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Research article

Complex interval-value intuitionistic fuzzy sets: Quaternion number representation, correlation coefficient and applications

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Abstract: Complex interval-valued intuitionistic fuzzy sets not only consider uncertainty and periodicity semantics at the same time but also choose to express the information value with an interval value to give experts more freedom and make the solution to the problem more reasonable. In this study, we used the interval quaternion number space to generalize and extend the utility of complex interval-valued intuitionistic fuzzy sets, analyze their order relation, and offer new operations based on interval quaternion numbers. We proposed a new score function and correlation coefficient under interval quaternion representation. We applied the interval quaternion representation and correlation coefficient to a multi-criterion decision making model and applied the model to enterprise decision-making.

Keywords: complex interval-valued intuitionistic fuzzy sets; interval quaternion number; score function; correlation coefficient; multi-criteria decision making
Mathematics Subject Classification: 08A72, 26E50, 03E72

1. Introduction

A classical set has unambiguous boundaries, i.e., $a \in M$ or $a \notin M$ excludes any other possibility. However, there is a lot of vague and imprecise information in real life. Therefore, as an extension of the classical set, Zadeh [1] introduced the concept of fuzzy sets (FS). By adding the nonmembership part to the notion of a fuzzy set, Atanassov [2] proposed the theory of intuitionistic fuzzy sets (IFSs) in 1986. It takes into account information about membership degree, nonmembership degree, and hesitation degree simultaneously and is more adaptable to simulate imprecision and ambiguity phenomena. For IFS theory, Wan et al. [3] proposed a new intuitionistic fuzzy best-worst method for group decision-making with intuitionistic fuzzy preference relations. Interval-valued intuitionistic fuzzy sets (IVIFSs) as extensions of IFSs were proposed by Atanassov et al. [4, 5] in 1989. Dong et al. [6–8] proposed some new methods on IVIFSs, such as a new consistency definition for interval-valued intuitionistic multiplicative preference relations and a new concept for type-2 IVIFSs, which makes the IVIFSs much more abundant. In recent years, scholars have paid great attention to various aggregation operators of IVIFSs [9–14]. Furthermore, IVIFSs have shown extensive application value in decision-making [15–18] and pattern recognition [19–21].

Complex fuzzy sets (CFSs) were first introduced by Ramot et al. in 2002 [22]. CFSs are a suitable mathematical instrument that can simultaneously describe the information's periodicity and uncertainty semantics, which results in reduced data loss and more rational solutions to problems. In 2023, Gong and Wang [23] initiated (r, θ)-cut sets of CFSs, as well as decomposition theorems and extension principles. It enriches the CFS. In 2012, Alkouri et al. [24, 25] introduced complex intuitionistic fuzzy sets (CIFSs) as a combination of CFSs and IFSs. In this case, the complex-valued membership function $\mu_A(x)$ and the nonmembership function $\nu_A(x)$ are represented in the following form, respectively, $\mu_A(x) = r_A(x)e^{i\omega_{r_A}(x)}, \nu_A(x) = k_A(x)e^{i\omega_{k_A}(x)}, i^2 = -1$, $r_A(x), k_A(x)$ are real-valued functions, and $r_A(x), k_A(x) \in [0, 1]$ such that $0 \leq r_A(x) + k_A(x) \leq 1$, $\omega_{r_A}(x), \omega_{k_A}(x)$ are real-valued functions. For CIFSs, Gong and Wang [26] defined and studied the (α, β)-equalities in 2023. In many application scenarios, people often face the challenge of understanding the nature of things, especially when making decisions and evaluations. In order to ensure the scientificity and rationality of decision-making, reduce the loss of information, and reduce the volatility and fuzziness of decision-making information, people usually use interval values to express the characteristics that need to be considered when making judgments. Therefore, Garg and Rani [27] proposed complex interval-valued intuitionistic fuzzy sets (CIVIFSs) in 2019. This kind of CIVIFS has high practicability. For example, one company needs to install biometric-based attendance devices at branches across the country. The company plans to buy from suppliers that offer both the model number of the attendance equipment and the production date, with the task of selecting the best model of the equipment that is synchronized with the production date. Enterprises can choose to use CIVIFSs to deal with these two aspects of information provided by suppliers. The amplitude term represents the decision information of the equipment model, and the phase term represents the decision information of the production date.

