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Research article

A novel modeling and prediction approach using Caputo derivative: An economical review via multi-deep assessment methodology

Nisa Özge Önal Tuğrul[1,*], Kamil Karaçuha[2], Esra Ergün[1], Vasil Tabatadze[1] and Ertuğrul Karaçuha[1]

[1] Informatics Institute, Istanbul Technical University, Istanbul 34467, Turkiye
[2] Faculty of Electrical and Electronics Engineering, Istanbul Technical University, Istanbul 34467, Turkiye

* **Correspondence:** Email: onal16@itu.edu.tr; Tel: + 902122857391.

Abstract: In this study, we proposed a novel modeling and prediction method employing both fractional calculus and the multi-deep assessment methodology (M-DAM), utilizing multifactor analysis across the entire dataset from 2000 to 2019 for comprehensive data modeling and prediction. We evaluated and reported the performance of M-DAM by modeling various economic factors such as current account balance (% of gross domestic product (GDP)), exports of goods and services (% of GDP), GDP growth (annual %), gross domestic savings (% of GDP), gross fixed capital formation (% of GDP), imports of goods and services (% of GDP), inflation (consumer prices, annual %), overnight interbank rate, and unemployment (total). The dataset used in this study covered the years between 2000 and 2019. The Group of Eight (G-8) countries and Turkey were chosen as the experimental domain. Furthermore, to understand the validity of M-DAM, we compared the modeling performance with multiple linear regression (MLR) and the one-step prediction performance with a recurrent neural network, long short-term memory (LSTM), and MLR. The results showed that in 75.04% of the predictions, M-DAM predicted the factors with less than 10% error. For the order of predictability considering the years 2018 and 2019, Germany was the most predictable country; the second group consisted of Canada, France, the UK, and the USA; the third group included Italy and Japan; and the fourth group comprised Russia. The least predictable country was found to be Turkey. Comparison with LSTM and MLR showed that the three methods behave complementarily.

Keywords: Caputo fractional derivative; deep assessment methodology; mathematical modeling; time series prediction

Mathematics Subject Classification: 15A09, 26A33, 44A10, 91-10

1. Introduction

The availability of computer technologies and high-performance computing opportunities over the last quarter-century has given momentum to studies involving the development of economic models and the prediction of economic factors. With the sudden impact of globalization, liberalization, and privatization across the globe, it has become even more critical to develop reliable models. Many studies in the literature focus on the modeling of economic and financial time series data [1–6]. In [2], the authors focus on reducing the influence of uninformative predictors and estimating the factors in the forecasting equation. In [3], an algorithm for probabilistic inference in the form of a nonlinear iterative filter is presented and tested with postwar U.S. data on real gross national product. In [4], improved prediction accuracy across various chaotic time series and stock datasets was demonstrated using an enhanced echo state networks (ESN) model. [5] applies the Bayesian evidence framework to least squares support vector machine (LS-SVM) regression, enabling the inference of nonlinear models for predicting financial time series and volatility. Lastly, [6] provides a practical guide to designing neural network forecasting models for economic time series data.

Economic prediction methods have long been a common research topic [7–10]. However, with the rise of machine learning, current economic prediction studies increasingly use new methods rather than stationary statistical models. Recent advancements in artificial intelligence (AI) have significantly impacted economic modeling and forecasting. Furman and Robert discuss how AI and robotics are increasing productivity growth but may also cause labor market upheavals, with mixed effects across different occupations and industries [11]. In the realm of financial forecasting, a hybrid model is proposed integrating generalized autoregressive conditional heteroskedasticity (GARCH) and artificial neural networks to forecast the volatility of cryptocurrencies, highlighting the growing significance of such technologies in financial strategy [12]. Similarly, the following study demonstrates the effectiveness of using long short-term memory (LSTM) recurrent neural networks for multinational trade forecasting, achieving higher accuracy compared to traditional time-series models [13]. In [14], a comprehensive review of data science applications in economics notes the superiority of hybrid models in various economic domains, including stock markets, e-commerce, and cryptocurrency. Besides, the transformative impact of machine learning on economic research, where automation in data analysis enhances the capability to identify causal effects and construct valid outcome metrics, is emphasized in [15]. Furthermore, in [16], reinforcement learning techniques in economics and finance are explored, presenting applications that solve complex behavioral problems through optimal control strategies. Apart from the other studies, [17] introduces a machine-learning method using genetic programming for constructing sentiment indicators, which improves economic forecasting accuracy for European economies. Similarly, Seck investigates the benefits of international technology diffusion for developing countries, highlighting the role of imports and foreign direct investments in enhancing productivity through research and development (R&D) spillovers [18]. With a different perspective, [19] provides a historical perspective on urban economics, emphasizing the influence of urbanization patterns on economic outcomes and the critical role of infrastructure development in shaping economic growth. Additionally, there is a growing number of studies focusing on the estimation of the cryptocurrency market [20,21] and energy economics [22,23].

Fractional calculus has also begun to be used in predictive studies due to its memorization attribute. The advancements in fractional calculus in recent years have increased its usage in academic literature [24–26]. It is a useful tool for modeling systems with memory and hereditary, which are a generalization of classical integer-order derivatives. This study uses fractional order derivatives for several important reasons. These are memory, hereditary properties, and modeling flexibility [27–30]. Studies conducted with this branch of science involve financial prediction [24–28]. For example, [29] proposed an economic interpretation of Caputo derivatives of non-integer orders based on the generalization of average and marginal values of economic indicators. Additionally, [30] introduced a discrete model using the generalized fractal derivative. Both [31] and [32] utilize fractional calculus to enhance the modeling of complex systems. In [33], the authors propose an approach for time series modeling and prediction by expressing a function with its previous values and derivatives, evaluating the method with GDP per capita. [34] considers fractional differential equations in macroeconomics and proposes an application to describe phenomena with power-law memory. Other studies, such as [33,35,36,37], model the economic growth of countries by expressing GDP per capita as a function of variables such as the country's land area, school attendance, and exports of goods and services. These studies conclude that fractional derivatives are effective in modeling economic growth. Pseudo phase plane (PPP) and fractional calculus (FC) are utilized to analyze global economic downturns and forecast future states based on historical data [38]. Similarly, [39] applies fractional derivatives to predict the economic growth of (group of 20) G20 countries, finding that fractional models outperform integer-order models in short-term GDP prediction. Furthermore, [40] introduces the fractal market hypothesis to model macroeconomic time series, employing nonstationary fractional dynamics to capture market behaviors. Also, [41] designs a robust sliding mode controller to make the states of the fractional-order financial system asymptotically stable. Economical modeling and prediction using time series and fractional calculus have been explored in recent research. Wang proposes a novel stock financial market stochastic volatility and pricing model incorporating the Taylor formula, principial component analysis (PCA), and artificial neural network (ANN) methods to analyze complex financial time series [42]. This model enhances prediction accuracy by employing a decomposition-reconstruction-integration approach based on fractional calculus equations. On the other hand, Pavlickova and Petras focus on time series data analysis using fractional calculus, demonstrating the manipulation of information content through fractional derivatives [43]. Besides, [43,44] explores the integration of fractional calculus with machine learning to enhance the modeling and prediction of complex dynamic systems. By leveraging fractional derivatives for data preprocessing, feature augmentation, and optimization, we highlight the potential for improved accuracy and robustness in machine learning applications. Key studies and practical recommendations illustrate the benefits of combining these two powerful methodologies. [45] introduces a model of a fractional hyperchaotic economic system, employing variable-order fractional derivatives to capture the intricate dynamics of economic variables. Additionally, a nonlinear model predictive controller is proposed for effectively controlling the hyperchaotic behavior exhibited by the economic system. The integration of variable-order fractional derivatives and nonlinear control techniques offers promising avenues for understanding and controlling complex economic dynamics.

Similar to the studies mentioned above, we have previously developed various fractional calculus-based mathematical models [46–48] and compared their performance to that of both linear and polynomial modeling techniques [49–51] and the (LSTM). In [46] and [47], we modeled the mobile and fixed broadband subscriptions, and the total number of subscribers of the Turkish mobile communications market, using a fractional approach. In [48] and [49], we modeled Telecommunication Revenues and Telecommunications Investments, respectively, by using FC. In all these settings, we compared our models to the classical polynomial approach. Further, [50] applied a fractional approach combined with the least squares method to model the telecommunication sector in Turkey, demonstrating improved performance in capturing the dynamics of GDP per capita. We followed a similar approach to analyze and predict children's development metrics such as weight, height, and body mass index (BMI) in [51]. We also focused on modeling, prediction, and analysis of COVID-19-confirmed, recovered, and death cases using the deep assessment methodology (DAM) and its variant based on the second derivative [52,53]. The proposed study differs from the literature and our previous studies by including all the following: the multifactor analysis, employing least squares error and deep assessment approach, and FC. Our study contrasts LSTM with our proposed methodology, which investigates deeper insight and relationships among economic indicators. The models we have developed thus far involved a single-input system, but in the current study, a mathematical model with multiple-input and deep assessment structure is suggested. Novel modeling and prediction methodologies were developed by plugging multiple factor effects on a single factor and historical data into the system. By employing FC and deep assessment techniques, we seek to unveil how data factors mutually influence each other.

In the current study, current account balance (% of GDP), exports of goods and services (% of GDP), GDP growth (annual %), gross domestic savings (% of GDP), gross fixed capital formation (% of GDP), imports of goods and services (% of GDP), inflation, consumer prices (annual %), overnight interbank rate, and unemployment (total) data of group of 8 (G-8) countries and Turkey from 2000 to 2019 are used. These economic indicators are considered utilizing novel methods in the literature such as FC and deep assessment of each factor [33]. We are proposing a novel, multifunctional modeling method which has not been considered in the literature before namely, we associate the factor we want to model with the past values of this factor and other factors that we determine affect the target factor. Our approach intrinsically finds every factor's interaction with the other factors' past values. While doing so, we use FC and deep assessment techniques and methodology [52,53]. In our study, we first find the optimum fractional order value of the derivative for each factor that will lead to successful modeling. Second, we model each factor with multi deep assessment methodology (M-DAM); this model includes each factor's impact on every other factor, including on itself, with values derived from the previous step. Lastly, we develop a formulation regarding the prediction of future steps.

In the next section, a problem formulation will be given for the modeling and prediction. The third section will provide example applications. The fourth section will include the results and figures of modeling and prediction implementations. In the fifth and last section, a general evaluation will take place.

2. Materials and methods

In this section, the mathematical approach is provided. To begin, a continuous and bounded function is expanded into a Taylor series. Then, fractional differential equations are proposed, with the solution assumed to model the given data.

2.1. Formulation of the factors modeling

We can express an analytic function, which is a well-known mathematical theory, with the series expansion below known as Taylor Series expansion:

$$g(x) = \sum_{n=0}^{\infty} a_n \, x^n. \tag{2.1}$$

Here a_n's are the constant unknown coefficients for the corresponding factor. For each economic factor, let us denote as $f^{(m)}(x)$, can be expressed in such a way. After expressing each economic factor in terms of the Taylor series which with unknown coefficients, fractional-order differential equations are proposed for the aforementioned factors. This is inspired by the first-order derivative of the functions.

