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Research article

An empirical assessment of Tukey combined extended exponentially weighted moving average control chart

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Abstract: Statistical process control (SPC) is a quality control method that enables the monitoring of processes using statistical methodologies. Nonparametric control charts, including the Tukey control chart (TCC), are a robust and effective instrument to assess a method since the actual distribution of the quality characteristic in question is indeterminate. The extended exponentially weighted moving average (EEWMA) control chart was employed to monitor the mean process because of its rapid detection of shifts. To maximize the benefits of both control charts, we developed a method known as EEWMA-TCC, which combines EEWMA with TCC. The efficacy of the proposed chart was evaluated under symmetrical distribution using various individual and aggregate performance metrics based on average run length (ARL) and percentage reduction in ARL (PD_{ARL}). Our findings indicated that the suggested chart outperforms control charts, including the TCC chart, the EWMA chart, the EEWMA chart, and the EWMA-TCC (mixed exponentially weighted moving average-Tukey) chart, in the quick identification of shifts. An application of the proposed designs in the crucial dimension of machined part data is demonstrated. The results indicated that they were consistent with the research findings. On the other hand, nonparametric control charts provide an alternate way to track the mean process.

Keywords: control chart; Tukey; extended exponentially weighted moving average; nonparametric; average run length
Mathematics Subject Classification: 62G05, 62P30

1. Introduction

One tool for statistical process control (SPC) is the control chart, which enables users to monitor a process variable across time. The Shewhart chart [1], exponentially weighted moving average chart (EWMA) [2], and moving average chart (MA) [3] are often used control charts for process monitoring. The efficient monitoring chart provides a swift and quick signal. In any case, a number of writers have suggested a variety of control chart mixes to improve procedure quality. A variety of hybrid exponentially weighted moving average (HEWMA) charts [4], a modified exponentially weighted moving average (MEWMA) chart [5,6], a double exponentially weighted moving average (DEWMA) chart [7,8], a triple exponentially weighted moving average (TEWMA) chart [9], and an extended exponentially weighted moving average (EEWMA) control chart [10] were all developed to improve the EWMA chart's capacity to detect small to moderate changes.

Several researchers have proposed integrated control charting methodologies to enhance the efficacy of control charts in detecting abrupt shifts. Typically, parametric control charts assume a normal distribution for the underlying process. Nonparametric or distribution-free control charts are being investigated as an alternate method for monitoring processes based on target values. In this study, we include considerations of various mixed control charts, encompassing both parametric and nonparametric types. For example, Abbas et al. [11] investigated the use of mixed control charts that used EWMA and CUSUM to track operations; their results showed that the suggested chart is much more sensitive to minor changes. A CUSUM mixed EWMA chart, as suggested by Zaman et al. [12], is more effective and robust in detecting moderate and minor fluctuations. In their performance evaluation, Sukparungsee et al. [13] utilized a combined EWMA-MA control chart in addition to other charts that indicated the average, median, and standard deviation of run length. According to Taboran et al. [14], who were looking for process changes with different distributions, the suggested chart—which combined MA and EWMA charts—performed better in terms of efficiency than the Shewhart chart, EWMA charts, and MA chart when utilizing average, median, and standard deviation of run length. The MEWMA-MA chart (MMEM) was developed by Talordphop et al. [15] with process mean observation, and the results demonstrated that the suggested chart outperforms traditional control charts, namely in detecting minor to moderate changes. The EEWMA-MA control chart was created by Naveed et al. [16] and includes supplementary data that measures performance based on average run length. According to the results, the proposed chart is the best way to spot changes in the process location parameter. To monitor process goal deviations, they implemented an EWMA-Sign control chart [17]. To better detect small changes, the researchers in [18] constructed a nonparametric CUSUM mean chart using the sign statistic. In [19], the author introduced a MEWMA-Sign, which was named after the modified exponentially weighted moving average control chart that contained the sign test. According to the results shown in [20,21], the EEWMA that utilizes signed-rank, and the sign test is more effective than the EWMA chart in detecting minute changes.

Alemi [22] created the Tukey's control chart (TCC) and is easy to use in cases where the workflow distribution is unclear or when non-normal occurrences occur and is helpful in monitoring the mean process. It has become popular for the tracking process means as an effective substitute for parametric control charts. In addition, research on creating enhanced or hybrid TCC designs is available in the literature. The EWMA-TCC for skewed distribution was introduced by Khaliq et al. [23], the MEC-TCC by Riaz et al. [24], the Tukey MA-EWMA and Tukey MA-DEWMA by Taboran et al. [25,26], the MDEWMA-TCC by Phantu et al. [27], and the Tukey MEWMA-MA by Talordphop et al. [28] to construct control charts where we can react rapidly to adjustments and apply them to varied situations with minimal restrictions. Nonetheless, Sukparungsee [29] also addressed the reliability of Tukey's control charts, discovering skew and non-skew procedures, and that the asymmetric Tukey's control chart performs better than the symmetric one. Under reasonable subgrouping, Khaliq et al. [30] developed Tukey-EWMA and median-based Tukey charts. Mahmood et al. [31] provided the TEWMA-Tukey control charts for both normal and non-normal processes utilizing repeat sampling schemes and single sample schemes.

