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Research article

A new H_∞ control method of switched nonlinear systems with persistent dwell time: H_∞ fuzzy control criterion with convergence rate constraints

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Abstract: This study aims to explore the problem of H_∞ fuzzy control with an adjustable convergence rate for switched nonlinear systems with time-varying delays under the persistent dwell time (PDT) switching. Compared to the widely studied dwell time (DT) switching or average dwell time (ADT) switching in existing literature, PDT switching provides a more comprehensive consideration of the switching frequency and has a broader range of applicability. Subsequently, by combining the interval stability definition, T-S fuzzy model, PDT technique, and Lyapunov-Krasovskii (L-K) functional, a new H_∞ fuzzy control criterion for adjusting the convergence rate of switched nonlinear systems with time-varying delays is proposed. This criterion enables the development of a novel method for constructing H_∞ fuzzy controllers, which can regulate the system's convergence rate and achieve the specified H_∞ performance. Combining the above methods, an algorithm is introduced to precisely control the convergence rate of the target system. Finally, the effectiveness of this method is validated through a control example of a single-link robot arm.

Keywords: switched T-S fuzzy systems; H_∞ control; convergence rate; persistent dwell time; time-varying delay
Mathematics Subject Classification: 34D20, 93B36, 93C42, 93D09, 93D20

1. Introduction

Switched systems are constituted of a series of discrete-time or continuous-time subsystems known as modes, and rules for coordinating the switching of modes. This system finds wide applications in diverse fields, including network control systems, robotic arms, and wireless communication systems [1]. However, the majority of current research is focused on switched linear systems [2, 3]. In practice, practical switched systems composed of electronic or mechanical components frequently exhibit nonlinear characteristics. Examples include mobile robots [4], cars [5], and DC converters [6, 7]. Therefore, research on switched nonlinear systems is essential. The nonlinearity of the system magnifies the complexity associated with analyzing and synthesizing control problems.

Takagi and Sugeno introduced the T-S fuzzy model in 1985, leveraging the principles of fuzzy logic [8]. Owing to its remarkable ability to approximate any smooth nonlinear systems, this model effectively bridges the gap between nonlinear systems and linear control [9, 10]. For nonlinear switched systems, each nonlinear subsystem is first approximated as a T-S model, namely a fuzzy subsystem [11]. By introducing switching rules, these fuzzy subsystems form the entire switched fuzzy system. This method provides practical approaches for synthesizing and analyzing controls in complex switched nonlinear systems [12, 13]. However, time-varying delay is commonly encountered in practical engineering applications, and it has a complex impact on the dynamics of the system [14]. Since delay is a significant factor that leads to system instability and performance degradation, conducting stability analysis and designing controllers for switched nonlinear systems with time-varying delay have always been one of the primary concerns for researchers [15–18].

Research shows that the switching mechanism profoundly influences the behavior of switched systems, and our goal is to find a satisfactory switching mechanism. In 1996, Morse and Hespanha proposed a switching strategy called dwell time switching regularity (DTSR) in [19]. Subsequently, average dwell time switching regularity (ADTSR) was studied in [20] to overcome the limitation in DTSR where each switching interval must be no less than the dwell time τ, but its switching frequency is still limited. To address the aforementioned issue, the reference [21] introduced the concept of persistent dwell time switching regularity (PDTSR), where the switching signal can be divided into a slow switching signal with a dwell time not less than τ and a fast switching signal with a switching period not greater than T. Therefore, PDT proves useful in describing switched systems that exhibit a combination of slow and fast switching characteristics. This includes systems that may face abrupt and sporadic failures [22].

It is worth noting that PDTSR distinguishes the T-part and τ-part by utilizing the dwell time τ instead of imposing restrictions on the intervals. As a result, PDTSR offers broader applicability compared to DTSR and ADTSR in terms of the limitations imposed by shorter intervals. Furthermore, previous research [21] has established that the ADT and DT are regarded as specific instances of PDT, thus emphasizing the widespread applicability of PDT. Recent years have witnessed a gradual growth in research concerning PDT switching strategies [23–25]. Nonetheless, limited research exists regarding controller design for PDT switching in continuous-time switched systems. This research gap serves as one of the motivations for conducting this paper.

Accurate control of the dynamic performance of a control system, particularly the rate of reaching equilibrium, is highly significant. Pole placement in linear systems allows for the assignment of eigenvalues to desired pole positions, resulting in enhanced system performance. Interval stability [26] is an extension of pole placement that indicates the convergence rate of a system. It holds significant importance in the realm of control systems. In recent years, the author has utilized the interval stability theorem to investigate the state feedback interval stabilization problem for different types of linear systems [27–30]. However, achieving precise control of nonlinear systems is exceptionally challenging. By using the T-S fuzzy model, we can bridge the gap between nonlinear systems and linear control, enabling the extension of the research discussed earlier to nonlinear systems. This is one of the motivations of the study. This paper examines the issue of adjustable convergence rate H_∞ control for nonlinear switched systems under PDT switching. This paper makes the following key contributions.

(1). This article employs the PDT switching strategy, which involves both fast and slow switching processes. It can be simplified into DT or ADT switching, making it a more versatile method. Additionally, when utilizing PDT switching signals, it is possible to obtain non-weighted H_∞ performance, which holds greater physical significance when contrasted with weighted outcomes [31].

(2). This paper presents the first introduction of H_∞ control with constraints on the convergence rate in a switched nonlinear system with time-varying delay. Using the interval stability and T-S fuzzy model, the H_∞ fuzzy control criterion with a convergence rate constraint is devised. The new H_∞ control criterion can accurately control the target system and has certain anti-interference abilities.

(3). According to the criterion above, a new construction approach of the H_∞ fuzzy controller with a convergence rate constraint is introduced. The H_∞ fuzzy controller not only ensures the stability of the system, but also precisely controls the convergence rate of the state, and satisfies specified performance requirements.

(4). Combined with the above methods, an algorithm is designed to realize the precise control of the convergence rate of the system state at the operation level.

The structure of the remaining sections in this paper is outlined below. Section II provides a problem description, necessary theorems and definitions, and illustrates the concept of PDT switching strategies. Section III applies the T-S fuzzy modeling technique, interval stability definition, and L-K functionals to derive stability criteria for the adjustable convergence rate of time-varying delayed switched nonlinear systems based on PDT, as well as non-weighted H_∞ performance. Section IV designs an H_∞ fuzzy controller with a constraint on the convergence rate based on the sufficient conditions mentioned above. Section V verifies the effectiveness of the methodology through a control example involving a single-link robot arm. Finally, Section VI summarizes the conclusions of this paper.

Notation: $\lambda_{min}(A)$ and $\lambda_{max}(A)$ respectively mark the minimum and maximum eigenvalues of matrix A; C_{-b}^{-a} is the strip domain representing $(-b, -a)$ on a complex plane; $\|\cdot\|$ expresses the Euclidean norm; and $\sigma(A)$: is the eigenvalue set of the matrix "A".

2. Problem description and preliminaries

Consider the switched nonlinear systems

$$\begin{cases} \dot{x}(t) = q_{\sigma(t)}(x(t), x(t-d(t)), u(t), w(t)), \\ y(t) = h_{\sigma(t)}(x(t), w(t)), \\ x(\beta) = \phi(\beta),\ \beta \in [-d, 0], \end{cases} \tag{2.1}$$

where $x(t) \in \mathbb{R}^m$ represents the state vector; $u(t) \in \mathbb{R}^n$ is a continuously controlled input; $y(t) \in \mathbb{R}^p$ means the measured output; and $w(t) \in \mathbb{R}^q$ represents a continuously disturbed input that belongs to $\mathcal{L}_2[0, \infty)$. The functions q and h are nonlinear and ensure the existence of a unique strong solution to the aforementioned system, adhering to the local Lipschitz condition. $\sigma(t)$: $[0, \infty) \longrightarrow \mathcal{M} = \{1, 2, \cdots, s\}$ means the switching signal, which represents a piecewise constant function with respect to time. $\phi(t)$ represents the initial condition defined on $[-d, 0]$, and $d(t)$ denotes the time-varying delay satisfying

$$\dot{d}(t) \leq \epsilon, \quad 0 \leq d(t) \leq d, \tag{2.2}$$

where ϵ and d are the upper bound of $d(t)$ and $\dot{d}(t)$. To maintain universality, $t_0 = 0$ is assumed in this article.

The switching nonlinear system (2.1) is expressed by the following fuzzy rules, and its i-th subsystem is represented as follows.

