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Research article

Highly dispersive gap solitons for conformable fractional model in optical fibers with dispersive reflectivity solutions using the modified extended direct algebraic method

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Abstract: We investigated the dynamics of highly dispersive nonlinear gap solitons in optical fibers with dispersive reflectivity, utilizing a conformable fractional derivative model. The modified extended direct algebraic method was employed to obtain various soliton solutions, including bright solitons and singular solitons, as well as hyperbolic and trigonometric solutions. The key findings demonstrated that the fractional derivative parameter (α) can effectively control the wave propagation, causing a shift in the wave signal while maintaining the same amplitude. This is a novel contribution, as the ability to control soliton properties through the conformable derivative is explored for the first time in this work. The results showcase the significant influence of fractional derivatives in shaping the characteristics of the soliton solutions, which is crucial for accurately modeling the dispersive and nonlocal effects in optical fibers. This research provides insights into the potential applications of fractional calculus in the design and optimization of photonic devices for optical communication systems.

Keywords: highly dispersive gap solitons; conformable fractional model; modified extended direct algebraic method
Mathematics Subject Classification: 26A33, 35C07, 35C08, 35C09

1. Introduction

The study of solitons in optical fibers has attracted significant attention due to their potential applications in high-speed communication systems. Solitons, which are self-sustaining wave packets, can propagate over long distances without undergoing significant distortion. They are characterized by their ability to maintain their shape and amplitude during propagation, making them ideal for transmitting information reliably. Numerous scholars have delved the study of soliton solutions, for example, the authors in [1] derived the soliton solutions for higher-order nonlinear Schrödinger's dynamical equation with fourth-order dispersion and cubic-quintic nonlinearity. The authors in [2] established the new solitons for higher order Sasa–Satsuma equation. The authors in [3] discussed the dynamics of soliton for the Vakhneno equation in an optical fiber system. The authors in [4] discussed the soliton resonance and burifcation for the Maccari system. The authors in [5] studied the interaction behavior between soliton and other hyprid forms for a short pulse equation. The authors in [6] established the soliton resonance for the Lakshmanan Daniel system. Soliton also plays an important rule in other sciences such as fluid dynamics, such as [7, 8]. The authors [9] studied the soliton solution in the fourth-order Schrödinger equation, which plays an important rule in optical communication. The authors in [10] showed the gaussian and super gaussian soliton solutions. The authors in [11, 12] studied the bright and dark soliton in optical fiber.

Soliton wave solutions have been derived with applying the more advanced techniques for acquiring precise and efficient results from non-linear models, such as an improved modified extended tanh function method [13, 14], extended F-expansion method [15], and Jacobi elliptic function expansion method [16].

One of the key factors affecting the behavior of solitons in optical fibers is dispersion. Dispersion refers to the phenomenon of propagating different wavelengths with different speeds, causing the pulse to broaden or narrow over time. While dispersion can be managed to some extent through the use of dispersion compensation techniques, it remains a critical challenge in the design and optimization of optical fiber systems [17]. In recent years, the study of highly dispersive gap solitons has emerged as an area of interest. Gap solitons refer to localized solutions that exist in the presence of periodic or quasi-periodic structures, such as fiber Bragg gratings or photonic crystals, within the optical fiber. These solitons can form in the spectral gaps of the linear dispersion relation, where no linear wave propagation is allowed. The interaction between the nonlinearity of the optical fiber and the dispersive reflectivity of the periodic structure gives rise to soliton solutions [18, 19].

In this research paper, we investigate highly dispersive gap solitons in optical fibers with dispersive reflectivity. To analyze the behavior of these solitons, we employ a conformable fractional model, which takes into account fractional derivatives to describe the nonlocal response of the system. The use of fractional derivatives allows for a more accurate representation of the dynamics of the soliton solutions and long-range interactions. The authors in [20] discussed the conformable fractional derivative properties on the telegraph equation and the third-order Kdv equation. Also, the conformable derivative was discussed in [21] on evolution equation in optical fiber systems to explore the dynamics of solitons. The authors in [22] discussed the conformable fractional derivative on modified Benjamin-Mahony equation. The paper [23] discussed the conformable fractional coupled type Boussinesq-Burger equation. There are many other types of fractional derivatives, such as beta derivatives [24, 25], that discussed the effect of fractional order on soliton solutions. In this work, we study the highly dispersive coupled system of fractional nonlinear schrödinger equation (NLSE) with a parabolic non-local law of nonlinearity. This model reads as [26]:

$$iD_t^{\alpha}\mathrm{q} + ia_1 r_x + a_2 r_{\mathrm{xx}} + ia_3 r_{\mathrm{xxx}} + a_4 r_{\mathrm{xxxx}} + \mathrm{i}a_5 r_{\mathrm{xxxxx}} + a_6 r_{\mathrm{xxxxxx}} + q\left(c_1 \left|q\right|^2 + d_1\right| r|^2\right) +$$

$$q\left(e_1\left|q\right|^4+f_1\left|q\right|^2|r|^2+g_1|r|^4\right)+q\left(L_1\left(\left|q\right|^2\right)_{xx}+m_1\left(\left|r\right|^2\right)_{xx}\right)+i\alpha_1 q_x+\beta_1 r+q^* r^2\sigma_1$$
$$-i\gamma_1\left(|q|^2 q\right)_x-\mathrm{i}\theta_1\left(|q|^2\right)_x q-i\mu_1|q|^2 q_x=0, \tag{1.1}$$

$$iD_t^{\alpha}\mathrm{r}+ib_1 q_x+b_2 q_{\mathrm{xx}}+ib_3 q_{\mathrm{xxx}}+b_4 q_{\mathrm{xxxx}}+ib_5 q_{\mathrm{xxxxx}}+b_6 q_{\mathrm{xxxxxx}}+r\left(c_2\left|r\right|^2+d_2\left|q\right|^2\right)+$$
$$r\left(e_2\left|r\right|^4+f_2\left|r\right|^2|q|^2+g_2|q|^4\right)+r\left(L_2\left(\left|r\right|^2\right)_{\mathrm{xx}}+m_2\left(\left|q\right|^2\right)_{\mathrm{xx}}\right)+i\alpha_2 r_x+\beta_2 q+q^2 r^*\sigma_2$$
$$-i\gamma_2\left(|r|^2 r\right)_x-i\theta_2\left(|r|^2\right)_x r-i\mu_2|r|^2 r_x=0. \tag{1.2}$$

