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*Research article*

# Neural network-based adaptive finite-time tracking control for multiple inputs uncertain nonlinear systems with positive odd integer powers and unknown multiple faults

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**Abstract:** This paper addresses the adaptive finite-time tracking control (FTTC) problem for multiple-input nonlinear systems (NSs). The system under consideration encompasses high-order nonlinear terms with positive odd integer powers, uncertain dynamics, parametric nonlinear dynamics, multiple unknown faults, and unknown control gains. The proposed adaptive FTTC strategy integrates the neural network (NN) approximation technique with the backstepping control approach. By employing the NN approximator, the challenge of approximating uncertain nonlinear dynamics and unknown nonlinear functions was effectively resolved. Concurrently, adaptive control laws for unknown parameters were formulated using the adaptive estimation method. Furthermore, to address unknown control coefficients arising from unknown faults and unknown control gains within the system, the Nussbaum gain function (NGF) was incorporated into the control design process. Subsequently, NN-based adaptive FTTC strategies were developed for inputs under various fault conditions. The designed control strategies ensured that all signals of the closed-loop system (ASCLS) with multiple faults maintain semi-global practical finite-time stability (SGPFS), and the tracking error of the system converges to a small neighborhood of zero within a finite time (SNZFT). Finally, the efficacy of the developed control method was validated through a simulation example.

**Keywords:** multiple-input nonlinear systems; finite-time control; unknown multiple faults; odd integer power; neural network

**Mathematics Subject Classification:** 93A30, 93C10, 97N40

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## 1. Introduction

To date, the control design and performance analysis of NSs continue to attract significant attention due to the inherent nonlinearities present in most practical engineering systems. It is evident that uncertain nonlinear dynamics and unknown nonlinearities pose inevitable challenges in the control design of NSs. Fuzzy logic systems (FLS) and NN methods have been extensively used to approximate these uncertain dynamics and unknown nonlinearities, owing to their capability to approximate any smooth nonlinear function with arbitrary accuracy [1,2]. Numerous control scenarios based on FLS or NN have been investigated for various NSs [3–6]. For instance, FLS was considered [7] to develop an adaptive fuzzy inverse optimal control scheme for the uncertain NSs with the strict-feedback form, while an adaptive fault-tolerant sliding mode controller with NN was designed [8] to solve the problem of automatic ships under the influence of actuator faults and input saturation.

Significant progress has been made in the control design of NSs, as evidenced by studies such as [9–15]. However, it is noteworthy that studies [9–13] primarily addressed the issue of exponential convergence, where the tracking error converges to a compact set only as time approaches infinity. In many practical applications, it is more practical to achieve the desired tracking performance within a finite time (FT), as highlighted in studies [16,17]. Consequently, the problem of FTTC problems for NSs has been extensively investigated in the literature. An FT consensus control scheme incorporating an extended state observer was proposed for nonlinear multi-agent systems with uncertain dynamics, where the observer was designed to estimate unavailable states and external disturbances [15]. Liu et al. [18] investigated fractional-order nonlinear multi-agent systems and developed an FTTC scheme with an event-triggered mechanism for the containment control issue. Moreover, an FT controller leveraging NNs [19] was developed for output-constrained NSs, in which the control design process was significantly simplified using an improved command filter. Additionally, to achieve FTTC for a specific class of performance-constrained NSs, an optimal FT adaptive performance constrain control scheme was formulated based on the reinforcement learning method [20]. Although the issues of FTTC have been extensively deliberated in the aforementioned works, the FTTC problem of NSs with multiple inputs has scarcely been discussed by researchers. It is evident that when the system is subjected to more than one input, the design of a controller to achieve FT control presents a topic worthy of discussion. This motivated the completion of this paper.

A review of the aforementioned literature reveals a common characteristic: The powers of the systems are uniformly equal to one. However, certain practical systems exhibit positive odd integer powers, such as the coupled inverted double pendulum system [21], the planar power integrator system [22], and underactuated mechanical systems [23]. These systems are classified as high-order nonlinear systems (HONSs), yet they have received relatively limited attention. Due to the presence of high-order terms, many control schemes derived from the NSs with powers equal to one cannot be directly applied to high-order cases. At present, some research results on HONSs have been proposed. In light of sensor fault and dead-zone fault, adaptive NN fault-tolerant control strategies were proposed to address the tracking challenges associated with HONSs [24,25]. The prescribed time control problem for HONSs with actuator faults was addressed [26], where the proposed strategy ensures that system states converge to zero within a predetermined time frame, independent of initial conditions and design parameters. Moreover, Lv et al. [27] achieved asymptotic tracking control of HONSs with odd rational powers from the perspective of prescribed performance. The event-triggered tracking control issues for HONSs with full-state constraints and input saturation were explored [28]. Additionally, concerning the FTTC of HONSs, Li et al. [29] developed an FTTC scheme utilizing a dual event-triggered mechanism, while fast and global FT adaptive stabilization

challenges were addressed [30–32]. However, it is important to note that all aforementioned HONSs consider only a single input. The challenge of ensuring the FT convergence for HONSs with multiple inputs represents a significant area for further investigation, which also motivated the completion of this paper.

In practical engineering applications, faults are both common and unavoidable. When a system experiences a fault, it often leads to degraded performance and can even cause instability within the closed-loop system. As a result, investigating control challenges for NSs with fault characteristics is of significant practical importance. Notably, the control issues related to NSs with actuator faults have been extensively explored, yielding numerous commendable outcomes [33–35]. Additionally, the controllers developed in prior studies have successfully maintained the control performance of NSs with sensor faults [36,37]. Moreover, the adaptive tracking control challenges for uncertain NSs affected by dead-zone faults have been examined [38,39]. Obviously, the aforementioned results primarily focus on single-input fault systems, while research on the control problem of multiple-input fault systems is rarely mentioned. Furthermore, there appears to be a notable gap in the existing literature regarding fault issues in the multiple-input NSs with positive odd integer powers, as well as issues of the control direction arising from unknown faults and parametric nonlinear dynamics. This further motivated the completion of this paper.

In light of the preceding discussion, the FT tracking issue for a class of multiple-input uncertain NSs with positive odd integer powers, parametric nonlinear dynamics, and unknown multiple faults is studied in this paper. The radial basis function neural network (RBFNN) and NGF technique are employed to handle unknown nonlinear dynamics and unknown control coefficients, respectively. Within the framework of the backstepping control technique, an NN-based adaptive FTTC strategy is designed for each input. The main contributions of this paper are summarized as follows.

(i) Unlike the systems studied in the previous papers [12,13,24–26], the system model presented in this paper incorporates high-order terms with positive odd integer powers, multiple inputs, multiple unknown faults, and unknown control coefficients. Obviously, the proposed model exhibits greater complexity and generality.

(ii) By employing the RBFNN approximation method, the problem of approximating uncertain nonlinear dynamics and unknown nonlinear functions is effectively resolved. Meanwhile, the unknown control gains arising from unknown control coefficients and unknown faults are handled using the NGF technique. The successful application of these methods significantly reduces the complexity of designing control strategies.

(iii) In contrast to the tracking control problems discussed in [29–32], this paper explores the FTTC problem for multiple-input NSs. Subsequently, NN-based adaptive FTTC strategies are developed for inputs subject to different faults.

(iv) The proposed control strategies ensure that ASCLS achieves SGPFS, and the tracking error converges to a SNZFT, regardless of the impacts of unknown control gains and multiple unknown faults.

The remainder of this paper is organized as follows: Section 2 presents the problem formulation and preliminaries, including fault models, RBFNN, and relevant lemmas. Section 3 details the main results of this paper, focusing on the design of FTTC strategies and the stability analysis. Section 4 provides simulation results to demonstrate the efficacy of the proposed control approach. Finally, Section 5 concludes the paper.

## 2. Problem description and preliminaries

### 2.1. System description

Consider the following multiple-input uncertain parametric NSs with positive odd integer powers and unknown multiple faults

$$\begin{aligned}\dot{x}_i &= x_{i+1}^{\beta_i} + \theta_i^T \gamma_i(\bar{x}_i) + f_i(\bar{x}_i), i = 1, \dots, n-1, \\ \dot{x}_n &= \sum_{j=1}^p g_{nj}(\bar{x}_n) u_j^F(t) + \theta_n^T \gamma_n(\bar{x}_n) + f_n(\bar{x}_n), \\ y &= x_1,\end{aligned}\tag{2.1}$$

where  $\bar{x}_i = [x_1, \dots, x_i]^T \in R^i$ ,  $i = 1, \dots, n$ , denotes state vectors,  $u_j^F(t) \in R$ ,  $j = 1, \dots, p$ , represents control inputs with  $p$  being the number of control inputs,  $y \in R$  denotes the system output,  $\beta_i \in R_{\text{odd}}^{\geq 1}$ ,  $i = 1, \dots, n-1$ , denotes the positive odd integer powers,  $\theta_i \in R^q$ ,  $\gamma_i(\bar{x}_i) \in R^q$ ,  $i = 1, \dots, n$ , represent unknown constant vectors and known smooth function vectors,  $f_i(\bar{x}_i)$ ,  $i = 1, \dots, n$ , represents uncertain nonlinear dynamics, and  $g_{nj}(\bar{x}_n) \neq 0$ ,  $j = 1, \dots, p$ , are unknown but bounded nonlinear functions representing control gains. For convenience,  $\gamma_i(\bar{x}_i)$ ,  $\gamma_n(\bar{x}_n)$ ,  $f_i(\bar{x}_i)$ ,  $f_n(\bar{x}_n)$ , and  $g_{nj}(\bar{x}_n)$  are abbreviated as  $\gamma_i$ ,  $\gamma_n$ ,  $f_i$ ,  $f_n$ ,  $g_i$ , and  $g_{nj}$ , respectively.