R_i^n : IF $\vartheta_{i1}(t)$ is M_{i1}^n and $\cdots$ and $\vartheta_{ie_i}(t)$ is $M_{ie_i}^n$,
THEN

$$\begin{cases} \dot{x}(t) = A_{in}x(t) + E_{in}x(t-d(t)) + H_{in}u_i(t) + B_{in}w(t), \\ y(t) = C_{in}x(t) + D_{in}w(t), \\ x(\beta) = \phi(\beta),\ \beta \in [-d, 0], \end{cases}$$

where $M_{i1}^n, \cdots, M_{ie_i}^n$ are fuzzy sets, $n \in \mathcal{R} = \{1, 2, \cdots, r_i\}$, $i \in \mathcal{M}$, where r_i represents the total number of IF-THEN rules. $\vartheta_i(t) = [\vartheta_{i1}(t), \vartheta_{i2}(t), \cdots, \vartheta_{ie_i}(t)]$ represents the premise variables. $A_{in}, E_{in}, H_{in}, B_{in}, C_{in}$, and D_{in} are real constant matrices.

Introduce

$$h_{in}(\vartheta_i(t)) = \frac{\prod_{k=1}^{e_i} M_{ik}^n(\vartheta_{ik}(t))}{\sum_{n=1}^{r_i}\prod_{k=1}^{e_i} M_{ik}^n M_{ik}^n(\vartheta_{ik}(t))} \geq 0,$$

where $M_{ik}^n(\vartheta_{ik}(t))$ represents the membership function grade of premise $\theta_{ik}(t)$ in M_{ik}^n. Then, for $n \in \mathcal{R}$, $i \in \mathcal{M}$, $h_{in}(\vartheta_i(t)) > 0$, $\sum_{n=1}^{r_i} h_{in}(\vartheta_i(t)) = 1$. By the T-S fuzzy inference method, system (2.1) is denoted as

$$\begin{cases} \dot{x}(t) = \sum_{n=1}^{r_{\sigma(t)}} h_{\sigma(t)n}(\vartheta_{\sigma(t)}(t))[A_{\sigma(t)n}x(t) + E_{\sigma(t)n}x(t-d(t)) + H_{\sigma(t)n}u_{\sigma(t)}(t) + B_{\sigma(t)n}w(t)], \\ y(t) = \sum_{n=1}^{r_{\sigma(t)}} h_{\sigma(t)n}(\vartheta_{\sigma(t)}(t))[C_{\sigma(t)n}x(t) + D_{\sigma(t)n}w(t)], \\ x(\beta) = \phi(\beta),\ \beta \in [-d, 0]. \end{cases} \tag{2.3}$$

In this study, our design methodology involves using a fuzzy controller for each individual fuzzy subsystem. Specifically, the fuzzy controller and system (2.3) share the same fuzzy inference premise variables.

R_{ic}^n : IF $\vartheta_{i1}(t)$ is M_{i1}^n and $\cdots$ and $\vartheta_{ie_i}(t)$ is $M_{ie_i}^n$,
THEN

$$u_i(t) = K_{in}x(t),\ (i = 1, 2, \cdots, m,\ n = 1, 2, \cdots, r_i),$$

where K_{in} represents the controller gain to be denoted for $\sigma(t) = i \in \mathcal{M}$ and $n \in \mathcal{R}$, and the overall control is defined as follows

$$u_{\sigma(t)}(t) = \sum_{n=1}^{r_{\sigma(t)}} h_{\sigma(t)n}(\vartheta_{\sigma(t)}(t))K_{\sigma(t)n}x(t).$$

Definition 1. *[21] For all $\tau > 0$, $T \in [0, \infty)$, the set $\mathcal{S}_{PDT}[\tau, T]$ is considered a PDT signal set when it includes an infinite number of disjoint intervals, each with a duration of at least τ, where $\sigma(t)$ remains constant. Additionally, consecutive intervals with this property must have a separation of no more than T. The constant τ is referred to as the persistent dwell time, while T represents the period of persistence.*

As depicted in Figure 1, the operational process of the system within the PDT scheme can be segmented into several stages. Each stage is composed of a τ-portion followed by a T-portion. In the T-portion, multiple switches will occur, with the interval between any two adjacent switches always being shorter than τ, and the total persistent of the T-portion does not surpass T. In the τ-portion, no switching occurs; instead, a subsystem is activated, which operates for a minimum duration of τ_D. When considering the system in the q-th stage, switches occur at t_{s_q}, $t_{s_q+1}, \cdots$, $t_{s_{q+1}}$. Here, t_{s_q+1} is the next switching time after t_{s_p} in the q-th stage, while $t_{s_{q+1}}$ is the switching time when the system enters the $(q+1)$-th stage. During the τ-portion of stage q, where $t \in [t_{s_q}, t_{s_q+1})$, the running time of a specific subsystem satisfies $\tau_q \geq \tau_D$. During the T-portion of the q-th stage, where $t \in [t_{s_q+1}, t_{s_{q+1}})$, the actual running time of the system t_q satisfies

$$T_q = \sum_{r=1}^{N_\sigma(t_{s_q+1}, t_{s_{q+1}})} T(t_{s_q+r}, t_{s_q+r+1}) \leq T,$$

where $T(t_m, t_n) = t_n - t_m$, and $N(t_m, t_n)$ represents the overall count of switching events that occur within the time interval $[t_m, t_n)$. For any $0 \leq t_m \leq t_s \leq t_n$, we have $N(t_m, t_n) = N(t_m, t_s) + N(t_s, t_n)$. Here, we assume that the switching frequency in the T-portion is f_q, which satisfies

$$f_q = \frac{N(t_{s_q+1}, t_{s_{q+1}})}{T(t_{s_q+1}, t_{s_{q+1}})},$$

and $f = max\{f_q\}$.

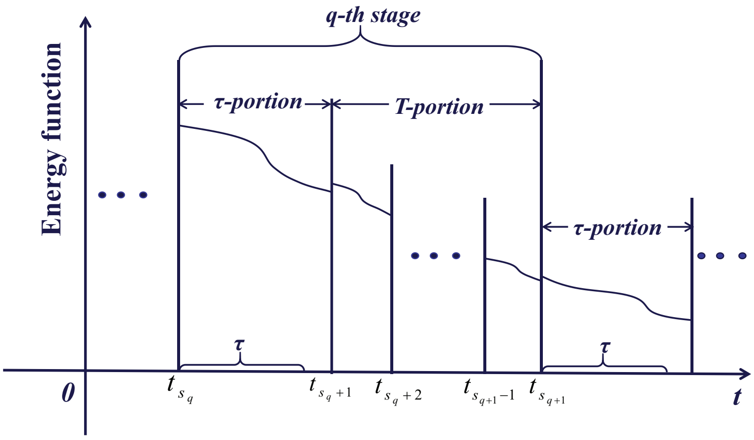

Figure 1. PDT switching strategy.

Remark 1. *The total number of switch events in the T-portion of the q-th stage is $N(t_{s_q+1}, t_{s_{q+1}}) = f_q T(t_{s_q+1}, t_{s_{q+1}}) \leq fT$. In a practical system, it is not possible to have an unlimited number of switches during a finite time period. In this paper, the switching frequency f_q is introduced to limit the number of switches in the T-portion.*

Remark 2. *The DTSR state that each interval's duration in the activated subsystem's running time should be greater than or equal to the dwell time τ, implying that DTSR should not encompass shorter time intervals. In order to balance these intervals, ADTSR introduces a relatively longer interval, but* *its switching frequency is still subject to significant limitations. Then, PDTSR concentrates all the relatively shorter intervals into the T-portion, thus overcoming the aforementioned problem. The remaining disjoint intervals are referred to as the τ-portion. PDTSR uses only the dwell time to distinguish between the τ-portion and T-portion, instead of imposing restrictions on the intervals. Therefore, from the perspective of the limitations of short intervals, PDTSR has a wider applicability compared to DTSR and ADTSR. Moreover, previous research has demonstrated the relationships between DT and ADT, classifying them as specific instances within the PDT framework. This classification results in a more general PDT switching rule.*

Definition 2. *[32] For $\forall x(0) \in \mathbb{R}^m$, the system (2.3) with $w(t) = 0$ is asymptotically stable (AS) if*

$$\lim_{t\to\infty} \|x(t)\|^2 = 0.$$

Definition 3. *[27] The systems (2.3) is asymptotically interval stable (AIS) in $(-b, -a)$ under a certain switching signal $\sigma(t)$ if system (2.3) is AS and*

$$\sigma(A_i) \subset C_{-b}^{-a} := \{\lambda| - b < Re(\lambda) < -a\}.$$

Definition 4. *[33] Let $\alpha > 0$ and $\gamma > 0$ be the positive constants. For the system (2.3), if there is a switching signal $\sigma(t)$ satisfies PDT, such that*
(1) when $w(t) = 0$, system (2.3) is AIS in $(-b, -a)$;
(2) the L_2-gain from the disturbance input $w(t)$ to $y(t)$ can be expressed as being smaller than γ in terms of the mean square, or in other words

$$\int_0^\infty y^T(s)y(s)ds \leq \gamma^2 \int_0^\infty w^T(s)w(s)ds, \tag{2.4}$$

for the zero initial condition $x(t) = 0,\ t \in [-d, 0]$ and any nonzero $w(t) \in L_2[0, \infty)$. Then the system (2.3) is AIS in $(-b, -a)$ with a non-weighted H_∞ performance γ.