The wave profiles of fiber Bragg gratings are described by the functions $q = q(x,t)$ and $r = r(x,t)$. The temporal evolution of the optical field is captured by the operators $D_{\mathrm{t}}^{\alpha}q$ and $D_{\mathrm{t}}^{\alpha}r$. Additionally, the complex conjugates of these functions are denoted as q^* and r^*, where $i^2 = -1$.

The system includes several coefficients representing different types of dispersion and nonlinear effects:

(1) Inter-modal dispersion coefficients: a_1 and b_1.

(2 Chromatic dispersion (CD) coefficients: a_2 and b_2.

(3) Third-order dispersion coefficients: a_3 and b_3.

(4) Fourth-order dispersion coefficients: a_4 and b_4.

(5) Fifth-order dispersion coefficients: a_5 and b_5.

(6) Sixth-order dispersion coefficients: a_6 and b_6.

(7) Self-Phase Modulation (SPM) - c_j and e_j:

- These coefficients quantify the self-phase modulation effect, where an optical pulse modifies its own phase due to nonlinear refractive index changes. SPM is essential in high-intensity pulse propagation.

(8) Cross-Phase Modulation (XPM) - d_j and g_j (where $j = 1,2$):

- These coefficients are involved in cross-phase modulation, where the phase of one beam is affected by the presence of another beam within the same medium. XPM is significant in multi-channel communication systems.

(9) Nonlinear Terms - f_j (where $j = 1,2$):

- The constants f_j define the strength of nonlinear effects that are not captured by SPM or XPM alone. These nonlinearities can lead to intricate behavior such as waveform distortions and the formation of solitons.

(10) Non-Local Law Terms - L_j and m_j (where $j = 1,2$):

- These coefficients describe interactions that are non-local, meaning the effect at a given point is influenced by the field at different locations. This is important in systems where long-range interactions occur.

(11) Inter-Modal Dispersion (IMD) - α_j(where $j = 1,2$):

- The α_j coefficients measure the dispersion between different modes within a system. IMD can cause temporal spreading and is a key factor in the design of fiber optic systems.

(12) Detuning Parameters - β_j(where $j = 1,2$):

- These coefficients represent detuning parameters that affect how closely a system can resonate at a given frequency. Detuning is crucial in maintaining system stability and performance.

(13) Four-Wave Mixing (4WM) - σ_j (where $j = 1,2$):

- Representing four-wave mixing, these coefficients are pivotal in processes where three interacting waves produce a fourth wave. 4WM is integral in wavelength conversion and parametric

amplification.
(14) Self-Steepening (SS) - γ_j (where $j = 1, 2$):
- The γ_j coefficients pertain to the self-steepening effect, leading to the steepening of pulse edges in high-power regimes. This term becomes important for ultra-short pulse applications.
(15) Nonlinear Dispersion Terms - θ_j and μ_j (where $j = 1, 2$):
- These coefficients are associated with the nonlinear dispersion terms, which affect the linear dispersion properties of a system in a nonlinear context. This influence is crucial for describing pulse propagation in highly nonlinear media.

Understanding and managing these coefficients allow for the precise design and optimization of advanced optical systems, from telecommunications to laser technology and beyond. Balancing these effects is key to achieving desired performance and ensuring system stability. The conformable fractional derivative, which is a generalization of the classical derivative, is defined as [27, 28].

$$D_t^{\alpha} f(t) = \lim_{\delta \to 0} \frac{f(t + \delta t^{1-\alpha}) - f(t)}{\delta}, \quad t > 0, \ 0 < \alpha \leq 1.$$

We discuss the highly dispersive coupled system of fractional nonlinear Schrödinger equation (NLSE) and illustrate the effect of fractional derivative on the wave. By varying the parameter α, which controls the fractional derivative, we can investigate the influence of fractional derivative on the soliton solutions and understand the impact of non-locality on the system dynamics. This model was studied in ref [26] as a classical derivative point of view. This study was conducted by implementing the extended auxiliary equation approach to derive bright, dark, and singular solitons. Authors in [29–31] discussed this method before and how to apply it.

In this research paper, we employ the modified extended direct algebraic method (MEDAM), a powerful analytical technique widely used in the study of nonlinear wave phenomena. This method enables us to derive various solitons and other exact solutions. This work is organized as follows. The proposed methodology is briefly explained in Section 2. In Section 3, the proposed method is implemented to derive exact solutions for the investigated model. Some of the extracted solutions are illustrated graphically in Section 4 to depict the characteristics of the propagating wave. In the final section, we conclude the work. While the present study offers valuable insights into the dynamics of highly dispersive nonlinear gap solitons in optical fibers using a conformable fractional derivative model, it is important to acknowledge several key limitations. The analysis is primarily based on theoretical modeling and analytical solutions derived using (MEDAM), which relies on specific functional forms for the soliton solutions. This approach may not fully capture the complex and real-world behavior of optical fibers, and its applicability is limited to certain types of NLPDE. Furthermore, we do not consider the potential impact of higher-order effects, such as third-order dispersion and self-steepening, on the conformable fractional soliton characteristics. The lack of experimental validation and the exclusion of these higher-order nonlinear terms limit the direct applicability of the findings to practical optical communication systems. Future research efforts will need to address these limitations by incorporating more comprehensive modeling approaches and conducting experimental investigations to bridge the gap between the theoretical predictions and the actual system performance.

