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Research article

Cohomology and its applications on multiplicative Hom-δ-Jordan Lie color triple systems

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Abstract: The theory of cohomologies on multiplicative Hom-δ-Jordan Lie color triple systems is given. As an application, deformations and extensions on the multiplicative Hom-δ-Jordan Lie color triple system are characterized in view of relevant cohomology.

Keywords: multiplicative Hom-δ-Jordan Lie color triple system; cohomology; deformation; abelian extension
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1. Introduction

Lie triple systems were introduced in connection with the symmetric space. In the study of constant curvature spaces and their tangent spaces, the Lie triple system is one of the important tools. We know also that Lie triple systems are closely related to the totally geodesic submani-fold and elementary theoretical physics [1,2]. The study on Lie triple systems has experienced a vigorous development. For example, the controlling cohomology, cohomology spaces, and several deformations and extensions on Lie triple systems were discussed [3–5]. As a promotion structure on Lie triple systems, that is, Lie supertriple systems, was studied [6]. In 1997, Okubo and Kamiya got a new kind of Lie supertriple system, that is, δ-Jordan Lie supertriple systems [7]. Later, they gave several meaningful results on δ-Jordan Lie supertriple systems in [8,9]. Recently, cohomologies on δ-Jordan Lie triple systems were developed [10]. Then, Wang, Zhang, and Guo obtained structures of the derivation and the deformation on δ-Jordan Lie supertriple systems [11].

Some Hom type algebraic structures have been determined [12–22]. For instance, Yau introduced the concept of Hom Lie triple systems [14]. In recent years, the authors studied the cohomology and several deformations on Hom Lie triple systems, respectively [16–18]. In 2023, Li and Ma obtained two kinds of extensions on Hom-δ-Jordan Lie supertriple systems [19]. Li and Ma also studied Lie color triple systems, and got relevant operators and extensions [23, 24]. This paper is a continuation based on [23, 24].

The organization of this paper is as follows: In Section 2, it is necessary to give definitions and examples concerning Hom-δ-Jordan Lie color triple systems. We define the representation ϑ and construct a kind of Hom-δ-Jordan Lie color triple system by the representation ϑ, then we obtain the suitable cohomology structure on Hom-δ-Jordan Lie color triple systems. Applying the results given in Section 2, we obtain that equivalent 1-parameter formal deformations $l_{\mathcal{L}}(t, p, q) = \sum_{i\geq 0} l_i(t, p, q)s^i$ and $l'_{\mathcal{L}}(t, p, q) = \sum_{i\geq 0} l'_i(t, p, q)s^i$ are in the same cohomology class in $H^3(T, T)$ in Section 3. In Section 4, we prove that there is the same representation between two equivalent abelian extensions; moreover, we also construct a kind of multiplicative Hom-δ-Jordan Lie color triple system using a 3-Hom-cocycle and obtain the necessary and sufficient condition for two equivalent abelian extensions.

Throughout this article, we study an algebraically closed field $\mathbf{F}$ of characteristic 0.

2. Basic definitions and cohomologies

Definition 2.1. *[25] Suppose that $\mathcal{G}$ is an abelian group. A bi-character on $\mathcal{G}$ is a map $\varepsilon : \mathcal{G} \times \mathcal{G} \to \mathbb{K} \setminus \{0\}$ and*

$$\varepsilon(g_1, g_2)\varepsilon(g_2, g_1) = 1,$$
$$\varepsilon(g_1, g_2 + g_3) = \varepsilon(g_1, g_2)\varepsilon(g_1, g_3),$$
$$\varepsilon(g_1 + g_3, g_2) = \varepsilon(g_1, g_2)\varepsilon(g_3, g_2),$$

where $g_1, g_2, g_3 \in \mathcal{G}$. It is obvious that

$$\varepsilon(g, 0) = \varepsilon(0, g) = 1, \quad \varepsilon(g, g) = \pm 1, \quad \forall g \in \mathcal{G}.$$

Definition 2.2. *[14] The Hom Lie triple system $(\mathcal{L}, [\cdot,\cdot,\cdot], \beta = (\beta_1, \beta_2))$ consists of an $\mathbf{F}$-vector space $\mathcal{L}$, a trilinear map $[\cdot,\cdot,\cdot] : \mathcal{L}\times\mathcal{L}\times\mathcal{L} \to \mathcal{L}$, with even linear maps $\beta_i : \mathcal{L} \to \mathcal{L}$ for $i = 1, 2$, which is called twisted maps, satisfying for all elements $t, p, q, u, v \in \mathcal{L}$,*

$$[t, t, p] = 0,$$

$$[t, p, q] + [p, q, t] + [q, t, p] = 0,$$

$$\begin{aligned}[\beta_1(u), \beta_2(v), [t, p, q]] =& [[u, v, t], \beta_1(p), \beta_2(q)] + [\beta_1(t), [u, v, p], \beta_2(q)] \\ &+ [\beta_1(t), \beta_2(p), [u, v, q]].\end{aligned}$$

If $|t|$ appears in some aspect, we usually think of t as a homogeneous element, meanwhile $|t|$ as the homogeneous degree of t.

Definition 2.3. *The Hom-δ-Jordan Lie color triple system $(\mathcal{L}, [\cdot,\cdot,\cdot], \beta = (\beta_1, \beta_2))$ consists of a $\mathcal{G}$-graded vector space $\mathcal{L} = \oplus_{g\in\mathcal{G}}\mathcal{L}_g$ over $\mathbf{F}$, a trilinear map $[\cdot,\cdot,\cdot] : \mathcal{L}\times\mathcal{L}\times\mathcal{L} \to \mathcal{L}$, with even linear maps $\beta_i : \mathcal{L} \to \mathcal{L}$ for $i = 1, 2$, which is called twisted maps, satisfying for all elements $t, p, q, u, v \in \mathcal{L}$, $\delta = \pm 1$,*

$$\varepsilon(|t|, |p|) = \varepsilon(|p|, |t|), \tag{2.1}$$

$$[t,p,q]=-\delta\varepsilon(|t|,|p|)[p,t,q], \tag{2.2}$$

$$\varepsilon(|t|,|q|)[t,p,q]+\varepsilon(|p|,|t|)[p,q,t]+\varepsilon(|q|,|p|)[q,t,p]=0, \tag{2.3}$$

$$\begin{aligned}[\beta_1(u),\beta_2(v),[t,p,q]] =&[[u,v,t],\beta_1(p),\beta_2(q)]+\varepsilon(|t|,|u|+|v|)[\beta_1(t),[u,v,p],\beta_2(q)]\\ &+\delta\varepsilon(|u|+|v|,|t|+|p|)[\beta_1(t),\beta_2(p),[u,v,q]]. \end{aligned}\tag{2.4}$$

Example 2.1. *Hom-δ-Jordan Lie triple systems are examples of Hom-δ-Jordan Lie color triple systems with $\mathcal{G}=\{0\}$, and $\varepsilon(|0|,|0|)=1$.*

Example 2.2. *Hom-δ-Jordan Lie supertriple systems are examples of Hom-δ-Jordan Lie color triple systems with $\mathcal{G}=\{\mathbb{Z}_2\}=\{\bar{0},\bar{1}\}$, and $\varepsilon(|p|,|q|)=(-1)^{ij}$ for any $i,j\in\mathbb{Z}_2$.*

Example 2.3. *Suppose $(\mathcal{L},[\cdot,\cdot])$ is a Hom-δ-Jordan Lie color algebra, thus $(\mathcal{L},[\cdot,\cdot,\cdot])$ defines a Hom-δ-Jordan Lie color triple system by $[t,p,q]=[[t,p],q]$, for any $t,p,q\in\mathcal{L}$.*

Example 2.4. *Suppose $(\mathcal{L},[\cdot,\cdot,\cdot])$ is a Hom-δ-Jordan Lie color triple system, and l is an indeterminate. Assume $\mathcal{L}'=\{\Sigma_{i\geq 0}y\otimes l^i|y\in\mathcal{L}\}$. Then $(\mathcal{L}',[\cdot,\cdot,\cdot]')$ is a Hom-δ-Jordan Lie color triple system satisfying a product $[\cdot,\cdot,\cdot]'$ defined using*

$$[p\otimes l^i,q\otimes l^j,m\otimes l^k]'=[p,q,m]\otimes l^{i+j+k},$$

where $p\otimes l^i,q\otimes l^j,m\otimes l^k\in\mathcal{L}'$, and $|p\otimes l^i|=|p|$.

The Hom-δ-Jordan Lie color triple system is called be multiplicative if $\beta_1=\beta_2=\beta$ and $\beta([t,p,q])=[\beta(t),\beta(p),\beta(q)]$.

The morphism $h:(\mathcal{L},[\cdot,\cdot,\cdot],\beta=(\beta_1,\beta_2))\to(\mathcal{L}',[\cdot,\cdot,\cdot]',\beta'=(\beta_1',\beta_2'))$ on multiplicative Hom-δ-Jordan Lie color triple system is a linear map such that $h([t,p,q])=[h(t),h(p),h(q)]'$ and $h\circ\beta_i=\beta_i'\circ h$, where $i=1,2$. The bijective morphism is called the isomorphism.

Definition 2.4. *Suppose that $(\mathcal{L},[\cdot,\cdot,\cdot])$ is a multiplicative Hom-δ-Jordan Lie color triple system. W is a $\mathcal{G}$-graded vector space over $\mathbf{F}$ and $\gamma\in\mathrm{End}(W)$. W is called a $(\mathcal{L},[\cdot,\cdot,\cdot])$-module relating to γ if there is a bilinear map $\vartheta:\mathcal{L}\times\mathcal{L}\to\mathrm{End}(W)$, $(t,t_1)\mapsto\vartheta(t,t_1)$ satisfying for all elements $t,t_1,t_2,t_3\in\mathcal{L}$,*

$$\vartheta(\beta(t),\beta(t_1))\circ\gamma=\gamma\circ\vartheta(t,t_1), \tag{2.5}$$

$$\begin{aligned}&\varepsilon(|t|+|t_1|,|t_2|+|t_3|)\vartheta(\beta(t_2),\beta(t_3))\vartheta(t,t_1)-\delta\varepsilon(|t|,|t_1|)\varepsilon(|t_3|,|t|+|t_2|)\vartheta(\beta(t_1),\beta(t_3))\vartheta(t,t_2)\\ &-\vartheta(\beta(t),[t_1,t_2,t_3])\circ\gamma+\varepsilon(|t|,|t_1|+|t_2|)D(\beta(t_1),\beta(t_2))\vartheta(t,t_3)=0,\end{aligned}\tag{2.6}$$

$$\begin{aligned}&\delta\varepsilon(|t|+|t_1|,|t_2|+|t_3|)\vartheta(\beta(t_2),\beta(t_3))D(t,t_1)-\delta D(\beta(t),\beta(t_1))\vartheta(t_2,t_3)\\ &+\vartheta([t,t_1,t_2],\beta(t_3))\circ\gamma+\delta\varepsilon(|t_2|,|t|+|t_1|)\vartheta(\beta(t_2),[t,t_1,t_3])\circ\gamma=0,\end{aligned}\tag{2.7}$$

$$\begin{aligned}&\delta\varepsilon(|t|+|t_1|,|t_2|+|t_3|)D(\beta(t_2),\beta(t_3))D(t,t_1)-D(\beta(t),\beta(t_1))D(t_2,t_3)\\ &+\delta D([t,t_1,t_2],\beta(t_3))\circ\gamma+\delta\varepsilon(|t_2|,|t|+|t_1|)D(\beta(t_2),[t,t_1,t_3])\circ\gamma=0,\end{aligned}\tag{2.8}$$

where $D(t,t_1)=\varepsilon(|t|,|t_1|)\vartheta(t_1,t)-\delta\vartheta(t,t_1)$.

Then ϑ is said to be the representation of $(\mathcal{L},[\cdot,\cdot,\cdot])$ on W relating to γ. When $\vartheta=0$, W is said to be the trivial $(\mathcal{L},[\cdot,\cdot,\cdot])$-module relating to γ.

In particular, set $W = \mathcal{L}$, and $\vartheta(t,p)(q) = \varepsilon(|q|,|t|+|p|)[q,t,p]$. Hence $D(t,p)(q) = \delta[t,p,q]$ and (2.5)–(2.8) hold. Under this situation, $\mathcal{L}$ is shown to be the adjoint $(\mathcal{L},[\cdot,\cdot,\cdot])$-module, and ϑ is said to be the adjoint representation of $(\mathcal{L},[\cdot,\cdot,\cdot])$.

