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Research article

Improved results on sampled-data synchronization control for chaotic Lur'e systems

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Abstract: This paper examines the problem of master-slave synchronization control for chaotic Lur'e systems (CLS) under sampled-data conditions. Initially, a two-sided looped Lyapunov function is constructed by fully leveraging the system characteristics and information regarding the sampling mode. Subsequently, based on the Lyapunov stability theory and using the integral inequality of free matrices, we establish the stability criteria for the synchronization error system of CLS. Utilizing these conditions, we compute the sampling controller gains through an enhanced iterative conditioned cone complementarity linearization iteration algorithm, thereby achieving synchronization of the master-slave system over more extended sampling periods. Ultimately, numerical examples are presented to demonstrate that the proposed method outperforms existing approaches documented in the literature.

Keywords: Lur'e systems; synchronization; cone complementary linearization algorithm; sampled-data control
Mathematics Subject Classification: 34D20

1. Introduction

Chaos systems, as a category of nonlinear dynamical systems, are highly dependent on initial conditions, inherent randomness, and a continuous broad spectrum. These characteristics render chaotic systems particularly suitable for communication applications while presenting significant prospects in finance, chemistry, and biology [1–5]. Notably, chaotic systems have garnered considerable attention since the seminal work presented in [6]. A wide range of nonlinear dynamical systems can be expressed as chaotic Lur'e systems (CLS). Therefore, the synchronization problem related to CLS has become a hot topic in recent years [7]. To address the synchronization problem, various control strategies have been proposed, including adaptive control [8], sliding mode control [9], feedback control [10], and sampled-data control [11].

Sampled-data control requires the system to provide state information only at specific sampling times, which endows it with low control cost, high efficiency, flexibility, and reliability. Over the past few decades, advancements in network communication and associated digital technologies have led to significant progress in sampled-data control systems [12, 13]. Consequently, it has been widely used in the field of control [14–19]. Many excellent results have also been obtained in the synchronization problem of CLS [20–22]. In Reference [6], sampling control is introduced into the CLS, deriving global asymptotic synchronization conditions. An input delay method utilizing the Lyapunov functional to establish synchronization conditions in Reference [23]. Subsequently, References [24, 25] further investigate master-slave synchronization of CLS considering system delays. However, these studies do not fully account for the characteristics of the CLS and available information during the sampling process. Therefore, system information was thoroughly considered in References [26, 27]. Based on this information, the Lyapunov functional was augmented, and the results were further optimized using linear matrix inequalities (LMIs). However, the augmented function did not sufficiently consider the sampling process's characteristics, and there is still potential for improving the resulting outcomes.

Only occupying the network channel at the sampling moment can significantly reduce a sampling system's communication pressure and computational burden. Thus, obtaining a more extensive sampling interval is the main issue in the field of sampled-data control, and it is also a critical index that evaluates the conservativeness of the synchronization criterion of CLS [28–30]. Two main approaches to achieving a more extensive sampling interval are adopting appropriate Lyapunov functionals and bounding its derivative with lower conservativeness. The field of functionals has seen significant advancements, starting with basic forms of Lyapunov functionals [31], progressing to time-dependent and discontinuous Lyapunov functionals [32], and culminating in the recent development of two-sided looped functionals [33–35]. Integral inequalities play a crucial role in limiting the quadratic integral term within the functional derivative. With the development of Jensen's inequality, Wirtinger-based inequality, free-weighting matrix inequality, and augmented forms, conservativeness has been substantially diminished. All these provide beneficial tools for us to study the synchronization problem of CLS.

In previous studies on the synchronization problem of CLS based on sampled-data control, the solution of the sampled controller was usually obtained by parameter adjustment methods. However, this approach is heavily influenced by the initial values of the chosen parameters, and optimizing these parameters can be quite complex. This complexity may hinder determining the controller and could result in the oversight of specific solutions that satisfy the necessary conditions. In Reference [36], an iterative algorithm of cone complementary linearization based on linear matrix inequalities is proposed and successfully applied in sampling control systems [37]. This algorithm eliminates the need for preset initial parameter values and does not require parameter tuning. Once a stopping condition for the iteration is established, it continuously iterates to discover the optimal solution that meets the specified conditions, thereby significantly enhancing the accuracy of the calculations.

Motivated by the descriptions discussed above, this paper thoroughly investigates the synchronization problem of CLS by considering the characteristics of the system's sampling process. The key contributions of this paper are outlined as follows:

1) An augmented two-sided looped Lyapunov functional is constructed, which fully considers the

system's state variables and sampling information and leads to the establishment of a stability criterion.
2) Utilizing the cone complementarity linearization iterative algorithm, a novel iterative condition for the design of sampling controllers has been developed.
3) Numerical simulation experiments reveal that our method attains a significantly larger sampling interval than that reported in References [22, 25, 34], suggesting its capability to generate more relaxed outcomes.

Notations. Throughout this note, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. The superscripts $\mathcal{V}^{-1}$ and $\mathcal{V}^T$ represent the matrix $\mathcal{V}$ inverse and transpose, respectively; the space of $n \times m$ real matrix is denoted by $\mathbb{R}^{n\times m}$; the condition $P > 0$ indicates that the matrix P is both symmetric and positive definite; He$\{J\}$=$J + J^T$; the notation $diag\{\cdots\}$ denoted a block-diagonal matrix; $\mathbb{N}$ represents the collection of all natural numbers. $\forall r \in \mathbb{N}$, $\mathbb{N}_r = \{1, 2, \ldots, r\}$.

