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Research article

Fixed-time consensus of second-order multi-agent systems based on event-triggered mechanism under DoS attacks

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Abstract: This paper investigates fixed-time consensus (FXTC) for second-order nonlinear multi-agent systems under denial of service (DoS) attacks using event-triggered control. First, consensus in second-order nonlinear multi-agent systems with directed topologies is studied under a static event-triggered mechanism. Building upon this, dynamic auxiliary variables are introduced, and a dynamic event-triggered mechanism is designed. Consensus control protocols are proposed for both leader-follower and leaderless scenarios. Using Lyapunov stability theory and algebraic graph theory, the fixed-time consensus of multi-agent systems with directed topologies under DoS attacks is analyzed. Furthermore, Zeno behavior is excluded. Finally, numerical examples are presented to validate the theoretical results.

Keywords: fixed-time consensus; multi-agent systems; second-order nonlinear dynamics; dynamic event-triggered mechanism; DoS attack
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1. Introduction

Cooperative control of multi-agent systems is a cutting-edge topic in control theory, that has attracted the attention of researchers across various fields. It has broad applications, including information control [1], clustering science [2], distributed sensor networks [3], and more. Meanwhile, synergy control holds significant potential for application prospects in engineering and technology fields, such as collaborative robot control [4] and aircraft formation control [5]. The literature [6] explores the relationship between networked dynamic systems and consensus problems across various applications, presenting simulation results that demonstrate the impact of the small-world effect on the convergence speed of consensus algorithms and on the cooperative control of multi-vehicle formations.

In collaborative control, each system requires coordination through interactions among multiple agents, and these interactions must be synchronized to achieve consensus.

The consensus problem is a fundamental challenge in cooperative control, with significant progress made in the study of multi-agent system consensus [7–9]. Early works, the literature [10] introduced a continuous control strategy that enables continuous information exchange. However, in practical applications, information transfer is often disrupted by various issues, and the continuous operation of multiple agents leads to frequent controller updates, which can result in resource exhaustion. To address this challenge, researchers have incorporated an event-triggered mechanism into periodic control. Specifically, reference [11] explored an event-triggered mechanism where the controller updates its state only when the sampled measurement meets a predefined condition, remaining inactive otherwise. Furthermore, reference [12] explored the consensus of multi-agent systems represented by undirected graphs, employing an event-triggered mechanism in conjunction with an apt control protocol. Notably, the static event-triggered mechanism used in these studies relies on a fixed threshold, which may not adequately respond to sudden environmental changes. Antoine's groundbreaking work revolutionized the field by integrating dynamic auxiliary variables into the event-triggered mechanism, leading to the development of the dynamic event-triggered mechanism [13]. This pivotal innovation significantly reduced the frequency of triggers, sparking a wave of subsequent research, as demonstrated by numerous studies [14–16]. To reduce communication frequency and minimize reliance on global information, literature [17] employed an adaptive dynamic event-triggered mechanism to investigate the consensus problem in multi-agent systems with general linear dynamics. Meanwhile, to further reduce communication, [18] developed an observer-based dynamic event-triggered (DET) mechanism that incorporating discontinuous nonlinear terms, specifically designed to address the event-triggered mechanism consensus problem in multi-agent systems (MASs) with switched topologies. Compared to the results in [11, 12], this paper argues that the directed topology graph under the dynamic event-triggered mechanism achieves faster stabilized and reduces resource wastage. It is worth noting, however, that while the event-triggered mechanism effectively minimizes the controller's update frequency, it does not explicitly specify the time needed to achieve consensus. This aspect remains an area of ongoing research and optimization efforts.

In real-world applications, systems that aim to achieve consensus within a specified time frame often prefer finite-time strategies due to their faster consensus rates, enabling quicker attainment of consensus [19, 20]. Nevertheless, the settling time in finite-time mechanisms is often dependent on the initial conditions. Recognizing this limitation, the stability of fixed-time strategies was first explored in [21], where it was show that the settling time under fixed-time control is independent of initial conditions, ensuring stabilization within a more precise timeframe. As a result, extensive research has been conducted on fixed-time control [22–24]. Compared to the results in [19, 20], this paper studies fixed-time consensus in multi-agent systems, which achieves faster convergence than finite-time consensus. For multi-agent systems, network security is crucial. With the rapid advancements of network technology, attacks, such as denial of service (DoS) attacks, spoofing and replay attacks, have become increasingly prevalent. Among these, DoS attacks present a significant challenge due to their ease of execution, difficulty in prevention, and anonymity, making them one of the most common and hard to address network threats [25, 26]. Consequently, DoS attacks have garnered significant attention from researchers. To mitigate their impact, some scholars have proposed dynamic event-triggered mechanisms, which, due to their low triggering frequency, effectively reduce vulnerability to such attacks [27–29]. In [30], the consensus of nonlinear multi-agent systems with switching topologies under DoS attacks was investigated. In [31], the observer-based event-triggered mechanism containment control problem for linear multi-agent systems (MASs) under denial of service attacks is studied. In [32], the secure event-triggered mechanism control problem for multi-agent systems (MASs) compromised by DoS attacks with multiple modes is investigated. Additionally, reference [33] delved into the fixed-time average consensus (FxTAC) of nonlinear multi-agent systems (MAS) under DoS attacks, proposing less conservative fixed-time criteria to address these challenges.

Building on the aforementioned findings, this paper investigates the fixed-time consensus of second-order multi-agent systems under DoS attacks, utilizing dynamic event-triggered mechanisms. The innovations of this paper, compared to existing research, are as follows:

(1) Directed Networks in Practical Applications: Compared to [25, 26, 29], this paper studies the consensus of multi-agent systems under directed connectivity topology, while also considering the impact of external factors during information transmission that may prevent the system from receiving information. This highlights the need to address multi-agent systems with directed networks under denial of service (DoS) attacks.

(2) Event-triggered Mechanism and Fixed-time Consensus: Inspired by [22], an event-triggered mechanism is designed using the hyperbolic tangent function to avoid the discontinuity in the symbolic function. The fixed-time consensus is studied, which does not depend on the initial velocity and achieves faster convergence. Compared to [22], a denial of service (DoS) attack is introduced in the second-order multi-agent system, and a dynamic event-triggered mechanism is employed to reduce the frequency of control protocol updates, saving resources and minimizing unnecessary waste.

(3) DoS Attacks in Fixed-time Systems: Inspired by [33], this paper explores the introduction of DoS attacks into fixed-time systems, which offers broader applications than [22, 33]. It investigates the fixed-time consensus of event-triggered mechanism second-order multi-agent systems under DoS attacks.

Notations. The following notations are used throughout this paper. Let $\mathbb{R}$ denote the set of real number set; $\mathbb{R}^n$ stands for the n-dimensional Euclidean space, and $\mathbb{R}^{N\times N}$ represents the $N \times N$ dimensional real matrices. Let A^T represent the transpose of matrix A. I_n is n-order identity matrix, $0_n \in \mathbb{R}^{N\times N}$is a vector with all the entries being 0, diag$\{\ldots\}$ represents a diagonal matrix, 1_N represents $(1, 1, \ldots, 1)^T$. $\lambda_{\min}(M)$ represents minimum eigenvalues of a symmetric matrix M. The norm of a vector x is defined as $\| x \|= \sqrt{x^T x}$ and the symbol $\otimes$ represents the Kronecker product.

2. Preliminaries

2.1. Graph theory

Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ be a directed graph where the set of nodes is $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, $\{v_i\}$ the set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ with non-negative adjacency elements a_{ij}. Let $N_i = \{v_j \subseteq V \mid (v_j, v_i) \in E\}$ represent the set of neighbors of vertex i. Define the matrix of the Laplacian matrix L as $L = [l_{ij}] \in \mathbb{R}^{N\times N}$, its element l_{ij} satisfies the following definition:

$$l_{ij} = \begin{cases} -a_{ij}, i \neq j, \\ \sum\limits_{j=1}^{N} a_{ij}, i = j. \end{cases}$$

A directed path from node v_i to v_j is a finite-ordered sequence of edges $(v_i, v_{k_1}), (v_{k_1}, v_{k_2}), \ldots, (v_{k_i}, v_j)$, with distinct nodes v_{k_m}, $m = 1, 2 \ldots, l$. In a directed graph, if there is a root node that can follow the edge to all other nodes, it is called the directed spanning tree of the graph. Denote D=diag$\{d_1, d_2, \ldots, d_N\}$ by the degree diagonal matrix, where $d_i = \sum_{j=1}^{N} a_{ij}$ for $i = 1, \ldots, N$. Adding a leader to the above multi-agent system, denoted by $\hat{D}$=diag$\{\hat{d}_1, \hat{d}_2, \ldots, \hat{d}_N\}$ the communication relationship between each follower i and the leader, where $d_i = 1$ if agent i can use the leader's information and $d_i = 0$, otherwise, the matrix $H = L + \hat{D}$.

2.2. Problem formulation

Consider a multi-agent system consisting of $N + 1$ agents, where one is the leader and the remaining N are followers. The leader is labeled as given as 0. The dynamics of the i-th follower are described by

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = f(x_i(t), v_i(t), t) + u_i(t), \end{cases} \tag{2.1}$$

where $x_i(t) \in \mathbb{R}^N, v_i(t) \in \mathbb{R}^N$, and $u_i(t) \in \mathbb{R}^N$ represent the position state, velocity state, and control input of the i-th agent, respectively. The function $f : \mathbb{R}^N \times \mathbb{R}^N \times \mathbb{R}^+ \longrightarrow \mathbb{R}^N$ describes a nonlinear continuous function governing the dynamics of the agent i. Furthermore, the leader's dynamics are governed by

$$\begin{cases} \dot{x}_0(t) = v_0(t), \\ \dot{v}_0(t) = f(x_0(t), v_0(t), t), \end{cases} \tag{2.2}$$

where $x_0(t) \in \mathbb{R}^N$ and $v_0(t) \in \mathbb{R}^N$ represent the position state and velocity state of the leader, respectively.

Lemma 2.1. *(see [34]) Let $c_1, c_2, \ldots, c_N$. Then*

$$\sum_{i=1}^{N} c_i^{\sigma} \geq \left(\sum_{i=1}^{N} c_i \right)^{\sigma}, \textit{if } 0 < \sigma < 1, \sum_{i=1}^{N} c_i^{\omega} \geq N^{1-\omega} \left(\sum_{i=1}^{N} c_i \right)^{\omega}, 1 < \omega < \infty. \tag{2.3}$$

Lemma 2.2. *(see [35]) For any $\varphi \in \mathbb{R}$, we have*

$$0 \leq |\varphi| - \varphi \tanh(\mu\varphi) \leq \frac{\iota}{\mu}, \tag{2.4}$$

where $\mu \gg 1$ and ι=0.2785.

Lemma 2.3. *(see [36]) For matrices Z_1, Z_2, Z_3, and Z_4, the Kronecker product is denoted by the $\otimes$ symbol. The following properties hold:*

$$\begin{aligned} (Z_1 \otimes Z_2)^T &= Z_1^T \otimes Z_2^T, \\ (Z_1 + Z_2) \otimes Z_3 &= Z_1 \otimes Z_3 + Z_2 \otimes Z_3, \\ (Z_1 \otimes Z_2)(Z_3 \otimes Z_4) &= (Z_1 Z_3) \otimes (Z_2 Z_4). \end{aligned}$$

Assumption 2.1. *The communication topology of the system, denoted by $\mathcal{G}$, is strongly connected.*

To facilitate a better analysis of DoS attacks, the following standard assumptions are made:

Assumption 2.2. *(see [33]) There exists $N_d(T_2, T_1) \leq n_0 + \varsigma(T_2 - T_1)$, where $n_0 > 0, \varsigma \in (0, 1), N_d(T_2, T_1)$ is the number of DoS attacks in $[T_1, T_2)$.*

Assumption 2.3. *(see [33]) There exists $\Xi_d(T_2, T_1) \leq n_1 + \Xi_a N_d(T_2, T_1)$, where $n_1 > 0, \Xi_a$ is the duration of each attack, and $\Xi_d(T_2, T_1)$ is the total attack duration in $[T_1, T_2)$. Let $\Xi_c(T_1, T_2)$ be the total control duration in $[T_1, T_2)$, so $\Xi_c(T_2, T_1) = \frac{\Xi_d(T_2, T_1)}{[T_1, T_2]}$, and*

$$\Xi_c(T_2, T_1) \geq \epsilon(T_2 - T_1) - T_\epsilon, \forall T_2 > T_1 \geq 0, \tag{2.5}$$

where $\epsilon = 1 - \varsigma\Xi_a$, $T_\epsilon = n_1 + \Xi_a n_0$, T_ϵ is called the elasticity number. Additionally, ϵ can be interpreted as the non-attack rate.

Lemma 2.4. *(see [33]) For the system (2.1), if there exists a Lyapunov function V(t) such that*

$$\begin{cases} \dot{V}(t) \leq b_3 V(t) - b_1 V(t)^\sigma - b_2 V(t)^\varrho, t \in [T_m, S_m), \\ \dot{V}(t) \leq b_4 V(t), t \in [S_m, T_{m+1}), \end{cases} \tag{2.6}$$

where $\varrho \in (1, \infty), \sigma \in (0, 1), b_k > 0$ $(k = 1, 2, 3, 4)$ satisfying $b_3 + \omega(1 - \epsilon) < \min\{b_1, b_2 \exp((1 - \varrho)\omega T_\epsilon)\}, b_4 - \omega\theta_1 < 0$ and $\omega > 0$. Then the above system is fixed-time stable, and the settling time T^* satisfies

$$T^* \leq \frac{1 + \left(\hat{b}_3 - \hat{b}_2\right)(1 - \varrho)T_\epsilon}{\left(\hat{b}_3 - \hat{b}_2\right)(1 - \varrho)\epsilon} + \frac{-1 + \left(\hat{b}_3 - b_1\right)(1 - \sigma)T_\epsilon}{\left(\hat{b}_3 - b_1\right)(1 - \sigma)\epsilon}, \tag{2.7}$$

where $\hat{b}_3 = b_3 + \omega(1 - \epsilon), \hat{b}_2 = b_2 \exp\{(1 - \varrho)\omega T_\epsilon\}$.