Proposition 2.1. *Suppose that ϑ is a representation of a multiplicative Hom-δ-Jordan Lie color triple system $(\mathcal{L},[\cdot,\cdot,\cdot])$ on W relating to γ. Define the calculation $[\cdot,\cdot,\cdot]_W : (\mathcal{L}\oplus W)\times(\mathcal{L}\oplus W)\times(\mathcal{L}\oplus W) \to \mathcal{L}\oplus W$ by*

$$\begin{aligned}[(t,w),(t_1,w_1),(t_2,w_2)]_W =&([t,t_1,t_2],\varepsilon(|t|,|t_1|+|t_2|)\vartheta(t_1,t_2)(w)\\ &-\delta\varepsilon(|t_1|,|t_2|)\vartheta(t,t_2)(w_1)+\delta D(t,t_1)(w_2)),\end{aligned}$$

and assume the twisted map $\beta+\gamma : \mathcal{L}\oplus W \to \mathcal{L}\oplus W$ by

$$(\beta+\gamma)(t,w) = (\beta(t),\gamma(w)),$$

thus $\mathcal{L}\oplus W$ is a multiplicative Hom-δ-Jordan Lie color triple system.

Proof. By $D(t,t_1) = \varepsilon(|t|,|t_1|)\vartheta(t_1,t) - \delta\vartheta(t,t_1)$, it follows that

$$\begin{aligned}&[(t,w),(t_1,w_1),(t_2,w_2)]_W\\ =&([t,t_1,t_2],\varepsilon(|t|,|t_1|+|t_2|)\vartheta(t_1,t_2)(w)-\delta\varepsilon(|t_1|,|t_2|)\vartheta(t,t_2)(w_1)+\delta D(t,t_1)(w_2))\\ =&-\delta\varepsilon(|t|,|t_1|)([t_1,t,t_2],\varepsilon(|t_1|,|t|+|t_2|)\vartheta(t,t_2)(w_1)\\ &-\delta\varepsilon(|t|,|t_2|)\vartheta(t_1,t_2)(w)-\varepsilon(|t|,|t_1|)D(t,t_1)(w_2))\\ =&-\delta\varepsilon(|t|,|t_1|)([t_1,t,t_2],\varepsilon(|t_1|,|t|+|t_2|)\vartheta(t,t_2)(w_1)\\ &-\delta\varepsilon(|t|,|t_2|)\vartheta(t_1,t_2)(w)+\delta D(t_1,t)(w_2))\\ =&-\delta\varepsilon(|t|,|t_1|)[(t_1,w_1),(t,w),(t_2,w_2)]_W,\end{aligned}$$

and

$$\begin{aligned}&\varepsilon(|t|,|t_2|)[(t,w),(t_1,w_1),(t_2,w_2)]_W+\varepsilon(|t_1|,|t|)[(t_1,w_1),(t_2,w_2),(t,w)]_W\\ &+\varepsilon(|t_2|,|t_1|)[(t_2,w_2),(t,w),(t_1,w_1)]_W\\ =&(\varepsilon(|t|,|t_2|)[t,t_1,t_2],\varepsilon(|t|,|t_1|)\vartheta(t_1,t_2)(w)-\delta\varepsilon(|t_2|,|t|+|t_1|)\vartheta(t,t_2)(w_1)\\ &+\delta\varepsilon(|t|,|t_2|)D(t,t_1)(w_2))+(\varepsilon(|t_1|,|t|)[t_1,t_2,t],\varepsilon(|t_1|,|t_2|)\vartheta(t_2,t)(w_1)\\ &-\delta\varepsilon(|t|,|t_1|+|t_2|)\vartheta(t_1,t)(w_2)+\delta\varepsilon(|t_1|,|t|)D(t_1,t_2)(w))\\ &+(\varepsilon(|t_2|,|t_1|)[t_2,t,t_1],\varepsilon(|t_2|,|t|)\vartheta(t,t_1)(w_2)-\delta\varepsilon(|t_1|,|t|+|t_2|)\vartheta(t_2,t_1)(w)\\ &+\delta\varepsilon(|t_2|,|t_1|)D(t_2,t)(w_1))\\ =&(0,\varepsilon(|t|,|t_1|)\vartheta(t_1,t_2)(w)-\delta\varepsilon(|t_1|,|t|+|t_2|)\vartheta(t_2,t_1)(w)+\delta\varepsilon(|t_1|,|t|)D(t_1,t_2)(w)\\ &+\varepsilon(|t_1|,|t_2|)\vartheta(t_2,t)(w_1)-\delta\varepsilon(|t_2|,|t|+|t_1|)\vartheta(t,t_2)(w_1)+\delta\varepsilon(|t_2|,|t_1|)D(t_2,t)(w_1)\\ &+\varepsilon(|t_2|,|t|)\vartheta(t,t_1)(w_2)-\delta\varepsilon(|t|,|t_1|+|t_2|)\vartheta(t_1,t)(w_2)+\delta\varepsilon(|t|,|t_2|)D(t,t_1)(w_2))\\ =&(0,0).\end{aligned}$$

By (2.6)–(2.8), it is clear that

$$[[(t,w),(t_1,w_1),(t_2,w_2)]_W,(\beta+\gamma)(t_3,w_3),(\beta+\gamma)(t_4,w_4)]_W$$

$$
\begin{aligned}
=&[([t,t_1,t_2],\varepsilon(|t|,|t_1|+|t_2|)\vartheta(t_1,t_2)(w)-\delta\varepsilon(|t_1|,|t_2|)\vartheta(t,t_2)(w_1)+\delta D(t,t_1)(w_2)),\\
&(\beta(t_3),\gamma(w_3)),(\beta(t_4),\gamma(w_4))]_W\\
=&([[t,t_1,t_2],\beta(t_3),\beta(t_4)],\varepsilon(|t|+|t_1|+|t_2|,|t_3|+|t_4|)\vartheta(\beta(t_3),\beta(t_4))(\varepsilon(|t|,|t_1|\\
&+|t_2|)\vartheta(t_1,t_2)(w)-\delta\varepsilon(|t_1|,|t_2|)\vartheta(t,t_2)(w_1)+\delta D(t,t_1)(w_2))\\
&-\delta\varepsilon(|t_3|,|t_4|)\vartheta([t,t_1,t_2],\beta(t_4))(\gamma(w_3))\\
&+\delta D([t,t_1,t_2],\beta(t_3))(\gamma(w_4))),
\end{aligned}
$$

$$
\begin{aligned}
&\varepsilon(|t_2|,|t|+|t_1|)[(\beta+\gamma)(t_2,w_2),[(t,w),(t_1,w_1),(t_3,w_3)]_W,(\beta+\gamma)(t_4,w_4)]_W\\
=&\varepsilon(|t_2|,|t|+|t_1|)[(\beta(t_2),\gamma(w_2)),([t,t_1,t_3],\varepsilon(|t|,|t_1|+|t_3|)\vartheta(t_1,t_3)(w)\\
&-\delta\varepsilon(|t_1|,|t_3|)\vartheta(t,t_3)(w_1)+\delta D(t,t_1)(w_3)),(\beta(t_4),\gamma(w_4))]_W\\
=&\varepsilon(|t_2|,|t|+|t_1|)([\beta(t_2),[t,t_1,t_3],\beta(t_4)],\varepsilon(|t_2|,|t|+|t_1|+|t_3|\\
&+|t_4|)\vartheta([t,t_1,t_3],\beta(t_4))(\gamma(w_2))-\delta\varepsilon(|t|+|t_1|+|t_3|,|t_4|)\vartheta(\beta(t_2),\beta(t_4))(\varepsilon(|t|,|t_1|\\
&+|t_3|)\vartheta(t_1,t_3)(w)-\delta\varepsilon(|t_1|,|t_3|)\vartheta(t,t_3)(w_1)+\delta D(t,t_1)(w_3))\\
&+\delta D(\beta(t_2),[t,t_1,t_3])(\gamma(w_4))),
\end{aligned}
$$

$$
\begin{aligned}
&\delta\varepsilon(|t|+|t_1|,|t_2|+|t_3|)[(\beta+\gamma)(t_2,w_2),(\beta+\gamma)(t_3,w_3),[(t,w),(t_1,w_1),(t_4,w_4)]_W]_W\\
=&\delta\varepsilon(|t|+|t_1|,|t_2|+|t_3|)[(\beta(t_2),\gamma(w_2)),(\beta(t_3),\gamma(w_3)),([t,t_1,t_4],\varepsilon(|t|,|t_1|\\
&+|t_4|)\vartheta(t_1,t_4)(w)-\delta\varepsilon(|t_1|,|t_4|)\vartheta(t,t_4)(w_1)+\delta D(t,t_1)(w_4))]_W\\
=&\delta\varepsilon(|t|+|t_1|,|t_2|+|t_3|)([\beta(t_2),\beta(t_3),[t,t_1,t_4]],\varepsilon(|t_2|,|t_3|+|t|+|t_1|\\
&+|t_4|)\vartheta(\beta(t_3),[t,t_1,t_4])(\gamma(w_2))-\delta\varepsilon(|t_3|,|t|+|t_1|+|t_4|)\vartheta(\beta(t_2),[t,t_1,t_4])(\gamma(w_3))\\
&+\delta D(\beta(t_2),\beta(t_3))(\varepsilon(|t|,|t_1|+|t_4|)\vartheta(t_1,t_4)(w)\\
&-\delta\varepsilon(|t_1|,|t_4|)\vartheta(t,t_4)(w_1)+D(t,t_1)(w_4))),
\end{aligned}
$$

$$
\begin{aligned}
&[(\beta+\gamma)(t,w),(\beta+\gamma)(t_1,w_1),[(t_2,w_2),(t_3,w_3),(t_4,w_4)]_W]_W\\
=&[(\beta(t),\gamma(w)),(\beta(t_1),\gamma(w_1)),([t_2,t_3,t_4],\varepsilon(|t_2|,|t_3|+|t_4|)\vartheta(t_3,t_4)(w_2)\\
&-\delta\varepsilon(|t_3|,|t_4|)\vartheta(t_2,t_4)(w_3)+\delta D(t_2,t_3)(w_4))]_W\\
=&([\beta(t),\beta(t_1),[t_2,t_3,t_4]],\varepsilon(|t|,|t_1|+|t_2|+|t_3|+|t_4|)\vartheta(\beta(t_1),[t_2,t_3,t_4])(\gamma(w))\\
&-\delta\varepsilon(|t_1|,|t_2|+|t_3|+|t_4|)\vartheta(\beta(t),[t_2,t_3,t_4])(\gamma(w_1))+\delta D(\beta(t),\beta(t_1))(\varepsilon(|t_2|,|t_3|\\
&+|t_4|)\vartheta(t_3,t_4)(w_2)-\delta\varepsilon(|t_3|,|t_4|)\vartheta(t_2,t_4)(w_3)+D(t_2,t_3)(w_4))).
\end{aligned}
$$

The results above show that (2.2)–(2.4) hold.

Since $\beta+\gamma$ is an even linear map and in view of (2.5), it follows that

$$
\begin{aligned}
&(\beta+\gamma)[(t,w),(t_1,w_1),(t_2,w_2)]_W\\
=&(\beta+\gamma)([t,t_1,t_2],\varepsilon(|t|,|t_1|+|t_2|)\vartheta(t_1,t_2)(w)-\delta\varepsilon(|t_1|,|t_2|)\vartheta(t,t_2)(w_1)+\delta D(t,t_1)(w_2))\\
=&(\beta([t,t_1,t_2]),\gamma\circ(\varepsilon(|t|,|t_1|+|t_2|)\vartheta(t_1,t_2)(w)-\delta\varepsilon(|t_1|,|t_2|)\vartheta(t,t_2)(w_1)+\delta D(t,t_1)(w_2)))\\
=&([\beta(t),\beta(t_1),\beta(t_2)],\varepsilon(|t|,|t_1|+|t_2|)\vartheta(\beta(t_1),\beta(t_2))\gamma(w)-\delta\varepsilon(|t_1|,|t_2|)\vartheta(\beta(t),\beta(t_2))\gamma(w_1)
\end{aligned}
$$

$$\begin{aligned}&+\delta D(\beta(t),\beta(t_1))\gamma(w_2))\\=&[(\beta(t),\gamma(w)),(\beta(t_1),\gamma(w_1)),(\beta(t_2),\gamma(w_2))]_W\\=&[(\beta+\gamma)(t,w),(\beta+\gamma)(t_1,w_1),(\beta+\gamma)(t_2,w_2)]_W.\end{aligned}$$

Moreover, $(\mathcal{L}\oplus W,[\cdot,\cdot,\cdot]_W)$ is a multiplicative Hom-δ-Jordan Lie color triple system.