In real engineering applications, systems are often subject to multiple simultaneous faults. Hence, this paper considers the coexistence of actuator faults and dead-zone faults within the system. To maintain generality, the actuator fault is modeled as

$$u_j^F(t) = h_{1j} u_j(t) + o_{1j}(t), j = 1, \dots, d,\tag{2.2}$$

and the dead-zone fault is denoted as

$$u_j^F(t) = \begin{cases} h_{2j} (u_j(t) - a_{jr}) & a_{jr} \leq u_j(t), \\ 0 & -a_{jl} < u_j(t) < a_{jr}, j = d+1, \dots, p, \\ h_{2j} (u_j(t) + a_{jl}) & u_j(t) \leq -a_{jl}. \end{cases}\tag{2.3}$$

In (2.2),  $0 \leq h_{1j} \leq 1$ ,  $o_{1j}(t)$  is the bounded bias signal, and  $u_j(t)$ ,  $j = 1, \dots, d$ , represents the control input of the  $j$ th actuator; in (2.3),  $h_{2j} > 0$  is the slope of the dead-zone fault,  $a_{jl} > 0$  and  $a_{jr} > 0$  represent the left and right breakpoints of the dead-zone fault, and  $u_j(t)$ ,  $j = d+1, \dots, p$ , is the control input of the  $j$ th dead-zone. In this paper, we assume  $h_{1j}$ ,  $o_{1j}(t)$ ,  $h_{2j}$ ,  $a_{jl}$ , and  $a_{jr}$  are unknown but bounded.

Noting (2), the following actuator operation modes are included:

- 1) For the case  $h_{1j} = 0$  and  $o_{1j}(t) \neq 0$ , it implies that the actuator is undergoing the bias fault.
- 2) For the case  $0 < h_{1j} < 1$  and  $o_{1j}(t) = 0$ , it implies that the actuator works in the partial loss of effectiveness.
- 3) For the case  $0 < h_{1j} < 1$  and  $o_{1j}(t) \neq 0$ , it indicates that the actuator operates under conditions of both partial loss of effectiveness and simultaneous bias fault.

4) For the case  $h_{1j} = 1$  and  $o_{1j}(t) = 0$ , it signifies that the actuator functions normally.

In addition, the dead-zone fault (2.3) can also be rewritten as [4,39]

$$u_j^F(t) = h_{2j}u_j(t) + o_{2j}(t), \quad j = d+1, \dots, p, \quad (2.4)$$

with

$$o_{2j}(t) = \begin{cases} -h_{2j}a_{jr} & a_{jr} \geq u_j(t), \\ -h_{2j}u_j(t) & -a_{jl} < u_j(t) < a_{jr}, \\ h_{2j}a_{jl} & u_j(t) \leq -a_{jl}. \end{cases} \quad (2.5)$$

The primary control objective of this paper is to develop adaptive FTTC strategies for the system (2.1) with multiple faults, ensuring that ASCLS remain bounded and that the tracking error converges to a SNZFT.

The paper operates under the following assumption:

**Assumption 2.1.** The signs of  $g_{nj}(\bar{x}_n)$ ,  $j = 1, \dots, p$ , are unknown, and there are unknown constants  $g_{nj,M} > 0$  such that  $0 < |g_{nj}(\bar{x}_n)| \leq g_{nj,M}$ . Without loss of generality, let  $0 < g_{nj}(\bar{x}_n) \leq g_{nj,M}$ .

**Remark 2.1.** In fact, Assumption 2.1 has been widely considered and applied as one of the stability conditions of controllable system (2.1) [12,25,29,31–33]. Given that  $g_{nj}(\bar{x}_n) \neq 0$  and the direction is unknown, the NGF technique can be introduced to realize the design of the control strategy.

**Remark 2.2.** Observing the system (2.1), it can be transformed into different systems when  $\beta_i = 1$  and  $p = 1$ , which has been discussed in studies [11,13,20,36]. However, when  $\beta_i \neq 1$  and  $p \neq 1$ , the control strategies proposed in studies [11,13,20,36] are no longer applicable to the system (2.1). Although the NSs with positive odd integer or odd rational powers have been studied [25–30], without exception, these systems have only one input, and only some research works have considered the impact of input fault. In comparison, this paper not only considers multiple inputs but also takes into account the effects of actuator fault or dead-zone fault on each input. Obviously, the system considered in this paper is more general.

**Remark 2.3.** In contrast to the systems considered in studies [25–30], the system discussed in this paper is entirely different, which also means that existing control methods cannot be directly applied to this study. Therefore, it is of great significance to design the corresponding control strategy for the system and achieve the expected tracking performance. This paper achieves this goal from another angle.

## 2.2. RBFNN

An RBFNN  $W^T\psi(X)$  is used to handle any unknown nonlinear function  $\mathcal{Y}(X)$  [6,24], that is

$$\mathcal{Y}(X) = W^T\psi(X), \quad (2.6)$$

where  $X \in \Omega_X \subset R^n$  stands for the input vector,  $W = [w_1, \dots, w_l]^T \in R^l$  represents the weight vector,  $\psi(X) = [\psi_1(X), \dots, \psi_l(X)]^T \in R^l$  denotes the basis function vector where  $\psi_i(X)$  is selected as the Gaussian functions as  $\psi_i(X) = \exp\left(-\frac{(X - \ell_i)^T(X - \ell_i)}{2b_i^2}\right)$ ,  $i = 1, \dots, l$  and  $\ell_i = [\ell_{i1}, \dots, \ell_{in}]^T$  represent the center vector, and  $b_i$  stands for the width.

Then, for the unknown nonlinear function  $\mathcal{L}(X)$  over the compact set,  $\Omega_X \subset R^n$  can be

approximated as

$$\mathcal{L}(X) = (W^*)^T \psi(X) + \varepsilon(X), \quad (2.7)$$

where  $\varepsilon(X)$  represents the approximation error and satisfies  $|\varepsilon(X)| \leq \Xi$  with  $\Xi > 0$ , and the ideal weight vector  $W^*$  is defined as

$$W^* := \arg \min_{W \in R^l} \left\{ \sup_{X \in \Omega_X} |\mathcal{L}(X) - W^T \psi(X)| \right\}. \quad (2.8)$$

### 2.3. Useful definitions and lemmas

**Definition 2.1.** [10] A smooth function  $\mathcal{N}(\chi)$  that satisfies the following properties

$$\begin{cases} \limsup_{r \rightarrow \infty} \frac{1}{r} \int_0^r \mathcal{N}(\chi) d\chi = +\infty, \\ \liminf_{r \rightarrow \infty} \frac{1}{r} \int_0^r \mathcal{N}(\chi) d\chi = -\infty, \end{cases} \quad (2.9)$$

is called the Nussbaum-type function. There are several functions that serve as Nussbaum functions such as  $\chi^2 \sin(\chi)$ ,  $\chi^2 \cos(\chi)$ , and  $\exp(\chi^2) \cos(\pi\chi/2)$ . The Nussbaum-type function  $\mathcal{N}(\chi) = \exp(\chi^2) \cos(\pi\chi/2)$  is selected in this paper.

**Lemma 2.1.** [40] Let  $\chi_i(t)$  be a smooth function on  $[0, t_f)$ ,  $V(t)$  be a positive definite function, and  $\mathcal{N}_i(\chi_i)$  be Nussbaum-type function. If there exists

$$V(t) \leq \xi_0 + e^{(-\xi_1 t)} \sum_{i=1}^n \int_0^t e^{(\xi_1 \tau)} (G_i(\tau) \mathcal{N}_i(\chi_i) + 1) \dot{\chi}_i d\tau, \quad (2.10)$$

where  $\xi_0$  and  $\xi_1$  are positive constants, and  $G_i(t)$  is non-zero but a bounded time-varying parameter, then  $V(t)$  and  $\chi_i(t)$  are bounded on  $[0, t_f)$ .

**Lemma 2.2.** [41] Consider the nonlinear system  $\dot{x} = f(x)$ ; if there is a positive definite and smooth function  $V(x)$  and constants  $\alpha > 0$ ,  $0 < \rho < 1$ , and  $C_0 > 0$  such that  $\dot{V}(x) \leq -\alpha V^\rho(x) + C_0$  for  $t \geq 0$ , then  $\dot{x} = f(x)$  is SGPFSS and satisfies

$$V^\rho(x) \leq \frac{C_0}{(1-\lambda)\alpha}, \quad \forall t \geq T_s, \quad (2.11)$$

and the reach time  $T_s$  is

$$T_s \leq \frac{1}{(1-\rho)\lambda\alpha} \left[ V^{1-\rho}(x(0)) - \left( \frac{C_0}{(1-\lambda)\alpha} \right)^{\frac{1-\rho}{\rho}} \right], \quad (2.12)$$

where  $0 < \lambda < 1$ , and  $x(0)$  is the initial state of the system.