Recently, Tamir et al. [28] introduced the complex number representation of IFSs and obtained significant properties. For the first time, Ngan et al. [29] introduced the quaternion representation of complex intuitionistic fuzzy sets (CIFS-Q). Pan et al. [30] presented a quaternion model of Pythagorean fuzzy sets and its distance measure. The quaternion number theory was initially put forth by William Rowan Hamilton in 1837. The expression quaternion refers to elements of the type $\alpha = a + bi + cj + dk$, where the coefficients a, b, c, and d are real numbers and the symbols i, j, and k are formal symbols termed as fundamental units. The fundamental tenets of the product of quaternions are shown in Figure 1. Figure 2 can help you remember the quaternion operations, which are multiplying in a clockwise direction to get $ij = k$ and in a counterclockwise direction to produce $ji = -k$. The properties of the interval quaternion number were first introduced by Moura et al. [31]. Interval quaternion numbers are elements of the form $\mathbb{I}(\mathbb{H}) = A + iB + jC + kD$, where A, B, C, and $D \in \mathbb{I}(\mathbb{R})$. Combining CIVIFS and interval quaternion theory, we proposed an interval quaternion representation of complex interval-valued intuitionistic fuzzy sets (CIVIFS-IQ). Quaternion number and interval quaternion number representations not only extend the description space of fuzzy information to a four-dimensional space but also make the transmission of fuzzy information more intuitive and effective. Furthermore, the interval quaternion number representation can provide experts with more flexibility and a greater degree of information loss reduction. In particular, we obtain the degree interval of real membership, imaginary membership, real nonmembership, and imaginary nonmembership by combining the degree interval of complex membership with complex nonmembership using the notion of an interval quaternion number. For example, suppose a business decides to install new data processing and analytics software. To this end, the company consulted experts who provided information on (a) different alternatives to the software and (b) versions of the corresponding software. The company wants to choose both the best software alternative and the best version. The CIVIFS-IQ theory can be chosen here to express the information provided by the experts. The real membership and real nonmembership of CIVIFS-IQ can be used to give the company's decision about software alternatives, and the imaginary membership and imaginary nonmembership can be used to represent the company's decision about software versions.

H	1	i	j	k
1	1	i	j	k
i	i	-1	k	-j
j	j	-k	-1	i
k	k	j	-i	-1

Figure 1. The fundamental tenets of the product of quaternion.

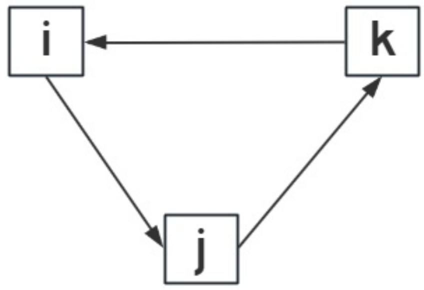

Figure 2. Schematic diagram of the basic principle of quaternion product.

One significant component of decision theory is multi-criteria decision making (MCDM). The main problem with the MCDM approach is determining how to effectively assess the alternatives under the multiple criteria and choose the best alternatives from the alternative set. However, due to the lack of knowledge and uncertainty of information, decision-makers often face difficulties under standard assessment. Therefore, decision-makers assess alternatives in accordance with the criteria using techniques for uncertainty and ambiguity. In the process of MCDM, one of the best strategies to evaluate alternatives is to use CIVIFSs. At present, we still need to figure out how to compare MCDM described by CIVIFSs. A well-designed ranking system has the power to influence decision-makers' choices. In order to address the corporate decision problem, Garg and Rani [27] created a few new operators using CIVIF information. In order to handle the evaluation in a CIVIF environment, Zindani et al. [32] presented the CIVIFS-TODIM (an acronym in Portuguese for interactive and multi-criteria decision making). Khan et al. [33] proposed applying the Aczel-Alsina operator of CIVIFS to solve the decision-making problem of medical institutions in public hospitals. Although there are various sorting methods, no sorting method based on the correlation coefficient of CIVIFS has been found. On the other hand, the correlation coefficient can be used to transfer information between CIVIFSs. Therefore, we established correlation coefficients based on CIVIFS and applied them to MCDM studies.