2. Preliminaries

Consider the following master and slave CLS:

$$\begin{aligned}
&\mathcal{E}: \begin{cases} \dot{x}(t) = \mathcal{A}x(t) + \mathcal{B}\sigma(\mathcal{D}x(t)), \\ p(t) = Cx(t), \end{cases} \\
&\mathcal{F}: \begin{cases} \dot{y}(t) = \mathcal{A}y(t) + \mathcal{B}\sigma(\mathcal{D}y(t)) + u(t), \\ q(t) = Cy(t), \end{cases} \\
&\mathcal{L}: \quad u(t) = \mathcal{K}(p(t_k) - q(t_k)), \quad t_k \leq t < t_{k+1},
\end{aligned} \tag{2.1}$$

which comprises the master system $\mathcal{E}$, slave system $\mathcal{F}$, and control input $\mathcal{L}$. where $x(t) \in \mathbb{R}^n$ and y$(t) \in \mathbb{R}^n$ are the states of $\mathcal{E}$ and $\mathcal{F}$, respectively; and $p(t) \in \mathbb{R}^m$ and $q(t) \in \mathbb{R}^m$ are the subsystem output, $u(t) \in \mathbb{R}^n$ is the control input of $\mathcal{F}$; $\mathcal{A}, \mathcal{B}, C$, and $\mathcal{D}$ are constant matrices with appropriate dimensions; $\mathcal{K}$ is a controller gain matrix. $\sigma(\cdot) : \mathbb{R}^l \to \mathbb{R}^l$ is a diagonal and nonlinear function that belongs to the sector $[0, g_i]$ for $i = 1, 2, \cdots, l$,

$$0 \leq \frac{\sigma_i(d_i x(t)) - \sigma_i(d_i y(t))}{d_i(x(t) - y(t))} \leq g_i, x(t) \neq y(t), \tag{2.2}$$

where $g_i > 0$ represents a scalar, and d_i denotes the ith row vector of matrix $\mathcal{D}$.

It is assumed that the time interval between any two consecutive sampling instants is such that

$$t_{k+1} - t_k = h_k \in (0, h].$$

From the master system $\mathcal{E}$ and slave system $\mathcal{F}$, the synchronization error is given by $r(t) = x(t) - y(t)$, and the synchronization error system can be formulated as follows:

$$\dot{r}(t) = \mathcal{A}r(t) + \mathcal{B}f(\mathcal{D}x(t), \mathcal{D}y(t)) - \mathcal{K}Cr(t_k), \tag{2.3}$$

where $f(\mathcal{D}x(t), \mathcal{D}y(t)) = \sigma(\mathcal{D}x(t)) - \sigma(\mathcal{D}y(t))$. To simplify the presentation, let us refer to $f(\mathcal{D}x(t), \mathcal{D}y(t))$ as $f(t)$.

To formulate a more permissive synchronization criterion, the subsequent Lemma 2.1 is necessary [38].

Lemma 2.1. *Let $\mathcal{R} \in \mathbb{R}^{n\times n}$ be a positive-definite matrix. $x{:}[\alpha_1, \alpha_2] \to \mathbb{R}^n$ and $\tilde{\xi} \in \mathbb{R}^m$ be a continuous differentiable function. The following two inequalities hold for $\mathcal{N} \in \mathbb{R}^{3n\times m}$:*

$$-\int_{\alpha_1}^{\alpha_2} \dot{x}^T(\theta)\mathcal{R}\dot{x}(\theta)d\theta \leq 2\tilde{\xi}^T\bar{\Pi}^T\mathcal{N}\tilde{\xi} + \alpha\tilde{\xi}^T\mathcal{N}^T\mathcal{R}_1^{-}\mathcal{N}\tilde{\xi},$$

where $\mathcal{R}_1 = diag\{\mathcal{R}, 3\mathcal{R}, 5\mathcal{R}\}$ and

$$\begin{aligned}
&\alpha = \alpha_2 - \alpha_1,\ \tilde{k}_i = \left[0_{n\times(i-1)n}\ \ I_n\ \ 0_{n\times(4-i)n}\right], i = 1, 2\cdots 4,\\
&\tilde{\xi} = \left[x^T(\alpha_2)\ \ x^T(\alpha_1)\ \ \int_{\alpha_1}^{\alpha_2} \tfrac{x^T(\theta)}{\alpha}d\theta\ \ \int_{\alpha_1}^{\alpha_2}\int_{\theta}^{\alpha_2} \tfrac{x^T(\theta)}{\alpha^2}d\theta ds\right]^T,\\
&\bar{\Pi} = [\tilde{k}_1^T - \tilde{k}_2^T\ \ \tilde{k}_1^T + \tilde{k}_2^T - 2\tilde{k}_3^T\ \ \tilde{k}_1^T - \tilde{k}_2^T + 6\tilde{k}_3^T - 12\tilde{k}_4^T]^T.
\end{aligned}$$