2. The applied method and its application

2.1. The methodology: MEDAM

In this section, MEDAM is briefly discussed [32, 33].

Assume the following NLPDE :

$$M(u, D_t^{\alpha} u, u_x, u_{xx}, u_{tt}, ...) = 0. \tag{2.1}$$

To address this nonlinear partial differential equation (NLPDE) using the proposed methodology, the following steps are taken:

Step (A): Wave transformation

We begin by transforming the NLPDE in Eq (2.1) into an ordinary differential equation (ODE) through the wave transformation:

$$u(x,t) = A(\xi)e^{i\psi}, \quad \xi = \eta\left(x - \frac{\lambda t^{\alpha}}{\alpha}\right), \quad \psi = \phi - kx + \frac{\omega t^{\alpha}}{\alpha},$$

where k represents the wave speed. Applying this transformation to Eq (2.1) results in the following ODE:

$$G(A, A', A'', A^{(3)}, A^{(4)}, \dots) = 0. \tag{2.2}$$

Step (B): Solution Representation

Next, we express the solution to the transformed ODE as:

$$A(\xi) = \sum_{n=0}^{N} c_n \mu^n(\xi) + \sum_{n=-1}^{-N} d_{-n} \mu^n(\xi), \tag{2.3}$$

where $\mu(\xi)$ satisfies the differential equation:

$$\mu'(\xi) = \sqrt{q_0 + q_1\mu(\xi) + q_2\mu^2(\xi) + q_3\mu^3(\xi) + q_4\mu^4(\xi) + q_6\mu^6}. \tag{2.4}$$

Step (C): Determine Integer N

Then, we determine the integer N in the series expansion by applying the balancing rule to the transformed ODE in Eq (2.2).

Step (D): Nonlinear algebraic equations

We substitute Eqs (2.3) and (2.4) into the transformed ODE (Eq (2.2)), collecting the coefficients of $\mu^m(\xi)$ for $m = 0, 1, 2, \dots$ and setting them to zero. This yields a set of nonlinear algebraic equations.

Step (E): Solve algebraic system

To determine the values of c_n, d_{-n} and k, we employ software programs like Mathematica to solve the nonlinear algebraic equations system at hand.

Step (F): Generating solutions

By varying the numerical values of $q_0, q_1, q_2, q_3, q_4, q_6$, we can generate various solutions. Here are some specific cases:

Case 1: $q_0 = q_1 = q_3 = q_6 = 0$

$$\mu(\xi) = \sqrt{-\frac{q_2}{q_4}}\operatorname{sech}(\sqrt{q_2}\xi), \quad q_2 > 0, q_4 < 0,$$

$$\mu(\xi) = \sqrt{-\frac{q_2}{q_4}}\sec(\sqrt{-q_2}\xi), \quad q_2 < 0, q_4 > 0,$$

$$\mu(\xi) = \sqrt{-\frac{q_2}{q_4}}\csc(\sqrt{-q_2}\xi), \quad q_2 < 0, q_4 > 0.$$

Case 2: $q_1 = q_3 = q_6 = 0, q_0 = \frac{q_2^2}{4q_4}$

$$\mu(\xi) = \sqrt{\frac{-q_2}{2q_4}}\tanh(\sqrt{\frac{-q_2}{2}}\xi), \quad q_2 < 0, q_4 > 0,$$

$$\mu(\xi) = \sqrt{\frac{q_2}{2q_4}}\tan(\sqrt{\frac{q_2}{2}}\xi), \quad q_2 > 0, q_4 > 0.$$

Case 3: $q_0 = q_1 = q_6 = 0$

$$\mu(\xi) = \frac{-q_2\operatorname{sech}^2(\sqrt{\frac{q_2}{2}}\xi)}{2\sqrt{q_2q_4}\tanh(\sqrt{\frac{q_2}{2}}\xi) - q_3}, \quad q_2 > 0, q_3^2 \neq 4q_2q_4,$$

$$\mu(\xi) = \frac{-q_2}{q_3}(\tanh(\sqrt{\frac{q_2}{2}}\xi) + 1), \quad q_2 > 0, q_4 = \frac{q_3^2}{4q_2}.$$

Case 4: $q_1 = q_3 = 0$

$$\mu(\xi) = \sqrt{\frac{2q_2\operatorname{sech}^2(\sqrt{q_2}\xi)}{2\sqrt{q_4^2 - 4q_2q_6} - \left(\sqrt{q_4^2 - 4q_2q_6} + q_4\right)\operatorname{sech}^2(\sqrt{q_2}\xi)}},$$

$$\mu(\xi) = \sqrt{\frac{2q_2\sec^2\left(\sqrt{-q_2}\xi\right)}{2\sqrt{q_4^2 - 4q_2q_6} - \left(\sqrt{q_4^2 - 4q_2q_6} - q_4\right)\sec^2\left(\sqrt{-q_2}\xi\right)}},$$

$$\mu(\xi) = \sqrt{\frac{8q_2\tanh^2\left(\sqrt{-\frac{q_3}{3}}\xi\right)}{3q_4\left(\tanh^2\left(\sqrt{-\frac{q_3}{3}}\xi\right) + 3\right)}},$$

$$\mu(\xi) = \sqrt{\frac{8q_2\tan^2\left(\sqrt{\frac{q_3}{3}}\xi\right)}{3q_4\left(3 - \tan^2\left(\sqrt{\frac{q_3}{3}}\xi\right)\right)}}.$$

Step (G): Final solutions

Finally, by substituting the constants c_n, d_{-n} obtained in Step (E) and the general solutions of Eq (2.4) into Eq (2.3), various solutions for Eq (2.2) are generated. This method offers a structured approach to elucidate complex nonlinear phenomena in partial differential equations.