With (2.1) in mind, r is to be the total number of factors we want to include in modeling calculations which can be any parameters affecting the data aimed to be modeled. Now it is better to move on with the assumption (2.2) below [52,53].

$$\frac{\partial^{\alpha_m} f^{(m)}(x)}{\partial x^{\alpha_m}} \triangleq \sum_{n=1}^{\infty} a_n^{(m)} (n\alpha_m) \, x^{n\alpha_m - 1}, \tag{2.2}$$

where α_m is the derivative order and ranges between 0 and 1, while $m = 1,2,\ldots,r$. As seen from (2.2), each factor is assumed to satisfy the corresponding fractional-order differential equation.

Before moving on with the formulation, let us define Caputo's fractional derivative [52,53].

$${}_0^C D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\frac{df(s)}{ds}}{(t-s)^{\alpha}} ds \ , \ \ 0 \le \alpha < 1. \tag{2.3}$$

It is important to note that $f(t)$ denotes the factor we are modeling, while D_t^{α} corresponds to the fractional derivative of order α with respect to t $\left(\frac{\partial^{\alpha} \mathrm{f}(t)}{\partial t^{\alpha}}\right)$. Also, $\Gamma(1-\alpha)$ is the Gamma function. After proposing (2.2), the question of how to solve this fractional order differential equation is raised. The approach in the present article is to reduce the fractional differential equation (2.2) into an algebraic equation by employing the *Laplace transform* $(\mathcal{L})$. The Laplace transform of the fractional differential equation provided in (2.2) can be found as follows while taking into account the fractional derivative definition given in (2.3) [52,53]:

$$\mathcal{L}\left[f^{(m)}(x)\right] = F^{(m)}(s) = \int_0^{\infty} f^{(m)}(x) e^{-st} \, dx \, , \tag{2.4.1}$$

$$s^{(\alpha_m)} F^{(m)}(s) - s^{(\alpha_m - 1)} f^{(m)}(0) = \sum_{n=1}^{\infty} a_n^{(m)} (n\alpha_m) \frac{\Gamma(n\alpha_m)}{s^{n\alpha_m}}. \tag{2.4.2}$$

(2.5) is obtained by taking the *inverse Laplace transform* $(\mathcal{L}^{-1})$ of (2.4.2) again. Notice that, the infinite summation is reduced to M for numerical calculation. By taking the direct and the inverse

Laplace transform including the properties for the fractional derivative terms, the unknown function $f^{(m)}(x)$ can be expressed as a summation which is the solution of the proposed fractional differential equation given in (2.2) [53].

$$f^{(m)}(x) \cong f^{(m)}(0) + \sum_{n=1}^{M} a_n^{(m)} C_n^{(m)}(x), \tag{2.5}$$

where,

$$C_n^{(m)}(x) = \frac{\Gamma(n\alpha_m + 1)}{\Gamma(n\alpha_m + \alpha_m)} (x)^{(n\alpha_m + \alpha_m - 1)}. \tag{2.6}$$

After this point, one needs to find the unknown coefficients $a_n^{(m)}$ for each factor. To fit the proposed $f^{(m)}$ for each factor, the *least-squares method* is employed. To apply the least-squares method to find the optimum $a_n^{(m)}$ coefficients and optimum fractional derivative order for each economical factor function, we can define the sum of squared errors as (2.7).

$$\begin{aligned} \epsilon_T^2 = \sum_{i=z}^{t} \epsilon_i^2 &= \sum_{i=z}^{t} \left[\left(P_i^{(m)} - f^{(m)}(x_i) \right)^2 \right] \\ &= \sum_{i=z}^{t} \left(P_i^{(m)} - f^{(m)}(0) - \sum_{n=1}^{M} a_n^{(m)} C_n^{(m)}(x_i) \right)^2. \end{aligned} \tag{2.7}$$

Here,

m: The numbers [1, 2, ..., 9] represent each factor, corresponding to current account balance, exports of goods and services, GDP growth, gross domestic savings, gross fixed capital formation, imports of goods and services, inflation, overnight interbank rate, and unemployment rate, respectively.

x_i: The numbers [1,2, …, 20] represent each year as a numerical value. For example, for the year 2000, $x_i = 1$; for year 2002, $x_i = 3$; for the year 2019, $x_i = 20$.

$P_i^{(m)}$: $\left[P_1^{(1)}, P_2^{(1)}, \dots, P_{20}^{(1)}, P_1^{(2)}, P_2^{(2)}, \dots, P_{20}^{(2)}, \dots, P_1^{(9)}, P_2^{(9)}, \dots, P_{20}^{(9)}\right]$ the value of m^{th} factor in i^{th} year for the target country, for example, $P_2^{(3)}$ denotes the gross domestic savings data from 2001 for the country of interest.

z and t values are shown in Figure 1 namely, the first data point in the target region (range) for the model is $x = z$ while the last is $x = t$.

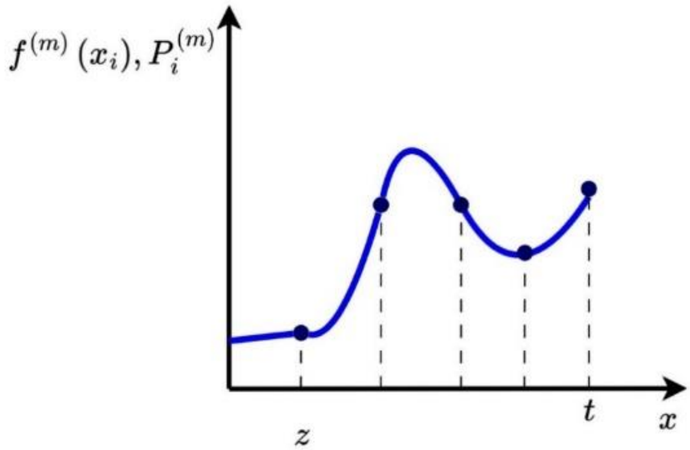

Figure 1. Modeling of the dataset.

To obtain the proposed function $f^{(m)}(x)$ with the minimum error, the least squares method is employed [44]. In other words, the derivative of the square of the total error's sum with respect to unknown coefficients such as $f^{(m)}(0)$ and $a_p^{(m)}$ should be equal to 0 as given in (2.8). Then, the system of linear algebraic equations with the numbers of $M+1$ is obtained. Here, $f^{(m)}(0)$ comes from the Laplace transform and $a_p^{(m)}$ comes out of the expansion of the proposed function as the infinite sum.

$$\frac{\partial \epsilon_T^2}{\partial f^{(m)}(0)} = 0, \qquad \frac{\partial \epsilon_T^2}{\partial\left[a_p^{(m)}\right]} = 0, \quad p = 1,2,\ldots,M, \tag{2.8}$$

$$\sum_{i=z}^{t} P_i^{(m)} = \sum_{i=z}^{t}\left(f^{(m)}(0) + \sum_{n=1}^{M} a_n^{(m)} C_n^{(m)}(x_i)\right), \tag{2.9}$$

$$\sum_{i=z}^{t} P_i^{(m)} C_p^{(m)}(x_i) = f^{(m)}(0)\sum_{i=z}^{t} C_p^{(m)}(x_i) + \sum_{i=z}^{t}\left(\sum_{n=1}^{M} a_n^{(m)} C_n^{(m)}(x_i)\right) C_p^{(m)}(x_i).$$

From Eq (2.9), $M+1$ numbers of equation sets are obtained for every m value by changing the fractional order α_m (grid search) in the equation set to a value between the range of (0,1), However the optimal $a_n^{(m)}, f^m(0)$, and α_m are found by the least squares approach where the error is defined in (2.7). Then, the continuous curve representing the data with the minimum error can be achieved. By (2.8), a system of linear algebraic equations (SLAE) given in (2.10) is obtained where the matrix contains all unknowns $a_n^{(m)}$, and $f^m(0)$ [53] similar to the $[A][B] = [C]$ matrix system. By inversion, the unknowns can be obtained.

$$\begin{bmatrix} t & \sum_{i=1}^{t} C_1^{(m)} & \sum_{i=1}^{t} C_2^{(m)} & \cdots & \sum_{i=1}^{t} C_M^{(m)} \\ \sum_{i=1}^{t} C_1^{(m)} & \sum_{i=1}^{t} C_1^{(m)} C_1^{(m)} & \sum_{i=1}^{t} C_2^{(m)} C_1^{(m)} & \cdots & \sum_{i=1}^{t} C_M^{(m)} C_1^{(m)} \\ \sum_{i=1}^{t} C_2^{(m)} & \sum_{i=1}^{t} C_1^{(m)} C_2^{(m)} & \sum_{i=1}^{t} C_2^{(m)} C_2^{(m)} & \cdots & \sum_{i=1}^{t} C_M^{(m)} C_2^{(m)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \sum_{i=1}^{t} C_M^{(m)} & \sum_{i=1}^{t} C_1^{(m)} C_M^{(m)} & \sum_{i=1}^{t} C_2^{(m)} C_M^{(m)} & \cdots & \sum_{i=1}^{t} C_M^{(m)} C_M^{(m)} \end{bmatrix} \begin{bmatrix} f(0) \\ a_1^{(m)} \\ a_2^{(m)} \\ a_3^{(m)} \\ \vdots \\ a_M^{(m)} \end{bmatrix} = \begin{bmatrix} \sum_{i=z}^{t} P_i^{(m)} \\ \sum_{i=z}^{t} P_i^{(m)} C_1^{(m)} \\ \sum_{i=z}^{t} P_i^{(m)} C_2^{(m)} \\ \sum_{i=z}^{t} P_i^{(m)} C_3^{(m)} \\ \vdots \\ \sum_{i=z}^{t} P_i^{(m)} C_M^{(m)} \end{bmatrix}. \tag{2.10}$$

2.2. Modeling via multifactors

After expressing the modeling of an economic factor using the Taylor expansion approach, we apply the following procedure. According to our model, an economic factor is influenced by other factors and their previous values. Therefore, it is formulated as the weighted distribution of the other factors along with the previous values. This will lead us to the M-DAM, where the mathematical expression of the m^{th} factor will consist of various other affecting factors. This is shown in (2.11). Apart from the previous approach, here the proposed function to model each factor is assumed to be the functions of previous values of all factors with unknown weighted coefficients as given below.

$$f^{(m)}(x) = \sum_{k=1}^{l} \sum_{j=1}^{r} \alpha_k^{(j)} f^{(j)}(x-k). \tag{2.11}$$

Here, m is the factor index. If we are formulating the 1st factor by using other factors, we set $m = 1$. For any factor index j, we can obtain the equation below by using (2.1) and (2.2). Then, again inspiration from Taylor expansion is considered to express the corresponding function.

$$f^{(j)}(x-k) = \sum_{n=0}^{M} a_n^{(j)} (x-k)^{n\alpha_j}. \tag{2.12}$$

We can express (2.11) as (2.13) below.

$$f^{(m)}(x) = \sum_{k=1}^{l} \sum_{j=1}^{r} \sum_{n=0}^{M} \alpha_k^{(j)} a_n^{(j)} (x-k)^{n\alpha_j}. \tag{2.13}$$