Remark 2.1. *Given a representation ϑ of the multiplicative Hom-δ-Jordan Lie color triple system $(\mathcal{L},[\cdot,\cdot,\cdot])$ on W, define the calculation $[\cdot,\cdot,\cdot]_W:(\mathcal{L}\oplus W)\times(\mathcal{L}\oplus W)\times(\mathcal{L}\oplus W)\to\mathcal{L}\oplus W$, we construct a kind of multiplicative Hom-δ-Jordan Lie color triple system and regard Proposition 2.1 as an example of multiplicative Hom-δ-Jordan Lie color triple systems.*

Suppose that ϑ is a representation of $(\mathcal{L},[\cdot,\cdot,\cdot])$ on W relating to γ. Assume that an n-linear map $h:\underbrace{\mathcal{L}\times\cdots\times\mathcal{L}}_{n\text{ times}}\to W$ satisfies

$$h(t_1,\cdots,t,p,t_n)=-\delta\varepsilon(|t|,|p|)h(t_1,\cdots,p,t,t_n),$$

$$\begin{aligned}&\varepsilon(|t|,|q|)h(t_1,\cdots,t_{n-3},t,p,q)+\varepsilon(|p|,|t|)h(t_1,\cdots,t_{n-3},p,q,t)\\&+\varepsilon(|q|,|p|)h(t_1,\cdots,t_{n-3},q,t,p)=0,\end{aligned}$$

thus h is called an n-cochain on $\mathcal{L}$. Denote by $C^n_\gamma(\mathcal{L},W)$ the set of all n-cochains, $\forall n\geq 1$.

Definition 2.5. *For $n=1,2,3,4$, the definition of the coboundary operator $d^n:C^n_\gamma(\mathcal{L},W)\to C^{n+2}_\gamma(\mathcal{L},W)$ is given as follows:*

- *If $h\in C^1(\mathcal{L},W)$, then*

$$\begin{aligned}&d^1h(w_1,w_2,w_3)\\=&\varepsilon(|h|+|w_1|,|w_2|+|w_3|)\vartheta(w_2,w_3)h(w_1)-\delta\varepsilon(|w_2|,|w_3|)\varepsilon(|h|,|w_1|+|w_3|)\\&\vartheta(w_1,w_3)h(w_2)+\delta\varepsilon(|h|,|w_1|+|w_2|)D(w_1,w_2)h(w_3)-h([w_1,w_2,w_3]).\end{aligned}$$

- *If $h\in C^2(\mathcal{L},W)$, then*

$$\begin{aligned}&d^2h(y,w_1,w_2,w_3)\\=&\varepsilon(|h|+|y|+|w_1|,|w_2|+|w_3|)\vartheta(\beta(w_2),\beta(w_3))h(y,w_1)\\&-\delta\varepsilon(|w_2|,|w_3|)\varepsilon(|h|+|y_1|,|w_1|+|w_3|)\vartheta(\beta(w_1),\beta(w_3))h(y,w_2)\\&+\delta\varepsilon(|h|+|y|,|w_1|+|w_2|)D(\beta(w_1),\beta(w_2))h(y,w_3)\\&-h(\beta(y),[w_1,w_2,w_3]).\end{aligned}$$

- *If $h\in C^3(\mathcal{L},W)$, then*

$$\begin{aligned}&d^3h(w_1,w_2,w_3,w_4,w_5)\\=&\varepsilon(|h|+|w_1|+|w_2|+|w_3|,|w_4|+|w_5|)\vartheta(\beta(w_4),\beta(w_5))h(w_1,w_2,w_3)\\&-\delta\varepsilon(|h|+|w_1|+|w_2|,|w_3|+|w_5|)\varepsilon(|w_4|,|w_5|)\vartheta(\beta(w_3),\beta(w_5))h(w_1,w_2,w_4)\\&-\delta\varepsilon(|h|,|w_1|+|w_2|)D(\beta(w_1),\beta(w_2))h(w_3,w_4,w_5)\end{aligned}$$

$$+ \varepsilon(|h| + |w_1| + |w_2|, |w_3| + |w_4|)D(\beta(w_3), \beta(w_4))h(w_1, w_2, w_5)$$
$$+ h([w_1, w_2, w_3], \beta(w_4), \beta(w_5)) + \varepsilon(|w_3|, |w_1| + |w_2|)h(\beta(w_3), [w_1, w_2, w_4], \beta(w_5))$$
$$+ \delta\varepsilon(|w_1| + |w_2|, |w_3| + |w_4|)h(\beta(w_3), \beta(w_4), [w_1, w_2, w_5])$$
$$- h(\beta(w_1), \beta(w_2), [w_3, w_4, w_5]).$$

- *If* $h \in C^4(\mathcal{L}, W)$, *then*

$$\begin{aligned}
&d^4h(y, w_1, w_2, w_3, w_4, w_5)\\
=&\varepsilon(|h| + |y| + |w_1| + |w_2| + |w_3|, |w_4| + |w_5|)\vartheta(\beta^2(w_4), \beta^2(w_5))h(y, w_1, w_2, w_3)\\
&- \delta\varepsilon(|h| + |y| + |w_1| + |w_2|, |w_3| + |w_5|)\varepsilon(|w_4|, |w_5|)\vartheta(\beta^2(w_3), \beta^2(w_5))h(y, w_1,\\
&w_2, w_4) - \delta\varepsilon(|h| + |y|, |w_1| + |w_2|)D(\beta^2(w_1), \beta^2(w_2))h(y, w_3, w_4, w_5)\\
&+ \varepsilon(|h| + |y| + |w_1| + |w_2|, |w_3| + |w_4|)D(\beta^2(w_3), \beta^2(w_4))h(y, w_1, w_2, w_5)\\
&+ h(\beta(y), [w_1, w_2, w_3], \beta(w_4), \beta(w_5))\\
&+ \varepsilon(|w_3|, |w_1| + |w_2|)h(\beta(y), \beta(w_3), [w_1, w_2, w_4], \beta(w_5))\\
&+ \delta\varepsilon(|w_1| + |w_2|, |w_3| + |w_4|)h(\beta(y), \beta(w_3), \beta(w_4), [w_1, w_2, w_5])\\
&- h(\beta(y), \beta(w_1), \beta(w_2), [w_3, w_4, w_5]).
\end{aligned}$$

Theorem 2.1. *About the coboundary operator* d^n *defined above, we have* $d^{n+2}d^n = 0$, $n = 1, 2$.

Proof. Using Definition 2.5, it is obvious immediately that $d^3d^1 = 0$ implies $d^4d^2 = 0$. Then we only need to prove $d^3d^1 = 0$. In fact, by (2.5)–(2.8), we obtain

$$\begin{aligned}
&d^3(d^1h)(w_1, w_2, w_3, w_4, w_5)\\
=&\varepsilon(|h| + |w_1| + |w_2| + |w_3|, |w_4| + |w_5|)\vartheta(\beta(w_4), \beta(w_5))(d^1h)(w_1, w_2, w_3)\\
&- \delta\varepsilon(|h| + |w_1| + |w_2|, |w_3| + |w_5|)\varepsilon(|w_4|, |w_5|)\vartheta(\beta(w_3), \beta(w_5))(d^1h)(w_1, w_2, w_4)\\
&- \delta\varepsilon(|h|, |w_1| + |w_2|)D(\beta(w_1), \beta(w_2))(d^1h)(w_3, w_4, w_5)\\
&+ \varepsilon(|h| + |w_1| + |w_2|, |w_3| + |w_4|)D(\beta(w_3), \beta(w_4))(d^1h)(w_1, w_2, w_5)\\
&+ (d^1h)([w_1, w_2, w_3], \beta(w_4), \beta(w_5)) + \varepsilon(|w_3|, |w_1| + |w_2|)(d^1h)(\beta(w_3), [w_1, w_2, w_4], \beta(w_5))\\
&+ \delta\varepsilon(|w_1| + |w_2|, |w_3| + |w_4|)(d^1h)(\beta(w_3), \beta(w_4), [w_1, w_2, w_5])\\
&- (d^1h)(\beta(w_1), \beta(w_2), [w_3, w_4, w_5])\\
=&\varepsilon(|h| + |w_1| + |w_2| + |w_3|, |w_4| + |w_5|)\vartheta(\beta(w_4), \beta(w_5))(\varepsilon(|h| + |w_1|, |w_2| + |w_3|)\vartheta(w_2, w_3)h(w_1)\\
&- \delta\varepsilon(|w_2|, |w_3|)\varepsilon(|h|, |w_1| + |w_3|)\vartheta(w_1, w_3)h(w_2) + \delta\varepsilon(|h|, |w_1| + |w_2|)D(w_1, w_2)h(w_3)\\
&- h([w_1, w_2, w_3])) - \delta\varepsilon(|h| + |w_1| + |w_2|, |w_3| + |w_5|)\varepsilon(|w_4|, |w_5|)\vartheta(\beta(w_3), \beta(w_5))\\
&(\varepsilon(|h| + |w_1|, |w_2| + |w_4|)\vartheta(w_2, w_4)h(w_1)\\
&- \delta\varepsilon(|w_2|, |w_4|)\varepsilon(|h|, |w_1| + |w_4|)\vartheta(w_1, w_4)h(w_2) + \delta\varepsilon(|h|, |w_1| + |w_2|)D(w_1, w_2)h(w_4)\\
&- h([w_1, w_2, w_4]))\\
&- \delta\varepsilon(|h|, |w_1| + |w_2|)D(\beta(w_1), \beta(w_2))(\varepsilon(|h| + |w_3|, |w_4| + |w_5|)\vartheta(w_4, w_5)h(w_3)\\
&- \delta\varepsilon(|w_4|, |w_5|)\varepsilon(|h|, |w_3| + |w_5|)\vartheta(w_3, w_5)h(w_4) + \delta\varepsilon(|h|, |w_3| + |w_4|)D(w_3, w_4)h(w_5)\\
&- h([w_3, w_4, w_5]))\\
&+ \varepsilon(|h| + |w_1| + |w_2|, |w_3| + |w_4|)D(\beta(w_3), \beta(w_4))(\varepsilon(|h| + |w_1|, |w_2| + |w_5|)\vartheta(w_2, w_5)h(w_1)
\end{aligned}$$

