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Research article

Adaptive fuzzy consensus tracking control of multi-agent systems with predefined time

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Abstract: This article concentrated on practically predefined-time consensus tracking control of multi-agent systems (MASs) exhibiting strict feedback dynamics, and attaining a predefined level of accuracy. Specifically, a sufficient condition was derived to decide whether the consensus error converges into a predefined region in a certain predefined time that can be appointed beforehand irrespective of initial conditions. By the established stability criterion, a distributed robust fuzzy controller was designed, whose primary aim is to ensure the cooperative stability of the consensus output tracking errors. Command filters were utilized to obtain the estimations of virtual inputs and their derivatives. More notably, the followers can track a designated trajectory structure guided by the leader within a predefined time and tracking errors can be arbitrarily small, which provides a theoretical criterion for the consensus tracking problem of MASs. Finally, an example was utilized to indicate the effectiveness and practicality of the proposed approach.

Keywords: multi-agent; predefined time; stability criterion; fuzzy control; command filter
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1. Introduction

Over the last several decades, leader-following consensus control of multi-agent systems (MASs) has received growing attention. At the beginning, most related works were devoted to studying linear MASs [1], or low-order (i.e., first-order or second-order) nonlinear MASs [2–4]. Recently, some methods have been proposed for high-order MASs, e.g., [5, 6]. In [5], a global optimal consensus topic for the MAS was revisited by designing a bounded local controller, where each of the agents is described by the dynamics of the integrator chain as well as has an objective function known only by itself, and all agents reach the consensus while minimizing the sum of their objective functions. By using ellipsoidal regional investigation and Lyapunov redesign, a distributed consensus controller was proposed in [6] for MASs with smooth nonlinear functions. It should be emphasized that uncertainties including uncertain terms and external disturbances are not considered in the above-mentioned literature. In fact, the existence of uncertainties is inevitable in the actual control systems, which will reduce the control performance, or even cause equipment breakdown and result in system instability. Up to now, a great quantity of effective control methods have been developed to handle this matter, including sliding mode control [7], observer-based neural network control [8], adaptive fuzzy control [9], and so on. Unfortunately, it is worth noting that numerous existing findings, including the ones highlighted earlier, predominantly concentrate on aspects such as asymptotic stability of tracking errors between leaders and followers. These outcomes imply that the state or output of the followers can only align with the ones of the leader over an indefinitely extended period. Note that settling time is a key index for the consensus tracking of the MASs, because faster convergence speed has better flexibility and stronger robustness against sudden obstacles, structural transformations, and other complex environments.

Bearing this objective in mind, the concept of finite-time stability emerges as a valuable consideration. Finite-time stability not only enhances convergence speed under specific conditions but also brings forth advantages such as robustness against uncertainties and a robust defense against interference. These attributes are inherent within the realm of traditional optimal control theory. As a typical nonlinear control tool, the finite-time stability theory was first applied to MASs in [10], where the formation message is separated into the global message and the local message. Afterward, the investigation on finite-time cooperative consensus and formation control of MASs has been reported, e.g., in [11, 12]. Reference [11] proposed a finite-time controler for MASs with dead-zone input, actuator fault, and unknown control directions, in which a sufficiently small positive number is devised to address the singularity phenomenon produced by utilizing the finite-time backstepping procedure. In [12], a finite-time backstepping controller was implemented for uncertain MASs by employing barrier Lyapunov functions to guarantee system performances. A leaderless consensus control scheme for MASs constructed by some mechanical systems that suffer from parametric uncertainties under an undirected graph was considered in [13], where both the finite-time distributed control technique and the transient characteristics were explored from the perspectives of convergence rate and time. However, these existing finite-time consensus control results are incapable of making the settling time independent of the system initial value, i.e., when the system initial value increases, the convergence time may be infinitely prolonged, and this confines the promotion of the finite-time technique in some actual dynamics with relatively large initial values.

To avert the aforementioned issues, fixed-time stability was introduced, by which the settling time can be prescribed in advance, free from the influence of the initial conditions. Therefore, some fixed-time cooperative consensus control methods have been applied for MASs, and some results have been emerged [14–22]. The fixed-time formation-containment control issue of general MASs under time-varying output was considered in [14], in which the considered MASs comprise one virtual leader, multiple leaders, and followers. Reference [15] focused on the formation tracking problem for uncertain MASs without the leader's velocity information, where a fixed-time formation control approach was combined with the nonsmooth backstepping technique in view of the presented cascaded leader state observer and radial basis functions. A novel robust optimization algorithm of distributed MASs which ensures fixed-time convergence was proposed in [16], and the method was developed under the Lyapunov analysis. In [17], the fixed-time consensus problem of MASs subject to external disturbances was handled by using an improved observer. Note that the existing fixed-time consensus works can achieve the convergence objective within the desired time; however, the convergence time will inevitably be overestimated and the relevance of settling time and system parameters is indistinct. Thus, the settling-time-derived stability analysis is usually much larger than the true convergence time and it will be an extremely difficult mission to realize consensus at the time the user expects it.

In the sense of the aforementioned problems, another scheme exhibiting predefined-time stability was first proposed in [23–32], where convergence time can be artificially pre-set and avoid the over-valuation of the convergence time boundary. For instance, the predefined-time distributed optimization scheme of linear MASs subject to equality constraints under directed and connected topologies was studied in [24]. Reference [25] solved the containment control of MASs with directed topologies, where a sampled-data-based agreement was analyzed by utilizing motion-planning approaches. The predefined-time optimization problem of homogeneous and heterogeneous MASs was considered in [26], where two distributed approaches for the studied MASs based on the time-based generator technique and output feedback were proposed to minimize the global cost function. In [28], a control algorithm based on a prescribed-time observer was proposed by introducing a time-varying function, which solves the predefined-time tracking problem of the second-order multi-agent network. However, these results are applicable for linear systems and do not consider the impact of nonparametric uncertainties and external disturbances. In [31], based on the assumption that the nonlinear system function satisfies the Lipschitz condition and the external disturbance satisfies the state-dependent condition, a consensus control strategy was proposed for a class of first-order nonlinear multi-agent systems to ensure that the tracking error approaches zero within a predefined time. More notably, these results all require to achieve zero steady-state error, which is difficult to guarantee in the actual system due to the influence of the external complex environments or emergencies on the MASs, and it is no exaggeration to say that if the existence of adverse factors is ignored, the system may fail to be controlled. In contrast, the controller design that ensures sufficient accuracy can promote the practical applications of MASs, which motivates the so-called practically predefined-time control, in which the steady-state error tends into a sufficiently small area near the origin with a predefined time. However, when devising a predefined-time stabilizing controller, two main difficulties demand to be considered. One is that utilizing mathematical tools to achieve predefined time and accuracy control while avoiding singularity issues is a challenging task. Another aspect is that selecting an expected Lyapunov function and designing a systematic construction method to meet predefined-time stability conditions is not easy. Hence, further research on the predefined-time control method for strict-feedback MASs is necessary.

Motivated by the above observations, a general Lyapunov criterion that is feasible to decide whether the consensus error converges into an arbitrarily small set within a user-set time is desired in this paper. Under the derived lemma, an adaptive fuzzy predefined-time control for nonlinear MASs is implemented via the backstepping control technique. The command filtering technology is employed to address the classical "explosion of complexity" problem. Compared with some prior works, the main distinguishing features of our work are summarized as follows.

- This article considers the practically predefined-time stability problem of a class of uncertain high-order MASs. The assumptions are looser than those in [30, 31], and this article does not require the unknown function to satisfy the Lipschitz condition and the external disturbance to satisfy the state-dependence condition. Instead, it only requires that the unknown function is a smooth function and the external disturbance is bounded.

- Based on the application of the developed practically predefined-time stability criterion in this article and the piecewise function in Lemma 7, the proposed predefined-time controller in this article contains both the predefined-time parameter and the predefined-range parameter. The difference from the fixed-time controllers in [14–16, 22] is that the control effect can achieve the tracking error entering different predefined ranges within the predefined time by setting these two parameters, while the fixed-time control methods in [14–16, 22] cannot achieve it.
- The practically predefined-time adaptive fuzzy control strategy developed in this article effectively avoids the problem of complexity explosion in [20] due to the use of first-order filters. Moreover, this strategy also overcomes the computational complexity in [20, 21].

The rest of the paper is organized as follows. Section 2 introduces some lemmas, theories, and fundamental information about fuzzy logic systems (FLSs). The consensus control problem is stated in Section 3. The implementation of distributed fuzzy controllers for each individual of the MASs and the systematic stability analysis are presented in Section 4. Simulation examples and the conclusion are included in Sections 5 and 6, respectively.

2. Preliminaries

Several basic definitions and results are presented in advance to facilitate the subsequent analysis. Especially, the basic graph theory and fuzzy logic systems (FLSs) will be introduced, which play a vital role in getting the main results of this paper.

2.1. Predefined-time stability

First, a fundamental definition on practically predefined-time stability is given as follows.

Definition 2.1. *Let*

$$\dot{\boldsymbol{y}} = \boldsymbol{g}(t, \boldsymbol{y}),\ \boldsymbol{y}(0) = \boldsymbol{y}_0, \tag{2.1}$$

be a nonlinear system, where $\boldsymbol{y} \in \mathbb{R}^n$ *is the system state, and* $\boldsymbol{g} : R^+ \times \mathbb{R}^n \to \mathbb{R}^n$ *is a continuous function with* $\boldsymbol{g}(0, 0) = 0$*. If there exists a positive constant* ε *and a settling time* $\mathcal{T}_p > 0$ *such that* $\|\boldsymbol{y}\| \le \varepsilon$ *for* $t > \mathcal{T}_p$*, then the system* (2.1) *is said to have practically predefined-time stability.*

In order to better assess the practically predefined-time stability of the system (2.1), the following lemmas are derived.

Lemma 2.1. *Consider the system* (2.1)*, and let* $\Lambda(y)$ *be a Lyapunov function,*

$$\dot{\Lambda}(y) \le -\frac{3}{\nu\mathcal{T}}\Big(\Lambda^{1-\frac{\nu}{2}}(y) + 2\Lambda(y) + \Lambda^{1+\frac{\nu}{2}}(y)\Big) + \rho, \tag{2.2}$$

where $\nu \in (0, 1)$*,* $\mathcal{T} > 0$*, and* $\rho > 0$*. Then the nonlinear system* (2.1) *has practically predefined-time stability and the predefined time is* $\mathcal{T}$*.*

Proof. Denote $\delta = \frac{\nu\rho\mathcal{T}}{4}$, $\Omega_y = \{y|\Lambda(y) \le \delta\}$, and $\bar{\Omega}_y = \{y|\Lambda(y) > \delta\}$. Without loss of generality, assume that $\Lambda(y(0)) > \delta$. If $y \in \bar{\Omega}_y$, it holds that

$$\frac{1}{\nu\mathcal{T}}\Big(\Lambda^{1-\frac{\nu}{2}}(y) + 2\Lambda(y) + \Lambda^{1+\frac{\nu}{2}}(y)\Big) \ge \frac{4}{\nu\mathcal{T}}\Lambda(y) > \frac{4\delta}{\nu\mathcal{T}} = \rho. \tag{2.3}$$

It is known from (2.2) and (2.3) that

$$\begin{aligned}\dot{\Lambda}(y) &\le -\frac{2}{\nu\mathcal{T}}\Big(\Lambda^{1-\frac{\nu}{2}}(y) + 2\Lambda(y) + \Lambda^{1+\frac{\nu}{2}}(y)\Big) - \frac{1}{\nu\mathcal{T}}\Big(\Lambda^{1-\frac{\nu}{2}}(y) + 2\Lambda(y) + \Lambda^{1+\frac{\nu}{2}}(y)\Big) + \rho \\ &\le -\frac{2}{\nu\mathcal{T}}\Big(\Lambda^{1-\frac{\nu}{2}}(y) + 2\Lambda(y) + \Lambda^{1+\frac{\nu}{2}}(y)\Big).\end{aligned} \tag{2.4}$$

Let $\mathcal{T}_*$ be the time for y to arrive at the region Ω_y from y_0. It follows from (2.4) that

$$\int_0^{\mathcal{T}_*} \frac{d\Lambda(y)}{\Lambda^{1-\frac{\nu}{2}}(y) + 2\Lambda(y) + \Lambda^{1+\frac{\nu}{2}}(y)} \le -\int_0^{\mathcal{T}_*} \frac{2}{\nu\mathcal{T}}dt,$$

and as a result

$$\begin{aligned}\mathcal{T}_* &\le -\mathcal{T}\int_0^{\mathcal{T}_*} \frac{d\Lambda^{\frac{\nu}{2}}(y)}{(1 + \Lambda^{\frac{\nu}{2}}(y))^2} \\ &\le \mathcal{T}\int_{\Lambda(y(0))}^{\delta} d\Big(\frac{1}{1 + \Lambda^{\frac{\nu}{2}}(y)}\Big) \\ &= \mathcal{T}\Big(\frac{1}{1 + \delta^{\frac{\nu}{2}}} - \frac{1}{1 + \Lambda^{\frac{\nu}{2}}(0)}\Big) \\ &< \mathcal{T}.\end{aligned}$$

This indicates that once y enters into the region Ω_y, it will remain in the region. According to Definition 2.1, $\mathcal{T}$ is the predefined time.