2.2. Applying to studied model

To derive exact solutions in the following manner for Eqs (1.1) and (1.2), We assume the following transformation:

$$q(x,t) = Q(z)e^{i\phi} \quad ,z = \eta\left(x - \frac{\nu t^{\alpha}}{\alpha}\right) \quad \textbf{and} \quad \phi = \theta - kx + \frac{\omega t^{\alpha}}{\alpha}. \tag{2.5}$$

Assuming that $r(x,t) = A\ q(x,t)$, where A is constant and $A \neq 0\ or\ 1$.

By substituting Eq (2.5) into Eq (1.1), the fractional derivative NLPDE in Eq (1.1) is transformed into a complete derivative ODE, yielding the following real and imaginary components:

$$\begin{aligned}
&Q(z)\left(a_6(-A)k^6 + a_5Ak^5 + a_4Ak^4 - a_3Ak^3 - a_2Ak^2 + a_1Ak + 2A^2\eta^2m_1Q'(z)^2 + A\beta_1 + \alpha_1k + 2\eta^2L_1Q'(z)^2 - \omega\right)\\
&+ A\eta^2\left(15a_6k^4Q''(z) - 10a_5k^3Q''(z) - 15a_6\eta^2k^2Q^{(4)}(z) - 6a_4k^2Q''(z) + 5a_5\eta^2kQ^{(4)}(z) + 3a_3kQ''(z)\right.\\
&\quad \left.+a_6\eta^4Q^{(6)}(z) + a_4\eta^2Q^{(4)}(z) + a_2Q''(z)\right) + Q(z)^3\left(A^2d_1 + A^2\delta_1 + c_1 - \gamma_1k - k\mu_1\right)\\
&+ \eta Q(z)^2\left(2A^2\eta m_1Q''(z) + 2\eta L_1Q''(z)\right) + Q(z)^5\left(A^4g_1 + A^2f_1 + e_1\right) = 0,
\end{aligned} \tag{2.6}$$

$$\begin{aligned}
&\eta\left(A\eta\left(20a_6\eta k^3Q^{(3)}(z) - 10a_5\eta k^2Q^{(3)}(z) - 6a_6\eta^3kQ^{(5)}(z) - 4a_4\eta kQ^{(3)}(z) + a_5\eta^3Q^{(5)}(z) + a_3\eta Q^{(3)}(z)\right)\right.\\
&\quad \left.-Q'(z)\left(6a_6Ak^5 - 5a_5Ak^4 - 4a_4Ak^3 + 3a_3Ak^2 + 2a_2Ak - a_1A - \alpha_1 + \nu\right)\right)\\
&- \eta Q(z)^2\left(3\gamma_1Q'(z) + 2\theta_1Q'(z) + \mu_1Q'(z)\right) = 0.
\end{aligned} \tag{2.7}$$

Equating the coefficients of Eq (2.7) to zero yields:

$$\begin{aligned}
&k = \frac{a_5}{6a_6},\\
&\nu = -6a_6Ak^5 + 5a_5Ak^4 + 4a_4Ak^3 - 3a_3Ak^2 - 2a_2Ak + a_1A + \alpha_1,\\
&3\gamma_1 + 2\theta_1 + \mu_1 = 0,\\
&-20a_6k^3 + 10a_5k^2 + 4a_4k - a_3 = 0.
\end{aligned} \tag{2.8}$$

By substituting Eq (2.5) into Eq (1.2), the fractional derivative NLPDE is converted into a complete derivative ODE, resulting in the following real and imaginary parts:

$$\begin{aligned}
&Q(z)\left(Q'(z)^2\left(2A^3\eta^2L_2 + 2A\eta^2m_2\right) + \alpha_2Ak - A\omega - b_6k^6 + b_5k^5 + b_4k^4 - b_3k^3 - b_2k^2 + b_1k + \beta_2\right)\\
&+Q(z)^2Q''(z)\left(2A^3\eta^2L_2 + 2A\eta^2m_2\right)\\
&+Q(z)^3\left(A^2c_2 + A\left(A^2\gamma_2(-k) - A^2k\mu_2 + d_2 + \delta_2\right)\right)\\
&+AQ(z)^5\left(A^4e_2 + A^2f_2 + g_2\right) + Q^{(4)}(z)\left(b_4\eta^4 - 15b_6\eta^4k^2 + 5b_5\eta^4k\right)\\
&+\left(b_2\eta^2 + 15b_6\eta^2k^4 - 10b_5\eta^2k^3 - 6b_4\eta^2k^2 + 3b_3\eta^2k\right)Q''(z) + b_6\eta^6Q^{(6)}(z) = 0,
\end{aligned} \tag{2.9}$$

$$Q(z)^2Q'(z)\left(3A^3\gamma_2\eta + 2A^3\eta\theta_2 + A^3\eta\mu_2\right)$$

$$+\eta Q'(z)\left(-\alpha_2 A + A\nu + 6b_6k^5 - 5b_5k^4 - 4b_4k^3 + 3b_3k^2 + 2b_2k - b_1\right)$$
$$+Q^{(3)}(z)\left(10b_5\eta^3\mathrm{i}k^2 - b_3\eta^3 - 20b_6\eta^3k^3 + 4b_4\eta^3k\right)$$
$$+Q^{(5)}(z)\left(6b_6\eta^5k - b_5\eta^5\right) = 0. \tag{2.10}$$

By setting the coefficients of $Q(z)$ to zero, we derive the following equations:

$$-\alpha_2 A + A\nu + 6b_6k^5 - 5b_5k^4 - 4b_4k^3 + 3b_3k^2 + 2b_2k - b_1 = 0,$$
$$3A^3\gamma_2 + 2A^3\theta_2 + A^3\mu_2 = 0,$$
$$-b_3 - 20b_6k^3 + 10b_5k^2 + 4b_4k = 0,$$
$$6b_6k - b_5 = 0,$$