Assumption 2.4. *For any $x, y, v, u \in \mathbb{R}^N$ and $t \geq 0$, there exist two nonnegative constants ρ_1 and ρ_2 such that*

$$\|f(x, v, t) - f(y, u, t)\| \leq \rho_1 \|x - y\| + \rho_2 \|v - u\|.$$

Lemma 2.5. *(see [37]) Suppose that L is irreducible. Then, $L1_N = 0$, and there is a positive vector $\xi = (\xi_1, \xi_2, \ldots, \xi_N)^T$ satisfies $\sum_{i=1}^{N} \xi_i = 1$ such that $\xi^T L = 0$. In addition, there exists a positive-definite diagonal matrix $M = diag(\xi_1, \xi_2, \ldots, \xi_N)$ such that $\hat{L} = \frac{\left(ML + L^T M\right)}{2}$ is symmetric, and $\sum_{i=1}^{N} \hat{L}_{ij} = \sum_{j=1}^{N} \hat{L}_{ji} = 0$ for all $i, j = 1, 2, \ldots, N$.*

Definition 2.1. *(see [37]) For a strongly connected network with Laplacian matrix L, the general algebraic connectivity is defined by*

$$a(L) = \min_{x^T\xi = 0, x \neq 0} \frac{x^T \hat{L} x}{x^T M x} > 0,$$

where $\hat{L} = \frac{\left(ML + L^T M\right)}{2}$, $M = diag\,(\xi_1, \xi_2, \ldots, \xi_N)$, $\xi = (\xi_1, \xi_2, \ldots, \xi_N)^T > 0$, and $\xi^T L = 0$, $\sum_{i=1}^{N} \xi_i = 1$.

Lemma 2.6. *(see [38]) Given $x, y \in \mathbb{R}$, for any $k \in \mathbb{R} > 0, |xy| \leq \frac{x^2}{2k} + \frac{ky^2}{2}, |xy| \leq \frac{1}{4}x^2 + y^2$.*

Lemma 2.7. *(see [39]) Let L be the Laplacian matrix of the graph $\mathcal{G}$, $H = \begin{pmatrix} 0 \\ I_{N-1} \end{pmatrix} \in \mathbb{R}^{N\times N-1}$. If the graph $\mathcal{G}$ is strongly connected, then $H^T L^T LH$ is a positive definite matrix.*

Definition 2.2. *(see [40]) The consensus for second-order MAS described by (2.1) is said to be reached if, for any initial conditions,*

$$\lim_{t\to\infty} \| x_i(t) - x_j(t) \|= 0, \lim_{t\to\infty} \| v_i(t) - v_j(t) \|= 0,$$

where $i, j = 1, 2, \ldots, N$.

3. Results

3.1. Centralized static event-triggered mechanism consensus analysis

In this section, we study the consensus problem of a centralized static event-triggered mechanism multi-agent system with a directed topology. Using Lyapunov stability theory and inequality scaling , Zeno behavior is excluded. Finally, the validity of the theoretical results is verified through numerical examples.

First, the following control protocols are examined:

$$u_i(t) = -\alpha \sum_{j=1}^{N} a_{ij}(x_i(t_k) - x_j(t_k)) - \beta \sum_{j=1}^{N} a_{ij}(v_i(t_k) - v_j(t_k)), \tag{3.1}$$

where, t_k represents the k-th trigger time of the agent. Define the position and velocity errors of the i-th agent as

$$\begin{aligned} e_i(t) =& \alpha \sum_{j=1}^{N} a_{ij}(x_i(t_k) - x_j(t_k)) - \alpha \sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t)) \\ &+ \beta \sum_{j=1}^{N} a_{ij}(v_i(t_k) - v_j(t_k)) - \beta \sum_{j=1}^{N} a_{ij}(v_i(t) - v_j(t)). \end{aligned} \tag{3.2}$$

For subsequent description, let $y_i(t) = \sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t)), z_i(t) = \sum_{j=1}^{N} a_{ij}(v_i(t) - v_j(t))$. These are obtained from (3.2), and then we obtain

$$u(t) = -e(t) - \alpha y(t) - \beta z(t). \tag{3.3}$$

Let $x(t) = (x_1^T(t), x_2^T(t), \ldots, x_N^T(t))^T$, $v(t) = (v_1^T(t), v_2^T(t), \ldots, v_N^T(t))^T$, $y(t) = (y_1^T(t), y_2^T(t), \ldots, y_N^T(t))^T$, $z(t) = (z_1^T(t), z_2^T(t), \ldots, z_N^T(t))^T$ and $e(t) = (e_1^T(t), e_2^T(t), \ldots, e_N^T(t))^T$. Then, $y(t) = Lx(t), z(t) = Lv(t)$, it is obtained by (2.1) and (3.3),

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = f(x_i(t), v_i(t), t) - \sum\limits_{i=1}^{N} e_i(t) - \alpha \sum\limits_{i=1}^{N} y_i(t) - \beta \sum\limits_{i=1}^{N} z_i(t), \end{cases} \tag{3.4}$$

where $f(x(t), v(t), t) = (f(x_1(t), v_1(t), t), f(x_2(t), v_2(t), t) \dots f(x_N(t), v_N(t), t))$. Therefore,

$$\begin{cases} \dot{y}_i(t) = z_i(t), \\ \dot{z}_i(t) = Lf(y_i(t), z_i(t), t) - L\sum_{i=1}^{N} e_i(t) - \alpha L \sum_{i=1}^{N} y_i(t) - \beta L \sum_{i=1}^{N} z_i(t). \end{cases} \tag{3.5}$$

The consensus for the second-order MAS (2.1) is said to be reached if, for any initial conditions,

$$\lim_{t\to\infty} \parallel y_i(t) - y_j(t) \parallel = 0, \lim_{t\to\infty} \parallel z_i(t) - z_j(t) \parallel = 0.$$

Let $\varphi(t) = \left(y^T(t), z^T(t)\right)^T$ and it can be rewritten as

$$\dot{\varphi}(t) = L\left[F(y(t), z(t), t) - (B \otimes I_n)\varphi(t)\right] - Le(t), \tag{3.6}$$

where $F(y(t), z(t), t) = \begin{pmatrix} 0_{nN} \\ (I_N - 1_{N\times N} \otimes I_n) f(y(t), z(t), t) \end{pmatrix}, B = \begin{pmatrix} 0_N & I_N \\ \alpha I_N & \beta I_N \end{pmatrix}$.

Theorem 3.1. *Suppose that the digraph $\mathcal{G}$ is connected and Assumption 2.1 holds. Then, second-order consensus in system (3.5) is achieved if the following conditions are satisfied:*

$$\mathbf{N}^2 > \frac{-\lambda_* \left(a_L \lambda_{\max}(B) - 2\right)}{2l^2 \xi_{\max}^2}, \tag{3.7}$$

event-triggered mechanism satisfies the following conditions:

$$\parallel e \parallel \leq \frac{\sigma \Delta_1}{2\lambda_* \parallel M \parallel \parallel L \parallel} \parallel \varphi \parallel, \tag{3.8}$$

where $\mathbf{N}$ is a positive constant, $\Delta_1 = \lambda_*(a_L \lambda_{\max}(B) - 2) + 2\mathbf{N}^2 l^2 \xi_{\max}^2 > 0$, $\xi_{\max} = \max\{\xi_1, \xi_2, \dots, \xi_N\}$, $i = 1, 2, \dots N$, $l_1 = \rho_1 \cdot \mid l_{ii} \mid, l_2 = \rho_2 \cdot \mid l_{ii} \mid$, $l^2 = 2\max\{l_1^2, l_2^2\}$, $\lambda_* = \lambda_{\min}(H^T L^T L H)$. Under protocol (3.1), multiple agents can reach agreement.

Proof. Consider the following Lyapunov function:

$$V(t) = \frac{1}{2}\varphi^T(t) M \varphi(t). \tag{3.9}$$

Taking the derivative of V(t), we conclude

$$\begin{aligned} \dot{V}(t) &= \varphi^T(t) M \dot{\varphi}(t) \\ &= \varphi^T(t) M[L(F(y(t), z(t), t) - (B \otimes I_n)\varphi(t)) - Le(t)] \\ &\leq \varphi^T(t) MLF(y(t), z(t), t) - \frac{1}{2}\varphi^T(t) B\left(ML + L^T M\right)\varphi(t) + \varphi^T(t) MLe(t) \\ &\leq \varphi^T(t) MLF(y(t), z(t), t) - \frac{1}{2} a_L \lambda_{\max}(B) \parallel \varphi(t) \parallel^2 + \parallel \varphi(t) \parallel \parallel M \parallel \parallel L \parallel \parallel e(t) \parallel, \end{aligned} \tag{3.10}$$

where $F(y(t), z(t), t) = (f^T(y_1(t), z_1(t), t), f^T(y_2(t), z_2(t), t), \dots, f^T(y_N(t), z_N(t), t))^T$.

Let $P = \varphi^T(t) MLF(y(t), z(t), t)$, then we know from Assumption 2.4,

$$P = \varphi^T(t) ML[f(y_i(t), z_i(t), t) - f(y_j(t), z_j(t), t)]$$

$$
\begin{aligned}
&= \sum_{i=1}^{N} \varphi_i(t)\xi_i \sum_{j=1}^{N} l_{ij}(f(y_i(t), z_i(t), t) - f(y_j(t), z_j(t), t)) \\
&\leq \sum_{i=1}^{N}\sum_{j=1}^{N} l_1\xi_i|\varphi_i(t)| \cdot |y_i(t) - y_j(t)| + \sum_{i=1}^{N}\sum_{j=1}^{N} l_2\xi_i|\varphi_i(t)| \cdot |z_i(t) - z_j(t)|. \qquad (3.11)
\end{aligned}
$$

This is given by Lemma 2.6,

$$
\begin{aligned}
P \leq &\left(l_1\xi_{\max} \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{1}{2\mathbf{N}l_1\xi_{\max}}|\varphi_i(t)|^2 + \frac{\mathbf{N}l_1\xi_{\max}}{2}|y_i(t) - y_j(t)|^2\right) \\
&+ \left(l_2\xi_{\max} \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{1}{2\mathbf{N}l_2\xi_{\max}}|\varphi_i(t)|^2 + \frac{\mathbf{N}l_2\xi_{\max}}{2}|z_i(t) - z_j(t)|^2\right) \\
\leq &\frac{1}{2} \parallel \varphi(t) \parallel^2 + \frac{\mathbf{N}^2 l_1^2 \xi_{\max}^2}{2} \sum_{j=1}^{N}(y_i(t) - y_j(t))^2 + \frac{1}{2} \parallel \varphi(t) \parallel^2 + \frac{\mathbf{N}^2 l_2^2 \xi_{\max}^2}{2} \sum_{j=1}^{N}(z_i(t) - z_j(t))^2. \qquad (3.12)
\end{aligned}
$$

This can be obtained by Eqs (3.10) and (3.12),

$$
\begin{aligned}
\dot{V}(t) \leq &- \frac{a_L\lambda_{\max}(B) - 2}{2} \parallel \varphi(t) \parallel^2 + \frac{\mathbf{N}^2 l_1^2 \xi_{\max}^2}{2} \sum_{j=1}^{N}(y_i(t) - y_j(t))^2 \\
&+ \frac{\mathbf{N}^2 l_2^2 \xi_{\max}^2}{2} \sum_{j=1}^{N}(z_i(t) - z_j(t))^2 + \parallel \varphi^T(t) \parallel\parallel M \parallel\parallel L \parallel\parallel e(t) \parallel . \qquad (3.13)
\end{aligned}
$$

From Lemma 2.7, we deduce matrix $H^T L^T LH$ is a positive definite matrix, so $\delta_i^T H^T L^T LH\ \delta_i \geq \lambda_* \parallel \delta_i \parallel^2$, $\delta_i = (\varphi_2 - \varphi_i, \varphi_3 - \varphi_i, \ldots, \varphi_j - \varphi_i)^T$. On the basis of $\lambda_* = \lambda_{min}(H^T L^T LH)$, we can get $\parallel \delta_i \parallel^2 \leq \frac{\|\varphi\|^2}{\lambda_*}$, so

$$
\dot{V}(t) \leq -\frac{\lambda_*(a_L\lambda_{\max}(B) - 2) + 2\mathbf{N}^2 l^2 \xi_{\max}^2}{2\lambda_*} \parallel \varphi(t) \parallel^2 + \parallel \varphi^T \parallel\parallel M \parallel\parallel L \parallel\parallel e(t) \parallel . \qquad (3.14)
$$

From formula (3.8), one has

$$
\dot{V}(t) \leq (\sigma - 1)\frac{\Delta_1}{2\lambda_*} \parallel \varphi(t) \parallel^2 . \qquad (3.15)
$$

According to the above Eq (3.8), when $0 < \sigma < 1$ and satisfy $\mathbf{N}^2 > \frac{-\lambda_*(a_L\lambda_{\max}(B)-2)}{2l^2\xi_{\max}^2}$ can guarantee $\dot{V}(t) \leq 0$, then we can obtain all the agents to reach consensus. Therefore, we will propose that Zeno behavior can be excluded. □

Remark 3.1. *Literature [40, 41] studies the consensus of multi-agent systems under event-triggered mechanism protocols. However, this work is all focused on first-order multi-agent systems. Compared to the first-order multi-agent system, the consensus problem for second-order multi-agent systems under the event-triggered control protocol is more challenging. An important question is how to design an appropriate event-triggered mechanism protocol for such systems.*

Theorem 3.2. *Assume that the topology of the second-order multi-agent system (2.1) under the consensus protocol (3.1) is directed and strongly connected, and that the triggered conditions are satisfied. Then, the time interval* $t^i_{k+1} - t^i_k$ *between two consecutive events is not less than*

$$\tau = \frac{-\left(\alpha^2+\beta^2\right)(2\lambda_* \parallel M \parallel\parallel L \parallel)}{\omega_2\left(\sigma\Delta_1 + \sqrt{\alpha^2+\beta^2}2\lambda_* \parallel M \parallel\parallel L \parallel\right)}, \tag{3.16}$$

where $\omega_1 = \frac{\rho_1+\rho_2}{\sqrt{\lambda_*}} + \parallel B \parallel$, $\omega_1 > \parallel L \parallel$, $\omega_2 = \omega_1\sqrt{\alpha^2+\beta^2}$. $Q = (\alpha I_N, \beta I_N)$, $\dot{e}(t) = (Q \otimes I_N)\dot{\varphi}(t)$.