The ability to use nonparametric control charts with data of any distribution type is their main strength. To address this, we introduce the EEWMA-TCC nonparametric control chart, which integrates the EEWMA control chart with a TCC for process mean observation. We examine the average run length (ARL) to evaluate the quality of the control charts used in Monte Carlo simulations and ensure that the results are accurate using a percentage reduction in ARL (PD_{ARL}). We implement the EEWMA-TCC chart in a real-world scenario and compare it with control charts to demonstrate its practical significance. This paper is structured as follows: In Section 2, we detail the study's methodology and materials. In Section 3, we present the simulation findings and an example. In Section 4, we present our conclusions.

2. Materials and methods

In this section, we outline the design components of the control chart and evaluate its efficacy. In Section 2.1, we present five varieties of control charts: Exponentially weighted moving average (EWMA), Tukey control chart (TCC), extended exponentially weighted moving average (EEWMA), EWMA-Tukey control chart (EWMA-TCC), and the proposed EEWMA-Tukey control chart (EEWMA-TCC). In Section 2.2, we delineate the measurement methodology.

2.1. Control charts

Let $X_1, X_2, ..., X_k, ...$ be a set of normally distributed, independently occurring random variables with a mean of μ and a standard deviation of σ. The following is a general outline of the control chart format.

2.1.1. Exponentially weighted moving average control chart (EWMA)

Roberts [2] first proposed the EWMA statistic and implemented it using a smoothing parameter $\eta \left(0 < \eta \le 1\right)$, as shown in Eq (1).

$$Z_k = \eta X_k + (1 - \mu) Z_{k-1}. \tag{1}$$

The baseline value for the first time point is $Z_0 = \mu$. The mean and asymptotic variance are displayed below:

$$E(Z_k) = \mu \tag{2}$$

$$V(Z_k) = \sigma^2 \left[\frac{\eta}{2 - \eta} \right]. \tag{3}$$

The control limits, with G representing the control limit coefficient for the process considered to be in control, are outlined as follows:

$$UCL / LCL = \mu \pm G\sigma\sqrt{\frac{\eta}{2-\eta}}. \tag{4}$$

2.1.2. Tukey control chart (TCC)

Alemi [22] presented a non-parametric control chart that is applicable when the data distribution is unspecified, with the upper and lower control limits of TCC defined as follows:

$$\begin{aligned} UCL &= Q_3 + G_1(IQR) \\ LCL &= Q_1 - G_1(IQR) \end{aligned} \tag{5}$$

where G_1 is a control limit coefficient for the process considered to be in control of the TCC control chart. IQR is the interquartile range, where $(Q_3 - Q_1)$, Q_1, and Q_3 are the first and third quartiles.

2.1.3. Extended Exponentially Weighted Moving Average (EEWMA) Control Chart

Naveed et al. [10] developed the EEWMA control chart to detect a fast change in the mean. The smoothing parameters are η_1 and η_2, each ranging from 0 to 1, with $0 < \eta_1 \le 1$ and $0 \le \eta_2 < \eta_1$, respectively. The EEWMA statistic is outlined:

$$E_k = \eta_1 X_k - \eta_2 X_{k-1} + (1-\eta_1+\eta_2)E_{k-1}. \tag{6}$$

The baseline value for the first time point of E_0 and X_0 are taken as the target mean. Therefore, the average and the asymptotic variance of E_k are

$$E(E_k) = \mu, \tag{7}$$

$$V(E_k) = \sigma^2\left[\frac{\eta_1^2+\eta_2^2-2\eta_1\eta_2(1-\eta_1+\eta_2)}{2(\eta_1-\eta_2)-(\eta_1-\eta_2)^2}\right]. \tag{8}$$

The control limits of the EEWMA chart having G_2 represent the control limit coefficient for the process considered to be in control are outlined as follows:

$$UCL / LCL = \mu \pm G_2\sigma\sqrt{\frac{\eta_1^2+\eta_2^2-2\eta_1\eta_2(1-\eta_1+\eta_2)}{2(\eta_1-\eta_2)-(\eta_1-\eta_2)^2}}\ . \tag{9}$$

2.1.4. EWMA-Tukey control chart (EWMA-TCC)

Khaliq et al. [23] developed the EWMA-TCC through the combination of the EWMA and TCC control charts. The statistics are presented in the form of the EWMA statistic, and the asymptotic control limit of EWMA-TCC is represented as follows:

$$UCL = Q_3 + G_3\left(IQR\right)\sqrt{\frac{\eta}{2-\eta}}$$
$$LCL = Q_1 - G_3\left(IQR\right)\sqrt{\frac{\eta}{2-\eta}} \tag{10}$$

where G_3 represents the control limit coefficient for the process considered to be in control.