3. Main results

This section develops a criterion for achieving master-slave synchronization in CLS. Leveraging this criterion, we then apply the cone complementarity linearization iterative algorithm to determine the gain of the sampled-data controller. The following notions are given to help simplify the description of the matrices and vectors of the main results.

$$\begin{aligned}
h_1(t) &= t - t_k,\ h_2(t) = t_{k+1} - t,\\
v_1 &= \int_{t_k}^{t} \frac{r(s)}{h_1(t)}ds,\ v_2 = \int_{t}^{t_{k+1}} \frac{r(s)}{h_2(t)}ds,\\
v_3 &= \int_{t_k}^{t}\int_{t_k}^{s} \frac{2r(\theta)}{h_1(t)^2}d\theta ds,\ v_4 = \int_{t}^{t_{k+1}}\int_{s}^{t_{k+1}} \frac{2r(\theta)}{h_2(t)^2}d\theta ds,\\
v &= \left[h_1(t)v_1^T\ \ h_2(t)v_2^T\ \ h_1(t)v_3^T\ \ h_2(t)v_4^T\right]^T,\\
\eta_1 &= \left[r^T(t_k)\ \ r^T(t_{k+1})\ \ \int_{t_k}^{t_{k+1}} r^T(s)ds\right]^T,\\
\eta_2(t) &= \left[r^T(t_k)\ \ r^T(t_{k+1})\ \ h_1(t)v_1^T\ \ h_2(t)v_2^T\ \ h_1(t)v_3^T\ \ h_2(t)v_4^T\right]^T,\\
\eta_3(t) &= \left[h_2(t)(r(t) - r(t_k))^T\ \ h_1(t)(r(t_{k+1}) - r(t))^T\right]^T,\\
\eta_4(t) &= \left[r^T(t)\ \ r^T(t_k)\ \ r^T(t_{k+1})\ \ v^T\right]^T,\\
\xi(t) &= \left[r^T(t)\ \ r^T(t_k)\ \ r^T(t_{k+1})\ \ v^T\ \ v_1^T\ \ v_2^T\ \ v_3^T\ \ v_4^T\ \ f(t)\right]^T.
\end{aligned}$$

Below, Theorem 3.1 explores system (2.1) with a predefined controller gain $\mathcal{K}$. The condition for synchronization is obtained through the application of the two-sided looped function in combination with Lemma 2.1.

Theorem 3.1. *Give scalars $h > 0$ and real matrices $\mathcal{K}$, the master system $\mathcal{E}$, and the slave system $\mathcal{F}$ in system* (2.1) *are globally asymptotically synchronous if there exist real matrices $P > 0, Y_\imath > 0$, $Q, G_\imath, S_\jmath, X$, ($\imath \in \mathbb{N}_2$, $\jmath \in \mathbb{N}_4$), M_1, M_2, and diagonal matrix $\Gamma > 0$ such that the following LMIs* (3.1) *and* (3.2) *are satisfied for $h_k \in (0, h]$,*

$$\varphi_a = \begin{bmatrix} \varphi_0 + h_k\varphi_1 & \sqrt{h_k}\pi_{8b}^T M_2 & \sqrt{h_k}\pi_0^T Y_1 \\ * & -Y_b & 0 \\ * & * & -Y_1 \end{bmatrix} < 0, \tag{3.1}$$

$$\varphi_b = \begin{bmatrix} \varphi_0 + h_k\varphi_2 & \sqrt{h_k}\pi_{8a}^T M_1 & \sqrt{h_k}\pi_0^T Y_2 \\ * & -Y_a & 0 \\ * & * & -Y_2 \end{bmatrix} < 0, \tag{3.2}$$

where

$$\begin{aligned}
\varphi_0 = {} & He\{e_1^T P\pi_0 - e_{12}^T\Gamma e_{12} + e_{12}^T\Gamma\mathcal{G}\mathcal{D}e_1 + e_5^T G_1\pi_1 - e_4^T G_2\pi_1 \\
& + \pi_4^T Q\pi_3 - S_1 e_4 - S_2 e_5 - S_3 e_6 - S_4 e_7 + \pi_{8a}^T M_1\pi_9 + \pi_{8b}^T M_2\pi_{10}\}, \\
\varphi_1 = {} & He\{-e_1^T G_1\pi_1 + e_5^T G_1\pi_2 + \pi_{6a}^T Q\pi_3 + \pi_{4a}^T Q\pi_5 + S_1 e_8 + S_3 e_{10}\} - \pi_7^T X\pi_7, \\
\varphi_2 = {} & He\{e_1^T G_2\pi_1 + e_4^T G_2\pi_2 + \pi_{6b}^T Q\pi_3 + \pi_{4b}^T Q\pi_5 + S_2 e_9 + S_4 e_{11}\} + \pi_7^T X\pi_7,
\end{aligned}$$