Lemma 2.2. *[33] Suppose that* $\kappa_1, \kappa_2, \cdots, \kappa_L \ge 0$, $h_1 \in (0, 1]$, *and* $h_2 > 1$. *Then it holds that*

$$\sum_{\eta=1}^{L} \kappa_\eta^{h_1} \ge \Big(\sum_{\eta=1}^{L} \kappa_\eta\Big)^{h_1}, \sum_{\eta=1}^{L} \kappa_\eta^{h_2} \ge L^{1-h_2}\Big(\sum_{\eta=1}^{L} \kappa_\eta\Big)^{h_2}.$$

Lemma 2.3. *[34] Assume* $\zeta \in \mathbb{R}$, $\omega > 0$, *and it holds that*

$$0 \le |\zeta| - \zeta \tanh\Big(\frac{\zeta}{\omega}\Big) \le 0.2785\omega.$$

Lemma 2.4. *[35] Assume* ϕ *and* $\hat{\phi}$ *are two scalar functions and* $\tilde{\phi} = \phi - \hat{\phi}$. *It holds that*

$$\tilde{\phi}\hat{\phi}^{\upsilon} \le w_1\phi^{\upsilon+1} - w_2\tilde{\phi}^{\upsilon+1},$$

where $\upsilon = \frac{c_1}{c_2} \in (0, 1]$, c_1 *and* c_2 *are odd integers,* $w_1 = \frac{1}{1+\upsilon}(1 + \frac{\upsilon}{1+\upsilon} + \frac{2^{\upsilon}(1-\upsilon^2)}{1+\upsilon} - 2^{\upsilon-1})$, *and* $w_2 = \frac{2^{\upsilon-1}}{1+\upsilon}(1 - 2^{\upsilon(\upsilon-1)})$.

Lemma 2.5. *[36] Let χ be a positive constant and $y < \chi$. It holds that*

$$(\chi - y)^q y \le \chi^{q+1} - y^{q+1},$$

where $q = \frac{q_1}{q_2} > 1$, and q_1 and q_2 are positive odd numbers.

Lemma 2.6. *Consider the following differential equation.*

$$\dot{\zeta}(t) = -\lambda_1\zeta^{1-\nu}(t) - \lambda_2\zeta^{1+\nu}(t) - \lambda_3\zeta(t) + r(t), \tag{2.5}$$

where $\zeta(t) \in \mathbb{R}$, $\nu = \frac{k_1}{k_2} < 1$, and k_1 and k_2 are, respectively, an odd positive number and an even positive number. λ_1, λ_2, and λ_3 are positive constants and $r(t)$ is a positive function. If $\zeta(t_0) \ge 0$, then $\zeta(t) \ge 0$ for all $t > t_0$.

Proof. From (2.5), one knows that $\dot{\zeta}(t) > -\lambda_1\zeta^{1-\nu}(t) - \lambda_2\zeta^{1+\nu}(t) - \lambda_3\zeta(t)$. Let $\dot{\eta}(t) = -\lambda_1\eta^{1-\nu}(t) - \lambda_2\eta^{1+\nu}(t) - \lambda_3\eta(t)$, $\eta(t_0) = \zeta(t_0)$, and one can use Lyapunov function $V = \frac{1}{2}\eta^2(t)$ to prove that $\eta(t)$ reaches zero in finite time for any initial value $\eta(t_0)$. Assuming $\eta(t_0) > 0$ and $t_\alpha = \min\{t|\eta(t) = 0\}$, if there exists a moment $t_\beta \in (t_0, t_\alpha)$ such that $\eta(t_\beta) < 0$, then $\eta(t_0)\eta(t_\beta) < 0$. It is known from the properties of a continuous function in a closed interval $[t_0, t_\beta]$ that there exists $t_\gamma \in (t_0, t_\beta)$ such that $\eta(t_\gamma) = 0$, which contradicts the definition of t_α. Therefore, $\eta(t) \ge 0$ holds for all $t \ge t_0$. Furthermore, the principle of comparison indicates that $\zeta(t) \ge \eta(t) \ge 0$, $t \ge t_0$.

Lemma 2.7. *[37] The following function $\Xi(y)$ and its derivative $\frac{\mathrm{d}\Xi(y)}{\mathrm{d}y}$ are continuous for $\forall y \in \mathbb{R}$.*

$$\Xi(y) = \begin{cases} |y|^{1-\alpha}\mathrm{sign}(y), & \text{if } |y| \ge \alpha, \\ m_1|y|^{2-\alpha}\mathrm{sign}(y) + m_2\alpha^{|y|}y, & \text{if } |y| < \alpha, \end{cases}$$

$$\frac{\mathrm{d}\Xi(y)}{\mathrm{d}y} = \begin{cases} (1-\alpha)|y|^{-\alpha}, & \text{if } |y| \ge \alpha, \\ m_1(2-\alpha)|y|^{1-\alpha} + m_2(|y|\ln\alpha + 1)\alpha^{|\alpha|}, & \text{if } |y| < \alpha, \end{cases}$$

where $\alpha \in (0, \frac{1}{e})$, $m_1 = \frac{-1-\ln\alpha}{1-\alpha-\alpha\ln\alpha}$, $m_2 = \frac{\alpha^{-2\alpha}}{1-\alpha-\alpha\ln\alpha}$, and $\Xi(y)$, $\frac{\mathrm{d}\Xi(y)}{\mathrm{d}y}$ satisfy the following inequalities:

$$|\Xi(y)| \le a_1|y|, \quad |\frac{\mathrm{d}\Xi(y)}{\mathrm{d}y}| \le a_2,$$

where $a_1 = \frac{\alpha^{-2\alpha}-1-\ln\alpha}{1-\alpha-\alpha\ln\alpha}$ and $a_2 = \frac{\alpha^{1-\alpha}(\alpha^{-\alpha-1}-2+\alpha)-alpha^{1-\alpha}(2-\alpha+\alpha^{-\alpha})ln\alpha}{1-\alpha-\alpha\ln\alpha}$.

Remark 2.1. *In [37], the function $\Xi(y)$ was used to design the terminal sliding mode surface for uncertain robot manipulators. Under the assumption of known upper bounds on external disturbances and internal uncertainties, the tracking errors can reach the sliding mode surface within a finite time, and then the tracking errors reach the small predefined range along the sliding mode surface within a finite time. This article will use $\Xi(y)$ to a design control scheme that enables the state errors between the leader and followers to reach a predefined range within a predefined time under unknown external disturbances and system functions.*

Remark 2.2. *In [38], if the Lyapunov function $\Lambda(y)$ satisfies $\dot{\Lambda}(y) \le -c_1\Lambda^{\lambda_1}(y) + \rho_1$, where $c_1 > 0$, $0 < \lambda_1 < 1$, and $\rho_1 > 0$, then the system (2.1) has practically finite-time stability. Denote $\Omega_{\bar{y}} =$*

$\{y|\Lambda(y) \leq \frac{\rho_1}{c_1(1-\gamma)}\}$, $\gamma \in (0,1)$. If $y \notin \Omega_{\bar{y}}$, then a settling time $T_r = \frac{1}{(1-\lambda_1)\gamma c_1}(\Lambda^{1-\lambda_1}(y(0)) - (\frac{\rho_1}{(1-\gamma)c_1})^{\frac{1-\lambda_1}{\lambda_1}})$ can be obtained and it is ensured that $y \in \Omega_{\bar{y}}$ for $t \geq T_r$. Obviously, compared with the practically finite-time stability convergence time T_r, the proposed predefined time $\mathcal{T}$ in Lemma 2.1 is independent of the initial value. In [22], if the Lyapunov function $\Lambda(y)$ satisfies $\dot{\Lambda}(y) \leq -c_1\Lambda^{\lambda_1}(y) - c_2\Lambda^{\lambda_2}(y) + \rho_2$, where $c_1 > 0, c_2 > 0$, $0 < \lambda_1 < 1$, $\lambda_2 > 1$, and $\rho_2 > 0$, then the system (2.1) has practically fixed-time stability. Denote $\Omega_{\bar{\bar{y}}} = \{y|\Lambda(y) \leq \min\{(\frac{\rho_2}{c_1(1-\gamma)})^{\frac{1}{\lambda_1}}, (\frac{\rho_2}{c_2(1-\gamma)})^{\frac{1}{\lambda_2}}\}\}$, $\gamma \in (0,1)$. If $y \notin \Omega_{\bar{\bar{y}}}$, then a fixed time $T_s = \frac{1}{(1-\lambda_1)\gamma c_1} + \frac{1}{(\lambda_2-1)\gamma c_2}$ can be obtained and it is ensured that $y \in \Omega_{\bar{\bar{y}}}$ for $t \geq T_s$. By comparing the predefined time $\mathcal{T}$ in Lemma 2.1 and the fixed time T_s, one can find that the practically predefined-time stability can establish the relationship between system parameters and stabilization time, but fixed-time stability cannot achieve this.

2.2. Description of the FLS

To dispose of the uncertainty existing in the MASs, the basic theoretical knowledge of FLS is recalled here.

An FLS consists of four parts: a set of fuzzy rules, center-average defuzzifer, a fuzzifier, and product inference. Let the ath rule be (suppose that there are n_a fuzzy rules) $\mathbb{R}^{(a)}$: if y_1 is B_1^r and $\cdots$ y_n is B_n^r, then F_y is C^a, with $\boldsymbol{y} = [y_1, y_2, \cdots, y_n]^T$ being the FLS input, F_y being the FLS output, and B_i^a and C^a being fuzzy sets. The fuzzy functions $\mu_{B_i^a}(y_i)$ and $\mu_{C^a}(F_y)$ are with respect to B_i^a and C^a.

