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Correction

Correction: Legendre spectral collocation method for solving nonlinear fractional Fredholm integro-differential equations with convergence analysis

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A correction on

Legendre spectral collocation method for solving nonlinear fractional Fredholm integro-differential equations with convergence analysis
by A. H. Tedjani, A. Z. Amin, Abdel-Haleem Abdel-Aty, M. A. Abdelkawy, and Mona Mahmoud. AIMS Mathematics, 2024, 9(4): 7973–8000. DOI: 10.3934/math.2024388
The author would like to make the following corrections to the published paper [1].

- In the abstract of our paper [1], we stated: "In addition, we provide some numerical test cases to demonstrate that the approach can preserve the non-smooth solution of the underlying problem". Acutully, this statement requires further clarification. Specifically, we address the challenge posed by non-smooth solutions, which can significantly degrade the performance of numerical schemes, particularly their order of convergence. To overcome this limitation, we employed fractional-order Legendre functions, denoted as $\mathcal{L}_\varepsilon(\lambda^\gamma)$, in our numerical approach. This methodology was applied specifically in Examples 6 and 7, as mentioned in the conclusion section of [1].

- The corrected form of Eqs (1.1) and (1.2) are

$$D^{\alpha_1}\mathcal{Y}(s) = \phi(s) + \int_0^1 \eta(s,t)G(\mathcal{Y}(t))dt, \qquad 0 < \alpha_1 < 2, \tag{0.1}$$

with the initial conditions;

$$\mathcal{Y}^{(\beta)}(0) = 0, \beta = 0, 1, \tag{0.2}$$

where D^{α_1} denotes the fractional derivative of order α_1, and $0 < \alpha_1 < 2$ (This modification must be applied consistently across the paper).

- Using the following transformations $t = 2\lambda - 1$, $s = 2\varrho - 1$, $\mathcal{Y}(2\varrho - 1) = \mathcal{Z}(\varrho)$, $\phi(2\varrho - 1) = \phi(\varrho)$, $2G(\mathcal{Y}(2\lambda - 1) = F(\mathcal{Z}(\lambda))$, and $\eta(2\varrho - 1, 2\lambda - 1) = \sigma(\varrho, \lambda)$ we obtain, Eq (3.12) in [1].
- The initial conditions (3.12) must change to be

$$\mathcal{Z}^{(\beta)}(-1) = 0, \beta = 0, 1. \tag{0.3}$$

- Equation (3.17) must change to be

$$\int_{-1}^{1} I_{\varrho,\nu_1} I_{\lambda,\nu_1}[\sigma(\varrho,\lambda)F(\mathcal{Z}(\lambda))]d\lambda = \sum_{\varepsilon=0}^{\nu_1}\sum_{\iota=0}^{\nu_1} e_{\varepsilon\iota}\mathcal{L}_\varepsilon(\varrho)\int_{-1}^{1}\mathcal{L}_\iota(\lambda)d\lambda = \sum_{\varepsilon=0}^{\nu_1} e_{\varepsilon,0}\mathcal{L}_\varepsilon(\varrho), \tag{0.4}$$

where

$$e_{\varepsilon,0} = \frac{2\varepsilon + 1}{2}\sum_{|a|_\infty \le N}\sum_{|b|_\infty \le N} \varpi_a\varpi_b\sigma(\varrho_a, \lambda_b)F(\mathcal{Z}(\lambda_b))\mathcal{L}_\iota(\varrho_a).$$

- The system of $(\nu_1 + 1)$ algebraic equations derived in Eq (3.23) constitutes a nonlinear system.
- **Lemma 3.** Consider $e(x) = \mathcal{Z}(\varrho) - \mathcal{Z}N(\varrho)$ to represent the error function of the solution. The subsequent inequality is applicable in this context:

$$||e|| \le \sum_{\ell=1}^{3} ||B_\ell|| \tag{0.5}$$

where
$B_1 = I_{\varrho,N}D^{\alpha_1}\mathcal{Z}(\varrho) - D^{\alpha_1}\mathcal{Z}(\varrho)$
$B_2 = I_{\varrho,N}\int_{-1}^{1}(I - I_{\lambda,N})\Big[\sigma(\varrho,\lambda)F(\mathcal{Z}(\lambda))\Big]d\lambda$
$B_3 = I_{\varrho,N}\int_{-1}^{1} I_{\lambda,N}\Big[\sigma(\varrho,\lambda)F(\mathcal{Z}(\lambda)) - \sigma(\varrho,\lambda)F(\mathcal{Z}_N(\lambda))\Big]d\lambda.$

Proof. By using the Caputo definition, we write the equation of non-FFIDEs as follows:

$$D^{\alpha_1}\mathcal{Z}(\varrho) = I_{\varrho,N}\phi(\varrho) + I_{\varrho,N}\int_{-1}^{1}\sigma(\varrho,\lambda)F(\mathcal{Z}(\lambda))d\lambda, \qquad 0 < \alpha_1 < 1 \tag{0.6}$$

and when utilizing the approximate solution we have,

$$I_{\varrho,N}D^{\alpha_1}\mathcal{Z}(\varrho) = I_{\varrho,N}\phi(\varrho) + \int_{-1}^{1} I_{\varrho,N}I_{\lambda,N}[\sigma(\varrho,\lambda)F(\mathcal{Z}_N(\lambda))]d\lambda. \tag{0.7}$$