It is observed that the Eq (2.9) have the same form of Eq (2.6) under the following conditions:

$$b_6 = a_6A,$$
$$A\left(-15a_6k^2 - 5a_5k + a_4\right) = -15b_6k^2 - 5b_5k + b_4,$$
$$A\left(15a_6k^4 - 10a_5k^3 - 6a_4k^2 + 3a_3k + a_2\right) = 15b_6k^4 - 10b_5k^3 - 6b_4k^2 + 3b_3k + b_2,$$
$$A^2m_1 + L_1 = A\left(A^2L_2 + m_2\right),$$
$$A\left(-a_6k^6 + a_5k^5 + a_4k^4 - a_3k^3 - a_2k^2 + a_1k + \beta_1\right) + \alpha_1k - \omega$$
$$= A\left(\alpha_2k - \omega\right) - b_6k^6 + b_5k^5 + b_4k^4 - b_3k^3 - b_2k^2 + b_1k + \beta_2,$$
$$A^2\left(d_1 + \sigma_1\right) + c_1 - k\left(\gamma_1 + \mu_1\right) = A\left(A^2c_2 + d_2 - \mathrm{k}A^2\left(\gamma_2 + \mu_2\right) + \sigma_2\right),$$
$$A^4g_1 + A^2f_1 + e_1 = AA^4e_2 + A^2f_2 + g_2. \tag{2.11}$$

To determine the integer N required for the proposed technique, we equate $Q^{(6)}$ with Q^5 in Eq (2.6) and find $N = \frac{3}{2}$. To obtain an integer value for N, we perform the following mathematical transformation:

$$Q(z) = U^{3/2}(z). \tag{2.12}$$

This will convert Eq (2.6) to the following equation:

$$U(z)^5\left(U^4(z)\left(96a_4A\eta^4 - 1440a_6A\eta^4k^2 + 480a_5A\eta^4k\right) + 96a_6A\eta^6U^6(z) + M_{11}U''(z)\right)$$
$$+U(z)^2\left(720a_6A\eta^6U^3(z)U'(z)^3 + 1620a_6A\eta^6U'(z)^2U''(z)^2 + M_6U'(z)^4\right)$$
$$+U(z)^3\left(U'(z)^2\left(M_7U''(z) - 360a_6A\eta^6U^4(z)\right) - 360a_6A\eta^6U''(z)^3 - 1440a_6A\eta^6U^3(z)U'(z)U''(z)\right)$$
$$+U(z)^4(U'(z)\left(288a_6A\eta^6U^5(z) + M_{10}U^3(z)\right) + 480a_6A\eta^6U^3(z)^2 + 720a_6A\eta^6U^4(z)U''(z)$$
$$+M_9U''(z)^2 + M_8U'(z)^2) + 315a_6A\eta^6U'(z)^6 - 1350a_6A\eta^6U(z)U'(z)^4U''(z) + M_5U(z)^8U''(z)$$
$$+M_4U(z)^7U'(z)^2 + M_3U(z)^{12} + M_2U(z)^9 + M_1U(z)^6 = 0, \tag{2.13}$$

where

$$M_1 = -64\left(a_6Ak^6 - a_5Ak^5 - a_4Ak^4 + a_3Ak^3 + a_2Ak^2 - a_1Ak - A\beta_1 - \alpha_1k + \omega\right),$$

$$\begin{aligned}
M_2 &= 64\left(A^2 d_1 + A^2\delta_1 + c_1 - \gamma_1 k - k\mu_1\right),\\
M_3 &= 64\left(A^4 g_1 + A^2 f_1 + e_1\right),\\
M_4 &= 384\eta^2\left(A^2 m_1 + L_1\right),\\
M_5 &= 192\eta^2\left(A^2 m_1 + L_1\right),\\
M_6 &= 36a_4A\eta^4 - 540a_6A\eta^4k^2 + 180a_5A\eta^4k,\\
M_7 &= -144a_4A\eta^4 + 2160a_6A\eta^4k^2 - 720a_5A\eta^4k,\\
M_8 &= 48a_2A\eta^2 + 720a_6A\eta^2k^4 - 480a_5A\eta^2k^3 - 288a_4A\eta^2k^2 + 144a_3A\eta^2k,\\
M_9 &= 144a_4A\eta^4 - 2160a_6A\eta^4k^2 + 720a_5A\eta^4k,\\
M_{10} &= 192a_4A\eta^4 - 2880a_6A\eta^4k^2 + 960a_5A\eta^4k,\\
M_{11} &= 96a_2A\eta^2 + 1440a_6A\eta^2k^4 - 960a_5A\eta^2k^3 - 576a_4A\eta^2k^2 + 288a_3A\eta^2k,\\
M_{12} &= 96a_4A\eta^4 - 1440a_6A\eta^4k^2 + 480a_5A\eta^4k.
\end{aligned}$$

By balancing U^{12} with $U^5U^{(6)}$, we find $N = 1$. The solution of the resulting (ODE) can be expressed as follows:

$$U(z) = s_0 + s_1\lambda(z) + \frac{s_2}{\lambda(z)}. \tag{2.14}$$

$$\lambda'(z) = \sqrt{p_0 + p_1\lambda(z) + p_2\lambda^2(z) + p_3\lambda^3(z) + p_4\lambda^4(z) + p_6\lambda^6(z)}. \tag{2.15}$$

By Substituting Eqs (2.14) and (2.15) into the expression for Eq (2.13) and setting the coefficients of $\lambda(z)$ to zero, a system of nonlinear algebraic equations is generated. To solve this system, Mathematica software packages are employed, resulting in the subsequent results:

Case(1): $p_0 = p_1 = p_3 = p_6 = 0$

$$\begin{aligned}
s_0 &= 0,\\
s_2 &= 0,\\
M_1 &= -53361a_6A\eta^6p_2^3,\\
M_8 &= 22377a_6A\eta^6p_2^2,\\
M_4 &= 0,\\
p_2 &= -\frac{M_6}{1611a_6A\eta^6},\\
p_4 &= \frac{\sqrt[3]{-\frac{1}{5005}}\sqrt[3]{M_3}s_1^2}{3\sqrt[3]{a_6}\sqrt[3]{A}\eta^2}.
\end{aligned}$$