Through the technique in [39], the output F_y can be expressed as

$$F_y(\boldsymbol{y}) = \frac{\sum_{a=1}^{n_a} \bar{F}_a[\prod_{i=1}^{n} \mu_{B_i^a}(y_i)]}{\sum_{a=1}^{n_a}[\prod_{i=1}^{n} \mu_{B_i^a}(y_i)]}, \tag{2.6}$$

with $\bar{F}_a = \max_{F_y \in R} \mu_{C^a}(F_y)$. So, the fuzzy basis function can be defined as

$$\boldsymbol{\varphi}_a(\boldsymbol{y}) = \frac{\prod_{i=1}^{n} \mu_{B_i^a}(y_i)}{\sum_{a=1}^{n_a}[\prod_{i=1}^{n} \mu_{B_i^a}(y_i)]}. \tag{2.7}$$

Denote $\boldsymbol{\theta}^T = [\bar{F}_1, \bar{F}_2, \cdots, \bar{F}_{n_a}] = [\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \cdots, \boldsymbol{\theta}_{n_a}]$, $\boldsymbol{\varphi}(\boldsymbol{y}) = [\boldsymbol{\varphi}_1(\boldsymbol{y}), \boldsymbol{\varphi}_2(\boldsymbol{y}), , \cdots, \boldsymbol{\varphi}_{n_a}(\boldsymbol{y})]^T$, and then the output F_y in (2.6) can be rewritten as

$$F_y(\boldsymbol{y}) = \boldsymbol{\theta}^T\boldsymbol{\varphi}(\boldsymbol{y}). \tag{2.8}$$

Lemma 2.8. *[36] If $f(\boldsymbol{y})$ is a continuous function on the compact set Ω_y, then for any positive constant σ, there exists an FLS (2.8) such that*

$$\sup_{\boldsymbol{y}\in\Omega_y} |f(\boldsymbol{y}) - \boldsymbol{\theta}^T\boldsymbol{\varphi}(\boldsymbol{y})| \leq \sigma. \tag{2.9}$$

2.3. Basic concepts on graph thoery

To describe the control problem, some basic notions about graph theory are mentioned here as in [40].

Consider the MAS comprising a leader and N followers. Let message exchanges among multiple followers be denoted through an undirected diagram $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, with $\mathcal{V} = (v_1, \cdots, v_n)$, $\mathcal{E}$ being the set of nodes, edges, respectively, and $\mathcal{A} = [a_{j,k}] \in \mathbb{R}^{N\times N}$ being an adjacency matrix where $a_{j,k} \geq 0$. The node collection is a part of a countable index set $\mathcal{I} = \{1, \cdots, N\}$. In addition, $\mathcal{E}$ is described as $(v_j, v_k) \in \mathcal{E}$ $(j \neq k)$ representing a road leading from the jth follower to the kth follower, that is, the follower j can deliver information to the follower k. The matrix $\mathcal{A}$ is given by $a_{j,k} = a_{k,j}$, $a_{j,j} = 0$, and $a_{j,k} > 0$ if $(v_j, v_k) \in \mathcal{E}$ $(j \neq k)$. The neighbor of node j is $N_j = \{k|(v_k, v_j) \in \mathcal{E}\}$. Besides, let $\mathcal{D}$ be the in-degree matrix with $d_j = \sum_{v_k\in N_j} a_{j,k}$ for $\forall j \in \mathcal{I}$. Then, let $\mathcal{L} = [l_{j,k}] = \mathcal{D} - \mathcal{A} \in \mathbb{R}^{N\times N}$.

One can use another graphic $\overline{\mathcal{G}} = \{\overline{\mathcal{V}}, \overline{\mathcal{E}}\}$ to portray the communication topology between the leader v_0 with N followers $v_j \in \mathcal{E}$, $j \in \mathcal{I}$. Let $\mathcal{B} = \text{diag}\{b_1, \cdots, b_N\} \in \mathbb{R}^{N\times N}$ be the leader adjacency matrix, with $b_j > 0$ iff v_j is contacted with the node v_0 across the communication (v_j, v_0) and $b_j = 0$ otherwise. In particular, suppose that the topology is undirected and fixed.

3. Problem statement

Consider the following MAS having N followers, in which the jth follower can be presented as

$$\begin{cases} \dot{x}_{j,i} = f_{j,i}(\overline{\boldsymbol{x}}_{j,i}) + x_{j,i+1} + \tau_{j,i}(\overline{\boldsymbol{x}}_{j,i}, t), \\ \dot{x}_{j,n} = f_{j,n}(\overline{\boldsymbol{x}}_{j,n}) + u_j + \tau_{j,n}(\overline{\boldsymbol{x}}_{j,n}, t), \\ y_j = x_{j,1}, \end{cases} \tag{3.1}$$

with $i \in [1, n-1]$, $j \in [1, \cdots, N]$. $\overline{\boldsymbol{x}}_{j,i} = [x_{j,1}, \cdots, x_{j,i}]^T \in \mathbb{R}^i$ is a measurable state, $u_j \in \mathbb{R}$ is the control input, $y_j \in \mathbb{R}$ is the output signal, $f_{j,i}(\cdot) : \mathbb{R}^i \to \mathbb{R}$ is an unknown smooth function, and $\tau_{j,i}(\cdot) \in \mathbb{R}$ represents an unknown external disturbance.

Remark 3.1. *Note that the considered model* (3.1) *has a strict-feedback form, which can express many actual nonlinear systems, such as quadrotor unmanned aerial vehicles, aircrafts, and hypersonic flight vehicles. Therefore, studying its stability has important practical significance, and some prior works can be seen in [12, 41–43]. However, the scholars in references [12, 41–43] only studied finite-time stability. Therefore, this article will discuss the practically predefined-time stability of such systems, by which the convergence time is not limited by initial conditions and can be formulated as needed.*

Let the leader (denoted by 0) be

$$\begin{cases} \dot{x}_{0,i} = x_{0,i+1}, \\ \dot{x}_{0,n} = f_0(\overline{\boldsymbol{x}}_0, t), \\ y_0 = x_{0,1}, \end{cases} \tag{3.2}$$

with $\overline{\boldsymbol{x}}_0 = [x_{0,1}, \cdots, x_{0,n}]^T \in \mathbb{R}^n$ being the measurable and bounded state vector, $y_0 \in \mathbb{R}$ being the leader's output, and $f_0(\cdot) \in \mathbb{R}$ being an unknown smooth function. Note that the states of the leader and the jth follower are represented as $x_{0,i}$ and $x_{j,i}$, respectively, and the right subscript i refers to the ith component of the state vectors $\overline{\boldsymbol{x}}_0 \in \mathbb{R}^n$ and $\overline{\boldsymbol{x}}_{j,n} \in \mathbb{R}^n$.

Let the tracking error for the jth follower be

$$z_{j,1} = \sum_{k\in N_j} a_{j,k}(y_k - y_j) + b_j(y_0 - y_j) = -(d_j + b_j)y_j + \sum_{k\in N_j} a_{j,k}y_k + b_j y_0, \tag{3.3}$$

$$z_{j,i} = x_{j,i} - \zeta_{j,i-1}, \tag{3.4}$$

where

$$\dot{\zeta}_{j,i-1} = -\varpi_{j,i-1}(\zeta_{j,i-1} - \alpha_{j,i-1}), \tag{3.5}$$

with $\varpi_{j,i-1} > 0$ being a filter parameter, $\alpha_{j,i-1}$ being the filter input which will be defined later, and the initial condition being designed as $\zeta_{j,i-1}(0) = \alpha_{j,i-1}(0)$. Denote $\xi_{j,i-1} = \zeta_{j,i-1} - \alpha_{j,i-1}$ as the filter error. Assume that $\dot{\alpha}_{j,i-1}$ is bounded, and then there exists a positive constant $\beta_{j,i-1}$ such that $|\dot{\alpha}_{j,i-1}| \leq \beta_{j,i-1}$. By using (3.5), one has $\dot{\xi}_{j,i-1} = -\varpi_{j,i-1}\xi_{j,i-1} + \dot{\alpha}_{j,i-1}$. So, $|\xi_{j,i-1}| = | \int_0^t \dot{\alpha}_{j,i-1} e^{-\varpi_{j,i-1}(t-s)} \mathrm{d}s| \leq \frac{\beta_{j,i-1}}{\varpi_{j,i-1}}$. Therefore, for a given constant $\bar{\xi}_{j,i-1} > 0$, there exists an appropriate positive real number $\varpi_{j,i-1} > 0$ such that $\frac{\beta_{j,i-1}}{\varpi_{j,i-1}} \leq \bar{\xi}_{j,i-1}$, that is,

$$|\xi_{j,i-1}| = |\zeta_{j,i-1} - \alpha_{j,i-1}| \leq \bar{\xi}_{j,i-1}. \tag{3.6}$$

The goal of this article is to develop a consensus tracking protocol based on Lemma 2.1 by using suitable adaptive fuzzy strategy and the backstepping technique. The developed consensus tracking protocol ensures the predefined-time consensus tracking between the leader and followers, as well as the boundedness of state variables in the MAS. To achieve this goal, some reasonable assumptions should be stated.

Assumption 3.1. *All external disturbances $\tau_{j,i}(\cdot)$ are bounded.*

For the unknown function $\Gamma_{j,i}$ (whose specific expression can be found in Section 4), by using Lemma 2.8, there is an FLS $\boldsymbol{\theta}_{j,i}^T \boldsymbol{\varphi}_{j,i}$ such that

$$\Gamma_{j,i} = \boldsymbol{\theta}_{j,i}^T \boldsymbol{\varphi}_{j,i} + \sigma_{j,i}, \tag{3.7}$$

where $\sigma_{j,i}$ is the fuzzy approximation error. Obviously, $\sigma_{j,i}$ is bounded, i.e., there exists a constant $\underline{\sigma}_{j,i} > 0$ such that $|\sigma_{j,i}| \leq \underline{\sigma}_{j,i}$.

Remark 3.2. *Assumption 3.1 is not a constraint condition, as external disturbances in actual systems are bounded. Compared to some existing results that do not consider external disturbances [36, 44], Assumption 3.1 is reasonable. A similar Assumption can be found in [45, 46].*

Remark 3.3. *The consensus issue of MASs has captured extensive interest resulting from their widespread applications in various fields, including autonomous driving, energy distribution, formation control, trajectory planning, and so on. However, in practical applications, the system is inevitably affected by external environments, such as noise or disturbances that can affect system performance. Therefore, in order to ensure consistent performance, one can consider the impact of uncertainties while formatting control schemes. Therefore, this article adopts the FLS to handle the uncertainties in the system.*

Remark 3.4. *In this paper, the filter* (3.5) *is utilized to estimate $\alpha_{j,i-1}$ and its derivative, so the conventional "explosion of complexity" problem can be avoided. In addition, the filter error $\xi_{j,i-1}$ will keep it bounded by setting $\varpi_{j,i-1}$ as a larger constant according to [47]. Since $\varpi_{j,i-1}$ needs to select a large positive real number, this article sets $\varpi_{j,1} = \varpi_{j,2} = \circ\circ\circ = \varpi_{j,n-1} = \varpi$. It implies that Assumption 3.1 is less stringent so that the command filtered backstepping technique is more suitable for some actual dynamics models in which the higher-order derivatives are difficult to obtain such as land vehicle systems. Meanwhile, in consensus control, a leader can be seen as a command generator to deliver instructions to its followers, and all followers must understand the instructions to follow, so the condition that $\overline{\boldsymbol{x}}_0$ can be measured is reasonable.*

4. Main results

For the convenience of subsequent theoretical derivation, the functions $f_{j,i}(\overline{\boldsymbol{x}}_{j,i})$ and $\tau_{j,i}(\overline{\boldsymbol{x}}_{j,i}, t)$ are simplified as $f_{j,i}$ and $\tau_{j,i}$.

4.1. Controller design

In the subsequent backsepping design process, effective virtual controllers and the actual controller of the jth follower, as well as the corresponding adaptive laws, will be provided.

