



Research article

A proposed non-parametric triple generally weighted moving average sign chart

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Abstract: In practical applications of statistical process control (SPC), the distribution in which the sample data follow often remains unknown. Non-parametric control charts are necessary in process monitoring in such cases. A triple generally weighted moving average (TGWMA) sign chart is proposed in this study for monitoring the process when the underlying distribution is unknown. Simulation is used to compare the performance of the TGWMA sign chart with the existing double generally weighted moving average (DGWMA) sign chart, for both steady state (SS) and zero state (ZS) cases. From the comparison, the TGWMA sign chart shows superior sensitivity in identifying small shifts in the process proportion for both ZS and SS cases. Lastly, we demonstrate the application of the TGWMA sign chart through a practical example and compare it to the DGWMA sign chart in detecting process shifts, further showing the effectiveness of using the TGWMA sign chart.

Keywords: average run length; average sign chart; non-parametric control charts; steady state; triple generally weighted moving; zero state

Mathematics Subject Classification: 62P30, 90B25

1. Introduction

In order to satisfy or surpass consumers' expectations, it is imperative for the process performance to be consistent. SPC is a useful approach for improving product quality and enhancing process stability through the reduction of variability. The control chart is the most practical tool in SPC and is frequently employed in process monitoring. Many new types of control charts have been developed since the inception of the Shewhart chart. The cumulative sum (CUSUM), exponentially weighted moving average (EWMA), and generally weighted moving average (GWMA) charts were developed to facilitate the rapid detection of small process shifts as the Shewhart chart is inefficient in this regard. Most of the above-mentioned charts assume that the samples being observed are normally distributed. However, in actual process monitoring situations, the normality assumption is usually violated and practitioners do not have any knowledge about the distribution of the underlying process being monitored. An effective approach to address the aforementioned issue is the use of non-parametric control charts.

In this paragraph, some non-parametric charts are evaluated. Amin et al. [1] developed non-parametric Shewhart and CUSUM charts to identify fluctuations in the mean or standard deviation of a process by utilizing sign-test statistics computed for each sample. Chakraborti and Eryilmaz [2] utilized the Wilcoxon signed rank statistic and several runs type constraints to develop Shewhart-type non-parametric charts for known in-control median of a continuous distribution. Zhou et al. [3] developed a non-parametric chart that employs the Mann-Whitney statistic to identify changes in the mean, which has been modified for repetitive sequential use. In detecting an increase in process variation, Khilare and Shirke [4] proposed synthetic charts that were non-parametric and side-sensitive for fraction nonconforming. Tapang et al. [5] expanded the application of non-parametric charts in real-world contexts by applying ranked set sampling techniques to several well-known non-parametric tests, while exploring alternative sampling methods. Li et al. [6] utilized a run test to extend non-parametric monitoring in a multivariate context, adapting it to multiple quality characteristics simultaneously. The efficiency of monitoring complex processes was enhanced by Boroomandi and Kharrati-Kopaei [7], who devised four non-parametric variance estimators and three control charts based on partially rank ordered set (PROS) sampling. This further advanced innovative sampling approaches. Lastly, Malela-Majika [8] introduced a precedence chart that employed repetitive sampling, thereby demonstrating an application-oriented perspective on the design of non-parametric control charts and offering practical insights for their implementation.

The EWMA method is essential in the development of non-parametric control charts. The accuracy and efficiency in process monitoring were improved by Malela-Majika et al. [9], who introduced an EWMA chart that applies the Wilcoxon rank-sum statistic and recurrent sampling technique. Tang et al. [10] proposed a non-parametric adaptive EWMA chart that is based on the sign statistic, which offers improved flexibility in adapting to process changes over time. Alevizakos et al. [11] advanced the use of the EWMA chart by proposing a triple EWMA chart that tracks the location parameter of an unknown continuous distribution using the sign statistic. Furthermore, an EWMA sign chart with dynamic probability control limits that can accommodate both fixed and variable sample sizes was presented by Haq [12].

The EWMA approach has been significantly improved by researchers in recent years to handle real-world process monitoring issues. Ng et al. [13] suggested an EWMA t chart with variable sampling intervals and auxiliary information that reduces the impact of estimation error. By developing a non-parametric adaptive EWMA chart for monitoring multivariate time-between-events and amplitude data, Xue et al. [14] extended the EWMA approach and showed its increasing usefulness in intricate

processes. Sheu and Lin [15] showed that the EWMA chart may be viewed as a particular instance of the GWMA chart when the latter employs a parameter value of $\gamma = 1$. Likewise, the double EWMA (DEWMA) and triple EWMA (TEWMA) charts are special cases of the DGWMA and TGWMA charts when the last two charts adopt the parameter values $\alpha = \beta = 1$ and $\alpha = \beta = \gamma = 1$, respectively [16]. Details about the adjustable parameters α , β , and γ will be explained in Sections 2 and 3. The GWMA chart and its extended counterparts enhance the sensitivity of the EWMA charts in detecting small shifts as a result of having more adjustable parameters.

Several existing GWMA-type charts have been reviewed in the literature, each of which has contributed to the development of non-parametric methods. Lu [17] developed a non-parametric GWMA sign chart for an enhanced detection of small process shifts. Chakraborty et al. [18] extended this concept by proposing a non-parametric GWMA chart that employs the Wilcoxon signed-rank statistic, thereby providing increased sensitivity in detecting process shifts. A non-parametric DGWMA sign chart was introduced by Lu [19] which further enhanced the GWMA sign chart's ability to detect small shifts. In addition, Alevizakos et al. [20] provided a non-parametric DGWMA chart that utilizes the signed-rank statistic to monitor location parameters, thereby presenting an alternative method in process monitoring. Mabude et al. [21] further contributed to the field by developing a non-parametric Phase-II composite Shewhart-GWMA chart that employs the Mann-Whitney statistic. In the interim, Godase and Mahadik [22] introduced non-parametric combined Shewhart-CUSUM charts that incorporate the sign statistic to enhance process monitoring. In addition, Mabude et al. [23] developed two non-parametric mixed charts by combining the GWMA and CUSUM charts, which are useful in real-world applications.

Although the current EWMA type, GWMA type, and their hybrid charts have made substantial progress in detecting small shifts, these charts continue to encounter problems in detecting extremely small and progressively changing process shifts. The high complexity of current hybrid charts, such as the GWMA-CUSUM chart, can result in increased computational costs. Additionally, there is a notable lack of research in the SS performance, as the majority of current non-parametric charts concentrate significantly in the ZS performance. The double moving weighting mechanism of the DGWMA sign chart renders it exceptionally appropriate for monitoring intricate, noisy industrial processes with minimal shift amplitudes, showcasing distinct value and potential for application. Furthermore, there is no "triple" type chart for non-parametric monitoring in the existing literature besides the proposed TGWMA sign chart. The TGWMA sign chart retains the advantages of the DGWMA sign chart, in terms of noise smoothing, dynamic balancing of false alarm rates and sensitivity, flexible parameter adjustment, robustness towards small sample monitoring, and progressive stability for long-term monitoring. These advantages are challenging to be replicated by other existing non-parametric control charts. The proposed TGWMA sign chart in this study also offers enhanced monitoring capabilities, particularly for very small shifts. In addition to the ZS performance, this paper focuses on the SS performance to align more closely with real-life production processes.

The remaining sections of this paper are organized as follows: In Section 2, an overview of the DGWMA sign chart is given. Section 3 presents the proposed TGWMA sign chart, while Section 4 evaluates the developed TGWMA sign chart by comparing its performance with that of the existing DGWMA sign chart. An example of an application for the TGWMA sign chart is given in Section 5. Conclusions are drawn and suggestions for further research are given in Section 6.

2. An overview of the DGWMA sign chart

Lu [19] developed the DGWMA sign chart for monitoring small process shifts when the

distribution of the quality characteristic of the process is unknown. Let X denote a quality characteristic from a continuous distribution with a known value of the target median μ_0 . In addition, let

$Y = X - \mu_0$ and the process proportion $p = P(Y > 0)$. Note that if

$$\begin{cases} p = 0.5, & \text{the process is in-control} \\ p \neq 0.5, & \text{the process is out-of-control} \end{cases}$$

Suppose that a sample of size n is taken at time t . Then, $Y_{it} = X_{it} - \mu_0$ for $i = 1, 2, \dots, n$. Let

$$I_{it} = \begin{cases} 1, & \text{if } Y_{it} > 0 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

and $S_t = \sum_{i=1}^n I_{it}$. S_t follows the binomial $\text{Bin}(n, 0.5)$ distribution when the process is in-control. The GWMA sign chart's statistic is (Lu [19])

$$G_t = \sum_{j=1}^t P(H_1 = j)S_{t-j+1} + P(H_1 > t)G_0. \quad (2)$$

In Eq (2), $P(H_1 = 1)$, $P(H_1 = 2)$, ..., $P(H_1 = t)$ are used to weight S_t , S_{t-1} , ..., S_1 , respectively, and $P(H_1 > t)$ is used to weight the starting value of G_t , i.e., G_0 . Note that $G_0 = n/2$ because $E(G_t) = E(S_t) = n/2$ when the process is in-control (Lu [19]). In Eq (2), $P(H_1 = j) = q_1^{(j-1)\alpha} - q_1^{j\alpha}$ and $P(H_1 > t) = q_1^{t\alpha}$ for $j = 1, 2, \dots, t$, where q_1 is a constant satisfying $0 \leq q_1 < 1$ and α is an adjustable parameter.

To obtain the DGWMA sign chart's statistic, i.e., DG_t , we weight G_t in Eq (2) in the same way that we weight S_t , hence, giving

$$\begin{aligned} DG_t &= P(H_2 = 1)G_t + P(H_2 = 2)G_{t-1} + \dots + P(H_2 = t)G_1 + P(H_2 > t)G_0 \\ &= P(H_2 = 1)(P(H_1 = 1)S_t + P(H_1 = 2)S_{t-1} + \dots + P(H_1 = t)S_1 + P(H_1 > t)G_0) \\ &\quad + P(H_2 = 2)(P(H_1 = 1)S_{t-1} + P(H_1 = 2)S_{t-2} + \dots + P(H_1 = t-1)S_1 + P(H_1 > t-1)G_0) \\ &\quad + \dots + P(H_2 = t)(P(H_1 = 1)S_1 + P(H_1 > 1)G_0) + P(H_2 > t)G_0 \\ &= M_1S_t + M_2S_{t-1} + \dots + M_tS_1 + (1 - \sum_{i=1}^t M_i)G_0 \\ &= \sum_{i=1}^t M_iS_{t-i+1} + (1 - \sum_{i=1}^t M_i)G_0, \end{aligned} \quad (3)$$

where

$$\begin{aligned} M_t &= \sum_{i=1}^t P(H_1 = i)P(H_2 = t - i + 1) \\ &= \sum_{i=1}^t (q_1^{(i-1)\alpha} - q_1^{i\alpha})(q_2^{(t-i)\beta} - q_2^{(t-i+1)\beta}). \end{aligned} \quad (4)$$

In Eq (4), α and β are adjustable constants and $0 \leq q_k < 1$ (for $k = 1, 2$). According to Lu [19], the expectation and variance of the DGWMA sign chart's statistic, DG_t , when the process is in-control are given in Eqs (5) and (6), respectively.

$$\begin{aligned} E(DG_t) &= E\left(M_1S_t + M_2S_{t-1} + \dots + M_tS_1 + \left(1 - \sum_{i=1}^t M_i\right)G_0\right) \\ &= E(M_1S_t + M_2S_{t-1} + \dots + M_tS_1) + E\left((1 - \sum_{i=1}^t M_i)G_0\right) \end{aligned}$$

$$= \frac{n}{2} \sum_{i=1}^t M_i + \frac{n}{2} (1 - \sum_{i=1}^t M_i) = n/2 \quad (5)$$

and

$$\begin{aligned} \text{Var}(DG_t) &= \text{Var}\left(M_1 S_t + M_2 S_{t-1} + \cdots + M_t S_1 + \left(1 - \sum_{i=1}^t M_i\right) G_0\right) \\ &= \text{Var}(M_1 S_t + M_2 S_{t-1} + \cdots + M_t S_1) + \text{Var}\left(\left(1 - \sum_{i=1}^t M_i\right) G_0\right) \\ &= \frac{n}{4} \sum_{i=1}^t M_i^2. \end{aligned} \quad (6)$$

Consequently, the time-varying limits of the DGWMA sign chart are

$$UCL = n/2 + L_{DG} \sqrt{\frac{n}{4} \sum_{i=1}^t M_i^2} \quad (7a)$$

$$CL = n/2 \quad (7b)$$

and

$$LCL = n/2 - L_{DG} \sqrt{\frac{n}{4} \sum_{i=1}^t M_i^2}. \quad (7c)$$

The asymptotic variance of the statistic DG_t is obtained as

$$\lim_{t \rightarrow \infty} \text{Var}(DG_t) = \frac{n}{4} \sum_{t=1}^{\infty} \left(\sum_{i=1}^t \left(q_1^{(i-1)\alpha} - q_1^{i\alpha} \right) \left(q_2^{(t-i)\beta} - q_2^{(t-i+1)\beta} \right) \right)^2, \quad (8)$$

where α and β are adjustable constants and $0 \leq q_k < 1$ (for $k = 1, 2$). Let

$$R_t = \sum_{t=1}^{\infty} \left(\sum_{i=1}^t \left(q_1^{(i-1)\alpha} - q_1^{i\alpha} \right) \left(q_2^{(t-i)\beta} - q_2^{(t-i+1)\beta} \right) \right)^2. \quad (9)$$

Then, the asymptotic limits of the DGWMA sign chart are

$$UCL = n/2 + L_{DG} \sqrt{\frac{nR_t}{4}} \quad (10a)$$

$$CL = n/2 \quad (10b)$$

and

$$LCL = n/2 - L_{DG} \sqrt{\frac{nR_t}{4}}. \quad (10c)$$

where L_{DG} is the width constant of the DGWMA sign chart's limits.