Proof. When $\frac{\|e(t)\|}{\|\varphi(t)\|} = \frac{\sigma\Delta_1}{2\lambda_*\|M\|\|L\|}$, an immediate system trigger, we take the derivative of $\frac{\|e(t)\|}{\|\varphi(t)\|}$ with respect to time t:

$$\begin{aligned}
\frac{d}{dt}\left(\frac{\parallel e(t)\parallel}{\parallel \varphi(t)\parallel}\right) &= \frac{d}{dt}\frac{\left(e^T(t)e(t)\right)^{\frac{1}{2}}}{(\varphi^T(t)\varphi(t))^{\frac{1}{2}}} \\
&= \frac{\left(e^T(t)(e(t)\right)^{-\frac{1}{2}} \cdot e^T(t)\dot{e}(t) \parallel \varphi \parallel - (\varphi^T(t)\varphi(t))^{-\frac{1}{2}} \cdot \varphi^T(t)\dot{\varphi}(t) \parallel e(t) \parallel}{[(\varphi^T(t)\varphi(t))^{\frac{1}{2}}]^2} \\
&= \frac{\dot{e}(t)e^T(t)}{\parallel e(t)\parallel\parallel \varphi(t)\parallel} - \frac{\varphi^T(t)\dot{\varphi}(t)\parallel e(t)\parallel}{\parallel \varphi(t)\parallel^3} \\
&\le \frac{\parallel \dot{e}(t)\parallel}{\parallel \varphi(t)\parallel} + \frac{\parallel \dot{\varphi}(t)\parallel\parallel e(t)\parallel}{\parallel \varphi(t)\parallel^2} \\
&\le \frac{\parallel Q \parallel\parallel \dot{\varphi}(t)\parallel}{\parallel \varphi(t)\parallel} + \frac{\parallel \dot{\varphi}(t)\parallel\parallel e(t)\parallel}{\parallel \varphi(t)\parallel^2} \\
&\le \sqrt{\alpha^2+\beta^2}\frac{\parallel \dot{\varphi}(t)\parallel}{\parallel \varphi(t)\parallel}\left(1 + \frac{\parallel e(t)\parallel}{\sqrt{\alpha^2+\beta^2}\parallel \varphi(t)\parallel}\right) \\
&\le \sqrt{\alpha^2+\beta^2}\left(\frac{\rho_1+\rho_2}{\sqrt{\lambda_*}} + \parallel B \parallel + \parallel L \parallel\frac{\parallel e(t)\parallel}{\parallel \varphi(t)\parallel}\right)\left(1 + \frac{\parallel e(t)\parallel}{\sqrt{\alpha^2+\beta^2}\parallel \varphi(t)\parallel}\right) \\
&\le \sqrt{\alpha^2+\beta^2}\omega_1\left(1 + \frac{\parallel e(t)\parallel}{\sqrt{\alpha^2+\beta^2}\parallel \varphi(t)\parallel}\right)^2 \\
&= \omega_2\left(1 + \frac{\parallel e(t)\parallel}{\sqrt{\alpha^2+\beta^2}\parallel \varphi(t)\parallel}\right)^2.
\end{aligned} \tag{3.17}$$

Let $\mu = \frac{\|e(t)\|}{\|\varphi(t)\|}$, we can obtain $\dot{\mu} \le \omega_2\left(1 + \frac{1}{\sqrt{\alpha^2+\beta^2}}\mu\right)^2$, and satisfy $\mu(t) \le \mu(t, \mu_0), \mu(t, \mu_0)$ is the solution to $\dot{\mu}(t) = \omega_2\left(1 + \frac{1}{\sqrt{\alpha^2+\beta^2}}\mu\right)^2, \mu(0, \mu_0) = \mu_0$. We know from the triggered conditions $\mu(\tau, 0) = \frac{\sigma\Delta_1}{2\lambda_*\|M\|\|L\|}$, solve this equation as

$$\tau = \frac{-(\alpha^2+\beta^2)(2\lambda_* \parallel M \parallel\parallel L \parallel)}{\omega_2\left(\sigma\Delta_1 + \sqrt{\alpha^2+\beta^2}2\lambda_* \parallel M \parallel\parallel L \parallel\right)}. \tag{3.18}$$

The proof of Theorem 3.2 is finished. □

3.2. Fixed-time consensus for dynamic event-triggered mechanism under DoS attacks.

First, the consensus of second-order nonlinear multi-agent systems with a directed topology is studied under a static event-triggered mechanism. Second, dynamic auxiliary variables are introduced, and a dynamic event-triggered mechanism is designed. A consensus control protocol is proposed for both leader and leaderless cases. Using Lyapunov stability theory and algebraic graph theory, the fixed-time consensus of a multi-agent system under a DoS attack with directed topology is analyzed. In addition, Zeno behavior is excluded, and the validity of the theoretical results is verified through numerical examples.

3.2.1. Leaderless multi-agent consensus.

Under the dynamic event-triggered control (ETC) strategy, the control input of agent i is first constructed as

$$\begin{aligned} u_i(t) =& -\alpha_1 \sum_{i=1}^{N} a_{ij}(x_i(t_k^i) - x_j(t_k^i))^p - \beta_1 \sum_{i=1}^{N} a_{ij}(v_i(t_k^i) - v_j(t_k^i))^p \\ & -\alpha_2 \sum_{i=1}^{N} a_{ij}(x_i(t_k^i) - x_j(t_k^i)) - \beta_2 \sum_{i=1}^{N} a_{ij}(v_i(t_k^i) - v_j(t_k^i)) \\ & -\alpha_3 \sum_{i=1}^{N} a_{ij}\tanh(\mu(x_i(t_k^i) - x_j(t_k^i))) - \beta_3 \sum_{i=1}^{N} a_{ij}\tanh(\mu(v_i(t_k^i) - v_j(t_k^i))), \end{aligned} \tag{3.19}$$

where $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3 > 0, p \in (1, \infty)$ is the ratio of positive odd numbers, $t \in [t_k^i, t_{k+1}^i)$, t_k^i is the latest triggering instant of agent i. The measurement error $e_i(t)$ is designed as

$$\begin{aligned} e_i(t) =& \alpha_1 \sum_{i=1}^{N} a_{ij}(x_i(t_k^i) - x_j(t_k^i))^p + \beta_1 \sum_{i=1}^{N} a_{ij}(v_i(t_k^i) - v_j(t_k^i))^p \\ & + \alpha_2 \sum_{i=1}^{N} a_{ij}(x_i(t_k^i) - x_j(t_k^i)) + \beta_2 \sum_{i=1}^{N} a_{ij}(v_i(t_k^i) - v_j(t_k^i)) \\ & + \alpha_3 \sum_{i=1}^{N} a_{ij}\tanh(\mu(x_i(t_k^i) - x_j(t_k^i))) + \beta_3 \sum_{i=1}^{N} a_{ij}\tanh(\mu(v_i(t_k^i) - v_j(t_k^i))) \\ & - \alpha_1 \sum_{i=1}^{N} a_{ij}(x_i(t) - x_j(t))^p - \beta_1 \sum_{i=1}^{N} a_{ij}(v_i(t) - v_j(t))^p \\ & - \alpha_2 \sum_{i=1}^{N} a_{ij}(x_i(t) - x_j(t)) - \beta_2 \sum_{i=1}^{N} a_{ij}(v_i(t) - v_j(t)) \\ & - \alpha_3 \sum_{i=1}^{N} a_{ij}\tanh(\mu(x_i(t) - x_j(t))) - \beta_3 \sum_{i=1}^{N} a_{ij}\tanh(\mu(v_i(t) - v_j(t))). \end{aligned} \tag{3.20}$$

Then, we have

$$u_i(t) = -\sum_{i=1}^{N} e_i(t) - \alpha_1 \sum_{i=1}^{N} a_{ij}(x_i(t) - x_j(t))^p - \beta_1 \sum_{i=1}^{N} a_{ij}(v_i(t) - v_j(t))^p$$

$$
\begin{aligned}
&- \alpha_2 \sum_{i=1}^{N} a_{ij}(x_i(t) - x_j(t)) - \beta_2 \sum_{i=1}^{N} a_{ij}(v_i(t) - v_j(t)) \\
&- \alpha_3 \sum_{i=1}^{N} a_{ij} \tanh(\mu(x_i(t) - x_j(t))) - \beta_3 \sum_{i=1}^{N} a_{ij} \tanh(\mu(v_i(t) - v_j(t))).
\end{aligned}
\tag{3.21}
$$

Let $y_i(t) = \sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t)), z_i(t) = \sum_{j=1}^{N} a_{ij}(v_i(t) - v_j(t)), \varphi(t) = (y^T(t), z^T(t))^T$. To describe whether the DoS attacks occur or not; the attack signal $\Gamma_{attrack}$ is defined as follows:

$$
\Gamma_{attrack} = \begin{cases} 0, & [T_m, S_m), \\ 1, & [S_m, T_{m+1}), \end{cases} \tag{3.22}
$$

the dynamics of system (2.1) is written as

$$
\begin{cases} \dot{y}_i(t) = z_i(t), \\ \dot{z}_i(t) = L[f(x_i(t), v_i(t), t) + (1 - \Gamma_{attrack})u_i(t)], \end{cases} \tag{3.23}
$$

$$
\begin{aligned}
\dot{\varphi}(t) =& L[F(y(t), z(t), t) - (1 - \Gamma_{attrack})((B_1 \otimes I_n) \left(y^{pT}(t) \quad z^{pT}(t) \right)^T \\
&+ (B_2 \otimes I_n)\varphi(t) - (B_3 \otimes I_n) \tanh(\mu\varphi(t)) - e(t))],
\end{aligned}
\tag{3.24}
$$

where $F(y(t), z(t), t) = \begin{pmatrix} 0_{nN} \\ (I_N - 1_{N\times N} \otimes I_n) f(y(t), z(t), t) \end{pmatrix}$, $B_1 = \begin{pmatrix} 0_N & 0_N \\ \alpha_1 I_N & \beta_1 I_N \end{pmatrix}$, $B_2 = \begin{pmatrix} 0_N & I_N \\ \alpha_2 I_N & \beta_2 I_N \end{pmatrix}$, $B_3 = \begin{pmatrix} 0_N & 0_N \\ \alpha_3 I_N & \beta_3 I_N \end{pmatrix}$. And the dynamic triggered function of each agent is designed as follows:

$$
\psi_i(t) = \|e_i(t)\| + \theta \left(k_1 \|\varphi_i(t)\|^p + k_2 \|\varphi_i(t)\| - k_3 \right), \tag{3.25}
$$

where $k_1, k_2, k_3 \in (0, 1)$ and $\theta > 0$. The event-triggered condition is given as follows:

$$
t_{k+1}^i = \inf \left\{ t > t_k^i | \psi_i(t) \geq \chi_i(t) \right\}, \tag{3.26}
$$

where $\chi_i(t)$ is a dynamic variable and designed as follows:

$$
\begin{aligned}
\dot{\chi}_i(t) =& \delta \|\varphi_i(t)\| (-\theta k_1 \|\varphi_i(t)\|^p - \theta k_2 \|\varphi_i(t)\| + \theta k_3 - \|e_i(t)\|) \\
&- k_4 \chi_i^{\frac{p+1}{2}}(t) - k_5 \chi_i^{\frac{1}{2}}(t) - k_6 \chi_i^2(t) + \chi_i(t) - \frac{\iota N \|\hat{L}\| \|B_3\|}{\mu},
\end{aligned}
\tag{3.27}
$$

where $\chi_i(0) > 0, \delta \in (0, 1), k_4, k_5$ and k_6 are positive constants. Then, based on (3.26) and (3.27), we have $\dot{\chi}_i(t) \geq -\delta \|\varphi_i(t)\| \chi_i(t) - k_4 \chi_i^{\frac{p+1}{2}}(t) - k_5 \chi_i^{\frac{1}{2}}(t) - k_6 \chi_i^2(t) + \chi_i(t) - \frac{\iota N \|\hat{L}\| \|B_3\|}{\mu}$. Thus, we have $\chi_i(t) \geq e^{\int_0^t \eta_i(s) ds} \chi_i(0) > 0$ with $\eta_i(t) = -\delta \|\varphi_i(t)\| - k_4 \chi_i^{\frac{p-1}{2}}(t) - k_5 \chi_i^{-\frac{1}{2}}(t) - k_6 \chi_i(t) + 1$ for all $t > 0$.