**Lemma 2.3.** [24] For all  $\mathcal{A}_1 \in R$  and  $\mathcal{A}_2 \in R$ , there is an odd integer  $\beta \geq 1$  such that

$$|\mathcal{A}_1^\beta - \mathcal{A}_2^\beta| \leq \beta |\mathcal{A}_1 - \mathcal{A}_2| (\mathcal{A}_1^{\beta-1} + \mathcal{A}_2^{\beta-1}). \quad (2.13)$$

**Lemma 2.4.** [25] For  $\mathcal{O}_1 \in \mathbb{R}$  and  $\mathcal{O}_2 \in \mathbb{R}$ , and any positive constants  $a_1$ ,  $a_2$ , and  $a_3$ , there is

$$|\mathcal{O}_1|^{a_1} |\mathcal{O}_2|^{a_2} \leq \frac{a_1}{a_1 + a_2} a_3 |\mathcal{O}_1|^{a_1 + a_2} + \frac{a_2}{a_1 + a_2} a_3^{\frac{a_1}{a_2}} |\mathcal{O}_2|^{a_1 + a_2}. \quad (2.14)$$

**Lemma 2.5.** [27] For any positive constants  $\mathcal{Z}$  and  $\eta$ , there is

$$0 \leq |\mathcal{Z}| - \frac{\mathcal{Z}^2}{\sqrt{\mathcal{Z}^2 + \eta^2}} \leq \eta. \quad (2.15)$$

**Lemma 2.6.** [41] For  $0 < \rho \leq 1$ ,  $\mathcal{K}_i \in \mathbb{R}$ , and  $i = 1, \dots, n$ , there is

$$\left( \sum_{i=1}^n |\mathcal{K}_i| \right)^\rho \leq \sum_{i=1}^n |\mathcal{K}_i|^\rho \leq n^{1-\rho} \left( \sum_{i=1}^n |\mathcal{K}_i| \right)^\rho. \quad (2.16)$$

### 3. Main results

#### 3.1. Adaptive FTTC strategy design

In this subsection, the NN approximate method and backstepping control technique will be applied to design the adaptive FTTC strategy, aiming to achieve the control objective. Consider the following coordinate transformation

$$\begin{aligned} z_1 &= x_1 - y_r, \\ z_i &= x_i - v_{i-1}, i = 2, \dots, n, \end{aligned} \quad (3.1)$$

where  $z_i$  represents the tracking error,  $y_r$  represents the reference trajectory, and  $v_i$ ,  $i = 1, \dots, n-1$ , represent virtual control laws.

**Step 1.** Considering (2.1) and (3.1), the derivative of  $z_1$  is

$$\dot{z}_1 = v_1^{\beta_1} + (x_2^{\beta_1} - v_1^{\beta_1}) + \theta_1^T \gamma_1 + \mathcal{F}_1, \quad (3.2)$$

where  $\mathcal{F}_1 = f_1 - \dot{y}_r$ .

For the unknown nonlinear function  $\mathcal{F}_1$  in (3.2), a RBFNN is considered to approximate it, that is

$$\mathcal{F}_1 = (W_1^*)^T \psi_1(X_1) + \varepsilon_1(X_1), |\varepsilon_1(X_1)| \leq \Xi_1, \quad (3.3)$$

where  $X_1 = [\bar{x}_1, \dot{y}_r]^T$ ,  $\Xi_1$  is a positive constant.

Let  $V_{11} = 1/2 z_1^2$  and considering (3.2) and (3.3), then one has

$$\dot{V}_{11} = z_1 v_1^{\beta_1} + z_1 (x_2^{\beta_1} - v_1^{\beta_1}) + z_1 \theta_1^T \gamma_1 + z_1 (W_1^*)^T \psi_1(X_1) + z_1 \varepsilon_1(X_1). \quad (3.4)$$

Considering Lemmas 2.3–2.5, we have

$$|z_1(x_2^{\beta_1} - v_1^{\beta_1})| \leq \beta_1 |z_1| |x_2 - v_1| (x_2^{\beta_1-1} + v_1^{\beta_1-1}) \leq \beta_1^2 z_1^2 + \frac{1}{4} z_2^2 (x_2^{\beta_1-1} + v_1^{\beta_1-1})^2, \quad (3.5)$$

$$|z_1(W_1^*)^T \psi_1(X_1)| \leq \mu_1 z_1^2 \Lambda_1 \Upsilon_1 + \frac{1}{4\mu_1}, \quad (3.6)$$

$$|z_1 \varepsilon_1(X_1)| \leq \Xi_1 |z_1| \leq \frac{\Xi_1 z_1^2}{\sqrt{z_1^2 + \eta^2}} + \Xi_1 \eta, \quad (3.7)$$

where  $\Lambda_1 = (W_1^*)^T W_1^*$ ,  $\Upsilon_1 = (\psi_1(X_1))^T \psi_1(X_1)$ ,  $\mu_1 > 0$  is the design parameter.

Selecting the following candidate Lyapunov function as

$$V_1 = V_{11} + \frac{1}{2\sigma_1} \tilde{\theta}_1^T \tilde{\theta}_1 + \frac{1}{2\delta_1} \tilde{\Lambda}_1^2 + \frac{1}{2\lambda_1} \tilde{\Xi}_1^2, \quad (3.8)$$

where  $\sigma_1 > 0$ ,  $\delta_1 > 0$ , and  $\lambda_1 > 0$  are design parameters,  $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$ ,  $\tilde{\Lambda}_1 = \Lambda_1 - \hat{\Lambda}_1$ , and  $\tilde{\Xi}_1 = \Xi_1 - \hat{\Xi}_1$ ,  $\hat{\theta}_1$ ,  $\hat{\Lambda}_1$  and  $\hat{\Xi}_1$  are the estimates of  $\theta_1$ ,  $\Lambda_1$  and  $\Xi_1$ , respectively.

Combining (3.4) and (3.7), the derivative of  $V_1$  is

$$\begin{aligned} \dot{V}_1 &= \dot{V}_{11} - \frac{1}{\sigma_1} \tilde{\theta}_1^T \dot{\hat{\theta}}_1 - \frac{1}{\delta_1} \tilde{\Lambda}_1 \dot{\hat{\Lambda}}_1 - \frac{1}{\lambda_1} \tilde{\Xi}_1 \dot{\hat{\Xi}}_1 \\ &\leq z_1 v_1^{\beta_1} + \beta_1^2 z_1^2 + z_1 \theta_1^T \gamma_1 + \mu_1 z_1^2 \Lambda_1 \Upsilon_1 + \frac{\Xi_1 z_1^2}{\sqrt{z_1^2 + \eta^2}} - \frac{1}{\sigma_1} \tilde{\theta}_1^T \dot{\hat{\theta}}_1 - \frac{1}{\delta_1} \tilde{\Lambda}_1 \dot{\hat{\Lambda}}_1 - \frac{1}{\lambda_1} \tilde{\Xi}_1 \dot{\hat{\Xi}}_1 \\ &\quad + \frac{1}{4} z_2^2 (x_2^{\beta_1-1} + v_1^{\beta_1-1})^2 + \left( \frac{1}{4\mu_1} + \Xi_1 \eta \right). \end{aligned} \quad (3.9)$$

Design parameter adaptive control laws  $\hat{\theta}_1$ ,  $\hat{\Lambda}_1$ , and  $\hat{\Xi}_1$ , and the virtual control law  $v_1$  as

$$\dot{\hat{\theta}}_1 = \sigma_1 z_1 \gamma_1 - \varsigma_1 \hat{\theta}_1, \quad (3.10)$$

$$\dot{\hat{\Lambda}}_1 = \delta_1 \mu_1 z_1^2 \Upsilon_1 - \varphi_1 \hat{\Lambda}_1, \quad (3.11)$$

$$\dot{\hat{\Xi}}_1 = \frac{\lambda_1 z_1^2}{\sqrt{z_1^2 + \eta^2}} - \vartheta_1 \hat{\Xi}_1, \quad (3.12)$$

$$v_1 = - \left( k_1 z_1^{2\rho-1} + \beta_1^2 z_1 + \frac{1}{2} c_1 z_1 + \hat{\theta}_1^T \gamma_1 + \mu_1 z_1 \hat{\Lambda}_1 \Upsilon_1 + \frac{\hat{\Xi}_1 z_1}{\sqrt{z_1^2 + \eta^2}} \right)^{\frac{1}{\beta_1}}, \quad (3.13)$$

where  $0 < \rho < 1$ ,  $\varsigma_1 > 0$ ,  $\varphi_1 > 0$ ,  $\vartheta_1 > 0$ ,  $k_1 > 0$ , and  $c_1 > 0$  are design parameters.

Substituting (3.10)–(3.13) into (3.9), one gets



$$\dot{V}_1 \leq -k_1 z_1^{2\rho} - \frac{1}{2} c_1 z_1^2 + \frac{\zeta_1}{\sigma_1} \tilde{\theta}_1^T \hat{\theta}_1 + \frac{\varphi_1}{\delta_1} \tilde{\Lambda}_1 \hat{\Lambda}_1 + \frac{\varrho_1}{\lambda_1} \tilde{\Xi}_1 \hat{\Xi}_1 + \frac{1}{4} z_2^2 (x_2^{\beta_1-1} + \nu_1^{\beta_1-1})^2 + \left( \frac{1}{4\mu_1} + \Xi_1 \eta \right). \quad (3.14)$$

**Step  $i$**  ( $i = 2, \dots, n-1$ ). Considering (2.1) and (3.1), the derivative of  $z_i$  is

$$\dot{z}_i = \nu_i^{\beta_i} + (x_{i+1}^{\beta_i} - \nu_i^{\beta_i}) + \theta_i^T \gamma_i + f_i - \dot{\nu}_{i-1}. \quad (3.15)$$

Let  $V_{i1} = V_{i-1} + 1/2 z_i^2$  and considering (3.15), then we have

$$\dot{V}_{i1} = \dot{V}_{i-1} + z_i \nu_i^{\beta_i} + z_i (x_{i+1}^{\beta_i} - \nu_i^{\beta_i}) + z_i \theta_i^T \gamma_i + z_i (f_i - \dot{\nu}_{i-1}). \quad (3.16)$$

Given the result of the  $(i-1)$ th step, we get

$$\begin{aligned} \dot{V}_{i-1} \leq & -\sum_{m=1}^{i-1} k_m z_m^{2\rho} - \frac{1}{2} \sum_{m=1}^{i-1} c_m z_m^2 + \sum_{m=1}^{i-1} \frac{\zeta_m}{\sigma_m} \tilde{\theta}_m^T \hat{\theta}_m + \sum_{m=1}^{i-1} \frac{\varphi_m}{\delta_m} \tilde{\Lambda}_m \hat{\Lambda}_m + \sum_{m=1}^{i-1} \frac{\varrho_m}{\lambda_m} \tilde{\Xi}_m \hat{\Xi}_m \\ & + \frac{1}{4} z_i^2 (x_i^{\beta_{i-1}-1} + \nu_{i-1}^{\beta_{i-1}-1})^2 + \sum_{m=1}^{i-1} \left( \frac{1}{4\mu_m} + \Xi_m \eta \right). \end{aligned} \quad (3.17)$$