Step 1. For the jth follower, differentiating the tracking error $z_{j,1}$ in (3.3) produces

$$
\begin{aligned}
\dot{z}_{j,1} &= -(d_j + b_j)(f_{j,1} + x_{j,2} + \tau_{j,1}) + \sum_{k\in N_j} a_{j,k}(f_{k,1} + x_{k,2} + \tau_{k,1}) + b_j x_{0,2} \\
&= \Big[-(d_j + b_j) f_{j,1} + \sum_{k\in N_j} a_{j,k} f_{k,1} \Big] + \Big[-(d_j + b_j)\tau_{j,1} + \sum_{k\in N_j} a_{j,k}\tau_{k,1} \Big] \\
&\quad - (d_j + b_j)(z_{j,2} + \zeta_{j,1} - \alpha_{j,1} + \alpha_{j,1}) + \sum_{k\in N_j} a_{j,k} x_{k,2} + b_j x_{0,2} \\
&= \overline{f}_{j,1} + \bar{\tau}_{j,1} - (d_j + b_j) z_{j,2} - (d_j + b_j)\xi_{j,1} - (d_j + b_j)\alpha_{j,1} + \Lambda_{j,1},
\end{aligned} \tag{4.1}
$$

where $\overline{f}_{j,1} = -(d_j + b_j) f_{j,1} + \sum_{k\in N_j} a_{j,k} f_{k,1}$, $\bar{\tau}_{j,1} = -(d_j + b_j)\tau_{j,1} + \sum_{k\in N_j} a_{j,k}\tau_{k,1}$, and $\Lambda_{j,1} = \sum_{k\in N_j} a_{j,k} x_{k,2} + b_j x_{0,2}$. By using the FLS (3.7), one has

$$
\overline{f}_{j,1} = \boldsymbol{\theta}_{j,1}^T \boldsymbol{\varphi}_{j,1} + \sigma_{j,1}. \tag{4.2}
$$

Due to the boundedness of fuzzy estimation error $\sigma_{j,1}$ and Assumption 3.1, there exists an unknown positive constant $\bar{\kappa}_{j,1}$ such that $|\sigma_{j,1} + \bar{\tau}_{j,1}| \leq \bar{\kappa}_{j,1}$. So, by applying Lemma 2.3, for any $\omega > 0$, one obtains

$$
\begin{aligned}
z_{j,1}(\overline{f}_{j,1} + \bar{\tau}_{j,1}) &= z_{j,1}(\boldsymbol{\theta}_{j,1}^T \boldsymbol{\varphi}_{j,1} + \sigma_{j,1} + \bar{\tau}_{j,1}) \\
&\leq |z_{j,1}|(\|\boldsymbol{\theta}_{j,1}\| \|\boldsymbol{\varphi}_{j,1}\| + \bar{\kappa}_{j,1}) \\
&\leq |z_{j,1}| \chi_{j,1} \psi_{j,1} \\
&\leq z_{j,1} \chi_{j,1} \psi_{j,1} \tanh\Big(\frac{z_{j,1}\psi_{j,1}}{\omega}\Big) + 0.2785\omega \chi_{j,1},
\end{aligned} \tag{4.3}
$$

where $\chi_{j,1} = \max\{\|\boldsymbol{\theta}_{j,1}\|, \bar{\kappa}_{j,1}\}$ and $\psi_{j,1} = \|\boldsymbol{\varphi}_{j,1}\| + 1$.

By using (3.6), one has

$$
-z_{j,1}(d_j + b_j)\xi_{j,1} \leq \frac{1}{2} z_{j,1}^2 + \frac{1}{2}(d_j + b_j)^2 \overline{\xi}_{j,1}^2. \tag{4.4}
$$

Let $\hat{\chi}_{j,1}$ be the estimation of $\chi_{j,1}$. Construct the following Lyapunov function

$$
V_{j,1} = \frac{1}{2} z_{j,1}^2 + \frac{1}{2} \tilde{\chi}_{j,1}^2, \tag{4.5}
$$

where $\tilde{\chi}_{j,1} = \chi_{j,1} - \hat{\chi}_{j,1}$.

Substituting (4.1), (4.3), and (4.4) into $\dot{V}_{j,1}$, one can further obtain

$$
\dot{V}_{j,1} \leq z_{j,1}\chi_{j,1}\psi_{j,1} \tanh\Big(\frac{z_{j,1}\psi_{j,1}}{\omega}\Big) - z_{j,1}(d_j + b_j)\alpha_{j,1} - (d_j + b_j) z_{j,1} z_{j,2}
$$

$$
\begin{aligned}
&+\frac{1}{2}z_{j,1}^2+z_{j,1}\Lambda_{j,1}-\tilde{\chi}_{j,1}\dot{\hat{\chi}}_{j,1}+\rho_{j,1}\\
&=z_{j,1}\tilde{\chi}_{j,1}\psi_{j,1}\tanh\Big(\frac{z_{j,1}\psi_{j,1}}{\omega}\Big)+z_{j,1}\hat{\chi}_{j,1}\psi_{j,1}\tanh\Big(\frac{z_{j,1}\psi_{j,1}}{\omega}\Big)\\
&-z_{j,1}(d_j+b_j)\alpha_{j,1}-(d_j+b_j)z_{j,1}z_{j,2}\\
&+\frac{1}{2}z_{j,1}^2+z_{j,1}\Lambda_{j,1}-\tilde{\chi}_{j,1}\dot{\hat{\chi}}_{j,1}+\rho_{j,1}\\
&=z_{j,1}\hat{\chi}_{j,1}\psi_{j,1}\tanh\Big(\frac{z_{j,1}\psi_{j,1}}{\omega}\Big)-z_{j,1}(d_j+b_j)\alpha_{j,1}-(d_j+b_j)z_{j,1}z_{j,2}\\
&+\frac{1}{2}z_{j,1}^2+z_{j,1}\Lambda_{j,1}+\tilde{\chi}_{j,1}\Big(z_{j,1}\psi_{j,1}\tanh\Big(\frac{z_{j,1}\psi_{j,1}}{\omega}\Big)-\dot{\hat{\chi}}_{j,1}\Big)+\rho_{j,1},
\end{aligned}\tag{4.6}
$$

where $\rho_{j,1}=0.2785\omega\psi_{j,1}+\frac{1}{2}(d_j+b_j)^2\bar{\xi}_{j,1}^2$. In order to ensure the continuity of the virtual controller at the boundary points of the predefined region, according to Lemma 2.7, the virtual controller $\alpha_{j,1}$ is designed as follows.

$$
\alpha_{j,1}=\frac{\frac{\mu_1}{2}z_{j,1}+\mu_2\Xi(z_{j,1})+\mu_3 z_{j,1}^{1+\nu}+\hat{\chi}_{j,1}\psi_{j,1}\tanh\Big(\frac{z_{j,1}\psi_{j,1}}{\omega}\Big)+\Omega_{j,1}}{d_j+b_j},\tag{4.7}
$$

where $\nu=\frac{k_1}{k_2}\in(0,\frac{1}{e})$ is a predefined tracking error boundary value, k_1 is a positive even number, k_2 is a positive odd number, $\Omega_{j,1}=\frac{1}{2}z_{j,1}+\Lambda_{j,1}$, and

$$
\Xi(z_{j,1})=\begin{cases}|z_{j,1}|^{1-\nu}\mathrm{sign}(z_{j,1}), & \text{if } |z_{j,1}|\geq\nu,\\ m_1|z_{j,1}|^{2-\nu}\mathrm{sign}(z_{j,1})+m_2\nu^{|z_{j,1}|}z_{j,1}, & \text{if } |z_{j,1}|<\nu,\end{cases}\tag{4.8}
$$

where $m_1=\frac{-1-\ln\nu}{1-\nu-\nu\ln\nu}, m_2=\frac{\nu^{-2\nu}}{1-\nu-\nu\ln\nu}$. The adaptive law is given as

$$
\dot{\hat{\chi}}_{j,1}=z_{j,1}\psi_{j,1}\tanh\Big(\frac{z_{j,1}\psi_{j,1}}{\omega}\Big)-\mu_1\hat{\chi}_{j,1}-\frac{\mu_2}{K_2}\hat{\chi}_{j,1}^{1-\nu}-\mu_3\hat{\chi}_{j,1}^{1+\nu},\tag{4.9}
$$

where μ_1,μ_2,μ_3, and K_2 will be given later.

Case 1. For $|z_{j,1}|\geq\nu$, substituting (4.7) and (4.9) into (4.6) yields

$$
\begin{aligned}
\dot{V}_{j,1}\leq&-\frac{\mu_1}{2}z_{j,1}^2-\mu_2\Big(z_{j,1}^2\Big)^{1-\frac{\nu}{2}}-\mu_3\Big(z_{j,1}^2\Big)^{1+\frac{\nu}{2}}-(d_j+b_j)z_{j,1}z_{j,2}\\
&+\mu_1\tilde{\chi}_{j,1}\hat{\chi}_{j,1}+\frac{\mu_2}{K_2}\tilde{\chi}_{j,1}\hat{\chi}_{j,1}^{1-\nu}+\mu_3\tilde{\chi}_{j,1}\hat{\chi}_{j,1}^{1+\nu}+\rho_{j,1}.
\end{aligned}\tag{4.10}
$$

Then, by using Lemma 2.4, one can derive that

$$
\begin{aligned}
\frac{\mu_2}{K_2}\tilde{\chi}_{j,1}\hat{\chi}_{j,1}^{1-\nu}&\leq\frac{\mu_2K_1}{K_2}\chi_{j,1}^{2-\nu}-\frac{\mu_2K_2}{K_2}\tilde{\chi}_{j,1}^{2-\nu}\\
&=\frac{\mu_2K_1}{K_2}\chi_{j,1}^{2-\nu}-\mu_2\Big(\tilde{\chi}_{j,1}^2\Big)^{1-\frac{\nu}{2}},
\end{aligned}\tag{4.11}
$$

where $K_1=\frac{1}{1+\nu}(1+\frac{\nu}{1+\nu}+\frac{2^\nu(1-\nu^2)}{1+\nu}-2^{\nu-1})$ and $K_2=\frac{2^{\nu-1}}{1+\nu}(1-2^{\nu(\nu-1)})$.

From Lemma 2.6, one knows that $\hat{\chi}_{j,1} \geq 0$ if initial value $\hat{\chi}_{j,1}(t_0) \geq 0$. Therefore, one gets $\tilde{\chi}_{j,1} = \chi_{j,1} - \hat{\chi}_{j,1} \leq \chi_{j,1}$. Applying Lemma 2.5, it can be obtained that

$$\begin{aligned}\mu_3\tilde{\chi}_{j,1}\hat{\chi}_{j,1}^{1+\nu} &= \mu_3\tilde{\chi}_{j,1}(\chi_{j,1} - \tilde{\chi}_{j,1})^{1+\nu} \\ &\leq \mu_3\chi_{j,1}^{2+\nu} - \mu_3\left(\tilde{\chi}_{j,1}^2\right)^{1+\frac{\nu}{2}}.\end{aligned} \tag{4.12}$$

For $\mu_3\tilde{\chi}_{j,1}\hat{\chi}_{j,1}$, by using the Young inequality, one has

$$\begin{aligned}\mu_1\tilde{\chi}_{j,1}\hat{\chi}_{j,1} &= \mu_1\tilde{\chi}_{j,1}(\chi_{j,1} - \tilde{\chi}_{j,1}) \\ &\leq \frac{\mu_1}{2}\chi_{j,1}^2 - \frac{\mu_1}{2}\tilde{\chi}_{j,1}^2.\end{aligned} \tag{4.13}$$

Substituting (4.11), (4.12), and (4.13) into (4.10) yields

$$\begin{aligned}\dot{V}_{j,1} \leq& -\frac{\mu_1}{2}\left(z_{j,1}^2 + \tilde{\chi}_{j,1}^2\right) - \mu_2\left((z_{j,1}^2)^{1-\frac{\nu}{2}} + (\tilde{\chi}_{j,1}^2)^{1-\frac{\nu}{2}}\right) - \mu_3\left((z_{j,1}^2)^{1+\frac{\nu}{2}} + (\tilde{\chi}_{j,1}^2)^{1+\frac{\nu}{2}}\right) \\ &- (d_j + b_j)z_{j,1}z_{j,2} + \bar{\rho}_{j,1},\end{aligned} \tag{4.14}$$

where $\bar{\rho}_{j,1} = \rho_{j,1} + \frac{\mu_1}{2}\chi_{j,1}^2 + \frac{\mu_2 K_1}{K_2}\chi_{j,1}^{2-\nu} + \mu_3\chi_{j,1}^{2+\nu}$.