2.1.5. EEWMA-Tukey control chart (EEWMA-TCC)

The proposed chart, EEWMA-TCC, integrates the advantages of the EEWMA and TCC control charts, with the former being responsive to minor shifts and the latter being independent of distribution parameters. The statistics will be presented in the form of the EEWMA statistic, and the asymptotic control limit of EEWMA-TCC is represented as follows:

$$UCL = Q_3 + G_4\left(IQR\right)\sqrt{\frac{\eta_1^2+\eta_2^2-2\eta_1\eta_2\left(1-\eta_1+\eta_2\right)}{2\left(\eta_1-\eta_2\right)-\left(\eta_1-\eta_2\right)^2}}$$
$$LCL = Q_1 - G_4\left(IQR\right)\sqrt{\frac{\eta_1^2+\eta_2^2-2\eta_1\eta_2\left(1-\eta_1+\eta_2\right)}{2\left(\eta_1-\eta_2\right)-\left(\eta_1-\eta_2\right)^2}} \tag{11}$$

where G_4 represents the control limit coefficient for the process considered to be in control.

2.2. Performance measures

Common control chart symbols are UCL, CL, and LCL, which are three straight lines. A control diagram procedure is considered to be in control when its charting statistic values fall between the upper and lower control limits; nevertheless, for it to be in an out-of-control declaration, a single point must be drawn beyond these bounds. The ARL recall from the average run length (ARL) denotes the anticipated number of runs before the control chart delivers a message indicating that the activity has deviated from the control. Provided the issue is under control, the reading of ARL_0 should be raised, whereas ARL_1 will remain relatively low. Various methods for analyzing the ARL have been published in scholarly works [32,33]. The optimal efficacy of the control chart is demonstrated by the minimal value, as indicated by the subsequent ARL constructions.

$$ARL = \frac{\sum_{k=1}^{N} RL_k}{N}. \tag{12}$$

Moreover, the standard deviation of the ARL is

$$SDRL = \sqrt{\frac{\sum_{k=1}^{N}(RL_k - ARL)^2}{N-1}}. \tag{13}$$

The evaluations of the control chart are conducted based on the percentage reduction in ARL, which can be calculated as PD_{ARL} [34], as delineated in the following equation:

$$PD_{ARL} = \left(\frac{ARL_0 - ARL_1}{ARL_0} \right) \times 100 \,. \tag{14}$$

However, we employ the Monte Carlo method to compute the simulation results under control, $ARL_0 = 370$, utilizing 100,000 replications and between 5 and 10 subgroups, to provide the most advantageous run length investigated outcomes. The procedures of the simulation study are conducted as follows:

- Generate n samples of random data following a specified distribution with the mean adjusted from μ to $\mu_1 ; \mu_1 = \mu + \delta\sigma$. Formulate 5 and 10 such subgroups.
- Evaluate the suggested observing statistic and examine coefficient "G" at $ARL_0 = 370$.
- Obtain the statistical values and the control limit for each control chart.
- Reiterate the three steps to compute the Average Run Length (ARL_1) and the standard deviation of run length (SDRL) 100,000 times.
- Assess the percentage decrease in ARL as shown by PD_{ARL} in Eq (14).
- Evaluate the efficacy of the control chart.

3. Results and discussion

3.1. Simulation results

We examine the efficacy of the proposed chart in identifying shifts in the process mean utilizing EWMA, EEWMA, EWMA-TCC, and TCC control charts for Normal(0,1), Laplace(0,1), and logistic(6,2) distributions. The values of the parameters are established at $ARL_0 = 370,$ with constant shifts of (0.05, 0.1, 0.25, 0.5, 0.75, 1, and 1.5, 2), although the sensitive parameters for EWMA are set at 0.1 and 0.25, and for EEWMA, parameters $\eta_1 = 0.1, \ \eta_2 = 0.03$ and $\eta_1 = 0.25, \ \eta_2 = 0.10$ are utilized. The bold outcome of the all tables shows that the control chart with the smallest ARL_1 is the most efficient.

The simulation results show that in all distributions, the control limit constants of the mixed nonparametric control charts (EWMA-TCC and EEWMA-TCC) significantly exceed those of the respective control charts, while the control limit constants show an increasing pattern as the size of the subgroups increases. Nevertheless, we observe that when the smoothing value of the parameters climb, the coefficient control limit of parametric control charts rise, whereas that of nonparametric control charts diminish.

The size of the subgroup influences the performance of ARL_1; as the subgroup size increases, ARL_1 decreases. However, with an increase in the smoothing value, ARL_1 increases.

Tables 1–6 categorize the efficacy of the control charts regarding the ARL criterion by subgroup and distribution. When there are small to large changes in the normal distribution, the suggested chart performs better than the other designs. When there is a big change (shift = 3), the EWMA-TCC chart performs better than the observed chart for both smoothing parameters set 1 and set 2 (Table 1). Table 2 shows that different subgroups produce identical results. Figures 1–3 display the ARL curves from various outcomes of the simulation study.