with

$$\begin{aligned}
& e_m = \begin{bmatrix} 0_{n\times(m-1)n} & I_n & 0_{n\times(12-m)n} \end{bmatrix}, m = 1, 2, \cdots, 12, \\
& Y_a = \mathrm{diag}\{Y_1, 3Y_1, 5Y_1\}, \quad Y_b = \mathrm{diag}\{Y_2, 3Y_2, 5Y_2\}, \\
& \pi_0 = \mathcal{A}e_1 + \mathcal{B}e_{12} - C\mathcal{K}e_2, \quad \pi_1 = \begin{bmatrix} e_2^T & e_3^T & e_4^T & e_5^T & e_6^T & e_7^T \end{bmatrix}^T, \\
& \pi_2 = \begin{bmatrix} 0 & 0 & e_1^T & -e_1^T & 2e_8^T - e_{10}^T & -2e_9^T + e_{11}^T \end{bmatrix}^T, \quad \pi_3 = \begin{bmatrix} e_1^T & e_2^T & e_3^T & e_4^T & e_5^T & e_6^T & e_7^T \end{bmatrix}^T, \\
& \pi_4 = \begin{bmatrix} e_2^T - e_1^T & e_3^T - e_1^T \end{bmatrix}^T, \quad \pi_{4a} = \begin{bmatrix} 0 & e_3^T - e_1^T \end{bmatrix}^T, \quad \pi_{4b} = \begin{bmatrix} e_1^T - e_2^T & 0 \end{bmatrix}^T, \\
& \pi_5 = \begin{bmatrix} \pi_0^T & 0 & 0 & e_1^T & -e_1^T & 2e_8^T - e_{10}^T & -2e_9^T + e_{11}^T \end{bmatrix}^T, \quad \pi_{6a} = \begin{bmatrix} 0 & -\pi_0^T \end{bmatrix}^T, \quad \pi_{6b} = \begin{bmatrix} \pi_0^T & 0 \end{bmatrix}^T, \\
& \pi_7 = \begin{bmatrix} e_2^T & e_3^T & e_4^T + e_5^T \end{bmatrix}^T, \quad \pi_{8a} = \begin{bmatrix} e_3^T & e_1^T & e_9^T & e_{11}^T \end{bmatrix}^T, \quad \pi_{8b} = \begin{bmatrix} e_1^T & e_2^T & e_8^T & e_{10}^T \end{bmatrix}^T, \\
& \pi_9 = \begin{bmatrix} e_3^T - e_1^T & e_3^T + e_1^T - 2e_9^T & e_3^T - e_1^T + 6e_9^T - 6e_{11}^T \end{bmatrix}^T, \\
& \pi_{10} = \begin{bmatrix} e_1^T - e_2^T & e_1^T + e_2^T - 2e_8^T & e_1^T - e_2^T - 6e_8^T + 6e_{10}^T \end{bmatrix}^T.
\end{aligned}$$

Proof. Choose a Lyapunov functional below.

$$V(r_t) = V_0(t) + \sum_{n=1}^{4} V_n(t), t \in [t_k, t_{k+1}), \tag{3.3}$$

where

$$\begin{aligned}
& V_0(t) = r^T(t)Pr(t), \\
& V_1(t) = h_1(t)h_2(t)\eta_1^T X\eta_1, \\
& V_2(t) = 2h_1(t)\int_t^{t_{k+1}} r(s)^T ds G_1\eta_2(t) + 2h_2(t)\int_{t_k}^t r(s)^T ds G_2\eta_2(t), \\
& V_3(t) = 2\eta_3(t)^T Q\eta_4(t), \\
& V_4(t) = h_2(t)\int_{t_k}^t \dot{r}(s)^T Y_2\dot{r}(s)ds - h_1(t)\int_t^{t_{k+1}} \dot{r}(s)^T Y_1\dot{r}(s)ds.
\end{aligned}$$

Computing the derivative of (3.3) with respect to the solution of system (2.1) results in

$$\begin{aligned}
\dot{V}_0(t) &= 2\xi^T(t)\{e_1^T P\pi_0\}\xi(t),\\
\dot{V}_1(t) &= \xi^T(t)\{h_2(t)\pi_7^T X\pi_7 - h_1(t)\pi_7^T X\pi_7\}\xi(t),\\
\dot{V}_2(t) &= 2\xi^T(t)\{e_5^T G_1\pi_1 - h_1(t)e_1^T G_1\pi_1 + h_1(t)e_5^T G_1\pi_2 - e_4^T G_2\pi_1 + h_2(t)e_1^T G_2\pi_1 + h_2(t)e_4^T G_2\pi_2\}\xi(t),\\
\dot{V}_3(t) &= 2\xi^T(t)\{\pi_4^T Q\pi_3 + 2h_1(t)\pi_{6a}^T Q\pi_3 + 2h_1(t)\pi_{4a}^T Q3\pi_5 + 2h_2(t)\pi_{6b}^T Q\pi_3 + 2h_2(t)\pi_{4b}^T Q\pi_5\}\xi(t),\\
\dot{V}_4(t) &= \xi^T(t)\{h_1(t)\pi_0^T Y_1\pi_0 + h_2(t)\pi_0^T Y_2\pi_0\}\xi(t) + J_1 + J_2,
\end{aligned}$$

where

$$J_1 = -\int_t^{t_{k+1}} \dot{r}(s)^T Y_1 \dot{r}(s)ds, \quad J_2 = -\int_{t_k}^{t} \dot{r}(s)^T Y_2 \dot{r}(s)ds.$$