3. A proposed TGWMA sign chart

To obtain the TGWMA sign chart's statistic, i.e., TG_t , we weight DG_t in Eq (3) in the same way that we weight G_t or S_t . Thus,

$$\begin{aligned}
TG_t &= P(H_3 = 1)DG_t + P(H_3 = 2)DG_{t-1} + \cdots + P(H_3 = t)DG_1 + P(H_3 > t)G_0 \\
&= P(H_3 = 1)(M_1S_t + M_2S_{t-1} + \cdots + M_tS_1 + (1 - \sum_{i=1}^t M_i)G_0) \\
&\quad + P(H_3 = 2)(M_1S_{t-1} + M_2S_{t-2} + \cdots + M_{t-1}S_1 + (1 - \sum_{i=1}^{t-1} M_i)G_0) + \cdots \\
&\quad + P(H_3 = t)(M_1S_1 + (1 - M_1)G_0) + P(H_3 > t)G_0 \\
&= P(H_3 = 1)M_1S_t + [P(H_3 = 1)M_2 + P(H_3 = 2)M_1]S_{t-1} + \cdots \\
&\quad + [P(H_3 = 1)M_t + P(H_3 = 2)M_{t-1} + \cdots + P(H_3 = t)M_1]S_1 \\
&\quad + P(H_3 = 1)(1 - \sum_{i=1}^t M_i)G_0 + P(H_3 = 2)(1 - \sum_{i=1}^{t-1} M_i)G_0 + \cdots + P(H_3 = t)(1 - M_1)G_0 + P(H_3 > t)G_0 \\
&= N_1S_t + N_2S_{t-1} + \cdots + N_tS_1 + Z,
\end{aligned} \tag{11}$$

where

$$\begin{aligned}
N_t &= \sum_{j=1}^t P(H_3 = j)M_{t-j+1} \\
&= \sum_{j=1}^t \left(q_3^{(j-1)^Y} - q_3^{j^Y} \right) M_{t-j+1}
\end{aligned} \tag{12}$$

and

$$\begin{aligned}
Z &= P(H_3 = 1)(1 - \sum_{i=1}^t M_i)G_0 + P(H_3 = 2)(1 - \sum_{i=1}^{t-1} M_i)G_0 \\
&\quad + \cdots + P(H_3 = t)(1 - M_1)G_0 + P(H_3 > t)G_0 \\
&= (P(H_3 = 1) + P(H_3 = 2) + \cdots + P(H_3 = t) + P(H_3 > t))G_0 \\
&\quad - (P(H_3 = 1)\sum_{i=1}^t M_i + P(H_3 = 2)\sum_{i=1}^{t-1} M_i + \cdots + P(H_3 = t)M_1)G_0 \\
&= G_0 - (P(H_3 = 1)(M_1 + M_2 + \cdots + M_t) + P(H_3 = 2)(M_1 + M_2 + \cdots + M_{t-1}) \\
&\quad + \cdots + P(H_3 = t)M_1)G_0 \\
&= G_0 - (P(H_3 = 1)M_1 + P(H_3 = 1)M_2 + P(H_3 = 2)M_1 \\
&\quad + \cdots + (P(H_3 = 1)M_t + P(H_3 = 2)M_{t-1} + \cdots + P(H_3 = t)M_1))G_0 \\
&= G_0 - (N_1 + N_2 + \cdots + N_t)G_0 \\
&= \left(1 - \sum_{j=1}^t N_j \right) G_0.
\end{aligned} \tag{13}$$

From Eqs (11) and (13),

$$TG_t = \sum_{j=1}^t N_j S_{t-j+1} + \left(1 - \sum_{j=1}^t N_j \right) G_0. \tag{14}$$

In Eqs (11)–(13), $P(H_3 = j)$ and $P(H_3 > t)$ for $j = 1, 2, \dots, t$ are defined below Eq (2). The expectation and variance of the TGWMA sign chart's statistic, TG_t , when the process is in-control are given in Eqs (15) and (16), respectively.

$$\begin{aligned}
E(TG_t) &= E\left(\sum_{j=1}^t N_j S_{t-j+1} + \left(1 - \sum_{j=1}^t N_j\right) G_0\right) \\
&= E\left(\sum_{j=1}^t N_j S_{t-j+1}\right) + E\left(\left(1 - \sum_{j=1}^t N_j\right) G_0\right) \\
&= \frac{n}{2} \sum_{j=1}^t N_j + \frac{n}{2} \left(1 - \sum_{j=1}^t N_j\right) \\
&= n/2
\end{aligned} \tag{15}$$

and

$$\begin{aligned}
Var(TG_t) &= Var\left(\sum_{j=1}^t N_j S_{t-j+1} + \left(1 - \sum_{j=1}^t N_j\right) G_0\right) \\
&= Var\left(\sum_{j=1}^t N_j S_{t-j+1}\right) + Var\left(\left(1 - \sum_{j=1}^t N_j\right) G_0\right) \\
&= \sum_{j=1}^t N_j^2 \cdot Var(S_{t-j+1}) \\
&= \frac{n}{4} \sum_{j=1}^t N_j^2.
\end{aligned} \tag{16}$$

The time-varying limits of the TGWMA sign chart are

$$UCL = n/2 + L_{TG} \sqrt{\frac{n}{4} \sum_{j=1}^t N_j^2} \tag{17a}$$

$$CL = n/2 \tag{17b}$$

and

$$LCL = n/2 - L_{TG} \sqrt{\frac{n}{4} \sum_{j=1}^t N_j^2}, \tag{17c}$$

where L_{TG} is the width constant of the TGWMA sign chart's limits.

The asymptotic variance of the statistic TG_t is obtained as

$$\lim_{t \rightarrow \infty} Var(TG_t) = \frac{n}{4} \sum_{t=1}^{\infty} \left(\sum_{j=1}^t \left(q_3^{(j-1)\gamma} - q_3^{j\gamma} \right) M_{t-j+1} \right)^2, \tag{18}$$

where γ is an adjustable constant and $0 \leq q_3 < 1$. Let

$$R_2 = \sum_{t=1}^{\infty} \left(\sum_{j=1}^t \left(q_3^{(j-1)\gamma} - q_3^{j\gamma} \right) M_{t-j+1} \right)^2. \tag{19}$$

Consequently, the asymptotic limits of the TGWMA sign chart are

$$UCL = n/2 + L_{TG} \sqrt{\frac{nR_2}{4}} \quad (20a)$$

$$CL = n/2 \quad (20b)$$

and

$$LCL = n/2 - L_{TG} \sqrt{\frac{nR_2}{4}}. \quad (20c)$$

4. Performance evaluation

This study will exclusively compare the proposed TGWMA sign chart with the DGWMA sign chart, as the latter has already been evaluated against other high-performing non-parametric control charts, demonstrating superior monitoring capabilities. Due to the presence of the six parameters ($q_1, q_2, q_3, \alpha, \beta, \gamma$), the design of the TGWMA sign chart appears somewhat complex. To facilitate the design of this chart, $q_1 = q_2 = q_3 = q$ and $\alpha = \beta = \gamma$ are considered. Values of the sample size $n \in \{5, 10, 15\}$, process proportion $p \in \{0.1, 0.4, 0.6, 0.9\}$, constant $q \in \{0.5, 0.7, 0.9\}$, and adjustable constant $\alpha \in \{0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.2, 1.5\}$ are considered in the analyses for the ZS and SS conditions, respectively. To further show the effectiveness of the proposed TGWMA sign chart in detecting small shifts in the process proportion, additional values of p in the neighborhood of 0.5, i.e., $p \in \{0.44, 0.48, 0.52, 0.56\}$, are considered for the ZS and SS conditions. The asymptotic limits of the two charts are adopted. The ARL values for the ZS and SS based TGWMA sign and DGWMA sign charts are computed for various shift sizes in the process proportion by using the MATLAB software for simulation. Each ARL value is computed based on 10000 simulation trials. The ARL measures the average number of samples required until a chart signals an out-of-control. The ARL is denoted as ARL_0 when the process is in-control and ARL_1 when the process is out-of-control. The ZS condition assumes that the process shift occurs at the beginning of process monitoring. On the contrary, under the SS condition, the process does not start as out-of-control, but rather it runs for a certain time before becoming out-of-control. Hence, the SS condition is more consistent with an actual production process.

As $ARL_0 = 370$ is adopted for the ZS and SS conditions, the values of the limit constants, L_{DG} (Eqs (10a) and (10c)) and L_{TG} (Eqs (20a) and (20c)) of the DGWMA and TGWMA sign charts, respectively, are computed using the binary search method in MATLAB, so that the said ARL_0 value is attained. The ARL_1 performance of the proposed TGWMA sign chart will be compared with that of the existing DGWMA sign chart. For the ZS condition, the following step-by-step procedure explains the computation of the ARL_1 value of the proposed TGWMA sign chart by using the MATLAB software for simulation.

Step 1. Compute the value of R_2 using Eq (19) based on $t = 500$ by considering practical implications.

Step 2. Generate the dataset using the `binornd` function. Then, use the binary search method to obtain the value of L_{TG} ($0 < L_{TG} < 3$) in Eqs (20a) and (20c) for the various (n, q, α) combinations to attain an ARL_0 value as close as possible to the desired value of 370. Note that $p = 0.5$ is adopted here as the process proportion is assumed to be in-control in computing L_{TG} , followed by UCL and LCL.

Step 3. Next, compute the ARL_1 values for the out-of-control process proportions ($p \neq 0.5$) for the various (n, q, α) combinations.

The above-mentioned procedure can also be used to compute the ZS ARL of the DGWMA sign

chart. Moreover, the above-mentioned procedure can be used in computing the SS ARLs of the TGWMA and DGWMA sign charts, but by assuming that the first M samples of the process are in-control and the process becomes out-of-control from the $(M + 1)$ th sample onwards, where $M = 100$ is adopted in the analyses. In the SS condition, the run length of a chart until an out-of-control signal is detected is counted starting from the $(M + 1)$ th sample, meaning the first M samples are not included.

Table 1 provides the values of R_2 of the TGWMA sign chart computed using Eq (19), while Tables 2 and 3 provide the values of the width constant, L_{TG} , of the said chart for the ZS and SS conditions, respectively, for various combinations of (n, q, α) considered in the analyses. Note that for any (q, α) pair, the value of R_2 remains the same, irrespective of the sample size n and ZS or SS condition. In Table 2, it is seen that for the ZS condition, the L_{TG} value lies between 1 and 3 for the TGWMA sign chart to attain $ARL_0 = 370$. However, for the SS condition in Table 3, the L_{TG} values when $q = 0.9$ and $\alpha < 1.5$ are less than unity. In addition, it is observed that for the same combination of (n, q, α) , the L_{TG} value of the TGWMA sign chart under the ZS condition (Table 2) is greater than that under the SS condition (Table 3). For example, when $(n, q, \alpha) = (5, 0.5, 0.5)$, the L_{TG} values are 2.747 and 2.091 in Tables 2 and 3, respectively, where the former is larger than the latter.

Table 1. R_2 values of the TGWMA sign chart for various (q, α) combinations.

α	0.5	0.6	0.7	0.8	0.9	1	1.2	1.5
R_2 ($q = 0.5$)	0.0525	0.068	0.0846	0.1017	0.1189	0.1358	0.1679	0.2102
R_2 ($q = 0.7$)	0.0127	0.0208	0.0308	0.0421	0.0545	0.0676	0.0949	0.1359
R_2 ($q = 0.9$)	0.001	0.0027	0.0053	0.0090	0.0139	0.0198	0.0343	0.0610

Table 2. L_{TG} values of the TGWMA sign chart under the ZS condition, for various (n, q, α) combinations, based on $ARL_0 = 370$.

q	α	n		
		5	10	15
0.5	0.5	2.747	2.763	2.765
	0.6	2.736	2.742	2.750
	0.7	2.725	2.739	2.746
	0.8	2.725	2.742	2.746
	0.9	2.731	2.75	2.753
	1.0	2.737	2.757	2.762
	1.2	2.750	2.779	2.786
	1.5	2.772	2.814	2.822
0.7	0.5	2.293	2.291	2.286
	0.6	2.326	2.323	2.323
	0.7	2.369	2.372	2.371
	0.8	2.418	2.422	2.423
	0.9	2.467	2.47	2.473
	1.0	2.511	2.519	2.521
	1.2	2.588	2.601	2.602
	1.5	2.673	2.695	2.699
0.9	0.5	1.088	1.085	1.089
	0.6	1.265	1.260	1.264
	0.7	1.491	1.489	1.492

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q	α	n		
		5	10	15
0.9	0.8	1.689	1.686	1.687
	0.9	1.858	1.852	1.850
	1.0	1.993	1.986	1.986
	1.2	2.198	2.199	2.200
	1.5	2.407	2.412	2.415

Table 3. L_{TG} values of the TGWMA sign chart under the SS condition, for various (n, q, α) combinations, based on $ARL_0 = 370$.

q	α	n		
		5	10	15
0.5	0.5	2.091	2.132	2.149
	0.6	2.014	2.047	2.053
	0.7	1.967	1.986	1.995
	0.8	1.935	1.959	1.961
	0.9	1.927	1.939	1.946
	1.0	1.926	1.942	1.943
	1.2	1.946	1.960	1.962
	1.5	2.003	2.023	2.025
0.7	0.5	1.305	1.365	1.368
	0.6	1.363	1.295	1.295
	0.7	1.291	1.274	1.275
	0.8	1.271	1.281	1.285
	0.9	1.279	1.306	1.308
	1.0	1.591	1.343	1.343
	1.2	1.433	1.438	1.436
	1.5	1.342	1.594	1.598
0.9	0.5	0.475	0.474	0.473
	0.6	0.450	0.452	0.453
	0.7	0.493	0.492	0.490
	0.8	0.546	0.547	0.547
	0.9	0.605	0.608	0.607
	1.0	0.673	0.671	0.671
	1.2	0.810	0.810	0.810
	1.5	1.027	1.028	1.027

Table 4 gives the computed ARL_1 values for the DGWMA and TGWMA sign charts for various shift sizes in the process proportion p under the ZS condition. In Table 4, it is seen that when the shift in the process proportion is large, such as $p = 0.1$ or 0.9 , the ARL_1 of the TGWMA sign chart is larger than that of the DGWMA sign chart, hence the latter detects a large shift quicker than the former. However, for smaller shifts, such as $p = 0.4$ or 0.6 , the superiority of the TGWMA sign chart to the DGWMA sign chart begins to show up, especially for smaller values of q , such as $q = 0.5$ and 0.7 . When $q = 0.5$ and $p \in \{0.4, 0.6\}$, regardless of the values of the sample size (n) and $\alpha \in \{0.5, 0.6, \dots, 1.0, 1.2, 1.5\}$, all the ARL_1 s of the TGWMA sign chart are smaller than that of the DGWMA sign chart, indicating that the former is quicker in detecting shifts. For example, in Table 4 when $n = 5$, $q = 0.5$, $p = 0.6$, and $\alpha \in \{0.5, 0.6, \dots, 1.5\}$, $ARL_1 \in \{33.977, 36.291, \dots, 70.892\}$ and $\{40.935, 43.636, \dots$

81.477} for the TGWMA and DGWMA sign charts, respectively, where the former has smaller ARL_{1s} . Moreover, if $p \in \{0.4, 0.6\}$ and $q = 0.7$, (i) for $n = 5$, the ARL_{1s} of the TGWMA sign chart are smaller than those of the DGWMA sign chart; (ii) for $n = 10$ and $\alpha \geq 0.8$, the ARL_{1s} of the TGWMA sign chart are lower than that of the DGWMA sign chart; and (iii) for $n = 15$ and $\alpha \geq 1.0$, the TGWMA sign chart has smaller ARL_{1s} than the corresponding ARL_{1s} of the DGWMA sign chart. However, when $q = 0.9$, for the same shift size $p \in \{0.4, 0.6\}$, the TGWMA sign chart has almost no advantage compared to the DGWMA sign chart as the latter has smaller ARL_{1s} than the former (see Table 4). In Table 4, it is also found that the charts' performance improves (ARL_1 decreases) as the sample size n increases.

