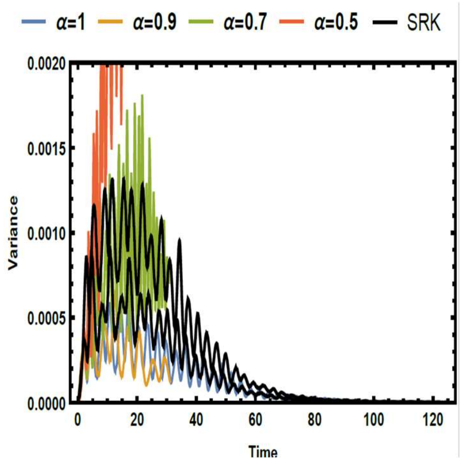
$$y_{1,k}^{i+1} = \text{-}1.9\ \text{h}\,{t_i}^{\alpha-1}\ \mho_{0,k}^{i}\text{-}1.9\ \text{h}\,{t_i}^{\alpha-1}\left(\mho_{0,k}^{i}\right)^2 - \text{h}\,{t_i}^{\alpha-1}\left(1+\left(\mho_{0,k}^{i}\right)\right)^2 y_{0,k}^{i}$$
$$-0{,}1\mho_{0,k}^{i}\left(1+\left(\mho_{0,k}^{i}\right)\right)\left(\omega_{i+1,k}-\omega_{i,k}\right), \tag{31e}$$

$$y_{n,k}^{i+1} = \text{-}1.9\ \text{h}\,{t_i}^{\alpha-1}\ \mho_{n-1,k}^{i}\text{-}1.9\ \text{h}\,{t_i}^{\alpha-1}\left(\mho_{n-1,k}^{i}\right)^2 - \text{h}\,{t_i}^{\alpha-1}\left(1+\left(\mho_{n-1,k}^{i}\right)\right)^2 y_{n-1,k}^{i}$$
$$-0{,}1{t_i}^{\alpha-1}\mho_{n-1,k}^{i}\left(1+\left(\mho_{n-1,k}^{i}\right)\right)\left(\omega_{i+1,k}-\omega_{i,k}\right), n{:}\ 2,3. \tag{31f}$$

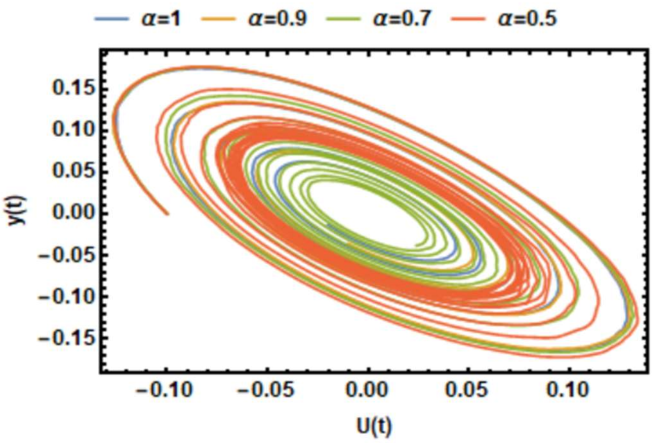
Over the interval $t \in [0,125]$ with $\Delta t = 0.025$ and initial conditions $\big(\mho(0), y(0)\big) = (-0.1,0)$, as detailed in [25], Figure 3 demonstrates that the approximate trajectories generated by the proposed scheme (31) stay near the origin. This closely mirrors the behavior of the exact solution, indicating that the method effectively captures the system's dynamics and provides a reliable numerical approximation consistent with the theoretical solution. In [38], Nouri et al. investigated a stochastic system model through a Ginzburg–Landau approach, emphasizing the accurate tracking of trajectories in stochastic differential equations (SDEs) similar to the present example. Figure 4 illustrates the comparison of the mean (a) and variance (b) for the Brusselator system, evaluated using the CFDTAM method at α = 1 and the stochastic Runge–Kutta (SRK) method. Their work highlights the reliability of numerical methods in modeling complex systems with random parameters. In particular, the ability of the proposed scheme to replicate the solution's behavior is consistent with the observations made in their study of stochastic processes, reinforcing the applicability and robustness of this method in solving stochastic population models.



(a)

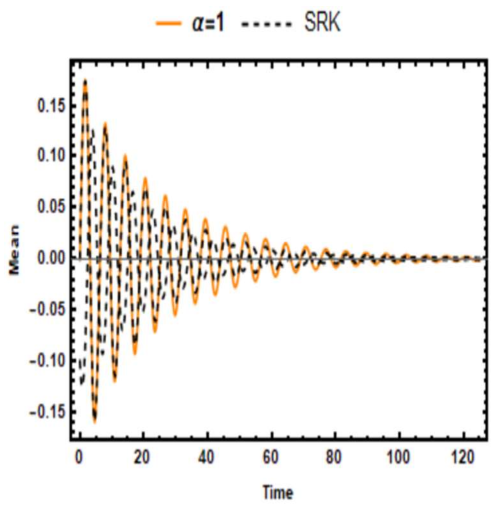


(b)

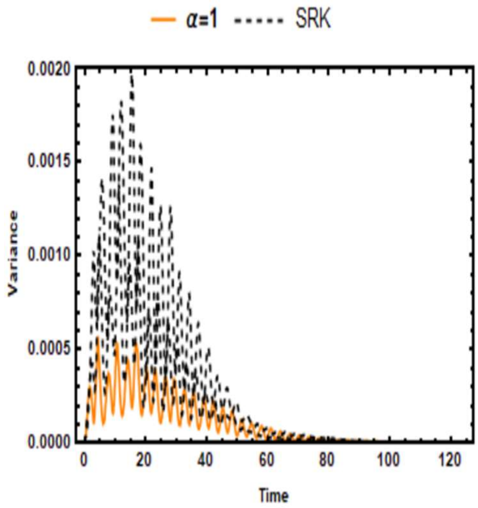


(c)

Figure 3. Mean (a), variance (b), and solution (c) for the stochastic Brusselator system at $\alpha = 0.4, 0.6, 0.8, 1$.