Case 2. For $|z_{j,1}| < \nu$, substituting (4.7) and (4.9) into (4.6), one has

$$\begin{aligned}\dot{V}_{j,1} \leq& -\frac{\mu_1}{2}z_{j,1}^2 - \mu_2 m_1|z_{j,1}|^{3-\nu} - \mu_2 m_2\nu^{|z_{j,1}|}z_{j,1}^2 - \mu_3\left(z_{j,1}^2\right)^{1+\frac{\nu}{2}} - (d_j + b_j)z_{j,1}z_{j,2} \\ &+ \mu_1\tilde{\chi}_{j,1}\hat{\chi}_{j,1} + \frac{\mu_2}{K_2}\tilde{\chi}_{j,1}\hat{\chi}_{j,1}^{1-\nu} + \mu_3\tilde{\chi}_{j,1}\hat{\chi}_{j,1}^{1+\nu} + \rho_{j,1} \\ \leq& -\frac{\mu_1}{2}z_{j,1}^2 - \mu_3\left(z_{j,1}^2\right)^{1+\frac{\nu}{2}} - (d_j + b_j)z_{j,1}z_{j,2} + \rho_{j,1} \\ &+ \mu_1\tilde{\chi}_{j,1}\hat{\chi}_{j,1} + \frac{\mu_2}{K_2}\tilde{\chi}_{j,1}\hat{\chi}_{j,1}^{1-\nu} + \mu_3\tilde{\chi}_{j,1}\hat{\chi}_{j,1}^{1+\nu}.\end{aligned} \tag{4.15}$$

For $\mu_1\tilde{\chi}_{j,1}\hat{\chi}_{j,1}$, $\frac{\mu_2}{K_2}\tilde{\chi}_{j,1}\hat{\chi}_{j,1}^{1-\nu}$, and $\mu_3\tilde{\chi}_{j,1}\hat{\chi}_{j,1}^{1+\nu}$, substituting the same analysis results (4.11)–(4.13) into (4.15) yields

$$\begin{aligned}\dot{V}_{j,1} \leq& -\frac{\mu_1}{2}\left(z_{j,1}^2 + \tilde{\chi}_{j,1}^2\right) - \mu_2(\tilde{\chi}_{j,1}^2)^{1-\frac{\nu}{2}} - \mu_3\left((z_{j,1}^2)^{1+\frac{\nu}{2}} + (\tilde{\chi}_{j,1}^2)^{1+\frac{\nu}{2}}\right) \\ &- (d_j + b_j)z_{j,1}z_{j,2} + \bar{\rho}_{j,1},\end{aligned} \tag{4.16}$$

where $\bar{\rho}_{j,1}$ is the same as in (4.14). Notice that $\left(\nu^2\right)^{1-\frac{\nu}{2}} - \left(z_{j,1}^2\right)^{1-\frac{\nu}{2}} > 0$ for $|z_{j,i}| < \nu < 1$, and one can rewrite (4.16) as

$$\begin{aligned}\dot{V}_{j,1} \leq& -\frac{\mu_1}{2}\left(z_{j,1}^2 + \tilde{\chi}_{j,1}^2\right) - \mu_2\left((z_{j,1}^2)^{1-\frac{\nu}{2}} + \tilde{\chi}_{j,1}^2)^{1-\frac{\nu}{2}}\right) - \mu_3\left((z_{j,1}^2)^{1+\frac{\nu}{2}} + (\tilde{\chi}_{j,1}^2)^{1+\frac{\nu}{2}}\right) \\ &- (d_j + b_j)z_{j,1}z_{j,2} + \bar{\bar{\rho}}_{j,1},\end{aligned} \tag{4.17}$$

where $\bar{\bar{\rho}}_{j,1} = \bar{\rho}_{j,1} + \mu_2\left(\nu^2\right)^{1-\frac{\nu}{2}}$.

Step (2 ≤ *i* ≤ *n* − 1). According to $z_{j,i} = x_{j,i} - \zeta_{j,i-1}$, the time derivative of $z_{j,i}$ can be obtained as

$$\dot{z}_{j,i} = f_{j,i} + x_{j,i+1} + \tau_{j,i} - \dot{\zeta}_{j,i-1}$$

$$
\begin{aligned}
&= f_{j,i} + (z_{j,i+1} + \zeta_{j,i} - \alpha_{j,i} + \alpha_{j,i}) + \tau_{j,i} - \dot{\zeta}_{j,i-1} \\
&= f_{j,i} + z_{j,i+1} + \xi_{j,i} + \alpha_{j,i} + \tau_{j,i} - \dot{\zeta}_{j,i-1} \\
&= \boldsymbol{\theta}_{j,i}^T \boldsymbol{\varphi}_{j,i} + \sigma_{j,i} + \tau_{j,i} + z_{j,i+1} + \xi_{j,i} + \alpha_{j,i} - \dot{\zeta}_{j,i-1},
\end{aligned} \tag{4.18}
$$

where $f_{j,i} = \boldsymbol{\theta}_{j,i}^T \boldsymbol{\varphi}_{j,i} + \sigma_{j,i}$. Since $\tau_{j,i}$ and $\sigma_{j,i}$ are bounded, i.e., there exists an unknown positive constant $\bar{\kappa}_{j,i}$ such that $|\sigma_{j,i} + \tau_{j,i}| \leq \bar{\kappa}_{j,i}$, then, according to Lemma 2.3, for any $\omega > 0$, one obtains

$$
\begin{aligned}
z_{j,i}(\boldsymbol{\theta}_{j,i}^T \boldsymbol{\varphi}_{j,i} + \sigma_{j,i} + \tau_{j,i}) &\leq |z_{j,i}|(\|\boldsymbol{\theta}_{j,i}\| \|\boldsymbol{\varphi}_{j,i}\| + \bar{\kappa}_{j,i}) \\
&\leq |z_{j,i}| \chi_{j,i} \psi_{j,i} \\
&\leq z_{j,i} \chi_{j,i} \psi_{j,i} \tanh\Big(\frac{z_{j,i}\psi_{j,i}}{\omega}\Big) + 0.2785\omega\chi_{j,i},
\end{aligned} \tag{4.19}
$$

where $\chi_{j,i} = \max\{\|\boldsymbol{\theta}_{j,i}\|, \bar{\kappa}_{j,i}\}$ and $\psi_{j,i} = \|\boldsymbol{\varphi}_{j,i}\| + 1$.

By using Young's inequality and (3.6), one has

$$
z_{j,i}\xi_{j,i} \leq \frac{1}{2} z_{j,i}^2 + \frac{1}{2}\bar{\xi}_{j,i}^2. \tag{4.20}
$$

Define the estimation error $\tilde{\chi}_{j,i} = \chi_{j,i} - \hat{\chi}_{j,i}$, where $\hat{\chi}_{j,i}$ is the estimation of $\chi_{j,i}$. Build the following Lyapunov function.

$$
V_{j,i} = \frac{1}{2} z_{j,i}^2 + \frac{1}{2}\tilde{\chi}_{j,i}^2. \tag{4.21}
$$

Similar to Step 1, differentiating $V_{j,i}$ yields

$$
\begin{aligned}
\dot{V}_{j,i} \leq{}& z_{j,i}\hat{\chi}_{j,i}\psi_{j,i} \tanh\Big(\frac{z_{j,i}\psi_{j,i}}{\omega}\Big) + \frac{1}{2} z_{j,i}^2 + z_{j,i} z_{j,i+1} + z_{j,i}\alpha_{j,i} \\
&- z_{j,i}\dot{\zeta}_{j,i-1} + \tilde{\chi}_{j,i}\Big(z_{j,i}\psi_{j,i} \tanh\Big(\frac{z_{j,i}\psi_{j,i}}{\omega}\Big) - \dot{\hat{\chi}}_{j,i}\Big) + \rho_{j,i},
\end{aligned} \tag{4.22}
$$

where $\rho_{j,i} = 0.2785\omega\chi_{j,i} + \frac{1}{2}\bar{\xi}_{j,i}^2$. Select the following virtual controller and adaptive law:

$$
\alpha_{j,i} = -\Big(\frac{\mu_1}{2} z_{j,i} + \mu_2 \Xi(z_{j,i}) + \mu_3 z_{j,i}^{1+\nu} + \hat{\chi}_{j,i}\psi_{j,i} \tanh\Big(\frac{z_{j,i}\psi_{j,i}}{\omega}\Big) + \Omega_{j,i}\Big), \tag{4.23}
$$

where $\Omega_{j,i} = \frac{1}{2} z_{j,i} + \iota z_{j,i-1} - \dot{\zeta}_{j,i-1}$, $\iota = -(d_j + b_j)$ for $i = 2$ and $\iota = 1$ for $i = 3, 4, \cdots, n-1$,

$$
\Xi(z_{j,i}) = \begin{cases} |z_{j,i}|^{1-\nu}\mathrm{sign}(z_{j,i}), & \text{if } |z_{j,i}| \geq \nu, \\ m_1 |z_{j,i}|^{2-\nu}\mathrm{sign}(z_{j,i}) + m_2 \nu^{|z_{j,i}|} z_{j,i}, & \text{if } |z_{j,i}| < \nu. \end{cases} \tag{4.24}
$$

The adaptive law is given as

$$
\dot{\hat{\chi}}_{j,i} = z_{j,i}\psi_{j,i} \tanh\Big(\frac{z_{j,i}\psi_{j,i}}{\omega}\Big) - \mu_1\hat{\chi}_{j,i} - \frac{\mu_2}{K_2}\hat{\chi}_{j,i}^{1-\nu} - \mu_3\hat{\chi}_{j,i}^{1+\nu}. \tag{4.25}
$$

By substituting (4.23) and (4.25) into (4.22), similar to the analysis in Step 1, we can obtain the following results.

$$
\dot{V}_{j,i} \leq -\frac{\mu_1}{2}\Big(z_{j,i}^2 + \tilde{\chi}_{j,i}^2\Big) - \mu_2\Big((z_{j,i}^2)^{1-\frac{\nu}{2}} + \tilde{\chi}_{j,i}^2)^{1-\frac{\nu}{2}}\Big) - \mu_3\Big((z_{j,i}^2)^{1+\frac{\nu}{2}} + (\tilde{\chi}_{j,i}^2)^{1+\frac{\nu}{2}}\Big)
$$

$$- \iota z_{j,i} z_{j,i-1} + z_{j,i} z_{j,i+1} + \bar{\rho}_{j,i}, \quad \text{for } |z_{j,i}| \geq \nu, \tag{4.26}$$

where $\bar{\rho}_{j,i} = \rho_{j,i} + \frac{\mu_1}{2}\chi_{j,i}^2 + \frac{\mu_2 2K_1}{K_2}\chi_{j,i}^{2-\nu} + \mu_3\chi_{j,i}^{2+\nu}$.

$$\begin{aligned} \dot{V}_{j,i} \leq & -\frac{\mu_1}{2}\Big(z_{j,i}^2 + \tilde{\chi}_{j,i}^2\Big) - \mu_2\Big((z_{j,i}^2)^{1-\frac{\nu}{2}} + \tilde{\chi}_{j,i}^2)^{1-\frac{\nu}{2}}\Big) - \mu_3\Big((z_{j,i}^2)^{1+\frac{\nu}{2}} + (\tilde{\chi}_{j,i}^2)^{1+\frac{\nu}{2}}\Big) \\ & - \iota z_{j,i} z_{j,i-1} + z_{j,i} z_{j,i+1} + \bar{\bar{\rho}}_{j,i}, \quad \text{for } |z_{j,i}| < \nu, \end{aligned} \tag{4.27}$$

where $\bar{\bar{\rho}}_{j,i} = \bar{\rho}_{j,i} + \mu_2\Big(\nu^2\Big)^{1-\frac{\nu}{2}}$.