As a result, Eq (1.1) permits the derivation of both bright soliton and periodic solutions.

$$q_1(x,t) = \frac{\sqrt[4]{5005}\left(\sqrt{-\frac{M_6}{a_6^{2/3}A^{2/3}\eta^4\sqrt[3]{M_3}}}\operatorname{sech}\left(\frac{\sqrt{-\frac{M_6}{a_6A\eta^4}}\left(x-\frac{\nu t^\alpha}{\alpha}\right)}{3\sqrt{179}}\right)\right)^{3/2}}{537^{3/4}}\exp\left(i\left(\theta - kx + \frac{\omega t^\alpha}{\alpha}\right)\right). \tag{2.16}$$

$$q_2(x,t)=\frac{\sqrt[4]{5005}\left(\sqrt{-\frac{M_6}{a_6^{2/3}A^{2/3}\eta^4\sqrt[3]{M_3}}}\sec\left(\frac{\sqrt{\frac{M_6}{a_6A\eta^4}}\left(x-\frac{\nu t^\alpha}{\alpha}\right)}{3\sqrt{179}}\right)\right)^{3/2}}{537^{3/4}}\exp\left(i\left(\theta-kx+\frac{\omega t^\alpha}{\alpha}\right)\right). \tag{2.17}$$

$$q_3(x,t)=\frac{\sqrt[4]{5005}\left(\sqrt{-\frac{M_6}{a_6^{2/3}A^{2/3}\eta^4\sqrt[3]{M_3}}}\csc\left(\frac{\sqrt{\frac{M_6}{a_6A\eta^4}}\left(x-\frac{\nu t^\alpha}{\alpha}\right)}{3\sqrt{179}}\right)\right)^{3/2}}{537^{3/4}}\exp\left(i\left(\theta-kx+\frac{\omega t^\alpha}{\alpha}\right)\right). \tag{2.18}$$

Case (2): $p_1=p_3=p_6=0, p_0=\frac{p_2^2}{4p_4}$

$$\begin{aligned}
&s_0=0,\\
&p_2=\frac{2p_4s_2}{s_1},\\
&M_1=\frac{3415104a_6A\eta^6p_4^3s_2^3}{s_1^3},\\
&M_8=\frac{358032a_6A\eta^6p_4^2s_2^2}{s_1^2}, M_4=0,\\
&M_6=\frac{6444a_6A\eta^6p_4s_2}{s_1},\\
&M_2=0,\\
&p_4=-\frac{\sqrt[3]{M_3}s_1^2}{3\sqrt[3]{5005}\sqrt[3]{a_6}\sqrt[3]{A}\eta^2}.
\end{aligned}$$

Then, the following singular soliton and singular periodic solutions are obtained

$$q_4(x,t)=2\sqrt{2}\left(\frac{s_2\operatorname{csch}\left(\frac{2\sqrt{\frac{\sqrt[3]{M_3}s_1s_2}{\sqrt[3]{a_6}\sqrt[3]{A}}}\left(x-\frac{\nu t^\alpha}{\alpha}\right)}{\sqrt{3}\sqrt[6]{5005}}\right)}{\sqrt{-\frac{s_2}{s_1}}}\right)^{3/2}\exp\left(i\left(\theta-kx+\frac{\omega t^\alpha}{\alpha}\right)\right), \tag{2.19}$$

$$q_5(x,t)=2\sqrt{2}\left(s_1\sqrt{\frac{s_2}{s_1}}\csc\left(\frac{2\sqrt{\frac{\sqrt[3]{-\frac{1}{5005}}\sqrt[3]{M_3}s_1s_2}{\sqrt[3]{a_6}\sqrt[3]{A}}}\left(x-\frac{\nu t^\alpha}{\alpha}\right)}{\sqrt{3}}\right)\right)^{3/2}\exp\left(i\left(\theta-kx+\frac{\omega t^\alpha}{\alpha}\right)\right). \tag{2.20}$$

Case(3): $p_0=p_1=p_6=0, p_3^2\neq 4p_2p_4$

$$s_0=0,$$

$$
\begin{aligned}
s_2 &= 0,\\
p_2 &= -\frac{p_3^2}{p_4},\\
M_1 &= \frac{52200 a_6 A \eta^6 p_3^6}{p_4^3},\\
M_8 &= \frac{24339 a_6 A \eta^6 p_3^4}{p_4^2}\\
M_4 &= -\frac{124740 a_6 A \eta^6 p_3 p_4}{s_1^3},\\
M_6 &= \frac{2394 a_6 A \eta^6 p_3^2}{p_4},\\
M_2 &= -\frac{377370 a_6 A \eta^6 p_3^3}{s_1^3},\\
M_3 &= -\frac{135135 a_6 A \eta^6 p_4^3}{s_1^6}.
\end{aligned}
$$

Then, the following soliton solution can be presented

$$
q_6(x,t) = \frac{29}{16}\sqrt{\frac{29}{2}}\left(-\frac{p_3 s_1 \operatorname{sech}^2\left(\frac{1}{8}\sqrt{\frac{29}{2}}\eta\sqrt{\frac{p_3^2}{p_4}}\left(x-\frac{\nu t^\alpha}{\alpha}\right)\right)}{p_4\left(4-\sqrt{58}\tanh\left(\frac{1}{8}\sqrt{\frac{29}{2}}\eta\sqrt{\frac{p_3^2}{p_4}}\left(x-\frac{\nu t^\alpha}{\alpha}\right)\right)\right)}\right)^{3/2} \times \exp\left(i\left(\theta - kx + \frac{\omega t^\alpha}{\alpha}\right)\right). \tag{2.21}
$$

$$
q_7(x,t) = \frac{29}{16}\sqrt{\frac{29}{2}}\left(-\frac{p_3 s_1 \sec^2\left(\frac{1}{8}\sqrt{\frac{29}{2}}\eta\sqrt{\frac{-p_3^2}{p_4}}\left(x-\frac{\nu t^\alpha}{\alpha}\right)\right)}{p_4\left(4-\sqrt{58}\tan\left(\frac{1}{8}\sqrt{\frac{29}{2}}\eta\sqrt{\frac{-p_3^2}{p_4}}\left(x-\frac{\nu t^\alpha}{\alpha}\right)\right)\right)}\right)^{3/2} \times \exp\left(i\left(\theta - kx + \frac{\omega t^\alpha}{\alpha}\right)\right). \tag{2.22}
$$