$$
\begin{aligned}
&- \delta\varepsilon(|w_2|,|w_5|)\varepsilon(|h|,|w_1|+|w_5|)\vartheta(w_1,w_5)h(w_2) + \delta\varepsilon(|h|,|w_1|+|w_2|)D(w_1,w_2)h(w_5)\\
&- h([w_1,w_2,w_5]))\\
&+ \varepsilon(|h|+|w_1|+|w_2|+|w_3|,|w_4|+|w_5|)(\vartheta(\beta(w_4),\beta(w_5))h([w_1,w_2,w_3])\\
&- \delta\varepsilon(|w_4|,|w_5|)\varepsilon(|h|,|w_1|+|w_2|+|w_3|+|w_5|)\vartheta([w_1,w_2,w_3],\beta(w_5))h(\beta(w_4))\\
&+ \delta\varepsilon(|h|,|w_1|+|w_2|+|w_3|+|w_4|)D([w_1,w_2,w_3],\beta(w_4))h(\beta(w_5)) - h([[w_1,w_2,w_3],\beta(w_4),\beta(w_5)]))\\
&+ \varepsilon(|w_3|,|w_1|+|w_2|)\varepsilon(|h|+|w_3|,|w_1|+|w_2|+|w_4|+|w_5|)(\vartheta([w_1,w_2,w_4],\beta(w_5))h(\beta(w_3))\\
&- \delta\varepsilon(|h|+|w_1|+|w_2|,|w_3|+|w_5|)\varepsilon(w_4|,|w_5|)\vartheta(\beta(w_3),\beta(w_5))h([w_1,w_2,w_4])\\
&+ \delta\varepsilon(|w_3|,|w_1|+|w_2|)\varepsilon(|h|,|w_1|+|w_2|+|w_3|+|w_4|)D(\beta(w_3),[w_1,w_2,w_4])h(\beta(w_5))\\
&- \varepsilon(|w_3|,|w_1|+|w_2|)h([\beta(w_3),[w_1,w_2,w_4],\beta(w_5)]))\\
&+ \delta\varepsilon(|w_1|+|w_2|,|w_3|+|w_4|)(\varepsilon(|h|+|w_3|,|w_1|+|w_2|+|w_4|+|w_5|)\vartheta(\beta(w_4),[w_1,w_2,w_5])h(\beta(w_3))\\
&- \delta\varepsilon(|w_4|,|w_1|+|w_2|+|w_5|)\varepsilon(|h|,|w_1|+|w_2|+|w_3|+|w_5|)\vartheta(\beta(w_3),[w_1,w_2,w_5])h(\beta(w_4))\\
&+ \delta\varepsilon(|h|,|w_3|+|w_4|)D(\beta(w_3),\beta(w_4))h([w_1,w_2,w_5]) - h([\beta(w_3),\beta(w_4),[w_1,w_2,w_5]]))\\
&- (\varepsilon(|h|+|w_1|,|w_2|+|w_3|+|w_4|+|w_5|)\vartheta(\beta(w_2),[w_3,w_4,w_5])h(\beta(w_1))\\
&- \delta\varepsilon(|h|,|w_1|+|w_3|+|w_4|+|w_5|)\varepsilon(|w_2|,|w_3|+|w_4|+|w_5|)\vartheta(\beta(w_1),[w_3,w_4,w_5])h(\beta(w_2))\\
&+ \delta\varepsilon(|h|,|w_1|+|w_2|)D(\beta(w_1),\beta(w_2))h([w_3,w_4,w_5]) - h([\beta(w_1),\beta(w_2),[w_3,w_4,w_5]]))\\
=&- h([[w_1,w_2,w_3],\beta(w_4),\beta(w_5)]) - \varepsilon(|w_3|,|w_1|+|w_2|)h([\beta(w_3),[w_1,w_2,w_4],\beta(w_5)])\\
&- \delta\varepsilon(|w_1|+|w_2|,|w_3|+|w_4|)h([\beta(w_3),\beta(w_4),[w_1,w_2,w_5]]) + h([\beta(w_1),\beta(w_2),[w_3,w_4,w_5]])\\
&+ \varepsilon(|h|+|w_1|+|w_2|+|w_3|,|w_4|+|w_5|)\varepsilon(|h|+|w_1|,|w_2|+|w_3|)\vartheta(\beta(w_4),\beta(w_5))\vartheta(w_2,w_3)h(w_1)\\
&- \delta\varepsilon(|h|+|w_1|+|w_2|,|w_3|+|w_5|)\varepsilon(|w_4|,|w_5|)\varepsilon(|h|+|w_1|,|w_2|+|w_4|)\\
&\vartheta(\beta(w_3),\beta(w_5))\vartheta(w_2,w_4)h(w_1)\\
&+ \varepsilon(|h|+|w_1|+|w_2|,|w_3|+|w_4|)\varepsilon(|h|+|w_1|,|w_2|+|w_5|)D(\beta(w_3),\beta(w_4))\vartheta(w_2,w_5)h(w_1)\\
&- \varepsilon(|h|+|w_1|,|w_2|+|w_3|+|w_4|+|w_5|)\vartheta(\beta(w_2),[w_3,w_4,w_5])h(\beta(w_1))\\
&- \delta\varepsilon(|h|+|w_1|+|w_2|+|w_3|,|w_4|+|w_5|)\varepsilon(|w_2|,|w_3|)\varepsilon(|h|,|w_1|+|w_3|)\vartheta(\beta(w_4),\beta(w_5))\\
&\vartheta(w_1,w_3)h(w_2)\\
&+ \varepsilon(|h|+|w_1|+|w_2|,|w_3|+|w_5|)\varepsilon(|w_4|,|w_2|+|w_5|)\varepsilon(|h|,|w_1|+|w_4|)\vartheta(\beta(w_3),\beta(w_5))\\
&\vartheta(w_1,w_4)h(w_2)\\
&- \delta\varepsilon(|h|+|w_1|+|w_2|,|w_3|+|w_4|)\varepsilon(|w_2|,|w_5|)\varepsilon(|h|,|w_1|+|w_5|)D(\beta(w_3),\beta(w_4))\vartheta(w_1,w_5)h(w_2)\\
&+ \delta\varepsilon(|h|,|w_1|+|w_3|+|w_4|+|w_5|)\varepsilon(|w_2|,|w_3|+|w_4|+|w_5|)\vartheta(\beta(w_1),[w_3,w_4,w_5])h(\beta(w_2))\\
&+ \delta\varepsilon(|h|+|w_1|+|w_2|+|w_3|,|w_4|+|w_5|)\varepsilon(|h|,|w_1|+|w_2|)\vartheta(\beta(w_4),\beta(w_5))D(w_1,w_2)h(w_3)\\
&- \delta\varepsilon(|h|,|w_1|+|w_2|)\varepsilon(|h|+|w_3|,|w_4|+|w_5|)D(\beta(w_1),\beta(w_2))\vartheta(w_4,w_5)h(w_3)\\
&+ \varepsilon(|w_3|,|w_1|+|w_2|)\varepsilon(|h|+|w_3|,|w_1|+|w_2|+|w_4|+|w_5|)\vartheta([w_1,w_2,w_4],\beta(w_5))h(\beta(w_3))\\
&+ \delta\varepsilon(|w_1|+|w_2|,|w_3|+|w_4|)\varepsilon(|h|+|w_3|,|w_1|+|w_2|+|w_4|+|w_5|)\vartheta(\beta(w_4),[w_1,w_2,w_5])h(\beta(w_3))\\
&- \varepsilon(|h|+|w_1|+|w_2|,|w_3|+|w_5|)\varepsilon(|w_4|,|w_5|)\varepsilon(|h|,|w_1|+|w_2|)\vartheta(\beta(w_3),\beta(w_5))D(w_1,w_2)h(w_4)\\
&+ \varepsilon(|h|,|w_1|+|w_2|)\varepsilon(|w_4|,|w_5|)\varepsilon(|h|,|w_3|+|w_5|)D(\beta(w_1),\beta(w_2))\vartheta(w_3,w_5)h(w_4)\\
&- \delta\varepsilon(|w_4|,|w_5|)\varepsilon(|h|,|w_1|+|w_2|+|w_3|+|w_5|)\vartheta([w_1,w_2,w_3],\beta(w_5))h(\beta(w_4))\\
&- \varepsilon(|w_1|+|w_2|,|w_3|+|w_4|)\varepsilon(|w_4|,|w_1|+|w_2|+|w_5|)\varepsilon(|h|,|w_1|+|w_2|+|w_3|+|w_5|)
\end{aligned}
$$

$$
\begin{aligned}
&\vartheta(\beta(w_3),[w_1,w_2,w_5])h(\beta(w_4))\\
&-\varepsilon(|h|,|w_1|+|w_2|+|w_3|+|w_4|)D(\beta(w_1),\beta(w_2))D(w_3,w_4)h(w_5)\\
&+\delta\varepsilon(|h|+|w_1|+|w_2|,|w_3|+|w_4|)\varepsilon(|h|,|w_1|+|w_2|)D(\beta(w_3),\beta(w_4))D(w_1,w_2)h(w_5)\\
&+\delta\varepsilon(|h|,|w_1|+|w_2|+|w_3|+|w_4|)D([w_1,w_2,w_3],\beta(w_4))h(\beta(w_5))\\
&+\delta\varepsilon(|w_3|,|w_1|+|w_2|)\varepsilon(|h|,|w_1|+|w_2|+|w_3|+|w_4|)D(\beta(w_3),[w_1,w_2,w_4])h(\beta(w_5))\\
&-\varepsilon(|h|+|w_1|+|w_2|+|w_3|,|w_4|+|w_5|)\vartheta(\beta(w_4),\beta(w_5))h([w_1,w_2,w_3])\\
&+\delta\varepsilon(|h|+|w_1|+|w_2|,|w_3|+|w_5|)\varepsilon(|w_4|,|w_5|)\vartheta(\beta(w_3),\beta(w_5))h([w_1,w_2,w_4])\\
&+\delta\varepsilon(|h|,|w_1|+|w_2|)D(\beta(w_1),\beta(w_2))h([w_3,w_4,w_5])\\
&-\varepsilon(|h|,|w_3|+|w_4|)\varepsilon(|w_1|+|w_2|,|w_3|+|w_4|)D(\beta(w_3),\beta(w_4))h([w_1,w_2,w_5])\\
&+\varepsilon(|h|+|w_1|+|w_2|+|w_3|,|w_4|+|w_5|)\vartheta(\beta(w_4),\beta(w_5))h([w_1,w_2,w_3])\\
&-\delta\varepsilon(|h|+|w_1|+|w_2|,|w_3|+|w_5|)\varepsilon(|w_4|,|w_5|)\vartheta(\beta(w_3),\beta(w_5))h([w_1,w_2,w_4])\\
&+\varepsilon(|h|,|w_3|+|w_4|)\varepsilon(|w_1|+|w_2|,|w_3|+|w_4|)D(\beta(w_3),\beta(w_4))h([w_1,w_2,w_5])\\
&-\delta\varepsilon(|h|,|w_1|+|w_2|)D(\beta(w_1),\beta(w_2))h([w_3,w_4,w_5])\\
=&0.
\end{aligned}
$$

Therefore, the proof is complete.

For $n = 1, 2, 3, 4$. If $d^n h = 0$, then $h \in C^n_\gamma(\mathcal{L}, W)$ is called an n-cocycle. The subspace $Z^n_\gamma(\mathcal{L}, W)$ spanned by n-cocycle, and set $B^n_\gamma(\mathcal{L}, W) = d^{n-2}C^{n-2}_\gamma(\mathcal{L}, W)$.

Since $d^{n+2}d^n = 0$, $B^n_\gamma(\mathcal{L}, W)$ is a subspace of $Z^n_\gamma(\mathcal{L}, W)$. Therefore, we can determine a cohomology space $H^n_\gamma(\mathcal{L}, W)$ of $(\mathcal{L}, [\cdot,\cdot,\cdot])$ as the factor space $Z^n_\gamma(\mathcal{L}, W)/B^n_\gamma(\mathcal{L}, W)$.

3. 1-parameter formal deformations on multiplicative Hom-δ-Jordan Lie color triple systems

Suppose that $(\mathcal{L}, [\cdot,\cdot,\cdot])$ is a multiplicative Hom-δ-Jordan Lie color triple system, and $\mathbf{F}[[s]]$ is the ring of formal power series over $\mathbf{F}$. Suppose that $\mathcal{L}[[s]]$ is the set of formal power series over $\mathcal{L}$. An $\mathbf{F}$-trilinear map $h : \mathcal{L}\times\mathcal{L}\times\mathcal{L} \to \mathcal{L}$ is extended to be an $\mathbf{F}[[s]]$-trilinear map $h : \mathcal{L}[[s]]\times\mathcal{L}[[s]]\times\mathcal{L}[[s]] \to \mathcal{L}[[s]]$ by

$$h\left(\sum_{i\geq 0} t_i s^i, \sum_{j\geq 0} p_j s^j, \sum_{k\geq 0} q_k s^k\right) = \sum_{i,j,k\geq 0} h(t_i, p_j, q_k)s^{i+j+k}.$$

Definition 3.1. *Suppose that $(\mathcal{L}, [\cdot,\cdot,\cdot])$ is a multiplicative Hom-δ-Jordan Lie color triple system over $\mathbf{F}$. A 1-parameter formal deformation on $(\mathcal{L}, [\cdot,\cdot,\cdot])$ is a formal power series $l_{\mathcal{L}} : \mathcal{L}[[s]] \times \mathcal{L}[[s]] \times \mathcal{L}[[s]] \to \mathcal{L}[[s]]$ of the form*

$$l_{\mathcal{L}}(t, p, q) = \sum_{i\geq 0} l_i(t, p, q)s^i = l_0(t, p, q) + l_1(t, p, q)t + l_2(t, p, q)s^2 + \cdots,$$

any l_i is an $\mathbf{F}$-trilinear map $l_i : \mathcal{L} \times \mathcal{L} \times \mathcal{L} \to \mathcal{L}$, which is extended to be an $\mathbf{F}[[s]]$-trilinear map, and $l_0(t, p, q) = [t, p, q]$, satisfying the following relations hold

$$l_{\mathcal{L}}(\beta(t), \beta(p), \beta(q)) = \beta l_{\mathcal{L}}(t, p, q), \tag{3.1}$$

$$l_{\mathcal{L}}(t, p, q) = -\delta\varepsilon(|t|, |p|)l_{\mathcal{L}}(p, t, q), \tag{3.2}$$

$$\varepsilon(|t|,|q|)l_{\mathcal{L}}(t,p,q)+\varepsilon(|p|,|t|)l_{\mathcal{L}}(p,q,t)+\varepsilon(|q|,|p|)l_{\mathcal{L}}(q,t,p)=0, \tag{3.3}$$

$$\begin{aligned} l_{\mathcal{L}}(\beta(z),\beta(u),l_{\mathcal{L}}(t,p,q)) =& l_{\mathcal{L}}(l_{\mathcal{L}}(z,u,t),\beta(p),\beta(q)) \\ &+\varepsilon(|t|,|z|+|u|)l_{\mathcal{L}}(\beta(t),l_{\mathcal{L}}(z,u,p),\beta(q)) \\ &+\delta\varepsilon(|z|+|u|,|t|+|p|)l_{\mathcal{L}}(\beta(t),\beta(p),l_{\mathcal{L}}(z,u,q)). \end{aligned} \tag{3.4}$$

Equations (3.2)–(3.4) are said to be the deformation equations of a multiplicative Hom-δ-Jordan Lie color triple system.