Step n. From $z_{j,n} = x_{j,n} - \zeta_{j,n-1}$, one can obtain

$$\begin{aligned} \dot{z}_{j,n} &= f_{j,n} + u_j + \tau_{j,n} - \dot{\zeta}_{j,n-1} \\ &= \boldsymbol{\theta}_{j,n}^T \boldsymbol{\varphi}_{j,n} + \sigma_{j,n} + \tau_{j,n} + u_j - \dot{\zeta}_{j,n-1}, \end{aligned} \tag{4.28}$$

where $f_{j,n} = \boldsymbol{\theta}_{j,n}^T \boldsymbol{\varphi}_{j,n} + \sigma_{j,n}$. One can conclude that there exists an unknown positive constant $\bar{\kappa}_{j,n}$ such that $|\sigma_{j,n} + \tau_{j,n}| \leq \bar{\kappa}_{j,n}$. Similarly, by using Lemma 2.3, for any $\omega > 0$, one can get

$$\begin{aligned} z_{j,n}(\boldsymbol{\theta}_{j,n}^T \boldsymbol{\varphi}_{j,n} + \sigma_{j,n} + \tau_{j,n}) &\leq |z_{j,n}|(\|\boldsymbol{\theta}_{j,n}\|\|\boldsymbol{\varphi}_{j,n}\| + \bar{\kappa}_{j,n}) \\ &\leq |z_{j,n}|\chi_{j,n}\psi_{j,n} \\ &\leq z_{j,n}\chi_{j,n}\psi_{j,n} \tanh\Big(\frac{z_{j,n}\psi_{j,n}}{\omega}\Big) + 0.2785\omega\chi_{j,n}, \end{aligned} \tag{4.29}$$

where $\chi_{j,n} = \max\{\|\boldsymbol{\theta}_{j,n}\|, \bar{\kappa}_{j,n}\}$ and $\psi_{j,n} = \|\boldsymbol{\varphi}_{j,n}\| + 1$.

Define the estimation error $\tilde{\chi}_{j,n} = \chi_{j,n} - \hat{\chi}_{j,n}$, where $\hat{\chi}_{j,n}$ is the estimation of $\chi_{j,n}$. Choose the following Lyapunov function.

$$V_{j,n} = \frac{1}{2}z_{j,n}^2 + \frac{1}{2}\tilde{\chi}_{j,n}^2. \tag{4.30}$$

Similar to Step 1, the time derivation of $V_{j,n}$ yields

$$\begin{aligned} \dot{V}_{j,n} \leq & z_{j,n}\hat{\chi}_{j,n}\psi_{j,n} \tanh\Big(\frac{z_{j,n}\psi_{j,n}}{\omega}\Big) + z_{j,n}u_{j,n} - z_{j,n}\dot{\zeta}_{j,n-1} \\ & + \tilde{\chi}_{j,n}\Big(z_{j,n}\psi_{j,n} \tanh\Big(\frac{z_{j,n}\psi_{j,n}}{\omega}\Big) - \dot{\hat{\chi}}_{j,n}\Big) + \rho_{j,n}, \end{aligned} \tag{4.31}$$

where $\rho_{j,n} = 0.2785\omega\chi_{j,n}$. We can select the virtual controller and adaptive law as follows:

$$u_j = -\Big(\frac{\mu_1}{2}z_{j,n} + \mu_2\Xi(z_{j,n}) + \mu_3 z_{j,n}^{1+\nu} + \hat{\chi}_{j,n}\psi_{j,n} \tanh\Big(\frac{z_{j,n}\psi_{j,n}}{\omega}\Big) + \Omega_{j,n}\Big), \tag{4.32}$$

where $\Omega_{j,n} = z_{n-1} - \dot{\zeta}_{j,n-1}$,

$$\Xi(z_{j,n}) = \begin{cases} |z_{j,n}|^{1-\nu}\text{sign}(z_{j,n}), & \text{if } |z_{j,n}| \geq \nu, \\ m_1|z_{j,n}|^{2-\nu}\text{sign}(z_{j,n}) + m_2\nu^{|z_{j,n}|}z_{j,n}, & \text{if } |z_{j,n}| < \nu. \end{cases} \tag{4.33}$$

The adaptive law is given as

$$\dot{\hat{\chi}}_{j,n} = z_{j,n}\psi_{j,n} \tanh\Big(\frac{z_{j,n}\psi_{j,n}}{\omega}\Big) - \mu_1\hat{\chi}_{j,n} - \frac{\mu_2}{K_2}\hat{\chi}_{j,n}^{1-\nu} - \mu_3\hat{\chi}_{j,n}^{1+\nu}. \tag{4.34}$$

By substituting (4.32) and (4.34) into (4.31), similar to the analysis in Step 1, we can obtain the following results.

$$\dot{V}_{j,n} \leq -\frac{\mu_1}{2}\Big(z_{j,n}^2 + \tilde{\chi}_{j,n}^2\Big) - \mu_2\Big((z_{j,n}^2)^{1-\frac{\nu}{2}} + \tilde{\chi}_{j,n}^2)^{1-\frac{\nu}{2}}\Big) - \mu_3\Big((z_{j,n}^2)^{1+\frac{\nu}{2}} + (\tilde{\chi}_{j,n}^2)^{1+\frac{\nu}{2}}\Big)$$
$$- z_{j,n}z_{j,n-1} + \bar{\rho}_{j,n}, \quad \text{for } |z_{j,n}| \geq \nu, \tag{4.35}$$

where $\bar{\rho}_{j,n} = \rho_{j,n} + \frac{\mu_1}{2}\chi_{j,n}^2 + \frac{\mu_2 K_1}{K_2}\chi_{j,n}^{2-\nu} + \mu_3\chi_{j,n}^{2+\nu}$.

$$\dot{V}_{j,n} \leq -\frac{\mu_1}{2}\Big(z_{j,n}^2 - \mu_2\Big((z_{j,n}^2)^{1-\frac{\nu}{2}} + \tilde{\chi}_{j,n}^2)^{1-\frac{\nu}{2}}\Big) - \mu_3\Big((z_{j,n}^2)^{1+\frac{\nu}{2}} + (\tilde{\chi}_{j,n}^2)^{1+\frac{\nu}{2}}\Big) + \tilde{\chi}_{j,n}^2\Big)$$
$$- z_{j,n}z_{j,n-1} + \bar{\bar{\rho}}_{j,n}, \quad \text{for } |z_{j,n}| < \nu, \tag{4.36}$$

where $\bar{\bar{\rho}}_{j,n} = \bar{\rho}_{j,n} + \mu_2\Big(\nu^2\Big)^{1-\frac{\nu}{2}}$.

4.2. Stability analysis

Theorem 4.1. *Consider the leader-follower MAS* (3.1) *and* (3.2) *with Assumption 3.1. Under the virtual controllers* (4.7) *and* (4.23)*, actual controller* (4.32)*, and the parameter adaption laws* (4.9)*,* (4.25)*, and* (4.34)*, it holds that:*

(i) All error variables $z_{j,i}$ have practically predefined-time stability.

(ii) The predefined-time consensus tracking is achieved. The error $z_{j,1}$ can enter into a predefined small neighborhood in the predefined time.

Proof. Choose the Lapunov function V_j as

$$V_j = V_{j,1} + V_{j,2} + \cdots + V_{j,n}. \tag{4.37}$$

When $|z_{j,i}| \geq \nu$, it can be inferred from (4.16), (4.26), and (4.35) that

$$\dot{V}_j \leq -\frac{\mu_1}{2}\sum_{i=1}^{n}\Big(z_{j,i}^2 + \tilde{\chi}_{j,i}^2\Big) - \mu_2\sum_{i=1}^{n}\Big((z_{j,i}^2)^{1-\frac{\nu}{2}} + (\tilde{\chi}_{j,i}^2)^{1-\frac{\nu}{2}}\Big)$$
$$- \mu_3\sum_{i=1}^{n}\Big((z_{j,i}^2)^{1+\frac{\nu}{2}} + (\tilde{\chi}_{j,i}^2)^{1+\frac{\nu}{2}}\Big) + r_j, \tag{4.38}$$

where $r_j = \sum_{i=1}^{n}\bar{\rho}_{j,i}$. Select $\mu_1 = \frac{6}{\nu\mathcal{T}}$, $\mu_2 = \frac{3}{\nu\mathcal{T}2^{1-\frac{\nu}{2}}}$, and $\mu_3 = \frac{3\cdot 2^{\frac{\nu}{2}}n^{\frac{\nu}{2}}}{\nu\mathcal{T}2^{1+\frac{\nu}{2}}}$, and by using Lemma 2.2, one gets

$$\dot{V}_j \leq -\mu_1\sum_{i=1}^{n}\Big(\frac{1}{2}z_{j,i}^2 + \frac{1}{2}\tilde{\chi}_{j,i}^2\Big) - \mu_2 2^{1-\frac{\nu}{2}}\sum_{i=1}^{n}\Big((\frac{1}{2}z_{j,i}^2)^{1-\frac{\nu}{2}} + (\frac{1}{2}\tilde{\chi}_{j,i}^2)^{1-\frac{\nu}{2}}\Big)$$
$$- \mu_3 2^{1+\frac{\nu}{2}}\sum_{i=1}^{n}\Big((\frac{1}{2}z_{j,i}^2)^{1+\frac{\nu}{2}} + (\frac{1}{2}\tilde{\chi}_{j,i}^2)^{1+\frac{\nu}{2}}\Big) + r_j$$
$$\leq -\mu_1\sum_{i=1}^{n}\Big(\frac{1}{2}z_{j,i}^2 + \frac{1}{2}\tilde{\chi}_{j,i}^2\Big) - \mu_2 2^{1-\frac{\nu}{2}}\Big(\sum_{i=1}^{n}(\frac{1}{2}z_{j,i}^2 + \frac{1}{2}\tilde{\chi}_{j,i}^2)\Big)^{1-\frac{\nu}{2}}$$

$$
\begin{aligned}
&-\frac{\mu_3 2^{1+\frac{\nu}{2}}}{(2n)^{\frac{\nu}{2}}}\Big(\sum_{i=1}^{n}(\frac{1}{2}z_{j,i}^2+\frac{1}{2}\tilde{\chi}_{j,i}^2)\Big)^{1+\frac{\nu}{2}}+r_j\\
&=-\mu_1 V_j-\mu_2 2^{1-\frac{\nu}{2}}V_j^{1-\frac{\nu}{2}}-\frac{\mu_3 2^{1+\frac{\nu}{2}}}{(2n)^{\frac{\nu}{2}}}V_j^{1+\frac{\nu}{2}}+r_j\\
&=-\frac{3}{\nu\mathcal{T}}\Big(V_j^{1-\frac{\nu}{2}}+2V_j+V_j^{1+\frac{\nu}{2}}\Big)+r_j. \qquad (4.39)
\end{aligned}
$$

From (4.39), one can conclude that the tracking errors of the jth follower have practically predefined-time stability. Denote $\delta_j = \frac{\nu r_j\mathcal{T}}{4}$ and $\Omega_j = \{z_{j,i}|V_f \le \delta_j\}$, and according to Lemma 2.1, one can obtain that the tracking errors will enter the region Ω_j within the predefined time $\mathcal{T}$, which means $z_{j,i} \le \sqrt{2\delta_j}$. This ends the proof.