Remark 3. *An asymptotic interval $(-b, -a)$ stability with H_∞ level offers the more precise characterization compared to the existing H_∞ level asymptotic mean-square stability. According to Definition 4, the stability of system (2.3) is ensured by providing a level of H_∞ disturbance attenuation, while also controlling the convergence rate to stability via the arrangement of eigenvalues in the system matrix.*

Lemma 1. *[27] Given any matrices $S > 0$ and R, there is*

$$-R^T S^{-1} Z R \leq S - R - R^T.$$

Lemma 2. *[34] For matrix $G > 0$ and given constants n and m, if the function x: $[n, m] \to \mathbb{R}^n$ is continuous differentiable, then the following inequality holds*

$$\int_n^m \dot{x}^T(s)G\dot{x}(s)ds \geq \frac{1}{m-n}(\int_n^m \dot{x}(s)ds)^T G(\int_n^m \dot{x}(s)ds).$$

3. Interval stability and L_2-gain performance

This section aims to explore interval stability and L_2-gain analysis issues concerning the system (2.3) with $u_{\sigma(t)}(t) = 0$ under PDT switching.

Theorem 1. *Consider the system (2.3), where $\alpha > 0$, $\epsilon > 0$, $\mu > 1$, and d are fixed constants. Given a prescribed period of frequency f and persistence T, assume the existence of matrices $Q_i > 0$, $P_i > 0$, $R_i > 0$, N_i, M_i, and T_i, $\forall i \in \mathcal{M}$, along with a scalar γ, satisfying the following matrix inequalities:*

$$\Theta = \begin{bmatrix} \Theta_{11} & P_i(-E_{in})^{\mathrm{T}} - M_i + N_i^{\mathrm{T}} & -M_i + T_i^T & P_i(-B_i) + C_{in}^T D_{ij} & d(-A_{in} - bI)^T R_i & C_{in}^T \\ * & \Theta_{22} & -N_i - T_i^T & 0 & d(-E_{in})^T R_i & 0 \\ * & * & \Theta_{33} & 0 & 0 & 0 \\ * & * & * & D_{in}^T D_{in} - \gamma^2 I & dB_{in} R_i & 0 \\ * & * & * & * & -dR_i & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{3.1}$$

$$\bar{\Theta} = \begin{bmatrix} \bar{\Theta}_{11} & P_i E_{in}^{\mathrm{T}} - M_i + N_i^{\mathrm{T}} & -M_i + T_i^T & P_i B_i + C_{in}^T D_{in} & d(A_{in} + aI)^T R_i & C_{in}^T \\ * & \Theta_{22} & -N_i - T_i^T & 0 & dE_{in}^T R_i & 0 \\ * & * & \Theta_{33} & 0 & 0 & 0 \\ * & * & * & D_{in}^T D_{in} - \gamma^2 I & dB_{in} R_i & 0 \\ * & * & * & * & -dR_i & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{3.2}$$

$$P_i \leq \mu P_l, Q_i \leq \mu Q_l, R_i \leq \mu R_l, \forall i, l \in \mathcal{M}, \tag{3.3}$$

where

$$\begin{aligned} \Theta_{11} &= (-A_{in} - bI)^{\mathrm{T}} P_i + P_i(-A_{in} - bI) + Q_i + \alpha P_i + M_i + M_i^{\mathrm{T}}, \\ \Theta_{22} &= -(1-\epsilon)e^{-\alpha d} Q_i - N_i - N_i^{\mathrm{T}}, \quad \Theta_{33} = \frac{e^{-\alpha d}}{d}(\hat{P}_i + \hat{P}_i^T) - \hat{T}_i - \hat{T}_i^T, \\ \bar{\Theta}_{11} &= (A_{in} + aI)^{\mathrm{T}} P_i + P_i(A_{in} + aI) + Q_i + \alpha P_i + M_i + M_i^{\mathrm{T}}, \end{aligned}$$

for any PDT switching signal satisfying

$$\tau_D > \tau_D^* = max\{\frac{(Tf + 1)ln\mu}{\alpha} - T, \frac{1}{f}\}, \tag{3.4}$$

and the system (2.3) is AIS in $(-b, -a)$ with an L_2-gain no greater than

$$\gamma_s = \sqrt{-\frac{\alpha\mu^{Tf+1}}{\varrho}}\gamma, \tag{3.5}$$

with

$$\varrho = \frac{(Tf + 1)ln\mu}{T + \tau_D} - \alpha.$$

Proof. The initial step is to demonstrate that when $w(t) = 0$ and $u(t) = 0$, the system (2.3) is AIS in $(-b, -a)$. According to Definition 3, if the following systems

$$\begin{cases} \dot{x}(t) = \sum_{n=1}^{r_i} h_{in}(\vartheta_i(t))[(-A_{in} - bI)x(t) - E_{in}x(t-d(t))], \\ y(t) = \sum_{n=1}^{r_i} h_{in}(\vartheta_i(t))C_{in}x(t), \\ x(\beta) = \phi(\beta),\ \beta \in [-d, 0], \end{cases} \tag{3.6}$$

and

$$\begin{cases} \dot{x}(t) = \sum_{n=1}^{r_i} h_{in}(\vartheta_i(t))[(A_{in} + aI)x(t) + E_{in}x(t-d(t))], \\ y(t) = \sum_{n=1}^{r_i} h_{in}(\vartheta_i(t))C_{in}x(t), \\ x(\beta) = \phi(\beta),\ \beta \in [-d, 0], \end{cases} \tag{3.7}$$

are AS, then system (2.3) is AIS in $(-b, -a)$.

Suppose the i-th subsystem operates within the time interval $[t_k, t_{k+1})$. Let us consider the selection of multiple Lyapunov-Krasovskii functionals for this subsystem

$$V_i(x(t)) = V_i^1(x(t)) + V_i^2(x(t)) + V_i^3(x(t)), \tag{3.8}$$

where

$$\begin{aligned} V_i^1(x(t)) &= x^{\mathrm{T}}(t)P_i x(t), \\ V_i^2(x(t)) &= \int_{t-d(t)}^{t} e^{\alpha(s-t)} x^{\mathrm{T}}(s)Q_i x(s)ds, \\ V_i^3(x(t)) &= \int_{-d}^{0}\int_{t+\theta}^{t} e^{\alpha(s-t)} \dot{x}^{\mathrm{T}}(s)R_i \dot{x}(s)dsd\theta. \end{aligned}$$

Next, we differentiate the multiple Lyapunov-Krasovskii functionals with respect to the system (3.6) to obtain the derivative:

$$\begin{aligned} \dot{V}_i^1(x(t)) =& \sum_{n=1}^{r_i} h_{in}(\vartheta_i(t))\{x^{\mathrm{T}}(t)[(-A_{in} - bI)^{\mathrm{T}}P_i + P_i(-A_{in} - bI)]x(t) + x^{\mathrm{T}}(t-d(t))(-E_{in})^{\mathrm{T}}P_i x(t) \\ &+ x^{\mathrm{T}}(t)P_i(-E_{in})x(t-d(t))\}, \\ \dot{V}_i^2(x(t)) \le& -\alpha \int_{t-d(t)}^{t} e^{\alpha(s-t)} x^{\mathrm{T}}(s)Q_i x(s)ds - (1-\epsilon)e^{-\alpha d}x^{\mathrm{T}}(t-d(t))Q_i x(t-d(t)) + x^{\mathrm{T}}(t)Q_i x(t), \\ \dot{V}_i^3(x(t)) \le& -\alpha \int_{-d}^{0}\int_{t+\theta}^{t} e^{\alpha(s-t)} \dot{x}^{\mathrm{T}}(s)R_i \dot{x}(s)dsd\theta + d\dot{x}^{\mathrm{T}}(t)R_i\dot{x}(t) - \frac{e^{-\alpha d}}{d}(\int_{t-d(t)}^{t} \dot{x}(s)ds)^{\mathrm{T}}R_i(\int_{t-d(t)}^{t} \dot{x}(s)ds). \end{aligned}$$

There exist the following relationship among the matrices M_i, T_i, and N_i:

$$2[x^{\mathrm{T}}(t)M_i + x^{\mathrm{T}}(t-d(t))N_i + \int_{t-d(t)}^{t} \dot{x}^{\mathrm{T}}(s)ds \cdot Ti] \times [x(t) - \int_{t-d(t)}^{t} \dot{x}(s)ds - x(t-d(t))] = 0. \tag{3.9}$$