In summary, CIVIFS contains features from both CIFS and IVIFS. CIVIFS membership and nonmembership degrees are complex-valued and are represented in polar coordinates. The degree of belongings or non-belongings of an object in a CIVIFS is indicated by the amplitude term corresponding to the membership (nonmembership) degree, and extra information, typically linked to periodicity, is indicated by the phase term connected with the membership (nonmembership) degree. In some cases, experts involved in solving problems in the field of decision making prefer to give approximate ranges rather than clear values. In this case, CIFS cannot be used to represent decision information, but CIVIFS is a better choice because it uses interval values to represent decision information, thus giving experts more freedom and avoiding information loss to a certain extent. While CIVIFS uses two-dimensional information to express ambiguity and uncertainty, IVIFS employs single-dimensional information. However, in the actual world, using two-dimensional information to explain ambiguity and uncertainty is easier and more effective. The correlation coefficient is now one of the most significant metrics that can be used to compare two data entities as well as determine the degree and direction of their link. We are aware that the correlation mesure in the previously stated studies are unable to handle the CIVIFS data. Motivated by this, the following are this paper's primary contributions:

- CIVIFSs are represented by interval quaternion numbers. A compact representation of a CIVIFS is provided. Order relations, set theory operations, and some other operations are defined.
- In the context of CIVIFSs-IQ, a new scoring function is proposed that can overcome the shortcomings of other scoring functions to a certain extent, be more loyal to the decision information, and more truly reflect the will of decision makers.
- In the CIVIFSs-IQ environment, the correlation coefficient and the MCDM method using the correlation coefficient are proposed, and the proposed method is applied to enterprise decision-making.

The rest of the text is as follows: In Section 2, we will briefly review some basic concepts and operations. In Section 3, we introduce a new CIVIFS representation based on interval quaternion (CIVIFS-IQ), which defines order relations and some operations. In Section 4, we present a new score function and correlation coefficient in the CIVIFS-IQ environment. In Section 5, we introduce a new MCDM model based on our proposed correlation coefficients in the CIVIFS-IQ environment and apply them to the enterprise decision problem, and, finally, the conclusions are discussed in Section 6.

2. Preliminaries

In this section, we recall basic definitions and related notions used in the paper.

Let X be a universe of discourse. A FS A in X is defined as $A = \{< x, \mu_A(x) > |x \in X\}$, where $\mu_A(x) \in [0, 1]$ is real-valued. For each $x \in X$, the value $\mu_A(x)$ represents the degree of membership of x in the FS A. An IFS B in X is defined as $B = \{< x, \mu_B(x), \nu_B(x) > |x \in X\}$, where $\mu_B(x)$ and $\nu_B(x)$ are the degrees of membership and nonmembership, respectively, such that $0 \leq \mu_B(x) + \nu_B(x) \leq 1$ for each $x \in X$.

Definition 2.1. [5] Let X be a nonempty finite set. An IVIFS A in X is expressed by

$$A = \{< x, [u_A^-(x), u_A^+(x)], [\upsilon_A^-(x), \upsilon_A^+(x)] > |x \in X\},$$

where $[u_A^-(x), u_A^+(x)] \subseteq [0, 1]$ and $[\upsilon_A^-(x), \upsilon_A^+(x)] \subseteq [0, 1]$, respectively, denote the interval-valued degrees of membership and nonmembership of the element $x \in X$ with the condition $u_A^+(x) + \upsilon_A^+(x) \leq 1$.

Definition 2.2. [22] Let X be a universe of discourse. A CFS A over X is formed by

$$A = \{\langle x, \eta_A(x)\rangle : x \in X\},$$

where the complex-valued membership function $\eta_A(x)$ has the form $r_A(x)e^{i\omega_{r_A}(x)}$, where $i = \sqrt{-1}$, $r_A(x) \in [0, 1]$, and $\omega_{r_A}(x)$ is real-valued. The value of $\eta_A(x)$ lies in a unit circle in the complex plane.