Table 1. Performance of average run length (standard deviation) on a normal distribution with $n=5$ and two different smoothing parameters (0.1 on the left and 0.25 on the right).

Shifts	EWMA	TCC	EEWMA	EWMA-TCC	EEWMA-TCC	EWMA	EEWMA	EWMA-TCC	EEWMA-TCC
	$G=2.70$	$G_1=1.72$	$G_2=1.94$	$G_3=6.57$	$G_4=7.56$	$G=2.90$	$G_2=2.15$	$G_3=4.35$	$G_4=5.06$
0	370.02	370.72	370.13	370.74	370.84	370.44	370.34	370.75	370.84
	(0.50)	(0.52)	(0.54)	(0.61)	(0.58)	(0.52)	(0.55)	(0.60)	(0.59)
0.25	312.27	360.32	307.77	300.17	**256.42**	314.32	308.21	300.21	**256.99**
	(0.31)	(0.30)	(0.29)	(0.28)	(0.28)	(0.31)	(0.31)	(0.30)	(0.30)
0.5	250.78	329.14	241.43	228.80	**207.40**	251.42	246.09	230.29	**209.08**
	(0.08)	(0.07)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)
0.75	129.88	286.53	126.76	97.28	**71.50**	137.22	135.26	99.17	**73.01**
	(0.04)	(0.03)	(0.03)	(0.03)	(0.02)	(0.04)	(0.03)	(0.02)	(0.02)
1	91.19	240.23	89.18	41.82	**30.19**	95.45	93.25	47.39	**33.27**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)
1.5	38.82	156.87	35.16	14.05	**7.90**	39.11	34.51	16.57	**12.01**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
2	11.96	99.83	12.84	6.72	**1.60**	10.07	12.91	8.77	**1.66**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
3	8.12	35.40	8.41	**0.23**	**0.23**	8.12	8.42	**0.31**	**0.31**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Table 2. Performance of average run length (standard deviation) on a normal distribution with $n=10$ and two different smoothing parameters (0.1 on the left and 0.25 on the right).

Shifts	EWMA	TCC	EEWMA	EWMA-TCC	EEWMA-TCC	EWMA	EEWMA	EWMA-TCC	EEWMA-TCC
	$G=2.71$	$G_1=1.73$	$G_2=2.71$	$G_3=6.58$	$G_4=7.57$	$G=2.91$	$G_2=2.91$	$G_3=4.36$	$G_4=5.08$
0	370.15	370.57	370.40	370.61	370.55	370.35	370.39	370.65	370.62
	(0.55)	(0.53)	(0.55)	(0.55)	(0.54)	(0.55)	(0.52)	(0.52)	(0.52)
0.25	305.26	353.08	301.23	291.68	**253.65**	307.78	306.44	298.33	**254.43**
	(0.33)	(0.32)	(0.32)	(0.33)	(0.33)	(0.34)	(0.34)	(0.33)	(0.34)
0.5	220.43	322.75	216.98	206.54	**202.83**	240.31	239.12	225.41	**207.67**
	(0.08)	(0.08)	(0.09)	(0.09)	(0.08)	(0.07)	(0.07)	(0.07)	(0.07)
0.75	121.88	261.46	119.55	81.82	**63.91**	130.03	128.45	90.07	**70.15**
	(0.04)	(0.05)	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)	(0.04)	(0.04)
1	83.21	227.81	79.06	38.51	**24.43**	89.93	88.85	44.56	**30.22**
	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.02)	(0.02)	(0.02)	(0.01)
1.5	32.21	144.33	31.92	12.33	**5.22**	33.97	33.32	14.31	**10.31**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
2	9.22	91.56	9.46	4.70	**1.35**	9.25	10.22	8.67	**1.33**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
3	5.01	29.27	5.09	**0.20**	**0.20**	5.24	6.65	**0.30**	**0.30**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

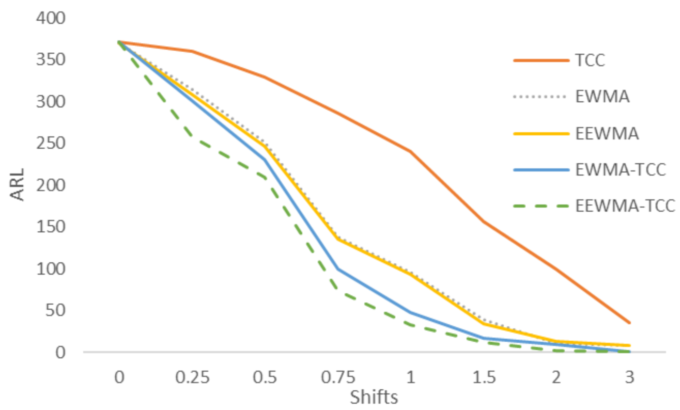

Figure 1. ARL curve in a normal distribution with n=5 and a smoothing value of 0.25.