Table 4. ARL_{1s} for the TGWMA and DGWMA sign charts for shift sizes in the process proportion, $p \in \{0.1, 0.4, 0.6, 0.9\}$ when $ARL_0 = 370$ under the ZS condition.

n	q	α	L_{DG}	L_{TG}	p							
					0.1		0.4		0.6		0.9	
					DGWMA	TGWMA	DGWMA	TGWMA	DGWMA	TGWMA	DGWMA	TGWMA
5	0.5	0.5	2.791	2.747	3.994	4.299	40.685	33.728	40.935	33.977	4.002	4.302
		0.6	2.804	2.736	3.905	4.223	43.344	36.193	43.636	36.291	3.909	4.226
		0.7	2.816	2.725	3.855	4.212	47.268	39.569	47.954	39.887	3.857	4.215
		0.8	2.823	2.725	3.708	4.177	51.799	43.708	52.721	44.079	3.709	4.182
		0.9	2.825	2.731	3.694	4.098	56.616	47.933	57.355	48.358	3.695	4.099
		1.0	2.825	2.737	3.703	4.046	61.599	52.323	62.168	52.783	3.703	4.047
		1.2	2.815	2.75	3.691	3.931	70.158	60.242	71.040	60.853	3.685	3.932
		1.5	2.804	2.772	3.637	3.917	80.442	70.064	81.477	70.892	3.640	3.922
	0.7	0.5	2.688	2.293	4.491	6.177	31.664	31.001	31.797	31.159	4.495	6.178
		0.6	2.673	2.326	4.786	6.275	31.893	30.500	32.047	30.708	4.786	6.279
		0.7	2.658	2.369	4.749	6.155	33.192	30.992	33.315	31.192	4.747	6.156
		0.8	2.654	2.418	4.517	5.974	35.260	32.535	35.436	32.611	4.511	5.976
		0.9	2.661	2.467	4.442	5.826	38.315	34.897	38.459	34.948	4.441	5.825
		1.0	2.671	2.511	4.427	5.676	41.683	37.849	42.029	37.630	4.428	5.676
		1.2	2.695	2.588	4.376	5.407	49.415	44.159	50.044	44.721	4.378	6.891
		1.5	2.74	2.673	4.090	5.033	61.551	54.259	62.271	55.172	4.097	5.038
	0.9	0.5	1.892	1.088	7.068	12.804	34.413	42.183	34.497	42.275	7.070	12.804
		0.6	1.87	1.265	7.942	14.312	33.962	41.932	34.078	42.050	7.945	14.317
		0.7	1.944	1.491	8.217	14.566	32.939	39.922	33.048	40.059	8.211	14.568
		0.8	2.033	1.689	8.273	14.085	31.824	37.461	32.000	37.628	8.268	14.087
		0.9	2.123	1.858	8.127	13.320	31.235	35.559	31.425	35.723	8.128	13.322
		1.0	2.202	1.993	7.882	12.480	31.314	34.410	31.481	34.559	7.887	12.478
		1.2	2.342	2.198	7.339	10.928	33.690	34.679	33.875	34.957	7.338	10.931
		1.5	2.502	2.407	6.555	9.239	40.690	39.459	40.586	39.456	6.556	9.240
10	0.5	0.5	2.870	2.763	2.432	3.011	22.634	19.314	22.757	19.410	2.435	3.017
		0.6	2.865	2.742	2.443	3.018	23.124	19.649	23.282	19.804	2.448	3.023
		0.7	2.862	2.739	2.430	3.010	24.227	20.621	24.386	20.747	2.436	3.013
		0.8	2.862	2.742	2.429	3.114	25.944	21.996	26.150	22.179	2.435	3.113
		0.9	2.858	2.75	2.346	3.087	27.755	23.718	28.145	23.954	2.351	3.087
		1.0	2.855	2.757	2.346	3.083	29.755	25.543	30.241	25.769	2.351	3.082
		1.2	2.856	2.779	2.342	3.072	34.076	29.291	34.386	29.518	2.347	3.070
		1.5	2.863	2.814	2.341	3.071	39.790	34.380	40.366	34.824	2.346	3.070
	0.7	0.5	2.709	2.291	3.117	4.394	19.344	19.938	19.440	19.993	3.124	4.400
		0.6	2.678	2.323	3.252	4.623	18.902	19.240	18.954	19.313	3.255	4.629
		0.7	2.665	2.372	3.299	4.705	18.812	18.838	18.868	18.926	3.303	4.712
		0.8	2.666	2.422	3.354	4.677	19.234	18.859	19.307	18.992	3.363	4.683
		0.9	2.671	2.47	3.289	4.611	20.118	19.256	20.205	19.410	3.295	4.617

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n	q	α	L_{DG}	L_{TG}	p							
					0.1		0.4		0.6		0.9	
					DGWMA	TGWMA	DGWMA	TGWMA	DGWMA	TGWMA	DGWMA	TGWMA
10		1.0	2.684	2.519	3.285	4.510	21.206	20.081	21.409	20.278	3.290	4.519
		1.2	2.714	2.601	3.211	4.316	24.351	22.440	24.525	22.660	3.214	4.320
		1.5	2.767	2.695	3.166	4.129	29.945	26.886	30.120	27.133	3.168	4.129
	0.9	0.5	1.889	1.085	4.846	9.741	22.861	30.690	22.893	30.759	4.846	9.738
		0.6	1.863	1.26	5.643	11.259	22.944	31.148	22.990	31.218	5.644	11.260
		0.7	1.936	1.489	6.117	11.763	22.320	29.945	22.408	30.009	6.121	11.760
		0.8	2.029	1.686	6.339	11.575	21.437	27.930	21.497	28.002	6.338	11.568
		0.9	2.117	1.852	6.353	11.104	20.545	25.945	20.600	26.017	6.356	11.103
		1.0	2.201	1.986	6.283	10.482	19.907	24.271	19.990	24.328	6.285	10.482
		1.2	2.344	2.199	6.023	9.333	19.670	22.396	19.839	22.524	6.025	9.336
		1.5	2.511	2.412	5.418	8.055	21.520	22.516	21.667	22.635	5.426	8.056
	0.5	0.5	2.89	2.765	1.865	2.309	15.970	14.124	15.980	14.187	1.864	2.302
		0.6	2.879	2.75	2.086	2.359	15.869	14.078	15.944	14.143	2.085	2.356
		0.7	2.873	2.746	2.086	2.565	16.177	14.394	16.334	14.426	2.085	2.567
		0.8	2.869	2.746	2.074	2.590	16.980	14.990	17.077	15.005	2.073	2.596
		0.9	2.866	2.753	2.072	2.591	17.932	15.795	18.052	15.788	2.071	2.596
		1.0	2.864	2.762	2.072	2.646	19.081	16.715	19.100	16.662	2.071	2.650
		1.2	2.866	2.786	2.071	2.621	21.617	18.762	21.634	18.840	2.070	2.620
		1.5	2.876	2.822	2.069	2.620	25.162	21.809	24.864	21.708	2.069	2.620
15	0.7	0.5	2.714	2.286	2.386	3.707	14.585	15.637	14.610	15.678	2.385	3.705
		0.6	2.683	2.323	2.606	4.014	14.142	15.060	14.197	15.099	2.610	4.011
		0.7	2.672	2.371	2.838	4.084	13.901	14.569	13.943	14.579	2.842	4.080
		0.8	2.669	2.423	2.845	4.096	13.873	14.294	13.931	14.297	2.849	4.091
		0.9	2.676	2.473	2.983	4.079	14.180	14.266	14.181	14.283	2.986	4.074
		1.0	2.686	2.521	2.982	4.057	14.642	14.526	14.674	14.511	2.984	4.055
		1.2	2.716	2.602	2.977	4.025	16.142	15.510	16.067	15.457	2.979	4.025
		1.5	2.775	2.699	3.012	3.924	19.171	17.843	19.228	17.718	3.012	3.927
	0.9	0.5	1.894	1.089	3.945	8.342	18.200	25.807	18.230	25.814	3.948	8.339
		0.6	1.866	1.264	4.695	9.862	18.485	26.594	18.527	26.598	4.695	9.858
		0.7	1.937	1.492	5.189	10.418	18.121	25.752	18.152	25.763	5.190	10.412
		0.8	2.026	1.687	5.423	10.345	17.398	24.068	17.440	24.083	5.424	10.344
		0.9	2.12	1.850	5.548	10.036	16.666	22.289	16.699	22.309	5.546	10.035
		1.0	2.202	1.986	5.530	9.521	15.982	20.697	15.999	20.724	5.529	9.520
		1.2	2.346	2.200	5.282	8.589	15.184	18.462	15.196	18.472	5.284	8.590
		1.5	2.515	2.415	5.033	7.346	15.579	17.270	15.520	17.281	5.031	7.346

As the TGWMA sign chart is generally found to be superior to the DGWMA sign chart towards small shifts in the process proportion, i.e., $p \in \{0.4, 0.6\}$, a further investigation is conducted by comparing the ARL_1 s of the two charts for $p \in \{0.44, 0.48, 0.52, 0.56\}$. Tables 5 and 6 present the computed ZS ARL_1 s for the two charts based on $p \in \{0.44, 0.48\}$ and $p \in \{0.52, 0.56\}$, respectively. In Tables 5 and 6, a negative (or positive) value of “% Diff” represents the percentage of decrease (or increase) in the ARL_1 value by using the TGWMA sign chart in place of the DGWMA sign chart. Consequently, $\% \text{ Diff} = \frac{ARL_1(\text{TGWMA}) - ARL_1(\text{DGWMA})}{ARL_1(\text{DGWMA})} \times 100\%$. In Tables 5 and 6 under the ZS condition, it is noticed that the TGWMA sign chart surpasses the DGWMA sign chart in terms of the ARL_1 criterion for $q = 0.5$ and 0.7 , as the former has smaller ARL_1 s than the latter. For instance, when $n = 10$, $p = 0.44$, $\alpha = 0.8$, and $q = 0.5$, the ARL_1 s are 59.578 and 71.891 for the TGWMA and DGWMA sign charts, respectively, and the former’s ARL_1 is 17.13% smaller relative to that of the latter (see Table 5). However, for $q = 0.9$, mixed performance is observed as sometimes the DGWMA sign chart

beats the TGWMA sign chart, and vice-versa. From the “% Diff” in Tables 5 and 6, the TGWMA sign chart shows the best performance over the DGWMA sign chart when p is 0.44 or 0.56, and this phenomenon is particularly evident when $q = 0.5$. For example, when $n = 5$, $p = 0.44$, and $q = 0.5$, the percentage of improvement in using the TGWMA sign chart in place of the DGWMA sign chart ranges from 8.43% to 18.41%.

From the results in Tables 5 and 6, the TGWMA sign chart outperforms the DGWMA sign chart in detecting small shifts in the process proportion, especially when q takes values of 0.5 and 0.7. When q takes the value of 0.9, the superiority of the TGWMA sign chart to the DGWMA sign chart is only evident when α takes larger values, such as 1.2 and 1.5, or when the shift is particularly small ($p \in \{0.48, 0.52\}$).

Table 5. ARL_{1s} for the TGWMA and DGWMA sign charts, for shift sizes in the process proportion $p \in \{0.44, 0.48\}$ when ARL₀ = 370 under the ZS condition.

n	q	α	L_{DG}	L_{TG}	p					
					0.44			0.48		
					DGWMA	TGWMA	% Diff	DGWMA	TGWMA	% Diff
5	0.5	0.5	2.791	2.747	95.776	78.142	-18.41	285.367	260.820	-8.60
		0.6	2.804	2.736	104.800	86.959	-17.02	294.006	273.362	-7.02
		0.7	2.816	2.725	115.441	97.025	-15.95	301.878	285.331	-5.48
		0.8	2.823	2.725	124.728	107.170	-14.08	308.082	292.988	-4.90
		0.9	2.825	2.731	134.911	117.338	-13.03	313.528	300.685	-4.10
		1.0	2.825	2.737	143.805	126.202	-12.24	319.195	304.397	-4.64
		1.2	2.815	2.750	156.476	140.464	-10.23	324.420	315.274	-2.82
		1.5	2.804	2.772	171.580	157.124	-8.43	331.808	324.645	-2.16
	0.7	0.5	2.688	2.293	66.588	61.447	-7.72	234.553	216.480	-7.71
		0.6	2.673	2.326	70.709	63.495	-10.20	248.261	230.015	-7.35
		0.7	2.658	2.369	77.329	68.543	-11.36	260.718	244.354	-6.28
		0.8	2.654	2.418	85.044	75.628	-11.07	272.001	258.275	-5.05
		0.9	2.661	2.467	93.918	83.962	-10.60	281.364	271.158	-3.63
		1.0	2.671	2.511	102.755	91.765	-10.70	290.391	278.292	-4.17
		1.2	2.695	2.588	119.378	106.926	-10.43	302.359	292.801	-3.16
		1.5	2.740	2.673	143.244	129.601	-9.52	319.739	309.438	-3.22
	0.9	0.5	1.892	1.088	62.594	69.256	10.64	199.449	198.789	-0.33
		0.6	1.870	1.265	61.699	68.106	10.38	201.461	199.679	-0.88
		0.7	1.944	1.491	60.990	65.670	7.67	208.406	203.299	-2.45
		0.8	2.033	1.689	61.243	64.049	4.58	217.804	211.053	-3.10
		0.9	2.123	1.858	63.264	64.581	2.08	229.728	222.067	-3.33
		1.0	2.202	1.993	67.052	66.612	-0.66	241.575	234.955	-2.74
		1.2	2.342	2.198	78.480	74.988	-4.45	262.528	254.086	-3.22
		1.5	2.502	2.407	98.284	91.967	-6.43	285.362	277.323	-2.82
10	0.5	0.5	2.870	2.763	56.531	44.535	-21.22	233.267	200.901	-13.88
		0.6	2.865	2.742	60.586	48.396	-20.12	245.522	216.739	-11.72
		0.7	2.862	2.739	66.000	53.630	-18.74	254.900	231.912	-9.02
		0.8	2.862	2.742	71.891	59.578	-17.13	264.164	244.420	-7.47
		0.9	2.858	2.750	77.589	65.737	-15.28	270.647	255.834	-5.47
		1.0	2.855	2.757	83.344	71.235	-14.53	276.855	263.289	-4.90

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n	q	α	L_{DG}	L_{TG}	p					
					0.44			0.48		
					DGWMA	TGWMA	% Diff	DGWMA	TGWMA	% Diff
10		1.2	2.856	2.779	93.612	80.749	-13.74	289.028	275.857	-4.56
		1.5	2.863	2.814	106.698	94.298	-11.62	299.891	290.508	-3.13
	0.7	0.5	2.709	2.291	40.599	38.246	-5.80	172.329	154.132	-10.56
		0.6	2.678	2.323	41.288	38.291	-7.26	185.063	167.276	-9.61
		0.7	2.665	2.372	43.317	39.817	-8.08	199.265	182.036	-8.65
		0.8	2.666	2.422	47.167	42.478	-9.94	214.139	197.262	-7.88
		0.9	2.671	2.470	51.556	46.109	-10.56	229.407	211.787	-7.68
		1.0	2.684	2.519	56.911	50.419	-11.41	242.247	227.441	-6.11
		1.2	2.714	2.601	67.598	60.700	-10.20	259.838	248.965	-4.18
		1.5	2.767	2.695	82.050	73.200	-10.79	276.994	266.652	-3.73
	0.9	0.5	1.889	1.085	41.417	49.067	18.47	142.487	144.623	1.50
		0.6	1.863	1.260	40.817	48.504	18.83	142.951	143.810	0.60
		0.7	1.936	1.489	39.735	46.392	16.75	146.877	145.349	-1.04
		0.8	2.029	1.686	38.833	43.906	13.06	154.802	150.547	-2.75
		0.9	2.117	1.852	38.720	42.264	9.15	164.584	158.916	-3.44
		1.0	2.201	1.986	39.573	41.737	5.47	177.120	168.667	-4.77
		1.2	2.344	2.199	43.832	43.936	0.24	202.043	191.874	-5.03
		1.5	2.511	2.412	54.656	51.862	-5.11	235.047	222.310	-5.42
15	0.5	0.5	2.890	2.765	40.137	32.268	-19.61	198.990	165.848	-16.65
		0.6	2.879	2.750	42.002	34.187	-18.61	209.390	182.731	-12.73
		0.7	2.873	2.746	45.307	37.212	-17.87	219.380	197.058	-10.17
		0.8	2.869	2.746	49.017	40.837	-16.69	228.770	211.204	-7.68
		0.9	2.866	2.753	53.278	44.800	-15.91	239.500	221.996	-7.31
		1.0	2.864	2.762	57.310	48.719	-14.99	247.760	230.831	-6.83
		1.2	2.866	2.786	65.219	56.198	-13.83	259.650	244.036	-6.01
		1.5	2.876	2.822	75.453	65.260	-13.51	270.990	258.570	-4.58
	0.7	0.5	2.714	2.286	30.502	29.537	-3.17	139.520	124.226	-10.96
		0.6	2.683	2.323	30.371	29.039	-4.39	151.120	133.818	-11.45
		0.7	2.672	2.371	31.331	29.361	-6.29	166.162	147.623	-11.16
		0.8	2.669	2.423	33.313	30.667	-7.94	180.443	162.789	-9.78
		0.9	2.676	2.473	36.050	32.638	-9.46	194.632	176.104	-9.52
		1.0	2.686	2.521	39.139	35.376	-9.62	205.380	189.902	-7.54
		1.2	2.716	2.602	46.258	41.479	-10.33	223.390	210.621	-5.72
		1.5	2.775	2.699	57.019	50.469	-11.49	246.323	232.050	-5.79
	0.9	0.5	1.894	1.089	32.833	40.788	24.23	116.583	119.683	2.66
		0.6	1.866	1.264	32.504	40.591	24.88	116.533	118.846	1.98
		0.7	1.937	1.492	31.505	38.682	22.78	118.651	118.815	0.14
		0.8	2.026	1.687	30.371	36.201	19.20	123.606	121.642	-1.59
		0.9	2.120	1.850	29.824	34.158	14.53	132.551	127.926	-3.49
		1.0	2.202	1.986	29.742	32.952	10.79	143.040	136.393	-4.65
		1.2	2.346	2.200	31.760	33.003	3.91	166.663	158.576	-4.85
		1.5	2.515	2.415	38.295	37.021	-3.33	198.606	187.344	-5.67