Let us apply the operations we conducted in Section 2.1 to Eq (2.13) once more. Here, we denote fractional derivative order as ν. If we take $F^{(m)}(s)$ as the Laplace transformation of $f^{(m)}(x)$ we can write,

$$\frac{\partial^{\nu} f^{(m)}(x)}{\partial x^{\nu}} \cong \sum_{k=1}^{l}\sum_{j=1}^{r}\sum_{n=1}^{M} \alpha_k^{(j)}\, a_n^{(j)} n\alpha_j (x-k)^{n\alpha_j - 1}, \tag{2.14}$$

$$F^{(m)}(s) = \frac{f^{(m)}(0)}{s} + \sum_{k=1}^{l}\sum_{j=1}^{r}\sum_{n=1}^{M} a_{kn}^{(j)} \Gamma\left(n\alpha_j + 1\right) \frac{e^{-ks}}{s^{n\alpha_j + \nu}}, \tag{2.15}$$

where, $a_{kn}^{(j)} = \alpha_k^{(j)} a_n^{(j)}$. If we take the inverse Laplace transformation of Eq (2.15),

$$f^{(m)}(x) = f^{(m)}(0) + \sum_{j=1}^{r}\sum_{k=1}^{l}\sum_{n=1}^{M} a_{kn}^{(j)}\, C_{kn}^{(j)}(x) \tag{2.16}$$

is achieved. Then,

$$C_{kn}^{(j)}(x) = \frac{\Gamma\left(n\alpha_j + 1\right)}{\Gamma\left(n\alpha_j + \nu\right)} (x-k)^{n\alpha_j + \nu - 1}$$

is obtained. Modeling of any factor will be expressed by using Eq (2.16) and by calculating the $a_{kn}^{(j)}$ coefficient with the least squared error method. The most important step here is changing the fractional order ν to a value ranging between (0,1) and finding the optimal ν value giving the least modeling error.

As mentioned earlier, during optimization, $a_{kn}^{(j)}$ coefficients and $f^{(m)}(0)$ are obtained by the least squares approach as given in Eqs (2.17) and (2.18) while the ν value is determined by the grid search.

$$\left(\epsilon_T^{(m)}\right)^2 = \sum_{i=z}^{t}\left(\epsilon_i^{(m)}\right)^2 = \sum_{i=z}^{t}\left[\left(P_i^{(m)} - f^{(m)}(x_i)\right)^2\right], \tag{2.17}$$

$$\frac{\partial\left(\epsilon_T^{(m)}\right)^2}{\partial f^{(m)}(0)} = 0 \text{ and } \frac{\partial \epsilon_T^2}{\partial a_{pw}^{(s)}} = 0, \quad s = 1,2,..r, \quad p = 1,2,\ldots l, \quad w = 1,2,..M. \tag{2.18}$$

When minimizing (2.17) by (2.18), $(1 + r * l * M)$ number of equations are obtained. We find the unknown coefficients by SLAE inversion. Then, the modeling process is completed by finding the ν with the least error and $f^{(m)}(0), a_{kn}^{(j)}$ values as given in (2.10).

2.3. A proposal for prediction

After optimizing the modeling of discrete economic factors with M-DAM, one can obtain the continuous curves $f^{(m)}$ representing each economic factor with minimum error. We can exploit this approach for predicting $x = x + 1$. Let us define the factor m we want to model with the target year and historical values below,

$$f^{(m)}(x) = \sum_{j=1}^{r}\sum_{k=1}^{l} \beta_k^{(j)} f^{(j)}(x-k) \tag{2.19}$$

$$f^{(m)}(x-1) = \sum_{j=1}^{r}\sum_{k=1}^{l} \beta_k^{(j)} f^{(j)}\big(x-(k+1)\big)$$

$$\vdots \qquad\qquad\qquad \vdots$$

$$f^{(m)}(x-l) = \sum_{j=1}^{r}\sum_{k=1}^{l} \beta_k^{(j)} f^{(j)}\big(x-(k+l)\big).$$

Here, $x > 2l$. Then square of the total error for the m[th] factor can be expressed as

$$\left(\epsilon_T^{(m)}\right)^2 = \sum_{i=z}^{t}\left(\epsilon_i^{(m)}\right)^2 = \sum_{i=z}^{t}\left[f^{(m)}(x-i) - \sum_{j=1}^{r}\sum_{k=1}^{l} \beta_k^{(j)} f^{(j)}\big(x-(k+i)\big)\right]^2. \tag{2.20}$$

After employing the least squares approach as given in (2.21), the unknowns are determined and (2.22) is obtained.

$$\frac{\partial\left(\epsilon_T^{(m)}\right)^2}{\partial \beta_y^{(s)}} = 0 \ , \qquad s = 1,2,..r, \quad y = 1,2,\dots l, \tag{2.21}$$

$$\sum_{i=z}^{t} f^{(m)}(x-i)\, f^{(s)}\big(x-(y+i)\big)$$
$$= \sum_{i=z}^{t}\sum_{j=1}^{r}\sum_{k=1}^{l} \beta_k^{(j)} f^{(j)}(x-(k+i))\, f^{(s)}(x-(y+i)). \tag{2.22}$$

$(l * r)$ numbers of equations are obtained from Eq (2.22). After finding the $\beta_k^{(j)}$ values from these equations as (2.10), future predictions of any factor can be made by using the first set of Eq (2.19). In other words, values in the next step are obtained by $x = x + 1$.

2.4. LSTM

In this study, we used a special type of recurrent neural network, LSTM, that is widely popular in predicting and modeling the content of time series for assessing the validity of M-DAM. There are four gates learned inside an LSTM cell: input, forget, output, and gate. Gate g is a hyperbolic tangent (tanh) and takes values between -1 and 1. All other gates are sigmoid functions and are between 0 and 1. LSTMs optionally inherit information from previous time steps with the help of gates. Gate equations are listed below in Eqs (2.23)–(2.28). Each gate learns its own set of parameters W's and b's. In Eqs (2.27) and (2.28), $\odot$ is the Hadamard product.

$$f_t = \sigma\left(W_f[h_{t-1}^l, h_t^{l-1}] + b_f\right). \tag{2.23}$$

$$i_t = \sigma\left(W_i[h_{t-1}^l, h_t^{l-1}] + b_i\right). \tag{2.24}$$

$$o_t = \sigma\left(W_o[h_{t-1}^l, h_t^{l-1}] + b_o\right). \tag{2.25}$$

$$g_t = \tanh\left(W_g[h_{t-1}^l, h_t^{l-1}] + b_g\right). \tag{2.26}$$

$$c_t^l = f \odot c_{t-1}^l + i \odot g. \tag{2.27}$$

$$h_t^l = o \odot \tanh\left(c_t^l\right). \tag{2.28}$$

An LSTM unit consists of one or multiple cells where each cell updates its state with the previous state c_{t-1}^l. The previous state information is used for computing the current state, only hidden state information h is fed to the following layers.

2.5. Multiple linear regression (MLR)

MLR is a statistical technique to analyze the relationship between two or more independent variables (predictors) and a dependent variable (response). Generally, the MLR model can be expressed as in Eq (2.29):

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + \dots + b_nx_n + \varepsilon. \tag{2.29}$$

Here,

- y is the dependent variable.
- $x_1, x_2, \dots x_n$ are the independent variables.
- b_0 is the intercept term (constant).
- $b_1, b_2, \dots b_n$ are the regression coefficients (also known as slope coefficients) corresponding to each independent variable.
- ε represents the error term, which captures the difference between the observed and predicted values of y.

The values of the coefficients are estimated using the least squares method. In this study, the MLR model is developed by using MATLAB standard functions ‘fitlm’ and ‘predict’ functions.

3. Model implementation

In this section, the developed approach will be implemented as an example. Nine economic factors from the G-8 countries and Turkey, covering the years 2000 to 2019, have been chosen as the implementation area. These economic factors are listed in Table 1. The overnight interbank rates were obtained from OECD data statistics [54], and the other data was sourced from the World Bank Databank [55].

Table 1. Economics factors of the dataset.

No	$f^{(m)}$	Series name
1	$f^{(1)}$	Current account balance (% of GDP)
2	$f^{(2)}$	Exports of goods and services (% of GDP)
3	$f^{(3)}$	GDP growth (annual %)
4	$f^{(4)}$	Gross domestic savings (% of GDP)
5	$f^{(5)}$	Gross fixed capital formation (% of GDP)
6	$f^{(6)}$	Imports of goods and services (% of GDP)
7	$f^{(7)}$	Inflation, consumer prices (annual %)
8	$f^{(8)}$	Overnight Interbank rate
9	$f^{(9)}$	Unemployment, total (ILO Estimated)

We will continue by explaining the process of our work step by step.

1) First, we find $a_n^{(m)}$ and α_m coefficients for every factor with Eqs (2.6) and (2.9).

2) To find the impact of all factors on one another, we find ν and $a_{kn}^{(j)}$ values by using (2.16) and (2.18) equation sets. This way, we model economic factors using past values.

3) We find $\beta_k^{(j)}$values by using Eqs (2.19) to (2.21). We then predicted $f^{(m)}(x+1)$ value of any $f^{(m)}(x)$ economic factor.

4. Results

4.1. Modeling results

Table 2 shows results obtained by using M-DAM, in which the impact of each factor between $f^{(1)}(x) - f^{(9)}(x)$ was considered for all countries. Table 2 and figures clearly show that all aforementioned economic factors have been modeled well. Modeling graphs belonging to each country are given in Figures 2–10. Modeling performance was measured using mean absolute percentage error (MAPE) in Eq (4.1). While $\mathrm{v}(i)$ represents a real value, $\tilde{\mathrm{v}}(i)$ represents the data modeled with M-DAM. $t-z+1$ is the number of sample data; since a value of $l=10$ is used, data between 2010–2019 are modeled, and thus $t-z+1$ value is taken as 10 in the current study.

$$MAPE = \frac{1}{t-z+1}\sum_{i=z}^{t}\left|\frac{\mathrm{v}(i)-\tilde{\mathrm{v}}(i)}{\mathrm{v}(i)}\right| \times 100. \tag{4.1}$$

Table 2. Modeling 2010–2019 years for $M = 10\ and\ l = 10$ values using economic factors data for G-8 countries and Turkey using 2000 to 2009 with M-DAM.