(a)



(b)

Figure 4. Comparison of the mean (a) and variance (b) for the stochastic Brusselator system, utilizing the CFDTAM method at α = 1 and the stochastic Runge–Kutta (SRK) method.

3.3. Discussion

This study's findings demonstrate the effectiveness of the CFDTAM as a reliable technique for solving fractional stochastic nonlinear differential equations. By combining the CFD, the suggested methodology effectively addresses some of the long-standing issues with fractional calculus, such as computing complexity and stochastic accuracy. Notably, the CFDTAM reduces computations while maintaining numerical approximation accuracy, as demonstrated by the examples of the stochastic population model and the Brusselator system presented here.

One of CFDTAM's primary strengths is its compatibility with both classical and fractional-order systems. The approach's versatility enables for seamless transition between integer-order and fractional-order scenarios, as evidenced by the consistency between CFDTAM findings and those obtained by the stochastic Runge–Kutta method for integer-order systems (α = 1). Furthermore, the CFDTAM framework takes advantage of CFD's compatibility with classical calculus operations, such as the product and chain rules, which greatly decreases processing costs when compared to standard fractional derivatives like the Caputo derivative.

The numerical trials carried out in this paper provide various insights. First, the variance amplitude attained with CFDTAM was consistently lower than with previous approaches, demonstrating its higher precision. This variance reduction is especially important for stochastic systems because small numerical errors can spread and jeopardize the trustworthiness of outputs. Furthermore, the iterative nature of CFDTAM allows for speedy convergence to accurate solutions with minimum computer resources, making it ideal for real-world applications that require fast and trustworthy results. The examples presented further highlight CFDTAM's adaptability to complicated stochastic systems. The stochastic population model accurately captured nonlinear quadratic losses and stochastic impacts, providing insights into the dynamics under different fractional orders (α).

Similarly, the application to the Brusselator system demonstrated CFDTAM's capacity to simulate complex oscillatory patterns found in nonlinear chemical systems. Despite these encouraging findings, certain limitations merit addressing. The method's convergence is dependent on the suitable choice of time step size (h), especially in systems with steep gradients or significant stochasticity. Although theoretical convergence conditions have been established, further investigation is required to refine these parameters for larger classes of fractional stochastic differential equations. Furthermore, while this study concentrated on Itô formulations, extending the CFDTAM framework to Stratonovich or other stochastic interpretations could broaden its application. Looking ahead, various areas of future research arise. First, CFDTAM's expansion to fractional stochastic partial differential equations may unlock its potential for modeling spatially distributed systems with memory effects. Second, combining CFDTAM with fuzzy logic or delay systems may result in a more comprehensive framework for dealing with uncertainty and temporal lags in real-world scenarios. Finally, comparisons with other cutting-edge fractional calculus methods will serve to benchmark CFDTAM's performance and find areas for development. The CFD offers distinct advantages in solving fractional stochastic differential equations, particularly in numerical analysis. Unlike traditional fractional derivatives such as Caputo and Riemann–Liouville, which involve complex integral definitions and kernel functions, CFD maintains a structure more aligned with classical calculus. This alignment allows it to preserve key mathematical properties, including the product, quotient, and chain rules, making it more computationally efficient. By integrating CFD into our proposed CFDTAM, we achieve improved numerical stability and accuracy. The method effectively bridges the gap between integer-order and fractional-order models, ensuring smoother computations and better convergence. Our numerical results further demonstrate that CFDTAM, when combined with CFD, minimizes variance and reduces computational costs compared to conventional approaches, making it a highly efficient tool for solving stochastic fractional differential equations.

4. Conclusions

This study introduces the CFDTAM, a unique and computationally efficient method for solving fractional stochastic nonlinear differential equations. By utilizing the CFD, CFDTAM bridges the gap between classical and fractional-order systems, providing a unifying framework for simplifying large stochastic problems while maintaining accuracy and lowering computational costs. The approach was used with two models: A stochastic population equation and a stochastic Brusselator system. The results produced using CFDTAM were compared to those obtained using Mathematica 12's stochastic Runge–Kutta method, demonstrating greater accuracy, efficiency, and variance reduction. These results demonstrate CFDTAM's robustness and adaptability in addressing complex nonlinear dynamics and real-world issues characterized by uncertainty and memory effects.

This study establishes a new standard in fractional calculus methodology and paves the road for additional research into fractional stochastic partial differential equations, systems with delays, and hybrid fuzzy systems. The novel combination of CFD and the Temimi–Ansari framework marks a paradigm change in modeling and computation, establishing CFDTAM as a foundation for future advances in stochastic differential equations and applied sciences.

Author contributions

Aisha F. Fareed: Conceptualization, supervision, investigation, validation, mathematical, writing original draft and formal analysis; Mourad S. Semary: Mathematical analysis, investigation, validation, formal analysis, writing original draft, writing review and editing; Emad A. Mohamed: Investigation and validation; Mokhtar Aly: Formal analysis, investigation, Literature review, review and editing. All authors have read and agreed to the published version of the manuscript.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflict of interest in this paper.

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