Since $\mathcal{L}[[s]]$ is a module over $\mathbf{F}[[s]]$ and $l_{\mathcal{L}}$ shows the trilinear on $\mathcal{L}[[s]]$, we get $\mathcal{L}_t=(\mathcal{L}[[s]],l_{\mathcal{L}})$ is a multiplicative Hom-δ-Jordan Lie color triple system. In the following, we study the deformation Eqs (3.2)–(3.4).

Equations (3.2) and (3.3) are equivalent to the following relations:

$$l_i(\beta(t),\beta(p),\beta(q))=\beta l_i(t,p,q), \tag{3.5}$$

$$l_i(t,p,q)=-\delta\varepsilon(|t|,|p|)l_i(p,t,q), \tag{3.6}$$

$$\varepsilon(|t|,|q|)l_i(t,p,q)+\varepsilon(|p|,|t|)l_i(p,q,t)+\varepsilon(|q|,|p|)l_i(q,t,p)=0, \tag{3.7}$$

respectively, for $i=0,1,2,\cdots$. The Eq (3.4) can be written as

$$\begin{aligned} &\sum_{i,j\geq 0} l_i(\beta(z),\beta(u),l_j(t,p,q)) \\ =&\sum_{i,j\geq 0} l_i(l_j(z,u,t),\beta(p),\beta(q))+\sum_{i,j\geq 0}\varepsilon(|t|,|z|+|u|)l_i(\beta(t),l_j(z,u,p),\beta(q)) \\ &+\delta\sum_{i,j\geq 0}\varepsilon(|z|+|u|,|t|+|p|)l_i(\beta(t),\beta(p),l_j(z,u,q)). \end{aligned}$$

Then

$$\begin{aligned} \sum_{i+j=n}\Big(& l_i(l_j(z,u,t),\beta(p),\beta(q))+\varepsilon(|t|,|z|+|u|)l_i(\beta(t),l_j(z,u,p),\beta(q)) \\ &+\delta\varepsilon(|z|+|u|,|t|+|p|)l_i(\beta(t),\beta(p),l_j(z,u,q)) \\ &-l_i(\beta(z),\beta(u),l_j(t,p,q))\Big)=0, \quad \forall n=0,1,2\cdots. \end{aligned}$$

Two $\mathbf{F}$-trilinear maps $f,g:\mathcal{L}\times\mathcal{L}\times\mathcal{L}\to\mathcal{L}$ which are extended to be $\mathbf{F}[[s]]$-trilinear, we show a map $f\circ g:\mathcal{L}[[s]]\times\mathcal{L}[[s]]\times\mathcal{L}[[s]]\times\mathcal{L}[[s]]\times\mathcal{L}[[s]]\to\mathcal{L}[[s]]$ using

$$\begin{aligned} f\circ g(z,u,t,p,q)=&f(g(z,u,t),\beta(p),\beta(q))+\varepsilon(|t|,|z|+|u|)f(\beta(t),g(z,u,p),\beta(q)) \\ &+\delta\varepsilon(|z|+|u|,|t|+|p|)f(\beta(t),\beta(p),g(z,u,q))-f(\beta(z),\beta(u),g(t,p,q)). \end{aligned}$$

Hence, the deformation Eq (3.4) can be obtained as

$$\sum_{i+j=n} l_i\circ l_j=0.$$

If $n=1$, then $l_0\circ l_1+l_1\circ l_0=0$.

If $n \geq 2$, then $-(l_0 \circ l_n + l_n \circ l_0) = l_1 \circ l_{n-1} + l_2 \circ l_{n-2} + \cdots + l_{n-1} \circ l_1$.

Section 2 gives that $\mathcal{L}$ is the adjoint $(\mathcal{L}, [\cdot,\cdot,\cdot], \beta)$-module by assuming $\vartheta(t,p)(q) = \varepsilon(|q|, |t| + |p|)[q,t,p]$. Under this situation, using (3.6) and (3.7), we get $l_i \in C^3(\mathcal{L},\mathcal{L})$. Considering the definition 2.5, we show $d^3 l_n = l_0 \circ l_n + l_n \circ_\beta l_0$, for $n = 0, 1, 2\cdots$. Therefore, the deformation Eq (3.4) can be shown as

$$d^3 l_1 = 0,$$
$$-d^3 l_n = l_1 \circ l_{n-1} + l_2 \circ l_{n-2} + \cdots + l_{n-1} \circ l_1.$$

Thus, l_1 is a 3-Hom-cocycle and is said to be the infinitesimal of $l_{\mathcal{L}}$.

Definition 3.2. *Suppose that $(\mathcal{L}, [\cdot,\cdot,\cdot])$ is a multiplicative Hom-δ-Jordan Lie color triple system. Assume that $l_{\mathcal{L}}(t,p,q) = \sum_{i\geq 0} l_i(t,p,q)s^i$ and $l'_{\mathcal{L}}(t,p,q) = \sum_{i\geq 0} l'_i(t,p,q)s^i$ are two 1-parameter formal deformations on $(\mathcal{L}, [\cdot,\cdot,\cdot])$. $l_{\mathcal{L}}$ and $l'_{\mathcal{L}}$ are known as equivalent, denoted by $l_{\mathcal{L}} \sim l'_{\mathcal{L}}$, if there is a formal isomorphism of* $\mathbf{F}[[s]]$*-modules*

$$\varphi_t(t) = \sum_{i\geq 0} \varphi_i(t)s^i : (\mathcal{L}[[s]], m_{\mathcal{L}}) \longrightarrow (\mathcal{L}[[s]], m'_{\mathcal{L}}),$$

any $\varphi_i : \mathcal{L} \to \mathcal{L}$ is an $\mathbf{F}$*-linear map that is extended to be an* $\mathbf{F}[[s]]$*-linear map, and φ_0 is the identity mapping of $\mathcal{L}$. We obtain*

$$\varphi_t \circ \beta = \beta \circ \varphi_t,$$
$$\varphi_t \circ l_{\mathcal{L}}(t,p,q) = l'_{\mathcal{L}}(\varphi_t(t), \varphi_t(p), \varphi_t(q)).$$

In the case $l_1 = l_2 = \cdots = 0$, $l_{\mathcal{L}} = l_0$ is known as the null deformation. If $l_{\mathcal{L}} \sim l_0$, then the 1-parameter formal deformation $l_{\mathcal{L}}$ is said to be trivial. If any 1-parameter formal deformation $l_{\mathcal{L}}$ is trivial, then the multiplicative Hom-δ-Jordan Lie color triple system $(\mathcal{L}, [\cdot,\cdot,\cdot])$ is known as analytically rigid.

Theorem 3.1. *Suppose that $l_{\mathcal{L}}(t,p,q) = \sum_{i\geq 0} l_i(t,p,q)s^i$ and $l'_{\mathcal{L}}(t,p,q) = \sum_{i\geq 0} l'_i(t,p,q)s^i$ are equivalent 1-parameter formal deformations on $(\mathcal{L}, [\cdot,\cdot,\cdot])$. Then l_1 and l'_1 belong to the same cohomology class in $H^3(\mathcal{L},\mathcal{L})$.*

Proof. Suppose that $\varphi_t(t) = \sum_{i\geq 0} \varphi_i(t)s^i$ is the formal $\mathbf{F}[[s]]$-module isomorphism, satisfying $\varphi_t \circ \beta = \beta \circ \varphi_t$, and

$$\sum_{i\geq 0} \varphi_i \left(\sum_{j\geq 0} l_j(t,p,q)s^j \right) s^i = \sum_{i\geq 0} l'_i \left(\sum_{k\geq 0} \varphi_k(t)s^k, \sum_{l\geq 0} \varphi_l(p)s^l, \sum_{l\geq 0} \varphi_m(q)s^m \right) s^i.$$

Then

$$\sum_{i+j=n} \varphi_i(l_j(t,p,q))s^{i+j} = \sum_{i+k+l+m=n} l'_i(\varphi_k(t), \varphi_l(p), \varphi_m(q))s^{i+k+l+m}.$$

In especial,

$$\sum_{i+j=1} \varphi_i(l_j(t,p,q)) = \sum_{i+k+l+m=1} l'_i(\varphi_k(t), \varphi_l(p), \varphi_m(q)),$$

thus,

$$l_1(t,p,q) + \varphi_1([t,p,q])$$

$$
\begin{aligned}
&=[\varphi_1(t), p, q] + [t, \varphi_1(p), q] + [t, p, \varphi_1(q)] + l'_1(t, p, q)\\
&=\varepsilon(|t|, |p| + |q|)\vartheta(p, q)\varphi_1(t) - \delta\varepsilon(|p|, |q|)\vartheta(t, q)\varphi_1(p) + \delta D(t, p)\varphi_1(q) + l'_1(t, p, q).
\end{aligned}
$$

We give that $l_1 - l'_1 = d^1\varphi_1 \in B^3(\mathcal{L}, \mathcal{L})$. □

Theorem 3.2. *Suppose that $(\mathcal{L}, [\cdot, \cdot, \cdot])$ is a multiplicative Hom-δ-Jordan Lie color triple system satisfying $H^3(\mathcal{L}, \mathcal{L}) = 0$. We obtain that $(\mathcal{L}, [\cdot, \cdot, \cdot])$ is analytically rigid.*

Proof. Suppose that $l_{\mathcal{L}}$ is a 1-parameter formal deformation on $(\mathcal{L}, [\cdot, \cdot, \cdot])$. Assume that $l_{\mathcal{L}} = l_0 + \sum_{i\geq n} l_i s^i$. Hence,

$$d^3 l_n = l_1 \circ l_{n-1} + l_2 \circ l_{n-2} + \cdots + l_{n-1} \circ l_1 = 0,$$

moreover, $l_n \in Z^3(\mathcal{L}, \mathcal{L}) = B^3(\mathcal{L}, \mathcal{L})$. It is proved that there is $h_n \in C^1(\mathcal{L}, \mathcal{L})$ satisfying $l_n = d^1 h_n$.

Assume that φ_t is even, and $\varphi_t = \mathrm{id}_{\mathcal{L}} - h_n s^n : (\mathcal{L}[[s]], l_{\mathcal{L}}) \longrightarrow (\mathcal{L}[[s]], l'_{\mathcal{L}})$. Notice that

$$\varphi_t \circ \sum_{i\geq 0} h_n^i s^{in} = \sum_{i\geq 0} h_n^i s^{in} \circ \varphi_t = \mathrm{id}_{\mathcal{L}[[s]]}.$$

It is easy to see that φ_t is a linear isomorphism, and $\varphi_t \circ \beta = \beta \circ \varphi_t$.