CANADA	ECONOMIC FACTORS	ν	MAPE (%)	JAPAN	ECONOMIC FACTORS	ν	MAPE (%)
$f^{(1)}$	Current account balance	0,654	2,98E-10	$f^{(1)}$	Current account balance	0,745	4,35E-06
$f^{(2)}$	Exports of goods and services	0,655	1,05E-10	$f^{(2)}$	Exports of goods and services	0,745	1,06E-06
$f^{(3)}$	GDP growth	0,654	5,61E-10	$f^{(3)}$	GDP growth	0,745	4,12E-05
$f^{(4)}$	Gross domestic savings	0,654	3,65E-11	$f^{(4)}$	Gross domestic savings	0,745	2,63E-07
$f^{(5)}$	Gross fixed capital formation	0,655	3,30E-11	$f^{(5)}$	Gross fixed capital formation	0,745	6,72E-08
$f^{(6)}$	Imports of goods and services	0,654	2,26E-11	$f^{(6)}$	Imports of goods and services	0,745	9,39E-07
$f^{(7)}$	Inflation, consumer prices	0,654	4,25E-10	$f^{(7)}$	Inflation, consumer prices	0,745	2,04E-05
$f^{(8)}$	Overnight Interbank rate	0,654	9,93E-06	$f^{(8)}$	Overnight Interbank rate	0,745	2,16E-05
$f^{(9)}$	Unemployment, total	0,654	1,64E-06	$f^{(9)}$	Unemployment, total	0,745	4,08E-07
FRANCE	ECONOMIC FACTORS	ν	MAPE (%)	RUSSIA	ECONOMIC FACTORS	ν	MAPE (%)
$f^{(1)}$	Current account balance	0,6	3.27E-04	$f^{(1)}$	Current account balance	0,663	7,62E-08
$f^{(2)}$	Exports of goods and services	0,6	8,04E-06	$f^{(2)}$	Exports of goods and services	0,663	5,71E-09
$f^{(3)}$	GDP growth	0,6	1,75E-04	$f^{(3)}$	GDP growth	0,663	2,30E-07
$f^{(4)}$	Gross domestic savings	0,6	5,59E-06	$f^{(4)}$	Gross domestic savings	0,663	3,86E-09
$f^{(5)}$	Gross fixed capital formation	0,6	1,30E-06	$f^{(5)}$	Gross fixed capital formation	0,663	3,54E-09
$f^{(6)}$	Imports of goods and services	0,6	6,18E-06	$f^{(6)}$	Imports of goods and services	0,472	4,19E-10
$f^{(7)}$	Inflation, consumer prices	0,6	3,11E-04	$f^{(7)}$	Inflation, consumer prices	0,663	1,59E-07
$f^{(8)}$	Overnight interbank rate	0,6	2,09E-04	$f^{(8)}$	Overnight interbank rate	0,663	1,84E-07
$f^{(9)}$	Unemployment, total	0,6	1,96E-06	$f^{(9)}$	Unemployment, total	0,663	1,37E-08
GERMANY	ECONOMIC FACTORS	ν	MAPE (%)	TURKEY	ECONOMIC FACTORS	ν	MAPE (%)
$f^{(1)}$	Current account balance	0,527	4,63E-07	$f^{(1)}$	Current account balance	0,981	1,28E-06
$f^{(2)}$	Exports of goods and services	0,527	1,53E-07	$f^{(2)}$	Exports of goods and services	0,963	1,62E-09
$f^{(3)}$	GDP growth	0,527	6,91E-06	$f^{(3)}$	GDP growth	0,509	5,52E-06
$f^{(4)}$	Gross domestic savings	0,527	3,34E-08	$f^{(4)}$	Gross domestic savings	0,509	2,72E-08
$f^{(5)}$	Gross fixed capital formation	0,527	5,85E-08	$f^{(5)}$	Gross fixed capital formation	0,981	1,63E-07
$f^{(6)}$	Imports of goods and services	0,527	1,34E-07	$f^{(6)}$	Imports of goods and services	0,80	9,72E-08
$f^{(7)}$	Inflation, consumer prices	0,527	6,84E-07	$f^{(7)}$	Inflation, consumer prices	0,963	7,36E-07
$f^{(8)}$	Overnight interbank rate	0,527	4,23E-06	$f^{(8)}$	Overnight interbank rate	0,509	1,58E-06
$f^{(9)}$	Unemployment, total	0,527	5,24E-08	$f^{(9)}$	Unemployment, total	0,981	1,92E-07
ITALY	ECONOMIC FACTORS	ν	MAPE (%)	UK	ECONOMIC FACTORS	ν	MAPE (%)
$f^{(1)}$	Current account balance	0,63	1,30E-05	$f^{(1)}$	Current account balance	0,827	1,12E-08
$f^{(2)}$	Exports of goods and services	0,63	6,91E-07	$f^{(2)}$	Exports of goods and services	0,827	5,68E-10
$f^{(3)}$	GDP growth	0,63	8,59E-06	$f^{(3)}$	GDP growth	0,863	4,24E-08
$f^{(4)}$	Gross domestic savings	0,63	3,96E-07	$f^{(4)}$	Gross domestic savings	0,827	7,40E-10
$f^{(5)}$	Gross fixed capital formation	0,63	1,12E-07	$f^{(5)}$	Gross fixed capital formation	0,827	2,89E-10
$f^{(6)}$	Imports of goods and services	0,63	9,59E-07	$f^{(6)}$	Imports of goods and services	0,827	1,69E-09
$f^{(7)}$	Inflation, consumer prices	0,63	1,54E-04	$f^{(7)}$	Inflation, consumer prices	0,818	6,81E-09
$f^{(8)}$	Overnight interbank rate	0,63	1,37E-05	$f^{(8)}$	Overnight interbank rate	0,827	6,20E-08
$f^{(9)}$	Unemployment, total	0,63	7,79E-07	$f^{(9)}$	Unemployment, total	0,9	3,88E-09
USA	ECONOMIC FACTORS	ν	MAPE (%)	USA	ECONOMIC FACTORS	ν	MAPE (%)
$f^{(1)}$	Current account balance	0,727	1,73E-06	$f^{(5)}$	Gross fixed capital formation	0,736	3,80E-07
$f^{(2)}$	Exports of goods and services	0,772	3,97E-06	$f^{(6)}$	Imports of goods and services	0,772	4,15E-06
$f^{(3)}$	GDP growth	0,727	2,56E-05	$f^{(7)}$	Inflation, consumer prices	0,727	4,06E-04
$f^{(4)}$	Gross domestic savings	0,727	3,92E-06	$f^{(8)}$	Overnight interbank rate	0,736	5,39E-05
				$f^{(9)}$	Unemployment, total	0,736	2,25E-06

In Table 3, the MAPE values for multi linear regression MLR are provided for all countries including all factors. It is very clear that M-DAM outperforms in all factors for all countries since the proposed approach introduces a new parameter as the fractional order for modeling to optimize the results. Furthermore, M-DAM and the fractional derivative offer the memory property of the data because the model itself presumes that for an arbitrarily chosen time, the value of any factor at that time is expressed as the weighted summation of its previous values of all factors as explained in the previous section. Moreover, the fractional derivative is not a local operator as in the case of the integer-valued derivative orders.

Table 3. Modeling MAPE (%) rates with MLR for values using economic factors data for G-8 countries and Turkey using 2000 to 2009.

$f^{(m)}$		CANADA	FRANCE	GERMANY	ITALY	JAPAN	RUSSIA	TURKEY	UK	USA
$f^{(1)}$	Current account balance (% of GDP)	9,66	55,78	9,66	17,67	3,45	4,52	9,08	14,3	2,01
$f^{(2)}$	Exports of goods and serv. (% of GDP)	0,43	0,30	0,43	0,38	0,38	0,77	0,71	0,32	0,35
$f^{(3)}$	GDP growth (annual %)	100,36	46,59	100,36	1040,8	457,04	42,17	57,2	19,7	16,41
$f^{(4)}$	Gross domestic savings (% of GDP)	0,87	0,55	0,87	0,76	0,54	1,34	0,90	0,58	0,49
$f^{(5)}$	Gross fixed capital form. (% of GDP)	0,77	0,64	0,77	0,51	0,56	1,70	1,20	0,52	0,44
$f^{(6)}$	Imports of goods and serv. (% of GDP)	0,53	0,34	0,53	0,44	0,46	1,01	0,72	0,28	0,29
$f^{(7)}$	Inflation, consumer prices (annual %)	23,24	39,16	23,24	92,92	435,51	20,55	20,2	18,1	57,49
$f^{(8)}$	Overnight interbank rate	69,15	142,63	69,15	143,74	1507,85	15,67	147,3	66,2	218,2
$f^{(9)}$	Unemployment, total	5,11	2,02	5,11	2,64	3,96	6,77	3,99	4,67	5,57

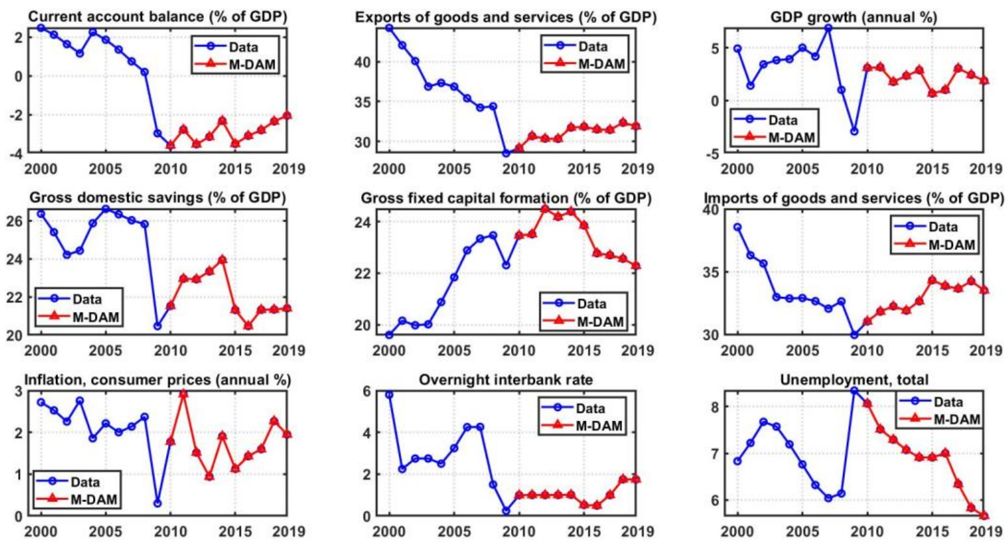

Figure 2. Modeling graphs with M=10 and l=10 between the years 2010 and 2019 for Canada (with M-DAM).

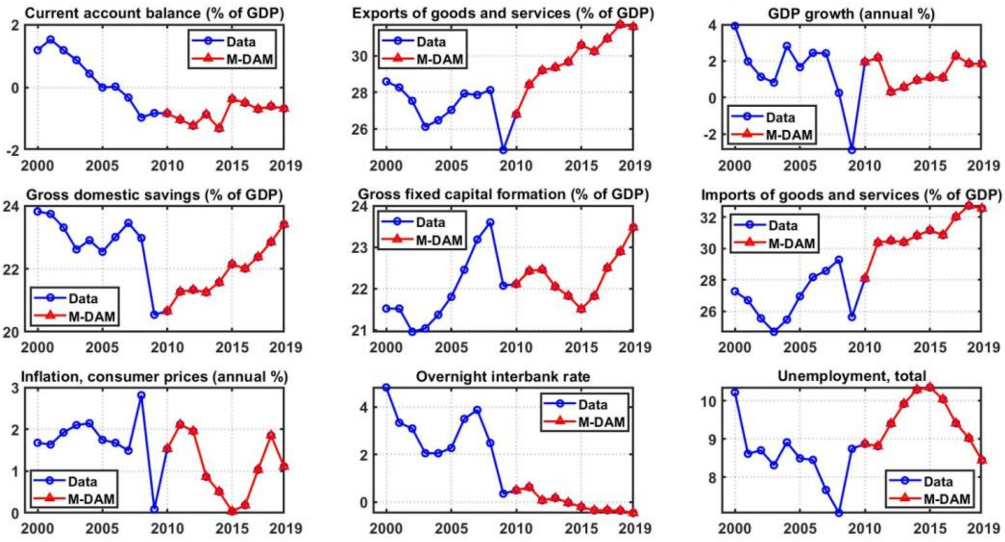

Figure 3. Modeling graphs with $M = 10$ and $l = 10$ between the years 2010 and 2019 for France (with M-DAM).