Remark 3.2. *Compared to the finite-time methods in [19] and [42], the convergence time achieved by the fixed-time consensus approach is independent of the initial conditions. This characteristic enables a more precise determination of the required time. Furthermore, the use of the hyperbolic tangent function helps mitigate discontinuities that may arise from the sign function, thus preventing unnecessary resource wastage.*

Theorem 3.3. *For the nonliner MASs (3.23), the dynamic event-triggered mechanism practical fixed-time consensus (FTC) can be achieved if the following inequalities and equality hold:*

$$\Delta > ||\hat{L}||||B_2|| + \frac{1}{2}(\lambda_{\min}(\hat{L}) + \delta)\delta\theta k_2 + \delta\theta k_2, ||\hat{L}||||B_3|| > \lambda_{\min}(\hat{L} + \delta + 1)\delta\theta k_3, \tag{3.28}$$

$$\lambda_{\max}(\hat{L}) + \delta = 2k_2k_6\delta\theta, \Delta + \frac{2}{k\delta} + 1 > 2\delta\theta k_2, \tag{3.29}$$

$$\hat{b}_3 + \omega(1-\epsilon) < \min\left\{\hat{b}_2, \hat{b}_1 exp((1-\varrho)\omega T_\epsilon)\right\}, \hat{b}_4 - \omega T_\epsilon < 0, \omega > 0. \tag{3.30}$$

Proof. Consider the following Lyapunov function

$$V(t) = V_1(t) + V_2(t),$$

where $V_1 = \frac{1}{2}\varphi^T(t)M\varphi(t)$ and $V_2 = \sum_{i=1}^{N}\chi_i(t)$. When $t \in [T_m, S_m)$, it yields

$$\begin{aligned}
\dot{V}(t) =&\dot{V}_1(t) + \dot{V}_2(t) \\
=&\varphi^T(t)M\dot{\varphi}(t) + \sum_{i=1}^{N}\dot{\chi}_i(t) \\
=&\varphi^T(t)ML[F(y(t), z(t), t) + u(t)] + \delta|\varphi^T(t)|(-\theta k_1||\varphi(t)||^p - \theta k_2||\varphi(t)|| + \theta k_3 \\
&- ||e(t)||) - k_4\sum_{i=1}^{N}\chi_i^{\frac{p+1}{2}}(t) - k_5\sum_{i=1}^{N}\chi_i^{\frac{1}{2}}(t) - k_6\chi_i^2(t) + \chi_i(t) - \frac{\iota N||\hat{L}||||B_3||}{\mu} \\
\leq&\varphi^T(t)ML[F(y(t), z(t), t) - (B_1 \otimes I_n)||\varphi(t)||^{\frac{p}{2}} - (B_2 \otimes I_n)||\varphi(t)|| \\
&- (B_3 \otimes I_n)\tanh(\mu\varphi(t)) - ||e(t)||] + \delta||\varphi(t)||(-\theta k_1||\varphi(t)||^p - \theta k_2||\varphi(t)|| + \theta k_3 \\
&- ||e(t)||) - k_4\sum_{i=1}^{N}\chi_i^{\frac{p+1}{2}}(t) - k_5\sum_{i=1}^{N}\chi_i^{\frac{1}{2}}(t) - k_6\chi_i^2(t) + \chi_i(t) - \frac{\iota N||\hat{L}||||B_3||}{\mu}.
\end{aligned} \tag{3.31}$$

From Theorem 3.1 above, it follows

$$\begin{aligned}
\dot{V}(t) \leq&\Delta \parallel \varphi(t) \parallel^2 - \frac{ML + L^TM}{2} \parallel B_1 \parallel ||\varphi||^{\frac{p+2}{2}} - \frac{ML + L^TM}{2} \parallel B_2 \parallel ||\varphi||^2 \\
&- \frac{ML + L^TM}{2} \parallel B_3 \parallel ||\varphi(t)||\tanh(\mu||\varphi(t)||) - \varphi^T(t)ML||e(t)|| \\
&- \delta\theta k_1||\varphi(t)||^{p+1} - \delta\theta k_2||\varphi(t)||^2 + \delta\theta k_3||\varphi(t)|| - \delta||\varphi(t)||||e(t)|| \\
&- k_4\sum_{i=1}^{N}\chi_i^{\frac{p+1}{2}}(t) - k_5\sum_{i=1}^{N}\chi_i^{\frac{1}{2}}(t) - k_6\chi_i^2(t) + \chi_i(t) - \frac{\iota N||\hat{L}||||B_3||}{\mu}.
\end{aligned} \tag{3.32}$$

According to Lemma 2.2, one has

$$\begin{aligned}
\dot{V}(t) \leq&\Delta||\varphi(t)||^2 - ||\hat{L}||||B_1||||\varphi(t)||^{\frac{p+2}{2}} - ||\hat{L}||||B_2||||\varphi(t)||^2 - ||\hat{L}||||B_3||||\varphi(t)|| \\
&- \delta\theta k_1||\varphi(t)||^{p+1} - \delta\theta k_2||\varphi(t)||^2 + \delta\theta k_3||\varphi(t)|| - k_4\sum_{i=1}^{N}\chi_i^{\frac{p+1}{2}}(t) - k_5\sum_{i=1}^{N}\chi_i^{\frac{1}{2}}(t)
\end{aligned}$$

$$- k_6\chi_i^2(t) + \chi_i(t) + (\lambda_{\max}(\hat{L}) + \delta)\|\varphi(t)\|\|e(t)\|, \tag{3.33}$$

where $\Delta = -\frac{\lambda_*(a_L\lambda_{\max}(B)-2)+2\mathbf{N}^2 l^2\xi_{\max}^2}{2\lambda_*}$, we use the event-triggered condition to obtain

$$\begin{aligned}\dot{V}(t) \leq& \Delta\|\varphi(t)\|^2 - \|\hat{L}\|\|B_1\|\|\varphi(t)\|^{\frac{p+2}{2}} - \|\hat{L}\|\|B_2\|\|\varphi(t)\|^2 - \|\hat{L}\|\|B_3\|\|\varphi(t)\| \\ & - \delta\theta k_1\|\varphi(t)\|^{p+1} - \delta\theta k_2\|\varphi(t)\|^2 + \delta\theta k_3\|\varphi(t)\| - k_4\sum_{i=1}^N \chi_i^{\frac{p+1}{2}}(t) \\ & - k_5\sum_{i=1}^N \chi_i^{\frac{1}{2}}(t) - k_6\chi_i^2(t) + \chi_i(t) + (\lambda_{\max}(\hat{L}) + \delta)(|\chi_i(t)|\|\varphi_i(t)\| \\ & - \delta\theta k_1\|\varphi(t)\|^{p+1} - \delta\theta k_2\|\varphi(t)\|^2 + \delta\theta k_3\|\varphi(t)\|).\end{aligned} \tag{3.34}$$

Then according to Lemmas 2.1 and 2.6, we have

$$\begin{aligned}\dot{V}(t) \leq& \left[\Delta - \|\hat{L}\|\|B_2\| - \frac{1}{2}(\lambda_{\min}(\hat{L}) + \delta)\delta\theta k_2 - \delta\theta k_2\right]\sum_{i=1}^N \varphi_i^2(t) \\ & - (\lambda_{\min}(\hat{L}) + \delta + 1)\delta\theta k_1 N^{\frac{1-p}{2}}\sum_{i=1}^N(\varphi_i^2(t))^{\frac{p+1}{2}} \\ & - \left[\|\hat{L}\|\|B_3\| - (\lambda_{\min}(\hat{L}) + \delta + 1)\delta\theta k_3\right]\sum_{i=1}^N(\varphi_i^2(t))^{\frac{1}{2}} \\ & - k_4 N^{\frac{1-p}{2}}\sum_{i=1}^N \chi_i^{\frac{p+1}{2}}(t) - k_5\sum_{i=1}^N \chi_i^{\frac{1}{2}}(t) + \sum_{i=1}^N \chi_i(t) \\ \leq& \frac{\Delta - \|\hat{L}\|\|B_2\| - \frac{1}{2}(\lambda_{\min}(\hat{L}) + \delta)\delta\theta k_2 - \delta\theta k_2}{\lambda_{\min}(M)}\sum_{i=1}^N \xi_i\varphi_i^2(t) \\ & - \frac{(\lambda_{\min}(\hat{L}) + \delta + 1)\delta\theta k_1 N^{\frac{1-p}{2}}}{\lambda_{\min}^{\frac{p+1}{2}}(M)}\sum_{i=1}^N\left(\xi_i\varphi_i^2(t)\right)^{\frac{p+1}{2}} \\ & - \frac{\|\hat{L}\|\|B_3\| - (\lambda_{\min}(\hat{L}) + \delta + 1)\delta\theta k_3}{\lambda_{\min}^{\frac{1}{2}}(M)}\sum_{i=1}^N\left(\xi_i\varphi_i^2(t)\right)^{\frac{1}{2}} \\ & - k_4 N^{\frac{1-p}{2}}\sum_{i=1}^N \chi_i^{\frac{p+1}{2}}(t) - k_5\sum_{i=1}^N \chi_i^{\frac{1}{2}}(t) + \sum_{i=1}^N \chi_i(t) \\ =& \frac{\Delta - \|\hat{L}\|\|B_2\| - \frac{1}{2}(\lambda_{\min}(\hat{L}) + \delta)\delta\theta k_2 - \delta\theta k_2}{\lambda_{\min}(M)}2V_1(t) \\ & - \frac{(\lambda_{\min}(\hat{L}) + \delta + 1)\delta\theta k_1 N^{\frac{1-p}{2}}}{\lambda_{\min}^{\frac{p+1}{2}}(M)}(2V_1(t))^{\frac{p+1}{2}} \\ & - \frac{\|\hat{L}\|\|B_3\| - (\lambda_{\min}(\hat{L}) + \delta + 1)\delta\theta k_3}{\lambda_{\min}^{\frac{1}{2}}(M)}(2V_1(t))^{\frac{1}{2}}\end{aligned}$$

$$
\begin{aligned}
& - k_4 N^{\frac{1-p}{2}} V_2^{\frac{p+1}{2}}(t) - k_5 V_2^{\frac{1}{2}}(t) + V_2(t) \\
\leq & \hat{b}_3 V(t) - \hat{b}_1 V^{\frac{p+1}{2}}(t) - \hat{b}_2 V^{\frac{1}{2}}(t),
\end{aligned} \tag{3.35}
$$

with $\hat{b}_1 = \min\left\{\frac{\lambda_{\min}(\hat{L}) + \delta + 1)\delta\theta k_1 N^{\frac{1-p}{2}}}{\lambda_{\min}^{\frac{p+1}{2}}(M)}, k_4 N^{\frac{1-p}{2}}\right\}$, $\hat{b}_2 = \min\left\{\frac{|\hat{L}|\|B_3\| - (\lambda_{\min}(\hat{L}) + \delta + 1)\delta\theta k_3}{\lambda_{\min}^{\frac{1}{2}}(M)}, k_5\right\}$, $\hat{b}_3 = \min\left\{\frac{\Delta - \|\hat{L}\|\|B_2\| - \frac{1}{2}(\lambda_{\min}(\hat{L}) + \delta)\delta\theta k_2 - \delta\theta k_2}{\lambda_{\min}(M)}, 1\right\}$.

When $t \in [S_m, T_{m+1})$, it yields

$$
\begin{aligned}
\dot{V}(t) = & \varphi^T(t) MLF(y(t), z(t), t) + \delta\|\varphi(t)\|(-\theta k_1 \|\varphi(t)\|^p - \theta k_2 \|\varphi(t)\| + \theta k_3 \\
& - \|e(t)\|) - k_4 \sum_{i=1}^{N} \chi_i^{\frac{p+1}{2}}(t) - k_5 \sum_{i=1}^{N} \chi_i^{\frac{1}{2}}(t) - k_6 \chi_i^2(t) + \chi_i(t) - \frac{\iota N\|\hat{L}\|\|B_3\|}{\mu} \\
\leq & \Delta \sum_{i=1}^{N} \|\varphi_i(t)\|^2 + \sum_{i=1}^{N} \chi_i(t) - 2\delta\theta k_2 \sum_{i=1}^{N} \|\varphi_i(t)\|^2 - k_6 \sum_{i=1}^{N} \chi_i^2(t) + \delta\|\varphi_i(t)\||\chi_i(t)| \\
& + (\delta\theta k_3)^2 + \sum_{i=1}^{N} \|\varphi_i(t)\|^2 - \frac{\iota N\|\hat{L}\|\|B_3\|}{\mu} \\
\leq & \left(\Delta - 2\delta\theta k_2 + \frac{2}{k\delta} + 1\right) \sum_{i=1}^{N} \|\varphi_i(t)\|^2 + \sum_{i=1}^{N} \chi_i(t) \leq \hat{b}_4 V(t),
\end{aligned} \tag{3.36}
$$

with $\hat{b}_4 = \max\left\{\Delta - 2\delta\theta k_2 + \frac{2}{k\delta} + 1, 1\right\}$, $\frac{\iota N\|\hat{L}\|\|B_3\|}{\mu} > (\delta\theta k_3)^2$ and $\frac{2}{k\delta} = k_6$.

