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Research article

Stability analysis for bidirectional associative memory neural networks: A new global asymptotic approach

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Abstract: This study employs specific and appropriate criteria to investigate the global stability of hybrid bidirectional associative memory (BAM) neural networks with time delays. We establish new and more general conditions for global asymptotic robust stability (GARS) in time-delayed BAM neural networks at the equilibrium point. This represents the primary objective and novelty of this paper. The derived conditions are independent of the system parameter delay in BAM neural networks. Finally, we provide numerical examples to illustrate the applicability and effectiveness of our conclusions with respect to network parameters.

Keywords: asymptotic stability; BAM; interval matrices; LKF; time-delay
Mathematics Subject Classification: 93D05, 34D23, 34D08, 68T07, 68T07

1. Introduction

In recent years, neural networks (NNs) have garnered significant attention due to their successful applications, and there has been a notable focus on the dynamical analysis of various kinds of NNs due to their importance as significant categories of non-linear mathematical models that can be used in addressing many categories of engineering challenges in optimization, image processing, and other engineering disciplines [1–4]. Numerous NNs are available for selection, such as cellular NNs, recurrent NNs, Hopfield NNs, Cohen-Grossberg NNs, and BAM NNs. We use NNs to solve technical difficulties such as signal processing, pattern recognition, and combinatorial optimization. However, a common challenge in the development and hardware implementation of NNs is the imprecision of NNs parameters, such as the inherent variability of network circuit parameters. Estimation errors occur during the network design process when looking at important data such as neuron firing rate, synaptic connection strength, and signal transmission delay, although it is possible to look at the ranges and limits of these parameters. Consequently, a successful model needs to possess certain attributes. As a result, certain resilience characteristics must be present in an effective model. Furthermore, a single equilibrium point plays a crucial role in the modification of the network model. Dynamical NNs are largely based on many types of equilibrium point stability analysis. Many researchers have looked at different ways to find stability, including GARS, full stability, and exponential stability of dynamic models with time delays in various works [5–9]. Researchers have already shown different stability analysis results for time-delayed NNs using Lyapunov and non-smooth analysis to look at stability and instability. In literature, the stability criteria of delayed NNs using delay-dependent results are discussed in [10–12]. Consequently, a significant issue is the analysis of GARS and control techniques of many multiple time-delayed BAM NNs. Many researchers have only recently focused on studying it [13, 14].

BAM is a significant NNs technique that was first presented by B. Kosko [15, 16]. The BAM NNs have two layers of neurons. A single layer of neurons lacks interconnectivity. The BAM NNs range from a monolayer auto-switching to a double-layer pattern-matched hetero junction chain that stores both forward and backward pattern pairs. Many researchers have studied in detail the dynamic characteristics and applications of BAM NNs to solve many real-time issues, including automatic control, optimization, signal processing, and pattern recognition. Some global asymptotic stability of BAM NNs with S-type distributed delays is discussed in [17]. The global stability analysis of fractional-order quaternion-valued BAM NNs is discussed in [18]. The global asymptotic stability of periodic solutions for neutral-type delayed BAM NNs by combining an abstract theorem of k-set contractive operator with the LMI method is discussed in [19]. The global asymptotic stability of periodic solutions for neutral-type BAM NNs with delays is discussed in [20]. The literature has also reported time delays in BAM NNs for GARS. In [21, 22], the issue of synchronization in uncertain delayed fractional order BAM NNs is examined with state feedback control and parameters. Taking into account cost efficiency and the longevity of equipment, infinite-time synchronization is not the optimal selection in engineering contexts such as secure communication and image encryption. Moreover, we can classify the result of BAM NNs into three distinct categories. Earlier research on BAM NNs identified only two types: models with time delays and models without time delays. In the literature, many authors have looked into the stability outcomes of the two types listed above. The hybrid form of the BAM NNs is a newer research topic. In this type, both the delay and the immediate signal occur simultaneously. A precise resolution is necessary for every conceivable initial condition in hybrid BAM NNs. From a mathematical perspective, this indicates that the GARS function caused a delay in the NNs reaching the time equilibrium point. Several authors have discussed the GARS of the hybrid BAM NNs [23–28]. The study of the global stability of synaptic connection matrices in NNs leads to interval matrix theory. This will result in a significant increase in computational capacity. So, there is much room left for us to investigate the GARS of hybrid BAM NNs with temporal delays.

The novelty of this paper deals with the GARS of hybrid BAM NNs that have time delays. The main objective of this research is to establish extensive criteria for the global asymptotic robustness of hybrid BAM NNs with delays. We also use the upper limit for the norms of interconnection matrices, Lyapunov-Krasovskii functionals (LKF), and certain activation functions to find results that make sure hybrid BAM NNs are stable. Finally, we implement a numerical example to demonstrate the efficacy of the proposed results.

Notations: The notations that will be utilized in this paper are as follows: $\mathbb{R}^n$ denotes n-dimensional Euclidean space, and $\mathbb{R}^{n\times m}$ is the set of all real matrices of $n \times m$. Define E as a matrix with elements e_{ij} for $n \times m$. The 2-norm of matrix E equals the square root of the maximum eigenvalue of $E^T E$. The absolute value of a matrix $E = (e_{ij})_{n\times m}$ with real numbers is equal to the absolute value of each entry in the matrix, denoted by $| E |= (| e_{ij} |)_{n\times m}$. A matrix is positive definite (semi-definite) when it $u^T Bu > 0(\geq 0)$ holds for all real vectors $u = (u_1, u_2, \cdots, u_n)^T \in \mathbb{R}^n$. Also that $\sum\limits_{i,j=1}^{n,m} = \sum\limits_{i=1}^{n}\sum\limits_{j=1}^{m}$ and $\sum\limits_{j,i=1}^{m,n} = \sum\limits_{j=1}^{m}\sum\limits_{i=1}^{n}$.

2. Preliminaries

Consider the system of NNs that includes delayed connections in the BAM as described below: [29]

$$\begin{cases} \dot{y}_j(t) = -\check{b}_j y_j(t) + \sum_{i=1}^{n} \check{g}_{ij}\phi_{1i}(w_i(t)) + \sum_{i=1}^{n} \check{g}^{\tau}_{ij}\phi_{1i}(w_i(t - \check{\sigma}_{ij})) + K_j, \ \forall j \\ \dot{w}_i(t) = -\check{a}_i w_i(t) + \sum_{j=1}^{m} \check{f}_{ji}\phi_{2j}(y_j(t)) + \sum_{j=1}^{m} \check{f}^{\tau}_{ji}\phi_{2j}(y_j(t - \check{\tau}_{ji})) + J_i, \ \forall i \end{cases} \tag{2.1}$$

where $w_i(t)$ and $y_j(t)$ represent the state of the i^{th} and j^{th} neurons in the vectors at time t. n and m represent the total number of neurons in the proposed hybrid BAM NNs (2.1). ϕ_{1i} and ϕ_{2j} indicate the activation functions of the neurons; $\check{f}_{ji}$, $\check{f}^{\tau}_{ji}$, $\check{g}_{ij}$, and $\check{g}^{\tau}_{ij}$ are the connection weight matrices; $\check{a}_i$ and $\check{b}_j$ stand for the neuron charging time constants; J_i and K_j, for every $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m$ are the inputs. For the stability of (2.1), the following several considerations have been made.

Assumption 2.1. $(A1)$. *Assume that there are certain $\check{l}_i > 0 < \check{h}_j$, such that the following specified conditions are satisfied:*
$0 \leq \dfrac{\phi_{1i}(\overline{x}) - \phi_{1i}(\overline{y})}{\overline{x} - \overline{y}} \leq \check{l}_i$, $0 \leq \dfrac{\phi_{2j}(\hat{x}) - \phi_{2j}(\hat{y})}{\hat{x} - \hat{y}} \leq \check{h}_j$, $\hat{x} \neq \hat{y}$, $\overline{x} \neq \overline{y}$ *for all* $\hat{x}$, $\hat{y}$, $\overline{x}$, $\overline{y} \in \mathbb{R}$.

Assumption 2.2. $(A2)$. *Assume there are positive constants $\check{M}_i$ and $\check{N}_j$ for which certain conditions are satisfied.* $|\phi_{1i}(w_1)| \leq \check{M}_i$ *and* $|\phi_{2j}(w_2)| \leq \check{N}_j$ *for all* $w_1, w_2 \in \mathbb{R}$, *where* $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m$. *Based on this assumption, the activation functions are limited in type.*

The matrices $\check{b}_j$, $\check{f}_{ji}$, $\check{f}^{\tau}_{ji}$, $\check{g}_{ij}$, $\check{g}^{\tau}_{ij}$, $\check{a}_i$, $\check{\tau}_{ji}$ and $\check{\sigma}_{ij}$ are assumed to be uncertain matrices. The usual approach to dealing with the delayed system includes modifying the synaptic strength connection matrices within a specific time frame in the following manner for $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m$.