Definition 2.3. [28] An IFS A over a universe X is characterized by the complex number function $z = \mu + i\nu$, where $\mu, \nu : X \to [0, 1]$ satisfying $\mu + \nu \in [0, 1]$ are the functions of membership and nonmembership, respectively. As a set of ordered pairs, the IFS A can be represented as the set of ordered pairs

$$A = \{(x, z) \mid x \in U, z = \mu(x) + i\nu(x)\}.$$

Definition 2.4. [29] Let X be a space, F_Q is the CIFS-Q on X with the quaternion function $Q = \alpha + i\beta + j\omega + k\gamma$, where i, j, k are complex roots, $i^2 = j^2 = k^2 = ijk = -1$,

$$F_Q = \{(x, Q(x)) : x \in X\},$$

for all $x \in X$, and the functions α, β, ω, and γ satisfy the following condition: $\alpha(x), \beta(x), \omega(x)$, and $\gamma(x) \in [0, 1], \alpha(x) + \beta(x) \leq 1, \omega(x) + \gamma(x) \leq 1, \alpha(x) + \omega(x) \leq 1, \beta(x) + \gamma(x) \leq 1$.

Definition 2.5. [27] Let X be a universe of discourse. A CIVIFS defined on X is a set given by

$$A = \{(x, [\mu_A^-(x), \mu_A^+(x)], [\nu_A^-(x), \nu_A^+(x)]) : x \in X\},$$

where $\mu_A^-(x), \mu_A^+(x)$ and $\nu_A^-(x), \nu_A^+(x)$ represent the degrees of lower and upper bound of the membership and nonmembership, respectively, which are defined as $\mu_A^-(x) = z_1^- = r_A^- e^{i\omega_{r_A}^-(x)}$ and $\mu_A^+(x) = z_1^+ = r_A^+ e^{i\omega_{r_A}^+(x)}$ such that$\mid z_1^- \mid\leq\mid z_1^+ \mid$, while $\nu_A^-(x) = z_2^- = k_A^-(x)e^{i\omega_{k_A}^-(x)}$ and $\nu_A^+(x) = z_2^+ = k_A^+(x)e^{i\omega_{k_A}^+(x)}$ be such that $\mid z_2^- \mid\leq\mid z_2^+ \mid$. The amplitude terms are $r_A^-, r_A^+, k_A^-, k_A^+ \in [0, 1]$ and satisfy the inequality $r_A^- \leq r_A^+, k_A^- \leq k_A^+$ and $r_A^+ + k_A^+ \leq 1, \forall x \in U$. On the other hand, the phase terms $\omega_{r_A}^-, \omega_{r_A}^+, \omega_{k_A}^-, \omega_{k_A}^+$ are real-valued which lie within the interval $[0, 2\pi]$ and satisfy the inequality $\omega_{r_A}^- \leq \omega_{r_A}^+, \omega_{k_A}^- \leq \omega_{k_A}^+$ and $\omega_{r_A}^+ + \omega_{k_A}^+ \leq 2\pi, \forall x \in X$. Therefore, mathematically, CIVIFS A defined on X can be represented as

$$A = \{(x, [r_A^-, r_A^+]e^{i[\omega_{r_A}^-, \omega_{r_A}^+]}, [k_A^-, k_A^+]e^{i[\omega_{k_A}^-, \omega_{k_A}^+]}\}.$$

Consider the set of closed intervals $\mathbb{I}(\mathbb{R}) = \{[a, b] : a \leq b, a, b \in \mathbb{R}\}$ endowed with the following arithmetic:
(1) $[a_1, b_1] + [a_2, b_2] = [a_1 + a_2, b_1 + b_2]$;
(2) $[a_1, b_1] - [a_2, b_2] = [a_1 - b_2, b_1 - a_2]$;
(3) $[a_1, b_1] \cdot [a_2, b_2] = [\min\{a_1a_2, a_1b_2, b_1a_2, b_1b_2\}, \max\{a_1a_2, a_1b_2, b_1a_2, b_1b_2\}]$;
(4) $[a_1, b_1]^{-1} = \frac{1}{[a_1,b_1]} = [\frac{1}{b_1}, \frac{1}{a_1}]$, if $0 \notin [a_1, b_1]$;
(5) $\frac{[a_2,b_2]}{[a_1,b_1]} = [a_2, b_2] \cdot [a_1, b_1]^{-1}$.