We shall consider $l'_{\mathcal{L}}(t, p, q) = \varphi_s^{-1} l_{\mathcal{L}}(\varphi_t(t), \varphi_t(p), \varphi_t(q))$. The following fact is clear to prove that $l'_{\mathcal{L}}$ is a 1-parameter formal deformation on $(\mathcal{L}, [\cdot, \cdot, \cdot])$. Indeed,

$$
\begin{aligned}
&l'_{\mathcal{L}}(t, p, q)\\
=&\varphi_s^{-1} l_{\mathcal{L}}(\varphi_t(t), \varphi_t(p), \varphi_t(q))\\
=&-\delta\varepsilon(|t|, |p|)\varphi_s^{-1} l_{\mathcal{L}}(\varphi_t(p), \varphi_t(t), \varphi_t(q))\\
=&-\delta\varepsilon(|t|, |p|) l'_{\mathcal{L}}(p, t, q).
\end{aligned}
$$

$$
\begin{aligned}
&\varepsilon(|t|, |q|) l'_{\mathcal{L}}(t, p, q) + \varepsilon(|p|, |t|) l'_{\mathcal{L}}(p, q, t) + \varepsilon(|q|, |p|) l'_{\mathcal{L}}(q, t, p)\\
=&\varepsilon(|t|, |q|)\varphi_s^{-1} l_{\mathcal{L}}(\varphi_t(t), \varphi_t(p), \varphi_t(q)) + \varepsilon(|p|, |t|)\varphi_s^{-1} l_{\mathcal{L}}(\varphi_t(p), \varphi_t(q), \varphi_t(t))\\
&+ \varepsilon(|q|, |p|)\varphi_s^{-1} l_{\mathcal{L}}(\varphi_t(q), \varphi_t(t), \varphi_t(p))\\
=&\varphi_s^{-1}(\varepsilon(|t|, |q|) l_{\mathcal{L}}(\varphi_t(t), \varphi_t(p), \varphi_t(q)) + \varepsilon(|p|, |t|) l_{\mathcal{L}}(\varphi_t(p), \varphi_t(q), \varphi_t(t))\\
&+ \varepsilon(|q|, |p|) l_{\mathcal{L}}(\varphi_t(q), \varphi_t(t), \varphi_t(p)))\\
=&0.
\end{aligned}
$$

$$
\begin{aligned}
&l'_{\mathcal{L}}(\beta(t_1), \beta(t_2), l'_{\mathcal{L}}(t, p, q))\\
=&l'_{\mathcal{L}}(\beta(t_1), \beta(t_2), \varphi_s^{-1} l_{\mathcal{L}}(\varphi_t(t), \varphi_t(p), \varphi_t(q)))\\
=&\varphi_s^{-1} l_{\mathcal{L}}(\varphi_t(\beta(t_1)), \varphi_t(\beta(t_2)), l_{\mathcal{L}}(\varphi_t(t), \varphi_t(p), \varphi_t(q)))\\
=&\varphi_s^{-1} l_{\mathcal{L}}(\beta(\varphi_t(t_1)), \beta(\varphi_t(t_2)), l_{\mathcal{L}}(\varphi_t(t), \varphi_t(p), \varphi_t(q))).
\end{aligned}
$$

$$
\begin{aligned}
&l'_{\mathcal{L}}(l'_{\mathcal{L}}(t_1, t_2, t), \beta(p), \beta(q))\\
=&l'_{\mathcal{L}}(\varphi_s^{-1} l_{\mathcal{L}}(\varphi_t(t_1), \varphi_t(t_2), \varphi_t(t)), \beta(p), \beta(q))
\end{aligned}
$$

$$=\varphi_s^{-1} l_{\mathcal{L}}(l_{\mathcal{L}}(\varphi_t(t_1), \varphi_t(t_2), \varphi_t(t)), \beta(\varphi_t(p)), \beta(\varphi_t(q))).$$

$$\begin{aligned}
&\varepsilon(|t|, |t_1| + |t_2|) l'_{\mathcal{L}}(\beta(t), l'_{\mathcal{L}}(t_1, t_2, p), \beta(q)) \\
=&\varepsilon(|t|, |t_1| + |t_2|) l'_{\mathcal{L}}(\beta(t), \varphi_s^{-1} l_{\mathcal{L}}(\varphi_t(t_1), \varphi_t(t_2), \varphi_t(p)), \beta(q)) \\
=&\varepsilon(|t|, |t_1| + |t_2|) \varphi_s^{-1} l_{\mathcal{L}}(\varphi_t(\beta(t)), l_{\mathcal{L}}(\varphi_t(t_1), \varphi_t(t_2), \varphi_t(p)), \varphi_t(\beta(q))) \\
=&\varepsilon(|t|, |t_1| + |t_2|) \varphi_s^{-1} l_{\mathcal{L}}(\beta(\varphi_t(t)), l_{\mathcal{L}}(\varphi_t(t_1), \varphi_t(t_2), \varphi_t(p)), \beta(\varphi_t(q))).
\end{aligned}$$

$$\begin{aligned}
&\delta\varepsilon(|t_1| + |t_2|, |t| + |p|) l'_{\mathcal{L}}(\beta(t), \beta(p), l'_{\mathcal{L}}(t_1, t_2, q)) \\
=&\delta\varepsilon(|t_1| + |t_2|, |t| + |p|) l'_{\mathcal{L}}(\beta(t), \beta(p), \varphi_s^{-1} l_{\mathcal{L}}(\varphi_t(t_1), \varphi_t(t_2), \varphi_t(q))) \\
=&\delta\varepsilon(|t_1| + |t_2|, |t| + |p|) \varphi_s^{-1} l_{\mathcal{L}}(\varphi_t(\beta(t)), \varphi_t(\beta(p)), l_{\mathcal{L}}(\varphi_t(t_1), \varphi_t(t_2), \varphi_t(q))) \\
=&\delta\varepsilon(|t_1| + |t_2|, |t| + |p|) \varphi_s^{-1} l_{\mathcal{L}}(\beta(\varphi_t(t)), \beta(\varphi_t(p)), l_{\mathcal{L}}(\varphi_t(t_1), \varphi_t(t_2), \varphi_t(q))).
\end{aligned}$$

The calculations above give that relations (3.2)–(3.4) hold. In view of Definition 3.2, we know $l_{\mathcal{L}} \sim l'_{\mathcal{L}}$. Assume that $l'_{\mathcal{L}} = \sum_{i \geq 0} l'_i s^i$. Hence

$$(\mathrm{id}_{\mathcal{L}} - h_n s^n)\left(\sum_{i\geq 0} l'_i(t, p, q) s^i\right) = \left(l_0 + \sum_{i \geq n} l_i s^i\right)(t - h_n(t)s^n, p - h_n(p)s^n, q - h_n(q)s^n),$$

i.e.,

$$\begin{aligned}
&\sum_{i\geq 0} l'_i(t, p, q)s^i - \sum_{i \geq 0} h_n \circ l'_i(t, p, q) s^{i+n} \\
=&[t, p, q] - ([h_n(t), p, q] + [t, h_n(p), q] + [t, p, h_n(q)])s^n \\
&+ ([h_n(t), h_n(p), q] + [t, h_n(p), h_n(q)] + [h_n(t), p, h_n(q)])s^{2n} - [h_n(t), h_n(p), h_n(q)]s^{3n} \\
&+ \sum_{i \geq n} l_i(t, p, q)s^i - \sum_{i\geq n}(l_i(h_n(t), p, q) + l_i(t, h_n(p), q) + l_i(t, p, h_n(q)))s^{i+n} \\
&+ \sum_{i\geq n}(l_i(h_n(t), h_n(p), q) + l_i(t, h_n(p), h_n(q)) + l_i(h_n(t), p, h_n(q)))s^{i+2n} \\
&- \sum_{i \geq n} l_i(h_n(t), h_n(p), h_n(q))s^{i+3n}.
\end{aligned}$$

Then, it follows $l'_1 = \cdots = l'_{n-1} = 0$ and

$$\begin{aligned}
&l'_n(t, p, q) - h_n([t, p, q]) \\
=& -([h_n(t), p, q] + [t, h_n(p), q] + [t, p, h_n(q)]) + l_n(t, p, q) \\
=& -\varepsilon(|t|, |p| + |q|)\vartheta(p, q)h_n(t) + \delta\varepsilon(|p|, |q|)\vartheta(t, q)h_n(p) - \delta D(t, p)h_n(q) + l_n(t, p, q).
\end{aligned}$$

Therefore, $l'_n = l_n - d^1 h_n = 0$ and $l'_{\mathcal{L}} = l_0 + \sum_{i \geq n+1} l'_i s^i$. By induction, we know $l_{\mathcal{L}} \sim l_0$, i.e., $(\mathcal{L}, [\cdot, \cdot, \cdot])$ is analytically rigid.

4. Abelian extensions on multiplicative Hom-δ-Jordan Lie color triple systems

A graded subspace I satisfying $[I, \mathcal{L}, \mathcal{L}] \subseteq I$, which is said to be a graded ideal on the multiplicative Hom-δ-Jordan Lie color triple system $\mathcal{L}$. If $[\mathcal{L}, I, I] = 0$, then the ideal I on a multiplicative Hom-δ-Jordan Lie color triple system is said to be an abelian ideal. Moreover, note that $[\mathcal{L}, I, I] = 0$ indicates $[I, \mathcal{L}, I] = 0$ and $[I, I, \mathcal{L}] = 0$.

Definition 4.1. *Suppose that $(\mathcal{L}, [\cdot,\cdot,\cdot]_{\mathcal{L}})$, $(W, [\cdot,\cdot,\cdot]_W)$, and $(\hat{\mathcal{L}}, [\cdot,\cdot,\cdot]_{\hat{\mathcal{L}}})$ are multiplicative Hom-δ-Jordan Lie color triple systems, and $f : W \to \hat{\mathcal{L}}$, $g : \hat{\mathcal{L}} \to \mathcal{L}$ are homomorphisms. The following sequence on multiplicative Hom-δ-Jordan Lie color triple systems is a short exact sequence if* $\mathrm{Im}(f) = \mathrm{Ker}(g)$, $\mathrm{Ker}(f) = 0$, *and* $\mathrm{Im}(g) = \mathcal{L}$,

$$0 \longrightarrow W \xrightarrow{f} \hat{\mathcal{L}} \xrightarrow{g} \mathcal{L} \longrightarrow 0. \tag{4.1}$$

Under this circumstance, we show $\hat{\mathcal{L}}$ an extension of $\mathcal{L}$ by W, which is denoted by $E_{\hat{\mathcal{L}}}$. It is termed an abelian extension if W is an abelian ideal of $\hat{\mathcal{L}}$, that is, $[w_1, w_2, \cdot]_{\hat{\mathcal{L}}} = [w_1, \cdot, w_2]_{\hat{\mathcal{L}}} = [\cdot, w_1, w_2]_{\hat{\mathcal{L}}} = 0$, for all $w_1, w_2 \in W$.

A section $\tau : \mathcal{L} \to \hat{\mathcal{L}}$ of $g : \hat{\mathcal{L}} \to \mathcal{L}$ is composed of the linear map $\tau : \mathcal{L} \to \hat{\mathcal{L}}$ satisfying $g \circ \tau = \mathrm{id}_{\mathcal{L}}$, and $\widehat{\beta} \circ \tau = \tau \circ \beta$.

Definition 4.2. *Two extensions on multiplicative Hom-δ-Jordan Lie color triple systems $E_{\hat{\mathcal{L}}} : 0 \longrightarrow W \xrightarrow{f} \hat{\mathcal{L}} \xrightarrow{g} \mathcal{L} \longrightarrow 0$ and $E_{\tilde{\mathcal{L}}} : 0 \longrightarrow W \xrightarrow{f'} \tilde{\mathcal{L}} \xrightarrow{g'} \mathcal{L} \longrightarrow 0$ are equivalent. If there is a multiplicative Hom-δ-Jordan Lie color triple system homomorphism $F : \hat{\mathcal{L}} \to \tilde{\mathcal{L}}$ satisfying the following diagram commutes*

$$\begin{array}{ccccccccc} 0 & \longrightarrow & W & \xrightarrow{f} & \hat{\mathcal{L}} & \xrightarrow{g} & \mathcal{L} & \longrightarrow & 0 \\ & & \downarrow{\scriptstyle \mathrm{id}} & & \downarrow{\scriptstyle F} & & \downarrow{\scriptstyle \mathrm{id}} & & \\ 0 & \longrightarrow & W & \xrightarrow{f'} & \tilde{\mathcal{L}} & \xrightarrow{g'} & \mathcal{L} & \longrightarrow & 0 \end{array}$$

Suppose that $\hat{\mathcal{L}}$ is an abelian extension on $\mathcal{L}$ by W, and an even linear mapping $\tau : \mathcal{L} \to \hat{\mathcal{L}}$ is a section. Assume maps $\mathcal{L} \otimes \mathcal{L} \to \mathrm{End}(W)$ by

$$D(u_1, u_2)(v) = \delta[\tau(u_1), \tau(u_2), v]_{\hat{\mathcal{L}}}, \tag{4.2}$$

$$\vartheta(u_1, u_2)(v) = \varepsilon(|v|, |u_1| + |u_2|)[v, \tau(u_1), \tau(u_2)]_{\hat{\mathcal{L}}}. \tag{4.3}$$

It is obvious that the following result holds, that is,

$$D(u_1, u_2)(v) = \varepsilon(|u_1|, |u_2|)\vartheta(u_2, u_1)(v) - \delta\vartheta(u_1, u_2)(v),$$

for all $(u_1, u_2) \in \mathcal{L} \otimes \mathcal{L}$, $v \in W$.