Focusing on Eqs (3.8) and (3.9) produces

$$\dot{V}_i(x(t)) + \alpha V_i(x(t)) \leq \sum_{n=1}^{r_i} h_{in}(\vartheta_i(t))\pi^{\mathrm{T}}(t)\Lambda_{in}\pi(t),$$

where

$$\begin{aligned}
&\pi^{\mathrm{T}}(t) = \begin{bmatrix} x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-d(t)) & (\int_{t-d(t)}^{t}\dot{x}(s)ds)^{\mathrm{T}} \end{bmatrix},\\
&\Lambda_{in} = \begin{bmatrix} \Lambda_{in11} & P_i(-E_{in})^{\mathrm{T}} + d(-A_{in}-bI)^{\mathrm{T}}R_i(-E_{in}) - M_i + N_i^{\mathrm{T}} & -M_{in} + T_{in}^{\mathrm{T}} \\ * & -(1-\epsilon)e^{-\alpha d}Q_i - d(-E_{in})^{\mathrm{T}}R_i(-E_{in}) - N_i - N_i^{\mathrm{T}} & -N_{in} - T_{in}^{\mathrm{T}} \\ * & * & -\frac{-\alpha d}{d}R_i - T_i - T_i^{\mathrm{T}} \end{bmatrix},\\
&\Lambda_{in11} = (-A_{in}-bI)^{\mathrm{T}}P_i + P_i(-A_{in}-bI) + Q_i + M_i + M_i^{\mathrm{T}} + d(-A_{in}-bI)^{\mathrm{T}}R_i(-A_{in}-bI) + \alpha P_i.
\end{aligned}$$

In terms of the Schur complement theorem, we can readily deduce the aforementioned inequality to

$$\dot{V}_i(t) = -\alpha V_i(t). \tag{3.10}$$

Moreover, according to Eq (3.3), $\forall(i,l) \in \mathcal{M}\times\mathcal{M}, i \neq l$, we have

$$V_i(x(t)) = \mu V_l(x(t)). \tag{3.11}$$

Taking into account (3.10) and (3.11), one gains

$$\begin{aligned}
V_{\sigma(t_{s_{q+1}})}(x(t_{s_{q+1}})) &\leq e^{-\alpha(t_{s_{q+1}}-t_{s_{q+1}-1})}V_{\sigma(t_{s_{q+1}})}(x(t_{s_{q+1}-1})) \leq \mu e^{-\alpha(t_{s_{q+1}}-t_{s_{q+1}-1})}V_{\sigma(t_{s_{q+1}-1})}(x(t_{s_{q+1}-1}))\\
&\leq \cdots \leq \mu^{N_{\sigma(t_{s_{q+1}},t_{s_{q+1}+1})}}e^{-\alpha(t_{s_{q+1}}-t_{s_q})}V_{\sigma(t_{s_q})}(x(t_{s_q})) \leq \mu^{T_qf+1}e^{-\alpha(\tau_q+T_q)}V_{\sigma(t_{s_q})}(x(t_{s_q}))\\
&\leq \mu^{T_qf+1}e^{-\alpha(\tau+T_q)}V_{\sigma(t_{s_q})}(x(t_{s_q})).
\end{aligned}$$

According to the above relation, it can be obtained that

$$\begin{aligned}
V_{\sigma(t_{s_q})}(x(t_{s_q})) &\leq \mu^{T_{q-1}f+1}e^{-\alpha(\tau+T_{q-1})}V_{\sigma(t_{s_{q-1}})}(x(t_{s_{q-1}}))\\
&\leq \mu^{T_{q-1}f+1}e^{-\alpha(\tau+T_{q-1})}\mu^{T_{q-2}f+1}e^{-\alpha(\tau+T_{q-2})}V_{\sigma(t_{s_{q-2}})}(x(t_{s_{q-2}}))\\
&\leq \cdots \leq \prod_{r=1}^{q-1}\mu^{T_rf+1}e^{-\alpha(\tau+T_r)}V_{\sigma(t_{s_1})}(x(t_{s_1})).
\end{aligned} \tag{3.12}$$

Then according to (3.8), it can be computed that

$$\rho_m\|x(t_{s_q})\|^2 \leq \rho_M\prod_{r=1}^{q-1}\mu^{T_rf+1}e^{-\alpha(\tau+T_r)}\|x(t_{s_1})\|^2,$$

where $\rho_M = \underset{i\in\mathcal{M}}{max}\{\lambda_{max}\{P_i\}\} + d\underset{i\in\mathcal{M}}{max}\{\lambda_{max}\{Q_i\}\} + \frac{d^2}{2}\underset{i\in\mathcal{M}}{max}\{\lambda_{max}\{R_i\}\}, \rho_m = \underset{i\in\mathcal{M}}{min}\{\lambda_{min}\{P_i\}\}$.

On the other hand, foll all $t \in [t_{s_q}, t_{s_{q+1}})$, we combine (3.10) and (3.11) to get

$$\rho_m\|x(t)\|^2 \leq \rho_M\mu^{T_qf}\|x(t_{s_q})\|^2.$$

Taking note of $t_{s_1} = t_0$, $\forall t \in [t_{s_q}, t_{s_{q+1}})$, we have

$$\rho_m \|x(t_{s_q})\|^2 \leq \rho_M \mu^{T_q f} \frac{\rho_M}{\rho_m} \prod_{r=1}^{q-1} \mu^{T_r f+1} e^{-\alpha(\tau+T_r)} \|x(t_0)\|^2.$$

Summarizing the above, it is possible to obtain

$$\|x(t)\| \leq \xi \prod_{r=1}^{q-1} \sqrt{\beta_r} \|x(t_0)\|, \tag{3.13}$$

where $\xi = \frac{\rho_M}{\rho_m} \sqrt{\mu^{T_q f}}$, $\beta_r = \mu^{T_r f+1} e^{-\alpha(\tau+T_r)}$.

First, let us consider the condition $f ln\mu - \alpha < 0$. It should be noted that (3.4) implies $\tau > \frac{1}{f}$, and we can conclude that

$$\beta_r \leq e^{(f ln\mu-\alpha)(T_r+\frac{1}{f})} < 1.$$

Additionally, in the case of $f ln\mu - \alpha \geq 0$, it can be observed that

$$\beta_r \leq e^{(Tf+1)ln\mu-\alpha(T+\tau)}. \tag{3.14}$$

As the PDT switching signal fulfills (3.4), we can infer from (3.14) that when $f ln\mu - \alpha \geq 0, \beta_r < 1$ still holds. Therefore, by taking into account Eq (3.13), it is possible to deduce that $\lim_{t\to\infty} \|x(t)\| = 0$. Thus, we can conclude that (3.6) is AS. Furthermore, the stability criterion for system (3.7) can be obtained using a similar approach. Consequently, it can be concluded that system (2.3) is AIS in $(-b, -a)$.

In the following, we will demonstrate that system (2.3) achieves the desired H_∞ inhibition level γ. Let $w(t)$ be a non-zero function and $w(t) \in \mathcal{L}_2[0, \infty)$, and we can define

$$\Upsilon(t) = y^{\mathrm{T}}(t)y(t) - \gamma^2 w^{\mathrm{T}}(t)w(t), \tag{3.15}$$

where we still select Eq (3.8) as the Lyapunov function. Assuming $\forall t \in [t_s, t_{s+1})$, $\sigma(t) = i \in \mathcal{M}$, one can deduce that

$$\dot{V}_i(x(t)) + \alpha V_i(x(t)) + \Upsilon(t) \leq \sum_{n=1}^{r_s} h_{in}(\vartheta_i(t))\zeta^{\mathrm{T}}(t)\Pi_{in}\zeta(t),$$

where

$$\zeta^{\mathrm{T}}(t) = \begin{bmatrix} x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-d(t)) & (\int_{t-d(t)}^{t} \dot{x}(s)ds)^{\mathrm{T}} & w^{\mathrm{T}}(t) \end{bmatrix},$$

$$\Pi_{in} = \begin{bmatrix} \Pi_{in11} & \Pi_{in12} & -M_{in} + T_{in}^{\mathrm{T}} & P_i(-B_{in}) + C_{in}^{\mathrm{T}} D_{in} + d(-A_{in} - bI)^{\mathrm{T}} R_i(-B_{in}) \\ * & \Pi_{in22} & -N_{in} - T_{in}^{\mathrm{T}} & d(-E_{in})^{\mathrm{T}} R_i(-B_{in}) \\ * & * & -\frac{-\alpha d}{d} R_i - T_i - T_i^{\mathrm{T}} & 0 \\ * & * & * & d(-B_{in})^{\mathrm{T}} R_i(-B_{in}) + D_{in}^{\mathrm{T}} D_{in} - \gamma^2 I \end{bmatrix},$$

$$\Pi_{in11} = (-A_{in} - bI)^{\mathrm{T}} P_i + P_i(-A_{in} - bI) + Q_i + d(-A_{in} - bI)^{\mathrm{T}} R_i(-A_{in} - bI) + \alpha P_i + C_{in}^{\mathrm{T}} C_{in} + M_i + M_i^{\mathrm{T}},$$
$$\Pi_{in12} = P_i(-E_{in})^{\mathrm{T}} + d(-A_{in} - bI)^{\mathrm{T}} R_i(-E_{in}) - M_i + N_i^{\mathrm{T}},$$
$$\Pi_{in22} = -(1-\epsilon)e^{-\alpha d} Q_i - d(-E_{il})^{\mathrm{T}} R_i(-E_{il}) - N_i - N_i^{\mathrm{T}}.$$