Definition 2.6. [34] For $a_t \in \mathbb{R}, t \in T, [a_t, b_t] \in \mathbb{I}(\mathbb{R})$, where $\bigvee\limits_{t\in T} a_t = \sup\{a_t : t \in T\}$, $\bigwedge\limits_{t\in T} a_t = \inf\{a_t : t \in T\}$. Then, there is

(1) $\bigvee_{t\in T}[a_t, b_t] = [\bigvee_{t\in T} a_t, \bigvee_{t\in T} b_t]$, $\bigwedge_{t\in T}[a_t, b_t] = [\bigwedge_{t\in T} a_t, \bigwedge_{t\in T} b_t]$;
(2) $[a_t, b_t]' = [b_t', a_t'] = [1 - b_t, 1 - a_t]$;
(3) $[a_1, a_2] = [b_1, b_2] \Leftrightarrow a_1 = b_1, a_2 = b_2$;
(4) $[a_1, a_2] \le [b_1, b_2] \Leftrightarrow a_1 \le b_1, a_2 \le b_2$.

Definition 2.7. [27] Let $A = \{x, [r_A^-(x), r_A^+(x)]e^{i[\omega_{r_A}^-(x), \omega_{r_A}^+(x)]}, [k_A^-(x), k_A^+(x)]e^{i[\omega_{k_A}^-(x), \omega_{k_A}^+(x)]}\}$ and $B = \{x, [r_B^-(x), r_B^+(x)]e^{i[\omega_{r_B}^-(x), \omega_{r_B}^+(x)]}[k_B^-(x), k_B^+(x)]e^{i[\omega_{k_B}^-(x), \omega_{k_B}^+(x)]}\}$ be any two CIVIFSs in X, then
(1) $A \subseteq B$ if, and only if, $r_A^-(x) \le r_B^-(x)$, $r_A^+(x) \le r_B^+(x)$, $\omega_{r_A}^-(x) \le \omega_{r_B}^-(x)$, $\omega_{r_A}^+(x) \le \omega_{r_B}^+(x)$, $k_A^-(x) \ge k_B^-(x)$, $k_A^+(x) \ge k_B^+(x)$, $\omega_{k_A}^-(x) \ge \omega_{k_B}^-(x)$, $\omega_{k_A}^+(x) \ge \omega_{k_B}^+(x)$.
(2) $A = B$ if, and only if, $A \subseteq B$ and $B \supseteq A$.
(3) $A^c = \{< x, [k_A^-(x), k_A^+(x)]e^{i[\omega_{k_A}^-(x), \omega_{k_A}^+(x)]}, [r_A^-(x), r_A^+(x)]e^{i[\omega_{r_A}^-(x), \omega_{r_A}^+(x)]} > | x \in X\}$.

3. Representing CIVIFS by interval quaternion

Definition 3.1. Let $\ddot{U}$ be a space, $\ddot{I}_{IQ}$ is the CIVIFS-IQ on $\ddot{U}$ with the interval quaternion function $IQ = \ddot{A} + i\ddot{B} + j\ddot{C} + k\ddot{D}$, where i, j, k are complex roots, $i^2 = j^2 = k^2 = ijk = -1$, $\ddot{A}, \ddot{B}, \ddot{C}, \ddot{D} \in \mathbb{I}(\mathbb{R})$, $\ddot{A} = [\ddot{A}^-, \ddot{A}^+]$, $\ddot{B} = [\ddot{B}^-, \ddot{B}^+]$, $\ddot{C} = [\ddot{C}^-, \ddot{C}^+]$, $\ddot{D} = [\ddot{D}^-, \ddot{D}^+]$. Here $\ddot{A}, \ddot{B}, \ddot{C}$, and $\ddot{D}$ are the interval function of real membership, imaginary membership, real nonmembership, and imaginary nonmembership, respectively. For each $x \in \ddot{U}$, the interval functions $\ddot{A}, \ddot{B}, \ddot{C}$, and $\ddot{D}$ satisfy the following conditions:
(1) $\ddot{A}(x), \ddot{B}(x), C\ddot{(}x), \ddot{D}(x) \subseteq [0, 1]$;
(2) $[\ddot{A}^-(x) + \ddot{C}^-(x), \ddot{A}^+(x) + \ddot{C}^+(x)] \subseteq [0, 1]$;
(3) $[\ddot{B}^-(x) + \ddot{D}^-(x), \ddot{B}^+(x) + \ddot{D}^+(x)] \subseteq [0, 1]$,