Table 6. ARL_{1s} for the TGWMA and DGWMA sign charts, for shift sizes in the process proportion $p \in \{0.52, 0.56\}$ when $ARL_0 = 370$ under the ZS condition.

n	q	α	L_{DG}	L_{TG}	p					
					0.52			0.56		
					DGWMA	TGWMA	% Diff	DGWMA	TGWMA	% Diff
5	0.5	0.5	2.791	2.747	283.967	263.320	-7.27	96.980	78.134	-19.43
		0.6	2.804	2.736	293.126	278.963	-4.83	105.678	88.081	-16.65
		0.7	2.816	2.725	302.425	290.412	-3.97	115.410	98.026	-15.06
		0.8	2.823	2.725	309.249	299.157	-3.26	126.058	108.886	-13.62
		0.9	2.825	2.731	315.325	305.724	-3.04	136.451	118.727	-12.99
		1.0	2.825	2.737	320.868	311.620	-2.88	145.862	128.081	-12.19
		1.2	2.815	2.750	328.365	318.839	-2.90	160.400	142.703	-11.03
		1.5	2.804	2.772	334.231	328.831	-1.62	174.800	160.782	-8.02
	0.7	0.5	2.688	2.293	234.930	219.378	-6.62	66.738	61.294	-8.16
		0.6	2.673	2.326	249.469	231.280	-7.29	70.899	63.669	-10.20
		0.7	2.658	2.369	265.338	244.38	-7.90	77.767	68.633	-11.75
		0.8	2.654	2.418	276.773	261.171	-5.64	85.869	75.928	-11.58
		0.9	2.661	2.467	286.094	275.310	-3.77	94.550	84.310	-10.83
		1.0	2.671	2.511	294.972	285.356	-3.26	104.187	92.159	-11.54
		1.2	2.695	2.588	305.457	297.385	-2.64	121.744	109.092	-10.39
		1.5	2.740	2.673	322.193	311.332	-3.37	146.307	131.298	-10.26
	0.9	0.5	1.892	1.088	202.562	201.499	-0.52	62.608	69.176	10.49
		0.6	1.870	1.265	204.994	203.024	-0.96	61.829	68.081	10.11
		0.7	1.944	1.491	210.850	205.604	-2.49	61.100	65.797	7.69
		0.8	2.033	1.689	219.033	213.720	-2.43	61.342	64.188	4.64
		0.9	2.123	1.858	230.279	223.825	-2.80	63.587	64.611	1.61
		1.0	2.202	1.993	242.174	234.969	-2.98	67.306	66.835	-0.70
		1.2	2.342	2.198	265.223	255.643	-3.61	78.536	74.994	-4.51
		1.5	2.502	2.407	291.267	283.520	-2.66	99.040	92.518	-6.59
10	0.5	0.5	2.870	2.763	233.529	200.364	-14.20	57.261	45.420	-20.68
		0.6	2.865	2.742	245.610	216.762	-11.75	61.489	49.589	-19.35
		0.7	2.862	2.739	256.077	232.882	-9.06	66.612	54.722	-17.85
		0.8	2.862	2.742	266.367	246.390	-7.50	72.805	60.223	-17.28
		0.9	2.858	2.750	272.460	257.957	-5.32	78.530	66.438	-15.40
		1.0	2.855	2.757	279.352	265.057	-5.12	84.403	72.101	-14.58
		1.2	2.856	2.779	290.680	276.848	-4.76	95.964	82.420	-14.11
		1.5	2.863	2.814	300.031	293.507	-2.17	108.731	95.733	-11.95
	0.7	0.5	2.709	2.291	173.121	155.374	-10.25	41.150	38.666	-6.04
		0.6	2.678	2.323	184.823	168.718	-8.71	41.836	38.719	-7.45
		0.7	2.665	2.372	198.869	184.804	-7.07	44.120	40.303	-8.65
		0.8	2.666	2.422	215.238	199.802	-7.17	47.737	42.980	-9.96
		0.9	2.671	2.470	229.176	213.460	-6.86	52.287	46.620	-10.84
		1.0	2.684	2.519	242.220	226.719	-6.40	57.788	51.219	-11.37
		1.2	2.714	2.601	259.415	249.026	-4.00	68.592	61.430	-10.44
		1.5	2.767	2.695	278.987	267.700	-4.05	83.861	74.214	-11.50
	0.9	0.5	1.889	1.085	143.967	147.073	2.16	41.624	49.274	18.38
		0.6	1.863	1.260	144.764	146.470	1.18	41.034	48.666	18.60

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n	q	α	L_{DG}	L_{TG}	p					
					0.52			0.56		
					DGWMA	TGWMA	% Diff	DGWMA	TGWMA	% Diff
10	0.9	0.7	1.936	1.489	148.709	147.230	-0.99	39.940	46.483	16.38
		0.8	2.029	1.686	156.248	152.390	-2.47	39.130	44.074	12.63
		0.9	2.117	1.852	167.194	160.457	-4.03	38.919	42.526	9.27
		1.0	2.201	1.986	179.464	171.374	-4.51	39.929	41.985	5.15
		1.2	2.344	2.199	204.765	195.575	-4.49	44.541	44.485	-0.13
		1.5	2.511	2.412	234.767	223.978	-4.60	55.326	52.162	-5.72
15	0.5	0.5	2.890	2.765	197.860	165.998	-16.10	39.679	32.140	-19.00
		0.6	2.879	2.750	209.590	180.485	-13.89	41.855	34.134	-18.45
		0.7	2.873	2.746	218.480	196.552	-10.04	45.072	37.058	-17.78
		0.8	2.869	2.746	227.370	209.369	-7.92	49.169	40.541	-17.55
		0.9	2.866	2.753	237.000	220.456	-6.98	53.247	44.477	-16.47
		1.0	2.864	2.762	244.640	228.585	-6.56	57.300	48.466	-15.42
		1.2	2.866	2.786	256.530	243.192	-5.20	65.083	56.060	-13.86
		1.5	2.876	2.822	268.580	257.659	-4.07	74.845	65.386	-12.64
	0.7	0.5	2.714	2.286	140.670	124.572	-11.44	30.388	29.468	-3.03
		0.6	2.683	2.323	151.136	134.686	-10.88	30.274	28.995	-4.23
		0.7	2.672	2.371	165.440	149.199	-9.82	31.270	29.316	-6.25
		0.8	2.669	2.423	179.435	165.367	-7.84	33.228	30.643	-7.78
		0.9	2.676	2.473	192.322	177.948	-7.47	35.976	32.691	-9.13
		1.0	2.686	2.521	203.683	190.367	-6.54	38.899	35.249	-9.38
		1.2	2.716	2.602	222.144	207.492	-6.60	45.731	41.098	-10.13
		1.5	2.775	2.699	243.483	230.177	-5.46	57.101	50.344	-11.83
	0.9	0.5	1.894	1.089	117.288	120.783	2.98	32.813	40.769	24.24
		0.6	1.866	1.264	117.594	120.063	2.10	32.522	40.570	24.75
		0.7	1.937	1.492	119.089	120.407	1.11	31.514	38.691	22.77
		0.8	2.026	1.687	123.913	122.778	-0.92	30.406	36.268	19.28
		0.9	2.120	1.850	133.505	128.391	-3.83	29.829	34.227	14.74
		1.0	2.202	1.986	145.206	137.325	-5.43	29.693	32.989	11.10
		1.2	2.346	2.200	168.248	159.435	-5.24	31.678	32.954	4.03
		1.5	2.515	2.415	198.805	188.456	-5.21	37.992	36.844	-3.02

After verifying that the TGWMA sign chart exhibits better sensitivity in detecting small shifts than the DGWMA sign chart, it is also essential to assess whether the TGWMA sign chart shows better run length stability. To this end, the standard deviation of the run length (SDRL) performance of these two charts for small shifts is compared, as the strength of the TGWMA sign chart lies in detecting small shifts. Table 7 illustrates that when the shift is small ($p \in \{0.44, 0.48, 0.52, 0.56\}$), all $SDRL_1$ values of the TGWMA sign chart are smaller than that of the DGWMA sign chart under the ZS condition. This outcome clearly illustrates that the TGWMA sign chart gives better $SDRL_1$ performance than the DGWMA sign chart when the shift is very small. For example, when $n = 10$, $q = 0.5$, $p = 0.44$, and $\alpha \in \{0.5, 0.6, \dots, 1.5\}$, $SDRL_1 \in \{34.38, 40.17, \dots, 90.98\}$ and $\{48.19, 53.76, \dots, 103.43\}$ for the TGWMA and DGWMA sign charts, respectively, where the former has smaller $SDRL_1$ values (see Table 7). This outcome aligns with our findings on the ARL_1 comparison between the two charts.

Table 7. SDRL_{1s} for the TGWMA and DGWMA sign charts, for shift sizes in the process proportion $p \in \{0.44, 0.48, 0.52, 0.56\}$ when $ARL_0 = 370$ under the ZS condition.

n	q	α	L_{DG}	L_{TG}	p							
					0.44		0.48		0.52		0.56	
					DGWMA	TGWMA	DGWMA	TGWMA	DGWMA	TGWMA	DGWMA	TGWMA
5	0.5	0.5	2.791	2.747	87.14	67.38	281.82	251.17	276.78	254.74	88.27	67.32
		0.6	2.804	2.736	98.31	78.84	293.20	264.38	284.83	271.36	98.33	79.86
		0.7	2.816	2.725	110.82	91.25	298.81	277.61	294.10	282.63	109.09	91.03
		0.8	2.823	2.725	121.06	102.97	304.82	288.34	302.90	292.96	121.31	103.72
		0.9	2.825	2.731	131.73	114.25	311.64	295.91	310.80	299.59	133.11	113.27
		1.0	2.825	2.737	140.68	123.73	318.76	296.58	316.21	306.33	141.95	123.84
		1.2	2.815	2.750	154.25	138.33	321.87	311.41	325.50	314.50	156.81	139.34
		1.5	2.804	2.772	168.16	154.36	330.10	320.55	336.37	325.20	173.30	158.69
	0.7	0.5	2.688	2.293	49.79	41.35	217.35	192.27	215.47	196.08	50.25	41.19
		0.6	2.673	2.326	57.73	47.04	236.41	214.47	235.85	211.85	57.57	47.34
		0.7	2.658	2.369	67.39	55.25	250.38	233.70	254.90	109.57	68.05	56.03
		0.8	2.654	2.418	76.48	65.23	262.22	248.46	268.18	252.75	77.91	66.16
		0.9	2.661	2.467	87.58	75.82	272.84	260.38	278.07	267.64	87.21	76.29
		1.0	2.671	2.511	97.50	84.68	285.03	268.38	288.57	278.92	97.48	83.88
		1.2	2.695	2.588	115.79	100.85	299.07	288.49	298.89	292.75	116.04	102.54
		1.5	2.74	2.673	141.07	126.13	315.75	304.49	317.71	306.02	143.22	126.69
	0.9	0.5	1.892	1.088	34.97	30.95	158.08	145.93	161.83	146.90	34.56	30.60
		0.6	1.87	1.265	35.06	31.02	165.08	151.25	169.75	155.51	34.80	30.65
		0.7	1.944	1.491	36.91	32.39	178.96	164.78	182.27	167.12	36.57	32.06
		0.8	2.033	1.689	40.29	35.53	194.77	179.87	196.43	183.11	40.19	35.33
		0.9	2.123	1.858	45.65	40.43	211.70	196.45	209.19	196.68	45.90	40.53
		1.0	2.202	1.993	52.38	46.36	228.51	215.87	224.79	211.58	52.54	46.51
		1.2	2.342	2.198	67.97	60.19	250.61	237.86	255.85	238.34	68.46	60.44
		1.5	2.502	2.407	91.05	81.70	278.76	264.26	284.99	273.53	91.05	82.15
10	0.5	0.5	2.87	2.763	48.19	34.38	226.59	192.32	228.12	190.66	48.71	35.29
		0.6	2.865	2.742	53.76	40.17	239.87	211.17	241.34	210.32	54.85	41.66
		0.7	2.862	2.739	60.55	46.96	251.53	228.19	253.07	230.24	61.44	48.21
		0.8	2.862	2.742	66.94	54.01	262.21	242.73	264.36	241.89	68.84	54.57
		0.9	2.858	2.750	73.80	61.10	268.78	254.21	270.07	254.56	75.36	62.25
		1.0	2.855	2.757	80.17	66.87	273.82	259.84	276.85	260.55	82.43	68.58
		1.2	2.856	2.779	89.80	76.67	288.92	274.40	289.55	272.76	94.35	80.25
		1.5	2.863	2.814	103.43	90.98	301.09	287.71	297.87	293.30	106.88	93.50
	0.7	0.5	2.709	2.291	27.11	21.97	153.21	130.74	153.94	128.71	27.53	22.59
		0.6	2.678	2.323	29.60	24.23	171.19	149.25	169.48	149.54	30.19	24.67
		0.7	2.665	2.372	33.44	27.94	189.48	168.97	187.96	169.85	34.29	28.61
		0.8	2.666	2.422	39.28	32.64	206.51	186.06	208.21	188.35	39.47	33.09
		0.9	2.671	2.470	44.82	37.79	225.48	202.36	224.43	203.53	45.32	38.00
		1.0	2.684	2.519	50.88	42.93	238.48	220.27	238.10	221.31	51.52	43.70
		1.2	2.714	2.601	63.14	54.81	257.91	245.70	254.31	245.11	64.26	55.62
		1.5	2.767	2.695	78.47	68.11	274.77	261.43	276.01	262.40	81.49	69.93
	0.9	0.5	1.889	1.085	19.90	17.53	105.09	94.46	103.60	95.85	20.01	17.52
		0.6	1.863	1.26	19.43	17.10	108.69	98.02	107.28	98.72	19.47	17.03