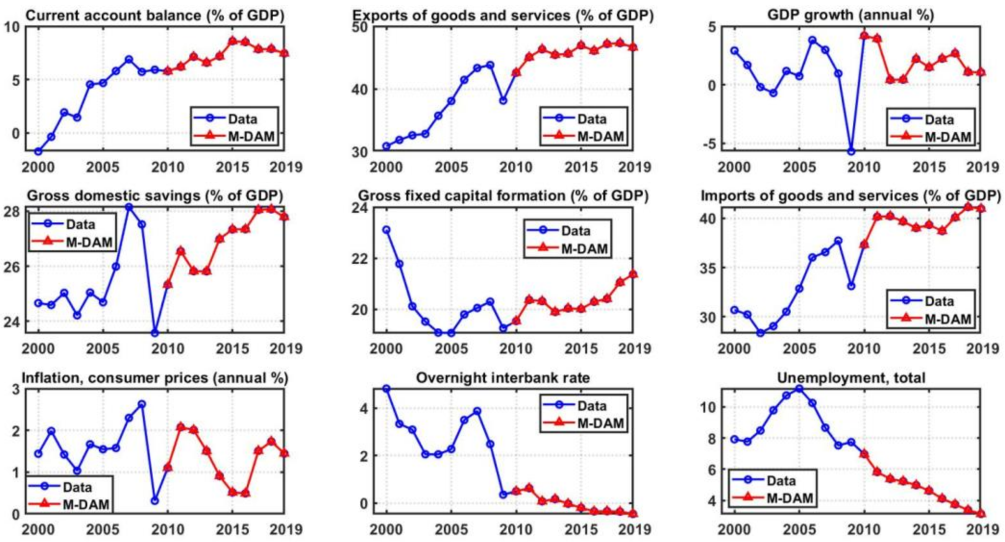

Figure 4. Modeling graphs with $M = 10$ and $l = 10$ between the years 2010 and 2019 for Germany (with M-DAM).

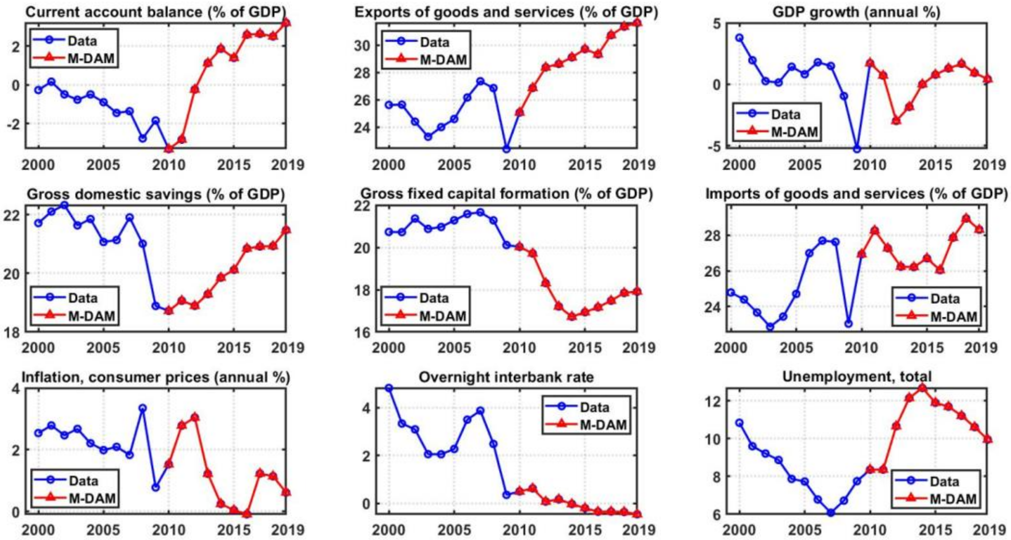

Figure 5. Modeling graphs with $M = 10$ and $l = 10$ between the years 2010 and 2019 for Italy (with M-DAM).

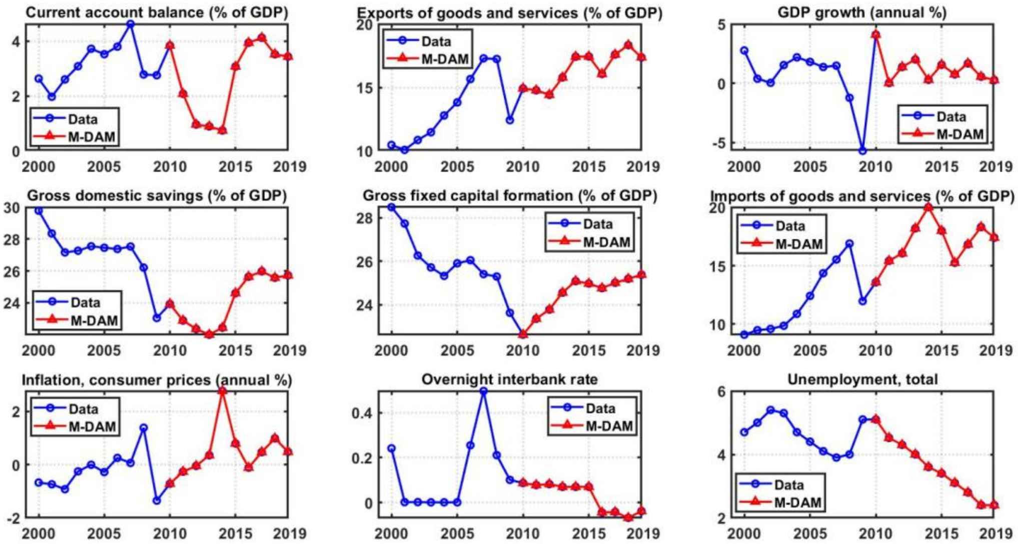

Figure 6. Modeling graphs with $M = 10$ and $l = 10$ between the years 2010 and 2019 for Japan (with M-DAM).

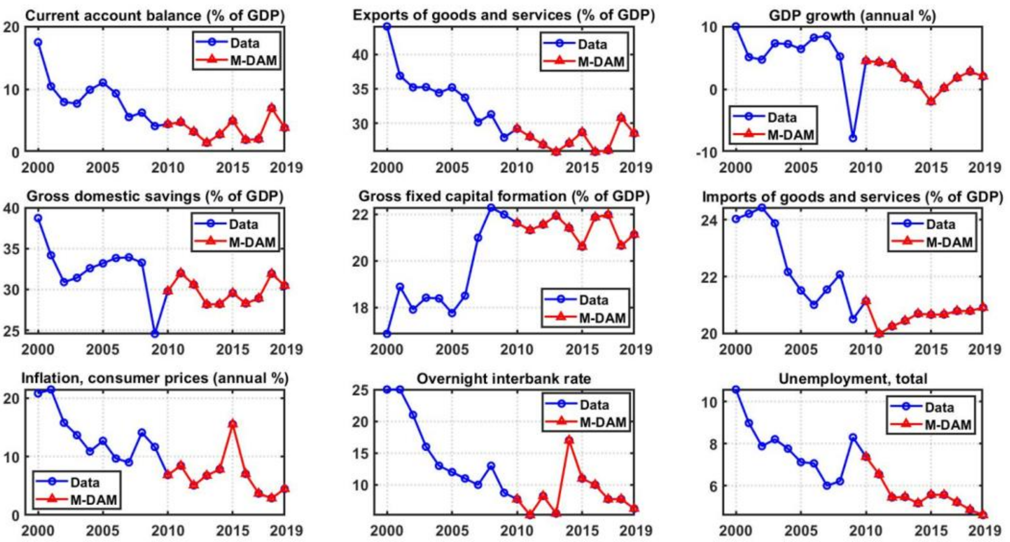

Figure 7. Modeling graphs with $M = 10$ and $l = 10$ between the years 2010 and 2019 for Russia (with M-DAM).

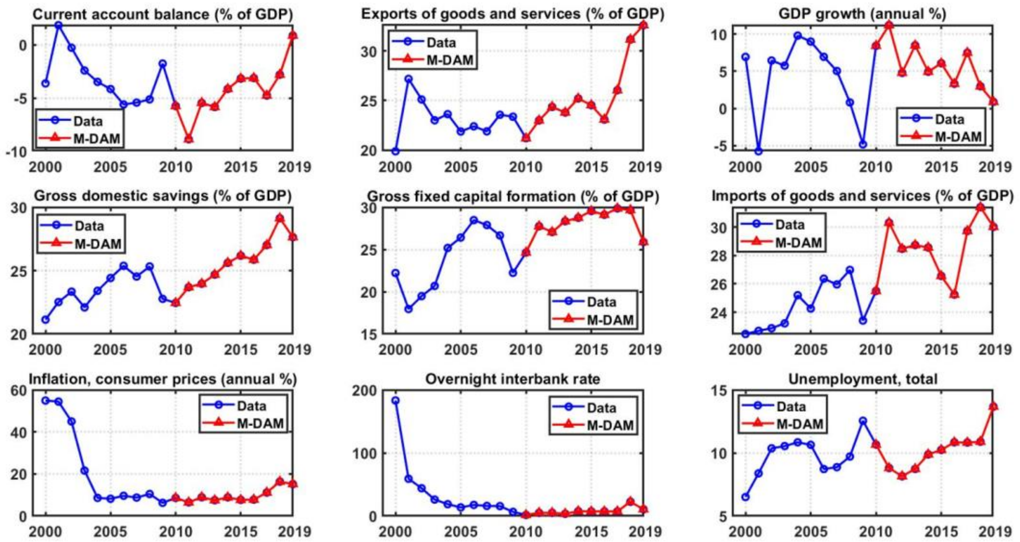

Figure 8. Modeling graphs with $M = 10$ and $l = 10$ between the years 2010 and 2019 for Turkey (with M-DAM).