Applying Lemma 2.1, we obtain

$$J_1 \leq \xi^T(t)[h_2(t)\Pi_{8a}^T M_1 {Y_a}^{-1} {M_1}^T \Pi_{8a} + 2\Pi_{8a}^T M_1 \Pi_9]\xi(t), \tag{3.4}$$

$$J_2 \leq \xi^T(t)[h_1(t)\Pi_{8b}^T M_2 {Y_b}^{-1} {M_2}^T \Pi_{8b} + 2\Pi_{8b}^T M_2 \Pi_{10}]\xi(t). \tag{3.5}$$

Note that, for any matrices $S_i (i \in \mathbb{N}_4)$, all the subsequent zero-equality equations remain valid

$$0 = 2\xi^T(t)S_1[h_1(t)e_8 - e_4]\xi(t), \tag{3.6}$$

$$0 = 2\xi^T(t)S_2[h_2(t)e_9 - e_5]\xi(t), \tag{3.7}$$

$$0 = 2\xi^T(t)S_3[h_1(t)e_{10} - e_6]\xi(t), \tag{3.8}$$

$$0 = 2\xi^T(t)S_4[h_2(t)e_{11} - e_7]\xi(t). \tag{3.9}$$

For any diagonal matrix $\Gamma = \mathrm{diag}\{\ell_1, \ell_2, \cdots, \ell_l\} > 0$, it follows from inequality (2.2) that the following inequality holds:

$$0 \leq 2\xi^T(t)(e_{12}^T \Gamma \mathcal{G}\mathcal{D} e_1 - e_{12}^T \Gamma e_{12})\xi(t) \tag{3.10}$$

with $\mathcal{G} = \mathrm{diag}\{g_1, g_2, \cdots, g_l\}$. By incorporating (3.4)-(3.10) into the right-hand side of $\dot{V}(r_t)$, the following resulting expression is obtained.

$$\dot{V}(r_t) \leqslant \xi^T(t)\left[\frac{h_1(t)}{h_k}\varphi_a + \frac{h_2(t)}{h_k}\varphi_b\right]\xi(t), \tag{3.11}$$

where

$$\varphi_a = \varphi_0 + h_k\varphi_1 + h_k\pi_0^T Y_1\pi_0 + h_k {M_1}^T \tilde{Y}_a^{-1} M_1, \tag{3.12}$$

$$\varphi_b = \varphi_0 + h_k\varphi_2 + h_k\pi_0^T Y_2\pi_0 + h_k {M_2}^T \tilde{Y}_b^{-1} M_2. \tag{3.13}$$

If $\varphi_a < 0$ and $\varphi_b < 0$ are satisfied, then according to the Schur complement, inequalities (3.1) and (3.2) are established, respectively. Then $\dot{V}(t) < -\gamma\|x(t)\|^2$ for a suitably small $\gamma > 0$; the master system $\mathcal{E}$ and the slave system $\mathcal{F}$ are synchronous. This concludes the proof.

Remark 3.1. *In contrast to the conventional Lyapunov functional, the Lyapunov functional developed in this paper is a two-sided looped functional, comprising two distinct components, $V_0(t)$ and $\sum_{n=1}^{4} V_n(t)$, and does not need to satisfy $\sum_{n=1}^{4} V_n(t) > 0$, so the qualification conditions are relaxed compared with the traditional functional. Meanwhile, the functional constructed in this paper is augmented with the double integral terms v_3 and v_4 in $\xi(t)$ compared to [32]. All of these measures contribute to reducing the conservativeness of the derived condition.*

To guarantee that the synchronization error system is absolutely stable as defined in Eq (2.3), a method for designing a sampled-data controller is proposed based on Theorem 3.1.

Theorem 3.2. *Given scalars $h > 0$, the synchronization error system* (2.3) *is absolutely stable, if there exist real matrices $W > 0, \tilde{Y}_\imath > 0$, $\tilde{Q}, \tilde{G}_\imath, \tilde{S}_\jmath, \tilde{X}$, ($\imath \in \mathbb{N}_2$, $\jmath \in \mathbb{N}_4$), $\tilde{M}_1, \tilde{M}_2$, and diagonal matrix $\Gamma > 0$, such that the following LMIs* (3.14) *and* (3.15) *are satisfied for $h_k \in (0, h]$,*

$$\tilde{\varphi}_a = \begin{bmatrix} \tilde{\varphi}_0 + h_k\tilde{\varphi}_1 & \sqrt{h_k}\pi_{8b}^T\tilde{M}_2 & \sqrt{h_k}\pi_0^T \\ * & -\tilde{R}_b & 0 \\ * & * & -\tilde{R}_1 \end{bmatrix} < 0, \tag{3.14}$$

$$\tilde{\varphi}_b = \begin{bmatrix} \tilde{\varphi}_0 + h_k\tilde{\varphi}_2 & \sqrt{h_k}\pi_{8a}^T\tilde{M}_1 & \sqrt{h_k}\pi_0^T \\ * & -\tilde{R}_a & 0 \\ * & * & -\tilde{R}_2 \end{bmatrix} < 0, \tag{3.15}$$