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n	q	α	L_{DG}	L_{TG}	p							
					0.44		0.48		0.52		0.56	
					DGWMA	TGWMA	DGWMA	TGWMA	DGWMA	TGWMA	DGWMA	TGWMA
10	0.9	0.7	1.936	1.489	19.64	17.27	117.80	106.97	117.22	105.84	19.77	17.20
		0.8	2.029	1.686	20.74	18.13	131.70	119.42	129.36	119.37	21.24	18.29
		0.9	2.117	1.852	22.91	19.95	146.20	133.96	146.53	131.76	23.36	20.46
		1.0	2.201	1.986	25.98	22.64	162.19	148.18	162.88	147.98	26.66	23.15
		1.2	2.344	2.199	33.77	29.64	190.25	176.61	193.77	179.04	34.69	30.47
		1.5	2.511	2.412	47.11	41.64	228.65	210.49	229.12	214.04	47.64	41.70
15	0.5	0.5	2.890	2.765	32.14	23.26	192.25	157.09	189.36	155.47	31.80	23.14
		0.6	2.879	2.750	35.36	26.57	205.11	176.04	203.53	171.91	35.89	26.61
		0.7	2.873	2.746	40.01	30.96	218.00	190.22	214.01	190.47	40.54	30.72
		0.8	2.869	2.746	44.28	35.47	227.80	205.80	224.26	205.35	45.20	35.45
		0.9	2.866	2.753	48.99	39.89	238.04	218.39	233.29	217.73	49.76	40.41
		1.0	2.864	2.762	53.55	44.15	247.07	228.24	241.77	224.44	54.43	44.62
		1.2	2.866	2.786	61.44	52.29	257.78	242.45	254.31	240.31	62.86	53.02
		1.5	2.876	2.822	73.04	61.54	268.24	256.07	267.93	255.49	73.17	62.85
	0.7	0.5	2.714	2.286	19.01	15.31	120.88	101.29	121.84	99.44	18.73	15.41
		0.6	2.683	2.323	20.14	16.35	137.68	116.07	137.28	116.06	19.77	16.37
		0.7	2.672	2.371	22.51	18.39	156.95	135.08	155.25	134.89	22.21	18.24
		0.8	2.669	2.423	25.81	21.35	172.68	153.20	170.47	155.44	25.71	21.09
		0.9	2.676	2.473	29.57	24.56	187.17	168.25	184.30	169.44	29.67	24.37
		1.0	2.686	2.521	33.62	28.34	199.43	181.86	197.61	184.98	33.43	28.04
		1.2	2.716	2.602	41.52	35.78	219.34	204.55	217.52	200.12	41.53	35.62
		1.5	2.775	2.699	52.91	45.30	245.23	229.19	238.33	224.17	53.95	45.81
	0.9	0.5	1.894	1.089	14.33	12.60	80.14	71.39	79.91	72.17	14.44	12.74
		0.6	1.866	1.264	13.71	12.11	83.21	73.85	83.55	74.89	13.89	12.22
		0.7	1.937	1.492	13.64	11.87	90.02	80.27	89.59	81.46	13.81	12.00
		0.8	2.026	1.687	14.03	12.11	100.80	90.37	98.93	90.50	14.23	12.34
		0.9	2.120	1.850	15.26	13.00	113.60	102.60	113.36	101.29	15.42	13.25
		1.0	2.202	1.986	16.87	14.61	127.64	115.55	128.75	114.63	16.94	14.81
		1.2	2.346	2.200	22.11	19.11	156.07	144.09	156.69	142.84	21.71	18.93
		1.5	2.515	2.415	31.26	26.90	190.31	177.77	190.76	178.74	31.13	26.58

Next, the ARL_1 comparison between the TGWMA and DGWMA sign charts under the SS condition will be made. Table 8 displays the SS ARL_1 s for these two charts based on $p \in \{0.1, 0.4, 0.6, 0.9\}$. Similar to the ZS condition, it is seen in Table 8 that for $p = 0.1$ and 0.9 , which indicates a larger shift in the process proportion, the ARL_1 of the DGWMA sign chart is smaller than that of the TGWMA sign chart. However, for a small shift, such as $p \in \{0.4, 0.6\}$, the superiority of the TGWMA sign chart to the DGWMA sign chart becomes evident. When $q = 0.5$ and $p \in \{0.4, 0.6\}$, the ARL_1 s of the TGWMA sign chart are less than that of the DGWMA sign chart. When $p \in \{0.4, 0.6\}$ and $q = 0.7$, the SS TGWMA sign chart beats the SS DGWMA sign chart for the following situations: (i) $n = 5$ and $\alpha \geq 0.6$, (ii) $n = 10$ and $\alpha \geq 1.2$, and (iii) $n = 15$ and $\alpha = 1.5$ (see Table 8). Nonetheless, similar to the findings under the ZS condition, when $q = 0.9$, the TGWMA sign chart has almost no advantage compared to the DGWMA sign chart, as the latter gives smaller ARL_1 s for all shift sizes. Since the performance trends of the TGWMA and DGWMA sign charts in Tables 4 (ZS condition) and 8 (SS condition) are the same, this paragraph provides a concise discussion regarding the SS performance of the two charts for $p \in \{0.1, 0.4, 0.6, 0.9\}$.

A further investigation on the SS performance of the TGWMA and DGWMA sign charts in detecting small shifts, $p \in \{0.44, 0.48, 0.52, 0.56\}$, is made, and the ARL_{1s} are shown in Tables 9 and 10. The trend in Tables 8 and 9 for the SS condition is the same as those for the ZS condition in Tables 5 and 6, where it is seen that when $p \in \{0.44, 0.56\}$ and $q \in \{0.5, 0.7\}$, the ARL_{1s} of the TGWMA sign chart are smaller than that of the DGWMA sign chart, irrespective of the sample size n . For even smaller shifts, such as $p \in \{0.48, 0.52\}$, all the ARL_{1s} of the TGWMA sign chart are less than that of the DGWMA sign chart, for any sample size n .

Table 8. ARL_{1s} for the TGWMA and DGWMA sign charts, for shift sizes in the process proportion $p \in \{0.1, 0.4, 0.6, 0.9\}$ when $ARL_0 = 370$ under the SS condition.

n	q	α	L_{DG}	L_{TG}	p							
					0.1		0.4		0.6		0.9	
					DGWMA	TGWMA	DGWMA	TGWMA	DGWMA	TGWMA	DGWMA	TGWMA
5	0.5	0.5	2.389	2.091	1.264	1.659	16.235	12.547	16.323	12.473	1.260	1.680
		0.6	2.341	2.014	1.339	1.769	17.275	12.799	17.328	12.831	1.343	1.795
		0.7	2.299	1.967	1.416	1.853	18.719	13.750	18.468	13.678	1.424	1.881
		0.8	2.265	1.935	1.491	1.916	20.850	14.732	20.663	14.654	1.503	1.944
		0.9	2.237	1.927	1.550	1.977	22.646	16.464	23.009	16.128	1.565	1.997
		1.0	2.222	1.926	1.611	2.020	25.391	18.426	25.407	18.102	1.625	2.027
		1.2	2.205	1.946	1.711	2.082	30.336	22.249	30.134	22.246	1.713	2.084
		1.5	2.217	2.003	1.823	2.149	38.921	28.914	38.260	28.653	1.830	2.146
	0.7	0.5	1.982	1.305	1.038	1.618	6.897	7.141	6.901	7.129	1.042	1.647
		0.6	1.859	1.363	1.090	1.794	6.903	6.812	6.914	6.748	1.093	1.800
		0.7	1.780	1.291	1.174	1.940	7.098	6.719	7.111	6.685	1.182	1.952
		0.8	1.730	1.271	1.278	2.046	7.536	6.864	7.451	6.853	1.277	2.067
		0.9	1.708	1.279	1.376	2.123	8.201	7.045	8.071	7.054	1.378	2.163
		1.0	1.702	1.591	1.461	2.211	8.971	7.609	8.882	7.553	1.476	2.232
		1.2	1.722	1.433	1.611	2.341	11.452	9.226	11.421	9.110	1.621	2.360
		1.5	1.807	1.342	1.776	2.478	16.879	13.616	16.775	13.015	1.774	2.487
	0.9	0.5	1.083	0.475	1.021	1.828	3.834	4.573	3.918	4.638	1.024	1.836
		0.6	0.932	0.450	1.062	1.814	3.425	3.948	3.458	4.001	1.064	1.826
		0.7	0.884	0.493	1.132	1.867	3.324	3.825	3.338	3.876	1.136	1.868
		0.8	0.877	0.546	1.232	1.996	3.386	3.885	3.358	3.886	1.235	1.979
		0.9	0.894	0.605	1.350	2.165	3.515	4.260	3.558	4.206	1.355	2.174
		1.0	0.925	0.673	1.468	2.364	3.737	4.589	3.765	4.612	1.487	2.358
		1.2	1.019	0.810	1.672	2.661	4.224	5.185	4.267	5.195	1.687	2.673
		1.5	1.193	1.027	1.938	3.091	5.421	6.061	5.402	6.064	1.971	3.066
10	0.5	0.5	2.504	2.132	1.017	1.201	9.349	7.397	9.262	7.378	1.015	1.204
		0.6	2.434	2.047	1.030	1.310	9.478	7.387	9.424	7.386	1.028	1.318
		0.7	2.369	1.986	1.050	1.405	9.673	7.459	9.549	7.424	1.046	1.411
		0.8	2.327	1.959	1.079	1.483	10.209	7.756	10.202	7.723	1.071	1.494
		0.9	2.288	1.939	1.111	1.547	10.776	8.146	10.897	8.157	1.101	1.553
		1.0	2.265	1.942	1.147	1.608	11.596	8.749	11.465	8.736	1.144	1.614
		1.2	2.245	1.960	1.230	1.695	13.539	10.044	13.307	10.103	1.228	1.699
		1.5	2.247	2.023	1.360	1.795	16.741	12.476	16.397	12.757	1.359	1.786
	0.6	0.5	2.031	1.365	1.000	1.258	4.125	4.761	4.124	4.773	1.000	1.266
		0.6	1.888	1.295	1.002	1.404	4.133	4.652	4.117	4.626	1.002	1.428

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n	q	α	L_{DG}	L_{TG}	p							
					0.1		0.4		0.6		0.9	
					DGWMA	TGWMA	DGWMA	TGWMA	DGWMA	TGWMA	DGWMA	TGWMA
10	0.7	0.7	1.801	1.274	1.013	1.549	4.265	4.630	4.287	4.628	1.011	1.560
		0.8	1.746	1.281	1.037	1.681	4.422	4.630	4.435	4.664	1.039	1.680
		0.9	1.718	1.306	1.079	1.769	4.634	4.715	4.660	4.786	1.083	1.783
		1.0	1.709	1.343	1.136	1.851	4.877	4.884	4.844	4.847	1.137	1.866
		1.2	1.731	1.438	1.278	1.988	5.662	5.336	5.600	5.367	1.283	1.998
		1.5	1.817	1.594	1.472	2.133	7.610	6.746	7.567	6.688	1.491	2.136
	0.9	0.5	1.095	0.474	1.000	1.258	2.466	4.761	2.486	4.773	1.000	1.266
		0.6	0.939	0.453	1.003	1.404	2.357	4.652	2.383	4.626	1.005	1.428
		0.7	0.888	0.490	1.020	1.549	2.377	4.630	2.403	4.628	1.024	1.560
		0.8	0.878	0.547	1.069	1.681	2.479	4.630	2.532	4.664	1.071	1.680
		0.9	0.892	0.607	1.150	1.769	2.620	4.715	2.668	4.786	1.154	1.783
		1.0	0.926	0.671	1.241	1.851	2.814	4.884	2.837	4.847	1.246	1.866
		1.2	1.020	0.810	1.425	1.988	3.186	5.336	3.166	5.367	1.432	1.998
		1.5	1.195	1.027	1.664	2.133	3.717	6.746	3.718	6.688	1.670	2.136
15	0.5	0.5	2.537	2.149	1.001	1.052	6.372	5.498	6.416	5.497	1.000	1.052
		0.6	2.455	2.053	1.001	1.119	6.356	5.433	6.384	5.427	1.002	1.121
		0.7	2.391	1.995	1.004	1.199	6.418	5.430	6.482	5.420	1.003	1.202
		0.8	2.343	1.961	1.008	1.274	6.579	5.499	6.647	5.504	1.008	1.280
		0.9	2.304	1.946	1.013	1.346	6.838	5.651	6.901	5.672	1.013	1.360
		1.0	2.278	1.943	1.022	1.415	7.161	5.878	7.353	5.845	1.021	1.426
		1.2	2.250	1.962	1.052	1.530	8.163	6.553	8.157	6.553	1.049	1.539
		1.5	2.256	2.025	1.122	1.652	9.898	8.030	10.001	7.956	1.119	1.652
	0.7	0.5	2.046	1.368	1.000	1.124	3.033	3.790	3.016	3.800	1.000	1.122
		0.6	1.899	1.295	1.000	1.249	3.065	3.783	3.099	3.796	1.000	1.253
		0.7	1.806	1.275	1.001	1.376	3.234	3.862	3.221	3.804	1.001	1.381
		0.8	1.750	1.285	1.003	1.490	3.350	3.905	3.331	3.848	1.004	1.493
		0.9	1.725	1.308	1.012	1.595	3.490	3.915	3.462	3.910	1.012	1.591
		1.0	1.713	1.343	1.032	1.677	3.617	3.988	3.622	4.004	1.033	1.668
		1.2	1.737	1.436	1.108	1.797	4.030	4.213	4.003	4.234	1.111	1.810
		1.5	1.818	1.598	1.280	1.935	5.011	4.876	4.954	4.903	1.289	1.938
	0.9	0.5	1.096	0.473	1.000	1.402	1.944	3.020	1.922	2.977	1.000	1.413
		0.6	0.941	0.453	1.000	1.510	1.953	2.816	1.949	2.763	1.000	1.511
		0.7	0.885	0.490	1.004	1.571	2.009	2.694	1.997	2.766	1.004	1.553
		0.8	0.878	0.547	1.020	1.670	2.127	2.932	2.124	2.888	1.023	1.671
		0.9	0.892	0.607	1.065	1.799	2.272	3.135	2.277	3.100	1.065	1.813
		1.0	0.927	0.671	1.140	1.968	2.427	3.394	2.437	3.370	1.138	1.945
		1.2	1.019	0.810	1.302	2.220	2.720	3.811	2.757	3.838	1.307	2.224
		1.5	1.194	1.027	1.531	2.561	3.157	4.349	3.168	4.412	1.531	2.575

Table 9. ARL_{1s} for the TGWMA and DGWMA sign charts, for shift sizes in the process proportion $p \in \{0.44, 0.48\}$ when ARL₀ = 370 under the SS condition.