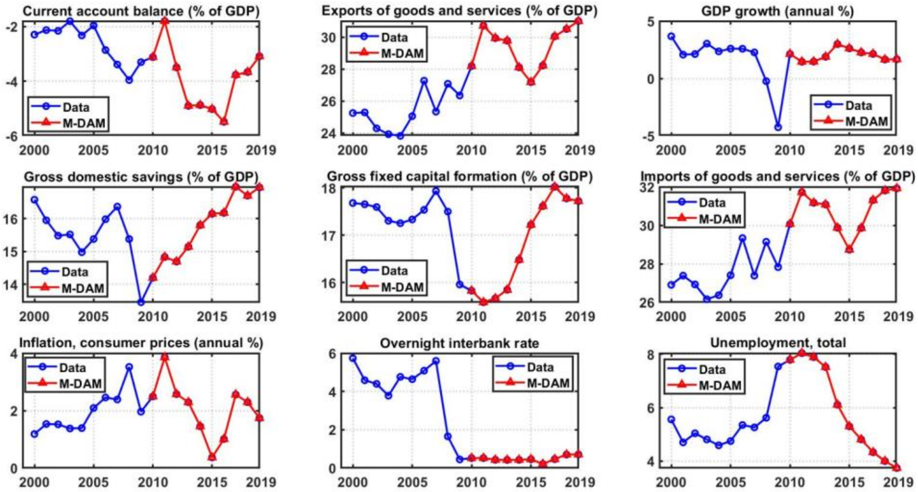

Figure 9. Modeling graphs with $M = 10$ and $l = 10$ between the years 2010 and 2019 for the UK (with M-DAM).

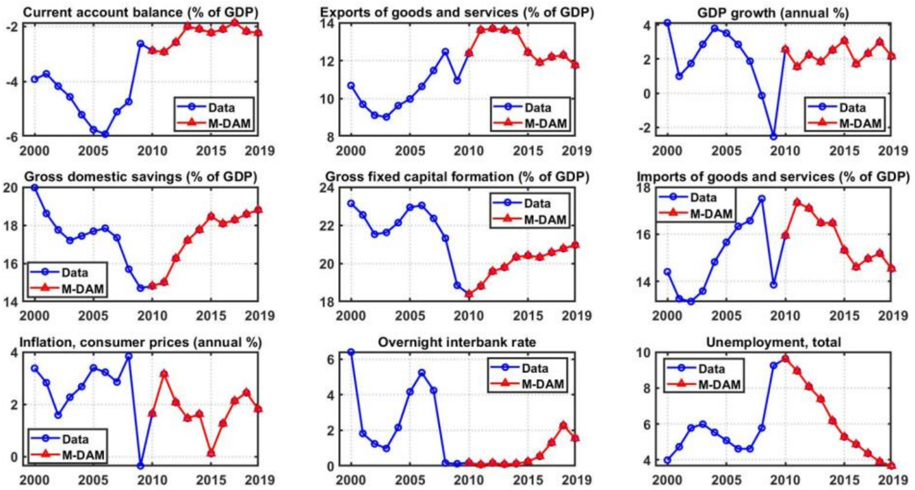

Figure 10. Modeling graphs with $M = 10$ and $l = 10$ between the years 2010 and 2019 for the USA (with M-DAM).

4.2. Prediction results

The accuracy of the predictions is undoubtedly crucial, and better predictions can be made for data that remain stable. To evaluate the validity of our proposed method, M-DAM, a deep neural network is trained, and the results are reported here. The generalization capacity of deep neural networks heavily depends on the number of samples used for training. To increase the number of samples, monthly data values are generated from the original dataset, covering the years 2000–2019, using the DAM.

In the LSTM model, since the predictions are made yearly, monthly training data is used with a 12-step time frame sampling scheme during training. Data is split into training and test sets. The number of past time steps used for prediction (l) is determined by grid search. To predict the next value of $f^{(1)}$ the model uses all other factors from l previous year.

All LSTM models reported in this section have two stacked layers, each having 64 neurons. The model is implemented in the PyTorch environment and optimized with the Adam optimizer. For each of the predictions and each of the factors $f^{(i)}$, a different LSTM model is trained.

Table 4 reports the performance of LSTM, M-DAM, and MLR predictions in terms of MAPE rates. According to the M-DAM predictions, France's gross domestic savings factor was predicted with the least error, at 0.01%. When prediction values below 10% are considered in terms of predictability of the countries, first place for predictability goes to Germany; in second place were Canada, France, the UK, and the USA; the third most predictable countries were Italy and Japan; the fourth was Russia; and last was Turkey. Thus, it can be concluded that the most predictable country is Germany, while the least predictable country appears to be Turkey.

In assessing Canada's economic indicators (see Table 4), both the LSTM and M-DAM models exhibit differing predictive performances. While LSTM generally demonstrates lower MAPE values, indicating better predictive accuracy, M-DAM showcases commendable performance in certain areas. For instance, M-DAM portrays relatively good predictions in metrics such as gross domestic savings and gross fixed capital formation, yielding MAPE values ranging from 0.10% to 0.64%. In contrast, LSTM consistently outperforms M-DAM in critical areas such as current account balance, exports, inflation, and unemployment, with MAPE values from 0.06% to 1.65%. When MLR is considered, M-DAM outperforms MLR except for current account balance, exports of goods and service inflation for 2018, and consumer prices for 2019.

Table 4. Canada reel and prediction values, l value, and MAPE for the years 2018 and 2019.

Year	$f^{(m)}$	Economic Factors of Canada	M-DAM		LSTM		MLR
			l	MAPE (%)	l	MAPE (%)	MAPE (%)
2018	$f^{(1)}$	Current account balance (% of GDP)	2	25,67	4	0,60	8,99
	$f^{(2)}$	Exports of goods and serv. (% of GDP)	8	0,89	2	0,06	0,50
	$f^{(3)}$	GDP growth (annual %)	7	4,56	4	0,59	51,28
	$f^{(4)}$	Gross domestic savings (% of GDP)	2	0,64	2	0,13	1,61
	$f^{(5)}$	Gross fixed capital form. (% of GDP)	4	0,10	2	0,009	0,84
	$f^{(6)}$	Imports of goods and serv. (% of GDP)	7	0,39	3	0,002	0,55
	$f^{(7)}$	Inflation, consumer prices (annual %)	8	28,81	3	1,65	34,11
	$f^{(8)}$	Overnight interbank rate	5	40,53	2	7,31	44,69
	$f^{(9)}$	Unemployment, total	3	6,50	2	1,72	10,11
2019	$f^{(1)}$	Current account balance (% of GDP)	4	1,08	4	2,12	16,27
	$f^{(2)}$	Exports of goods and serv. (% of GDP)	5	0,48	2	0,01	0,72
	$f^{(3)}$	GDP growth (annual %)	6	2,16	4	1,33	31,21
	$f^{(4)}$	Gross domestic savings (% of GDP)	4	0,37	2	0,23	0,66
	$f^{(5)}$	Gross fixed capital form. (% of GDP)	3	0,48	2	0,002	1,15
	$f^{(6)}$	Imports of goods and serv. (% of GDP)	4	0,95	3	0,039	0,74
	$f^{(7)}$	Inflation, consumer prices (annual %)	6	2,56	3	1,36	1,15
	$f^{(8)}$	Overnight interbank rate	4	4,13	2	6,95	20,83
	$f^{(9)}$	Unemployment, total	7	6,12	2	19,11	11,30

In examining the economic indicators for France (see Table 5) using M-DAM, LSTM, and MLR models for 2018 and 2019, the performance comparison reveals contrasting strengths between the two methods. While LSTM generally presents lower MAPE values, signifying higher predictive accuracy, M-DAM showcases competitive results in certain points. M-DAM notably excels in predicting gross domestic savings and gross fixed capital formation for both 2018 and 2019, demonstrating MAPE values below 1%. Conversely, LSTM consistently outperforms M-DAM across various key indicators like current account balance, exports, inflation, and unemployment. LSTM outputs MAPE values ranging from 0.14% to 5.09% in 2018 and 0.14% to 4.29% in 2019. Despite LSTM's overall superior performance in most indicators, M-DAM proves to be particularly reliable in predicting certain economic factors with exceptionally low MAPE values. The results demonstrate M-DAM's strength in specific domains such as savings and capital formation. Besides, the MAPE results for MLR in the case of France are also examined in the same table. When MLR is considered, M-DAM outperforms MLR almost for all indicators of France in 2019. However, in 2018, factors including exports of goods and services, gross fixed capital formation, imports of goods and services, and total unemployment are better predicted by MLR.

Table 5. France reel and prediction values, $\boldsymbol{l}$ value, and MAPE for the years 2018 and 2019.

Year	$f^{(m)}$	Economic Factors of France	M-DAM		LSTM		MLR
			l	MAPE (%)	l	MAPE (%)	MAPE (%)
2018	$f^{(1)}$	Current account balance (% of GDP)	2	3,67	3	5,09	46,58
	$f^{(2)}$	Exports of goods and serv. (% of GDP)	7	0,24	2	3,36	0,08
	$f^{(3)}$	GDP growth (annual %)	2	3,65	3	0,78	10,80
	$f^{(4)}$	Gross domestic savings (% of GDP)	7	0,07	2	0,06	0,57
	$f^{(5)}$	Gross fixed capital form. (% of GDP)	6	0,57	2	2,45	1,53
	$f^{(6)}$	Imports of goods and serv. (% of GDP)	6	0,35	2	5,79	0,00
	$f^{(7)}$	Inflation, consumer prices (annual %)	6	39,45	4	14,36	32,58
	$f^{(8)}$	Overnight interbank rate	5	0,51	3	0,92	362,39
	$f^{(9)}$	Unemployment, total	6	5,33	4	0,14	4,37
2019	$f^{(1)}$	Current account balance (% of GDP)	2	10,57	3	1,41	12,45
	$f^{(2)}$	Exports of goods and serv. (% of GDP)	9	0,59	2	4,29	0,58
	$f^{(3)}$	GDP growth (annual %)	2	0,19	3	0,14	72,23
	$f^{(4)}$	Gross domestic savings (% of GDP)	2	0,01	2	0,32	0,30
	$f^{(5)}$	Gross fixed capital form. (% of GDP)	2	0,15	2	2,84	2,38
	$f^{(6)}$	Imports of goods and serv. (% of GDP)	8	0,5	2	5,43	0,88
	$f^{(7)}$	Inflation, consumer prices (annual %)	9	10,23	4	0,17	68,75
	$f^{(8)}$	Overnight interbank rate	7	0,89	3	2,68	294,27

While analyzing Germany as given in Table 6, specifically, for the gross domestic savings (% of GDP), M-DAM showcased a MAPE of 0.33% in 2018, while LSTM had a higher error rate of 0.51%. Additionally, regarding the overnight ınterbank rate, M-DAM predicted a MAPE of 0.21% in 2018, contrasting LSTM's higher error at 0.51%. In 2019, M-DAM continued its superior performance in forecasting, particularly evident in GDP growth (annual %). Here, M-DAM achieved a lower MAPE of 1.72%, while LSTM displayed a significantly higher error rate of 23.50%. Moreover, in forecasting inflation and, consumer prices (annual %), M-DAM once again outperformed LSTM with a lower MAPE of 0.17%, while LSTM had a higher MAPE of 0.78%. These results indicate the consistent effectiveness of M-DAM over LSTM in capturing and predicting key economic indicators for Germany across both years. Apart from the LSTM, the performance of the M-DAM surpasses MLR for almost all economical factors of Germany including 2018 and 2019.

Table 6. Germany reel and prediction values, l value, and MAPE for the years 2018 and 2019.