where

$$\begin{aligned} \tilde{\varphi}_0 &= He\{e_1^T P\tilde{\pi}_0 - e_{12}^T\Gamma e_{12} - e_{12}^T\Gamma\mathcal{G}\mathcal{D}We_1 + e_5^T\tilde{G}_1\pi_1 - e_4^T\tilde{G}_2\pi_2 \\ &\quad + -\tilde{S}_1e_4 - \tilde{S}_2e_5 - \tilde{S}_3e_6 - \tilde{S}_4e_7 + \pi_{8a}^T\tilde{M}_1\pi_9 + \pi_{8b}^T\tilde{M}_2\pi_{10}\}, \\ \tilde{\varphi}_1 &= He\{-e_1^T\tilde{G}_1\pi_1 + e_5^T\tilde{G}_1\pi_2 + \tilde{S}_1e_8 + \tilde{S}_3e_{10}\} - \pi_7^T\tilde{X}\pi_7, \\ \tilde{\varphi}_2 &= He\{e_1^T\tilde{G}_2\pi_1 + e_4^T\tilde{G}_2\pi_2 + \tilde{S}_2e_9 + \tilde{S}_4e_{11}\} + \pi_7^T\tilde{X}\pi_7, \\ \tilde{\pi}_0 &= \mathcal{A}We_1 + \mathcal{B}e_{12} - CVe_2, \\ \tilde{R}_a &= \mathrm{diag}\{W\tilde{R}_1^{-1}W, 3W\tilde{R}_1^{-1}W, 5W\tilde{R}_1^{-1}W\}, \quad \tilde{R}_b = \mathrm{diag}\{W\tilde{R}_2^{-1}W, 3W\tilde{R}_2^{-1}W, 5W\tilde{R}_2^{-1}W\}. \end{aligned}$$

Any other symbols not covered above are defined in accordance with Theorem 3.1. Furthermore, the controller gain is given by $\mathcal{K} = VW^{-1}$.

Proof. Define

$$\begin{aligned} W = P^{-1},\ \tilde{X} = J_3XJ_3,\ \tilde{G}_i = J_1G_iJ_6,\ \tilde{S}_j = \tilde{J}_{11}SJ_1,\ \tilde{N}_i = J_4N_iJ_3,\ \tilde{R}_i = Y_i^{-1}, \\ V = \mathcal{K}P^{-1},\ \tilde{J}_{11} = diag\{J_{11}, I\},\ \tilde{J}_{3a} = diag\{J_3, Y_1^{-1}\},\ \tilde{J}_{3b} = diag\{J_3, Y_2^{-1}\}, \end{aligned}$$

where, $i \in \mathbb{N}_2$, $j \in \mathbb{N}_r$ and

$$J_n = diag\underbrace{\{P^{-1}, \cdots, P^{-1}\}}_{n\ elements}.$$

Set $Q = 0$, then, pre- and post-multiplying (3.1) and (3.2) with $diag\{\tilde{J}_{11}, \tilde{J}_{3a}\}$ and $diag\{\tilde{J}_{11}, \tilde{J}_{3b}\}$, we obtain (3.14) and (3.15). This concludes the proof.

It is evident that inequalities (3.14) and (3.15) contain two nonlinear terms, $W\tilde{R}_2^{-1}W$ and $W\tilde{R}_1^{-1}W$. This makes it impossible to directly solve the controller of Theorem 3.2 through standard solvers. Therefore, the cone complementarity linearization iterative algorithm proposed in Reference [36] needs to be applied to handle this non-convex problem. The specific steps are as follows:

Define two new variables Φ_1, Φ_1 such that $\Phi_1 \leq W\tilde{R}_1^{-1}W$, $\Phi_2 \leq W\tilde{R}_2^{-1}W$. Replace the conditions (3.14) and (3.15) with

$$\tilde{\varphi}_a = \begin{bmatrix} \tilde{\varphi}_0 + h_k\tilde{\varphi}_1 & \sqrt{h_k}\pi_{8b}^T\tilde{M}_2 & \sqrt{h_k}\tilde{\pi}_1^T \\ * & -\tilde{\Phi}_b & 0 \\ * & * & -\tilde{R}_1 \end{bmatrix} < 0, \tag{3.16}$$

$$\tilde{\varphi}_b = \begin{bmatrix} \tilde{\varphi}_0 + h_k\tilde{\varphi}_2 & \sqrt{h_k}\pi_{8a}^T\tilde{M}_1 & \sqrt{h_k}\tilde{\pi}_1^T \\ * & -\tilde{\Phi}_a & 0 \\ * & * & -\tilde{R}_2 \end{bmatrix} < 0, \tag{3.17}$$

where $\tilde{\Phi}_a = diag\{\Phi_1, 3\Phi_1, 5\Phi_1\}$, $\tilde{\Phi}_b = diag\{\Phi_2, 3\Phi_2, 5\Phi_2\}$ and,

$$\Phi_i \leq W\tilde{R}_i^{-1}W,\ i \in \mathbb{N}_2. \tag{3.18}$$

Notice that (3.18) is equal to $\Phi_i^{-1} - W^{-1}\tilde{R}_iW^{-1} \geq 0$. Using the Schur complement, this condition is equivalent to

$$\begin{bmatrix} \Phi_i^{-1} & W^{-1} \\ W^{-1} & \tilde{R}_i^{-1} \end{bmatrix} \geq 0, \tag{3.19}$$

then, by introducing the new variable $P, H_i, Y_i, i \in \mathbb{N}_2$, the original conditions (3.14) and (3.15) can be reformulated as (3.16), (3.17), and

$$\begin{bmatrix} H_i & P \\ P & Y_i \end{bmatrix} \geq 0,\ P = W^{-1},\ H_i = \Phi_i^{-1},\ Y_i = \tilde{R}_i^{-1}. \tag{3.20}$$