Remark 4.1. *Note that a switched control function*

$$
F(z_{j,i}) = \begin{cases} \frac{z_{j,i}}{|z_{j,i}|^2}, & |z_{j,i}| \ne 0,\\ 0, & |z_{j,i}| = 0, \end{cases}
$$

was employed in [48], which will bring about the singularity phenomenon when $|z_{j,i}|$ is very small. For this reason, the predefined parameter ν and the function $\Xi(z_{j,i})$ are provided in this article to solve the singularity problem.

Remark 4.2. *The control parameters in this article include the predefined boundary parameter ν, the predefined time parameter $\mathcal{T}$, the adaptive adjustment parameter ω, and the filter parameter ϖ. When parameters $\mathcal{T}$, ω, ϖ, and ν are selected appropriately, the performance of tracking errors can be well displayed. One can refer to the simulation section for detailed and intuitive results.*

Remark 4.3. *The selection guidance for the parameters is displayed as follows. 1) The parameter $\varpi_{j,i-1}$ in the filter* (3.5) *should be chosen as large as possible to ensure a preeminent approximation effect. However, excessive value will lead to parameter drift. 2) In order to achieve fast stability, a small $\mathcal{T}$ should be devised. Meanwhile, a small ν can realize a high control accuracy. However, the required control cost will correspondingly increase. Therefore, a favorable balance is required to be grasped between control accuracy and cost.*

5. Simulation results

In this section, consensus tracking of a MAS with a leader and four followers is studied. The communication graph is shown in Figure 1. From Figure 1, the corresponding adjacency matrix and Laplacian matrix are shown as

$$
A = \begin{bmatrix} 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0 \end{bmatrix}, L = \begin{bmatrix} 1 & -1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & -1 & 1 & 0\\ 0 & 0 & -1 & 1 \end{bmatrix}.
$$

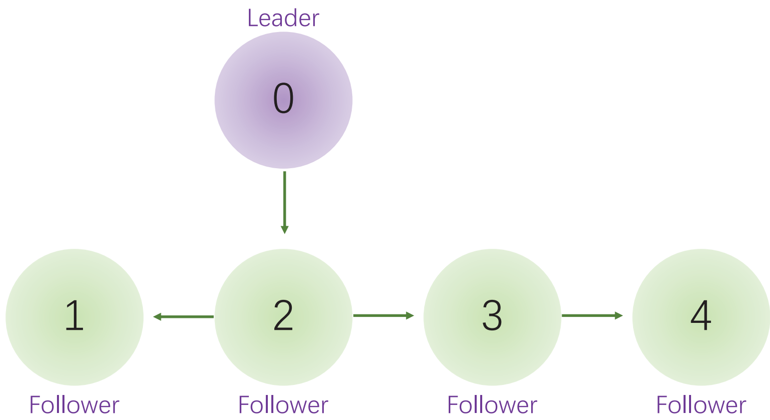


Figure 1. Communication graph.

Example. The dynamics of four followers are described as

$$\begin{cases} \dot{x}_{j,1} = (1 + 0.1(j-1))\cos(x_{j,1}) + \dfrac{x_{j,1}^2}{1 + x_{j,1}^2} + 0.25\sin(t) + x_{j,2}, \\ \dot{x}_{j,2} = x_{j,2}x_{j,1}^2 + (1 + 0.1(j-1))\cos(x_{j,1})x_{j,2}^2 + 0.25\cos(t) + u_j, \\ y_j = x_{j,1}. \end{cases} \tag{5.1}$$

Let $f_{j,1} = (1+0.1(j-1))\cos(x_{j,1}) + \frac{x_{j,1}^2}{1+x_{j,1}^2}$, $f_{j,2} = x_{j,2}x_{j,1}^2 + (1+0.1(j-1))\cos(x_{j,1})x_{j,2}^2$, $\tau_{j,1} = 0.25\sin(t)$, and $\tau_{j,2} = 0.25\cos(t)$. The FLSs are employed to estimate unknown functions of the system (5.1). Select the fuzzy set as [-5,5] and partitioning points as $-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$. The fuzzy functions are chosen as

$$\mu_{B_{j,i}^s} = \exp\Big[-\Big(\frac{x_{j,i} - 6 + s}{2}\Big)^2\Big],$$

where $j = 1, 2, 3, 4$, $i = 1, 2$, $s = 1, 2, \cdot\cdot\cdot, 11$. The initial values are chosen as $[x_{1,1}(0), x_{1,2}(0)]^T = [-1.20, 1.00]^T$, $[x_{2,1}(0), x_{2,2}(0)]^T = [0.60, -0.90]^T$, $[x_{3,1}(0), x_{3,2}(0)]^T = [-1.00, 0.80]^T$, and $[x_{4,1}(0), x_{4,2}(0)]^T = [1.40, -0.60]^T$, $\hat{\chi}_{j,i}(0) = 0$. Let the output of the leader be $y_0 = -\cos(1.5t) + \sin(0.5t)$. The values of the control parameters are $\mathcal{T} = 2$, $\omega = 0.01$, $\varpi = 100$, and $\nu = \frac{16}{121}$.

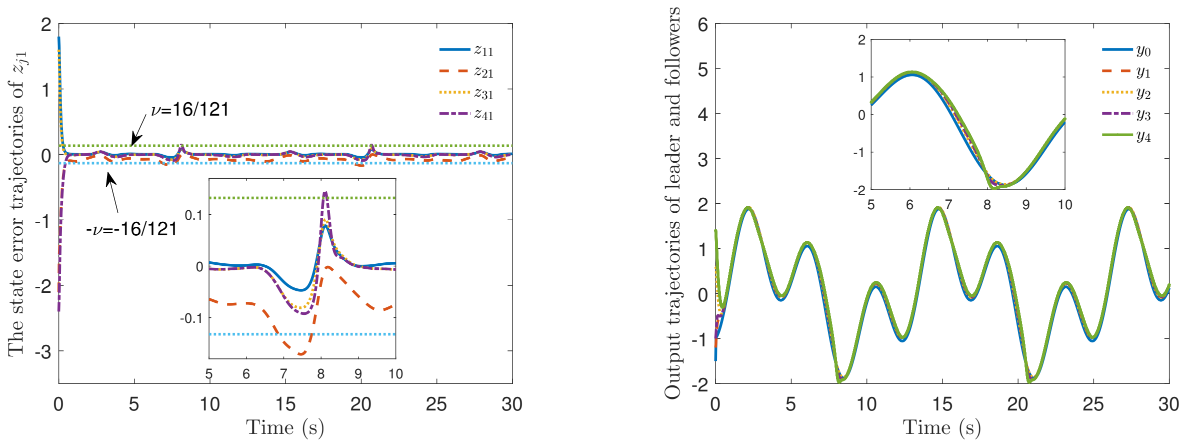
In order to compare the control performance of consensus tracking, a practically predefined-time control method for the follower systems (5.1) is first presented. Corresponding controllers and adaptive laws are designed as follows.

$$\begin{cases} \alpha_{j,1} = \dfrac{\mu_2 z_{j,1}^{1-\nu} + \mu_3 z_{j,1}^{1+\nu} + \hat{\chi}_{j,1}\psi_{j,1}\tanh\left(\frac{z_{j,1}\psi_{j,1}}{\omega}\right) + \Omega_{j,1}}{d_j + b_j}, \\ u_j = -\Big(\mu_2 z_{j,2}^{1-\nu} + \mu_3 z_{j,2}^{1+\nu} + \hat{\chi}_{j,2}\psi_{j,2}\tanh\Big(\dfrac{z_{j,2}\psi_{j,2}}{\omega}\Big) + \Omega_{j,2}\Big), \\ \dot{\hat{\chi}}_{j,1} = z_{j,1}\psi_{j,1}\tanh\Big(\dfrac{z_{j,1}\psi_{j,1}}{\omega}\Big) - \dfrac{\mu_2}{K_2}\hat{\chi}_{j,1}^{1-\nu} - \mu_3\hat{\chi}_{j,1}^{1+\nu}, \\ \dot{\hat{\chi}}_{j,2} = z_{j,2}\psi_{j,2}\tanh\Big(\dfrac{z_{j,2}\psi_{j,2}}{\omega}\Big) - \dfrac{\mu_2}{K_2}\hat{\chi}_{j,2}^{1-\nu} - \mu_3\hat{\chi}_{j,2}^{1+\nu}, \end{cases} \tag{5.2}$$

where $z_{j,2} = x_{j,2} - \zeta_{j,1}$, $\dot{\zeta}_{j,1} = -\varpi_{j,1}(\zeta_{j,1} - \alpha_{j,1})$, $\Omega_{j,1} = \sum_{k=1}^{4} a_{j,k}x_{k,2} + b_j\dot{y}_0 + \frac{1}{2}z_{j,1}$, $\Omega_{j,2} = -(d_j + b_j)z_1 - \dot{\zeta}_{j,1}$. By taking the derivative of function V_j in (4.37), one can obtain

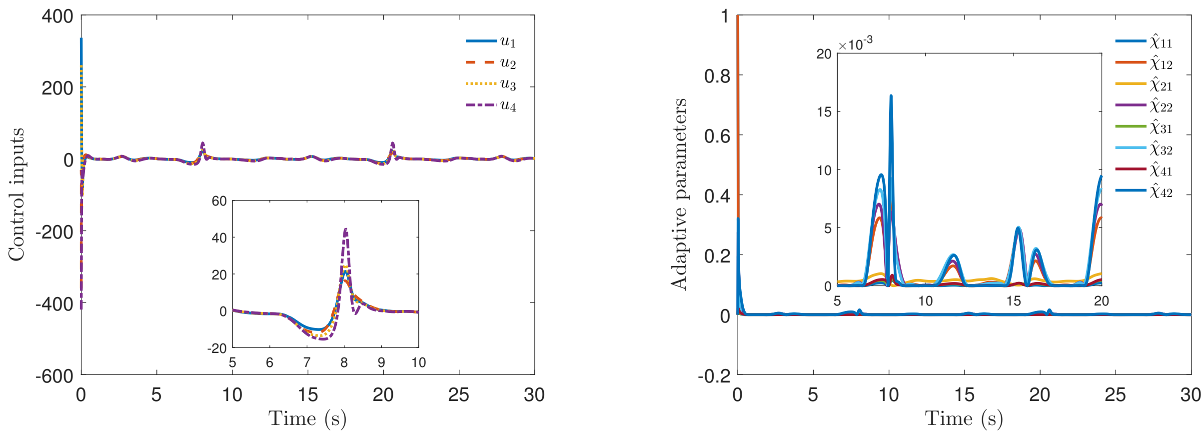
$$\dot{V}_j \leq -\frac{\pi}{\nu\mathcal{T}}\Big(V_j^{1-\frac{\nu}{2}} + V_j^{1+\frac{\nu}{2}}\Big) + \Delta_2, \tag{5.3}$$