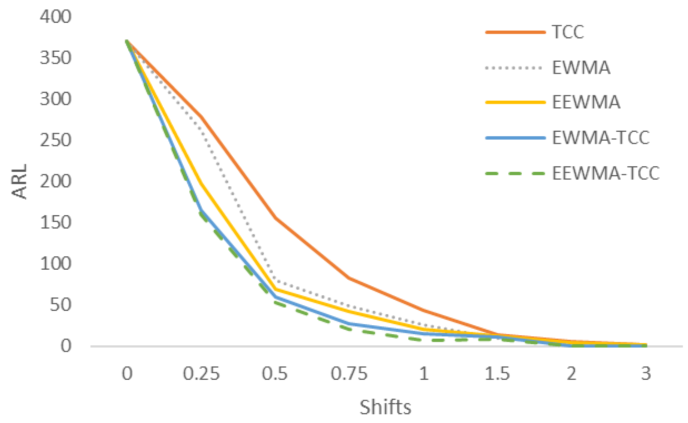

Figure 2. ARL curve in a Laplace distribution with n=10 and a smoothing value of 0.1.

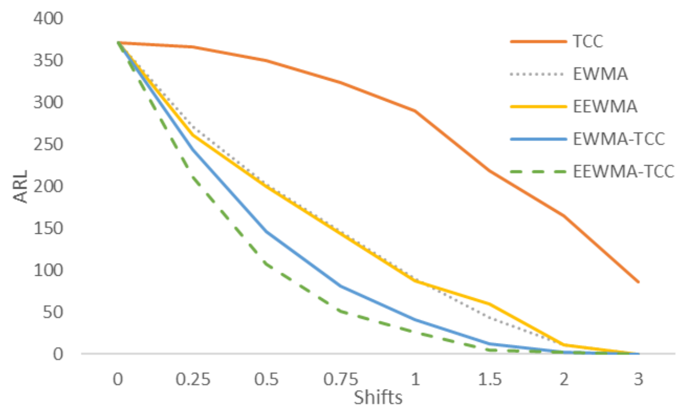

Figure 3. ARL curve in a Logistic distribution with n=5 and a smoothing value of 0.1.

Compared to the other charts for all shifts, the EEWMA-TCC displays lower ARL_1 values for the Laplace distribution, as presented in Tables 3 and 4. Moreover, the results for the subgroup n = 10 remain unchanged in the data set that includes outputs.

According to Tables 5 and 6, which display the overall findings of the Logistic distribution, the suggested chart outperforms its competitors—TCC, EWMA, and EWMA-TCC—for shifts ranging from 0.25 to 3.00. On the other hand, the EWMA-TCC chart also performs well for shifts 2.00 and 3.00.

Table 3. Performance of average run length (standard deviation) on a Laplace distribution with $n=5$ and two different smoothing parameters (0.1 on the left and 0.25 on the right).

Shifts	EWMA $G=4.01$	TCC $G_1=3.75$	EEWMA $G_2=2.96$	EWMA-TCC $G_3=7.27$	EEWMA-TCC $G_4=8.71$	EWMA $G=4.70$	EEWMA $G_2=3.67$	EWMA-TCC $G_3=5.782$	EEWMA-TCC $G_4=7.28$
0	370.81	370.40	370.08	370.62	370.58	370.88	370.35	370.51	370.60
	(0.58)	(0.59)	(0.58)	(0.57)	(0.57)	(0.58)	(0.58)	(0.57)	(0.57)
0.25	262.78	280.61	200.43	166.54	**160.03**	269.09	251.51	167.74	**161.16**
	(0.34)	(0.34)	(0.34)	(0.33)	(0.33)	(0.34)	(0.34)	(0.34)	(0.33)
0.5	80.04	156.21	71.67	61.14	**55.21**	80.68	136.80	62.89	**56.46**
	(0.09)	(0.09)	(0.09)	(0.08)	(0.08)	(0.09)	(0.09)	(0.08)	(0.08)
0.75	47.93	83.67	44.82	28.93	**21.52**	52.89	51.88	30.45	**27.35**
	(0.05)	(0.05)	(0.05)	(0.04)	(0.04)	(0.05)	(0.05)	(0.04)	(0.04)
1	25.62	45.79	23.79	17.88	**8.44**	38.95	38.10	18.09	**7.99**
	(0.03)	(0.03)	(0.03)	(0.03)	(0.02)	(0.03)	(0.03)	(0.03)	(0.02)
1.5	9.62	15.05	13.66	12.31	**10.05**	14.19	14.84	14.80	**11.78**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
2	5.13	6.48	5.32	**0.56**	**0.56**	5.21	5.59	**1.12**	**1.12**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
3	1.86	2.66	2.01	**0.11**	**0.11**	1.90	2.12	**0.22**	**0.22**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Table 4. Performance of average run length (standard deviation) on a Laplace distribution with $n=10$ and two different smoothing parameters (0.1 on the left and 0.25 on the right).