Case(4): $p_3 = p_1 = 0$

$$
\begin{aligned}
s_0 &= 0,\\
p_4 &= \frac{p_0 s_1^2}{s_2^2},\\
p_2 &= \frac{2 p_0 s_1}{s_2},\\
p_6 &= 0,\ M_1 = \frac{3415104 a_6 A \eta^6 p_0^3 s_1^3}{s_2^3},\\
M_8 &= \frac{358032 a_6 A \eta^6 p_0^2 s_1^2}{s_2^2},
\end{aligned}
$$

$$M_4 = 0,$$
$$M_6 = \frac{6444 a_6 A \eta^6 p_0 s_1}{s_2},$$
$$M_2 = 0,$$
$$M_3 = -\frac{135135 a_6 A \eta^6 p_0^3}{s_2^6}.$$

Then, Eq (1.1) can have the following hyperbolic solutions

$$q_8(x,t) = \frac{\left(s_1\left(\cosh\left(2\sqrt{2}\eta\sqrt{\frac{p_0 s_1}{s_2}}\left(x-\frac{\nu t^\alpha}{\alpha}\right)\right)+3\right)\sqrt{\frac{s_2}{s_1\left(\cosh\left(2\sqrt{2}\eta\sqrt{\frac{p_0 s_1}{s_2}}\left(x-\frac{\nu t^\alpha}{\alpha}\right)\right)-1\right)}}\right)^{3/2}}{2\sqrt{2}}$$
$$\times\exp\left(i\left(\theta - kx + \frac{\omega t^\alpha}{\alpha}\right)\right). \tag{2.23}$$

$$q_9(x,t) = \frac{\left(s_1\left(9\coth^2\left(\frac{\eta\sqrt{-p_3}\left(x-\frac{\nu t^\alpha}{\alpha}\right)}{\sqrt{3}}\right)+19\right)\sqrt{\frac{s_1 s_2}{3\coth^2\left(\frac{\eta\sqrt{-p_3}\left(x-\frac{\nu t^\alpha}{\alpha}\right)}{\sqrt{3}}\right)+1}}\right)^{3/2}}{8\ 3^{3/4}}$$
$$\times\exp\left(i\left(\theta - kx + \frac{\omega t^\alpha}{\alpha}\right)\right). \tag{2.24}$$

and a trigonometric solutions are obtained:

$$q_{10}(x,t) = \frac{\left(\frac{\sqrt{\frac{p_0 s_2}{p_0 s_1 \cos\left(2\sqrt{2}\eta\sqrt{-\frac{p_0 s_1}{s_2}}\left(x-\frac{\nu t^\alpha}{\alpha}\right)\right)+p_0 s_1}}\left(\sqrt{p_0^2 s_1^4}\cos\left(2\sqrt{2}\eta\sqrt{-\frac{p_0 s_1}{s_2}}\left(x-\frac{\nu t^\alpha}{\alpha}\right)\right)+5p_0 s_1^2\right)}{p_0 s_1}\right)^{3/2}}{2\sqrt{2}}$$
$$\times\exp\left(i\left(\theta - kx + \frac{\omega t^\alpha}{\alpha}\right)\right). \tag{2.25}$$

$$q_{11}(x,t) = \frac{\left(\left(9\cot^2\left(\frac{\eta\sqrt{p_3}\left(x-\frac{\nu t^\alpha}{\alpha}\right)}{\sqrt{3}}\right)+13\right)\sqrt{\frac{s_1 s_2}{3\cot^2\left(\frac{\eta\sqrt{p_3}\left(x-\frac{\nu t^\alpha}{\alpha}\right)}{\sqrt{3}}\right)-1}}\right)^{3/2}}{8\ 3^{3/4}}$$
$$\times\exp\left(i\left(\theta - kx + \frac{\omega t^\alpha}{\alpha}\right)\right). \tag{2.26}$$

3. Graphical simulations

Graphical simulations illustrating selected solutions aim to highlight key features of the results. The following figures depict a bright soliton solution of Eq (2.16) with parameters $\eta = 0.19$, $\nu = 1.23$,

$M_6 = -1.68$, $M_3 = 0.54$, $a_6 = 1.7$, $A = 0.32$, and $t = 1.2$. This bright soliton exhibits a localized, bell-shaped profile with a distinct amplitude and narrow width.

Remarkably, these soliton waves can travel long distances while retaining their characteristic shape and speed. This remarkable stability is attributed to the intricate interplay between the dispersive and nonlinear effects in the optical fiber. The dispersive forces, which tend to stretch and weaken the pulse over time, are precisely counterbalanced by the self-focusing nonlinear effects. This delicate balance between these two opposing forces is what enables the solitons to propagate with minimal distortion or decay, maintaining their compact and localized form.

The ability to control and manipulate the soliton characteristics through the fractional derivative parameter α opens new avenues for the design and optimization of optical communication systems and photonic devices. The influence of the fractional derivative on the soliton solutions, as demonstrated by the graphical illustrations, shows the importance of incorporating fractional calculus in the modeling of these dispersive and nonlocal systems.

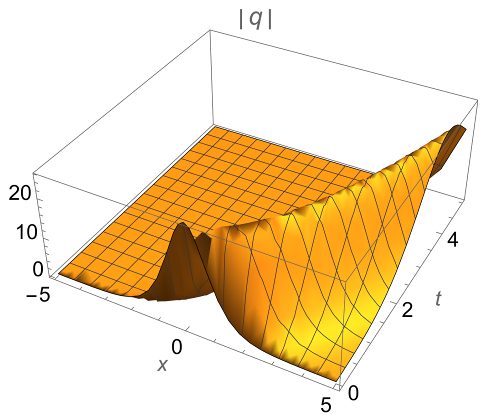

Figure 1. $\alpha = 0.55$.