Substituting (3.17) into (3.16), we obtain

$$\begin{aligned} \dot{V}_{i1} \leq & -\sum_{m=1}^{i-1} k_m z_m^{2\rho} - \frac{1}{2} \sum_{m=1}^{i-1} c_m z_m^2 + \sum_{m=1}^{i-1} \frac{\zeta_m}{\sigma_m} \tilde{\theta}_m^T \hat{\theta}_m + \sum_{m=1}^{i-1} \frac{\varphi_m}{\delta_m} \tilde{\Lambda}_m \hat{\Lambda}_m + \sum_{m=1}^{i-1} \frac{\varrho_m}{\lambda_m} \tilde{\Xi}_m \hat{\Xi}_m \\ & + z_i \nu_i^{\beta_i} + z_i (x_{i+1}^{\beta_i} - \nu_i^{\beta_i}) + z_i \theta_i^T \gamma_i + z_i \mathcal{F}_i + \sum_{m=1}^{i-1} \left( \frac{1}{4\mu_m} + \Xi_m \eta \right), \end{aligned} \quad (3.18)$$

where  $\mathcal{F}_i = f_i - \dot{\nu}_{i-1} + \frac{1}{4} z_i (x_i^{\beta_{i-1}-1} + \nu_{i-1}^{\beta_{i-1}-1})^2$ .

For the unknown nonlinear function  $\mathcal{F}_i$  in (3.18), a RBFNN is used to approximate it, that is

$$\mathcal{F}_i = (W_i^*)^T \psi_i(X_i) + \varepsilon_i(X_i), |\varepsilon_i(X_i)| \leq \Xi_i, \quad (3.19)$$

where  $X_i = [\bar{x}_i, y_r, \dot{y}_r]^T$ ,  $\Xi_i$  is a positive constant.

Considering Lemmas 2.3–2.5, we have

$$\left| z_i (x_{i+1}^{\beta_i} - \nu_i^{\beta_i}) \right| \leq \beta_i^2 z_i^2 + \frac{1}{4} z_{i+1}^2 (x_{i+1}^{\beta_i-1} + \nu_i^{\beta_i-1})^2, \quad (3.20)$$

$$\left| z_i (W_i^*)^T \psi_i(X_i) \right| \leq \mu_i z_i^2 \Lambda_i \Upsilon_i + \frac{1}{4\mu_i}, \quad (3.21)$$

$$\left| z_i \varepsilon_i(X_i) \right| \leq \frac{\Xi_i z_i^2}{\sqrt{z_i^2 + \eta^2}} + \Xi_i \eta, \quad (3.22)$$

where  $\Lambda_i = (W_i^*)^T W_i^*$ ,  $\Upsilon_i = (\psi_i(X_i))^T \psi_i(X_i)$ ,  $\mu_i > 0$  is the design parameter.

Selecting the following candidate Lyapunov function as

$$V_i = V_{i1} + \frac{1}{2\sigma_i} \tilde{\theta}_i^T \tilde{\theta}_i + \frac{1}{2\delta_i} \tilde{\Lambda}_i^2 + \frac{1}{2\lambda_i} \tilde{\Xi}_i^2, \quad (3.23)$$

where  $\sigma_i > 0$ ,  $\delta_i > 0$ , and  $\lambda_i > 0$  are design parameters,  $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ ,  $\tilde{\Lambda}_i = \Lambda_i - \hat{\Lambda}_i$  and  $\tilde{\Xi}_i = \Xi_i - \hat{\Xi}_i$ ,  $\hat{\theta}_i$ ,  $\hat{\Lambda}_i$  and  $\hat{\Xi}_i$  are the estimate of  $\theta_i$ ,  $\Lambda_i$  and  $\Xi_i$ , respectively.

Considering (3.18) and (3.20)–(3.22), the derivative of  $V_i$  is

$$\begin{aligned} \dot{V}_i &= \dot{V}_{i1} - \frac{1}{\sigma_i} \tilde{\theta}_i^T \dot{\tilde{\theta}}_i - \frac{1}{\delta_i} \tilde{\Lambda}_i \dot{\tilde{\Lambda}}_i - \frac{1}{\lambda_i} \tilde{\Xi}_i \dot{\tilde{\Xi}}_i \\ &\leq -\sum_{m=1}^{i-1} k_m z_m^{2\rho} - \frac{1}{2} \sum_{m=1}^{i-1} c_m z_m^2 + \sum_{m=1}^{i-1} \frac{\zeta_m}{\sigma_m} \tilde{\theta}_m^T \dot{\tilde{\theta}}_m + \sum_{m=1}^{i-1} \frac{\varphi_m}{\delta_m} \tilde{\Lambda}_m \dot{\tilde{\Lambda}}_m + \sum_{m=1}^{i-1} \frac{\vartheta_m}{\lambda_m} \tilde{\Xi}_m \dot{\tilde{\Xi}}_m - \frac{1}{\sigma_i} \tilde{\theta}_i^T \dot{\tilde{\theta}}_i \\ &\quad - \frac{1}{\delta_i} \tilde{\Lambda}_i \dot{\tilde{\Lambda}}_i - \frac{1}{\lambda_i} \tilde{\Xi}_i \dot{\tilde{\Xi}}_i + z_i \nu_i^{\beta_i} + \beta_i^2 z_i^2 + z_i \theta_i^T \gamma_i + \mu_i z_i^2 \Lambda_i \Upsilon_i + \frac{\Xi_i z_i^2}{\sqrt{z_i^2 + \eta^2}} \\ &\quad + \frac{1}{4} z_{i+1}^2 (x_{i+1}^{\beta_i-1} + \nu_i^{\beta_i-1})^2 + \sum_{m=1}^i \left( \frac{1}{4\mu_m} + \Xi_m \eta \right). \end{aligned} \quad (3.24)$$

Design parameter adaptive control laws  $\dot{\hat{\theta}}_i$ ,  $\dot{\hat{\Lambda}}_i$ , and  $\dot{\hat{\Xi}}_i$ , and the virtual control law  $\nu_i$  as

$$\dot{\hat{\theta}}_i = \sigma_i z_i \gamma_i - \varsigma_i \hat{\theta}_i, \quad (3.25)$$

$$\dot{\hat{\Lambda}}_i = \delta_i \mu_i z_i^2 \Upsilon_i - \varphi_i \hat{\Lambda}_i, \quad (3.26)$$

$$\dot{\hat{\Xi}}_i = \frac{\lambda_i z_i^2}{\sqrt{z_i^2 + \eta^2}} - \vartheta_i \hat{\Xi}_i, \quad (3.27)$$

$$\nu_i = - \left( k_i z_i^{2\rho-1} + \beta_i^2 z_i + \frac{1}{2} c_i z_i + \hat{\theta}_i^T \gamma_i + \mu_i z_i \hat{\Lambda}_i \Upsilon_i + \frac{\hat{\Xi}_i z_i}{\sqrt{z_i^2 + \eta^2}} \right)^{\frac{1}{\beta_i}}, \quad (3.28)$$

where  $0 < \rho < 1$ ,  $\varsigma_i > 0$ ,  $\varphi_i > 0$ ,  $\vartheta_i > 0$ ,  $k_i > 0$ , and  $c_i > 0$  are design parameters.

Substituting (3.25)–(3.28) into (3.24), one gets

$$\begin{aligned} \dot{V}_i &\leq -\sum_{m=1}^i k_m z_m^{2\rho} - \frac{1}{2} \sum_{m=1}^i c_m z_m^2 + \sum_{m=1}^i \frac{\zeta_m}{\sigma_m} \tilde{\theta}_m^T \dot{\tilde{\theta}}_m + \sum_{m=1}^i \frac{\varphi_m}{\delta_m} \tilde{\Lambda}_m \dot{\tilde{\Lambda}}_m + \sum_{m=1}^i \frac{\vartheta_m}{\lambda_m} \tilde{\Xi}_m \dot{\tilde{\Xi}}_m \\ &\quad + \frac{1}{4} z_{i+1}^2 (x_{i+1}^{\beta_i-1} + \nu_i^{\beta_i-1})^2 + \sum_{m=1}^i \left( \frac{1}{4\mu_m} + \Xi_m \eta \right). \end{aligned} \quad (3.29)$$

**Step  $n$ .** Similar to the previous  $n-1$  steps, and considering (2.1) and (3.1), the derivative of  $z_n$  is

$$\dot{z}_n = \sum_{j=1}^p g_{nj} u_j^F(t) + \theta_n^T \gamma_n + f_n - \dot{\nu}_{n-1}. \quad (3.30)$$

Let  $V_{n1} = V_{n-1} + 1/2z_n^2$  and considering (2.2), (2.4), and (3.30), then we have

$$\dot{V}_{n1} = \dot{V}_{n-1} + z_n \sum_{j=1}^p \dot{h}_j u_j(t) + z_n \theta_n^T \gamma_n + z_n (f_n - \dot{v}_{n-1} + G_0), \quad (3.31)$$

where  $G_0 = \sum_{j=1}^d g_{nj} o_{1j}(t) + \sum_{j=d+1}^p g_{nj} o_{2j}(t)$ ,  $\dot{h}_j = g_{nj} \dot{h}_{1j}$  for  $j=1, \dots, d$ , and  $\dot{h}_j = g_{nj} \dot{h}_{2j}$  for  $j=d+1, \dots, p$ .