Combining (3.35) and (3.36)

$$
\begin{cases}
\dot{V}(t) \leq \hat{b}_3 V(t) - \hat{b}_1 V(t)^{\frac{p+1}{2}} - \hat{b}_2 V(t)^{\frac{1}{2}}, t \in [T_m, S_m), \\
\dot{V}(t) \leq \hat{b}_4 V(t), t \in [S_m, T_{m+1}).
\end{cases} \tag{3.37}
$$

From Lemma 2.4, if $t \geq T^*$, where T^* satisfies

$$
T^* \leq \frac{-2 + \left(\hat{b}_3 - \hat{b}_2\right) T_\epsilon}{\left(\hat{b}_3 - \hat{b}_2\right)\epsilon} + \frac{1 + \left(\hat{b}_3 - \hat{b}_1\right)(1 - \hat{\sigma}) T_\epsilon}{\left(\hat{b}_3 - \hat{b}_1\right)(1 - \hat{\sigma})\epsilon}, \tag{3.38}
$$

where $\hat{\sigma} = \frac{p+1}{2}$.

The proof of Theorem 3.3 is finished. □

Remark 3.3. *As shown in [13,43,44], many similar dynamic models for multi-agent systems have been developed to address the consensus problem. This paper investigates the timing consensus problem under a directed topology and introduces dynamic auxiliary variables. The generalization from the first-order to the second-order increases the complexity due to the asymmetry of the Laplace matrix. Additionally, the case of instability under a DoS attack is also considered.*

Now, it is proved that there is no Zeno behavior in this dynamic event-triggered mechanism.

Proof. Evidence similar to that of literature [29], based on the above analysis , we have $\|\varphi_i(t)\| \leq \sqrt{2\lambda_{\max}(M)V_1(t)} \leq \sqrt{2\lambda_{\max}(M)V(t)}$ for $t \in [t_k^i, t_{k+1}^i)$, the definition of measurement error gives us

$$\begin{aligned} D^+|e(t)| \leq& \| \dot{e}(t) \| = \left[\| \bar{B}_1 \|\| \varphi(t) \|^p - \| \bar{B}_2 \|\| \varphi(t) \| - \| \bar{B}_3 \| \tanh(\mu \| \varphi(t) \|)\right]' \\ &\leq \left(p \| \bar{B}_1 \|\| \varphi(t) \|^{p-1} + \| \bar{B}_2 \| + \mu \| \bar{B}_3 \| (1 - \tanh^2(\mu \| \varphi(t) \|))\right) \| \dot{\varphi}(t) \|, \end{aligned} \tag{3.39}$$

where $\bar{B}_1 = \begin{pmatrix} 0_N & 0_N \\ \alpha_1 I_N & \beta_1 I_N \end{pmatrix}$, $\bar{B}_2 = \begin{pmatrix} 0_N & 0_N \\ \alpha_2 I_N & \beta_2 I_N \end{pmatrix}$, $\bar{B}_3 = \begin{pmatrix} 0_N & 0_N \\ \alpha_3 I_N & \beta_3 I_N \end{pmatrix}$, then one can obtain

$$\begin{aligned} D^+|e(t)| \leq& \left(p\|\bar{B}_1\|\|\varphi(t)\|^{p-1} + \|\bar{B}_2\| + \mu\|\bar{B}_3\|(1 - \tanh^2(\mu\|\varphi(t)\|))\right) \\ &\left(\sum_{i=1}^{N} a_{ij}(f(y_i(t), z_i(t), t) - f(y_j(t), z_j(t), t)) + \sum_{i=1}^{N} a_{ij}(u_i(t) - u_j(t))\right) \\ \leq& \left(p\|\bar{B}_1\|\|\varphi(t)\|^{p-1} + \|\bar{B}_2\| + \mu\|\bar{B}_3\|\right)\left(-\frac{\lambda_*(a_L\lambda_{max}(B) - 2) + 2\mathbf{N}^2 l^2 \xi_{max}^2}{2\lambda_*} \| \varphi(t) \|\right) \\ &+ \sum_{i=1}^{N} a_{ij}(u_i(t) - u_j(t)) \\ \leq& \left[U - \frac{\lambda_*(a_L\lambda_{max}(B) - 2) + 2\mathbf{N}^2 l^2 \xi_{max}^2}{2\lambda_*} \| \varphi(t) \|\right]\left(p\|\bar{B}_1\|\|\varphi(t)\|^{p-1} + \|\bar{B}_2\| + \mu\|\bar{B}_3\|\right), \end{aligned} \tag{3.40}$$

where $U = \max_{t\in[t_k^i, t_{k+1}^i)} \{\| u_i(t) \|\}$, one has

$$\| \dot{e}(t) \| \leq \left[U + A\sqrt{2\lambda_{\max}(M)V(t)}\right] C_1, \tag{3.41}$$

where $A = -\frac{\lambda_*(a_L\lambda_{\max}(B)-2)+2\mathbf{N}^2 l^2 \xi_{\max}^2}{2\lambda_*}$, $C_1 = p\|\bar{B}_1\|\|\varphi(t)\|^{p-1} + \|\bar{B}_2\| + \mu\|\bar{B}_3\|$. $t_{k'}^i$ is the latest triggering time of agent i. Since $e(t_{k'}^i) = 0$, it follows that $|e(t)| \leq \int_{t_k^i}^{t} \left[U + A\sqrt{2\lambda_{\max}(M)V(t)}\right] C_1 ds + |e(t_{k'}^i)|$. According to the dynamics of the event-triggered condition, we have $|e(t_{k+1}^i)| \leq \int_{t_k^i}^{t_{k+1}^i} \left[U + A\sqrt{2\lambda_{\max}(M)V(t)}\right] C_1 ds$, which yields $t_{k+1}^i - t_k^i \geq \frac{\theta k_3}{\left[U + A\sqrt{2\lambda_{\max}(M)V(t)}\right] C_1}$. □

3.2.2. Leader multi-agent consensus

The control protocol for the dynamic event-triggered mechanism is given for systems (2.1) and (2.2) as follows:

$$\begin{aligned} u_i(t) =& -\alpha_1 \sum_{i=1}^{N} a_{ij}(x_i(t_k^i) - x_j(t_k^i))^p - \beta_1 \sum_{i=1}^{N} a_{ij}(v_i(t_k^i) - v_j(t_k^i))^p \\ &- \alpha_2 \sum_{i=1}^{N} a_{ij}(x_i(t_k^i) - x_j(t_k^i)) - \beta_2 \sum_{i=1}^{N} a_{ij}(v_i(t_k^i) - v_j(t_k^i)) \\ &- \alpha_3 \sum_{i=1}^{N} a_{ij} \tanh(\mu(x_i(t_k^i) - x_j(t_k^i))) - \beta_3 \sum_{i=1}^{N} a_{ij} \tanh(\mu(v_i(t_k^i) - v_j(t_k^i))) \end{aligned}$$

$$
\begin{aligned}
&- \alpha_1 \sum_{i=1}^{N} a_{ij} d_i (x_i(t_k^i) - x_0(t))^p - \beta_1 \sum_{i=1}^{N} a_{ij} d_i (v_i(t_k^i) - v_0(t))^p \\
&- \alpha_2 \sum_{i=1}^{N} a_{ij} d_i (x_i(t_k^i) - x_0(t)) - \beta_2 \sum_{i=1}^{N} a_{ij} d_i (v_i(t_k^i) - v_0(t)) \\
&- \alpha_3 \sum_{i=1}^{N} a_{ij} d_i \tanh(\mu(x_i(t_k^i) - x_0(t))) - \beta_3 \sum_{i=1}^{N} a_{ij} d_i \tanh(\mu(v_i(t_k^i) - v_0(t))).
\end{aligned} \tag{3.42}
$$

The measurement error $e_i(t)$ is designed as

$$
\begin{aligned}
e_i(t) =& \alpha_1 \sum_{i=1}^{N} a_{ij} (x_i(t_k^i) - x_j(t_k^i))^p + \beta_1 \sum_{i=1}^{N} a_{ij} (v_i(t_k^i) - v_j(t_k^i))^p \\
&+ \alpha_2 \sum_{i=1}^{N} a_{ij} (x_i(t_k^i) - x_j(t_k^i)) + \beta_2 \sum_{i=1}^{N} a_{ij} (v_i(t_k^i) - v_j(t_k^i)) \\
&+ \alpha_3 \sum_{i=1}^{N} a_{ij} \tanh(\mu(x_i(t_k^i) - x_j(t_k^i))) + \beta_3 \sum_{i=1}^{N} a_{ij} \tanh(\mu(v_i(t_k^i) - v_j(t_k^i))) \\
&- \alpha_1 \sum_{i=1}^{N} a_{ij} (x_i(t) - x_j(t))^p - \beta_1 \sum_{i=1}^{N} a_{ij} (v_i(t) - v_j(t))^p \\
&- \alpha_2 \sum_{i=1}^{N} a_{ij} (x_i(t) - x_j(t)) - \beta_2 \sum_{i=1}^{N} a_{ij} (v_i(t) - v_j(t)) \\
&- \alpha_3 \sum_{i=1}^{N} a_{ij} \tanh(\mu(x_i(t) - x_j(t))) - \beta_3 \sum_{i=1}^{N} a_{ij} \tanh(\mu(v_i(t) - v_j(t))).
\end{aligned} \tag{3.43}
$$

Combining (3.42) and (3.43), we can obtain

$$
\begin{aligned}
u_i(t) =& - \sum_{i=1}^{N} e_i(t) - \alpha_1 \sum_{i=1}^{N} a_{ij} (x_i(t) - x_j(t))^p - \beta_1 \sum_{i=1}^{N} a_{ij} (v_i(t) - v_j(t))^p \\
&- \alpha_2 \sum_{i=1}^{N} a_{ij} (x_i(t) - x_j(t)) - \beta_2 \sum_{i=1}^{N} a_{ij} (v_i(t) - v_j(t)) \\
&- \alpha_3 \sum_{i=1}^{N} a_{ij} \tanh(\mu(x_i(t) - x_j(t))) - \beta_3 \sum_{i=1}^{N} a_{ij} \tanh(\mu(v_i(t) - v_j(t))) \\
&- \alpha_1 \sum_{i=1}^{N} a_{ij} d_i (x_i(t_k^i) - x_0(t))^p - \beta_1 \sum_{i=1}^{N} a_{ij} d_i (v_i(t_k^i) - v_0(t))^p \\
&- \alpha_2 \sum_{i=1}^{N} a_{ij} d_i (x_i(t_k^i) - x_0(t)) - \beta_2 \sum_{i=1}^{N} a_{ij} d_i (v_i(t_k^i) - v_0(t)) \\
&- \alpha_3 \sum_{i=1}^{N} a_{ij} d_i \tanh(\mu(x_i(t_k^i) - x_0(t))) - \beta_3 \sum_{i=1}^{N} a_{ij} d_i \tanh(\mu(v_i(t_k^i) - v_0(t))).
\end{aligned} \tag{3.44}
$$

Let $\hat{y}_i(t) = \sum\limits_{i=1}^{N} a_{ij}(x_i(t) - x_0(t)), \hat{z}_i(t) = \sum\limits_{i=1}^{N} a_{ij}(v_i(t) - v_0(t)), \bar{y}(t) = \sum\limits_{i=1}^{N} a_{ij}x_i(t_k^i) - x(t_0), \bar{z}(t) = \sum\limits_{i=1}^{N} a_{ij}v_i(t_k^i) - v(t_0)$. The error dynamics system is then:

$$\begin{cases} \dot{\hat{y}}_i(t) = & \hat{z}_i(t), \\ \dot{\hat{z}}_i(t) = & L\left[f(\hat{y}_i(t), \hat{z}_i(t), t) - f(\hat{y}_0(t), \hat{z}_0(t), t)\right] - (1 - \Gamma_{attrack})[L \sum\limits_{i=1}^{N} e_i(t) - \alpha_1 L \\ & [\sum\limits_{i=1}^{N} a_{ij}(\hat{y}_i(t) - \hat{y}_j(t))^p + \sum\limits_{i=1}^{N} a_{ij} d_i \bar{y}_i^p(t)] - \beta_1 L[\sum\limits_{i=1}^{N} a_{ij}(\hat{z}_i(t) - \hat{z}_j(t))^p \\ & + \sum\limits_{i=1}^{N} a_{ij} d_i \bar{z}_i^p(t)] - \alpha_2 L[\sum\limits_{i=1}^{N} a_{ij}(\hat{y}_i(t) - \hat{y}_j(t)) + \sum\limits_{i=1}^{N} a_{ij} d_i \bar{y}_i(t)] - \beta_2 L \\ & [\sum\limits_{i=1}^{N} a_{ij}(\hat{z}_i(t) - \hat{z}_j(t)) + \sum\limits_{i=1}^{N} a_{ij} d_i \bar{z}(t)] - \alpha_3 L[\sum\limits_{i=1}^{N} a_{ij} \tanh(\mu(\hat{y}_i(t) - \hat{y}_j(t))) \\ & + \sum\limits_{i=1}^{N} a_{ij} d_i \tanh(\mu \bar{y}_i(t))] - \beta_3 L[\sum\limits_{i=1}^{N} a_{ij} \tanh(\mu(\hat{z}_i(t) - \hat{z}_j(t))) \\ & + \sum\limits_{i=1}^{N} a_{ij} d_i \tanh(\mu \bar{z}_i(t))]]. \end{cases}$$