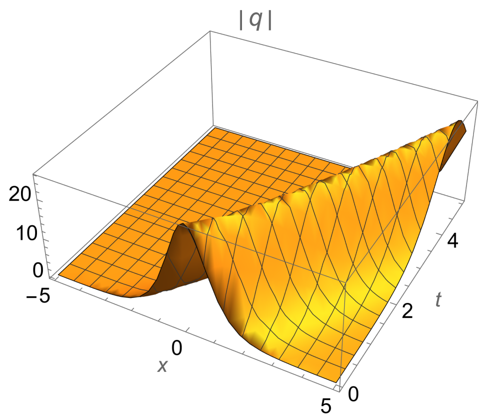

Figure 2. $\alpha = 0.7$.

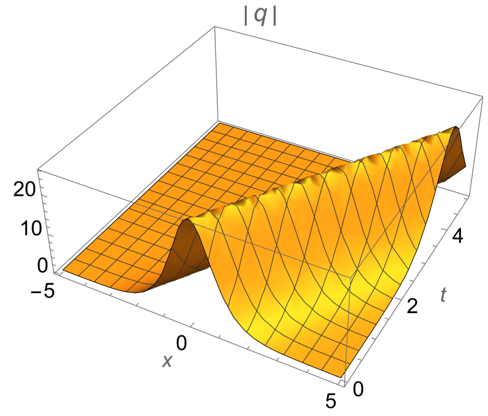

Figure 3. $\alpha = 1$.

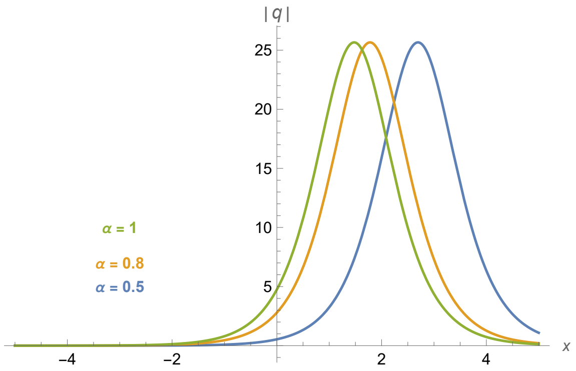

Figure 4. 2 D bright soliton at different values of α.

4. Comparison with other works

While we present the first application of the conformable fractional derivative approach to modeling nonlinear solitons in optical fibers, other researchers have investigated similar phenomena using the standard integer-order derivative model. The authors in [26] examined the dynamics of highly dispersive nonlinear gap solitons in optical fibers with dispersive reflectivity. Their work predicted the formation of bright solitons and singular solitons, which share certain characteristics with the soliton solutions obtained in this study. However, the conformable fractional derivative model introduced here introduces an additional parameter, the fractional order α, which allows for a more nuanced description of the highly dispersive nature of the optical fiber. Furthermore, the impact of the fractional order parameter α on the soliton characteristics, such as the soliton amplitude, width, and velocity, has been investigated in detail. This allows for a more comprehensive understanding of how the degree of dispersion in the optical fiber, as governed by the fractional order, influences the dynamics and propagation of the nonlinear solitons. By bridging the gap between the standard integer-order derivative models and the more generalized fractional derivative approach, this work provides a richer theoretical framework for analyzing the complex nonlinear phenomena in highly dispersive optical fiber systems. The conformable fractional derivative model offers additional flexibility and the potential to better capture the underlying physical processes, which may lead to improved designs and performance optimization in future optical communication and signal processing applications.

5. Conclusions

In this work, the MEDAM was implemented to investigate highly dispersive gap solitons in optical fibers with a conformable fractional derivative. We successfully derived soliton solutions, including bright, singular solitons and other solutions such as trigonometric and hyperbolic solutions. The key finding of this research is the significant impact of the fractional derivative parameter, denoted as α, on the characteristics of the soliton solutions.

The graphical illustrations presented in this work have clearly demonstrated the influence of the fractional derivative on the magnitude and behavior of the soliton waves. As the value of α is varied, the soliton solutions exhibit distinct changes in their amplitude, shape, and propagation dynamics. This observation underscores the importance of incorporating fractional calculus in the modeling of these dispersive and nonlocal systems, as it allows for a more accurate representation of the underlying physical processes.

The ability to control and manipulate the soliton characteristics through the fractional derivative parameter α opens new avenues for the design and optimization of optical communication systems and photonic devices. The findings of this research can potentially be extended to study other nonlinear wave phenomena in various physical systems, such as Bose-Einstein condensates, plasma physics, and biological systems, where the influence of fractional derivatives plays a crucial role. Future work could investigate the influence of higher-order effects, such as third-order dispersion and self-steepening, on the dynamics of conformable fractional solitons. Incorporating these higher-order terms would yield a more comprehensive understanding of the interplay between fractional derivatives and other dispersive and nonlinear factors, potentially leading to the discovery of novel soliton behaviors and applications. In summary, this research offers an invaluable understanding of the intricate and diverse characteristics displayed by solitons within the realm of fractional calculus. Such knowledge carries significant ramifications for both fundamental inquiry and pragmatic implementation purposes.

Author contributions

Mahmoud Soliman: Formal analysis, Software; Hamdy M. Ahmed: Validation, Methodology; Niveen Badra: Investigation, Writing-review & editing; Taher A. Nofal: Resources, Writing–review & editing; Islam Samir: Software, Writing- review & editing.

Use of AI tools declaration

The authors declare that they have not used Artificial Intelligence tools in the creation of this article.

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Conflict of interest

The authors declare that they have no conflicts of interest.

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