Given the result of the  $(n-1)$ th step, we get

$$\begin{aligned} \dot{V}_{n-1} \leq & -\sum_{m=1}^{n-1} k_m z_m^{2\rho} - \frac{1}{2} \sum_{m=1}^{n-1} c_m z_m^2 + \sum_{m=1}^{n-1} \frac{\zeta_m}{\sigma_m} \tilde{\theta}_m^T \hat{\theta}_m + \sum_{m=1}^{n-1} \frac{\varphi_m}{\delta_m} \tilde{\Lambda}_m \hat{\Lambda}_m + \sum_{m=1}^{n-1} \frac{\varrho_m}{\lambda_m} \tilde{\Xi}_m \hat{\Xi}_m \\ & + \frac{1}{4} z_n^2 (x_n^{\beta_{n-1}-1} + v_{n-1}^{\beta_{n-1}-1})^2 + \sum_{m=1}^{n-1} \left( \frac{1}{4\mu_m} + \Xi_m \eta \right). \end{aligned} \quad (3.32)$$

Substituting (3.32) into (3.31), we obtain

$$\begin{aligned} \dot{V}_{n1} \leq & -\sum_{m=1}^{n-1} k_m z_m^{2\rho} - \frac{1}{2} \sum_{m=1}^{n-1} c_m z_m^2 + \sum_{m=1}^{n-1} \frac{\zeta_m}{\sigma_m} \tilde{\theta}_m^T \hat{\theta}_m + \sum_{m=1}^{n-1} \frac{\varphi_m}{\delta_m} \tilde{\Lambda}_m \hat{\Lambda}_m + \sum_{m=1}^{n-1} \frac{\varrho_m}{\lambda_m} \tilde{\Xi}_m \hat{\Xi}_m \\ & + z_n \sum_{j=1}^p \dot{h}_j u_j(t) + z_n \theta_n^T \gamma_n + z_n \mathcal{F}_n + \sum_{m=1}^{n-1} \left( \frac{1}{4\mu_m} + \Xi_m \eta \right), \end{aligned} \quad (3.33)$$

where  $\mathcal{F}_n = f_n - \dot{v}_{n-1} + G_0 + \frac{1}{4} z_n^2 (x_n^{\beta_{n-1}-1} + v_{n-1}^{\beta_{n-1}-1})^2$ .

For the unknown nonlinear function  $\mathcal{F}_n$  in (3.33), a RBFNN is introduced to approximate it. Then, we have

$$\mathcal{F}_n = (W_n^*)^T \psi_n(X_n) + \varepsilon_n(X_n), \quad |\varepsilon_n(X_n)| \leq \Xi_n, \quad (3.34)$$

where  $X_n = [\bar{x}_n, y_r, \dot{y}_r]^T$ ,  $\Xi_n$  is a positive constant.

Considering Lemmas 2.3–2.5, we have

$$|z_n (W_n^*)^T \psi_n(X_n)| \leq \mu_n z_n^2 \Lambda_n \Upsilon_n + \frac{1}{4\mu_n}, \quad (3.35)$$

$$|z_n \varepsilon_n(X_n)| \leq \frac{\Xi_n z_n^2}{\sqrt{z_n^2 + \eta^2}} + \Xi_n \eta, \quad (3.36)$$

where  $\Lambda_n = (W_n^*)^T W_n^*$ ,  $\Upsilon_n = (\psi_n(X_n))^T \psi_n(X_n)$ ,  $\mu_n > 0$  is the design parameter.

Selecting the following candidate Lyapunov function as

$$V_n = V_{n1} + \frac{1}{2\sigma_n} \tilde{\theta}_n^T \tilde{\theta}_n + \frac{1}{2\delta_n} \tilde{\Lambda}_n^2 + \frac{1}{2\lambda_n} \tilde{\Xi}_n^2, \quad (3.37)$$

where  $\sigma_n > 0$ ,  $\delta_n > 0$ , and  $\lambda_n > 0$  are design parameters,  $\tilde{\theta}_n = \theta_n - \hat{\theta}_n$ ,  $\tilde{\Lambda}_n = \Lambda_n - \hat{\Lambda}_n$  and  $\tilde{\Xi}_n = \Xi_n - \hat{\Xi}_n$ ,  $\hat{\theta}_n$ ,  $\hat{\Lambda}_n$ , and  $\hat{\Xi}_n$  are the estimate of  $\theta_n$ ,  $\Lambda_n$ , and  $\Xi_n$ , respectively.

Considering (3.33), (3.35), and (3.36), the derivative of  $V_n$  is

$$\begin{aligned} \dot{V}_n &= \dot{V}_{n1} - \frac{1}{\sigma_n} \tilde{\theta}_n^T \dot{\hat{\theta}}_n - \frac{1}{\delta_n} \tilde{\Lambda}_n \dot{\hat{\Lambda}}_n - \frac{1}{\lambda_n} \tilde{\Xi}_n \dot{\hat{\Xi}}_n \\ &\leq -\sum_{m=1}^{n-1} k_m z_m^{2\rho} - \frac{1}{2} \sum_{m=1}^{n-1} c_m z_m^2 + \sum_{m=1}^{n-1} \frac{\varsigma_m}{\sigma_m} \tilde{\theta}_m^T \hat{\theta}_m + \sum_{m=1}^{n-1} \frac{\varphi_m}{\delta_m} \tilde{\Lambda}_m \hat{\Lambda}_m + \sum_{m=1}^{n-1} \frac{\varrho_m}{\lambda_m} \tilde{\Xi}_m \hat{\Xi}_m - \frac{1}{\sigma_n} \tilde{\theta}_n^T \dot{\hat{\theta}}_n \\ &\quad - \frac{1}{\delta_n} \tilde{\Lambda}_n \dot{\hat{\Lambda}}_n - \frac{1}{\lambda_n} \tilde{\Xi}_n \dot{\hat{\Xi}}_n + z_n \sum_{j=1}^p \hat{h}_j u_j(t) + z_n \theta_n^T \gamma_n + \mu_n z_n^2 \Lambda_n \Upsilon_n \\ &\quad + \frac{\Xi_n z_n^2}{\sqrt{z_n^2 + \eta^2}} + \sum_{m=1}^n \left( \frac{1}{4\mu_m} + \Xi_m \eta \right). \end{aligned} \quad (3.38)$$

Design parameter adaptive control laws  $\hat{\theta}_n$ ,  $\hat{\Lambda}_n$ ,  $\hat{\Xi}_n$ , and  $\chi_{nj}$ , and the adaptive FTTC law  $u_j(t)$  as

$$\dot{\hat{\theta}}_n = \sigma_n z_n \gamma_n - \varsigma_n \hat{\theta}_n, \quad (3.39)$$

$$\dot{\hat{\Lambda}}_n = \delta_n \mu_n z_n^2 \Upsilon_n - \varphi_n \hat{\Lambda}_n, \quad (3.40)$$

$$\dot{\hat{\Xi}}_n = \frac{\lambda_n z_n^2}{\sqrt{z_n^2 + \eta^2}} - \varrho_n \hat{\Xi}_n, \quad (3.41)$$

$$\dot{\chi}_{nj} = \frac{j}{\sum_{j=1}^p j} \left( k_n z_n^{2\rho} + \frac{1}{2} c_n z_n^2 + z_n \hat{\theta}_n^T \gamma_n + \mu_n z_n^2 \hat{\Lambda}_n \Upsilon_n + \frac{\hat{\Xi}_n z_n^2}{\sqrt{z_n^2 + \eta^2}} \right), \quad j=1, \dots, p, \quad (3.42)$$

$$u_j(t) = \frac{j}{\sum_{j=1}^p j} \mathcal{N}_{nj}(\chi_{nj}) \left( k_n z_n^{2\rho-1} + \frac{1}{2} c_n z_n + \hat{\theta}_n^T \gamma_n + \mu_n z_n \hat{\Lambda}_n \Upsilon_n + \frac{\hat{\Xi}_n z_n}{\sqrt{z_n^2 + \eta^2}} \right), \quad j=1, \dots, p, \quad (3.43)$$

where  $0 < \rho < 1$ ,  $\varsigma_n > 0$ ,  $\varphi_n > 0$ ,  $\varrho_n > 0$ ,  $k_n > 0$ , and  $c_n > 0$  are design parameters.

Substituting (3.39)–(3.43) into (3.38), one gets

$$\begin{aligned} \dot{V}_n &\leq -\sum_{m=1}^n k_m z_m^{2\rho} - \frac{1}{2} \sum_{m=1}^n c_m z_m^2 + \sum_{m=1}^n \frac{\varsigma_m}{\sigma_m} \tilde{\theta}_m^T \hat{\theta}_m + \sum_{m=1}^n \frac{\varphi_m}{\delta_m} \tilde{\Lambda}_m \hat{\Lambda}_m + \sum_{m=1}^n \frac{\varrho_m}{\lambda_m} \tilde{\Xi}_m \hat{\Xi}_m \\ &\quad + \sum_{j=1}^p (\hat{h}_j \mathcal{N}_{nj}(\chi_{nj}) + 1) \dot{\chi}_{nj} + \sum_{m=1}^n \left( \frac{1}{4\mu_m} + \Xi_m \eta \right). \end{aligned} \quad (3.44)$$

### 3.2. Stability analysis

Drawing from the aforementioned analysis, the principal findings of this paper can be encapsulated in the following theorem.

**Theorem 3.1.** Consider the multiple-input uncertain parametric NSs (2.1) with positive odd integer

powers and unknown multiple faults. Under Assumption 2.1, utilizing the virtual control laws (3.13) and (3.28), the adaptive control laws (3.10)–(3.12), (3.15)–(3.17), and (3.39)–(3.42), as well as the adaptive FTTC strategies (3.43), it can be assured that ASCLS are bounded and the tracking error can converge to a SNZFT.