Let $\hat{\varphi}(t) = (\hat{y}^T(t), \hat{z}^T(t))^T, \bar{\varphi}(t) = (\bar{y}^T(t), \bar{z}^T(t)), \hat{f}(\hat{y}_i(t), \hat{z}_i(t), t) = f(\hat{y}_i(t), \hat{z}_i(t), t) - f(\hat{y}_0(t), \hat{z}_0(t), t)$, so the above equation can be rewritten in the following form:

$$\begin{aligned} \dot{\hat{\varphi}}(t) =& L[\hat{F}(\hat{y}(t), \hat{z}(t), t) - (1 - \Gamma_{attrack})(B_1 \otimes I_n)(\hat{y}^{pT}(t), \hat{z}^{pT}(t))^T - (B_2 \otimes I_n)\hat{\varphi}(t) - (B_3 \otimes I_n) \\ & \tanh(\mu\hat{\varphi}(t)) - e(t) - (\hat{B}_1 \otimes I_n)(\bar{y}^{pT}(t), \bar{z}^{pT}(t)) - (\hat{B}_2 \otimes I_n)\bar{\varphi}(t) - (\hat{B}_3 \otimes I_n)\bar{\varphi}(t)], \end{aligned} \tag{3.45}$$

where B_1, B_2 and B_3 are the same as the above definition, $\hat{F}(\hat{y}(t), \hat{z}(t), t) = \begin{pmatrix} 0_{nN} \\ (I_N \otimes I_n)\hat{f}(\hat{y}(t), \hat{z}(t), t) \end{pmatrix}, \hat{B}_1 = \begin{pmatrix} 0_N & 0_N \\ \alpha_1\hat{D} & \beta_1\hat{D} \end{pmatrix}, \hat{B}_2 = \begin{pmatrix} 0_N & 0_N \\ \alpha_2\hat{D} & \beta_2\hat{D} \end{pmatrix}, \hat{B}_3 = \begin{pmatrix} 0_N & 0_N \\ \alpha_3\hat{D} & \beta_3\hat{D} \end{pmatrix}$. And the dynamic triggered function of each agent is designed as follows:

$$\Psi_i(t) = ||e_i(t)|| + \theta(k_1||\hat{\varphi}_i(t)||^p + k_2||\hat{\varphi}_i(t)|| - k_3) - \Upsilon_i^{\frac{1}{2}}(t), \tag{3.46}$$

where $k_1, k_2, k_3 \in (0, 1)$ and $\theta > 0$. The event-triggered condition is given as follows:

$$t_{k+1}^i = \inf\left\{t > t_k^i | \Psi_i(t) \geq \Upsilon_i(t)\right\}, \tag{3.47}$$

where $\Upsilon_i(t)$ is a dynamic variable and designed as follows:

$$\begin{aligned} \dot{\Upsilon}_i(t) =& \delta||\hat{\varphi}_i(t)||(-\theta k_1||\hat{\varphi}_i(t)||^p - \theta k_2||\hat{\varphi}_i(t)|| + \theta k_3 - ||e_i(t)||) \\ & - k_4\Upsilon_i^{\frac{p+1}{2}}(t) - k_5\Upsilon_i^{\frac{1}{2}}(t) - k_6\Upsilon_i^2(t) - \frac{\iota N||\hat{L}||||B_3||}{\mu} - \frac{\iota N||\hat{L}||||\hat{B}_3||}{\mu}, \end{aligned} \tag{3.48}$$

where $\Upsilon_i(0) > 0, \delta \in (0, 1), k_4, k_5$, and k_6 are positive constants. Then, based on (3.47) and (3.48), we have $\dot{\Upsilon}_i(t) \geq -k_4\Upsilon_i^{\frac{p+1}{2}}(t) - k_5\Upsilon_i^{\frac{1}{2}}(t) - k_6\Upsilon_i^2(t) - \delta||\hat{\varphi}_i(t)||\Upsilon_i^{\frac{1}{2}}(t) - \frac{\iota N||\hat{L}||||B_3||}{\mu} - \frac{\iota N||\hat{L}||||\hat{B}_3||}{\mu}$. Thus, $\Upsilon_i(t) \geq e^{\int_0^t \xi_i(s)ds}\Upsilon_i(0) > 0$ with $\xi_i(t) = -k_4\Upsilon_i^{\frac{p-1}{2}}(t) - k_5\Upsilon_i^{-\frac{1}{2}}(t) - k_6\Upsilon_i(t) - \delta||\hat{\varphi}_i(t)||\Upsilon_i^{-\frac{1}{2}}(t)$, for all $t > 0$.

Theorem 3.4. *For the MASs, practical fixed-time consensus can be achieved if the following inequalities are satisfied under the conditions imposed by the control inputs (3.44) and the dynamic event-triggered mechanism (3.47).*

$$\Delta + \frac{||\hat{L}||||\hat{B}_2||}{\lambda^2_{\min}(H)} + 2(\lambda_{\min}(\hat{L}) + \delta) > \lambda_{\min}(\hat{L})\theta k_2 + 2\delta\theta k_2 + ||\hat{L}||||B_2||, \tag{3.49}$$

$$||\hat{L}||||B_1|| + \lambda_{\min}(\hat{L})\theta k_1 + 2\delta\theta k_1 > \frac{||\hat{L}||||\hat{B}_1||}{\lambda^{p+1}_{\min}(H)}, ||\hat{L}||||B_3|| > \theta k_3 + \frac{||\hat{L}||||\hat{B}_3||}{\lambda_{\min}(H)} + 2\delta\theta k_3, \tag{3.50}$$

$$\Delta + 2\delta + 1 > 2\delta\theta k_2, \hat{c}_3 + \omega(1-\epsilon) < \min\{\hat{c}_2, \hat{c}_1 exp((1-\varrho)\omega T_\epsilon)\}, \hat{c}_4 - \omega T_\epsilon < 0. \tag{3.51}$$

Proof. Choose the Lyapunov function as

$$W(t) = W_1(t) + W_2(t),$$

where $W_1 = \frac{1}{2}\hat{\varphi}^T(t)M\hat{\varphi}(t)$ and $W_2 = \sum_{i=1}^{N}\Upsilon_i(t)$. When $t \in [T_m, S_m)$, we have

$$\begin{aligned}
\dot{W}(t) =& \dot{W}_1(t) + \dot{W}_2(t)\\
=& \hat{\varphi}^T(t)M\dot{\hat{\varphi}}(t) + \sum_{i=1}^{N}\dot{\Upsilon}_i(t)\\
=& \hat{\varphi}^T(t)ML[\hat{F}((t), z(t), t) + u(t)] + \delta|\hat{\varphi}^T(t)|(-\theta k_1||\hat{\varphi}(t)||^p - \theta k_2||\hat{\varphi}(t)|| + \theta k_3 - ||e(t)||)\\
&- k_4\sum_{i=1}^{N}\Upsilon_i^{\frac{p+1}{2}}(t) - k_5\sum_{i=1}^{N}\Upsilon_i^{\frac{1}{2}}(t) - Nk_6\sum_{i=1}^{N}\Upsilon_i^2(t) - \frac{\iota N||\hat{L}||||B_3||}{\mu} - \frac{\iota N||\hat{L}||||\hat{B}_3||}{\mu}\\
\leq& \hat{\varphi}^T(t)ML[\hat{F}(y(t), z(t), t) - (B_1\otimes I_n)||\hat{\varphi}(t)||^{\frac{p}{2}} - (B_2\otimes I_n)||\hat{\varphi}(t)||\\
&- (B_3\otimes I_n)\tanh(\mu||\hat{\varphi}(t)||) - ||e(t)|| + (\hat{B}_1\otimes I_n)||\bar{\varphi}(t)||^{\frac{p}{2}} - (\hat{B}_2\otimes I_n)||\bar{\varphi}(t)||\\
&- (\hat{B}_3\otimes I_n)\tanh(\mu||\bar{\varphi}(t)||)] + \delta|||\hat{\varphi}(t)||(-\theta k_1||\hat{\varphi}(t)||^p - \theta k_2||\hat{\varphi}(t)|| + \theta k_3\\
&- ||e(t)||) - k_4\sum_{i=1}^{N}\Upsilon_i^{\frac{p+1}{2}}(t) - k_5\sum_{i=1}^{N}\Upsilon_i^{\frac{1}{2}}(t) - k_6\sum_{i=1}^{N}\Upsilon_i^2(t) - \frac{\iota N||\hat{L}||||B_3||}{\mu} - \frac{\iota N||\hat{L}||||\hat{B}_3||}{\mu}.
\end{aligned} \tag{3.52}$$

From Theorem 3.1 and $||\ \bar{\varphi}(t)\ || \leq \frac{\sum_{i=1}^{N}||\ \hat{\varphi}(t)\ ||}{\lambda_{\min}(H)}$, then one can obtain

$$\begin{aligned}
\dot{W}(t) \leq& \Delta\ ||\ \hat{\varphi}(t)\ ||^2 - \frac{ML + L^TM}{2}\ ||\ B_1\ ||\ ||\hat{\varphi}(t)||^{\frac{p+2}{2}} - \frac{ML + L^TM}{2}\ ||\ B_2\ ||\ ||\hat{\varphi}(t)||^2\\
&- \frac{ML + L^TM}{2}\ ||\ B_3\ ||\ ||\hat{\varphi}(t)||\tanh(\mu||\hat{\varphi}(t)||) + ||\hat{\varphi}(t)||ML||e(t)||\\
&+ ||\hat{L}||||\hat{B}_1||||\frac{\sum_{i=1}^{N}||\ \hat{\varphi}(t)\ ||}{\lambda_{\min}(H)}||^{\frac{p+2}{2}} + ||\hat{L}||||\hat{B}_2||||\frac{\sum_{i=1}^{N}||\ \hat{\varphi}(t)\ ||}{\lambda_{\min}(H)}||^2 + ||\hat{L}||||\hat{B}_3|||\hat{\varphi}(t)||\tanh(\frac{\mu\sum_{i=1}^{N}||\ \hat{\varphi}(t)\ ||}{\lambda_{\min}(H)})\\
&- \delta\theta k_1||\hat{\varphi}(t)||^{p+1} - \delta\theta k_2||\hat{\varphi}(t)||^2 + \delta\theta k_3||\hat{\varphi}(t)|| + \delta||\hat{\varphi}(t)||||e(t)||
\end{aligned}$$

$$-k_4\sum_{i=1}^{N}\Upsilon_i^{\frac{p+1}{2}}(t)-k_5\sum_{i=1}^{N}\Upsilon_i^{\frac{1}{2}}(t)-k_6\sum_{i=1}^{N}\Upsilon_i^{2}(t)-\frac{\iota N\|\hat{L}\|\|B_3\|}{\mu}-\frac{\iota N\|\hat{L}\|\|\hat{B}_3\|}{\mu},\tag{3.53}$$

where $\Delta=-\frac{\lambda_*(a_L\lambda_{\max}(B)-2)+2\mathbf{N}^2l^2\xi_{\max}^2}{2\lambda_*}$. Under Definition 2.1, by Lemma 2.2, it follows that

$$\begin{aligned}\dot{W}(t)\leq&\Delta\parallel\hat{\varphi}(t)\parallel^2-\|\hat{L}\|\|B_1\|\|\hat{\varphi}(t)\|^{\frac{p+2}{2}}-\|\hat{L}\|\|B_2\|\|\hat{\varphi}(t)\|^2-\|\hat{L}\|\|B_3\|\|\hat{\varphi}(t)\|\\&+\lambda_{\max}(\hat{L})\|\hat{\varphi}(t)\|\|e(t)\|+\|\hat{L}\|\|\hat{B}_1\|\|\frac{\sum_{i=1}^{N}\parallel\hat{\varphi}(t)\parallel}{\lambda_{\min}(H)}\|^p+\|\hat{L}\|\|\hat{B}_2\|\|\frac{\sum_{i=1}^{N}\parallel\hat{\varphi}(t)\parallel}{\lambda_{\min}(H)}\|^2\\&+\|\hat{L}\|\|\hat{B}_3\|\|\frac{\sum_{i=1}^{N}\parallel\hat{\varphi}(t)\parallel}{\lambda_{\min}(H)}\|-\delta\theta k_1\|\hat{\varphi}(t)\|^{p+1}-\delta\theta k_2\|\hat{\varphi}(t)\|^2+\delta\theta k_3\|\hat{\varphi}(t)\|+\delta\|\hat{\varphi}(t)\|\|e(t)\|\\&-k_4\sum_{i=1}^{N}\Upsilon_i^{\frac{p+1}{2}}(t)-k_5\sum_{i=1}^{N}\Upsilon_i^{\frac{1}{2}}(t)-k_6\sum_{i=1}^{N}\Upsilon_i^{2}(t).\end{aligned}\tag{3.54}$$