Shifts	EWMA $G=4.28$	TCC $G_1=3.76$	EEWMA $G_2=4.69$	EWMA-TCC $G_3=7.28$	EEWMA-TCC $G_4=8.72$	EWMA $G=4.71$	EEWMA $G_2=5.03$	EWMA-TCC $G_3=5.79$	EEWMA-TCC $G_4=7.29$
0	370.56	370.45	370.34	370.39	370.49	370.18	370.61	370.49	370.61
	(0.58)	(0.58)	(0.58)	(0.58)	(0.57)	(0.58)	(0.58)	(0.58)	(0.57)
0.25	261.96	278.08	197.67	164.90	**159.22**	270.05	242.24	165.78	**160.23**
	(0.34)	(0.34)	(0.34)	(0.34)	(0.34)	(0.34)	(0.34)	(0.34)	(0.34)
0.5	80.63	155.33	69.32	60.03	**53.83**	130.03	129.81	62.20	**54.88**
	(0.09)	(0.09)	(0.09)	(0.09)	(0.08)	(0.09)	(0.09)	(0.09)	(0.08)
0.75	48.55	82.51	42.61	28.22	**20.31**	53.11	48.55	29.13	**25.62**
	(0.05)	(0.05)	(0.05)	(0.04)	(0.04)	(0.05)	(0.05)	(0.04)	(0.04)
1	26.34	44.34	21.44	15.56	**7.10**	39.17	35.32	16.73	**7.96**
	(0.03)	(0.03)	(0.03)	(0.03)	(0.02)	(0.03)	(0.03)	(0.03)	(0.02)
1.5	9.83	14.01	12.82	11.04	**9.23**	13.81	13.99	13.33	**10.64**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
2	4.22	6.21	4.41	**0.51**	**0.51**	4.65	5.01	**1.10**	**1.10**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
3	1.87	2.38	2.00	**0.10**	**0.10**	1.96	2.02	**0.20**	**0.20**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Table 5. Performance of average run length (standard deviation) on a Logistic distribution with $n=5$ and two different smoothing parameters (0.1 on the left and 0.25 on the right).

Shifts	EWMA $G=12.43$	TCC $G_1=2.50$	EEWMA $G_2=12.74$	EWMA-TCC $G_3=17.0$	EEWMA-TCC $G_4=23.09$	EWMA $G=12.59$	EEWMA $G_2=12.73$	EWMA-TCC $G_3=5.51$	EEWMA-TCC $G_4=7.13$
0	370.58	370.74	370.38	370.46	370.48	370.51	370.11	370.54	370.53
	(0.58)	(0.59)	(0.58)	(0.58)	(0.58)	(0.58)	(0.58)	(0.58)	(0.58)
0.25	270.42	365.32	260.95	242.98	**210.38**	299.61	297.54	255.07	**211.42**
	(0.34)	(0.34)	(0.34)	(0.34)	(0.33)	(0.34)	(0.34)	(0.34)	(0.33)
0.5	202.32	349.12	199.77	145.99	**107.79**	231.09	216.71	147.37	**115.64**
	(0.09)	(0.09)	(0.09)	(0.09)	(0.08)	(0.09)	(0.09)	(0.09)	(0.08)
0.75	145.81	323.45	143.39	80.59	**50.60**	147.74	146.39	85.69	**57.29**
	(0.05)	(0.05)	(0.05)	(0.05)	(0.04)	(0.05)	(0.05)	(0.05)	(0.04)
1	89.90	290.06	87.52	41.23	**25.55**	90.33	88.87	42.97	**27.15**
	(0.03)	(0.03)	(0.03)	(0.03)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)
1.5	43.87	218.89	59.89	11.96	**4.48**	45.61	60.13	12.33	**4.65**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
2	10.46	165.32	11.23	**2.43**	**2.43**	11.33	12.42	**2.57**	**2.57**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
3	**0.01**	85.41	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Table 6. Performance of average run length (standard deviation) on a Logistic distribution with $n=10$ and two different smoothing parameters (0.1 on the left and 0.25 on the right).

Shifts	EWMA $G=12.46$	TCC $G_1=2.51$	EEWMA $G_2=12.83$	EWMA-TCC $G_3=17.11$	EEWMA-TCC $G_4=23.15$	EWMA $G=12.63$	EEWMA $G_2=12.97$	EWMA-TCC $G_3=5.73$	EEWMA-TCC $G_4=7.25$
0	370.51	370.63	370.55	370.46	370.45	370.53	370.52	370.61	370.50
	(0.58)	(0.59)	(0.58)	(0.58)	(0.58)	(0.58)	(0.58)	(0.58)	(0.58)
0.25	271.77	364.44	258.35	240.66	**207.78**	299.94	293.38	251.18	**210.04**
	(0.34)	(0.34)	(0.34)	(0.34)	(0.34)	(0.34)	(0.34)	(0.34)	(0.34)
0.5	204.24	349.01	196.82	142.83	**105.22**	233.54	206.65	145.61	**110.43**
	(0.09)	(0.09)	(0.09)	(0.09)	(0.08)	(0.09)	(0.09)	(0.09)	(0.08)
0.75	146.68	322.72	140.23	75.42	**48.56**	148.11	144.90	83.66	**55.94**
	(0.05)	(0.05)	(0.05)	(0.05)	(0.04)	(0.05)	(0.05)	(0.05)	(0.04)
1	90.57	289.15	85.21	38.59	**22.31**	91.62	86.15	40.74	**24.22**
	(0.03)	(0.03)	(0.03)	(0.03)	(0.02)	(0.03)	(0.03)	(0.03)	(0.02)
1.5	44.82	218.17	57.62	10.41	**4.35**	46.73	60.04	12.09	**4.35**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
2	11.77	164.09	10.45	**2.40**	**2.40**	11.87	12.02	**2.50**	**2.50**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
3	**0.01**	83.53	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