*Proof.* Noting (3.44) and Lemma 2.4, we have

$$\frac{\varsigma_m}{\sigma_m} \tilde{\theta}_m^T \hat{\theta}_m \leq -\frac{\varsigma_m}{2\sigma_m} \tilde{\theta}_m^T \tilde{\theta}_m + \frac{\varsigma_m}{2\sigma_m} \theta_m^T \theta_m, \quad (3.45)$$

$$\frac{\varphi_m}{\delta_m} \tilde{\Lambda}_m \hat{\Lambda}_m \leq -\frac{\varphi_m}{2\delta_m} \tilde{\Lambda}_m^2 + \frac{\varphi_m}{2\delta_m} \Lambda_m^2, \quad (3.46)$$

$$\frac{\mathcal{G}_m}{\lambda_m} \tilde{\Xi}_m \hat{\Xi}_m \leq -\frac{\mathcal{G}_m}{2\lambda_m} \tilde{\Xi}_m^2 + \frac{\mathcal{G}_m}{2\lambda_m} \Xi_m^2. \quad (3.47)$$

Substituting (3.45)–(3.47) into (3.44), one gets

$$\begin{aligned} \dot{V}_n \leq & -\sum_{m=1}^n k_m z_m^{2\rho} - \varpi_1 \left( \sum_{m=1}^n \frac{1}{2\sigma_m} \tilde{\theta}_m^T \tilde{\theta}_m \right)^\rho - \varpi_2 \left( \sum_{m=1}^n \frac{1}{2\delta_m} \tilde{\Lambda}_m^2 \right)^\rho - \varpi_3 \left( \sum_{m=1}^n \frac{1}{2\lambda_m} \tilde{\Xi}_m^2 \right)^\rho \\ & + \varpi_1 \left( \sum_{m=1}^n \frac{1}{2\sigma_m} \tilde{\theta}_m^T \tilde{\theta}_m \right)^\rho + \varpi_2 \left( \sum_{m=1}^n \frac{1}{2\delta_m} \tilde{\Lambda}_m^2 \right)^\rho + \varpi_3 \left( \sum_{m=1}^n \frac{1}{2\lambda_m} \tilde{\Xi}_m^2 \right)^\rho \\ & - \frac{1}{2} \sum_{m=1}^n c_m z_m^2 - \sum_{m=1}^n \frac{\varsigma_m}{2\sigma_m} \tilde{\theta}_m^T \tilde{\theta}_m - \sum_{m=1}^n \frac{\varphi_m}{2\delta_m} \tilde{\Lambda}_m^2 - \sum_{m=1}^n \frac{\mathcal{G}_m}{2\lambda_m} \tilde{\Xi}_m^2 \\ & + \sum_{j=1}^p \left( \hbar_j \mathcal{N}_{nj}(\chi_{nj}) + 1 \right) \dot{\chi}_{nj} + \sum_{m=1}^n \left( \frac{1}{4\mu_m} + \Xi_m \eta \right) \\ & + \sum_{m=1}^n \frac{\varsigma_m}{2\sigma_m} \theta_m^T \theta_m + \sum_{m=1}^n \frac{\varphi_m}{2\delta_m} \Lambda_m^2 + \sum_{m=1}^n \frac{\mathcal{G}_m}{2\lambda_m} \Xi_m^2, \end{aligned} \quad (3.48)$$

where  $\varpi_1 > 0$ ,  $\varpi_2 > 0$ , and  $\varpi_3 > 0$  are design parameters.

Further, applying Lemma 2.4, we have

$$\varpi_1 \left( \sum_{m=1}^n \frac{1}{2\sigma_m} \tilde{\theta}_m^T \tilde{\theta}_m \right)^\rho \leq \sum_{m=1}^n \frac{\varpi_1}{2\sigma_m} \tilde{\theta}_m^T \tilde{\theta}_m + \varpi_1 (1-\rho) \rho^{\frac{\rho}{1-\rho}}, \quad (3.49)$$

$$\varpi_2 \left( \sum_{m=1}^n \frac{1}{2\delta_m} \tilde{\Lambda}_m^2 \right)^\rho \leq \sum_{m=1}^n \frac{\varpi_2}{2\delta_m} \tilde{\Lambda}_m^2 + \varpi_2 (1-\rho) \rho^{\frac{\rho}{1-\rho}}, \quad (3.50)$$

$$\varpi_3 \left( \sum_{m=1}^n \frac{1}{2\lambda_m} \tilde{\Xi}_m^2 \right)^\rho \leq \sum_{m=1}^n \frac{\varpi_3}{2\lambda_m} \tilde{\Xi}_m^2 + \varpi_3 (1-\rho) \rho^{\frac{\rho}{1-\rho}}. \quad (3.51)$$

Substituting (3.49)–(3.51) into (3.48) and considering Lemma 2.6, one has

$$\begin{aligned}
\dot{V}_n &\leq -\sum_{m=1}^n k_m z_m^{2\rho} - \varpi_1 \left( \sum_{m=1}^n \frac{1}{2\sigma_m} \tilde{\theta}_m^T \tilde{\theta}_m \right)^\rho - \varpi_2 \left( \sum_{m=1}^n \frac{1}{2\delta_m} \tilde{\Lambda}_m^2 \right)^\rho - \varpi_3 \left( \sum_{m=1}^n \frac{1}{2\lambda_m} \tilde{\Xi}_m^2 \right)^\rho \\
&\quad - \frac{1}{2} \sum_{m=1}^n c_m z_m^2 - \sum_{m=1}^n \frac{(\zeta_m - \varpi_1)}{2\sigma_m} \tilde{\theta}_m^T \tilde{\theta}_m - \sum_{m=1}^n \frac{(\varphi_m - \varpi_2)}{2\delta_m} \tilde{\Lambda}_m^2 - \sum_{m=1}^n \frac{(\vartheta_m - \varpi_3)}{2\lambda_m} \tilde{\Xi}_m^2 \\
&\quad + \sum_{j=1}^p (\hbar_j \mathcal{N}_{nj}(\chi_{nj}) + 1) \dot{\chi}_{nj} + \sum_{m=1}^n \left( \frac{1}{4\mu_m} + \Xi_m \eta \right) + \sum_{m=1}^n \frac{\zeta_m}{2\sigma_m} \theta_m^T \theta_m \\
&\quad + \sum_{m=1}^n \frac{\varphi_m}{2\delta_m} \Lambda_m^2 + \sum_{m=1}^n \frac{\vartheta_m}{2\lambda_m} \Xi_m^2 + (\varpi_1 + \varpi_2 + \varpi_3)(1-\rho) \rho^{\frac{\rho}{1-\rho}} \\
&\leq -\alpha_1 V_n^\rho - \alpha_2 V_n + \sum_{j=1}^p (\hbar_j \mathcal{N}_{nj}(\chi_{nj}) + 1) \dot{\chi}_{nj} + D_0,
\end{aligned} \tag{3.52}$$

where

$$\alpha_1 = \min \{ 2^\rho k_i, \varpi_1, \varpi_2, \varpi_3, i = 1, \dots, n \}, \tag{3.53}$$

$$\alpha_2 = \min \{ c_i, \zeta_i - \varpi_1, \varphi_i - \varpi_2, \vartheta_i - \varpi_3, i = 1, \dots, n \}, \tag{3.54}$$

$$\begin{aligned}
D_0 &= \sum_{m=1}^n \left( \frac{1}{4\mu_m} + \Xi_m \eta \right) + \sum_{m=1}^n \frac{\zeta_m}{2\sigma_m} \theta_m^T \theta_m + \sum_{m=1}^n \frac{\varphi_m}{2\delta_m} \Lambda_m^2 + \sum_{m=1}^n \frac{\vartheta_m}{2\lambda_m} \Xi_m^2 \\
&\quad + (\varpi_1 + \varpi_2 + \varpi_3)(1-\rho) \rho^{\frac{\rho}{1-\rho}},
\end{aligned} \tag{3.55}$$

and satisfy  $\zeta_i - \varpi_1 > 0$ ,  $\varphi_i - \varpi_2 > 0$ , and  $\vartheta_i - \varpi_3 > 0$  for  $i = 1, \dots, n$ .

Next, we will prove the conclusion of this paper through two steps.

**Step 1.** ASCLS remains bounded.

Considering (3.52), according to the definition of  $V_n$ , it can be obtained that  $\alpha_1 V_n^\rho > 0$  for  $\alpha_1 > 0$ . Hence, we get

$$\dot{V}_n \leq -\alpha_2 V_n + \sum_{j=1}^p (\hbar_j \mathcal{N}_{nj}(\chi_{nj}) + 1) \dot{\chi}_{nj} + D_0. \tag{3.56}$$

Multiplying inequality (3.56) by  $e^{\alpha_2 t}$  on both sides, and taking the integration over  $[0, t]$ , one has

$$\begin{aligned}
V_n(t) &\leq e^{-\alpha_2 t} \sum_{j=1}^p \int_0^t e^{\alpha_2 \tau} (\hbar_j \mathcal{N}_{nj}(\chi_{nj}) + 1) \dot{\chi}_{nj} d\tau + \frac{D_0}{\alpha_2} + \left( V_n(0) - \frac{D_0}{\alpha_2} \right) e^{-\alpha_2 t} \\
&\leq e^{-\alpha_2 t} \sum_{j=1}^p \int_0^t e^{\alpha_2 \tau} (\hbar_j \mathcal{N}_{nj}(\chi_{nj}) + 1) \dot{\chi}_{nj} d\tau + \frac{D_0}{\alpha_2} + V_n(0).
\end{aligned} \tag{3.57}$$

In view of Lemma 2.1, it is concluded that  $V_n(t)$  and  $\chi_{nj}$  are bounded.

Due to  $V_n(t)$  being bounded, it implies that  $z_i$ ,  $\tilde{\theta}_i$ ,  $\tilde{\Lambda}_i$  and  $\tilde{\Xi}_i$ ,  $i = 1, \dots, n$ , are bounded. Further, noting  $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ ,  $\tilde{\Lambda}_i = \Lambda_i - \hat{\Lambda}_i$ , and  $\tilde{\Xi}_i = \Xi_i - \hat{\Xi}_i$ , we can obtain that  $\hat{\theta}_i$ ,  $\hat{\Lambda}_i$ , and  $\hat{\Xi}_i$

are also bounded. Thus, the boundedness of  $v_m$  and  $u_j(t)$ ,  $m=1, \dots, n-1$ ,  $j=1, \dots, p$ , can remain. Given the boundedness of  $z_i$ ,  $y_r$ , and  $v_m$ , it can be obtained that  $x_i$  is also bounded. Hence, ASCLS remains bounded. The proof of this step is completed.