Substituting (3.47) into (3.54), one has

$$\begin{aligned}\dot{W}\leq&\Delta\parallel\hat{\varphi}(t)\parallel^2-\|\hat{L}\|\|B_1\|\|\hat{\varphi}(t)\|^{\frac{p+2}{2}}-\|\hat{L}\|\|B_2\|\|\hat{\varphi}(t)\|^2-\|\hat{L}\|\|B_3\|\|\hat{\varphi}(t)|\\&+\lambda_{\max}(\hat{L})\left[-\theta k_1\|\hat{\varphi}(t)\|^{p+1}-\theta k_2\|\hat{\varphi}(t)\|^2+\theta k_3\|\hat{\varphi}(t)\|+\|\hat{\varphi}(t)\|\|\Upsilon(t)\|+\|\hat{\varphi}(t)\|\|\Upsilon^{\frac{1}{2}}(t)\|\right]\\&+\|\hat{L}\|\|\hat{B}_1\|\|\frac{\sum_{i=1}^{N}\parallel\hat{\varphi}(t)\parallel}{\lambda_{\min}(H)}\|^{\frac{p+2}{2}}+\|\hat{L}\|\|\hat{B}_2\|\|\frac{\sum_{i=1}^{N}\parallel\hat{\varphi}(t)\parallel}{\lambda_{\min}(H)}\|^2+\|\hat{L}\|\|\hat{B}_3\|\|\frac{\sum_{i=1}^{N}\parallel\hat{\varphi}(t)\parallel}{\lambda_{\min}(H)}\|-\delta\theta k_1\|\hat{\varphi}(t)\|^{p+1}\\&-\delta\theta k_2\|\hat{\varphi}(t)\|^2+\delta\theta k_3\|\hat{\varphi}(t)\|-\delta\theta k_1\|\hat{\varphi}(t)\|^{p+1}-\delta\theta k_2\|\hat{\varphi}(t)\|^2+\delta\theta k_3\|\hat{\varphi}(t)\|\\&+\delta\|\hat{\varphi}(t)\|\|\Upsilon(t)\|+\delta\|\hat{\varphi}(t)\|\|\Upsilon^{\frac{1}{2}}(t)\|-k_4\sum_{i=1}^{N}\Upsilon_i^{\frac{p+1}{2}}(t)-k_5\sum_{i=1}^{N}\Upsilon_i^{\frac{1}{2}}(t)-k_6\sum_{i=1}^{N}\Upsilon_i^{2}(t).\end{aligned}\tag{3.55}$$

According to Lemma 2.6, it yields

$$\begin{aligned}\dot{W}\leq&\left[\Delta-\lambda_{\min}(\hat{L})\theta k_2+\frac{\|\hat{L}\|\|\hat{B}_2\|}{\lambda_{\min}^2(H)}-2\delta\theta k_2+2\lambda_{\min}(\hat{L})+2\delta-\|\hat{L}\|\|B_2\|\right]|\hat{\varphi}(t)|^2\\&-\left[\|\hat{L}\|\|B_1\|+\lambda_{\min}(\hat{L})\theta k_1-\frac{\|\hat{L}\|\|\hat{B}_1\|}{\lambda_{\min}^{p+1}(H)}+2\delta\theta k_1\right]|\hat{\varphi}^2(t)|^{\frac{p+1}{2}}\\&-\left[\|\hat{L}\|\|B_3\|-\theta k_3-\frac{\|\hat{L}\|\|\hat{B}_3\|}{\lambda_{\min}(H)}-2\delta\theta k_3\right]|\hat{\varphi}^2(t)|^{\frac{1}{2}}\\&-k_4\sum_{i=1}^{N}\Upsilon_i^{\frac{p+1}{2}}(t)-k_5\sum_{i=1}^{N}\Upsilon_i^{\frac{1}{2}}(t)+\frac{\lambda_{\max}(\hat{L})+\delta}{4}\Upsilon_i(t),\end{aligned}\tag{3.56}$$

where $k_6=\dfrac{\lambda_{\max}(\hat{L})+\delta}{4}$, according to Lemma 2.1 the following inequality can be obtained

$$\dot{W}(t)\leq\left[\Delta-\lambda_{\min}(\hat{L})\theta k_2+\frac{\|\hat{L}\|\|\hat{B}_2\|}{\lambda_{\min}^2(H)}-2\delta\theta k_2+2\lambda_{\min}(\hat{L})+2\delta-\|\hat{L}\|\|B_2\|\right]2W_1(t)$$

$$
\begin{aligned}
&-\left[\|\hat{L}\|\|B_1\|+\lambda_{\min}(\hat{L})\theta k_1-\frac{\|\hat{L}\|\|\hat{B}_1\|}{\lambda_{\min}^{p+1}(H)}+2\delta\theta k_1\right]\frac{2^{\frac{1-p}{2}}N^{\frac{1-p}{2}}}{\lambda_{\min}^{\frac{1+p}{2}}(M)}W_1^{\frac{p+1}{2}}\\
&-\left[\|\hat{L}\|\|B_3\|-\theta k_3-\frac{\|\hat{L}\|\|\hat{B}_3\|}{\lambda_{\min}(H)}-2\delta\theta k_3\right]\frac{2^{\frac{1}{2}}}{\lambda_{\min}^{\frac{1}{2}}(M)}W_1^{\frac{1}{2}}\\
&-k_4N^{\frac{1-p}{2}}W_2^{\frac{p+1}{2}}(t)-k_5W_2^{\frac{1}{2}}(t)+k_6W_2(t)\\
\leq&\hat{c}_3W(t)-\hat{c}_1W^{\frac{p+1}{2}}(t)-\hat{c}_2W^{\frac{1}{2}}(t),
\end{aligned}
\tag{3.57}
$$

with $\hat{c}_3=\min\left\{\Delta-\lambda_{\min}(\hat{L})\theta k_2+\frac{\|\hat{L}\|\|\hat{B}_2\|}{\lambda_{\min}^2(H)}-2\delta\theta k_2+2\lambda_{\min}(\hat{L})+2\delta-\|\hat{L}\|\|B_2\|,k_6\right\}$, $\hat{c}_2=\min\left\{\frac{2^{\frac{1}{2}}}{\lambda_{\min}^{\frac{1}{2}}(M)}\left[\|\hat{L}\|\|B_3\|-\theta k_3-\frac{\|\hat{L}\|\|\hat{B}_3\|}{\lambda_{\min}(H)}-2\delta\theta k_3\right],k_5\right\}$, $\hat{c}_1=\min\left\{\left[\|\hat{L}B_1\|+\lambda_{\min}(\hat{L})\theta k_1-\frac{\|\hat{L}\|\|\hat{B}_1\|}{\lambda_{\min}^{p+1}(H)}+2\delta\theta k_1\right]\frac{2^{\frac{1-p}{2}}N^{\frac{1-p}{2}}}{\lambda_{\min}^{\frac{1+p}{2}}(M)},k_4N^{\frac{1-p}{2}}\right\}$. When $t\in[S_m,T_{m+1})$, it yields

$$
\begin{aligned}
\dot{W}(t)=&\varphi^T(t)MLF(y(t),z(t),t)+\delta|\varphi^T(t)|(-\theta k_1\|\varphi(t)\|^p-\theta k_2\|\varphi(t)\|+\theta k_3\\
&-\|e(t)\|)-k_4\sum_{i=1}^N\Upsilon_i^{\frac{p+1}{2}}(t)-k_5\sum_{i=1}^N\Upsilon_i^{\frac{1}{2}}(t)-k_6\Upsilon_i^2(t)-\frac{\iota N\|\hat{L}\|\|B_3\|}{\mu}-\frac{\iota N\|\hat{L}\|\|\hat{B}_3\|}{\mu}\\
\leq&\Delta\|\sum_{i=1}^N\varphi_i(t)\|^2-\delta\theta k_1\|\sum_{i=1}^N\varphi_i(t)\|^{p+1}-\delta\theta k_2\|\sum_{i=1}^N\varphi_i(t)\|^2+\delta\theta k_3\|\sum_{i=1}^N\varphi_i(t)\|\\
&+\delta\|\varphi_i(t)\|\Upsilon_i(t)\|-\delta\theta k_1\|\sum_{i=1}^N\varphi_i(t)\|^{p+1}-\delta\theta k_2\|\sum_{i=1}^N\varphi_i(t)\|^2+\delta\theta k_3\|\sum_{i=1}^N\varphi_i(t)\|\\
&+\delta\|\varphi_i(t)\|\|\Upsilon_i(t)\|^{\frac{1}{2}}-k_4\sum_{i=1}^N\Upsilon_i^{\frac{p+1}{2}}(t)-k_5\sum_{i=1}^N\Upsilon_i^{\frac{1}{2}}(t)-k_6\Upsilon_i^2(t)-\frac{\iota N\|\hat{L}\|\|B_3\|}{\mu}-\frac{\iota N\|\hat{L}\|\|\hat{B}_3\|}{\mu}\\
\leq&\Delta\|\sum_{i=1}^N\varphi_i(t)\|^2-2\delta\theta k_2\|\sum_{i=1}^N\varphi_i^2(t)\|^2+2\delta\|\varphi_i(t)\|^2-k_6\sum_{i=1}^N\Upsilon_i^2(t)+\frac{\delta}{4}\|\Upsilon_i(t)\|^2+\frac{\delta}{4}\|\Upsilon_i(t)\|\\
&+(\delta\theta k_3)^2+\|\sum_{i=1}^N\varphi_i^2(t)\|-\frac{\iota N\|\hat{L}\|\|B_3\|}{\mu}-\frac{\iota N\|\hat{L}\|\|\hat{B}_3\|}{\mu}\\
\leq&(\Delta-2\delta\theta k_2+2\delta+1)\|\sum_{i=1}^N\varphi_i^2(t)\|+\frac{\delta}{4}\sum_{i=1}^N\Upsilon_i(t)\\
\leq&\hat{c}_4V(t),
\end{aligned}
\tag{3.58}
$$

with $\hat{c}_4=\max\left\{\Delta-2\delta\theta k_2+2\delta+1,\frac{\delta}{4}\right\}$, $\frac{\iota N\|\hat{L}\|\|B_3\|}{\mu}+\frac{\iota N\|\hat{L}\|\|\hat{B}_3\|}{\mu}>(\delta\theta k_3)^2$ and $\frac{\delta}{4}=k_6$.

Then the combination of (3.57) and (3.58), there holds

$$
\begin{cases}
\dot{W}(t)\leq\hat{c}_3W(t)-\hat{c}_1W(t)^{\frac{p+1}{2}}-\hat{c}_2W(t)^{\frac{1}{2}}, & t\in[T_m,S_m),\\
\dot{W}(t)\leq\hat{c}_4W(t), & t\in[S_m,T_{m+1}).
\end{cases}
\tag{3.59}
$$

From Lemma 2.4, if $t \geq T^*$, where T^* satisfies

$$T^* \leq \frac{-2 + (\hat{c}_3 - \hat{c}_2)\, T_\epsilon}{(\hat{c}_3 - \hat{c}_2)\, \epsilon} + \frac{1 + (\hat{c}_3 - \hat{c}_1)\,(1 - \hat{\sigma})T_\epsilon}{(\hat{c}_3 - \hat{c}_1)\,(1 - \hat{\sigma})\epsilon}, \tag{3.60}$$

where $\hat{\sigma} = \dfrac{p+1}{2}$. The proof of Theorem 3.4 is finished. □

Remark 3.4. *Compared to references [22, 33], this paper introduces a leader and considers fixed-time consensus under DoS attacks. The types of attacks are more relevant to real-world networks. Additionally, since nonlinear systems and second-order dynamics are prevalent in practical applications, we incorporate a nonlinear term, making the model more applicable to real-world scenarios.*

Next, we demonstrate that the control system, when subject to the control protocol and trigger function, is free from Zeno behavior. Similar to Theorem 3.3, we have $|e(t^i_{k+1})| \leq \int_{t^i_k}^{t^i_{k+1}} \left[U + A\sqrt{2\lambda_{\max}(M)W(t)}\right] C_2 ds$, which yields $t^i_{k+1} - t^i_k \geq \dfrac{\theta k_3}{\left[U + A\sqrt{2\lambda_{\max}(M)W(t)}\right] C_2} > 0$. where $C_2 = p\hat{B}_1\hat{\varphi}^{p-1}(t) + \hat{B}_2 + \mu\hat{B}_3$.

4. Numerical simulation

We will consider a second-order multi-agent with five agents, and the corresponding graph is shown in Figure 1. Based on Figure 1, the Laplacian matrix is given as follows:

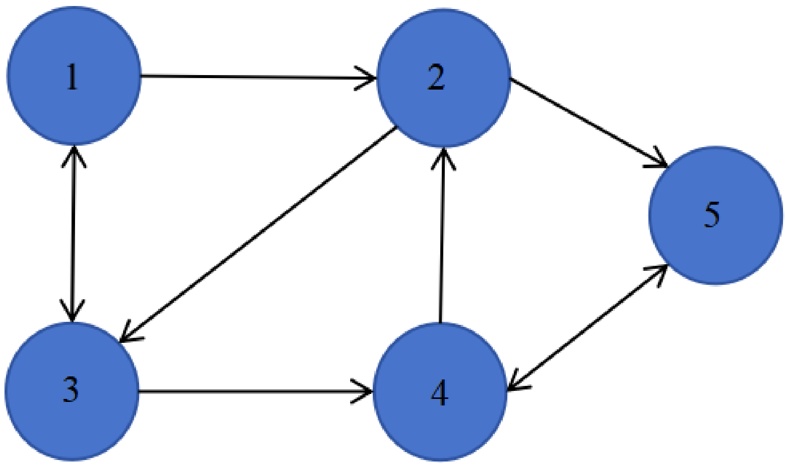
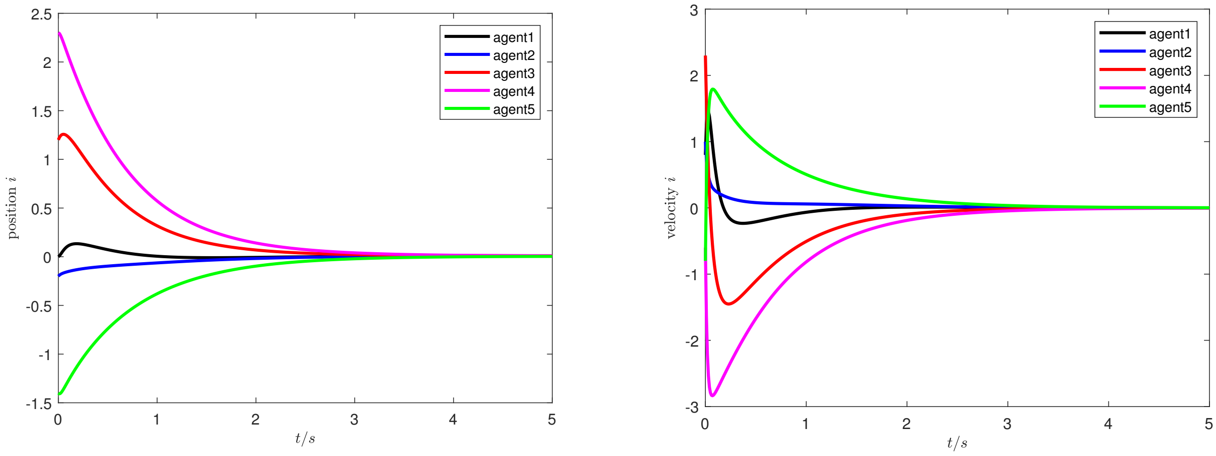


Figure 1. The communication graph.