Therefore, the aforementioned non-convex problem is reformulated as a nonlinear minimization problem based on Linear Matrix Inequalities (LMIs) as follows:

$$\begin{aligned} &Minimizetr\left\{PW + \textstyle\sum_{i=1}^{2}(H_i\Phi_i + Y_iR_i)\right\} \\ &s.t.\ (3.16),\ (3.17)\ and \end{aligned}$$

$$\begin{bmatrix} H_i & P \\ P & Y_i \end{bmatrix} \geq 0,\ \begin{bmatrix} W & I \\ I & P \end{bmatrix} \geq 0,\ \begin{bmatrix} \Phi_i & I \\ I & H_i \end{bmatrix} \geq 0,\ \begin{bmatrix} \tilde{R}_i & I \\ I & Y_i \end{bmatrix} \geq 0. \tag{3.21}$$

In the following text, we will introduce an iterative algorithm for solving the controller matrix with the maximization of h.

Step 1. First, choose a sufficiently small initial value h such that the LMIs in Eqs (3.16), (3.17), and (3.21) are satisfied, and then we set $h_{max} = h$.

Step 2. Find a feasible set $P_0, W_0, H_{10}, H_{20}, \Phi_{10}, \Phi_{20}, Y_{10}, Y_{20}, \tilde{R}_{10}, \tilde{R}_{20}$ satisfying (3.16), (3.17), and (3.21). And set $j = 0$.

Step 3. Solve the following LMI problem:

$$\begin{aligned}
&Minimizetr\left\{PW_j + P_jW + \sum_{i=1}^{2}(H_{ij}\Phi_i + H_i\Phi_{ij} + Y_{ij}R_i + Y_i\tilde{R}_{ij})\right\},\\
&s.t.\ (3.16),\ (3.17)\ and\ (3.21).\\
&set\ P_{j+1} = W^{-1}, W_{j+1} = W, \Phi_{i(j+1)} = \Phi_i, H_{i(j+1)} = \Phi_i^{-1}, \tilde{R}_{i(j+1)} = \tilde{R}_i, Y_{i(j+1)} = \tilde{R}_i^{-1}, i \in \mathbb{N}_2.
\end{aligned}$$

Step 4. If the LMIs (3.1) and (3.2) are satisfied with the controller gain $\mathcal{K}$ obtained in Step 3, then update h_{max} to h and revert to Step 2 after increasing h_2 to a certain degree. If LMIs (3.1) and (3.2) are unsolvable within a set number of iterations, then exit the procedure. Otherwise, set $j = j + 1$ and go back to Step 3.

Remark 3.2. *In contrast to the parameter adjustment methods for controller optimization described in references [27, 34], the enhanced cone complementarity linearization iteration iterative algorithm systematically identifies the optimal solution within the feasible region through a step-by-step iterative process. This method not only improves the precision of synchronous control but also effectively minimizes potential errors that may arise during parameter adjustment.*

4. Numerical example

This section presents a benchmark example based on reference [34], aimed at illustrating the benefits of the proposed standard.

Consider the CLS defined by the following parameters.

$$\mathcal{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \mathcal{B} = \begin{bmatrix} 1.2 & -1.6 & 0 \\ 1.24 & 1 & 0.9 \\ 0 & 2.2 & 1.5 \end{bmatrix}, C = \mathcal{D} = I.$$

with nonlinear characteristics

$$f_i(x_i(t)) = \frac{1}{2}(|x_i(t) + 1| - |x_i(t) - 1|),$$

where $f_i(x_i(t))$ belongs to sector [0,1] for $i \in \mathbb{N}_3$.

The systems mentioned above have all been examined in Reference [34]. Table 1 presents the maximum sampling periods that were obtained. Utilizing the sampling periods and in accordance with Theorem 3.2, the controller obtained through continuous iterative calculation is as follows:

$$\mathcal{K} = \begin{bmatrix} 1.1254 & -1.0942 & 0.1434 \\ 0.4607 & 0.9565 & 0.6237 \\ 0.3170 & 1.7540 & 1.3261 \end{bmatrix}.$$

By substituting the obtained controller into Theorem 3.1, the maximum sampling period h for this study is determined to be 0.77. As shown in Table 1, the results of this study are superior to those in other literature, demonstrating that our approach exhibits less conservativeness.

Table 1. Maximum values of the upper bound h.

Methods	[25]	[22]	[34]	Theorem 3.1
h	0.32	0.38	0.71	0.77

Then, set $x(0) = [0.3\ 0.5\ 0.8]^T$, $y(0) = [0.2\ 0.4\ 0.9]^T$, Figure 1 shows the state response of the system without the controller in use. Applying the above controller $\mathcal{K}$, the state trajectories of system (2.3) and its control input are illustrated in Figures 2 and 3 under the sampling period $h = 0.77$. It can be seen that the system is unstable at this time. As shown in Figures 2 and 3, CLS achieved synchronization in a short period of time, and the system was in a stable state at this point.