**Step 2.** The tracking error converges to a SNZFT.

Due to  $V_n(t)$  and  $\chi_{nj}$  being bounded, according to Assumption 2.1, (3.42), and (3.43), and the definition of  $\hat{h}_j$ ,  $j=1, \dots, p$ , it can be concluded that  $\sum_{j=1}^p (\hat{h}_j \mathcal{N}_{nj}(\chi_{nj}) + 1) \dot{\chi}_{nj}$  is bounded. Without loss of generality, let  $C_0 = \sum_{j=1}^p (\hat{h}_j \mathcal{N}_{nj}(\chi_{nj}) + 1) \dot{\chi}_{nj} + D_0$ , and note that  $\alpha_2 V_n > 0$  in (3.52) for  $\alpha_2 > 0$ , then we have

$$\dot{V}_n \leq -\alpha_1 V_n^\rho + C_0. \quad (3.58)$$

Now, let  $T_s = (1/(1-\rho)\tilde{\lambda}\alpha_1) \left[ V_n^{1-\rho}(x(0)) - (C_0/(1-\tilde{\lambda})\alpha_1)^{(1-\rho)/\rho} \right]$  with  $0 < \tilde{\lambda} < 1$  and  $V_n(x(0))$  being the initial value of  $V_n(x)$ . According to Lemma 2.2, it indicates that the inequality  $V_n^\rho(x) \leq C_0/(1-\tilde{\lambda})\alpha_1$  for  $\forall t \geq T_s$  holds. That is to say, ASCLS are SGPFs. Especially, we have

$$|z_1| = |x_1 - y_r| \leq 2 \left( \frac{C_0}{(1-\tilde{\lambda})\alpha_1} \right)^{1/2\rho}, \quad (3.59)$$

which implies that after the finite time  $T_s$ , the tracking error converges to a small neighborhood of zero. The proof of this step is completed.

Given the results of the above two steps, it is evident that Theorem 3.1 is completed.

**Remark 3.1.** Noting (3.59), the tracking error  $z_1$  can converge to an arbitrarily small value by adjusting the values of  $C_0$  and  $\alpha_1$ . Further, according to the definitions of  $C_0$  and  $\alpha_1$ , it can be found that the tracking error  $z_1$  will be affected by design parameters  $\sigma_i$ ,  $\zeta_i$ ,  $\delta_i$ ,  $\mu_i$ ,  $\varphi_i$ ,  $\lambda_i$ ,  $\vartheta_i$ , and  $k_i$ ,  $i=1, \dots, n$ . However, these design parameters also further affect the control signal. To achieve this, a balance needs to be struck between the tracking error and the control signal.

#### 4. Simulation analysis

This section presents a simulation case to demonstrate the efficacy of the proposed control method. A class of multiple-input uncertain parametric NSs with positive odd integer powers and multiple faults is given as

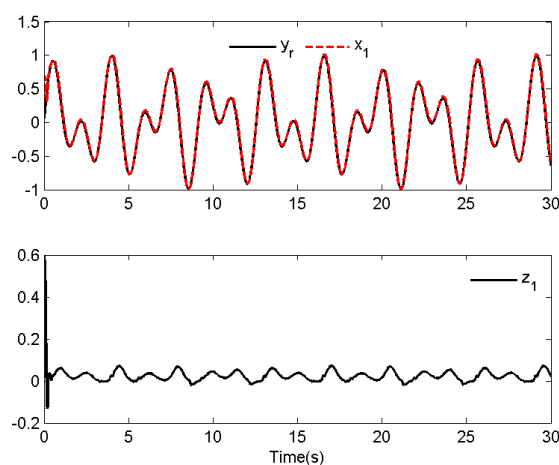
$$\begin{aligned} \dot{x}_1 &= x_2^{\beta_1} + \theta_1^T \gamma_1(\bar{x}_1) + f_1(\bar{x}_1), \\ \dot{x}_2 &= x_3^{\beta_2} + \theta_2^T \gamma_2(\bar{x}_2) + f_2(\bar{x}_2), \\ \dot{x}_3 &= g_{31}(\bar{x}_3)u_1^F(t) + g_{32}(\bar{x}_3)u_2^F(t) + g_{33}(\bar{x}_3)u_3^F(t) + \theta_3^T \gamma_3(\bar{x}_3) + f_3(\bar{x}_3), \end{aligned} \quad (4.1)$$

where  $\beta_1 = 3$ ,  $\beta_2 = 5$ ,  $\theta_1 = 2.0$ ,  $\theta_2 = [3.0, 2.5]^T$ ,  $\theta_3 = [1.0, 3.5, 2.0]^T$ ,  $\gamma_1(\bar{x}_1) = e^{-x_1^2}$ ,  $\gamma_2(\bar{x}_2) = \left[ e^{-x_2^2}, \sin(x_1 x_2) \right]^T$ ,  $\gamma_3(\bar{x}_3) = \left[ x_2, \cos(x_1 x_2), x_1 \sin(x_3) \right]^T$ ,  $f_1(\bar{x}_1) = 0.5 \sin(x_1)$ ,  $f_2(\bar{x}_2) = 2.5 x_1 x_2^2$ ,  $f_3(\bar{x}_3) = x_2 \cos(x_1 x_3)$ ,  $g_{31}(\bar{x}_3) = 1 + 0.5 \sin(x_2 x_3) e^{-0.5 x_1^2}$ ,  $g_{32}(\bar{x}_3) = 1 + 0.5 \sin(x_1 x_3) e^{-0.5 x_2^2}$ , and

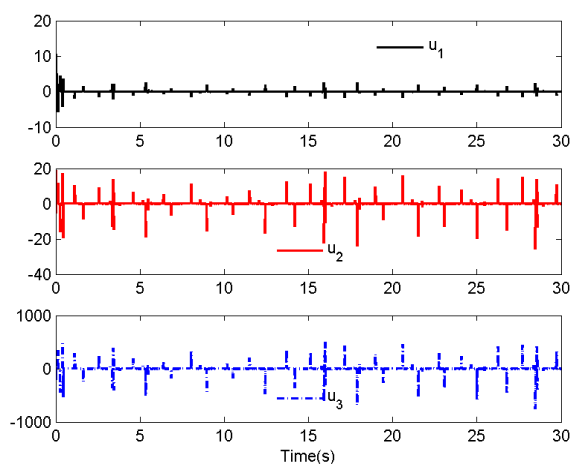
$$g_{33}(\bar{x}_3) = 1 + 0.5 \sin(x_1 x_2) e^{-0.5 x_3^2}.$$

In this paper, the number of inputs is  $p=3$ . Among them, it is assumed that the first and second inputs are subjected to actuator faults, while the third input is subjected to the dead-zone fault. For these two actuator faults, it is assumed that during the initial stage of the simulation,  $h_{11} = h_{12} = 1.0$  and  $o_{11} = o_{12} = 0.0$ , but after 10 seconds,  $h_{11} = 0.2$ ,  $h_{12} = 0.0$ ,  $o_{11} = 0.0$ , and  $o_{12} = 1 - 0.6 \sin(t)$ . For the dead-zone fault, let  $h_{23} = 1.7$ ,  $a_{3r} = 0.5$ , and  $a_{3l} = 0.4$ . In addition, the initial states of system (4.1) are set as  $x_1(0) = 0.6$ ,  $x_2(0) = 0.3$ , and  $x_3(0) = 0.1$ , the reference signal is given as  $y_r = 0.5 \sin(2t) + 0.5 \sin(3.5t)$ , and the simulation is selected as  $t = 30s$ .

The RBFNN used to approximate the unknown nonlinear functions  $\mathcal{F}_1$ ,  $\mathcal{F}_2$ , and  $\mathcal{F}_3$  contains 9 nodes, the centers of Gaussian functions are evenly spaced in the interval  $[-8, 8] \times [-8, 8]$ ,  $[-8, 8] \times [-8, 8] \times [-8, 8] \times [-8, 8]$ , and  $[-8, 8] \times [-8, 8] \times [-8, 8] \times [-8, 8] \times [-8, 8]$ , and the widths of Gaussian functions are selected as  $b_i = 1.5$  for  $i = 1, \dots, 9$ .