$$\begin{cases} \mathcal{B}_I = & \{\mathcal{B} = diag(\check{b}_j) : 0 < \underline{\mathcal{B}} \le \mathcal{B} \le \overline{\mathcal{B}},\ ie.,\ 0 < \underline{\check{b}}_j \le \check{b}_j \le \overline{\check{b}}_j\}, \forall \mathcal{B} \in \mathcal{B}_I \\ \mathcal{G}_I = & \{\mathcal{G} = (\check{g}_{ij}) : \underline{\mathcal{G}} \le \mathcal{G} \le \overline{\mathcal{G}},\ ie.,\ \underline{\check{g}}_{ij} \le \check{g}_{ij} \le \overline{\check{g}}_{ij}\}, \forall \mathcal{G} \in \mathcal{G}_I \\ \mathcal{G}_I^\tau = & \{\mathcal{G}^\tau = (\check{g}_{ij}^\tau) : \underline{\mathcal{G}}^\tau \le \mathcal{G}^\tau \le \overline{\mathcal{G}}^\tau,\ ie.,\ \underline{\check{g}}_{ij}^\tau \le \check{g}_{ij}^\tau \le \overline{\check{g}}_{ij}^\tau\}, \forall \mathcal{G}^\tau \in \mathcal{G}_I^\tau \\ \alpha_I = & \{\alpha = (\check{\sigma}_{ij}) : \underline{\alpha} \le \alpha \le \overline{\alpha},\ ie.,\ \underline{\check{\sigma}}_{ij} \le \check{\sigma}_{ij} \le \overline{\check{\sigma}}_{ij}\}, \forall \alpha \in \alpha_I, \\ \mathcal{A}_I = & \{\mathcal{A} = diag(\check{a}_i) : 0 < \underline{\mathcal{A}} \le \mathcal{A} \le \overline{\mathcal{A}}, i.e.,.,\ 0 < \underline{\check{a}}_i \le \check{a}_i \le \overline{\check{a}}_i\}, \forall \mathcal{A} \in \mathcal{A}_I \\ \mathcal{F}_I = & \{\mathcal{F} = (\check{f}_{ji}) : \underline{\mathcal{F}} \le \mathcal{F} \le \overline{\mathcal{F}},\ ie.,\ \underline{\check{f}}_{ji} \le \check{f}_{ji} \le \overline{\check{f}}_{ji}\}, \forall \mathcal{F} \in \mathcal{F}_I \\ \mathcal{F}_I^\tau = & \{\mathcal{F}^\tau = (\check{f}_{ji}^\tau) : \underline{\mathcal{F}}^\tau \le \mathcal{F}^\tau \le \overline{\mathcal{F}}^\tau,\ ie.,\ \underline{\check{f}}_{ji}^\tau \le \check{f}_{ji}^\tau \le \overline{\check{f}}_{ji}^\tau\}, \forall \mathcal{F}^\tau \in \mathcal{F}_I^\tau \\ \phi_I = & \{\phi = (\check{\tau}_{ji}) : \underline{\phi} \le \phi \le \overline{\phi},\ ie.,\ \underline{\check{\tau}}_{ji} \le \check{\tau}_{ji} \le \overline{\check{\tau}}_{ji}\}, \forall \phi \in \phi_I. \end{cases} \tag{2.2}$$

Next, we move the equilibrium point of (2.1) to the origin. To achieve this, we employ the subsequent alteration:

$$\check{x}_j(\cdot) = y_j(\cdot) - y_j^*,\ \check{u}_i(\cdot) = w_i(\cdot) - w_i^*,\ \text{for every } j = 1, 2, \cdots, m,\ i = 1, 2, \cdots, n.$$

Through the use of the transformation mentioned above, we change (2.1) into the form as shown below:

$$\begin{cases} \frac{d\check{x}_j(t)}{dt} = -\check{b}_j\check{x}_j(t) + \sum_{i=i}^{n} \check{g}_{ij}\chi_{1i}(\check{u}_i(t)) + \sum_{i=i}^{n} \check{g}_{ij}^\tau \chi_{1i}(\check{u}_i(t - \check{\sigma}_{ij})),\ \forall j, \\ \frac{d\check{u}_i(t)}{dt} = -\check{a}_i\check{u}_i(t) + \sum_{j=i}^{m} \check{f}_{ji}\chi_{2j}(\check{x}_j(t)) + \sum_{j=i}^{m} \check{f}_{ji}^\tau \chi_{2j}(\check{x}_j(t - \check{\tau}_{ji})),\ \forall i, \end{cases} \tag{2.3}$$

$$\text{where } \chi_{1i}(\check{u}_i(\cdot)) = \phi_{1i}(\check{u}_i(\cdot) + w_i^*) - \phi_{1i}(w_i^*),\ \chi_{1i}(0) = 0,$$
$$\chi_{2j}(\check{x}_j(\cdot)) = \phi_{2j}(\check{x}_j(\cdot) + y_j^*) - \phi_{2j}(y_j^*),\ \chi_{2j}(0) = 0,\ \text{for every } i, j.$$

Now, it is straightforward to verify that the functions χ_{1i} and χ_{2j} meet the requirements for ϕ_{1i} and ϕ_{2j}, meaning χ_{1i}, χ_{2j} satisfy both $(A1)$ and $(A2)$.

Definition 2.3. *[30] The BAM NNs* (2.3) *satisfying* (2.2) *is GARS if the origin of the unique equilibrium point of the BAM NNs* (2.3) *is globally asymptotically stable for all* $\mathcal{B} \in \mathcal{B}_I$, $\mathcal{G} \in \mathcal{G}_I$, $\mathcal{G}^\tau \in \mathcal{G}_I^\tau$, $\mathcal{A} \in \mathcal{A}_I$, $\mathcal{F} \in \mathcal{F}_I$, $\mathcal{F}^\tau \in \mathcal{F}_I^\tau$. *Regardless of the initial conditions, the solutions of (2.3) that converge to the origin of the unique equilibrium point constitute the system's global asymptotic stability.*

The identification and understanding of these following lemmas and facts are pivotal in establishing the prerequisites for conducting a thorough examination of global stability in (2.1).

Lemma 2.4. [31] If $\mathcal{F} \in \mathcal{F}_I$, then

$$\| \mathcal{F} \|_2 \le T(\mathcal{F}),$$

where $T(\mathcal{F}) = \sqrt{2 \||\ (\mathcal{F}^*)^T \mathcal{F}^*\ | + \mathcal{F}_*^T \mathcal{F}_* \|_2}$, $\mathcal{F}^* = \frac{1}{2}(\overline{\mathcal{F}} + \underline{\mathcal{F}})$, $\mathcal{F}_* = \frac{1}{2}(\overline{\mathcal{F}} - \underline{\mathcal{F}})$. Similarly, $\mathcal{G}^* = \frac{1}{2}(\overline{\mathcal{G}} + \underline{\mathcal{G}})$, $\mathcal{G}_* = \frac{1}{2}(\overline{\mathcal{G}} - \underline{\mathcal{G}})$. The matrices $\overline{\mathcal{F}}, \underline{\mathcal{F}}, \overline{\mathcal{G}}$ and $\underline{\mathcal{G}}$ are defined as in (2.2).

Lemma 2.5. [32] The following inequality holds for any two vectors $u = (u_1, u_2, \cdots, u_n)^T \in \mathbb{R}^n$ and $y = (y_1, y_2, \cdots, y_n)^T \in \mathbb{R}^n$.

$$2u^T y = 2y^T u \leq \beta u^T u + \frac{1}{\beta} y^T y, \ \forall \beta > 0.$$

Lemma 2.6. [33] For each matrix $\mathcal{F}$ in the interval $[\underline{\mathcal{F}}, \overline{\mathcal{F}}]$, the following inequality holds:

$$\| \mathcal{F} \|_2 \ \leq \| \mathcal{F}^* \|_2 + \| \mathcal{F}_* \|_2,$$

where $\mathcal{F}^* = \frac{1}{2}(\overline{\mathcal{F}} + \underline{\mathcal{F}}), \ \mathcal{F}_* = \frac{1}{2}(\overline{\mathcal{F}} - \underline{\mathcal{F}})$.

Remark 2.7. *The results described in Lemmas 2.4 and 2.6 are consistently applicable to any synaptic connection strength matrices defined as in* (2.1).

Consider the matrix $\mathcal{E}$, which satisfies Eq (2.2). Now, there exists a positive constant $T(\mathcal{E})$ that satisfies the following condition:

$$\| \mathcal{E} \|_2 \leq \ T(\mathcal{E}),$$

where $\mathcal{E}$ is any matrix as defined in (2.2).

3. Main results and proofs

In this section, we define specific generalized sufficient conditions for the GARS of the BAM NNs described by (2.1). Through the application of the assumption (A2), BAM NNs (2.1) that fulfill (2.2) possess the existence of the equilibrium point. Hence, demonstrating the uniqueness of the equilibrium point for the GARS of (2.1) is essential.

Theorem 3.1. *Assume the activation functions χ_{1i}, χ_{2j} fulfill conditions $(A1)$, $(A2)$, and there are positive constants, γ and δ, such that the conditions below are satisfied:*

$$\begin{aligned} \Psi_{1i} =& m\gamma\delta\underline{\check{a}}_i - \frac{1}{2}m\delta(\gamma^2 \check{l}_i^2 - \delta\gamma - m(f^{\tau^*})) - \frac{1}{2}n\gamma \check{l}_i^2 T^2(\mathcal{G}) > 0, \ \forall i = 1, 2, ..., n, \\ \Omega_{1j} =& n\gamma\delta\underline{\check{b}}_i - \frac{1}{2}n\delta(\gamma^2 \check{h}_j^2 - \delta\gamma - n(g^{\tau^*})) - \frac{1}{2}m\gamma \check{h}_j^2 T^2(\mathcal{F}) > 0, \ \forall j = 1, 2, ..., m, \end{aligned}$$

where $(f^{\tau^*}) = \sum_{j=1}^{m} (\check{f}_{ji}^{\tau^*})^2$, $(g^{\tau^*}) = \sum_{i=1}^{n} (\check{g}_{ij}^{\tau^*})^2$, $f_{ji}^{\tau^*} = max(| \underline{\check{f}}_{ji}^{\tau} |, | \overline{\check{f}}_{ji}^{\tau} |)$ *and* $g_{ij}^{\tau^*} = max(| \underline{\check{g}}_{ij}^{\tau} |, | \overline{\check{g}}_{ij}^{\tau} |)$.