According to the Schur complement theorem, for every $t \in [t_s, t_{s+1})$, one gets

$$\dot{V}_i(x(t)) + \alpha V_i(x(t)) + \Upsilon(t) < 0. \tag{3.16}$$

Then, from (3.11) and (3.16), we have

$$\begin{aligned}
&V_{\sigma(t_{s_{q+1}})}(x(t_{s_{q+1}}))\\
&\le \mu e^{-\alpha(t_{s_{q+1}}-t_{s_{q+1}-1})}V_{\sigma(t_{s_{q+1}-1})}(x(t_{s_{q+1}-1})) - \int_{t_{s_{q+1}-1}}^{t_{s_{q+1}}} e^{-\alpha(t_{s_{q+1}}-z)}\Upsilon(z)dz \le \cdots\\
&\le \mu^{N_\sigma(t_{s_1+1},t_{s_{q+1}})}e^{-\alpha(t_{s_{q+1}}-t_{s_1})}V_{\sigma(t_{s_1})}(x(t_{s_1})) - \mu^{N_\sigma(t_{s_1+1},t_{s_{q+1}})}e^{-\alpha(t_{s_{q+1}}-t_{s_1})}\int_{t_{s_1}}^{t_{s_1}+1} e^{-\alpha(t_{s_1+1}-z)}\Upsilon(z)dz\\
&\quad - \cdots - \int_{t_{s_{q+1}-1}}^{t_{s_{q+1}}} e^{-\alpha(t_{s_{q+1}}-z)}\Upsilon(z)dz.
\end{aligned}$$

It is important to note that at $t_{s_1} = t_0$, with a zero initial condition, $V_{\sigma(t_{s_1})}(x(t_{s_1}) = 0$. Hence, for all $t \in [t_{s_q}, t_{s_{q+1}})$, one can conclude

$$\int_{t_0}^{t} \mu^{N(z,t)}e^{-\alpha(t-z)}\Upsilon(z)dz \le 0. \tag{3.17}$$

In the q-th stage, the overall duration is determined by $\tau_q + T_q$, where τ_q and T_q indicate the time spent in the τ-portion and T-portion. Given that $T_q \le T$, $\tau_q \ge \tau_D$, and considering $\tau_D > \frac{1}{f}$, it is evident that $(f\tau_D - 1)(T_q - T) \le 0$. Assuming that $[z, t)$ falls within the q-th stage, it leads to the conclusion that

$$(\frac{t-z}{T_q+\tau} + 1)(T_q f + 1) \le (\frac{t-z}{T-\tau} + 1)(Tf + 1).$$

Furthermore, this relationship remains true for all stages, independent of the value of q. By adding up the stages within $[z, t)$, we can obtain

$$0 \le N(z,t) \le (\frac{t-z}{T-\tau} + 1)(Tf + 1). \tag{3.18}$$

Combining Eqs (3.17) and (3.18), we obtain

$$\int_{t_0}^{t} \mu^{N(z,t)}e^{-\alpha(t-z)}\Gamma(z)dz \le \int_{t_0}^{t} \mu^{(\frac{t-z}{T-\tau}+1)(Tf+1)}e^{-\alpha(t-z)}[y^{\mathrm{T}}(z)y(z) - \gamma^2 w^{\mathrm{T}}(z)w(z)]dz,$$

an this suggests that

$$\int_{t_0}^{t} e^{-\alpha(t-z)}y^{\mathrm{T}}(z)y(z)dz \le \gamma^2\mu^{Tf+1}\int_{t_0}^{t} e^{\varrho(t-z)}w^{\mathrm{T}}(z)w(z)dz, \tag{3.19}$$

where $\varrho = \frac{Tf+1}{T+\tau_D}ln\mu - \alpha$. It is important to note that $\varrho < 0$ can be ensured through inequality (3.4). The integral of expression (3.19) with respect to t from t_0 to ∞ yields

$$\int_{t_0}^{\infty}\int_{t_0}^{t} e^{-\alpha(t-z)}y^{\mathrm{T}}(z)y(z)dzdt \le \gamma^2\mu^{Tf+1}\int_{t_0}^{\infty}\int_{t_0}^{t} e^{\varrho(t-z)}w^{\mathrm{T}}(z)w(z)dzdt,$$

and we have

$$\int_{t_0}^{\infty}\int_{z}^{\infty} e^{-\alpha(t-z)}y^{\mathrm{T}}(z)y(z)dtdz \leq \gamma^2\mu^{Tf+1}\int_{t_0}^{\infty}\int_{z}^{\infty} e^{\varrho(t-z)}w^{\mathrm{T}}(z)w(z)dzdt.$$

Since

$$\int_{z}^{\infty} e^{-\alpha(t-z)}y^{\mathrm{T}}(z)y(z)dt = -\frac{1}{\alpha}y^{\mathrm{T}}(z)y(z),$$
$$\int_{z}^{\infty} e^{\varrho(t-z)}w^{\mathrm{T}}(z)w(z)dt = \frac{1}{\varrho}w^{\mathrm{T}}(z)w(z).$$

Therefore, we can obtain

$$\int_{t_0}^{\infty} y^{\mathrm{T}}(z)y(z)dz \leq \gamma_s^2\int_{t_0}^{\infty} w^{\mathrm{T}}(z)w(z)dz,$$

where $\gamma_s = \sqrt{\frac{-\alpha\mu^{Tf+1}}{\varrho}}\gamma$.

Hence, the system (3.6) is AS with its $\mathcal{L}_2$-gain limited to γ_s. By the same way, if $\bar{\Theta} < 0$, the system (3.7) is AS with an $\mathcal{L}_2$-gain not exceeding γ_s. Therefore, system (2.3) is AIS in $(-b, -a)$ with an $\mathcal{L}_2$-gain not exceeding γ_s. □

Remark 4. *Interval stability with H_∞ performance is a more precise stability condition than the general H_∞ stability. When $b = \infty$ and $a = 0$, according to Theorem 1, the classical stability condition can be obtained. In contrast, the general H_∞ stability criterion can only determine the system's stability under specific levels of disturbance attenuation, whereas the interval stability criterion can assess both the stability and the rate at which the system tends toward stability. Interval stability provides guidance for designing more accurate H_∞ controllers.*

When $a = 0, b = \infty$, the following corollary can be seen as a concrete instance of Theorem 1.

Corollary 1. *Given constants $\epsilon > 0$, $\mu > 1$, $\alpha > 0$, d, f, and T, there are matrices $Q_i > 0$, $P_i > 0$, $R_i > 0$, N_i, M_i, and T_i, $\forall i \in \mathcal{M}$, together with a scalar γ, satisfying the following matrix inequalities:*

$$\Xi = \begin{bmatrix} \Xi_{11} & P_iE_{in}^{\mathrm{T}} - M_i + N_i^{\mathrm{T}} & -M_i + T_i^T & P_iB_i + C_{in}^TD_{in} & dA_{in}^TR_i & C_{in}^T \\ * & \Xi_{22} & -N_i - T_i^T & 0 & dE_{in}^TR_i & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 \\ * & * & * & D_{in}^TD_{in} - \gamma^2I & dB_{in}^TR_i & 0 \\ * & * & * & * & -dR_i & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0,$$

$$P_i \leq \mu P_l, Q_i \leq \mu Q_l, R_i \leq \mu R_l, \forall i, l \in \mathcal{M},$$

where
$\Xi_{11} = A_{in}^{\mathrm{T}}P_i + P_iA_{in} + Q_i + \alpha P_i + M_i + M_i^{\mathrm{T}}$, $\Xi_{22} = -(1-\epsilon)e^{-\alpha d}Q_i - N_i - N_i^{\mathrm{T}}$, $\Xi_{33} = \frac{e^{-\alpha d}}{d}(\hat{P}_i + \hat{P}_i^T) - \hat{T}_i - \hat{T}_i^T$, *for any PDT switching signal that satisfies (3.4) and the system (2.3) is AS with an L_2-gain that does not exceed (3.5).*

4. Design of the H_∞ controller

In this section, the controller $u_{\sigma(t)}(t) = \sum_{n=1}^{r_{\sigma(t)}} h_{\sigma(t)n}(\vartheta_{\sigma(t)}(t))K_{\sigma(t)n}x(t)$ will be built to deal with the H_∞ control with interval stability constraints of system (2.3).