n	q	α	L_{DG}	L_{TG}	p					
					0.44			0.48		
					DGWMA	TGWMA	% Diff	DGWMA	TGWMA	% Diff
5	0.5	0.5	2.389	2.091	43.859	30.288	-30.94	206.928	166.350	-19.61
		0.6	2.341	2.014	49.624	33.027	-33.45	229.764	185.130	-19.43
		0.7	2.299	1.967	55.677	37.506	-32.64	247.635	203.183	-17.95
		0.8	2.265	1.935	62.597	41.694	-33.39	254.198	213.941	-15.84
		0.9	2.237	1.927	70.279	48.550	-30.92	264.116	231.882	-12.20
		1.0	2.222	1.926	75.878	55.098	-27.39	269.622	246.856	-8.44
		1.2	2.205	1.946	88.333	66.052	-25.22	286.585	259.882	-9.32
		1.5	2.217	2.003	107.235	84.920	-20.81	302.848	275.864	-8.91
	0.7	0.5	1.982	1.305	16.277	13.523	-16.92	95.150	61.710	-35.14
		0.6	1.859	1.363	16.479	12.851	-22.02	107.211	67.262	-37.26
		0.7	1.780	1.291	17.456	13.163	-24.59	123.888	80.903	-34.70
		0.8	1.730	1.271	19.412	14.298	-26.35	141.093	98.181	-30.41
		0.9	1.708	1.279	22.444	16.288	-27.43	157.615	118.169	-25.03
		1.0	1.702	1.591	26.594	18.844	-29.14	174.780	137.637	-21.25
		1.2	1.722	1.433	35.804	26.324	-26.48	204.987	173.632	-15.30
		1.5	1.807	1.342	54.600	41.247	-24.46	243.555	216.557	-11.08
	0.9	0.5	1.083	0.475	7.625	7.436	-2.47	30.577	21.678	-29.10
		0.6	0.932	0.450	6.223	5.938	-4.58	23.055	16.347	-29.09
		0.7	0.884	0.493	5.642	5.412	-4.09	20.987	14.856	-29.21
		0.8	0.877	0.546	5.619	5.491	-2.29	22.023	15.385	-30.14
		0.9	0.894	0.605	5.830	5.956	2.15	26.516	18.405	-30.59
		1.0	0.925	0.673	6.281	6.459	2.82	33.753	23.658	-29.91
		1.2	1.019	0.810	7.954	7.538	-5.23	59.188	41.791	-29.39
		1.5	1.193	1.027	13.197	11.112	-15.80	114.917	89.586	-22.04
10	0.5	0.5	2.504	2.132	26.935	17.490	-35.06	172.197	114.295	-33.63
		0.6	2.434	2.047	27.804	18.222	-34.46	183.439	129.920	-29.18
		0.7	2.369	1.986	29.593	19.150	-35.29	188.868	140.536	-25.59
		0.8	2.327	1.959	32.977	21.531	-34.71	201.378	159.515	-20.79
		0.9	2.288	1.939	34.960	23.386	-33.11	208.525	164.481	-21.12
		1.0	2.265	1.942	38.845	26.398	-32.04	217.502	178.574	-17.90
		1.2	2.245	1.960	46.172	32.571	-29.46	232.794	199.000	-14.52
		1.5	2.247	2.023	56.286	42.456	-24.57	250.453	220.997	-11.76
	0.7	0.5	2.031	1.365	9.662	8.719	-9.75	61.142	38.501	-37.03
		0.6	1.888	1.295	9.307	8.345	-10.34	63.997	38.869	-39.26
		0.7	1.801	1.274	9.653	8.331	-13.70	72.892	44.824	-38.51
		0.8	1.746	1.281	10.268	8.576	-16.48	83.152	54.095	-34.94
		0.9	1.718	1.306	11.275	9.014	-20.05	98.133	67.415	-31.30
		1.0	1.709	1.343	12.657	9.846	-22.21	109.922	79.157	-27.99
		1.2	1.731	1.438	16.617	12.894	-22.41	138.679	110.143	-20.58
		1.5	1.817	1.594	25.285	19.448	-23.08	176.707	151.183	-14.44
	0.9	0.5	1.095	0.474	4.915	5.371	9.28	20.281	15.720	-22.49
		0.6	0.939	0.452	4.226	4.490	6.26	15.791	11.714	-25.82

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n	q	α	L_{DG}	L_{TG}	p					
					0.44			0.48		
					DGWMA	TGWMA	% Diff	DGWMA	TGWMA	% Diff
		0.7	0.888	0.492	3.946	4.233	7.28	13.544	10.207	-24.64
10	0.9	0.8	0.878	0.547	3.941	4.279	8.59	13.860	10.534	-24.00
		0.9	0.892	0.608	4.146	4.721	13.85	15.535	11.578	-25.47
		1.0	0.926	0.671	4.357	5.060	16.16	18.614	13.987	-24.86
		1.2	1.020	0.810	5.059	5.763	13.92	29.797	22.015	-26.12
		1.5	1.195	1.028	6.988	7.061	1.05	64.014	47.708	-25.47
15	0.5	0.5	2.537	2.149	18.390	12.426	-32.43	139.083	85.675	-38.40
		0.6	2.455	2.053	18.616	12.526	-32.72	145.832	94.802	-34.99
		0.7	2.391	1.995	19.501	13.220	-32.21	152.573	105.469	-30.87
		0.8	2.343	1.961	21.047	14.142	-32.81	166.033	118.315	-28.74
		0.9	2.304	1.946	22.432	15.384	-31.42	171.438	129.249	-24.61
		1.0	2.278	1.943	24.345	16.919	-30.50	176.783	139.162	-21.28
		1.2	2.250	1.962	28.795	20.359	-29.30	191.834	159.782	-16.71
		1.5	2.256	2.025	35.273	26.633	-24.49	214.662	186.033	-13.34
	0.7	0.5	2.046	1.368	7.022	6.908	-1.62	44.609	28.655	-35.76
		0.6	1.899	1.295	6.809	6.577	-3.41	46.257	29.071	-37.15
		0.7	1.806	1.275	6.972	6.536	-6.25	51.171	32.185	-37.10
		0.8	1.750	1.285	7.260	6.654	-8.35	59.622	38.118	-36.07
		0.9	1.725	1.308	7.776	6.766	-12.99	69.432	45.571	-34.37
		1.0	1.713	1.343	8.422	7.252	-13.89	79.886	55.199	-30.90
		1.2	1.737	1.436	10.651	8.579	-19.46	105.700	78.353	-25.87
		1.5	1.818	1.598	15.592	12.344	-20.83	136.590	112.626	-17.54
	0.9	0.5	1.096	0.473	3.777	4.439	17.54	15.725	12.398	-21.15
		0.6	0.941	0.453	3.395	3.837	13.03	12.226	9.454	-22.67
		0.7	0.885	0.490	3.207	3.610	12.57	10.598	8.257	-22.09
		0.8	0.878	0.547	3.283	3.828	16.62	10.472	8.594	-17.93
		0.9	0.892	0.607	3.411	4.094	20.00	11.477	9.464	-17.54
		1.0	0.927	0.671	3.685	4.482	21.61	13.260	10.813	-18.45
		1.2	1.019	0.810	4.140	5.072	22.51	19.968	15.562	-22.06
		1.5	1.194	1.027	5.224	5.800	11.02	41.467	31.588	-23.82

Table 10. ARL_{1s} for the TGWMA and DGWMA sign charts, for shift sizes in the process proportion $p \in \{0.52, 0.56\}$ when ARL₀ = 370 under the SS condition.

n	q	α	L_{DG}	L_{TG}	p					
					0.52			0.56		
					DGWMA	TGWMA	% Diff	DGWMA	TGWMA	% Diff
5	0.5	0.5	2.389	2.091	209.226	166.647	-20.35	43.500	30.298	-30.35
		0.6	2.341	2.014	229.379	186.152	-18.85	49.661	33.589	-32.36
		0.7	2.299	1.967	241.981	206.289	-14.75	56.198	37.903	-32.55
		0.8	2.265	1.935	258.510	217.386	-15.91	63.211	42.328	-33.04
		0.9	2.237	1.927	266.651	230.145	-13.69	67.894	48.677	-28.30
		1.0	2.222	1.926	272.363	238.198	-12.54	76.114	54.874	-27.91
		1.2	2.205	1.946	280.510	259.583	-7.46	87.427	67.071	-23.28
		1.5	2.217	2.003	303.270	276.409	-8.86	105.355	83.311	-20.92
	0.7	0.5	1.982	1.305	94.476	63.667	-32.61	16.181	13.633	-15.74
		0.6	1.859	1.363	107.693	66.727	-38.04	16.129	12.771	-20.82
		0.7	1.780	1.291	121.746	80.677	-33.73	17.395	13.309	-23.49
		0.8	1.730	1.271	139.984	97.726	-30.19	19.687	14.213	-27.81
		0.9	1.708	1.279	160.618	117.059	-27.12	22.461	15.910	-29.17
		1.0	1.702	1.591	177.157	138.139	-22.02	26.357	18.742	-28.89
		1.2	1.722	1.433	211.368	174.282	-17.55	36.221	26.445	-26.99
		1.5	1.807	1.342	244.940	219.838	-10.25	53.780	40.870	-24.00
	0.9	0.5	1.083	0.475	30.831	21.476	-30.34	7.665	7.290	-4.89
		0.6	0.932	0.450	23.653	16.342	-30.91	6.218	5.959	-4.16
		0.7	0.884	0.493	21.010	14.596	-30.53	5.642	5.332	-5.49
		0.8	0.877	0.546	22.352	15.126	-32.33	5.611	5.475	-2.43
		0.9	0.894	0.605	26.435	18.039	-31.76	5.870	5.846	-0.40
		1.0	0.925	0.673	33.676	23.164	-31.21	6.256	6.311	0.88
		1.2	1.019	0.810	58.679	41.973	-28.47	7.829	7.529	-3.84
		1.5	1.193	1.027	113.888	88.482	-22.31	13.098	11.358	-13.28
10	0.5	0.5	2.504	2.132	171.698	113.141	-34.10	26.500	17.378	-34.42
		0.6	2.434	2.047	181.791	127.989	-29.60	27.851	18.072	-35.11
		0.7	2.369	1.986	192.217	140.131	-27.10	29.943	19.009	-36.51
		0.8	2.327	1.959	203.606	154.659	-24.04	32.840	20.987	-36.09
		0.9	2.288	1.939	210.117	164.940	-21.50	35.027	23.477	-32.98
		1.0	2.265	1.942	214.309	184.365	-13.97	38.700	26.905	-30.48
		1.2	2.245	1.960	237.088	198.176	-16.41	46.435	32.414	-30.20
		1.5	2.247	2.023	249.893	226.499	-9.36	55.671	42.291	-24.04
	0.7	0.5	2.031	1.365	61.643	38.092	-38.21	9.659	8.702	-9.91
		0.6	1.888	1.295	64.965	40.388	-37.83	9.396	8.265	-12.04
		0.7	1.801	1.274	73.471	45.753	-37.73	9.709	8.172	-15.83
		0.8	1.746	1.281	84.367	55.115	-34.67	10.311	8.332	-19.20
		0.9	1.718	1.306	98.852	66.096	-33.14	11.188	8.926	-20.22
		1.0	1.709	1.343	110.362	78.863	-28.54	12.533	9.761	-22.12
		1.2	1.731	1.438	139.238	110.958	-20.31	16.313	12.529	-23.20
		1.5	1.817	1.594	178.413	151.261	-15.22	25.381	18.705	-26.30
	0.9	0.5	1.095	0.474	20.707	15.159	-26.79	4.888	5.447	11.45
		0.6	0.939	0.452	15.645	11.589	-25.92	4.252	4.559	7.23

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n	q	α	L_{DG}	L_{TG}	p					
					DGWMA	TGWMA	% Diff	DGWMA	TGWMA	% Diff
10	0.9	0.7	0.888	0.492	13.583	10.180	-25.06	3.923	4.226	7.71
		0.8	0.878	0.547	13.695	10.391	-24.13	3.948	4.396	11.33
		0.9	0.892	0.608	15.164	11.668	-23.05	4.125	4.713	14.25
		1.0	0.926	0.671	18.158	13.793	-24.04	4.364	5.029	15.23
		1.2	1.020	0.810	30.562	21.254	-30.46	5.046	5.660	12.16
		1.5	1.195	1.028	64.556	46.848	-27.43	6.974	7.029	0.79
15	0.5	0.5	2.537	2.149	140.084	87.115	-37.81	18.169	12.305	-32.27
		0.6	2.455	2.053	144.999	93.707	-35.37	18.505	12.461	-32.66
		0.7	2.391	1.995	154.711	105.488	-31.82	19.503	13.133	-32.66
		0.8	2.343	1.961	164.023	117.697	-28.24	21.090	13.935	-33.93
		0.9	2.304	1.946	167.140	130.851	-21.71	22.473	15.331	-31.78
		1.0	2.278	1.943	180.418	138.107	-23.45	24.170	16.923	-29.98
		1.2	2.250	1.962	190.454	158.358	-16.85	28.601	20.450	-28.50
		1.5	2.256	2.025	211.809	180.815	-14.63	35.432	26.249	-25.92
	0.7	0.5	2.046	1.368	45.530	28.873	-36.58	6.986	6.794	-2.74
		0.6	1.899	1.295	45.744	29.116	-36.35	6.763	6.454	-4.58
		0.7	1.806	1.275	52.094	32.461	-37.69	6.948	6.447	-7.21
		0.8	1.750	1.285	59.696	37.995	-36.35	7.224	6.487	-10.21
		0.9	1.725	1.308	69.395	45.373	-34.62	7.747	6.820	-11.96
		1.0	1.713	1.343	79.332	56.029	-29.37	8.360	7.205	-13.81
		1.2	1.737	1.436	106.480	77.171	-27.53	10.570	8.482	-19.76
		1.5	1.818	1.598	138.830	114.277	-17.69	15.444	12.389	-19.78
	0.9	0.5	1.096	0.473	16.054	12.379	-22.89	3.769	4.439	17.77
		0.6	0.941	0.453	12.421	9.460	-23.84	3.387	3.806	12.39
		0.7	0.885	0.490	10.545	8.391	-20.42	3.203	3.689	15.16
		0.8	0.878	0.547	10.476	8.585	-18.05	3.256	3.788	16.32
		0.9	0.892	0.607	11.464	9.323	-18.68	3.423	4.094	19.59
		1.0	0.927	0.671	13.515	10.598	-21.58	3.632	4.425	21.84
		1.2	1.019	0.810	20.138	15.514	-22.96	4.102	5.006	22.03
		1.5	1.194	1.027	42.334	31.521	-25.54	5.214	5.858	12.33

Table 11 compares the $SDRL_1$ values of the TGWMA and DGWMA sign charts under the SS condition. The comparison indicates that, when the shift is small ($p \in \{0.44, 0.48, 0.52, 0.56\}$), all $SDRL_1$ values of the TGWMA sign chart are smaller than that of the DGWMA sign chart, with the exception of some cases when $q = 0.9$. For instance, when $n = 5$, $q = 0.5$, $p = 0.56$, and $\alpha \in \{0.5, 0.6, \dots, 1.5\}$, $SDRL_1 \in \{25.09, 29.98, \dots, 82.98\}$ and $\{39.45, 46.31, \dots, 103.83\}$ for the TGWMA and DGWMA sign charts, respectively, where the former has smaller $SDRL_1$ s (see Table 11). These findings align with our conclusion regarding the ARL_1 performance of the two charts.