3.1.1. EEWMA-TCC versus TCC

The proposed EEWMA-TCC chart adjusts more rapidly than the TCC chart across all shifts, smoothing settings, subgroups, and distribution. According to PD_{ARL}, the EEWMA-TCC chart exhibits the highest percentage of ARL. For example, the EEWMA-TCC chart diminishes the ARL by 68.04%, while the TCC chart reduces it by 36.52% when the case subgroup is 5 and the smoothing value is 0.10 in a normal distribution. Other instances exhibit consequences analogous to PD_{ARL}. The EEWMA-TCC chart may be a viable alternative to the TCC chart.

3.1.2. EEWMA-TCC versus EEWMA

The EEWMA-TCC chart surpasses the EEWMA chart for shifts, smoothing parameters, and subgroups in both distributions. In logistics distribution, the EEWMA-TCC promptly detects significant large shifts (shift = 3). The PD_{ARL} in EEWMA, with a subgroup size of 10 and a smoothing parameter of 0.25 in a logistic distribution, reduces the ARL by 60.35%, whereas the EEWMA-TCC chart results in a decrease of 73.72%. Other scenarios yield identical outcomes to PD_{ARL}. The results illustrate the benefits of the suggested chart compared to the EEWMA chart.

3.1.3. EEWMA-TCC versus EWMA

The EEWMA-TCC chart surpasses the EWMA chart for shifts, smoothing parameters, and subgroups in all distributions. The PD_{ARL} in EWMA, with a subgroup size of 5 and a smoothing parameter of 0.25 in a normal distribution, reduces the ARL by 58.57%, whereas the EEWMA-TCC chart results in a decrease of 67.66%. Other scenarios yield identical outcomes to PD_{ARL}. The results demonstrate the advantages of the proposed chart relative to the EWMA chart.

3.1.4. EEWMA-TCC versus EWMA-TCC

Tables 1–6 demonstrate that the EEWMA-TCC chart exhibits superior detection speed compared to the EWMA-TCC chart across all parameter setups; nevertheless, for substantial shifts (shift >=2), the results are equivalent. As reported by PDARL, when the relevant subgroup is 5 and the smoothing parameter is 0.1 in a Laplace distribution, the EEWMA-TCC chart reduces ARL by 78.83%, while the EWMA-TCC chart reduces ARL by 70.76%. The outcomes produced by PDARL are conditionally independent. According to the data, the recommended chart has more advantages than the EWMA-TCC chart.

3.2. An illustrated example

In this section, we evaluate the efficacy of the proposed chart in contrast to the control chart for detecting alterations in the critical dimension of a machined part [35]. The data conforms to a normal distribution, and the p-value of 0.62 suggests statistical significance. The estimated values of the control chart statistics are found at certain levels of the charting parameters. Then, the TCC, EEWMA, EWMA-TCC, and EEWMA-TCC control charts with lower and upper control limits are plotted, as shown in Figures 4 to 7. Figure 4 illustrates the graphed tracking statistics for the TCC control charts that do not indicate the initial out-of-control alarm. Nevertheless, the EEWMA chart demonstrates efficacy at sample number 7, corresponding to the EWMA-TCC chart, as illustrated in Figures 5 and 6.

Figure 7 demonstrates that the EEWMA-TCC recognizes the initial out-of-control signal at sample number 4. Nevertheless, the signal at sample number 5 remains uncontrolled, confirming the process deregulation. These data corroborate the advantages of the suggested control chart over its competitors, aligning with the comparable average run length characteristics.

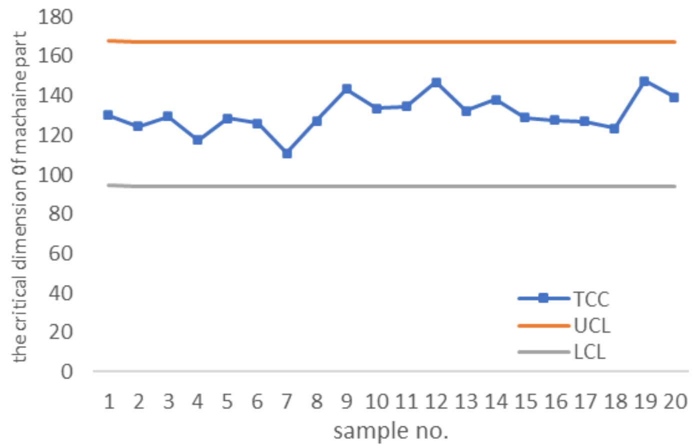

Figure 4. TCC control chart for the critical dimension of machined part data.