Theorem 2. *Given constants $\alpha > 0$, $\epsilon > 0$, $\mu > 1$, d, f, and T, there are matrices $\hat{Q}_i > 0$, $\hat{P}_i > 0$, $\hat{R}_i > 0$, $\hat{N}_i$, $\hat{M}_i$, $\hat{T}_i$, and L_{im}, $\forall i \in \mathcal{M}$, along with a scalar γ, satisfying the following matrix inequalities:*

$$\Delta = \begin{bmatrix} \Delta_{11} & (-E_{in})^T\hat{P}_i - \hat{M}_i\hat{+}N_i^T & -\hat{M}_i + \hat{T}_i^T & -B_{in} + \hat{P}_iC_{in}^TD_{in} & \Delta_{15} & \hat{P}_iC_{in}^T \\ * & -(1-\epsilon)e^{-\alpha d} - \hat{N}_i - \hat{N}_i^T & -\hat{N}_i - \hat{T}_i^T & 0 & d\hat{P}_i(-E_{in}^T) & 0 \\ * & * & \Delta_{33} & 0 & 0 & 0 \\ * & * & * & D_{in}^TD_{in} - \gamma^2 I & d(-B_{in})^T & 0 \\ * & * & * & * & -d\hat{R}_i & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{4.1}$$

$$\bar{\Delta} = \begin{bmatrix} \bar{\Delta}_{11} & E_{in}^T\hat{P}_i - \hat{M}_i\hat{+}N_i^T & -\hat{M}_i + \hat{T}_i^T & B_{in} + \hat{P}_iC_{in}^TD_{in} & \bar{\Delta}_{15} & \hat{P}_iC_{in}^T \\ * & -(1-\epsilon)e^{-\alpha d} - \hat{N}_i - \hat{N}_i^T & -\hat{N}_i - \hat{T}_i^T & 0 & d\hat{P}_iE_{in}^T & 0 \\ * & * & \Delta_{33} & 0 & 0 & 0 \\ * & * & * & D_{in}^TD_{in} - \gamma^2 I & dB_{in}^T & 0 \\ * & * & * & * & -d\hat{R}_i & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{4.2}$$

$$\hat{P}_i \leq \mu\hat{P}_l, \hat{Q}_i \leq \mu\hat{Q}_l, \hat{R}_i \leq \mu\hat{R}_l, \forall i, l \in \mathcal{M}, \tag{4.3}$$

where

$$\begin{aligned} \Delta_{11} &= \hat{P}_i(-A_{in} - bI)^T - L_{im}^TH_{in}^T + (-A_{in} - bI)\hat{P}_i - H_{in}L_{im} + \alpha\hat{P}_i + \hat{Q}_i + \hat{M}_i + \hat{M}_i^T, \\ \Delta_{15} &= d\hat{P}_i(-A_{in} - bI)^T - dL_{im}^TH_{in}^T, \quad \Delta_{33} = \frac{e^{-\alpha d}}{d}\hat{P}_i - \frac{e^{-\alpha d}}{d}(\hat{P}_i + \hat{P}_i^T) - \hat{T}_i - \hat{T}_i^T, \\ \bar{\Delta}_{11} &= \hat{P}_i(A_{in} + aI)^T + L_{im}^TH_{in}^T + (A_{in} + aI)\hat{P}_i + H_{in}L_{im} + \alpha\hat{P}_i + \hat{Q}_i + \hat{M}_i + \hat{M}_i^T, \\ \bar{\Delta}_{15} &= d\hat{P}_i(A_{in} + aI)^T + dL_{im}^TH_{in}^T, \end{aligned}$$

for any PDT satisfying (3.4), and the system (2.3) is AIS in $(-b, -a)$, while ensuring an L_2-gain that does not surpass (3.5). Additionally, the gains of the H_∞ fuzzy controller are given by the following formula:

$$K_{im} = L_{im}\hat{P}_i^{-1}. \tag{4.4}$$

Proof. In the context of system (2.3), replace A_{in} with $A_{in} + H_{in}K_{in}$ as stipulated by Theorem 1, and define $\hat{P}_i = P_i^{-1}$. By multiplying (3.1) by $diag\{P_i^{-1}, P_i^{-1}, P_i^{-1}, I, P_i^{-1}, I\}$ while introducing

$$\hat{Q}_i = \hat{P}_iQ_i\hat{P}_i,\ \hat{M}_i = \hat{P}_iM_i\hat{P}_i,\ \hat{N}_i = \hat{P}_iN_i\hat{P}_i,\ \hat{T}_i = \hat{P}_iT_i\hat{P}_i,\ L_{im} = K_{im}\hat{P}_i,\ \hat{R}_i = R_i^{-1},$$

the following form is easy to represent:

$$\hat{\Delta} = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & \Delta_{14} & \Delta_{15} & \hat{P}_iC_{in}^T \\ * & \Delta_{22} & \Delta_{23} & 0 & \Delta_{25} & 0 \\ * & * & \hat{\Delta}_{33} & 0 & 0 & 0 \\ * & * & * & \Delta_{44} & \Delta_{45} & 0 \\ * & * & * & * & -d\hat{R}_i & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{4.5}$$

where $\hat{\Delta}_{33} = \frac{e^{-\alpha d}}{d}\hat{P}_i\hat{R}_i^{-1}\hat{P}_i - \hat{T}_i - \hat{T}_i^T$ and others are identical to the definition in Theorem 1.

Furthermore, Theorem 1 leads to the derivation of LMI (4.1). Similarly, (4.2) can also be generated. Hence, the system (2.3) with $u(t) = \sum_{n=1}^{r_{\sigma(t)}} h_{\sigma(t)n}(\theta_{\sigma(t)}(t))K_{\sigma(t)n}x(t)$ is AIS in $(-b, -a)$ and achieves a specific level of H_∞ disturbance attenuation denoted by γ_s. □

Remark 5. *Leveraging Theorem 2 as a foundation, it becomes feasible to design H_∞ fuzzy controllers with varying convergence rates through the manipulation of parameters a and b. This expansion enhances the performance capabilities of general H_∞ controllers. Furthermore, the core concept of this new technology shares similarities with the pole assignment technique, but the new technology is applicable to more complex systems. The straightforwardness of the solution method using LMIs in Theorem 2 effectively minimizes the computational efficiency burden caused by the pole assignment method in the system.*

Remark 6. *In reference [35], the H_∞ control problem for switched fuzzy systems is studied, where the weighted H_∞ performance is obtained by using the ADT switching scheme. In contrast, this paper employs a PDT switching strategy to derive the non-weighted H_∞ performance, which has more physical significance than the weighted H_∞ performance. In addition, the H_∞ controller design method proposed in this paper can also effectively solve the convergence rate constraint problem of switched linear systems.*

Corollary 2. *Given constants $\alpha > 0$, $\epsilon > 0$, $\mu > 1$, d, f, and T, there are matrices $\hat{Q}_i > 0$, $\hat{P}_i > 0$, $\hat{R}_i > 0$, $\hat{N}_i$, $\hat{M}_i$, $\hat{T}_i$, and L_{im}, $\forall i \in \mathcal{M}$, as well as a scalar γ, such that*

$$\Upsilon = \begin{bmatrix} \Upsilon_{11} & E_{ij}^T\hat{P}_i - \hat{M}_i\hat{+}N_i^T & -\hat{M}_i + \hat{T}_i^T & B_{ij} + \hat{P}_iC_{ij}^TD_{ij} & d\hat{P}_i(A_{ij})^T + dL_{im}^TH_{ij}^T & \hat{P}_iC_{ij}^T \\ * & \Upsilon_{22} & -\hat{N}_i - \hat{T}_i^T & 0 & d\hat{P}_iE_{ij}^T & 0 \\ * & * & \Upsilon_{33} & 0 & 0 & 0 \\ * & * & * & D_{il}^TD_{ij} - \gamma^2 I & dB_{ij}^T & 0 \\ * & * & * & * & -d\hat{R}_i & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{4.6}$$

$$\hat{P}_i \leq \mu\hat{P}_l, \hat{Q}_i \leq \mu\hat{Q}_l, \hat{R}_i \leq \mu\hat{R}_l, \forall i, l \in \mathcal{M}, \tag{4.7}$$

where

$$\Upsilon_{11} = \hat{P}_i(A_{ij})^T + L_{im}^TH_{ij}^T + (A_{ij})\hat{P}_i + H_{ij}L_{im} + \alpha\hat{P}_i + \hat{Q}_i + \hat{M}_i + M_i^T,$$
$$\Upsilon_{22} = -(1-\epsilon)e^{-\alpha d}\hat{Q}_i - \hat{N}_i - \hat{N}_i^T,$$
$$\Upsilon_{33} = \frac{e^{-\alpha d}}{d}\hat{P}_i - \frac{e^{-\alpha d}}{d}(\hat{P}_i + \hat{P}_i^T) - \hat{T}_i - \hat{T}_i^T,$$

for any PDT switching signal satisfying (3.4), and the system (2.3) is AS with an L_2-gain that does not exceed (3.5). Additionally, the gains of the H_∞ controller can be represented in the following format: $K_{im} = L_{im}\hat{P}_i^{-1}$.