Then, the BAM NNs defined by (2.3) *with network parameters that meet* (2.2) *have GARS at their origin.*

Proof. This proof will be shown through a two-step process. In Step 1, we show that its origin is the only equilibrium point of (2.3). On the flip side, we show that BAM NNs (2.3) whose origin is GARS. Step 1. Assume that the equilibrium points of (2.3) are $(\check{u}_1^*, ..., \check{u}_n^*)^T = \check{u}^* \neq 0$ and $(\check{x}_1^*, ..., \check{x}_m^*)^T = \check{x}^* \neq 0$. The points that satisfy the equations stated below are the equilibrium points of (2.3).

$$\check{a}_i \check{u}_i^* + \sum_{j=i}^{m} \check{f}_{ji} \chi_{2j}(\check{x}_j^*) + \sum_{j=i}^{m} \check{f}_{ji}^{\tau} \chi_{2j}(\check{x}_j^*) = 0, \ \forall i, \tag{3.1}$$

$$\check{b}_j\check{x}_j^* + \sum_{i=i}^{n}\check{g}_{ij}\chi_{1j}(\check{u}_i^*) + \sum_{i=i}^{n}\check{g}_{ij}^{\tau}\chi_{1j}(\check{u}_i^*) = 0,\ \forall j. \tag{3.2}$$

Multiplying (3.1) by $2m\check{u}_i^*$ and (3.2) by $2n\check{x}_j^*$, then addition of the resulting equations,

$$\begin{aligned}
0 =& -2m\check{a}_i\check{u}_i^{*2} + \sum_{i,j=1}^{n,m} 2m\check{u}_i^*\check{f}_{ji}\chi_{2j}(\check{x}_j^*) + \sum_{j,i=1}^{m,n} 2m\check{u}_i^*\check{f}_{ji}^{\tau}\chi_{2j}(\check{x}_j^*) - 2n\check{b}_j\check{x}_j^{*2} + 2n\sum_{j,i=1}^{m,n}\check{x}_j^*\check{g}_{ij}\chi_{1j}(\check{u}_i^*)\\
&+ 2n\sum_{i,j=1}^{n,m}\check{x}_j^*\check{g}_{ij}^{\tau}\chi_{1j}(\check{u}_i^*),\\
0 =& -2m\check{a}_i\check{u}_i^{*2} + \sum_{j,i=1}^{m,n} 2m\check{u}_i^*\check{f}_{ji}\chi_{2j}(\check{x}_j^*) + \sum_{i,j=1}^{n,m} 2m\check{u}_i^*\check{f}_{ji}^{\tau}\chi_{2j}(\check{x}_j^*) - 2n\check{b}_j\check{x}_j^{*2} + 2n\sum_{j,i=1}^{m,n}\check{x}_j^*\check{g}_{ij}\chi_{1j}(\check{u}_i^*)\\
&+ 2n\sum_{j,i=1}^{m,n}\check{x}_j^*\check{g}_{ij}^{\tau}\chi_{1j}(\check{u}_i^*) + \frac{1}{\gamma}\sum_{i,j=1}^{n,m} m^2(\check{f}_{ji}^{\tau})^2\chi_{2j}^2(\check{x}_j^*) - \frac{1}{\gamma}\sum_{i,j=1}^{n,m} m^2(\check{f}_{ji}^{\tau})^2\chi_{2j}^2(\check{x}_j^*)\\
&+ \frac{1}{\gamma}\sum_{j,i=1}^{m,n} n^2(\check{g}_{ij}^{\tau})^2\chi_{1j}^2(\check{u}_i^*) - \frac{1}{\gamma}\sum_{j,i=1}^{m,n} n^2(\check{g}_{ij}^{\tau})^2\chi_{1j}^2(\check{u}_i^*),
\end{aligned}$$

$$\begin{aligned}
\leq& -2m\check{a}_i\check{u}_i^{*2} + \sum_{i,j=1}^{n,m} 2m\check{u}_i^*\check{f}_{ji}\chi_{2j}(\check{x}_j^*) + \sum_{i,j=1}^{n,m} 2m\check{u}_i^*\check{f}_{ji}^{\tau}\chi_{2j}(\check{x}_j^*) - 2n\check{b}_j\check{x}_j^{*2} + 2n\sum_{j,i=1}^{m,n}\check{x}_j^*\check{g}_{ij}\chi_{1j}(\check{u}_i^*)\\
&+ 2n\sum_{j,i=1}^{m,n}\check{x}_j^* g_{ij}^{\tau}\chi_{1j}(\check{u}_i^*) + \frac{1}{\gamma}\sum_{i,j=1}^{n,m} m^2(\check{f}_{ji}^{\tau})^2\check{h}_j^2(\check{x}_j^{*2}) - \frac{1}{\gamma}\sum_{i,j=1}^{n,m} m^2(\check{f}_{ji}^{\tau})^2\chi_{2j}^2(\check{x}_j^*)\\
&+ \frac{1}{\gamma}\sum_{j,i=1}^{m,n} n^2(\check{g}_{ij}^{\tau})^2\check{l}_i^2(\check{u}_i^{*2}) - \frac{1}{\gamma}\sum_{j,i=1}^{m,n} n^2(\check{g}_{ij}^{\tau})^2\chi_{1j}^2(\check{u}_i^*).
\end{aligned} \tag{3.3}$$

Take into account the forthcoming inequalities:

$$\begin{aligned}
\sum_{i,j=1}^{n,m} 2m\check{u}_i^*(t)\check{f}_{ji}\chi_{2j}(\check{x}_j^*) &= 2m\check{u}^{*T}\mathcal{F}\mathcal{S}(\check{x}^*)\\
&\leq m\delta\check{u}^{*T}\check{u}^* + m\frac{1}{\delta}\mathcal{S}^T(\check{x}^*)\mathcal{F}^T\mathcal{F}\mathcal{S}(\check{x}^*)\\
&\leq m\delta\check{u}^{*T}\check{u}^* + m\frac{1}{\delta}\parallel \mathcal{F}\parallel_2^2\parallel \mathcal{S}(\check{x}^*)\parallel_2^2\\
&\leq m\delta\sum_{i=1}^{n}\check{u}_i^{*2} + m\frac{1}{\delta}\parallel \mathcal{F}\parallel_2^2\sum_{j=1}^{m}\check{h}_j^2\check{x}_j^{*2},
\end{aligned} \tag{3.4}$$

$$\sum_{i,j=1}^{n,m} 2n\check{x}_j^*\check{g}_{ij}\chi_{1j}(\check{u}_i^*) = 2n\check{x}^{*T}\mathcal{G}\mathcal{S}(\check{u}^*)$$

$$\leq n\delta \check{x}^{*T}\check{x}^{*} + n\frac{1}{\delta}\mathcal{S}^{T}(\check{u}^{*})\mathcal{G}^{T}\mathcal{G}\mathcal{S}(\check{u}^{*})$$
$$\leq n\delta \check{x}^{*T}\check{x}^{*} + n\frac{1}{\delta}\parallel \mathcal{S}\parallel_2^2\parallel \mathcal{G}(u^{*})\parallel_2^2$$
$$\leq n\delta\sum_{j=1}^{m}\check{x}_j^{*2} + n\frac{1}{\delta}\parallel \mathcal{G}\parallel_2^2\sum_{i=1}^{n}\check{l}_i^2\check{u}_i^{*2}, \tag{3.5}$$

$$\sum_{i,j=1}^{n,m} 2m\check{u}_i^{*}\check{f}_{ji}^{\tau}\chi_{2j}(\check{x}_j^{*}) \leq \sum_{j,i=1}^{m,n}\frac{1}{\gamma}m^2(\check{f}_{ji}^{\tau})^2(\check{u}_i^{*})^2 + \sum_{i,j=1}^{n,m}\gamma\chi_{2j}^2(\check{x}_j^{*})$$
$$=\frac{1}{\gamma}m^2\sum_{j,i=1}^{m,n}(\check{f}_{ji}^{\tau})^2(\check{u}_i^{*})^2 + n\gamma\sum_{j=1}^{m}\check{h}_j^2((\check{x}_j^{*})^2), \tag{3.6}$$

$$\sum_{j,i=1}^{m,n} 2n\check{x}_j^{*}\check{g}_{ij}^{\tau}\chi_{2j}(\check{u}_i^{*}) \leq \sum_{i,j=1}^{n,m}\frac{1}{\gamma}n^2(\check{g}_{ij}^{\tau})^2(\check{x}_j^{*})^2 + \sum_{j,i=1}^{m,n}\gamma\chi_{1i}^2(\check{u}_i^{*})$$
$$=\frac{1}{\gamma}n^2\sum_{i,j=1}^{n,m}(\check{g}_{ij}^{\tau})^2(\check{x}_j^{*})^2 + m\gamma\sum_{i=1}^{n}\check{l}_i^2\Big((\check{u}_i^{*})^2\Big). \tag{3.7}$$

By applying the results (3.4)–(3.7) in (3.3), we have

$$0 \leq -\sum_{i=1}^{n}2m\check{a}_i(\check{u}_i^{*})^2 + m\delta\sum_{i=1}^{n}(\check{u}_i^{*})^2 + m\frac{1}{\delta}\|\mathcal{F}\|_2^2\sum_{j=1}^{m}\check{h}_j^2(\check{x}_j^{*})^2$$
$$-\sum_{j=1}^{m}2n\check{b}_j(\check{x}_j^{*})^2 + n\delta\sum_{j=1}^{m}(\check{x}_j^{*})^2 + n\frac{1}{\delta}\|\mathcal{G}\|_2^2\sum_{i=1}^{n}\check{l}_i^2(\check{u}_i^{*})^2$$
$$+\frac{1}{\gamma}n^2\sum_{i,j=1}^{n,m}(\check{g}_{ij}^{\tau})^2(\check{x}_j^{*})^2 + m\gamma\sum_{i=1}^{n}\check{l}_i^2\Big((\check{u}_i^{*})^2\Big)$$
$$+\frac{1}{\gamma}m^2\sum_{j,i=1}^{m,n}(\check{f}_{ji}^{\tau})^2(\check{u}_i^{*})^2 + n\gamma\sum_{j=1}^{m}\check{h}_j^2\Big((\check{x}_j^{*})^2\Big).$$