Table 11. SDRL₁s for the TGWMA and DGWMA sign charts, for shift sizes in the process proportion $p \in \{0.44, 0.48, 0.52, 0.56\}$ when $ARL_0 = 370$ under the SS condition.

n	q	α	L_{DG}	L_{TG}	p							
					0.44		0.48		0.52		0.56	
					DGWMA	TGWMA	DGWMA	TGWMA	DGWMA	TGWMA	DGWMA	TGWMA
5	0.5	0.5	2.389	2.091	39.65	25.38	202.21	168.00	209.16	167.41	39.45	25.09
		0.6	2.341	2.014	46.33	28.78	232.81	187.48	226.75	186.74	46.31	29.98
		0.7	2.299	1.967	52.82	35.02	251.29	208.61	240.99	210.02	52.89	34.64
		0.8	2.265	1.935	60.22	38.77	253.23	217.47	260.73	219.58	62.15	40.02
		0.9	2.237	1.927	68.86	47.27	262.70	234.67	265.58	235.47	66.81	46.95
		1.0	2.222	1.926	73.80	54.24	270.77	250.54	273.22	239.96	75.58	54.18
		1.2	2.205	1.946	87.91	64.56	288.01	263.26	281.69	265.09	87.22	66.52
		1.5	2.217	2.003	106.42	85.22	304.36	277.18	300.35	279.07	103.83	82.98
	0.7	0.5	1.982	1.305	11.99	10.44	88.99	61.59	88.38	64.23	11.80	10.23
		0.6	1.859	1.363	12.52	10.11	106.50	72.04	106.75	73.11	12.27	9.89
		0.7	1.780	1.291	13.98	10.41	125.69	91.97	126.18	90.82	13.66	10.66
		0.8	1.730	1.271	16.30	11.94	144.87	114.70	145.23	112.94	16.69	11.90
		0.9	1.708	1.279	19.92	14.29	160.08	136.75	165.15	134.27	20.05	14.32
		1.0	1.702	1.591	24.34	17.62	180.52	159.75	183.13	157.26	24.54	17.86
		1.2	1.722	1.433	34.60	26.22	210.87	198.16	215.10	194.78	34.81	27.07
		1.5	1.807	1.342	54.31	42.08	250.10	234.79	255.61	238.02	53.32	41.30
	0.9	0.5	1.083	0.475	6.47	8.15	28.70	27.01	28.95	27.10	6.43	7.97
		0.6	0.932	0.450	5.47	6.89	24.02	22.47	24.25	22.12	5.41	6.84
		0.7	0.884	0.493	4.94	6.32	23.01	20.70	22.59	20.52	4.93	6.25
		0.8	0.877	0.546	4.78	6.20	24.88	21.40	24.64	21.25	4.78	6.20
		0.9	0.894	0.605	4.85	6.29	31.02	26.17	30.87	25.48	4.84	6.27
		1.0	0.925	0.673	5.15	6.41	41.90	33.67	40.85	33.57	5.08	6.38
		1.2	1.019	0.810	6.53	6.91	74.13	60.78	73.76	61.93	6.62	7.03
		1.5	1.193	1.027	12.95	11.00	137.34	124.92	137.57	123.44	12.53	11.24
10	0.5	0.5	2.504	2.132	22.83	12.78	170.68	110.45	172.55	111.46	22.43	12.64
		0.6	2.434	2.047	24.06	14.23	178.71	131.59	179.20	126.63	24.58	14.25
		0.7	2.369	1.986	26.93	15.60	187.69	139.35	188.46	140.45	27.07	15.69
		0.8	2.327	1.959	30.96	18.26	201.68	158.37	201.29	156.04	30.20	18.15
		0.9	2.288	1.939	32.73	21.03	209.84	168.85	205.45	167.54	32.97	21.30
		1.0	2.265	1.942	36.74	24.33	216.36	179.57	216.46	186.87	36.73	25.08
		1.2	2.245	1.960	44.80	30.80	235.61	204.78	235.65	200.55	44.20	30.25
		1.5	2.247	2.023	55.90	41.23	248.33	223.54	253.55	231.50	55.27	40.95
	0.7	0.5	2.031	1.365	6.58	6.28	52.69	35.20	54.32	35.10	6.55	6.26
		0.6	1.888	1.295	6.36	5.91	60.24	38.52	60.34	39.82	6.43	5.78
		0.7	1.801	1.274	6.68	5.85	70.56	48.03	70.31	48.58	6.78	5.77
		0.8	1.746	1.281	7.39	6.09	82.88	58.20	85.38	60.16	7.38	5.90
		0.9	1.718	1.306	8.57	6.60	97.24	75.23	101.62	74.02	8.41	6.63
		1.0	1.709	1.343	10.19	7.62	114.20	88.26	112.90	89.77	10.24	7.60
		1.2	1.731	1.438	14.51	11.18	140.21	123.56	143.41	125.22	14.36	10.91
		1.5	1.817	1.594	24.27	18.55	180.57	160.83	181.01	163.89	24.14	17.71
	0.9	0.5	1.095	0.474	3.96	5.49	18.44	18.63	18.56	18.25	3.92	5.49
		0.6	0.939	0.452	3.43	4.88	15.26	15.24	15.29	15.13	3.41	4.87

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n	q	α	L_{DG}	L_{TG}	p							
					0.44		0.48		0.52		0.56	
					DGWMA	TGWMA	DGWMA	TGWMA	DGWMA	TGWMA	DGWMA	TGWMA
10	0.9	0.7	0.888	0.492	3.13	4.59	13.79	13.67	13.81	13.57	3.12	4.57
		0.8	0.878	0.547	3.06	4.54	14.16	13.60	14.00	13.40	3.05	4.59
		0.9	0.892	0.608	3.13	4.68	16.13	14.78	15.90	14.73	3.11	4.68
		1.0	0.926	0.671	3.21	4.75	20.18	17.54	20.19	17.15	3.19	4.70
		1.2	1.020	0.810	3.58	4.76	35.21	29.59	35.71	28.99	3.52	4.73
		1.5	1.195	1.028	5.46	5.51	75.75	64.23	75.97	62.43	5.30	5.43
15	0.5	0.5	2.537	2.149	14.62	8.50	135.64	80.41	135.92	82.76	14.45	8.39
		0.6	2.455	2.053	15.51	8.81	143.65	92.89	140.54	93.07	15.23	9.00
		0.7	2.391	1.995	16.68	10.07	150.64	105.54	154.17	103.02	16.87	9.98
		0.8	2.343	1.961	18.37	11.43	167.85	117.48	162.55	117.36	18.74	11.18
		0.9	2.304	1.946	20.03	12.94	173.87	127.80	168.63	130.99	20.54	12.84
		1.0	2.278	1.943	22.64	14.67	175.07	141.07	183.16	141.15	22.16	14.63
		1.2	2.250	1.962	27.46	18.46	189.80	162.95	190.35	160.08	27.06	18.73
		1.5	2.256	2.025	33.92	25.09	212.15	189.28	216.74	184.52	34.41	24.77
	0.7	0.5	2.046	1.368	4.67	4.66	37.10	25.13	37.36	25.11	4.52	4.50
		0.6	1.899	1.295	4.43	4.41	41.36	26.49	39.99	26.76	4.34	4.30
		0.7	1.806	1.275	4.54	4.27	47.82	32.29	48.73	33.02	4.42	4.21
		0.8	1.750	1.285	4.80	4.30	58.73	40.02	58.29	40.09	4.66	4.21
		0.9	1.725	1.308	5.37	4.41	68.68	49.29	68.44	49.29	5.15	4.41
		1.0	1.713	1.343	6.17	4.98	79.23	61.16	79.28	61.90	6.03	4.86
		1.2	1.737	1.436	8.70	6.54	105.78	85.66	108.50	85.37	8.59	6.52
		1.5	1.818	1.598	13.96	10.81	139.49	122.74	140.42	123.26	13.65	10.62
	0.9	0.5	1.096	0.473	2.93	4.36	13.93	14.59	14.11	14.62	2.87	4.36
		0.6	0.941	0.453	2.63	3.96	11.64	12.09	11.79	11.97	2.61	3.93
		0.7	0.885	0.490	2.42	3.79	10.44	10.79	10.33	10.82	2.40	3.80
		0.8	0.878	0.547	2.38	3.85	10.28	10.71	10.26	10.64	2.35	3.84
		0.9	0.892	0.607	2.43	3.96	11.17	11.32	11.14	11.18	2.41	3.94
		1.0	0.927	0.671	2.51	4.04	13.41	12.67	13.57	12.33	2.51	4.01
		1.2	1.019	0.810	2.65	4.03	21.68	18.95	22.13	19.18	2.65	4.00
		1.5	1.194	1.027	3.42	4.04	47.24	40.05	48.58	40.18	3.36	4.03

By comparing the ZS and SS performances of the charts, it is found that the magnitude in which the TGWMA sign chart surpasses the DGWMA sign chart in detecting small shifts is larger under the SS condition. This is reflected by the smaller ARL_1 s for the TGWMA sign chart compared to that of the DGWMA sign chart for all (n, q, α) combinations, even when $q = 0.9$. For example, when $p = 0.56$, $n = 5$, $q = 0.5$, and $\alpha = 0.7$, $ARL_1 = 115.410$ and 98.026 for the DGWMA and TGWMA sign charts, respectively, under the ZS condition, where the latter beats the former by 15.06% (see Table 6). For the same (p, n, q, α) combination but under the SS condition, the latter ($ARL_1 = 37.903$) beats the former ($ARL_1 = 56.198$) by 32.55% (see Table 10).

From the findings for the ZS and SS conditions, the following outcomes can be drawn: (1) For very small shifts in the process proportion ($p \in \{0.48, 0.52\}$), the TGWMA sign chart signals an out-of-control quicker than the DGWMA sign chart for both the ZS and SS conditions, irrespective of the value of q . (2) For moderately small shifts ($p \in \{0.44, 0.56\}$) and $q \in \{0.5, 0.7\}$, the TGWMA sign chart beats the DGWMA sign chart under both the ZS and SS conditions. (3) The superiority of the TGWMA sign chart to the DGWMA sign chart in detecting small shifts is more pronounced under the SS condition.

5. An example of application

To illustrate the implementation of the TGWMA sign chart, we use the data generated from the beta distribution, which are shown in Table 12. The observations of the first 30 samples (Phase-I), each of size 10, are generated from an in-control process where the two shape parameters of the beta distribution, say v_1 and v_2 are set to have a value of 0.5. Observations in the last 8 samples (Phase-II), each of size 10, are generated for the out-of-control process, where the shape parameters of the beta distribution are set as $v_1 = 2$ and $v_2 = 1$. The in-control process mean is computed as $\frac{v_1}{v_1+v_2} = 0.5$, while the out-of-control mean is computed as 0.667. Therefore, the out-of-control samples 31–38 represent the out-of-control process with an increasing process mean.

Table 12. Data from beta distribution for the example of application.

Sample,	X_{1t}	X_{2t}	X_{3t}	X_{4t}	X_{5t}	X_{6t}	X_{7t}	X_{8t}	X_{9t}	X_{10t}	S_t	TG_t	DG_t
1	0.570	0.0801	0.8845	0.6294	0.6825	0.9999	0.5792	0.7196	0.1024	0.2754	7	5.2500	5.5000
2	0.503	0.7026	0.7163	0.9572	0.9987	0.9738	0.8015	0.0932	0.4710	0.9966	8	5.7135	6.2013
3	0.997	0.0099	0.6221	0.1640	0.5071	0.0035	0.6010	0.2238	0.1793	0.1614	4	5.7142	5.7671
4	0.022	0.9918	0.8299	0.0003	0.9984	0.7867	0.6164	0.4991	0.7321	0.4766	6	5.7358	5.7734
5	0.002	0.7840	0.1454	0.9021	0.8500	0.5226	0.9881	0.0258	0.9102	0.2172	6	5.7765	5.8260
6	0.419	0.0599	0.8491	0.9793	0.4959	0.8305	0.4966	0.0167	0.7635	0.0462	4	5.5728	5.3769
7	0.831	0.0181	0.1706	0.4158	0.7909	0.9996	0.0095	0.4369	0.9753	0.0751	4	5.2772	4.9649
8	0.325	0.9188	0.2579	0.9519	0.7523	0.6978	0.9707	0.0090	0.0993	0.0275	5	5.1065	4.9028
9	0.486	0.1240	0.4065	0.8768	0.7640	0.7977	0.0133	0.8636	0.0967	0.1282	4	4.8958	4.6586
10	0.427	0.2337	0.5386	0.1306	0.3301	0.8889	0.7537	0.9757	0.3545	0.9814	5	4.8145	4.7037
11	0.021	0.3061	0.2492	0.0321	0.0126	0.9990	0.0055	0.7182	0.1173	0.8493	3	4.5559	4.2795
12	0.560	0.5740	0.9958	0.1685	0.9895	0.3019	0.6761	0.9728	0.9422	0.0103	7	4.7412	4.8946
13	0.995	0.0295	0.0014	0.5842	0.8523	0.3288	0.3556	0.4099	0.5343	0.5306	5	4.8703	5.0071
14	0.626	0.0034	0.6669	0.9214	0.0125	0.9973	0.6112	0.0001	0.2796	0.5704	6	5.0696	5.2821
15	0.076	0.4074	0.0250	0.8413	0.9966	0.5883	0.2643	0.9618	0.2809	0.8561	5	5.1520	5.2573
16	0.469	0.0032	0.2489	0.9944	3.55E-	0.6440	0.9845	0.1910	0.3805	0.3057	3	4.9160	4.6950
17	0.824	0.0186	0.9999	0.8650	0.3863	0.9208	0.9998	0.5184	0.0085	0.0699	6	4.9349	4.9361
18	0.054	0.2233	0.4345	0.8723	0.6366	0.7185	0.7502	0.0198	0.3740	0.0204	4	4.8332	4.7290
19	0.004	0.8768	5.86E-	0.9730	0.0515	0.8437	0.3679	0.8840	0.6035	0.1541	5	4.8059	4.7676
20	0.376	0.2355	0.4572	0.9924	0.6600	0.1293	0.8499	0.0094	0.5208	0.7177	5	4.8200	4.8282
21	0.077	0.3700	0.9717	0.00111	0.8748	0.2553	0.5996	0.5380	0.7887	0.0407	5	4.8506	4.8804
22	0.351	0.4647	0.5917	0.2451	0.5480	0.0659	0.2733	0.5804	0.1872	0.0025	3	4.6338	4.4193
23	0.808	0.2625	0.0925	0.8255	0.3767	0.9330	0.2117	0.2049	0.9302	0.8982	5	4.5748	4.4952
24	0.730	0.9339	0.0560	0.3375	0.9340	0.7078	0.5963	0.3827	0.7343	0.8731	7	4.8557	5.1249
25	0.990	0.0366	0.0188	0.6348	0.0643	0.6576	0.7008	0.0531	0.9979	0.6519	6	5.1361	5.4397
26	0.152	0.2855	0.6655	0.9994	0.0946	0.9967	0.4131	0.7836	0.0059	0.4662	4	5.1209	5.1390
27	0.051	0.9831	0.4765	0.8123	0.0293	0.0767	0.9967	0.4619	0.0041	0.9831	4	4.9641	4.8162
28	0.966	0.9771	0.4041	0.9568	0.1992	0.9989	0.2199	0.5505	0.2745	0.7642	6	5.0172	5.0590
29	0.386	0.0527	0.6222	0.6106	0.8858	0.9388	0.3434	0.3541	0.3172	0.9605	5	5.0448	5.0749
30	0.040	0.3719	0.5778	0.6285	0.6814	0.9991	0.1053	0.9997	0.0436	0.2898	5	5.0516	5.0617
31	0.875	0.2565	0.4494	0.2390	0.2225	0.3343	0.1083	0.9959	0.8059	0.7943	4	4.9225	4.7951
32	0.939	0.9905	0.4428	0.5394	0.7370	0.7404	0.4525	0.2217	0.8231	0.6579	7	5.1199	5.3055

Continued on next page

Sample,	X_{1t}	X_{2t}	X_{3t}	X_{4t}	X_{5t}	X_{6t}	X_{7t}	X_{8t}	X_{9t}	X_{10t}	S_t	TG_t	DG_t
33	0.595	0.2299	0.6993	0.7108	0.9304	0.9323	0.9121	0.9138	0.8760	0.7989	9	5.7032	6.3023
34	0.915	0.7351	0.2272	0.1542	0.6596	0.7372	0.3519	0.6821	0.6742	0.5813	7	6.1397	6.6372
35	0.307	0.6946	0.9601	0.8805	0.3982	0.0988	0.9869	0.8991	0.7780	0.6517	7	6.4388	6.7986
36	0.712	0.2154	0.4185	0.4989	0.6470	0.9694	0.9183	0.6171	0.3018	0.6075	6	6.5116	6.6334
37	0.993	0.8723	0.7515	0.9735	0.7097	0.6876	0.0605	0.7752	0.4794	0.9719	8	6.7212	6.9549
38	0.610	0.6176	0.5627	0.3619	0.7872	0.8606	0.6775	0.9885	0.7284	0.9417	9	7.1023	7.5135

The number of observations larger than the grand average, $\bar{\bar{X}} = 0.5029$ in each sample, is counted and the count is recorded as S_t ($= \sum_{i=1}^n I_{it}$) (for $t = 1, 2, \dots, 38$) in the third to last column of Table 12. Note that $\bar{\bar{X}}$ is computed from the 30×10 in-control observations of the 30 Phase-I samples, and it is also taken as μ_0 in Eq (1). The TG_t values for the TGWMA sign chart, computed using Eq (11), for the 38 samples are given in the second to last column of Table 12. $ARL_0 = 370$, the parameter combination $(n, q, \alpha) = (10, 0.5, 0.9)$, and the ZS condition are considered. Then, $L_{TG} = 2.750$ is obtained from Table 2. The TG_t values for the 38 samples are plotted in Figure 1 as the TGWMA sign chart, based on the limits $UCL = 6.4993$, $CL = 5$, and $LCL = 3.5007$, computed using Eqs (20a)–(20c), respectively.