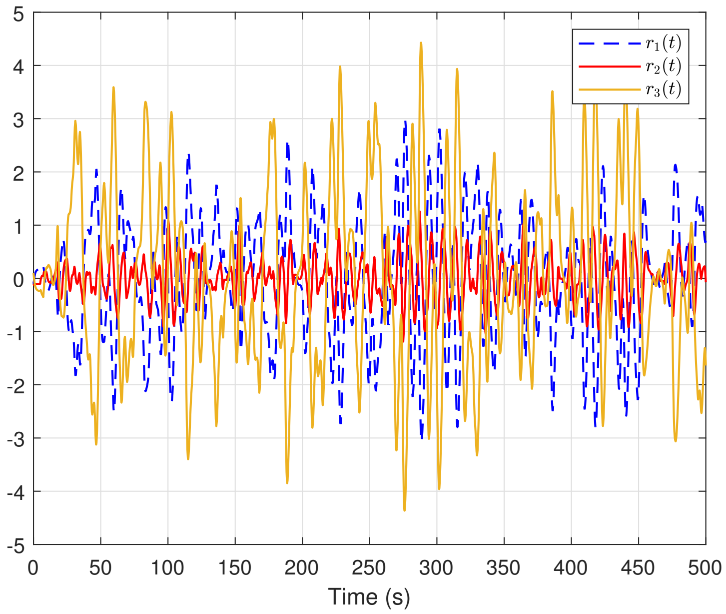

Figure 1. State response of the system with no controller.

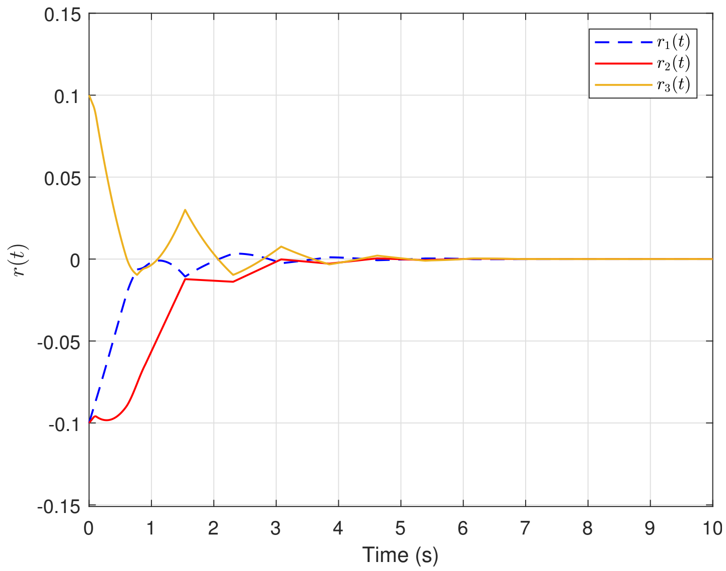

Figure 2. State response of the system.

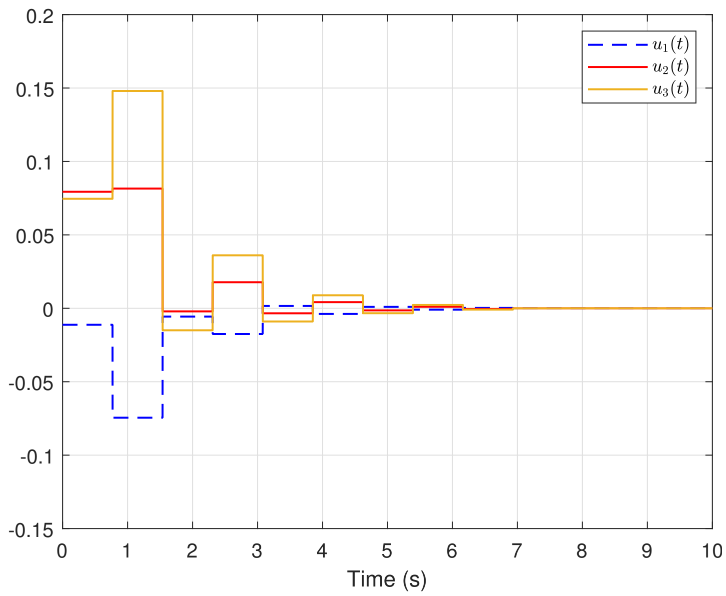

Figure 3. Control input u(t).

5. Conclusions

This paper focuses on examining CLS's synchronization issue. This study thoroughly considers the system's sampling process characteristics and constructs an improved augmented two-sided looped Lyapunov functional to derive the stability criterion. Subsequently, based on the derived conditions, the cone complementary linearization iteration algorithm is employed to design the sampling controller. Numerical simulation results demonstrate the effectiveness and superiority of the proposed approach.

Author contributions

Xin-Yu Li: writing-original draft, software, methodology, investigation. Wei Wang: Writing-review and editing, formal analysis, validation, conceptualization, supervision, funding acquisition. Jin-Ming Liang: Writing-review and editing.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence tools in the creation of this article.

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Data availability statement

The authors confirm that the data supporting the findings of this study are available within the article.

Conflict of interest

The authors declare no conflict of interest.

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