Since,

$$\parallel \mathcal{G}\parallel_2^2 \leq 2 \parallel\mid (\mathcal{G}^{*})^T\mathcal{G}^{*}\mid + \mathcal{G}_*^T\mathcal{G}_*\parallel_2 = T^2(\mathcal{G}),\ \parallel \mathcal{F}\parallel_2^2 \leq T^2(\mathcal{F}), (\check{f}_{ji}^{\tau})^2 \leq (f_{ji}^{\tau^*})^2,\ (\check{g}_{ij}^{\tau})^2 \leq (g_{ij}^{\tau^*})^2.$$

$$0 \leq \sum_{i=1}^{n}\Big\{m(-2\underline{\check{a}}_i + \gamma + \delta) + \frac{1}{\delta}n\check{l}_i^2\Big(2\parallel\mid (\mathcal{G}^{*})^T\mathcal{G}^{*}\mid + \mathcal{G}_*^T\mathcal{G}_*\parallel_2\Big) + \frac{1}{\gamma}n^2\check{l}_i^2\sum_{j=1}^{m}((f_{ij}^{\tau^*})^2)\Big\}\check{u}_i^{*2}$$
$$+\sum_{j=1}^{m}\Big\{n(-2\underline{\check{b}}_j + \gamma + \delta) + \frac{1}{\gamma}m^2\check{h}_j^2\sum_{i=1}^{n}((f_{ji}^{\tau^*})^2) + \frac{1}{\delta}m\check{h}_j^2\Big(2\parallel\mid (\mathcal{F}^{*})^T\mathcal{F}^{*}\mid + \mathcal{F}_*^T\mathcal{F}_*\parallel_2\Big)\Big\}\check{x}_j^{*2},$$

$$0 \leq \frac{-2}{\gamma\delta}\Bigg\{ \sum_{i=1}^{n} \Bigg\{ m\gamma\delta\underline{\check{a}}_i - \frac{1}{2}m\delta(\gamma^2\check{l}_i^2 - \delta\gamma - m(f^{\tau^*})) - \frac{1}{2}n\gamma\check{l}_i^2 T^2(\mathcal{G}) \Bigg\}\check{u}_i^{*2} +$$

$$\sum_{j=1}^{m} \Bigg\{ n\gamma\delta\underline{\check{b}}_i - \frac{1}{2}m\gamma\check{h}_j^2 T^2(\mathcal{F}) - \frac{1}{2}n\delta(\gamma^2\check{h}_j^2 - \delta\gamma - n(g^{\tau^*})) \Bigg\}\check{x}_j^{*2} \Bigg\},$$

$$0 \leq \frac{-2}{\gamma\delta}\Bigg\{ \sum_{i=1}^{n} \Psi_i \check{u}_i^2 + \sum_{j=1}^{m} \Omega_j \check{x}_j^2 \Bigg\}. \tag{3.8}$$

Since $\gamma > 0$ $\delta > 0$, $\Psi_i > 0$, $\Omega_j > 0$ for every i, j and $\check{x}^* \neq 0 \neq \check{u}^*$. But $\frac{-2}{\gamma\delta}\Big\{ \sum_{i=1}^{n} \Psi_i \check{u}_i^{*2} + \sum_{j=1}^{m} \Omega_j \check{x}_j^{*2} \Big\} < 0$. Here (3.8) contradicts the above result, and thus, we can deduce that the only equilibrium point is $\check{x}^* = 0 = \check{u}^*$. Therefore, the unique equilibrium point is the origin of (2.3).
Step 2. Let us examine the LKF provided below:

$$V(\check{x}(t), \check{u}(t)) = \sum_{i=1}^{n} m\check{u}_i^2(t) + \gamma \sum_{i,j=1}^{n,m} \int_{t-\check{\tau}_{ji}}^{t} \chi_{2j}^2(\check{x}_j(\eta))d\eta + \sum_{j=1}^{m} n\check{x}_j^2(t) + \gamma \sum_{j,i=1}^{m,n} \int_{t-\check{\sigma}_{ij}}^{t} \chi_{1i}^2 \check{u}_i(\xi)d\xi.$$

Obtaining $\dot{V}(\check{x}(t), \check{u}(t))$ in the trajectories of (2.3) and using Lemma 2.5 yields the following result:

$$\begin{aligned}
\dot{V}(\check{x}(t), \check{u}(t)) \leq\ & m\delta \sum_{i=1}^{n} \check{u}_i^2(t) - \sum_{i=1}^{n} 2m\check{a}_i\check{u}_i^2(t) + n\delta \sum_{j=1}^{m} x_j^2(t) + m\frac{1}{\delta} \parallel \mathcal{F} \parallel_2^2 \sum_{j=1}^{m} \check{h}_j^2 \check{x}_j^2(t) \\
& + n\frac{1}{\delta} \parallel \mathcal{G} \parallel_2^2 \sum_{i=1}^{n} \check{l}_i^2 \check{u}_i^2(t) - \sum_{j=1}^{m} 2n\check{b}_j \check{x}_j^2(t) + m\gamma \sum_{i=1}^{n} \check{l}_i^2 \check{u}_i^2(t) + n\gamma \sum_{j=1}^{m} \check{h}_j^2 \check{x}_j^2(t) \\
& + \frac{1}{\gamma} \sum_{i=1}^{n} \sum_{j=1}^{m} m^2 (\check{f}_{ji}^{\tau})^2 \check{u}_i^2(t) + \frac{1}{\gamma} \sum_{j=1}^{m} \sum_{i=1}^{n} n^2 (\check{g}_{ij}^{\tau})^2 \check{x}_j^2(t).
\end{aligned}$$

Since $\parallel \mathcal{G} \parallel_2^2 \leq T^2(\mathcal{G}), \parallel \mathcal{F} \parallel_2^2 \leq T^2(\mathcal{F})$, $(\check{f}_{ji}^{\tau})^2 \leq (f_{ji}^{\tau^*})^2$ and $(\check{g}_{ij}^{\tau})^2 \leq (g_{ij}^{\tau^*})^2$.

$$\begin{aligned}
\dot{V}(\check{x}(t), \check{u}(t)) \leq & \sum_{i=1}^{n} \Bigg\{ m(-2\underline{\check{a}}_i + \gamma l_i^2 + \delta) + \frac{1}{\delta} n\check{l}_i^2 \Big(T^2(\mathcal{G})\Big) + \frac{1}{\gamma} m^2 \sum_{j=1}^{m} ((f_{ji}^{\tau^*})^2) \Bigg\}\check{u}_i^2 \\
& + \sum_{j=1}^{m} \Bigg\{ n(-2\underline{\check{b}}_j + \gamma\check{h}_j^2 + \delta) + \frac{1}{\delta} m\check{h}_j^2 \Big(T^2(\mathcal{F})\Big) + \frac{1}{\gamma} n^2 \sum_{i=1}^{n} ((g_{ij}^{\tau^*})^2) \Bigg\}\check{x}_j^2, \\
= & \frac{-2}{\gamma\delta}\Bigg\{ \sum_{i=1}^{n} \Bigg\{ m\gamma\delta\underline{\check{a}}_i - \frac{1}{2}n\gamma\check{l}_i^2 T^2(\mathcal{G}) - \frac{1}{2}m\delta(\gamma^2\check{l}_i^2 - \delta\gamma - m(f^{\tau^*})) \Bigg\}\check{u}_i^{*2} \\
& + \sum_{j=1}^{m} \Bigg\{ n\gamma\delta\underline{\check{b}}_i - \frac{1}{2}m\gamma\check{h}_j^2 T^2(\mathcal{F}) - \frac{1}{2}n\delta(\gamma^2\check{h}_j^2 - \delta\gamma - n(g^{\tau^*})) \Bigg\}\check{x}_j^{*2} \Bigg\}, \\
\dot{V}(\check{x}(t), \check{u}(t)) \leq & \frac{-2}{\gamma\delta}\Bigg\{ \sum_{i=1}^{n} \Psi_i \check{u}_i^2 + \sum_{j=1}^{m} \Omega_j \check{x}_j^2 \Bigg\}.
\end{aligned}$$

Since $\gamma > 0$, $\delta > 0$, $\Psi_{1i} > 0$, and $\Omega_{1j} > 0$, $\forall i, j$, for all non-zero values of $\check{u}(t), \check{x}(t)$, $\dot{V}(\check{x}(t), \check{u}(t)) < 0$. Therefore, according to the theory of Lyapunov stability, the origin of (2.3) that satisfies (2.2) is GARS. □

The following theorem, which is obtained with the help of Lemmas 2.4 and 2.5, provides a different sufficient condition for GARS of the proposed system. Furthermore, this paper's numerical section discusses the effectiveness of the given results.