Year	$f^{(m)}$	Economic Factors of Germany	M-DAM		LSTM		MLR
			l	MAPE (%)	l	MAPE (%)	MAPE (%)
2018	$f^{(1)}$	Current account balance (% of GDP)	6	1,62	4	0,12	9,83
	$f^{(2)}$	Exports of goods and serv. (% of GDP)	5	0,43	4	0,009	1,37
	$f^{(3)}$	GDP growth (annual %)	8	80,86	4	0,33	277,34
	$f^{(4)}$	Gross domestic savings (% of GDP)	8	0,33	2	0,51	3,36
	$f^{(5)}$	Gross fixed capital form. (% of GDP)	3	2,11	3	0,06	0,21
	$f^{(6)}$	Imports of goods and serv. (% of GDP)	6	1,06	4	0,03	1,60
	$f^{(7)}$	Inflation, consumer prices (annual %)	6	9,12	2	0,99	6,87
	$f^{(8)}$	Overnight interbank rate	2	0,21	2	0,51	260,64
	$f^{(9)}$	Unemployment, total	7	6,38	4	1,95	21,66
2019	$f^{(1)}$	Current account balance (% of GDP)	4	4,24	4	0,48	5,34
	$f^{(2)}$	Exports of goods and serv. (% of GDP)	4	1,22	4	0,006	0,84
	$f^{(3)}$	GDP growth (annual %)	6	1,72	4	23,50	156,25
	$f^{(4)}$	Gross domestic savings (% of GDP)	7	0,76	2	0,04	2,28
	$f^{(5)}$	Gross fixed capital form. (% of GDP)	4	0,17	3	0,08	0,34
	$f^{(6)}$	Imports of goods and serv. (% of GDP)	6	0,03	4	0,03	0,98
	$f^{(7)}$	Inflation, consumer prices (annual %)	5	0,17	2	0,78	7,85
	$f^{(8)}$	Overnight interbank rate	7	2,56	2	1,79	133,35
	$f^{(9)}$	Unemployment, total	8	6,62	4	0,19	16,41

In 2018, M-DAM demonstrated lower MAPE values for several economic indicators in Italy compared to LSTM, as shown in Table 7. However, in 2019, the performance of M-DAM and LSTM varied. LSTM performed better in predicting certain indicators such as gross domestic savings, gross fixed capital formation, and GDP growth. Conversely, M-DAM exhibited superior performance for factors such as current account balance, imports and exports of goods and services, and inflation. Overall, LSTM displayed a lower error rate for most indicators in 2019, suggesting better predictive accuracy across these economic factors for Italy. In the case of MLR, again M-DAM and LSTM reveal better prediction results except for the factors, current account balance, and imports of goods and services in 2019.

Table 7. Italy reel and prediction values, l value, and MAPE for the years 2018 and 2019.

Year	$f^{(m)}$	Economic Factors of Italy	M-DAM		LSTM		MLR
			l	MAPE (%)	l	MAPE (%)	MAPE (%)
2018	$f^{(1)}$	Current account balance (% of GDP)	5	4,17	2	6,62	37,08
	$f^{(2)}$	Exports of goods and serv. (% of GDP)	7	0,12	2	4,97	0,81
	$f^{(3)}$	GDP growth (annual %)	3	71,74	2	1,15	223,14
	$f^{(4)}$	Gross domestic savings (% of GDP)	6	0,002	2	0,019	1,48
	$f^{(5)}$	Gross fixed capital form. (% of GDP)	6	0,58	3	0,13	0,60
	$f^{(6)}$	Imports of goods and serv. (% of GDP)	5	0,36	2	6,99	1,23
	$f^{(7)}$	Inflation, consumer prices (annual %)	7	22,09	3	6,14	62,59
	$f^{(8)}$	Overnight interbank rate	4	2,82	2	1,19	214,13
	$f^{(9)}$	Unemployment, total	6	1,39	3	0,07	8,74
2019	$f^{(1)}$	Current account balance (% of GDP)	7	14,68	2	35,59	6,11
	$f^{(2)}$	Exports of goods and serv. (% of GDP)	6	0,34	2	8,38	0,90
	$f^{(3)}$	GDP growth (annual %)	6	20,19	2	0,85	192,15
	$f^{(4)}$	Gross domestic savings (% of GDP)	2	0,33	2	0,15	1,62
	$f^{(5)}$	Gross fixed capital form. (% of GDP)	7	0,45	3	0,10	1,59
	$f^{(6)}$	Imports of goods and serv. (% of GDP)	7	2,36	2	5,92	0,97
	$f^{(7)}$	Inflation, consumer prices (annual %)	7	29,34	3	1,17	89,44
	$f^{(8)}$	Overnight interbank rate	7	6,14	2	21,72	202,25
	$f^{(9)}$	Unemployment, total	8	3,28	3	0,21	1,97

M-DAM and MLR exhibit weaker predictive performance in certain economic factors such as current account balance, exports, imports, inflation, and interbank rates. According to the Table 8, the results show M-DAM's promising improvements in predicting GDP growth and unemployment in Japan in 2019.

Table 8. Japan reel and prediction values, $\boldsymbol{l}$ value, and MAPE for the years 2018 and 2019.

Year	$f^{(m)}$	Economic Factors of Japan	M-DAM		LSTM		MLR
			l	MAPE (%)	l	MAPE (%)	MAPE (%)
2018	$f^{(1)}$	Current account balance (% of GDP)	4	1,24	2	0,45	4,72
	$f^{(2)}$	Exports of goods and serv. (% of GDP)	2	0,69	2	0,02	0,48
	$f^{(3)}$	GDP growth (annual %)	3	74,02	2	1,37	26,03
	$f^{(4)}$	Gross domestic savings (% of GDP)	8	0,003	3	0,13	0,39
	$f^{(5)}$	Gross fixed capital form. (% of GDP)	8	0,26	3	0,12	0,35
	$f^{(6)}$	Imports of goods and serv. (% of GDP)	2	0,98	4	0,04	0,53
	$f^{(7)}$	Inflation, consumer prices (annual %)	2	14,75	3	0,05	29,93
	$f^{(8)}$	Overnight interbank rate	7	41,90	7.	39,12	137,14
	$f^{(9)}$	Unemployment, total	2	20,76	2	9,06	12,49
2019	$f^{(1)}$	Current account balance (% of GDP)	3	4,12	2	0,24	0,54
	$f^{(2)}$	Exports of goods and serv. (% of GDP)	6	5,00	2	0,10	0,22
	$f^{(3)}$	GDP growth (annual %)	6	10,01	2	2,35	383,73
	$f^{(4)}$	Gross domestic savings (% of GDP)	8	0,09	3	0,02	0,48
	$f^{(5)}$	Gross fixed capital form. (% of GDP)	6	1,19	3	0,19	0,60
	$f^{(6)}$	Imports of goods and serv. (% of GDP)	2	4,16	4	0,05	0,21
	$f^{(7)}$	Inflation, consumer prices (annual %)	2	61,50	3	0,86	5,36
	$f^{(8)}$	Overnight interbank rate	5	5,08	7.	2,36	64,94
	$f^{(9)}$	Unemployment, total	7	4,70	2	1,24	10,14

For Russia's economic indicators, M-DAM and LSTM models were compared across various factors (Table 9). Notably, M-DAM showed superiority in predicting GDP growth with an accuracy of 0.06% compared to LSTM's 0.47%. However, this outcome wasn't consistent across all indicators. For instance, in forecasting the current account balance, M-DAM recorded a MAPE of 54.85% compared to LSTM's 2.31%. Similarly, in forecasting gross fixed capital formulation (2018), M-DAM recorded a MAPE of 0.08% compared to MLR's 0.89%.

As provided in Table 10 standing for Turkey's economic indicators, the comparison between M-DAM and LSTM models across various factors indicates a diverse performance between the two methods. In terms of predicting the current account balance as a percentage of GDP, M-DAM shows a significantly higher MAPE of 12.98% compared to LSTM's 0.53%. Similarly, M-DAM demonstrates higher MAPE values in predicting exports, GDP growth, gross domestic savings, gross fixed capital formation, imports, inflation, overnight interbank rates, and total unemployment compared to LSTM in both 2018 and 2019. However, LSTM outperforms M-DAM in predicting GDP growth in 2019, displaying a lower MAPE of 3.13% compared to M-DAM's 80.54%. When the MLR's performance is considered, MLR gives better results among all three approaches in the economical factors in 2018: exports of goods and services, gross domestic savings, and besides economic factors in 2019: current account balance.

Table 9. Russia reel and prediction values, l value, and MAPE for the years 2018 and 2019.

Year	$f^{(m)}$	Economic Factors of Russia	M-DAM		LSTM		MLR
			l	MAPE (%)	l	MAPE (%)	MAPE (%)
2018	$f^{(1)}$	Current account balance (% of GDP)	2	54,85	3	2,31	7,01
	$f^{(2)}$	Exports of goods and serv. (% of GDP)	3	14,28	2	0,85	3,25
	$f^{(3)}$	GDP growth (annual %)	5	0,06	4	0,47	176,22
	$f^{(4)}$	Gross domestic savings (% of GDP)	8	7,23	3	0,61	6,84
	$f^{(5)}$	Gross fixed capital form. (% of GDP)	2	0,17	2	0,08	0,89
	$f^{(6)}$	Imports of goods and serv. (% of GDP)	3	0,96	2	0,12	4,02
	$f^{(7)}$	Inflation, consumer prices (annual %)	5	30,81	6	4,37	349,87
	$f^{(8)}$	Overnight interbank rate	2	15,74	4	0,13	123,43
	$f^{(9)}$	Unemployment, total	7	2,66	2	2,66	54,82
2019	$f^{(1)}$	Current account balance (% of GDP)	8	1,76	3	0,62	18,10
	$f^{(2)}$	Exports of goods and serv. (% of GDP)	5	0,96	2	0,01	2,40
	$f^{(3)}$	GDP growth (annual %)	6	32,36	4	6,45	119,88
	$f^{(4)}$	Gross domestic savings (% of GDP)	3	1,32	3	0,03	3,25
	$f^{(5)}$	Gross fixed capital form. (% of GDP)	2	0,39	2	0,006	2,69
	$f^{(6)}$	Imports of goods and serv. (% of GDP)	4	0,39	2	0,09	1,21
	$f^{(7)}$	Inflation, consumer prices (annual %)	2	31,86	6	1,21	60,36
	$f^{(8)}$	Overnight interbank rate	6	16,39	4	0,13	60,59
	$f^{(9)}$	Unemployment, total	8	1,73	2	0,53	9,55

Table 10. Turkey reel and prediction values, $\boldsymbol{l}$ value, and MAPE for the years 2018 and 2019.