Based on Figure 1, the Laplacian matrix is given by the following

$$\mathbf{L} = \begin{pmatrix} 2 & -1 & -1 & 0 & 0 \\ 0 & 2 & 0 & -1 & -1 \\ -1 & 0 & 2 & -1 & 0 \\ 0 & -1 & 0 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{pmatrix}.$$

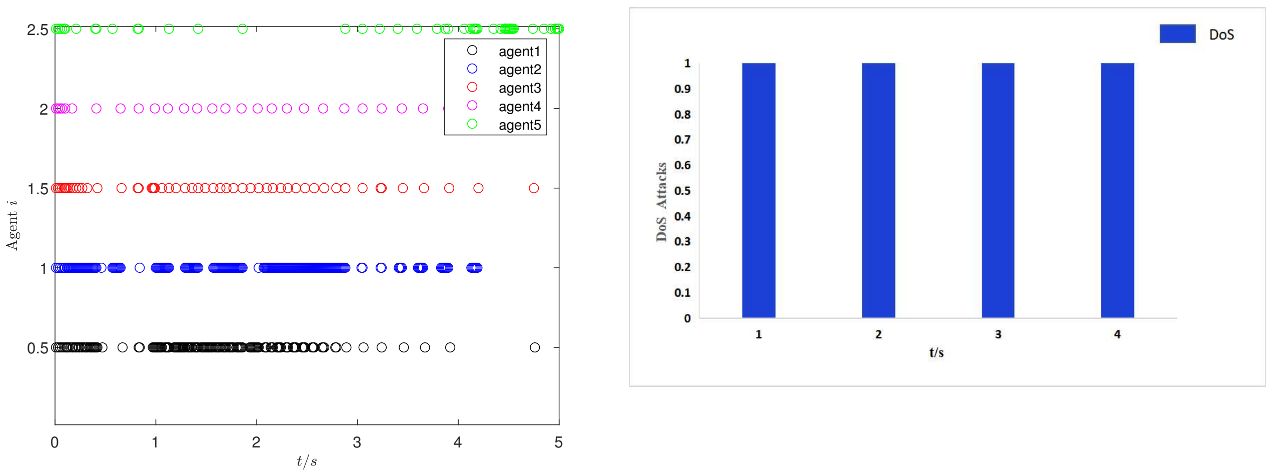
The nonlinear function is given by $f(x_i(t), v_i(t), t) = 0.05cos(x_i(t)) + 0.05sin(v_i(t))$. The initial conditions are $x_i(0) = (0, -0.2, 1.2, 2.3, -1.4)^T$ and $v_i(0) = (0.8, 1, 2.3, -0.6, -0.8)^T$. To guarantee the congditions of Theorem 3.1, simple calculations yield $\alpha = 20, \beta = 15$ and using Lemmas 2.7 and 2.5, we obtain $\lambda_* = 1.238, a_L = 0.191$, these values were calculated with $N^2 = 25$ and $\sigma = 0.3$. The position and velocity trajectories of the agents are shown in Figure 2, and the triggering instants for, event-triggered condition of the multi-agent system (MAS) are presented in Figure 3.



(a) position

(b) velocity

Figure 2. Trajectories of MAS.



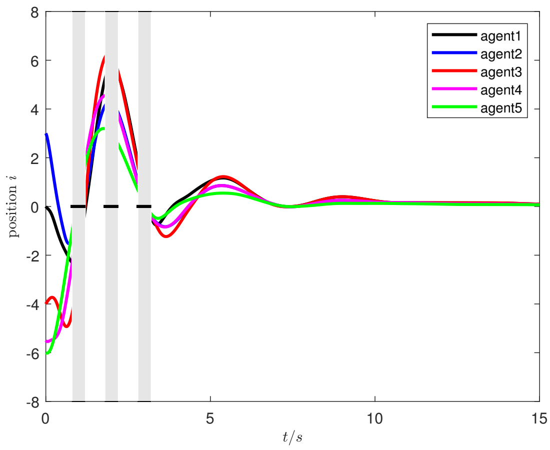
(a) event-triggered

(b) DoS attacks

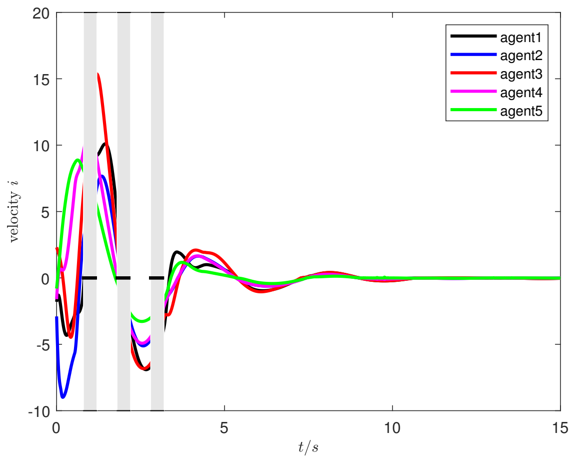
Figure 3. Trajectories of MAS with event-triggered mechanism and DoS attacks.

To satisfy Theorem 3.3, simulations are conducted using the topology shown in Figure 1. The initial conditions are set as $x_i(0) = (0, 3, -4, -5.5, -6)^T$ and $v_i(0) = (-1.8, -2.9, 2.3, -1.6, 0.8)^T$. The system's dynamic variables are defined accordingly. Under the dynamic event-triggered mechanism, the parameters are specified as: $p = \frac{7}{5}, \mu = 500, \theta = 0.8, k_1 = 0.05, k_2 = 0.1, k_3 = 0.1, k_4 = 0.05, k_5 =$

$0.001, k_6 0.031, \alpha_1 = 5, \alpha_2 = 0.5, \alpha_3 = 0.2, \beta_1 = 1, \beta_2 = 0.5, \beta_3 = 0.3$. The attack intervals are defined as $\bigcup_{m=0}^{+\infty} [S_m, T_{m+1})$ and the non-attack rate is set to $\theta = 0.8$, with attack intervals specified as $\bigcup_{g=0.8}^{+\infty} [g, g + 0.4]$, where g is the attack start time. The position and velocity trajectories of the agents are illustrated in Figure 4, respectively.



(a) position



(b) velocity

Figure 4. Trajectories of Leaderless MAS.

We consider the following topology with one leader and five followers, as shown in Figure 5. The initial conditions for the system are $x_i(0) = (0, 3, -4, -5.5, -6, -1)^T$ and $v_i(0) = (-1.8, -2.9, 2.3, -1.6, -0.8)^T$. The initial values of the dynamic variables are given by $\Upsilon_i(t) = (1, 1, 1, 1, 1)^T$. Under the dynamic event-triggered mechanism, the parameters are specified as $p = \frac{7}{5}, \mu = 5, \theta = 10, k_1 = 10, k_2 = 15, k_3 = 20, k_4 = 35, k_5 25, k_6 1, \alpha_1 = 0.5, \alpha_2 = 5, \alpha_3 = 1, \beta_1 = 5, \beta_2 = 1, \beta_3 = 3, \delta = 0.69$. The position and velocity trajectories, as well as event-triggered mechanisms for the agents, are shown in Figures 6 and 7, respectively.

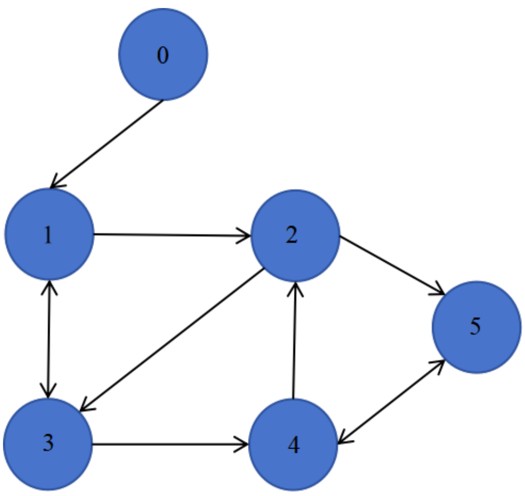
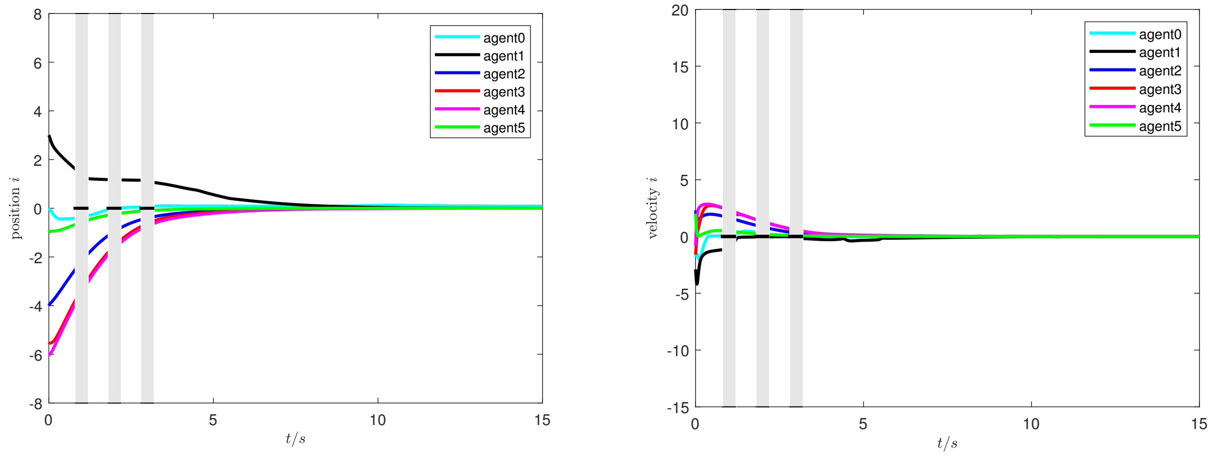
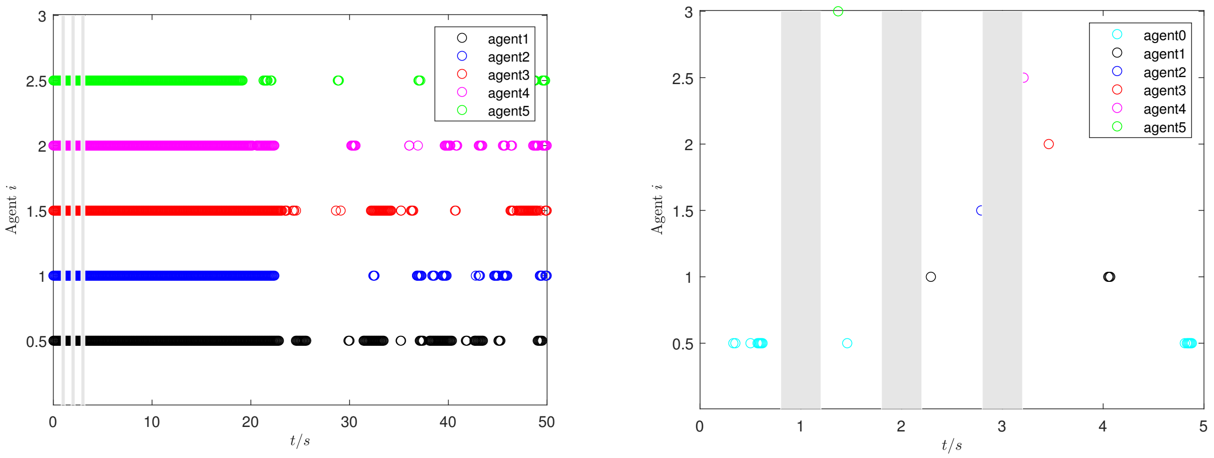


Figure 5. The communication graph.



(a) position (b) velocity

Figure 6. Trajectories of Leader MAS.



(a) leaderless (b) leader

Figure 7. Trajectories of MAS with event-triggered mechanism.

5. Conclusions

In this paper, we have proposed a fixed-time consensus protocol based on a dynamic event-triggered strategy. This protocol enables a nonlinear second-order leader-follower multi-agent system, operating under a directed topology, to achieve consensus in fixed-time. In contrast to finite-time consensus protocols, the fixed-time consensus protocol ensure that the consensus time is independent of the system's initial conditions. Finally, simulation examples demonstrate the feasibility of the proposed theory. Future work will focus on addressing computational complexity and extending the approach to higher-order multi-agent systems. In addition, other types of DoS attacks, such as random or protocol-aware DoS attacks, will be considered.

Author contributions

Jiaqi Liang: Writing–original draft, validation; Zhanheng Chen: Writing–review & editing, visualization, supervision; Zhiyong Yu: Conceptualization, supervision, writing–review & editing; Haijun Jiang: Supervision, conceptualization, formal analysis, resources. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflict of interest.

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