Theorem 3.2. *Assume the activation functions* χ_{1i}, χ_{2j} *fulfill conditions* $(A1)$, $(A2)$, *and there are positive constants* γ *and* δ, *such that the conditions below are satisfied:*

$$\Psi_{2i} = m\gamma\delta(2\underline{\check{a}}_i - \gamma - \delta) - n\check{l}_i^2(n\delta g^{\tau^*} + \gamma T^2(\mathcal{G})) > 0, \ \forall i = 1, 2, ..., n,$$
$$\Omega_{2j} = n\gamma\delta(2\underline{\check{b}}_j - \gamma - \delta) - m\check{h}_j^2(m\delta f^{\tau^*} + \gamma T^2(\mathcal{F})) > 0, \ \forall j = 1, 2, ..., m,$$

where $(f^{\tau^*}) = \sum_{j=1}^{m}(\check{f}_{ji}^{\tau^*})^2$, $(g^{\tau^*}) = \sum_{i=1}^{n}(\check{g}_{ij}^{\tau^*})^2$, $\check{f}_{ji}^{\tau^*} = max(|\ \underline{\check{f}}_{ji}^{\tau}\ |, |\ \overline{\check{f}}_{ji}^{\tau}\ |)$ *and* $\check{g}_{ij}^{\tau^*} = max(|\ \underline{\check{g}}_{ij}^{\tau}\ |, |\ \overline{\check{g}}_{ij}^{\tau}\ |)$. *Then the BAM NNs defined by* (2.3) *with network parameters that meet* (2.2) *have GARS at their origin.*

Proof. This proof will be shown through a two-step process. In Step 1, we show that its origin is the only equilibrium point of (2.3). On the flip side, we show that (2.3) has it origin in GARS.
Step 1. Assume that the equilibrium points of (2.3) are $(\check{u}_1^*, ..., \check{u}_n^*)^T = \check{u}^* \neq 0$ and $(\check{x}_1^*, ..., \check{x}_m^*)^T = \check{x}^* \neq 0$. The points that satisfy the equations stated below are the equilibrium points of (2.3).

$$\check{a}_i\check{u}_i^* + \sum_{j=i}^{m}\check{f}_{ji}\chi_{2j}(\check{x}_j^*) + \sum_{j=i}^{m}\check{f}_{ji}^{\tau}\chi_{2j}(\check{x}_j^*) = 0, \ \forall i, \tag{3.9}$$

$$\check{b}_j\check{x}_j^* + \sum_{i=i}^{n}\check{g}_{ij}\chi_{1j}(\check{u}_i^*) + \sum_{i=i}^{n}\check{g}_{ij}^{\tau}\chi_{1j}(\check{u}_i^*) = 0, \ \forall j. \tag{3.10}$$

Multiplying (3.1) by $2m\check{u}_i^*$ and (3.2) by $2n\check{x}_j^*$, then addition of the resulting equations,

$$\begin{aligned}0 =& -2m\check{a}_i\check{u}_i^{*2} + \sum_{i,j=1}^{n,m}2m\check{u}_i^*\check{f}_{ji}\chi_{2j}(\check{x}_j^*) + \sum_{j,i=1}^{m,n}2m\check{u}_i^*\check{f}_{ji}^{\tau}\chi_{2j}(\check{x}_j^*) - 2n\check{b}_j\check{x}_j^{*2} + 2n\sum_{j,i=1}^{m,n}\check{x}_j^*\check{g}_{ij}\chi_{1j}(\check{u}_i^*)\\ &+ 2n\sum_{j,i=1}^{m,n}\check{x}_j^*\check{g}_{ij}^{\tau}\chi_{1j}(\check{u}_i^*),\end{aligned}$$

$$\begin{aligned}0 =& -2m\check{a}_i\check{u}_i^{*2} + \sum_{i,j=1}^{n,m}2m\check{u}_i^*\check{f}_{ji}\chi_{2j}(\check{x}_j^*) + \sum_{i,j=1}^{n,m}2m\check{u}_i^*\check{f}_{ji}^{\tau}\chi_{2j}(\check{x}_j^*) - 2n\check{b}_j\check{x}_j^{*2} + 2n\sum_{j,i=1}^{m,n}\check{x}_j^*g_{ij}\chi_{1j}(\check{u}_i^*)\\ &+ 2n\sum_{j,i=1}^{m,n}\check{x}_j^*\check{g}_{ij}^{\tau}\chi_{1j}(\check{u}_i^*) + \frac{1}{\gamma}\sum_{i,j=1}^{n,m}m^2(\check{f}_{ji}^{\tau})^2\chi_{2j}^2(\check{x}_j^*) - \frac{1}{\gamma}\sum_{i,j=1}^{n,m}m^2(\check{f}_{ji}^{\tau})^2\chi_{2j}^2(\check{x}_j^*) + \frac{1}{\gamma}\sum_{j,i=1}^{m,n}n^2(\check{g}_{ij}^{\tau})^2\chi_{1j}^2(\check{u}_i^*)\end{aligned}$$

$$-\frac{1}{\gamma}\sum_{j,i=1}^{m,n} n^2(\breve{g}_{ij}^{\tau})^2\chi_{1j}^2(\breve{u}_i^*),$$

$$\leq -2m\breve{a}_i\breve{u}_i^{*2}+\sum_{i,j=1}^{n,m}2m\breve{u}_i^*\breve{f}_{ji}\chi_{2j}(\breve{x}_j^*)+\sum_{i,j=1}^{n,m}2m\breve{u}_i^*\breve{f}_{ji}^{\tau}\chi_{2j}(\breve{x}_j^*)-2n\breve{b}_j\breve{x}_j^{*2}+2n\sum_{j,i=1}^{m,n}\breve{x}_j^*\breve{g}_{ij}\chi_{1j}(\breve{u}_i^*)$$
$$+2n\sum_{j,i=1}^{m,n}\breve{x}_j^*g_{ij}^{\tau}\chi_{1j}(\breve{u}_i^*)+\frac{1}{\gamma}\sum_{i,j=1}^{n,m}m^2(\breve{f}_{ji}^{\tau})^2\breve{h}_j^2(\breve{x}_j^{*2})-\frac{1}{\gamma}\sum_{i,j=1}^{n,m}m^2(\breve{f}_{ji}^{\tau})^2\chi_{2j}^2(\breve{x}_j^*)+\frac{1}{\gamma}\sum_{j,i=1}^{m,n}n^2(\breve{g}_{ij}^{\tau})^2\breve{l}_i^2(\breve{u}_i^{*2})$$
$$-\frac{1}{\gamma}\sum_{j,i=1}^{m,n}n^2(\breve{g}_{ij}^{\tau})^2\chi_{1j}^2(\breve{u}_i^*). \tag{3.11}$$

Take into account the forthcoming inequalities:

$$\begin{aligned}\sum_{i,j=1}^{n,m}2m\breve{u}_i^*(t)\breve{f}_{ji}\chi_{2j}(\breve{x}_j^*) &= 2m\breve{u}^{*T}\mathcal{F}\mathcal{S}(\breve{x}^*)\\
&\leq m\delta\breve{u}^{*T}\breve{u}^*+m\frac{1}{\delta}\mathcal{S}^T(\breve{x}^*)\mathcal{F}^T\mathcal{F}\mathcal{S}(\breve{x}^*)\\
&\leq m\delta\breve{u}^{*T}\breve{u}^*+m\frac{1}{\delta}\parallel \mathcal{F}\parallel_2^2\parallel \mathcal{S}(\breve{x}^*)\parallel_2^2\\
&\leq m\delta\sum_{i=1}^{n}\breve{u}_i^{*2}+m\frac{1}{\delta}\parallel \mathcal{F}\parallel_2^2\sum_{j=1}^{m}\breve{h}_j^2\breve{x}_j^{*2},\end{aligned} \tag{3.12}$$

$$\begin{aligned}\sum_{j,i=1}^{m,n}2n\breve{x}_j^*\breve{g}_{ij}\chi_{1j}(\breve{u}_i^*) &=2n\breve{x}^{*T}\mathcal{G}\mathcal{S}(\breve{u}^*)\\
&\leq n\delta\breve{x}^{*T}\breve{x}^*+n\frac{1}{\delta}\mathcal{S}^T(\breve{u}^*)\mathcal{G}^T\mathcal{G}\mathcal{S}(\breve{u}^*)\\
&\leq n\delta\breve{x}^{*T}\breve{x}^*+n\frac{1}{\delta}\parallel \mathcal{S}\parallel_2^2\parallel \mathcal{G}(u^*)\parallel_2^2\\
&\leq n\delta\sum_{j=1}^{m}\breve{x}_j^{*2}+n\frac{1}{\delta}\parallel \mathcal{G}\parallel_2^2\sum_{i=1}^{n}\breve{l}_i^2\breve{u}_i^{*2},\end{aligned} \tag{3.13}$$

$$\begin{aligned}\sum_{i,j=1}^{n,m}2m\breve{u}_i^*\breve{f}_{ji}^{\tau}\chi_{2j}(\breve{x}_j^*) &\leq \sum_{i,j=1}^{n,m}\gamma\breve{u}_i^{*2}+\sum_{i,j=1}^{n,m}\frac{1}{\gamma}m^2(\breve{f}_{ji}^{\tau})^2\chi_{2j}^2(\breve{x}_j^*)\\
&= m\gamma\sum_{i=1}^{n}\breve{u}_i^{*2}+\sum_{i,j=1}^{n,m}\frac{1}{\gamma}m^2(\breve{f}_{ji}^{\tau})^2\chi_{2j}^2(\breve{x}_j^*),\end{aligned} \tag{3.14}$$

$$\sum_{j,i=1}^{m,n}2n\breve{x}_j^*\breve{g}_{ij}^{\tau}\chi_{2j}(\breve{u}_i^*) \leq \sum_{j,i=1}^{m,n}\gamma\breve{x}_j^*+\sum_{j,i=1}^{m,n}\frac{1}{\gamma}n^2(\breve{g}_{ij}^{\tau})^2\chi_{1i}^2(\breve{u}_i^*)$$

$$= n\gamma \sum_{j=1}^{m} \breve{x}_j^* + \sum_{j,i=1}^{m,n} \frac{1}{\gamma} n^2 (\breve{g}_{ij}^{\tau})^2 \chi_{1i}^2(\breve{u}_i^*). \tag{3.15}$$