Year	$f^{(m)}$	Economic Factors of Turkey	M-DAM		LSTM		MLR
			l	MAPE (%)	l	MAPE (%)	MAPE (%)
2018	$f^{(1)}$	Current account balance (% of GDP)	2	12,98	3	0,53	39,02
	$f^{(2)}$	Exports of goods and serv. (% of GDP)	7	14,89	2	22,19	3,65
	$f^{(3)}$	GDP growth (annual %)	2	27,14	2	0,29	96,85
	$f^{(4)}$	Gross domestic savings (% of GDP)	7	5,41	3	3,88	0,69
	$f^{(5)}$	Gross fixed capital form. (% of GDP)	5	0,02	2	1,99	1,97
	$f^{(6)}$	Imports of goods and serv. (% of GDP)	8	1,30	4	8,82	3,26
	$f^{(7)}$	Inflation, consumer prices (annual %)	3	38,50	3	4,31	24,31
	$f^{(8)}$	Overnight interbank rate	7	66,51	4	13,76	318,18
	$f^{(9)}$	Unemployment, total	2	0,30	2	0,15	11,51
2019	$f^{(1)}$	Current account balance (% of GDP)	4	274,17	3	185,76	103,47
	$f^{(2)}$	Exports of goods and serv. (% of GDP)	2	0,16	2	23,04	2,26
	$f^{(3)}$	GDP growth (annual %)	8	80,54	2	3,13	823,69
	$f^{(4)}$	Gross domestic savings (% of GDP)	9	3,28	3	0,01	2,27
	$f^{(5)}$	Gross fixed capital form. (% of GDP)	8	16,07	2	0,01	4,33
	$f^{(6)}$	Imports of goods and serv. (% of GDP)	8	0,40	4	4,56	2,95
	$f^{(7)}$	Inflation, consumer prices (annual %)	5	2,80	3	6,23	52,96
	$f^{(8)}$	Overnight interbank rate	7	31,05	4	8,72	486,54
	$f^{(9)}$	Unemployment, total	2	11,45	2	0,68	3,88

In 2018, LSTM demonstrated lower MAPE values for several economic indicators in UK compared to M-DAM, as shown in Table 11. However, in 2019, the performance of M-DAM and LSTM varied. LSTM performed better in predicting certain indicators such as current account balance, gross domestic savings, overnight interbank rate, unemployment and GDP growth. Conversely, M-DAM exhibited superior performance for factors such as imports and exports of goods and services. MLR shows strong performance in certain cases but is less consistent, particularly in 2019.

Table 11. United Kingdom reel and prediction values, $\boldsymbol{l}$ value, and MAPE for the years 2018 and 2019.

Year	$f^{(m)}$	Economic Factors of the UK	M-DAM		LSTM		MLR
			l	MAPE (%)	l	MAPE (%)	MAPE (%)
2018	$f^{(1)}$	Current account balance (% of GDP)	2	12,98	3	0,53	17,33
	$f^{(2)}$	Exports of goods and serv. (% of GDP)	7	14,89	2	22,19	0,07
	$f^{(3)}$	GDP growth (annual %)	2	27,14	2	0,29	58,90
	$f^{(4)}$	Gross domestic savings (% of GDP)	7	5,41	3	3,88	0,42
	$f^{(5)}$	Gross fixed capital form. (% of GDP)	5	0,02	2	1,99	0,98
	$f^{(6)}$	Imports of goods and serv. (% of GDP)	8	1,30	4	8,82	0,28
	$f^{(7)}$	Inflation, consumer prices (annual %)	3	38,50	3	4,31	22,28
	$f^{(8)}$	Overnight interbank rate	7	66,51	4	13,76	145,96
	$f^{(9)}$	Unemployment, total	2	0,30	2	0,15	18,51
2019	$f^{(1)}$	Current account balance (% of GDP)	4	274,17	3	185,76	19,80
	$f^{(2)}$	Exports of goods and serv. (% of GDP)	2	0,16	2	23,04	0,16
	$f^{(3)}$	GDP growth (annual %)	8	80,54	2	3,13	40,78
	$f^{(4)}$	Gross domestic savings (% of GDP)	9	3,28	3	0,01	0,30
	$f^{(5)}$	Gross fixed capital form. (% of GDP)	8	16,07	2	0,01	0,80
	$f^{(6)}$	Imports of goods and serv. (% of GDP)	8	0,40	4	4,56	0,01
	$f^{(7)}$	Inflation, consumer prices (annual %)	5	2,80	3	6,23	29,75
	$f^{(8)}$	Overnight interbank rate	7	31,05	4	8,72	118,36
	$f^{(9)}$	Unemployment, total	2	11,45	2	0,68	15,07

In the USA economic dataset, (Table 12), M-DAM demonstrates superior predictive accuracy with lower MAPE values in crucial aspects such as current account balance (% of GDP). In 2018, M-DAM showcased a notably lower MAPE value of 5.02% compared to LSTM's 14.15%. Similarly, in 2019, M-DAM continued this trend, revealing a significantly lower MAPE value of 0.48% compared to LSTM's 7.91% for the same indicator. Additionally, regarding unemployment, total, M-DAM consistently displayed lower MAPE values of 2.66% in 2018 and 4.68% in 2019 against LSTM's 8.10% and 13.63%, respectively. These consistently lower MAPE values highlight M-DAM's superior predictive accuracy in forecasting these critical economic indicators in the USA context. Among the three methods, MLR outperforms higher MAPE values for the following factors: current account balance, 2018 and 2019 gross domestic savings, and exports of goods and services.

Table 12. United States of America reel and prediction values, $\boldsymbol{l}$ value, and mape for the years 2018 and 2019.

Year	$f^{(m)}$	Economic Factors of USA	M-DAM		LSTM		MLR
			l	MAPE (%)	l	MAPE (%)	MAPE (%)
2018	$f^{(1)}$	Current account balance (% of GDP)	3	5,02	2	14,15	0,51
	$f^{(2)}$	Exports of goods and serv. (% of GDP)	7	0,17	4	0,28	0,09
	$f^{(3)}$	GDP growth (annual %)	3	3,33	2	1,20	4,17
	$f^{(4)}$	Gross domestic savings (% of GDP)	3	0,30	4	0,03	0,23
	$f^{(5)}$	Gross fixed capital form. (% of GDP)	8	0,87	2	0,03	0,35
	$f^{(6)}$	Imports of goods and serv. (% of GDP)	7	0,48	4	0,003	0,12
	$f^{(7)}$	Inflation, consumer prices (annual %)	5	2,42	2	0,03	18,00
	$f^{(8)}$	Overnight interbank rate	5	35,61	4	0,27	50,75
	$f^{(9)}$	Unemployment, total	4	2,66	4	8,10	19,08
2019	$f^{(1)}$	Current account balance (% of GDP)	8	0,48	2	7,91	2,78
	$f^{(2)}$	Exports of goods and serv. (% of GDP)	3	3,17	4	0,09	0,14
	$f^{(3)}$	GDP growth (annual %)	4	9,34	2	0,60	7,18
	$f^{(4)}$	Gross domestic savings (% of GDP)	2	0,64	4	0,36	0,32
	$f^{(5)}$	Gross fixed capital form. (% of GDP)	4	0,31	2	0,07	0,64
	$f^{(6)}$	Imports of goods and serv. (% of GDP)	3	3,04	4	0,01	0,13
	$f^{(7)}$	Inflation, consumer prices (annual %)	8	33,88	2	0,15	19,35
	$f^{(8)}$	Overnight interbank rate	3	45,44	4	0,69	6,18
	$f^{(9)}$	Unemployment, total	6	4,68	4	13,63	11,17

5. Conclusions

The current study aimed to model the data and make future predictions. Therefore, differential equations obtained by modeled data were generalized using a fractional derivative method. A novel and flexible approach, M-DAM was proposed. The dataset was modeled with the least margin of error with the least squares method, and other necessary goals were examined afterward.

In summary, first, a new mathematical model is proposed for the literature, by developing a multi-input deep assessment system via FC. This mathematical model is coded on MATLAB. As for application, current account balance (% of GDP), exports of goods and services (% of GDP), GDP growth (annual %), gross domestic savings (% of GDP), gross fixed capital formation (% of GDP), imports of goods and services (% of GDP), inflation, consumer prices (annual %), overnight interbank rate, and unemployment (total) data between 2000–2019 for Turkey and G-8 countries were chosen. The data was modeled and prediction studies were conducted and assessed.

As shown by the M-DAM modeling results in Table 2, modeling can be performed with an error lower than 0.0004%. Accurate results were obtained for modeling economic factors in the current study, such that the maximum error for the 81-factor model was found to be 0.0003112%, for France's inflation rate. Table 3 shows the MLR modeling results. According to the modeling results M-DAM outperforms in all factors for all countries. Regarding prediction Tables 4–12 show that, 75.4% of the errors were smaller than 10%, while only 24.6% of the errors were higher than 10% for a total of 162 predictions. Moreover, the most predictable country was found to be Germany. Following this, in the second position were Canada, France, the UK, and the USA. In the third position were Italy and Japan. In the fourth position was Russia, and lastly, Turkey was identified as the least predictable for both 2018 and 2019.

Across diverse economic indicators of different countries, the comparative analysis between M-DAM and LSTM models reveals a nuanced narrative. While LSTM often produces lower MAPE values, M-DAM showcases strength in specific points. M-DAM showcases strength in specific areas. M-DAM excels in predicting indicators like gross domestic savings and gross fixed capital formation in France and Germany, exhibiting MAPE values below 1%. However, LSTM frequently outperforms M-DAM in critical indicators such as current account balance, exports, inflation, and unemployment, indicating its robust forecasting capability. The USA's economic dataset highlights M-DAM's superior accuracy in predicting "current account balance (% of GDP)" and "unemployment, total" with notably lower MAPE values across both 2018 and 2019 compared to LSTM. This comparison emphasizes the need for a nuanced approach, where each model demonstrates strengths in particular economic indicators, contributing to a comprehensive and reliable predictive analysis of various countries' economies.

Our study shows that while LSTM often performs better than M-DAM in some cases, M-DAM works well even with less data. We use M-DAM to create monthly data, which improves LSTM's performance. So, even though LSTM may do better in several cases, it's important to recognize that M-DAM forms the basis of the data, highlighting its importance in economic predictions. When comparing the three approaches, each method has its own strengths and weaknesses for different economic factors. Therefore, using multiple methods in the analysis and prediction of economic factors is a good way to ensure reliable results.

Comparing MLR, LSTM, and the M-DAM shows that each method has its pros and cons depending on the context. LSTM generally did better for many indicators, especially for the UK and Italy in 2018, showing its ability to handle complex data. M-DAM excelled in predicting GDP growth and unemployment rates for Japan and Italy in 2019 because it can deeply assess many factors. Although MLR was often outperformed by the other models, it resulted in satisfactory results in specific cases, such as predicting exports, domestic savings, and imports for Turkey in 2018, and the current account balance in 2019. This shows the importance of choosing the right model based on the specific economic factor and context, as each method has unique strengths that can be useful in different scenarios.

For future studies, the addition of function derivatives would be an improvement to our novel modeling approach, the M-DAM model. While the current study has assessed each country by itself, the future study will explore the effects of economic factors of one country on other nations, and the degree to which a change in one country can affect another.

Author contributions

Ertuğrul Karaçuha: supervision, conceptualization, investigation, methodology, administration; Vasil Tabatadze: resources, supervision, validation; Nisa Özge Önal Tuğrul: investigation, administration, validation, writing; Esra Ergün: validation, writing, and editing; Kamil K. supported conceptualization, writing, editing. All authors have read and approved the final version of the manuscript for publication.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.

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