where values $\ddot{A}(x), \ddot{B}(x), \ddot{C}(x)$, and $\ddot{D}(x)$ are the degree interval of real membership, imaginary membership, real nonmembership and imaginary nonmembership, respectively, of x in $\ddot{I}_{IQ}$. Then, the CIVIFS $\ddot{I}_{IQ}$ represented by the interval quaternion function IQ is

$$\ddot{I}_{IQ} = \{(x, IQ(x)) | x \in \ddot{U}\}, \tag{3.1}$$

where IQ is called the characteristic interval quaternion function of CIVIFS-IQ. The interval quaternion function IQ is also written as follows:

$$IQ = \ddot{A} + i\ddot{B} + j\ddot{C} + k\ddot{D} = (\ddot{A} + i\ddot{B}) + j(\ddot{C} - i\ddot{D}) = \ddot{\mu} + j\ddot{\nu},$$

where $\ddot{\mu} = \ddot{A} + i\ddot{B}$ is the complex membership interval function, and $\ddot{\nu} = \ddot{C} - i\ddot{D}$ is the complex nonmembership interval function. For each $x \in U$, the values $\ddot{\mu}(x)$ and $\ddot{\nu}(x)$ are the complex membership and complex nonmembership degree intervals of x in $\ddot{I}_{IQ}$, respectively.

Remark 3.1. The following special cases are to be considered from Eq (3.1), which are summarized as follows:
(1) If $\ddot{A}^- = \ddot{A}^+, \ddot{B}^- = \ddot{B}^+, \ddot{C}^- = \ddot{C}^+, \ddot{D}^- = \ddot{D}^+$, then the CIVIFS-IQ reduces to CIFS-Q.
(2) If $\ddot{A}^- = \ddot{A}^+, \ddot{C}^- = \ddot{C}^+$, and $\ddot{B}^- = \ddot{B}^+ = \ddot{D}^- = \ddot{D}^+ = 0$, then the CIVIFS-IQ reduces to represent IFS by complex number.

Now, we consider the set IQ^* defined by $IQ^* = \{IQ = (\ddot{A}, \ddot{B}, \ddot{C}, \ddot{D}) = \ddot{A} + i\ddot{B} + j\ddot{C} + k\ddot{D} | \ddot{A}, \ddot{B}, \ddot{C}, \ddot{D}, [\ddot{A}^- + \ddot{C}^-, \ddot{A}^+ + \ddot{C}^+], [\ddot{B}^- + \ddot{D}^-, \ddot{B}^+ + \ddot{D}^+] \subseteq [0, 1]\}$. In the following, it is assumed that $IQ_t \in IQ^*$, and IQ_t has the representation $IQ_t = (\ddot{A}_t, \ddot{B}_t, \ddot{C}_t, \ddot{D}_t)$ or $IQ_t = \ddot{A}_t + i\ddot{B}_t + j\ddot{C}_t + k\ddot{D}_t (t = 1, 2, \ldots)$.

The order relation on $IQ_1, IQ_2 \in IQ^*$ is defined by
$IQ_1 \leq IQ_2 \Longleftrightarrow \ddot{A}_1 \leq \ddot{A}_2, \ddot{B}_1 \leq \ddot{B}_2, \ddot{C}_1 \geq \ddot{C}_2, \ddot{D}_1 \geq \ddot{D}_2$.