By applying the results (3.12)–(3.15) in (3.11), we have

$$\begin{aligned} 0 \leq & - \sum_{i=1}^{n} 2m\breve{a}_i \breve{u}_i^{*2} + m\delta \sum_{i=1}^{n} \breve{u}_i^{*2} + m\frac{1}{\delta} \parallel \mathcal{F} \parallel_2^2 \sum_{j=1}^{m} \breve{h}_j^2 \breve{x}_j^{*2} - \sum_{j=1}^{m} 2n\breve{b}_j \breve{x}_j^{*2} + n\delta \sum_{j=1}^{m} \breve{x}_j^{*2} \\ & + n\frac{1}{\delta} \parallel \mathcal{G} \parallel_2^2 \sum_{i=1}^{n} \breve{l}_i^2 \breve{u}_i^{*2} + m\gamma \sum_{i=1}^{n} \breve{u}_i^{*2} + n\gamma \sum_{j=1}^{m} \breve{x}_j^{*2} + \frac{1}{\gamma} \sum_{i,j=1}^{n,m} m^2 (\breve{f}_{ji}^{\tau})^2 \breve{h}_j^2 (\breve{x}_j^{*2}) \\ & + \frac{1}{\gamma} \sum_{j,i=1}^{m,n} n^2 (\breve{g}_{ij}^{\tau})^2 \breve{l}_i^2 (\breve{u}_i^{*2}). \end{aligned}$$

Since,

$\parallel \mathcal{G} \parallel_2^2 \leq 2 \parallel\mid (\mathcal{G}^*)^T \mathcal{G}^* \mid + \mathcal{G}_*^T \mathcal{G}_* \parallel_2 = T^2(\mathcal{G})$, $\parallel \mathcal{F} \parallel_2^2 \leq T^2(\mathcal{F})$, $(\breve{f}_{ji}^{\tau})^2 \leq (f_{ji}^{\tau^*})^2$, $(\breve{g}_{ij}^{\tau})^2 \leq (g_{ij}^{\tau^*})^2$.

$$\begin{aligned} 0 \leq & \sum_{i=1}^{n} \Big\{ -2m\underline{\breve{a}}_i + m(\gamma+\delta) + \frac{1}{\delta} n \breve{l}_i^2 \Big(T^2(\mathcal{G}) + \frac{1}{\gamma} n^2 \breve{l}_i^2 \sum_{j=1}^{m} ((g_{ij}^{\tau^*})^2) \Big\} \breve{u}_i^{*2} \\ & + \sum_{j=1}^{m} \Big\{ -2n\underline{\breve{b}}_j + n(\gamma+\delta) + \frac{1}{\gamma} m^2 \breve{h}_j^2 \sum_{i=1}^{n} ((f_{ji}^{\tau^*})^2) + \frac{1}{\delta} m \breve{h}_j^2 \Big(T^2(\mathcal{F}) \Big\} \breve{x}_j^{*2}, \end{aligned}$$

$$\begin{aligned} 0 \leq & - \sum_{i=1}^{n} \Big\{ m\gamma\delta(2\underline{\breve{a}}_i - \gamma - \delta) - n\breve{l}_i^2 (n\delta g^{\tau^*} + \gamma T^2(\mathcal{G})) \Big\} \breve{u}_i^{*2} \\ & - \sum_{j=1}^{m} \Big\{ n\gamma\delta(2\underline{\breve{b}}_j - \gamma - \delta) - m\breve{h}_j^2 (m\delta f^{\tau^*} + \gamma T^2(\mathcal{F})) \Big\} \breve{x}_j^{*2}, \\ 0 \leq & - \sum_{i=1}^{n} \Psi_{2i} \breve{u}_i^2 - \sum_{j=1}^{m} \Omega_{2j} \breve{x}_j^2. \end{aligned} \tag{3.16}$$

Given that $\Psi_{2i} > 0$ and $\Omega_{2j} > 0$, $\forall i, j$, $\breve{x}^* \neq 0 \neq \breve{u}^*$. But $-\sum_{i=1}^{n} \Psi_{2i}\breve{u}_i^{*2} - \sum_{j=1}^{m} \Omega_{2j}\breve{x}_j^{*2} < 0$. Here (3.16) contradicts the above result, and thus, we can deduce that the only equilibrium point is $\breve{x}^* = 0 = \breve{u}^*$. Therefore, the unique equilibrium point is the origin of (2.3).

Step 2. Let us examine the LKF provided below:

$$\begin{aligned} V(\breve{x}(t), \breve{u}(t)) = & \sum_{i=1}^{n} m\breve{u}_i^2(t) + \sum_{j=1}^{m} n\breve{x}_j^2(t) + \frac{1}{\gamma} \sum_{i,j=1}^{n,m} m^2 (\breve{f}_{ji}^{\tau})^2 \int_{t-\breve{\tau}_{ji}}^{t} \chi_{2j}^2(\breve{x}_j(\eta)) d\eta \\ & + \frac{1}{\gamma} \sum_{j,i=1}^{m,n} n^2 (\breve{g}_{ji}^{\tau})^2 \int_{t-\breve{\sigma}_{ij}}^{t} \chi_{1i}^2 \breve{u}_i(\xi) d\xi. \end{aligned}$$

Obtaining $\dot{V}(\check{x}(t), \check{u}(t))$ in the trajectories of (2.3) and using Lemma 2.5 yields the following result:

$$\begin{aligned}\dot{V}(\check{x}(t), \check{u}(t)) \leq & m\delta \sum_{i=1}^{n} \check{u}_i^2(t) - \sum_{i=1}^{n} 2m\check{a}_i\check{u}_i^2(t) + n\delta \sum_{j=1}^{m} x_j^2(t) + m\frac{1}{\delta} \parallel \mathcal{F} \parallel_2^2 \sum_{j=1}^{m} \check{h}_j^2\check{x}_j^2(t) \\ & + n\frac{1}{\delta} \parallel \mathcal{G} \parallel_2^2 \sum_{i=1}^{n} \check{l}_i^2\check{u}_i^2(t) - \sum_{j=1}^{m} 2n\check{b}_j\check{x}_j^2(t) + m\gamma \sum_{i=1}^{n} \check{u}_i^2(t) + n\gamma \sum_{j=1}^{m} \check{x}_j^2(t) \\ & + \frac{1}{\gamma} \sum_{j=1}^{m}\sum_{i=1}^{n} n^2(\check{f}_{ji}^{\tau})^2\check{l}_i^2\check{u}_i^2(t) + \frac{1}{\gamma}\sum_{i=1}^{n}\sum_{j=1}^{m} m^2(\check{g}_{ij}^{\tau})^2\check{h}_j^2\check{x}_j^2(t).\end{aligned}$$

Since $\parallel \mathcal{G} \parallel_2^2 \leq T^2(\mathcal{G})$, $\parallel \mathcal{F} \parallel_2^2 \leq T^2(\mathcal{F})$, $(\check{f}_{ji}^{\tau})^2 \leq (f_{ji}^{\tau^*})^2$ and $(\check{g}_{ij}^{\tau})^2 \leq (g_{ij}^{\tau^*})^2$.

$$\begin{aligned}\dot{V}(\check{x}(t), \check{u}(t)) \leq & \sum_{i=1}^{n}\Big\{-2m\underline{\check{a}}_i + m(\gamma+\delta) + \frac{1}{\delta}n\check{l}_i^2\big(T^2(\mathcal{G})\big) + \frac{1}{\gamma}n^2\check{l}_i^2\sum_{j=1}^{m}((g_{ij}^{\tau^*})^2)\Big\}\check{u}_i^{*2} \\ & + \sum_{j=1}^{m}\Big\{-2n\underline{\check{b}}_j + n(\gamma+\delta) + \frac{1}{\gamma}m^2\check{h}_j^2\sum_{i=1}^{n}((f_{ji}^{\tau^*})^2) + \frac{1}{\delta}m\check{h}_j^2\big(T^2(\mathcal{G})\big)\Big\}\check{x}_j^{*2}, \\ & \leq -\sum_{i=1}^{n}\Big\{m\gamma\delta(2\underline{\check{a}}_i - \gamma - \delta) - n\check{l}_i^2(n\delta g^{\tau^*} + \gamma T^2(\mathcal{G}))\Big\}\check{u}_i^{*2} \\ & -\sum_{j=1}^{m}\Big\{n\gamma\delta(2\underline{\check{b}}_j - \gamma - \delta) - m\check{h}_j^2(m\delta f^{\tau^*} + \gamma T^2(\mathcal{F}))\Big\}\check{x}_j^{*2}, \\ & = -\sum_{i=1}^{n}\Psi_{2i}\check{u}_i^2 - \sum_{j=1}^{m}\Omega_{2j}\check{x}_j^2.\end{aligned}$$

Given that $\Psi_{2i} > 0$ and $\Omega_{2j} > 0$, $\forall i, j$, for every non-zero values of $\check{u}(t), \check{x}(t)$, $\dot{V}(\check{x}(t), \check{u}(t)) < 0$. Therefore, according to the theory of Lyapunov stability, the origin of (2.3) that satisfies (2.2) is GARS. The proof is completed. □

4. Numerical examples

In this part, we demonstrate the contrast in outcomes of Theorems 3.1 and 3.2 through the following instances.