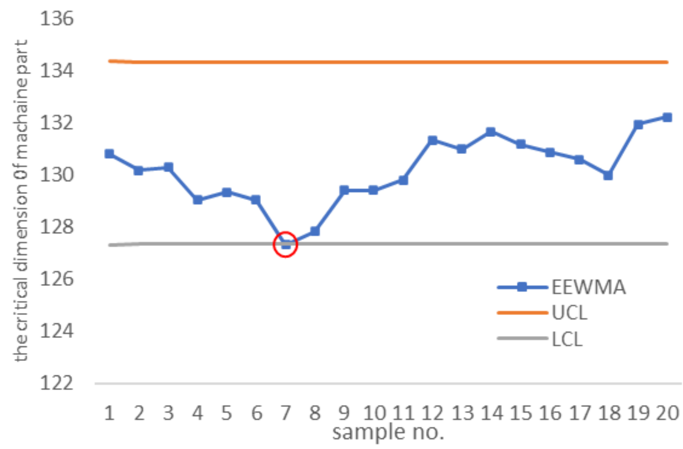

Figure 5. EEWMA control chart for the critical dimension of machined part data.

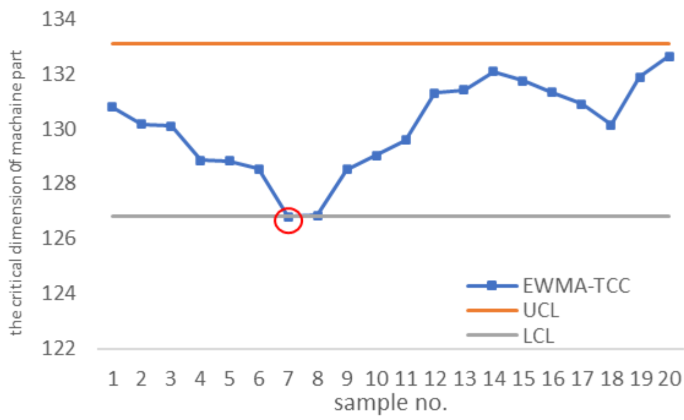

Figure 6. EWMA-TCC control chart for the critical dimension of machined part data.

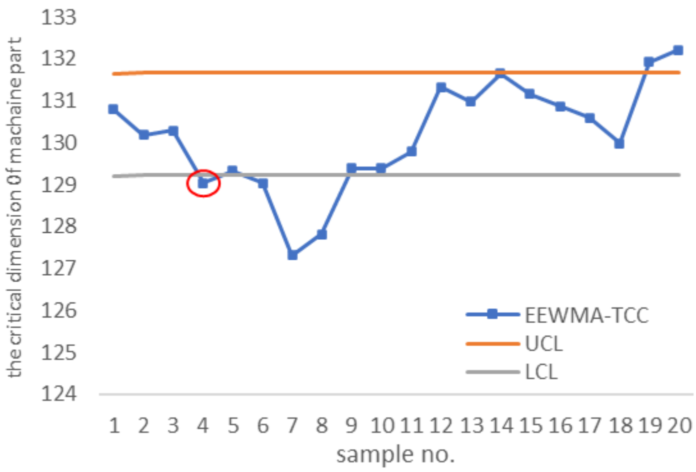

Figure 7. EEWMA-TCC control chart for the critical dimension of machined part data.

4. Conclusions

In the absence of knowledge of the actual distribution of a quality attribute, nonparametric control charts serve as a reliable and robust instrument for assessing a process. We present the combined EEWMA control chart without distribution, utilizing the Tukey statistic to detect alterations in the process mean. The effectiveness of control charts using the minimum ARL_1 is ascertained using Monte Carlo simulations predicated on symmetric distributions. Nonetheless, the PD_{ARL} is used to ensure the outcomes once more. The results show that different shifts are better detected by the proposed chart. In addition, a real-life example is provided to show how the proposed chart differs from previous fighting control charts in terms of practicality and ability to detect procedure changes. Following that, this study can only be used in situations where the data is normally distributed or symmetrical. It is also possible to undertake a thorough analysis to find the optimal span and smoothing factor values for certain shifts of interest. A further look into the practicality of the charting structure is necessary to monitor the dispersion of procedures.

Author contributions

Saowanit Sukparungsee: Conceptualization, methodology, writing—review and editing, visualization, project administration; Khanittha Talordphop: Software, formal analysis, investigation, data curation, writing—original draft preparation; Yupaporn Areepong: Validation, visualization. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflict of interest.

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