where $\mu_2 = \frac{\pi}{\nu\mathcal{T}}$, $\mu_3 = \frac{\pi 2^{\frac{\nu}{2}}4^{\frac{\nu}{2}}}{\nu\mathcal{T}2^{1+\frac{\nu}{2}}}$, and $\Delta_2 = \sum_{j=1}^{4}\Big(0.2785\omega\psi_{j,1} + \frac{1}{2}(d_j + b_j)^2\bar{\xi}_{j,1}^2 + 0.2785\omega\psi_{j,2} + \frac{\mu_2 K_1}{K_2}\chi_{j,1}^{2-\nu} + \frac{\mu_2 K_1}{K_2}\chi_{j,2}^{2-\nu} + \mu_3\chi_{j,1}^{2-\nu} + \mu_3\chi_{j,1}^{2-\nu}\Big)$. By using Lemma 3 in [49], one can conclude that tracking errors will enter a certain neighborhood within the predefined time $2\mathcal{T}$. The simulation results based on the practically predefined-time control method (5.2) are shown in Figures 2 and 3. Set the predefined interval as $[-\frac{16}{121}, \frac{16}{121}]$. Figure 2(a) shows that tracking errors $z_{1,1}$, $z_{2,1}$, $z_{3,1}$, and $z_{4,1}$ can enter a neighborhood of the origin within the predefined time $4s$, but tracking errors exhibit periodic jumps (see 7s to 9s). Figure 2(b) displays the output signals of the leader and followers. Figures 3(a) and 3(b) show the response curves of adaptive parameters and control inputs, respectively. It can be seen that the practically predefined-time control method (5.2) cannot weaken the impact of periodic jumps that occur in the system itself and limit tracking errors within the predefined interval $[-\frac{16}{121}, \frac{16}{121}]$.



(a) (b)

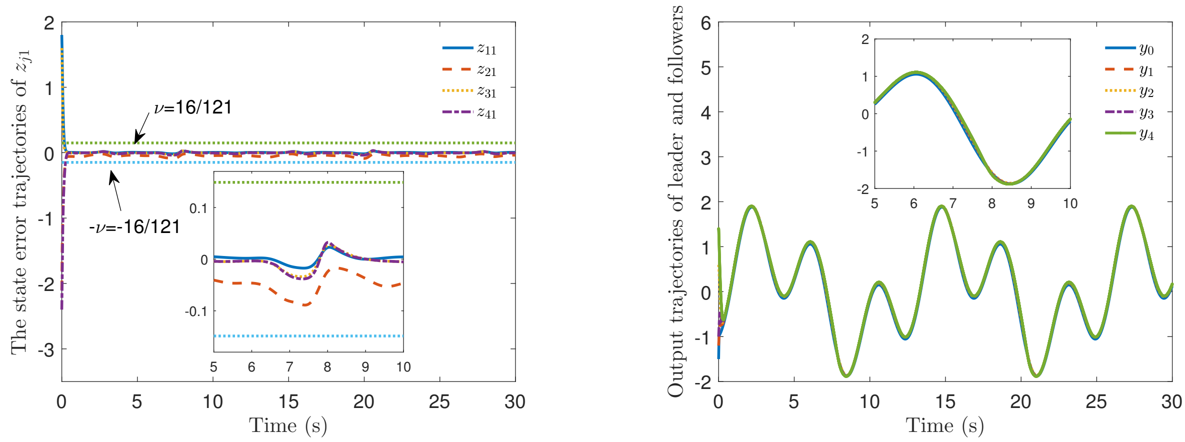
Figure 2. Simulation results based on the practically predefined-time control method (5.2): (a) The response trajectories of tracking errors; (b) the output trajectories of the leader and followers.



(a) (b)

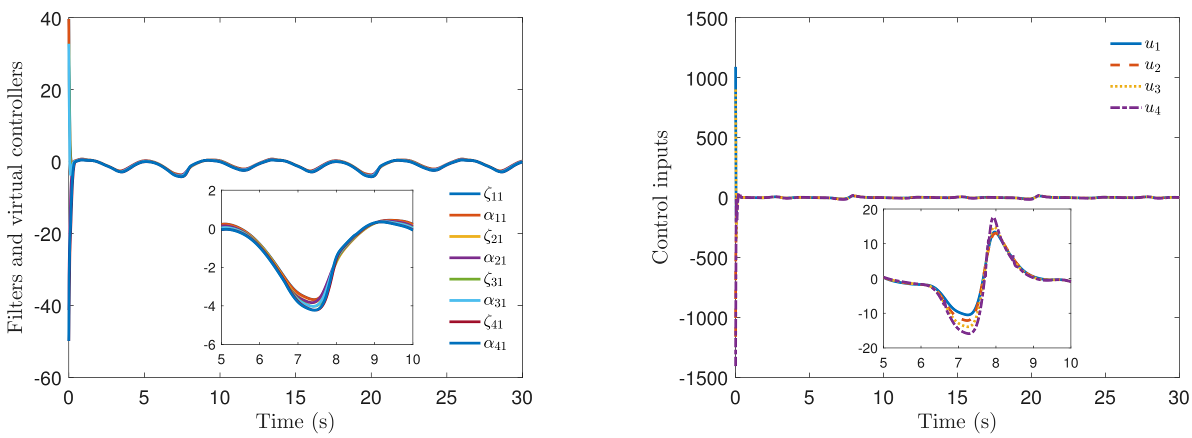
Figure 3. Simulation results based on the practically predefined-time control method (5.2): (a) The response trajectories of control inputs; (b) the response trajectories of the adaptive parameters.

Using the same control parameters and initial values, the simulation results of the proposed practically predefined-time control method (4.32) in this article are shown in Figures 4–7. Figure 4(a) shows all tracking errors $z_{1,1}, z_{2,1}, z_{3,1}$, and $z_{4,1}$ arrive at the predefined interval $[-\frac{16}{121}, \frac{16}{121}]$ within the predefined time $2s$ and the periodic jumping phenomenon is reduced. Comparing Figures 2 and 4, it can be concluded that the control effect of the proposed method (4.32) is better. Figure 5(a) shows that the first-order filter $\zeta_{j,1}$ can effectively estimate the virtual controller $\alpha_{j,1}$. Figure 5(b) shows that the control method (4.32) in this article requires more energy to overcome periodic jumps in the system itself. Figures 6(a) and 6(b) show that the followers' states and adaptive parameters are bounded. Figures 7(a) and (b) display that the consensus tracking performance can be improved by adjusting parameter ν in the proposed practically predefined-time control method (4.32). Obviously, the above simulation results verify the effectiveness and advantage of the proposed control approach.



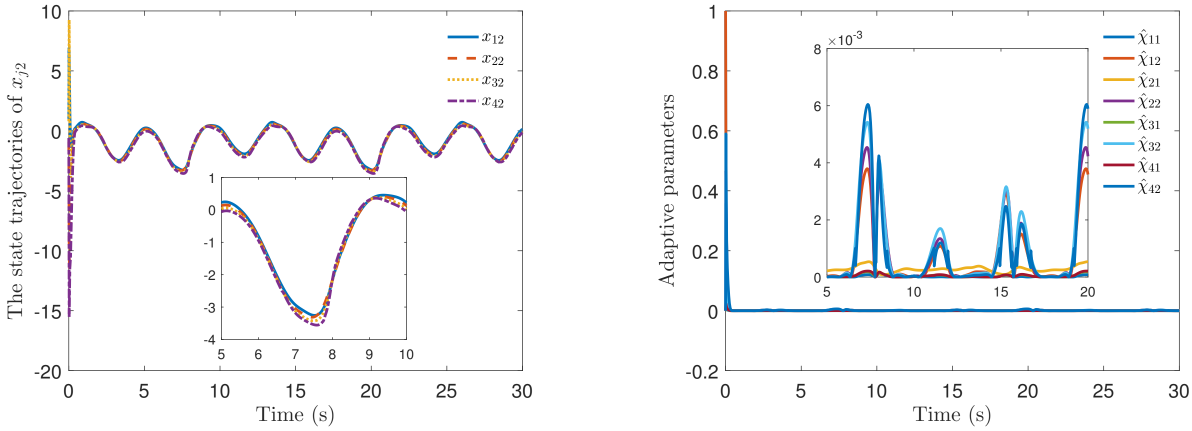
(a) (b)

Figure 4. Simulation results based on the proposed predefined-time control method (4.32) in this article: (a) The response trajectories of tracking errors; (b) the output trajectories of the leader and followers.



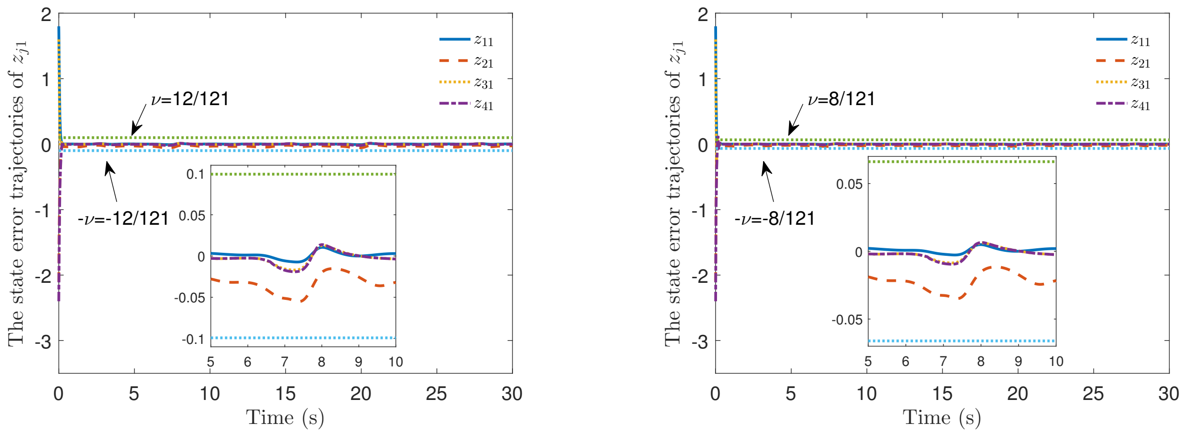
(a) (b)

Figure 5. Simulation results based on the proposed practically predefined-time control method (4.32) in this article: (a) The response trajectories of filters and virtual controllers; (b) the response trajectories of the input controllers.



(a) (b)

Figure 6. Simulation results based on the proposed practically predefined-time control method (4.32) in this article: (a) The response trajectories of states x_{j2}; (b) the response trajectories of the adaptive parameters.



(a) (b)

Figure 7. Simulation results based on the proposed practically predefined-time control method (4.32) in this article: (a) $\nu = \frac{12}{121}$; (b) $\nu = \frac{8}{121}$.

6. Conclusions

In this article, a cooperative output tracking control method was developed for uncertain MASs in strict-feedback form. It has been shown that the proposed adaptive fuzzy control approach guarantees practically predefined-time stability for the consensus tracking errors. The command-filtered-based backstepping recursive approach was utilized to realize the control objective and avoid the complexity explosion problem. Moreover, the consensus errors can be stable not only within a specified time but also in a sufficiently small predefined region. Compared to existing control methods for MASs, the proposed control method in this article only requires four control parameters, and the consensus tracking performance is directly associated with the predefined boundary parameter. A simulation example verified the effectiveness and advantage of the strategy proposed in this article. The direction of future work is to investigate practically predefined-time consensus tracking for nonlinear MASs subject to input saturation and tracking error constraints.

Author contributions

Fang Zhu:Writing-original draft. Pengtong Li: Writing-review and editing.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare that they have no conflicts of interest.

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