Example 4.1. Take into account the network parameters for the specified BAM NNs (2.1) that adhere to (2.2).

$$l_1 = l_2 = l_3 = \frac{1}{2},\ h_1 = h_2 = h_3 = \frac{1}{2},\ \gamma = \frac{1}{6},\ \delta = \frac{1}{6},$$

$$\underline{\mathcal{A}} = \mathcal{A} = \overline{\mathcal{A}} = \begin{bmatrix} 13 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & 13 \end{bmatrix} = \underline{\mathcal{B}} = \mathcal{B} = \overline{\mathcal{B}},\ \underline{\mathcal{G}} = \underline{\mathcal{F}} = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix},\ \overline{\mathcal{G}} = \overline{\mathcal{F}} = \frac{1}{2}\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 0 & 0 & 4 \end{bmatrix},$$

$$\mathcal{G}^* = \mathcal{F}^* = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & 0 & 2 \end{bmatrix}, \ \mathcal{G}_* = \mathcal{F}_* = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}, \ \underline{\mathcal{G}}^\tau = \begin{bmatrix} d_1 & d_1 & d_1 \\ d_1 & d_1 & d_1 \\ d_1 & d_1 & d_1 \end{bmatrix}, \ \overline{\mathcal{G}}^\tau = \frac{1}{8}\begin{bmatrix} d_1 & d_1 & d_1 \\ d_1 & d_1 & d_1 \\ d_1 & d_1 & d_1 \end{bmatrix} = \mathcal{G}^{\tau^*},$$

$$\underline{\mathcal{F}}^\tau = \frac{-1}{8}\begin{bmatrix} d_2 & d_2 & d_2 \\ d_2 & d_2 & d_2 \\ d_2 & d_2 & d_2 \end{bmatrix}, \ \overline{\mathcal{F}}^\tau = \frac{1}{8}\begin{bmatrix} d_2 & d_2 & d_2 \\ d_2 & d_2 & d_2 \\ d_2 & d_2 & d_2 \end{bmatrix} = \mathcal{F}^{\tau^*},$$

where $d_1 > 0, \ d_2 > 0$. We identify the norm in Lemma 2.4 in the following manner.

$$T^2(\mathcal{G}) = 2 \ ||| \ (\mathcal{G}^*)^T\mathcal{G}^* \ | + \mathcal{G}_*^T\mathcal{G}_* \ ||_2 = 8.1883 = T^2(\mathcal{F}) = 2 \ ||| \ (\mathcal{F}^*)^T\mathcal{F}^* \ | + \mathcal{F}_*^T\mathcal{F}_* \ ||_2 \ .$$

We exhibit the results of Theorem 3.1 for the upper bound T^2, we obtain

$$\Psi_{1i} = 1.0833 - 0.5118 - 0.0404d_2^2 = 0.5715 - 0.0404d_2^2.$$

Since $\Psi_{1i} > 0, \forall i = 1, 2, 3$. Therefore, $d_2^2 < 14.1460$.

$$\Omega_{1j} = 1.0833 - 0.5118 - 0.0404d_1^2 = 0.5715 - 0.0404d_1^2.$$

Since $\Omega_{1j} > 0, \forall j = 1, 2, 3$. Therefore, $d_1^2 < 14.1460$. Similarly, we exhibit the results of Theorem 3.2 for the upper bound T^2, we obtain

$$\Psi_{2i} = 2.1389 - 1.0236 - 0.0176d_1^2 = 1.1153 - 0.0176d_1^2.$$

Since $\Psi_{2i} > 0, \forall \ i = 1, 2, 3$. Therefore, $d_1^2 < 63.3693$.

$$\Omega_{2j} = 2.1389 - 1.0236 - 0.0176d_2^2 = 1.1153 - 0.0176d_2^2.$$

Since $\Omega_{2j} > 0, \forall \ j = 1, 2, 3$. Therefore, $d_2^2 < 63.3693$.

For this example, the Matlab simulation results from non-linear activation functions under the initial conditions $\check{x}(0) = [-0.5, 0.5, -0.2]^T$ and $\check{u}(0) = [0.3, -0.3, 0.1]^T$. The activation function used is a piecewise linear functions $f(\check{x}) = 0.5 \times (|\check{x} + 1| - |\check{x} - 1|)$ and $g(\check{u}) = 0.5 \times (|\check{u} + 1| - |\check{u} - 1|)$, which bounds neuron activations, ensuring stability. The state response graph shows the evolution of neuron activations over time for each layer ($\check{x}$ and $\check{u}$), influenced by time-delayed interactions. The responses converge smoothly to zero due to the damping effect in Figure 1.

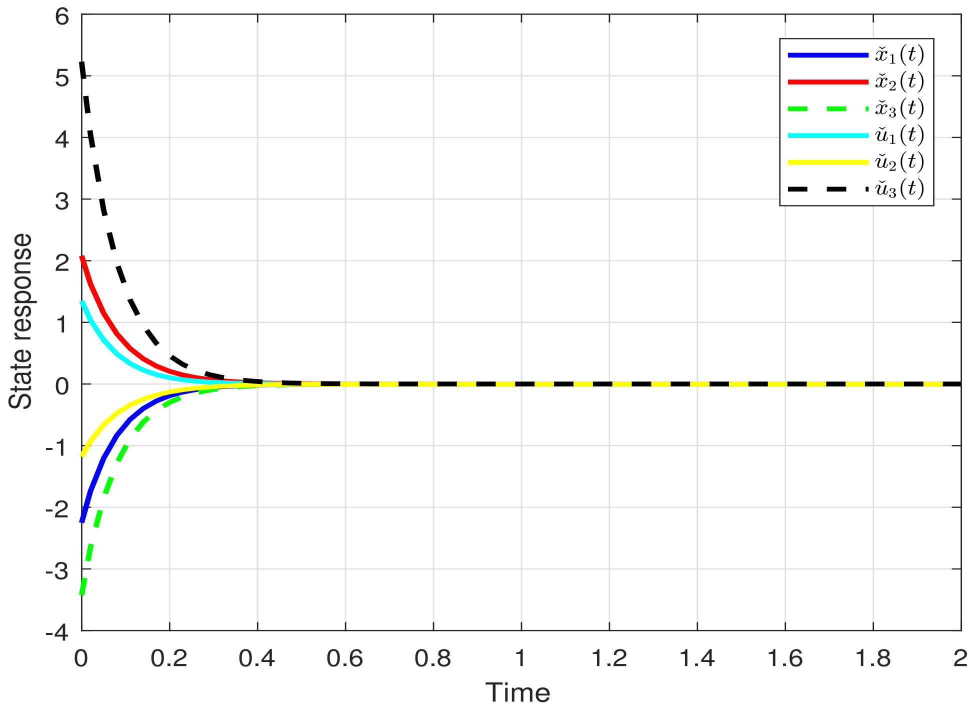

Figure 1. Response of $\check{x}$, $\check{u}$ among the different initial states.

Remark 4.1. *For Ψ_{2i} and $\Omega_{2j}, \forall i, j = 1,2,3$, d_1^2 and d_2^2 respectively, are valid in the domain,* $12.5869 < d_q^2 < 49.6193$, $q = 1,2$ *whereas Ψ_{1i} and $\Omega_{1j}, \forall\ i, j = 1,2,3$ are not valid in that domain. This is because of the new upper bound value T^2 norm and the sufficient conditions in 3.1 and 3.2. Hence, our new sufficient conditions in 3.2 will give better results when comparing 3.1 with the proposed BAM NNs.*

Remark 4.2. *In this example, we apply the existing norm in the literature as stated in Lemma 2.6, then* $T^2(\mathcal{G}) = (\| \mathcal{G}^* \|_2 + \| \mathcal{G}_* \|_2)^2 = (\| \mathcal{F}^* \|_2 + \| \mathcal{F}_* \|_2)^2 = T^2(\mathcal{F}) = 8.3284 > 8.1883 = 2 \|\| (\mathcal{G}^*)^T \mathcal{G}^* \mid + \mathcal{G}_*^T \mathcal{G}_* \|_2 = 2 \|\| (\mathcal{F}^*)^T \mathcal{F}^* \mid + \mathcal{F}_*^T \mathcal{F}_* \|_2$. *Therefore, the proposed results will give better domain of region as compared with their existing results. Hence, our results are efficient when compared with the existing results for this network parameters.*

Example 4.2. Take into account the network parameters for the specified BAM NNs (2.1) that adhere to (2.2).

$$l_1 = l_2 = l_3 = l_4 = \frac{1}{4},\ h_1 = h_2 = h_3 = h_4 = \frac{1}{4},\ \gamma = \frac{1}{5},\ \delta = \frac{1}{5},$$

$$\underline{\mathcal{A}} = \mathcal{A} = \overline{\mathcal{A}} = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix} = \underline{\mathcal{B}} = \mathcal{B} = \overline{\mathcal{B}},\ \underline{\mathcal{G}} = \underline{\mathcal{F}} = \frac{1}{25}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -6 & 0 & 0 & 0 \end{bmatrix},$$

$$\overline{\mathcal{G}} = \overline{\mathcal{F}} = \frac{1}{25}\begin{bmatrix} 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 \\ 0 & 0 & 0 & 12 \end{bmatrix}, \quad \mathcal{G}^* = \mathcal{F}^* = \frac{1}{25}\begin{bmatrix} 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \\ -3 & 0 & 0 & 6 \end{bmatrix},$$

$$\mathcal{G}_* = \mathcal{F}_* = \frac{1}{25}\begin{bmatrix} 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \\ 3 & 0 & 0 & 6 \end{bmatrix}, \quad \underline{\mathcal{G}}^{\tau} = \frac{-1}{5}\begin{bmatrix} e_1 & e_1 & e_1 & e_1 \\ e_1 & e_1 & e_1 & e_1 \\ e_1 & e_1 & e_1 & e_1 \\ e_1 & e_1 & e_1 & e_1 \end{bmatrix}, \quad \overline{\mathcal{G}}^{\tau} = \frac{1}{5}\begin{bmatrix} e_1 & e_1 & e_1 & e_1 \\ e_1 & e_1 & e_1 & e_1 \\ e_1 & e_1 & e_1 & e_1 \\ e_1 & e_1 & e_1 & e_1 \end{bmatrix},$$

$$\underline{\mathcal{F}}^{\tau} = \frac{-1}{5}\begin{bmatrix} e_2 & e_2 & e_2 & e_2 \\ e_2 & e_2 & e_2 & e_2 \\ e_2 & e_2 & e_2 & e_2 \\ e_2 & e_2 & e_2 & e_2 \end{bmatrix}, \overline{\mathcal{F}}^{\tau} = \frac{1}{5}\begin{bmatrix} e_2 & e_2 & e_2 & e_2 \\ e_2 & e_2 & e_2 & e_2 \\ e_2 & e_2 & e_2 & e_2 \\ e_2 & e_2 & e_2 & e_2 \end{bmatrix}.$$

where $e_1 > 0,\ e_2 > 0$. We identify the norm in Lemma 2.4 in the following manner.