Theorems 1 and 2 allow for controlling the real parts of the eigenvalues of the switched T-S fuzzy system, ensuring they fall within the appropriate interval. This approach can effectively regulate the convergence rate of the system to the steady state. Building upon this, algorithm 1 is designed as follows:

Algorithm 1 System eigenvalues adjustment algorithm

Require: $L = (-b, -a)$, $e = 0.01$;
S_{exp} = The expected convergence time of the closed-loop system to an equilibrium point;
S_{act} = The actual convergence time of the closed-loop system to an equilibrium point.
Ensure: [Theorem 2] and LMI toolbox to solve the gain matrix K_{im};
if $S_{act} < S_{exp} + e$ && $S_{act} > S_{exp} - e$ **then**
The system state's convergence rate trajectories are printed.
else if $S_{act} > S_{exp} + e$ && $S_{act} > S_{exp} + e$ **then**
$b++$;
$a++$;
Ensure: [Theorem 2] and LMI toolbox to solve the gain matrix K_{im};
else if $S_{act} < S_{exp} - e$ && $S_{act} < S_{exp} + e$ **then**
$b--$;
$a--$;
Ensure: [Theorem 2] and LMI toolbox to solve the gain matrix K_{im};
end if

5. Examples

This section aims to demonstrate the effectiveness of the devised fuzzy control method using a single-link mechanical arm system, which is inspired by [36]. Consider the continuous-time dynamic model of a single-link mechanical arm, characterized by its governing equations

$$\ddot{\vartheta}(t) = -\frac{gLM_i}{J_i} sin(\vartheta(t)) - \frac{R}{J_i}\dot{\vartheta}(t) + \frac{1}{J_i}u(t) + D\omega(t),$$

where $\dot{\theta}(t)$ and $\theta(t)$ are the angular velocity and angle of the robot arm, respectively. Additionally, M_i is the mass of the payload, g symbolizes the acceleration due to gravity, L signifies the length of the arm, J_i represents the moment of inertia, D represents the coefficient of viscous friction, and R denotes the damping coefficient. By defining $x_2(t) = \dot{\vartheta}(t)$ and $x_1(t) = \vartheta(t)$, and taking into account the time delay in the system, we can derive

$$\begin{aligned} \dot{x}_1(t) =& \ \varpi x_2(t) + (1-\varpi)x_2(t-d(t)). \\ \dot{x}_2(t) =& -\frac{gLM_i}{J_i} sin(x_1(t)) - \frac{(1-\varpi)R}{J_i}x_2(t-d(t)) - \frac{\varpi R}{J_i}x_2(t) + \frac{1}{J_i}u(t) + D\omega(t). \end{aligned} \tag{5.1}$$

Define $g = 9.81$ and $L = 0.5$, and suppose $R = 2$ and $\varpi = 0.85$. M_i and J_i have three distinct modes, which are displayed in Table 1. Applying the Euler approximation approach, we obtain the fuzzy basis functions for the T-S fuzzy system with a sampling period of $T = 0.1$, $l = 1, 2$, and $i = 1, 2, 3$ (refer to [37]):

$$h_{i1}(x_1(t)) = \begin{cases} \dfrac{sin(x_1(t)) - \rho x_1(t)}{(1-\rho)x_1(t)}, & x_1(t) \neq 0 \\ 1, & x_1(t) = 0 \end{cases}$$

$$h_{i2}(x_1(t)) = \begin{cases} \dfrac{x_1(t) - sin(x_1(t))}{(1-\rho)x_1(t)}, & x_1(t) \neq 0 \\ 0, & x_1(t) = 0 \end{cases}$$

and we have $x_1(t) \in (-\pi, \pi)$ and $\rho = \frac{10^{-2}}{\pi}$. A two-rule T-S fuzzy system can express the state space representation of a single-link robot arm.

Table 1. The modes of the parameters M_i and J_i.

mode θ_i	M_i	J_i
1	1	1
2	5	5
3	10	10

Model Rule 1: IF $x_1(t)$ is "about 0 rad,"
THEN

$$\begin{cases} \dot{x}(t) = A_{i1}x(t) + E_{i1}x(t-d(t)) + H_{i1}x(t) + B_{i1}w(t) \\ y(t) = C_{i1}x(t) + D_{i1}w(t) \end{cases}$$

Model Rule 2: IF $x_1(t)$ is "*about* $\pm\pi$ *rad,"*
THEN

$$\begin{cases} \dot{x}(t) = A_{i2}x(t) + E_{i2}x(t-d(t)) + H_{i2}x(t) + B_{i2}w(t) \\ y(t) = C_{i2}x(t) + D_{i2}w(t) \end{cases}$$

where $x(t) = \begin{bmatrix} x_1^T(t) & x_2^T(t) \end{bmatrix}^T$,

$$A_{i1} = \begin{bmatrix} 1 & T\varpi \\ -\frac{TgLM_i}{J_i} & 1 - \frac{T\varpi R}{J_i} \end{bmatrix}, A_{i2} = \begin{bmatrix} 1 & T\varpi \\ -\frac{\rho TgLM_i}{J_i} & 1 - \frac{T\varpi R}{J_i} \end{bmatrix}, E_{i1} = E_{i2} = \begin{bmatrix} 0 & T(1-\varpi) \\ 0 & -\frac{T(1-\varpi)D}{J_i}, \end{bmatrix}, H_{i1} = H_{i2} = \begin{bmatrix} 0 \\ \frac{1}{J_i}T \end{bmatrix},$$

$$B_{i1} = B_{i2} = \begin{bmatrix} 0 \\ T \end{bmatrix}, C_{i1} = C_{i2} = \begin{bmatrix} 1 & 0 \end{bmatrix}, D_{i1} = D_{i2} = 0.1.$$

Figure 2 depicts the state trajectories of the control-free system (5.1). Here, preset $\alpha = 0.91$, $\mu = 1.1$, $d = 0.2$, $\epsilon = 0.1$, the H_∞ disturbance attenuation level $\gamma = 0.31$, and assume that $T = 1s$, $f = 10s^{-1}$. According to Theorem 1, an admissible solution can be found with $\tau_D^* = 0.1520s$ and we can presume the PDT

$$\tau_D = 0.2564 > \tau_D^* = 0.1520. \tag{5.2}$$

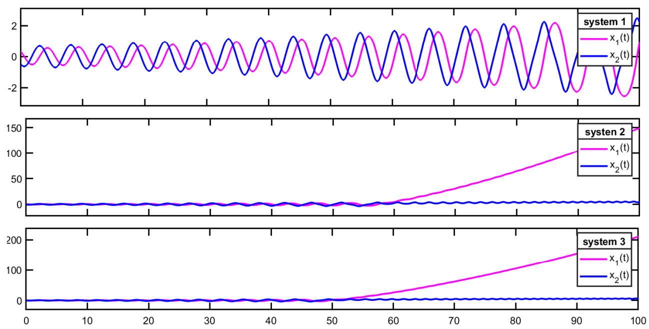

Figure 2. State trajectories of the robot arm system (5.1) without control.

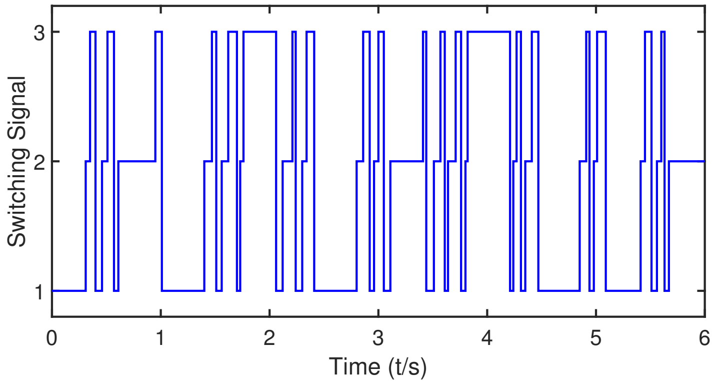

Figure 3. The switching signal $\sigma(t)$.