Subtracting (0.7) from (0.6) yields

$$e(\varrho) = I_{\varrho,N}D^{\alpha_1}\mathcal{Z}(\varrho) - D^{\alpha_1}\mathcal{Z}(\varrho) + I_{\varrho,N}\int_{-1}^{1}\Big[\sigma(\varrho,\lambda)F(\mathcal{Z}(\lambda)) - I_{\lambda,N}[\sigma(\varrho,\lambda)F(\mathcal{Z}_N(\lambda))]\Big]d\lambda \qquad (0.8)$$

hence

$$e(t) = I_{\varrho,N}D^{\alpha_1}\mathcal{Z}(\varrho) - D^{\alpha_1}\mathcal{Z}(\varrho) + I_{\varrho,N}\int_{-1}^{1}I_{\lambda,N}\Big[\sigma(\varrho,\lambda)F(\mathcal{Z}(\lambda)) - \sigma(\varrho,\lambda)F(\mathcal{Z}_N(\lambda))\Big]d\lambda. \qquad (0.9)$$

The desired result can be obtained directly from the above.

- In Theorem 1, Section 4.1, we compute B_1, instead of Eq (4.11), by using Lemma 3, Lemma (3-3) in [2], as

$$\| B_1 \|_{L^2(I)} \leq CN^{-\eta}|D^{\alpha_1}\mathcal{Z}|_{H^{m,N}_{\omega^c}(I)}. \qquad (0.10)$$

Accordingly, Eq (4.8) must be

$$\| E_N \|_{L^2(I)} \leq CN^{-\eta}|D^{\alpha_1}\mathcal{Z}|_{H^{\eta,N}(I)}. + c\sqrt{\frac{(N-\eta+1)!}{N!}}(N+\eta)^{-(\eta+1)/2}\Big[|F(\mathcal{Z}(\cdot))|_{H^1(I)} + |\mathcal{Z}|_{H^1(I)}\Big] + LM \| E_N \| . \qquad (0.11)$$

- In Eq (4.17), which L is Lipschitz condition, and $Max|\sigma(\varrho,\lambda)| \leq M$ and $L < 1/M$.
- The revised version of Figure 8 in [1] is included in Figure 1.

Theorem 1. Let $I_N\mathcal{Z}(\varrho)$ be the spectral approximate and let $\mathcal{Z}(\varrho)$ be the exact solution of the equation of non-FFIDEs and, F satisfies the Lipschitz condition with respect to its third argument with the Lipschitz constant $L < \frac{1}{M}$ and $Max|\sigma(\varrho,\lambda)| \leq M$.

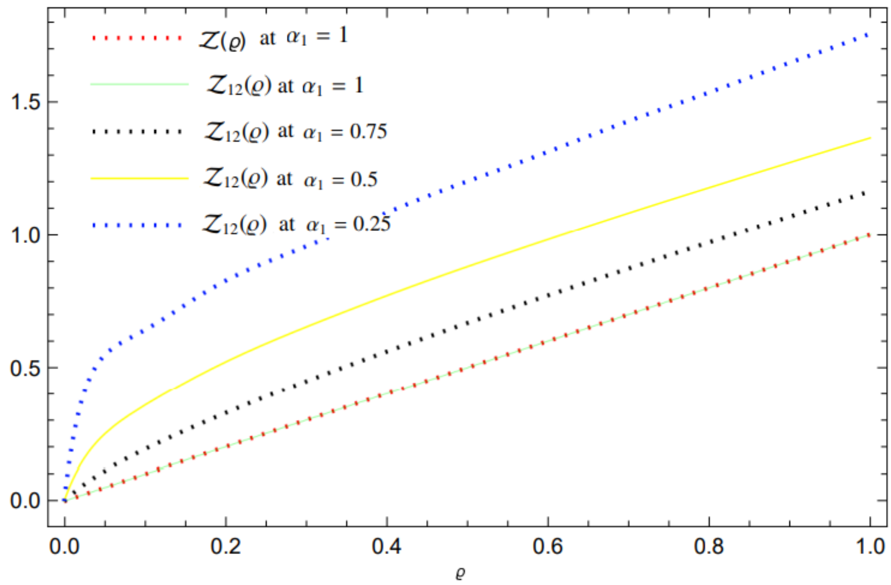

Figure 1. The approximate solutions for various values of α_1.

- In Example 6, while the solution is not smooth, then the order of convergence of the numerical scheme may deteriorate. However, this can be prevented by using fractional order Legendre functions $\mathcal{L}_\varepsilon(\lambda^\gamma)$. Then, in Figures 13 and 14, $\gamma = \frac{1}{2}$ and $\nu_1 = 8$. Also, we used fractional order Legendre functions $\mathcal{L}_\varepsilon(\lambda^\gamma)$ in Example 7 and then α_1 in Table 6 must be γ.

The changes have no material impact on the conclusion of this article. The original manuscript will be updated [1]. We apologize for any inconvenience caused to our readers by the changes.

Conflict of interest

The authors declare there is no conflict of interest.

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