$$T^2(\mathcal{G}) = T^2(\mathcal{F}) = 0.7778.$$

We exhibit the results of Theorem 3.1 for the upper bound T^2, we obtain

$$\Psi_{1i} = 1.6 - 0.0356 - 0.256e_2^2 = 1.5644 - 0.256e_2^2.$$

Since $\Psi_{1i} > 0, \forall i = 1, 2, 3, 4$. Therefore, $e_2^2 < 6.2576$.

$$\Omega_{1j} = 1.6 - 0.0356 - 0.256e_1^2 = 1.5644 - 0.256e_1^2.$$

Since $\Omega_{1j} > 0, \forall j = 1, 2, 3, 4$. Therefore, $e_1^2 < 6.2576$. Similarly, we exhibit the results of Theorem 3.2 for the upper bounds T^2, we obtain

$$\Psi_{2i} = 3.1360 - 0.0389 - 0.0320e_1^2 = 3.0971 - 0.0320e_1^2.$$

Since $\Psi_{2i} > 0, \forall\ i = 1, 2, 3, 4$. Therefore, $e_1^2 < 96.7844$.

$$\Omega_{2j} = 3.1360 - 0.0389 - 0.0320e_2^2 = 3.0971 - 0.0320e_2^2.$$

Since $\Omega_{2j} > 0, \forall\ j = 1, 2, 3, 4$. Therefore, $e_2^2 < 96.7844$.

For this example, the Matlab simulation results from non-linear activation functions under the initial conditions $\check{x}(0) = [-0.5, 0.5, -0.3, 0.4]^T$ and $\check{u}(0) = [0.3, -0.3, 0.2, -0.4]^T$. The activation function used is a piecewise linear function $f(\check{x}) = 0.5 \times (|\check{x} + 1| - |\check{x} - 1|)$ and $g(\check{u}) = 0.5 \times (|\check{u} + 1| - |\check{u} - 1|)$, which bounds neuron activations, ensuring stability. The state response graph shows the evolution of neuron activations over time for each layer ($\check{x}$ and $\check{u}$), influenced by time-delayed interactions. The responses converge smoothly to zero due to the damping effect in Figure 2.

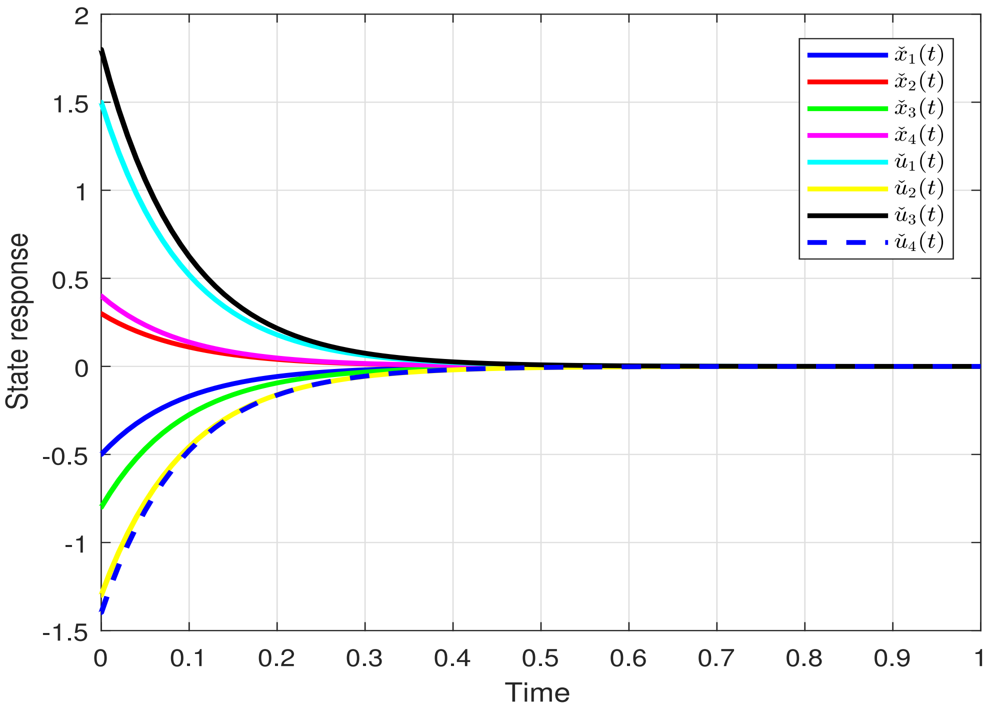

Figure 2. Response of $\breve{x}$, $\breve{u}$ among the different initial states.

Remark 4.3. *For Ψ_{2i} and $\Omega_{2j}, \forall i, j = 1, 2, 3, 4$, e_1^2 and e_2^2, respectively, are valid in the domain, $6.1035 < e_q^2 < 96.7186$, $q = 1, 2$ whereas Ψ_{1i} and $\Omega_{1j}, \forall\ i, j = 1, 2, 3, 4$ are not valid in that domain. This is because of the new upper bound value T^2 norm and the sufficient conditions in 3.1 and 3.2. Hence, our new sufficient conditions in 3.2 will give better results for the proposed BAM NNs. Moreover, Theorem 3.2 provides better results and is more robust compared to Theorem 3.1. It offers a significantly larger stability region $e_1^2, e_2^2 < 96.7186$, while Theorem 3.1 is limited to $e_1^2, e_2^2 < 6.1035$.*

Remark 4.4. *In this example, we apply the existing norm as stated in Lemma 2.6, then $T^2(\mathcal{G}) = (\| \mathcal{G}^* \|_2 + \| \mathcal{G}_* \|_2)^2 = (\| \mathcal{F}^* \|_2 + \| \mathcal{F}_* \|_2)^2 = T^2(\mathcal{F}) = 0.7811 > 0.7778 = 2 \||\ (\mathcal{G}^*)^T \mathcal{G}^* \mid + \mathcal{G}_*^T \mathcal{G}_* \|_2 = 2 \||\ (\mathcal{F}^*)^T \mathcal{F}^* \mid + \mathcal{F}_*^T \mathcal{F}_* \|_2$. Therefore, the proposed results will give better domain of region as compared with the existing results. Hence, our results are efficient when compared with the existing results for these network parameters.*

Remark 4.5. *The constant multiple time delays are employed in a range of real-time applications, such as network communication, and control systems, particularly in industrial processes, telecommunication operating systems, signal processing, and also gravitational lensing in astrophysics. The novel global asymptotic stability and dissipativity criteria of BAM NNs with delays have been discussed in [34]. The fractional-order uncertain BAM NNs with mixed time delays are discussed in [35]. Some new criteria on the stability of fractional-order BAM NNs with time delay is discussed in [36]. The concept of global the exponential stability via inequality technique for inertial BAM NNs with time delays is discussed in [37]. Also, the proposed results can be applied to the time-varying delay parameters of BAM NNs. Since we can find multiple constant time delays as the upper* *bounds for the time-varying delay parameters. The proposed results can be utilized for the GARS of time-varying delayed BAM NNs, and it will give better results for these network parameters.*

5. Conclusions

This research looks into the GARS of hybrid BAM NNs that deal with time delays and unknown parameters in great detail. We have defined the relevant conditions to ensure GARS in these hybrid BAM NNs. Also, the results for the GARS of these BAM NNs are given in 3.1 and 3.2. The proposed research work confers the following advantages: (1) The results stated in both 3.1 and 3.2 are different sufficient conditions. (2) From the numerical examples, it is clear that the results in 3.2 are valid in the larger domain, while the sufficient conditions in 3.1 are enforceable only in the smaller domain. (3) The domain of validity for the results in 3.2 is much larger compared to the larger network parameters given in 4.2. The new results stated in 3.1 and 3.2 are derived using the new upper bound and the suitable LKF. Our research has shown greater efficacy compared to some earlier findings. The two numerical instances demonstrate how our novel conditions yield effective outcomes. This suggested research could be expanded in further studies to incorporate complex-valued diffusion BAM NNs and the impulsive fractional-order quaternion-valued BAM NNs with real-time applications.

Author contributions

N. Mohamed Thoiyab: Writing-review & editing, writing-original draft, conceptualization; Mostafa Fazly: Writing-review & editing, visualization, investigation, funding acquisition; R. Vadivel: Writing-review & editing, methodology, validation; Nallappan Gunasekaran: Methodology, formal analysis, software, supervision. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors claim that they have no conflicts of interest in publishing this work.

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