Corollary 2 provides a method for obtaining the controller gain matrices by solving Eqs (4.6) and (4.7):

$$K_{11}^{1} = \begin{bmatrix} -278.62 & -7.98 \end{bmatrix}, K_{12}^{1} = \begin{bmatrix} -279.11 & -7.98 \end{bmatrix},$$
$$K_{21}^{1} = \begin{bmatrix} -1404.2 & -41.5 \end{bmatrix}, K_{22}^{1} = \begin{bmatrix} -1406.6 & -41.5 \end{bmatrix},$$
$$K_{31}^{1} = \begin{bmatrix} -2141.3 & -82.7 \end{bmatrix}, K_{32}^{1} = \begin{bmatrix} -2246.2 & -81.7 \end{bmatrix}.$$

Suppose the external disturbance $w(t) = \begin{bmatrix} 0.02e^{-4t} & 0.02e^{-4t} \end{bmatrix}^T$ and the initial state $x(0) = \begin{bmatrix} 0.31 & -0.51 \end{bmatrix}^T$. The obtained controller gain matrix is passed through fuzzy rules to obtain a fuzzy controller which is substituted into the nonlinear system (5.1). Figures 4 and 5 depict the trajectories of the state variables (x_1^1, x_2^1) for the resulting nonlinear system under PDT (3.4) with a dark blue dashed curve. From this, it can be seen that as $t \geq 1.20(s)$, $|x_1^1| < 5 \times 10^{-3}$, and when $t \geq 1.55(s)$, $|x_2^1| < 5 \times 10^{-2}$.

Remark 7. *The H_∞ fuzzy controller design solution obtained from Corollary 2 can only guarantee system stability but cannot regulate the rate at which the system tends to balance. However, the controller designed by Theorem 2 can accomplish the goal of adjusting the rate of system convergence toward equilibrium by modifying the interval $(-b, -a)$, while also ensuring system stability.*

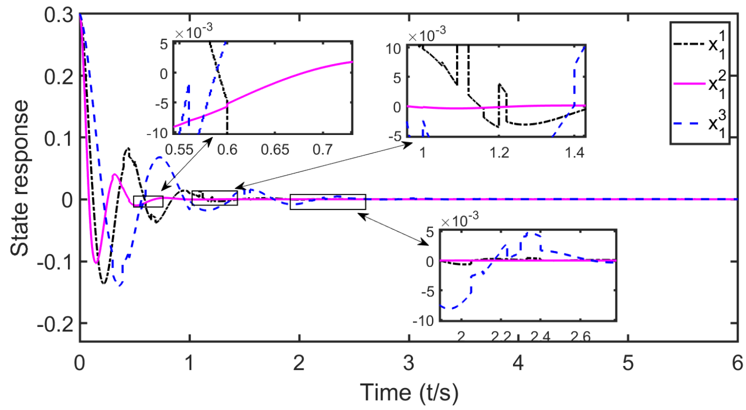

Figure 4. Response of system state $x_1(t)$.

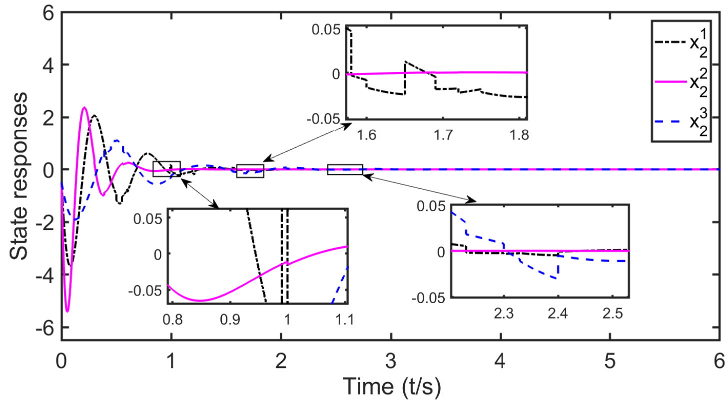

Figure 5. Response of system state $x_2(t)$.

Theorem 2 states that the convergence rate of the switched nonlinear system (5.1) can be controlled by adjusting the values of parameters a and b, thus allowing for the creation of different controllers. First, we adjust the parameters $b = 9$ and $a = 4$ to obtain the fast-converging controller parameters as follows:

$$K_{11}^2 = \begin{bmatrix} -653.4806 & -14.6533 \end{bmatrix}, K_{12}^2 = \begin{bmatrix} -653.9683 & -14.6533 \end{bmatrix},$$
$$K_{21}^2 = 10^3 \begin{bmatrix} -3.3624 & -0.0750 \end{bmatrix}, K_{22}^2 = 10^3 \begin{bmatrix} -3.3649 & -0.0750 \end{bmatrix},$$
$$K_{31}^2 = 10^3 \begin{bmatrix} -6.7249 & -0.1502 \end{bmatrix}, K_{32}^2 = 10^3 \begin{bmatrix} -6.7298 & -0.1502 \end{bmatrix}.$$

The obtained controller gain matrix is passed through fuzzy rules to obtain a fuzzy controller which is substituted into the nonlinear system (5.1). The red solid line in Figures 4 and 5 represents the trajectory of the state (x_1^2, x_2^2), as determined by the interval filter. From this, it can be seen that as $t \geq 0.60(s)$, $|x_1^2| < 5 \times 10^{-3}$, and when $t \geq 0.91(s)$, $|x_2^2| < 5 \times 10^{-2}$. Next, the slow-converging controller is obtained by setting $b = 2.5$ and $a = 0.5$, with its controller parameter values given below:

$$K_{11}^3 = \begin{bmatrix} -87.0343 & -5.4386 \end{bmatrix}, K_{12}^3 = \begin{bmatrix} -87.5229 & -5.4386 \end{bmatrix},$$
$$K_{21}^3 = \begin{bmatrix} -455.0888 & -28.8258 \end{bmatrix}, K_{22}^3 = \begin{bmatrix} -457.5319 & -28.8258 \end{bmatrix},$$
$$K_{31}^3 = \begin{bmatrix} -910.1796 & -57.8216 \end{bmatrix}, K_{32}^3 = \begin{bmatrix} -915.0652 & -57.8216 \end{bmatrix}.$$

The obtained controller gain matrix is passed through fuzzy rules to obtain a fuzzy controller which is substituted into the nonlinear system (5.1). The blue dashed line in Figures 4 and 5 represents the trajectory of the state (x_1^3, x_2^3), as determined by the interval filter. From this, it can be seen that as $t \geq 2.15(s)$, $|x_1^3| < 5 \times 10^{-3}$, and when $t \geq 2.15(s)$, $|x_2^3| < 5 \times 10^{-2}$.

Remark 8. *According to Figures 4, 5 and Tables 2, 3, the H_∞ fuzzy controller construction scheme proposed in Theorem 2 can control the convergence rate of the single-link robot arm system by adjusting the interval parameters $(-b, -a)$. The closer the position of the interval is to the right, the slower the system tends toward stability, while it becomes faster otherwise.*

Table 2. The convergence time of the system state x_1.

Methods	Corollary 2	Theorem 2	Theorem 2
Parameters		$(-9, -4)$	$(-2.5, -0.5)$
Time t at $\|x_1(t)\| = 5 \times 10^{-3}$	$1.2s$	$0.6s$	$2.15s$

Table 3. The convergence time of the system state x_2.

Methods	Corollary 2	Theorem 2	Theorem 2
Parameters		$(-9, -4)$	$(-2.5, -0.5)$
Time t at $\|x_2(t)\| = 5 \times 10^{-2}$	$1.55s$	$0.91s$	$2.15s$

6. Conclusions

This paper has explored the problem of adjustable convergence rate H_∞ control for switched nonlinear systems with time-varying delay under PDT. The PDT switching strategy has been considered, which has provided greater generality compared to regular DT or ADT switching strategies. Through the T-S fuzzy model, using a linear method combined with the definitions of interval stability, PDT technique, and L-K functionals, a construction method has been obtained for an H_∞ controller with convergence rate constraints for time-varying delay switched nonlinear systems. This approach has guaranteed system stability and H_∞ performance by limiting the rate at which the system achieves equilibrium. Finally, the effectiveness of this technique has been validated through a control example involving a single-link robot arm. In future work, the H_∞ control strategy with an adjustable convergence rate can be extended to switched positive systems.

Author contributions

Han Geng: Conceptualization, Writing-original draft, Software; Huasheng Zhang: Methodology, Formal analysis, Writing-review & editing. All authors have read and approved the final version of the manuscript for publication.

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Conflict of interest

The author declares that there are no conflicts of interest.

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