Theorem 4.1. *Using the above symbols, it follows that (W, ϑ) is a representation of $\mathcal{L}$, and ϑ is independent for the section τ. Furthermore, there is the same representation between equivalent abelian extensions.*

Proof. Suppose that another section $\tau' : \mathcal{L} \to \hat{\mathcal{L}}$, we obtain

$$p(\tau(u_i) - \tau'(u_i)) = u_i - u_i = 0 \Rightarrow \tau(u_i) - \tau'(u_i) \in W \Rightarrow \tau'(u_i) = \tau(u_i) + v_i,$$

for some $v_i \in W$.

Notice that $[w_1, w_2, \cdot]_{\hat{\mathcal{L}}} = 0 = [w_1, \cdot, w_2]_{\hat{\mathcal{L}}}$ for all $w_1, w_2 \in W$, this shows that

$$[v, \tau'(u_1), \tau'(u_2)]_{\hat{\mathcal{L}}} = [v, \tau(u_1), \tau(u_2)]_{\hat{\mathcal{L}}}.$$

Then, ϑ does not rely on the choice of the section τ.

Now, we will obtain that (W, ϑ) is a representation of $\mathcal{L}$.

By the equation

$$\begin{aligned}
&[\hat{\beta}(v), \hat{\beta}(\tau(t_1)), [\tau(u_1), \tau(u_2), \tau(u_3)]_{\hat{\mathcal{L}}}]_{\hat{\mathcal{L}}} \\
=&[[v, \tau(t_1), \tau(u_1)]_{\hat{\mathcal{L}}}, \hat{\beta}(\tau(u_2)), \hat{\beta}(\tau(u_3))]_{\hat{\mathcal{L}}} \\
&+ \varepsilon(|u_1|, |v| + |t_1|)[\hat{\beta}(\tau(u_1)), [v, \tau(t_1), \tau(u_2)]_{\hat{\mathcal{L}}}, \hat{\beta}(\tau(u_3))]_{\hat{\mathcal{L}}} \\
&+ \delta\varepsilon(|v| + |t_1|, |u_1| + |u_2|)[\hat{\beta}(\tau(u_1)), \hat{\beta}(\tau(u_2)), [v, \tau(t_1), \tau(u_3)]_{\hat{\mathcal{L}}}]_{\hat{\mathcal{L}}},
\end{aligned}$$

it follows that

$$\begin{aligned}
&\varepsilon(|t_1| + |u_1|, |u_2| + |u_3|)\vartheta(\beta(u_2), \beta(u_3))\vartheta(t_1, u_1)(v) \\
&- \delta\varepsilon(|t_1|, |u_1|)\varepsilon(|u_3|, |u_2| + |t_1|)\vartheta(\beta(u_1), \beta(u_3))\vartheta(t_1, u_2)(v) \\
&- \vartheta(\beta(t_1), [u_1, u_2, u_3])(\hat{\beta}(v)) + \varepsilon(|t_1|, |u_1| + |u_2|)D(\beta(u_1), \beta(u_2))\vartheta(t_1, u_3)(v) = 0.
\end{aligned}$$

Then we prove (2.6) holds.

Similarly, by the equation

$$\begin{aligned}
&[\hat{\beta}(\tau(t_1)), \hat{\beta}(\tau(t_2)), [v, \tau(u_1), \tau(u_2)]_{\hat{\mathcal{L}}}]_{\hat{\mathcal{L}}} \\
=&[[\tau(t_1), \tau(t_2), v]_{\hat{\mathcal{L}}}, \hat{\beta}(\tau(u_1)), \hat{\beta}(\tau(u_2))]_{\hat{\mathcal{L}}} \\
&+ \varepsilon(|v|, |t_1| + |t_2|)[\hat{\beta}(v), [\tau(t_1), \tau(t_2), \tau(u_1)]_{\hat{\mathcal{L}}}, \hat{\beta}(\tau(u_2))]_{\hat{\mathcal{L}}} \\
&+ \delta\varepsilon(|t_1| + |t_2|, |v| + |u_1|)[\hat{\beta}(v), \hat{\beta}(\tau(u_1)), [\tau(t_1), \tau(t_2), \tau(u_2)]_{\hat{\mathcal{L}}}]_{\hat{\mathcal{L}}},
\end{aligned}$$

we have

$$\begin{aligned}
&\delta\varepsilon(|t_1| + |t_2|, |u_1| + |u_2|)\vartheta(\beta(u_1), \beta(u_2))D(t_1, t_2)(v) - \delta D(\beta(t_1), \beta(t_2))\vartheta(u_1, u_2)(v) \\
&+ \vartheta([t_1, t_2, u_1], \beta(u_2))(\hat{\beta}(v)) + \delta\varepsilon(|u_1|, |t_1| + |t_2|)\vartheta(\beta(u_1), [t_1, t_2, u_2])(\hat{\beta}(v)) = 0,
\end{aligned}$$

by the equation

$$\begin{aligned}
&[\hat{\beta}(\tau(t_1)), \hat{\beta}(\tau(t_2)), [\tau(u_1), \tau(u_2), v]_{\hat{\mathcal{L}}}]_{\hat{\mathcal{L}}} \\
=&[[\tau(t_1), \tau(t_2), \tau(u_1)]_{\hat{\mathcal{L}}}, \hat{\beta}(\tau(u_2)), \hat{\beta}(v)]_{\hat{\mathcal{L}}} \\
&+ \varepsilon(|u_1|, |t_1| + |t_2|)[\hat{\beta}(\tau(u_1)), [\tau(t_1), \tau(t_2), \tau(u_2)]_{\hat{\mathcal{L}}}, \hat{\beta}(v)]_{\hat{\mathcal{L}}} \\
&+ \delta\varepsilon(|t_1| + |t_2|, |u_1| + |u_2|)[\hat{\beta}(\tau(u_1)), \hat{\beta}(\tau(u_2)), [\tau(t_1), \tau(t_2), v]_{\hat{\mathcal{L}}}]_{\hat{\mathcal{L}}},
\end{aligned}$$

we obtain

$$\begin{aligned}&\delta\varepsilon(|t_1|+|t_2|,|u_1|+|u_2|)D(\beta(u_1),\beta(u_2))D(t_1,t_2)(v)-D(\beta(t_1),\beta(t_2))D(u_1,u_2)(v)\\&+\delta D([t_1,t_2,u_1],\beta(u_2))(\hat{\beta}(v))+\delta\varepsilon(|u_1|,|t_1|+|t_2|)D(\beta(u_1),[t_1,t_2,u_2])(\hat{\beta}(v))=0.\end{aligned}$$

Moreover, we obtain that (2.7) and (2.8) hold. Consequently, we have that (W,ϑ) is a representation of $\mathcal{L}$.

Then, the following result will give that equivalent abelian extensions show the same ϑ.

Suppose that $E_{\hat{\mathcal{L}}}$ and $E_{\tilde{\mathcal{L}}}$ are equivalent abelian extensions, and $\mathcal{F}:\hat{\mathcal{L}}\to\tilde{\mathcal{L}}$ is the multiplicative Hom-δ-Jordan Lie color triple system homomorphism such that $\mathcal{F}\circ f=f'$, $g'\circ\mathcal{F}=g$. Considering linear sections τ and τ' of g and g', we know $g'\mathcal{F}\tau(u_i)=g\tau(u_i)=u_i=g'\tau'(u_i)$, thus $\mathcal{F}\tau(u_i)-\tau'(u_i)\in\mathrm{Ker}(g')\cong W$. Hence,

$$[v,\tau(u_1),\tau(u_2)]_{\hat{\mathcal{L}}}=[v,\mathcal{F}\tau(u_1),\mathcal{F}\tau(u_2)]_{\tilde{\mathcal{L}}}=[v,\tau'(u_1),\tau'(u_2)]_{\tilde{\mathcal{L}}},$$

completing the proof.

Assume that $\tau:\mathcal{L}\to\hat{\mathcal{L}}$ is a section of the abelian extension. Suppose that the map is as follows:

$$\omega(t_1,t_2,t_3)=[\tau(t_1),\tau(t_2),\tau(t_3)]_{\hat{\mathcal{L}}}-\tau([t_1,t_2,t_3]_{\mathcal{L}}),\tag{4.4}$$

for all $t_1,t_2,t_3\in\mathcal{L}$.

Theorem 4.2. *Suppose that* $0\longrightarrow W\longrightarrow\hat{\mathcal{L}}\longrightarrow\mathcal{L}\longrightarrow 0$ *is an abelian extension on* $\mathcal{L}$ *by* W*, and the representation* ϑ *is determined using (4.3). Thus* ω *given using (4.4) is a* 3*-Hom-cocycle of* $\mathcal{L}$ *with coefficients in* W*.*

Proof. Considering the equation

$$\begin{aligned}&[\hat{\beta}(\tau(u_1)),\hat{\beta}(\tau(u_2)),[\tau(t_1),\tau(t_2),\tau(t_3)]_{\hat{\mathcal{L}}}]_{\hat{\mathcal{L}}}\\=&[[\tau(u_1),\tau(u_2),\tau(t_1)]_{\hat{\mathcal{L}}},\hat{\beta}(\tau(t_2)),\hat{\beta}(\tau(t_3))]_{\hat{\mathcal{L}}}\\&+\varepsilon(|t_1|,|u_1|+|u_2|)[\hat{\beta}(\tau(t_1)),[\tau(u_1),\tau(u_2),\tau(t_2)]_{\hat{\mathcal{L}}},\hat{\beta}(\tau(t_3))]_{\hat{\mathcal{L}}}\\&+\delta\varepsilon(|u_1|+|u_2|,|t_1|+|t_2|)[\hat{\beta}(\tau(t_1)),\hat{\beta}(\tau(t_2)),[\tau(u_1),\tau(u_2),\tau(t_3)]_{\hat{\mathcal{L}}}]_{\hat{\mathcal{L}}}.\end{aligned}$$

The left side is equivalent to

$$\begin{aligned}&[\hat{\beta}(\tau(u_1)),\hat{\beta}(\tau(u_2)),[\tau(t_1),\tau(t_2),\tau(t_3)]_{\hat{\mathcal{L}}}]_{\hat{\mathcal{L}}}\\=&[\hat{\beta}(\tau(u_1)),\hat{\beta}(\tau(u_2)),\omega(t_1,t_2,t_3)+\tau([t_1,t_2,t_3]_{\mathcal{L}})]_{\hat{\mathcal{L}}}\\=&\delta D(\beta(u_1),\beta(u_2))\omega(t_1,t_2,t_3)+[\hat{\beta}(\tau(u_1)),\hat{\beta}(\tau(u_2)),\tau([t_1,t_2,t_3]_{\mathcal{L}})]_{\hat{\mathcal{L}}}\\=&\delta D(\beta(u_1),\beta(u_2))\omega(t_1,t_2,t_3)+\omega(\beta(u_1),\beta(u_2),[t_1,t_2,t_3]_{\mathcal{L}})+\tau([\beta(u_1),\beta(u_2),[t_1,t_2,t_3]_{\mathcal{L}}]_{\mathcal{L}}).\end{aligned}$$