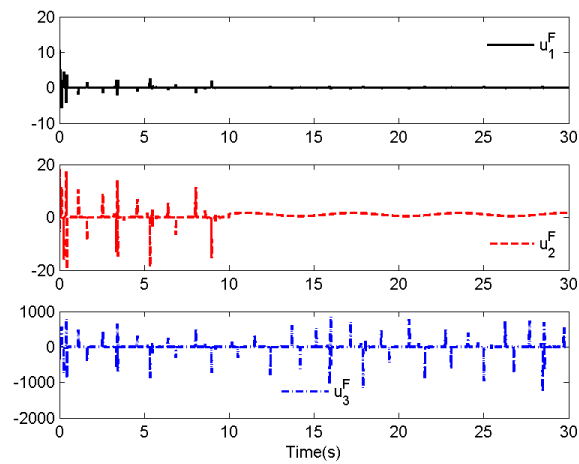


**Figure 1.** Tracking performance and tracking error  $z_1$ .

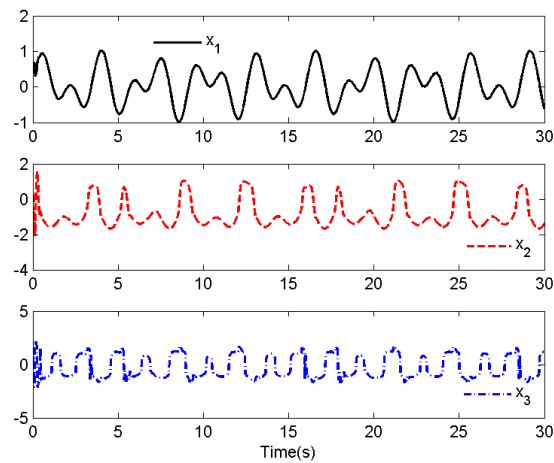


**Figure 2.** Designed control strategies  $u_i$  ( $i = 1, 2, 3$ ).

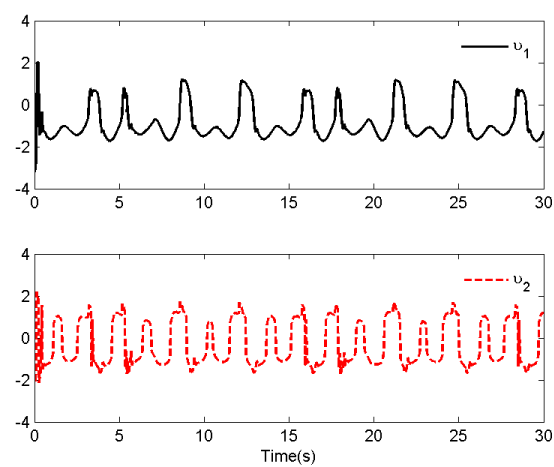




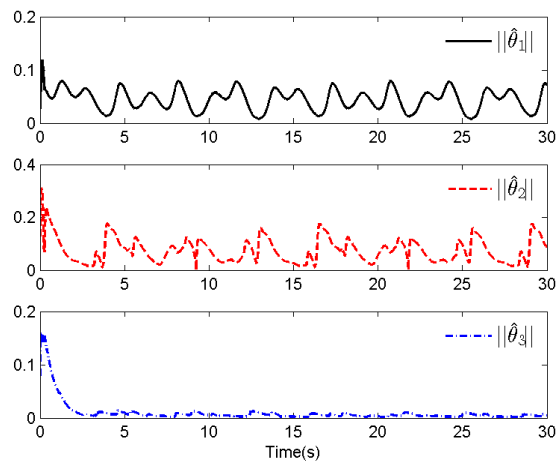
**Figure 3.** System input signals  $u_i^F$  ( $i=1,2,3$ ).



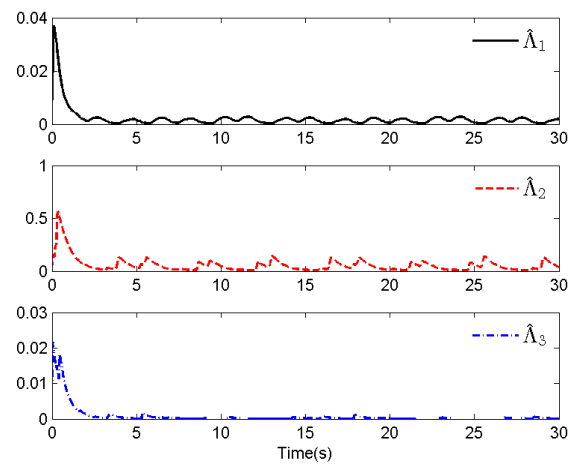
**Figure 4.** System states  $x_i$  ( $i=1,2,3$ ).



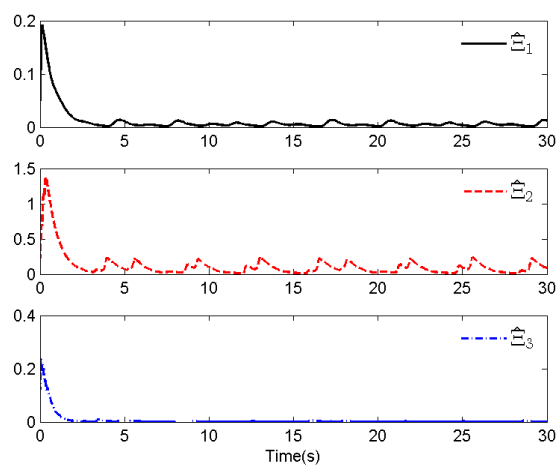
**Figure 5.** Virtual control laws  $v_i$  ( $i=1,2$ ).



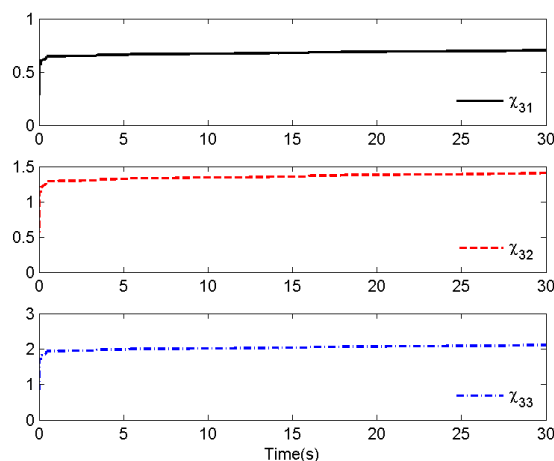
**Figure 6.** Adaptive control laws  $\|\hat{\theta}_i\|$  ( $i=1,2,3$ ).



**Figure 7.** Adaptive control laws  $\hat{\Lambda}_i$  ( $i=1,2,3$ ).



**Figure 8.** Adaptive control laws  $\hat{\Xi}_i$  ( $i=1,2,3$ ).



**Figure 9.** Adaptive control laws  $\chi_{3i}$  ( $i=1,2,3$ ).

The other parameters are  $\sigma_1 = 3.5$ ,  $\sigma_2 = 2.0$ ,  $\sigma_3 = 2.5$ ,  $\varsigma_1 = \varsigma_2 = \varsigma_3 = 1.5$ ,  $\delta_1 = 5.0$ ,  $\delta_2 = 4.0$ ,  $\delta_3 = 3.0$ ,  $\mu_1 = \mu_2 = \mu_3 = 1.5$ ,  $\varphi_1 = 2.5$ ,  $\varphi_2 = 1.5$ ,  $\varphi_3 = 2.0$ ,  $\lambda_1 = 8.5$ ,  $\lambda_2 = 3.5$ ,  $\lambda_3 = 4.5$ ,  $\vartheta_1 = \vartheta_2 = 1.5$ ,  $\vartheta_3 = 2.5$ ,  $k_1 = 30$ ,  $k_2 = 9.0$ ,  $k_3 = 3.5$ ,  $c_1 = 25$ ,  $c_2 = 12$ ,  $c_3 = 35$ ,  $\eta = 1.0$ , and  $\eta = 1.0$ . The initial values of all parameter adaptive control laws are set as zero. Figures 1–9 show the simulation results.

Figure 1 illustrates the trajectories of reference signal  $y_r$ , the system output  $x_1$ , and the tracking error  $z_1$ . As shown in Figure 1, the system exhibits excellent tracking performance under the proposed control strategies, with the tracking error converging to a very SNZFT.

Figures 2 and 3 depict the trajectories of designed control strategies  $u_i$  and the system input signals  $u_i^F$ ,  $i=1,2,3$ . Despite the presence of actuator faults and dead-zone faults affecting the input signals, Figure 1 indicates that the system's tracking performance remains largely unaffected. Obviously, Figures 2 and 3 further validate the effectiveness of the control method proposed in this paper.

Figures 4–9 display the trajectories of system states  $x_i$ , virtual control laws  $v_l$ , adaptive control laws  $\|\hat{\theta}_i\|$ ,  $\hat{\Lambda}_i$ ,  $\hat{\Xi}_i$ ,  $\chi_{3i}$ ,  $i=1,2,3$  and  $l=1,2$ . Combined with the results presented in Figures 1–3, it is evident that all signals remain bounded, thereby confirming the boundedness of ASCLS. In summary, the simulation results illustrated in Figures 1–9 validate the effectiveness of the proposed control approach.

From Figure 2, it is evident that the control signals exhibit a certain level of roughness. Referring to (3.43) and (3.59), it is clear that numerous coupled design parameters exist between the control strategies  $u_i$  and the tracking error  $z_1$ . This implies that adjustments to these parameters will not only affect the tracking error but also the control signals. As pointed out in Remark 3.1, it is crucial to strike a reasonable balance between the two. Clearly, the focus of this paper is tracking performance.

## 5. Conclusions

This paper investigates the FTTC issue for a class of multiple-input uncertain parametric NSs with positive odd integer powers and unknown multiple faults. The application of an NN approximator has successfully resolved the approximation challenges associated with uncertain

nonlinear dynamics and unknown nonlinear functions. Meanwhile, the NGF method has been introduced to handle unknown control gains arising from the system and unknown faults. Leveraging the backstepping control technique, adaptive FTTC strategies for different control inputs have been proposed to mitigate the effects of actuator faults and dead-zone faults while maintaining the desired tracking performance. It has been demonstrated that under the designed control method, ASCLS are bounded, and the tracking error can converge to an SNZFT.

This paper addresses the FTTC problem for multiple-input uncertain parametric NSs by integrating the NN control method, NGF technique, and backstepping control technique. However, the control problem under the time delay and the event-triggered mechanism has not been discussed. Therefore, our future research direction will concentrate on the tracking control problem of multiple-input NSs with input delays based on the event-triggered mechanism.

### Author contributions

Miao Xiao: Conceptualization, investigation, writing-original draft, funding acquisition; Zhe Lin: Project administration, formal analysis, investigation, software; Qian Jiang: Formal analysis, supervision, software; Dingcheng Yang: Methodology, supervision, writing-review & editing; Xiongfeng Deng: Investigation, software. All authors have read and approved the final version of the manuscript for publication.

### Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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### Conflict of interest

All authors declare no conflict of interest in this paper.

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