Figure 1 shows a plot of $TG_1, TG_2, \dots, TG_{30}$ (first 30 samples) within the limits UCL/LCL of the TGWMA sign chart. However, the TG_t statistic for the last 8 samples ($t = 31, 32, \dots, 38$) show an increasing trend, and the first out-of-control signal is detected at sample 36. In fact, TG_t (for $t = 36, 37$ and 38) is above the UCL of the TGWMA sign chart, hence the last 3 samples are out-of-control. Following the detection of these out-of-control signals, corrective actions are made to investigate the underlying process to bring the out-of-control process back into the in-control situation again.

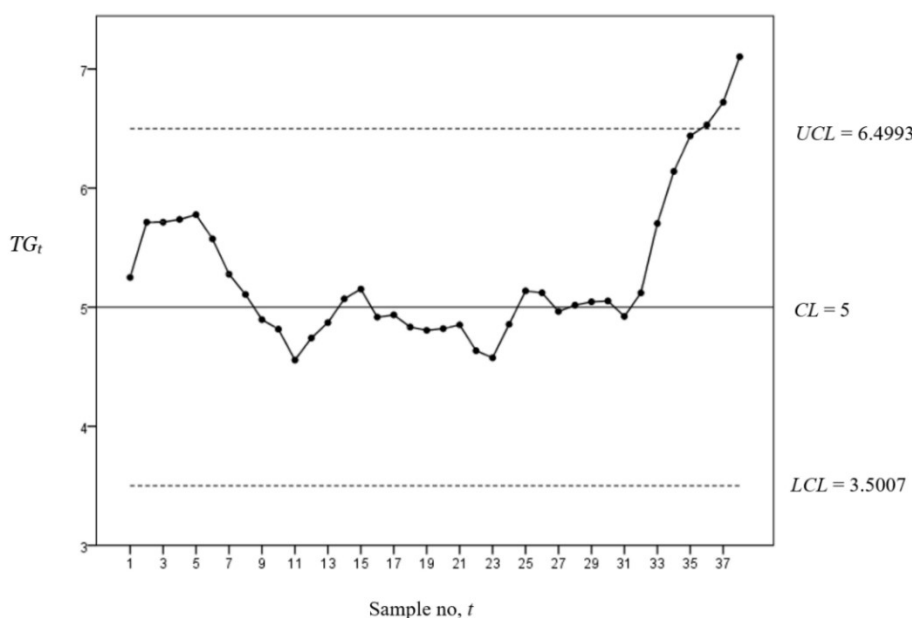


Figure 1. An example of application for the TGWMA sign chart, based on $(n, q, \alpha) = (10, 0.5, 0.9)$ and $ARL_0 = 370$.

In order to compare the TGWMA and DGWMA sign charts, we also construct the DGWMA sign chart in Figure 2 based on the chart's limits $UCL = 6.8535$, $CL = 5$, and $LCL = 3.1465$, computed using Eqs (10a)–(10c), respectively. Figure 2 shows that $DG_1, DG_2, \dots, DG_{30}$ (first 30 samples) are within

the limits UCL/LCL of the DGWMA sign chart. However, the DG_t statistic in the last 8 samples (i.e., $t = 31, 32, \dots, 38$) shows an increasing trend, and the first out-of-control signal is detected at sample 37 by the chart.

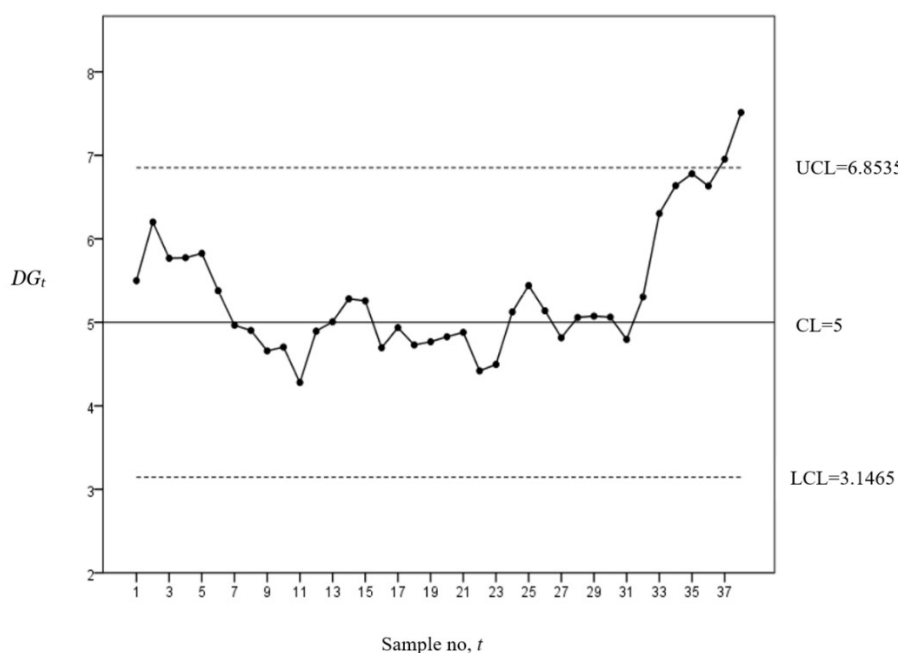


Figure 2. An example of application for the DGWMA sign chart, based on $(n, q, \alpha) = (10, 0.5, 0.9)$ and $ARL_0 = 370$.

By comparing Figures 1 and 2, it is found that the TGWMA sign chart detects the increasing shift in the mean slightly quicker than the DGWMA sign chart. The former signals at sample 36, while the latter is at sample 37.

6. Conclusions

The purpose of this study is to further enhance the ability of non-parametric control charts to detect small shifts, and to meet the real-world demands for high-performance control charts in achieving a stable process control scenario. The TGWMA sign chart's statistic (TG_t), expectation and variance of TG_t , and control limits of the TGWMA sign chart are derived in this paper. Through numerical simulations, the detection capability of the TGWMA and DGWMA sign charts, especially in detecting small shifts, is compared, in terms of the charts' ARL_1 s for both ZS and SS conditions. The findings show that regardless of the ZS or SS condition, and under most parameter combinations, the TGWMA sign chart is more effective in detecting small shifts in the process proportion than the DGWMA sign chart, and this superiority becomes more apparent as the shift becomes smaller. The implementation of the TGWMA sign chart is also demonstrated with an example. Given the excellent small shift detection effectiveness of the TGWMA sign chart, the said chart can be applied in actual production and manufacturing processes, especially when the data come from an unknown underlying distribution.

Practitioners in manufacturing and service industries can use the developed non-parametric TGWMA sign chart in process monitoring effectively, as tables providing the values of the constants L_{TG} and R_2 are given for various (n, q, α) combinations when $ARL_0 = 370$ to facilitate the computation of the chart's limits. This will enable practitioners to use the developed chart instantaneously for more efficient

process monitoring so that small shifts can be detected faster when the process distribution is unknown.

Further research could focus on investigating the performance of the proposed TGWMA sign chart when measurement errors exist in the process, as well as by considering the presence of autocorrelation in the underlying process. The use of advanced optimization techniques, including genetic algorithms or deep reinforcement learning, in computing the optimal parameters of the TGWMA sign chart could be a potential area for further research. An extension of the TGWMA sign control charting approach to its multivariate counterpart for monitoring multivariate non-parametric data could also be made. The fast initial response feature could also be incorporated into the TGWMA sign chart to improve the sensitivity of the chart in detecting shifts at process start-up.

Author contributions

Dongmei Cui: Conceptualization, formal analysis, investigation, methodology, software, visualization, writing-original draft; Michael B. C. Khoo: Data Curation, funding acquisition, project administration, supervision, writing - review & editing; Huay Woon You: Investigation and supervision; Sajal Saha: Software, visualization, supervision; Zhi Lin Chong: Supervision, writing - review & editing. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in producing this article.

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Conflict of interest

The authors declare no competing interests.

References

1. R. W. Amin, M. R. Reynolds Jr., S. Bakir, Nonparametric quality control charts based on the sign statistic, *Commun. Stat. Theory Methods*, **24** (1995), 1597–1623. <https://doi.org/10.1080/03610929508831574>
2. S. Chakraborti, S. Eryilmaz, A nonparametric Shewhart-type signed-rank control chart based on runs, *Commun. Stat. Simul. Comput.*, **36** (1995), 335–356. <http://doi.org/10.1080/03610910601158427>
3. C. Zhou, Y. Zhang, Z. Wang, Nonparametric control chart based on change-point model, *Stat. Pap.*, **50** (2009), 13–28. <https://doi.org/10.1007/s00362-007-0054-7>
4. S. K. Khilare, D. T. Shirke, Nonparametric synthetic control charts for process variation, *Qual. Reliab. Eng. Int.*, **28** (2012), 193–202. <https://doi.org/10.1002/qre.1233>

5. W. Tapang, A. Pongpullponsak, S. Sarikavanij, Three non-parametric control charts based on ranked set sampling, *Chiang Mai J. Sci.*, **43** (2016), 914–929. Available from: <https://www.thaiscience.info/journals/Article/CMJS/10982829.pdf>.
6. Y. Li, D. Pei, Z. Wu, A multivariate non-parametric control chart based on run test, *Comput. Ind. Eng.*, **149** (2020), 106839. <https://doi.org/10.1016/j.cie.2020.106839>
7. F. Boroomandi, M. Kharrati-Kopaei, Non parametric PROS quality control chart for monitoring the process mean, *Commun. Stat. Theory Methods*, **51** (2022), 1706–1723. <https://doi.org/10.1080/03610926.2020.1767139>
8. J. C. Malela-Majika, Nonparametric precedence chart with repetitive sampling, *Stat*, **12** (2023), e512. <https://doi.org/10.1002/sta4.512>
9. J. C. Malela-Majika, O. A. Adeoti, E. Rapoo, An EWMA control chart based on the Wilcoxon rank-sum statistic using repetitive sampling, *Int. J. Qual. Reliab. Manage.*, **35** (2018), 711–728. <https://doi.org/10.1108/IJQRM-10-2016-0181>
10. A. Tang, J. Sun, X. Hu, P. Castagliola, A new nonparametric adaptive EWMA control chart with exact run length properties, *Comput. Ind. Eng.*, **130** (2019), 404–419. <https://doi.org/10.1016/j.cie.2019.02.045>
11. V. Alevizakos, K. Chatterjee, C. Koukouvinos, The triple exponentially weighted moving average control chart, *Qual. Technol. Quant. Manag.*, **18** (2021), 326–354. <https://doi.org/10.1080/16843703.2020.1809063>
12. A. Haq, An EWMA sign chart for monitoring processes with fixed and variable sample sizes, *Stat*, **13** (2024), e652. <https://doi.org/10.1002/sta4.652>
13. P. S. Ng, M. B. C. Khoo, S. Saha, M. H. Lee, A variable sampling interval EWMA t chart with auxiliary information - A robustness study in the presence of estimation error, *Alex. Eng. J.*, **61** (2022), 6043–6059. <http://doi.org/10.1016/j.aej.2021.11.033>
14. L. Xue, L. An, S. Feng, Y. Liu, H. Wu, Q. Wang, A nonparametric adaptive EWMA control chart for monitoring multivariate time-between-events-and-amplitude data, *Comput. Ind. Eng.*, **193** (2024), 110250. <http://doi.org/10.1016/j.cie.2024.110250>
15. S. H. Sheu, T. C. Lin, The generally weighted moving average control chart for detecting small shifts in the process mean, *Qual. Eng.*, **16** (2023), 209–231. <https://doi.org/10.1081/QEN-120024009>
16. S. H. Sheu, Y. T. Hsieh, The extended GWMA control chart, *J. Appl. Stat.*, **36** (2009), 135–147. <https://doi.org/10.1080/02664760802443913>
17. S. L. Lu, An extended nonparametric exponentially weighted moving average sign control chart, *Qual. Reliab. Eng. Int.*, **31** (2015), 3–13. <https://doi.org/10.1002/qre.1673>
18. N. Chakraborty, S. Chakraborti, S. W. Human, N. Balakrishnan, A generally weighted moving average signed-rank control chart, *Qual. Reliab. Eng. Int.*, **32** (2016), 2835–2845. <https://doi.org/10.1002/qre.1968>
19. S. L. Lu, Non parametric double generally weighted moving average sign charts based on process proportion, *Commun. Stat. Theory Methods*, **47** (2018), 2684–2700. <https://doi.org/10.1080/03610926.2017.1342832>
20. V. Alevizakos, C. Koukouvinos, K. Chatterjee, A nonparametric double generally weighted moving average signed-rank control chart for monitoring process location, *Qual. Reliab. Eng. Int.*, **36** (2020), 2441–2458. <https://doi.org/10.1002/qre.2706>
21. K. Mabude, J. C. Malela-Majika, M. Aslam, Z. L. Chong, S. C. Shongwe, Distribution-free composite Shewhart-GWMA Mann-Whitney charts for monitoring the process location, *Qual. Reliab. Eng. Int.*, **37** (2021), 1409–1435. <https://doi.org/10.1002/qre.2804>

22. D. G. Godase, S. B. Mahadik, The combined Shewhart-CUSUM sign charts, *Commun. Stat. Simul. Comput.*, **53** (2021), 357–366. <https://doi.org/10.1080/03610918.2021.2020287>
23. K. Mabude, J. C. Malela-Majika, P. Castagliola, S. C. Shongwe, Distribution-free mixed GWMA-CUSUM and CUSUM-GWMA Mann-Whitney charts to monitor unknown shifts in the process location, *Commun. Stat. Simul. Comput.*, **51** (2022), 6667–6690. <https://doi.org/10.1080/03610918.2020.1811331>



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