On the other hand, the right side shows that

$$\begin{aligned}&[[\tau(u_1),\tau(u_2),\tau(t_1)]_{\hat{\mathcal{L}}},\hat{\beta}(\tau(t_2)),\hat{\beta}(\tau(t_3))]_{\hat{\mathcal{L}}}\\&+\varepsilon(|t_1|,|u_1|+|u_2|)[\hat{\beta}(\tau(t_1)),[\tau(u_1),\tau(u_2),\tau(t_2)]_{\hat{\mathcal{L}}},\hat{\beta}(\tau(t_3))]_{\hat{\mathcal{L}}}\\&+\delta\varepsilon(|u_1|+|u_2|,|t_1|+|t_2|)[\hat{\beta}(\tau(t_1)),\hat{\beta}(\tau(t_2)),[\tau(u_1),\tau(u_2),\tau(t_3)]_{\hat{\mathcal{L}}}]_{\hat{\mathcal{L}}}\end{aligned}$$

$$
\begin{aligned}
&= [\omega(u_1,u_2,t_1)+\tau([u_1,u_2,t_1]_{\mathcal{L}}),\hat{\beta}(\tau(t_2)),\hat{\beta}(\tau(t_3))]_{\hat{\mathcal{L}}}\\
&+ \varepsilon(|t_1|,|u_1|+|u_2|)[\hat{\beta}(\tau(t_1)),\omega(u_1,u_2,t_2)+\tau([u_1,u_2,t_2]_{\mathcal{L}}),\hat{\beta}(\tau(t_3))]_{\hat{\mathcal{L}}}\\
&+ \delta\varepsilon(|u_1|+|u_2|,|t_1|+|t_2|)[\hat{\beta}(\tau(t_1)),\hat{\beta}(\tau(t_2)),\omega(u_1,u_2,t_3)+\tau([u_1,u_2,t_3]_{\mathcal{L}})]_{\hat{\mathcal{L}}}\\
&= [\omega(u_1,u_2,t_1),\hat{\beta}(\tau(t_2)),\hat{\beta}(\tau(t_3))]_{\hat{\mathcal{L}}} + [\tau([u_1,u_2,t_1]_{\mathcal{L}}),\hat{\beta}(\tau(t_2)),\hat{\beta}(\tau(t_3))]_{\hat{\mathcal{L}}}\\
&+ \varepsilon(|t_1|,|u_1|+|u_2|)[\hat{\beta}(\tau(t_1)),\omega(u_1,u_2,t_2),\hat{\beta}(\tau(t_3))]_{\hat{\mathcal{L}}}\\
&+ \varepsilon(|t_1|,|u_1|+|u_2|)[\hat{\beta}(\tau(t_1)),\tau([u_1,u_2,t_2]_{\mathcal{L}}),\hat{\beta}(\tau(t_3))]_{\hat{\mathcal{L}}}\\
&+ \delta\varepsilon(|u_1|+|u_2|,|t_1|+|t_2|)[\hat{\beta}(\tau(t_1)),\hat{\beta}(\tau(t_2)),\omega(u_1,u_2,t_3)]_{\hat{\mathcal{L}}}\\
&+ \delta\varepsilon(|u_1|+|u_2|,|t_1|+|t_2|)[\hat{\beta}(\tau(t_1)),\hat{\beta}(\tau(t_2)),\tau([u_1,u_2,t_3]_{\mathcal{L}})]_{\hat{\mathcal{L}}}\\
&= [\omega(u_1,u_2,t_1),\tau(\beta(t_2)),\tau(\beta(t_3))]_{\hat{\mathcal{L}}} + [\tau([u_1,u_2,t_1]_{\mathcal{L}}),\tau(\beta(t_2)),\tau(\beta(t_3))]_{\hat{\mathcal{L}}}\\
&+ \varepsilon(|t_1|,|u_1|+|u_2|)[\tau(\beta(t_1)),\omega(u_1,u_2,t_2),\tau(\beta(t_3))]_{\hat{\mathcal{L}}}\\
&+ \varepsilon(|t_1|,|u_1|+|u_2|)[\tau(\beta(t_1)),\tau([u_1,u_2,t_2]_{\mathcal{L}}),\tau(\beta(t_3))]_{\hat{\mathcal{L}}}\\
&+ \delta\varepsilon(|u_1|+|u_2|,|t_1|+|t_2|)[\tau(\beta(t_1)),\tau(\beta(t_2)),\omega(u_1,u_2,t_3)]_{\hat{\mathcal{L}}}\\
&+ \delta\varepsilon(|u_1|+|u_2|,|t_1|+|t_2|)[\tau(\beta(t_1)),\tau(\beta(t_2)),\tau([u_1,u_2,t_3]_{\mathcal{L}})]_{\hat{\mathcal{L}}}\\
&= \varepsilon(|u_1|+|u_2|+|t_1|,|t_2|+|t_3|)\vartheta(\beta(t_2),\beta(t_3))\omega(u_1,u_2,t_1)\\
&+ \tau([[u_1,u_2,t_1]_{\mathcal{L}},\beta(t_2),\beta(t_3)]_{\mathcal{L}}) + \omega([[u_1,u_2,t_1]_{\mathcal{L}},\beta(t_2),\beta(t_3)]_{\mathcal{L}})\\
&- \delta\varepsilon(|t_1|,|u_1|+|u_2|)\varepsilon(|t_3|,|u_1|+|u_2|+|t_2|)\vartheta(\beta(t_1),\beta(t_3))\omega(u_1,u_2,t_2)\\
&+ \varepsilon(|t_1|,|u_1|+|u_2|)\omega(\beta(t_1),[u_1,u_2,t_2]_{\mathcal{L}},\beta(t_3))\\
&+ \varepsilon(|t_1|,|u_1|+|u_2|)\tau([\beta(t_1),[u_1,u_2,t_2]_{\mathcal{L}},\beta(t_3)]_{\mathcal{L}})\\
&+ \varepsilon(|u_1|+|u_2|,|t_1|+|t_2|)D(\beta(t_1),\beta(t_2))\omega(u_1,u_2,t_3)\\
&+ \delta\varepsilon(|u_1|+|u_2|,|t_1|+|t_2|)\omega(\beta(t_1),\beta(t_2),[u_1,u_2,t_3]_{\mathcal{L}})\\
&+ \delta\varepsilon(|u_1|+|u_2|,|t_1|+|t_2|)\tau([\beta(t_1),\beta(t_2),[u_1,u_2,t_3]_{\mathcal{L}}]_{\mathcal{L}}).
\end{aligned}
$$

Thus, it follows that

$$
\begin{aligned}
&\omega([u_1,u_2,t_1]_{\mathcal{L}},\beta(t_2),\beta(t_3)) + \varepsilon(|t_1|,|u_1|+|u_2|)\omega(\beta(t_1),[u_1,u_2,t_2]_{\mathcal{L}},\beta(t_3))\\
&+ \delta\varepsilon(|u_1|+|u_2|,|t_1|+|t_2|)\omega(\beta(t_1),\beta(t_2),[u_1,u_2,t_3]_{\mathcal{L}})\\
&+ \varepsilon(|u_1|+|u_2|+|t_1|,|t_2|+|t_3|)\vartheta(\beta(t_2),\beta(t_3))\omega(u_1,u_2,t_1)\\
&- \delta\varepsilon(|u_1|+|u_2|,|t_1|+|t_3|)\varepsilon(t_2,t_3)\vartheta(\beta(t_1),\beta(t_3))\omega(u_1,u_2,t_2)\\
&+ \varepsilon(|u_1|+|u_2|,|t_1|+|t_2|)D(\beta(t_1),\beta(t_2))\omega(u_1,u_2,t_3)\\
&- \omega(\beta(u_1),\beta(u_2),[t_1,t_2,t_3]_{\mathcal{L}}) - \delta D(\beta(u_1),\beta(u_2))\omega(t_1,t_2,t_3)\\
&= 0.
\end{aligned}
$$

Consequently, this proves the result.

Theorem 4.3. *Suppose that $\mathcal{L}$ is a multiplicative Hom-δ-Jordan Lie color triple system, (W,ϑ) is a $\mathcal{L}$-module, and ω is a 3-Hom-cocycle. We have $\mathcal{L}\oplus W$ is a multiplicative Hom-δ-Jordan Lie color triple system together with the following operation:*

$$
\begin{aligned}
&[u_1+x_1,u_2+x_2,u_3+x_3]_{\omega}\\
&= [u_1,u_2,u_3] + \omega(u_1,u_2,u_3) + \delta D(u_1,u_2)(x_3)
\end{aligned}
$$

$$-\delta\varepsilon(|u_2|,|u_3|)\vartheta(u_1,u_3)(x_2)+\varepsilon(|u_1|,|u_2|+|u_3|)\vartheta(u_2,u_3)(x_1).$$

And

$$(\beta+\gamma)(u+x)=\beta(u)+\gamma(x).$$

Proof. By a direct computation, the inclusion is straightforward.

Theorem 4.4. *Two abelian extensions on multiplicative Hom-δ-Jordan Lie color triple systems* $E_{\hat{\mathcal{L}}}: 0\longrightarrow W\xrightarrow{f}\hat{\mathcal{L}}\xrightarrow{g}\mathcal{L}\longrightarrow 0$ *and* $E_{\tilde{\mathcal{L}}}: 0\longrightarrow W\xrightarrow{f'}\tilde{\mathcal{L}}\xrightarrow{g'}\mathcal{L}\longrightarrow 0$ *are equivalent if and only if* ω *and* ω' *belong to the same cohomology class.*

Proof. $\Rightarrow$) Suppose that $\mathcal{H}:\mathcal{L}\oplus_\omega W\to\mathcal{L}\oplus_{\omega'} W$ is the corresponding homomorphism. Then

$$\mathcal{H}[u_1,u_2,u_3]_\omega=[\mathcal{H}(u_1),\mathcal{H}(u_2),\mathcal{H}(u_3)]_{\omega'}. \tag{4.5}$$

Since $\mathcal{H}$ is an equivalence of extensions, it follows $\rho:\mathcal{L}\to W$ satisfying

$$\mathcal{H}(u_i+x_i)=u_i+\rho(u_i)+x_i,\quad i=1,2,3. \tag{4.6}$$

The left side of (4.5) is equivalent to

$$\mathcal{H}([u_1,u_2,u_3]+\omega(u_1,u_2,u_3))=[u_1,u_2,u_3]+\omega(u_1,u_2,u_3)+\rho([u_1,u_2,u_3]),$$

and the right side of (4.5) shows that

$$\begin{aligned}&[u_1+\rho(u_1),u_2+\rho(u_2),u_3+\rho(u_3)]_{\omega'}\\=&[u_1,u_2,u_3]+\omega'(u_1,u_2,u_3)+\delta D(u_1,u_2)\rho(u_3)-\delta\varepsilon(|u_2|,|u_3|)\vartheta(u_1,u_3)\rho(u_2)\\&+\varepsilon(|u_1|,|u_2|+|u_3|)\vartheta(u_2,u_3)\rho(u_1).\end{aligned}$$

Hence, we have

$$\begin{aligned}&(\omega-\omega')(u_1,u_2,u_3)\\=&\delta D(u_1,u_2)\rho(u_3)-\delta\varepsilon(|u_2|,|u_3|)\vartheta(u_1,u_3)\rho(u_2)\\&+\varepsilon(|u_1|,|u_2|+|u_3|)\vartheta(u_2,u_3)\rho(u_1)-\rho([u_1,u_2,u_3]).\end{aligned}$$

Therefore, $\omega-\omega'=d^1\rho$, we obtain ω and ω' are in the same cohomology class.

$\Leftarrow$) On the other hand, we can assume that $\omega-\omega'=d^1\rho$, moreover, $\mathcal{H}$ defined by (4.6) is an equivalence.

5. Conclusions

In this paper, we define the multiplicative Hom-δ-Jordan Lie color triple system and give its semidirect product. Then the representation and cohomology are characterized in order to discuss structures of deformations and extensions. Finally, we obtain that two equivalent 1-parameter formal deformations are in the same cohomology class and prove that there is the same representation between two equivalent abelian extensions. Moreover, it is shown that two abelian extensions on Hom-δ-Jordan Lie color triple systems are equivalent $\Leftrightarrow$ 3-Hom-cocycles that construct Hom-δ-Jordan Lie color triple systems are in the same cohomology class.

Author contributions

L. Ma: Conceptualization, methodology, writing-original draft; Q. Li: Conceptualization, methodology, writing-review and editing. All authors have read and agreed to the published version of the manuscript.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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