Definition 3.2. Let $IQ_t = (\ddot{A}_t, \ddot{B}_t, \ddot{C}_t, \ddot{D}_t)(t = 1, 2) \in IQ^*$ and $\ddot{I}_{IQ_1}, \ddot{I}_{IQ_2}$ be two CIVIFSs-IQ on $\ddot{U}$, where $\ddot{A}_t, \ddot{B}_t, \ddot{C}_t, \ddot{D}_t(t = 1, 2)$ are the interval functions of real membership, imaginary membership, real nonmembership and imaginary nonmembership, respectively. The set $\ddot{I}_{IQ_1}$ is defined to be a subset of $\ddot{I}_{IQ_2}$, expressed as $\ddot{I}_{IQ_1} \subseteq \ddot{I}_{IQ_2}$, if, and only if, $IQ_1 \leq IQ_2$. If the set $\ddot{I}_{IQ_1}$ is defined to be equal to set $\ddot{I}_{IQ_2}$, denoted as $\ddot{I}_{IQ_1} = \ddot{I}_{IQ_2}$, if, and only if, $\ddot{I}_{IQ_1} \subseteq \ddot{I}_{IQ_2}$ and $\ddot{I}_{IQ_1} \supseteq \ddot{I}_{IQ_2}$.

Definition 3.3. The hesitation interval of a CIVIFS-IQ is defined as

$$[\pi^-_{IQ}, \pi^+_{IQ}] = [2 - (\ddot{A}^+ + \ddot{B}^+ + \ddot{C}^+ + \ddot{D}^+), 2 - (\ddot{A}^- + \ddot{B}^- + \ddot{C}^- + \ddot{D}^-)],$$

where the real hesitation degree interval is $[1 - (\ddot{A}^+ + \ddot{C}^+), 1 - (\ddot{A}^- + \ddot{C}^-)]$ and imaginary hesitation degree interval is $[1 - (\ddot{B}^+ + \ddot{D}^+), 1 - (\ddot{B}^- + \ddot{D}^-)]$.

Definition 3.4. Given $IQ_1 = (\ddot{A}_1, \ddot{B}_1, \ddot{C}_1, \ddot{D}_1) \in IQ^*$, $IQ_2 = (\ddot{A}_2, \ddot{B}_2, \ddot{C}_2, \ddot{D}_2) \in IQ^*$, and $\ddot{I}_{IQ_1}, \ddot{I}_{IQ_2}$ are two CIVIFSs-IQ on $\ddot{U}$, the addition and multiplication operations are defined as:
(1) Addition: $\ddot{I}_{IQ_1} + \ddot{I}_{IQ_2} = (\ddot{A}_1 + \ddot{A}_2, \ddot{B}_1 + \ddot{B}_2, \ddot{C}_1 + \ddot{C}_2, \ddot{D}_1 + \ddot{D}_2)$.
(2) Multiplication: $\ddot{I}_{IQ_1} \cdot \ddot{I}_{IQ_2} = [(\ddot{A}_1\ddot{A}_2 - \ddot{B}_1\ddot{B}_2 - \ddot{C}_1\ddot{C}_2 - \ddot{D}_1\ddot{D}_2), (\ddot{A}_1\ddot{B}_2 + \ddot{B}_1\ddot{A}_2 + \ddot{C}_1\ddot{D}_2 - \ddot{D}_1\ddot{C}_2), (\ddot{A}_1\ddot{C}_2 - \ddot{B}_1\ddot{D}_2 + \ddot{C}_1\ddot{A}_2 + \ddot{D}_1\ddot{B}_2), (\ddot{A}_1\ddot{D}_2 + \ddot{B}_1\ddot{C}_2 - \ddot{C}_1\ddot{B}_2 + \ddot{D}_1\ddot{A}_2)]$.

Definition 3.5. Let $\ddot{I}_{IQ_1}$ and $\ddot{I}_{IQ_2}$ be two CIVIFSs-IQ on $\ddot{U}$ with $IQ_1 = \ddot{A}_1 + i\ddot{B}_1 + j\ddot{C}_1 + k\ddot{D}_1$ and $IQ_2 = \ddot{A}_2 + i\ddot{B}_2 + j\ddot{C}_2 + k\ddot{D}_2$, respectively. Thus, $\ddot{I}_{IQ_1} \bigcup \ddot{I}_{IQ_2}, \ddot{I}_{IQ_1} \bigcap \ddot{I}_{IQ_2}$, and $\ddot{I}_{IQ_1^C}$ are CIVIFSs-IQ on $\ddot{U}$ functions as follows:

$$\begin{aligned}\ddot{I}_{IQ_1} \bigcup \ddot{I}_{IQ_2} = {} & [\max\{\ddot{A}^-_1, \ddot{A}^-_2\}, \max\{\ddot{A}^+_1, \ddot{A}^+_2\}] + i[\max\{\ddot{B}^-_1, \ddot{B}^-_2\}, \max\{\ddot{B}^+_1, \ddot{B}^+_2\}] + j[\min\{\ddot{C}^-_1, \ddot{C}^-_2\}, \min\{\ddot{C}^+_1, \ddot{C}^+_2\}] \\ & + k[\min\{\ddot{D}^-_1, \ddot{D}^-_2\}, \min\{\ddot{D}^+_1, \ddot{D}^+_2\}];\end{aligned}$$

$$\begin{aligned}\ddot{I}_{IQ_1} \bigcap \ddot{I}_{IQ_2} = {} & [\min\{\ddot{A}^-_1, \ddot{A}^-_2\}, \min\{\ddot{A}^+_1, \ddot{A}^+_2\}] + i[\min\{\ddot{B}^-_1, \ddot{B}^-_2\}, \min\{\ddot{B}^+_1, \ddot{B}^+_2\}] + j[\max\{\ddot{C}^-_1, \ddot{C}^-_2\}, \max\{\ddot{C}^+_1, \ddot{C}^+_2\}] \\ & + k[\max\{\ddot{D}^-_1, \ddot{D}^-_2\}, \max\{\ddot{D}^+_1, \ddot{D}^+_2\}];\end{aligned}$$

$$\ddot{I}_{IQ_1^C} = \ddot{C}_1 + i\ddot{D}_1 + j\ddot{A}_1 + k\ddot{B}_1 = [\ddot{C}^-_1, \ddot{C}^+_1] + i[\ddot{D}^-_1, \ddot{D}^+_1] + j[\ddot{A}^-_1, \ddot{A}^+_1] + k[\ddot{B}^-_1, \ddot{B}^+_1].$$

Example 3.1. Let $\ddot{I}_{IQ_1} = [0.5, 0.7] + i[0.3, 0.6] + j[0.1, 0.3] + k[0.1, 0.2]$ and $\ddot{I}_{IQ_2} = [0.3, 0.5] + i[0.4, 0.6] + j[0.1, 0.2] + k[0.2, 0.3]$. We can obtain:
$\ddot{I}_{IQ_1} \bigcup \ddot{I}_{IQ_2} = [0.5, 0.7] + i[0.4, 0.6] + j[0.1, 0.2] + k[0.1, 0.2]$.
$\ddot{I}_{IQ_1} \bigcap \ddot{I}_{IQ_2} = [0.3, 0.5] + i[0.3, 0.6] + j[0.1, 0.3] + k[0.2, 0.3]$.
$\ddot{I}_{IQ_1^C} = [0.1, 0.3] + i[0.1, 0.2] + j[0.5, 0.7] + k[0.3, 0.6]$.

4. Score function and correlation coefficient

In this section, we present a new score function and correlation coefficient that are based on the CIVIFSs-IQ.

Garg and Rani [27] defined the scoring function and the accuracy functions to be used in ranking the CIVIFSs. The scoring function $\mathcal{S}(\beta)$ and accuracy function $\mathcal{H}(\beta)$ of a given CIVIFS β are defined as

$$\mathcal{S}(\beta) = \frac{1}{2}[\ddot{A}^- + \ddot{A}^+ + \ddot{B}^- + \ddot{B}^+ - \ddot{C}^- - \ddot{C}^+ - \ddot{D}^- - \ddot{D}^+],$$

$$\mathcal{H}(\beta) = \frac{1}{2}[\ddot{A}^- + \ddot{A}^+ + \ddot{B}^- + \ddot{B}^+ + \ddot{C}^- + \ddot{C}^+ + \ddot{D}^- + \ddot{D}^+].$$

For two CIVIFSs β and γ,
(1) If $\mathcal{S}(\beta) < \mathcal{S}(\gamma)$, then $\beta < \gamma$;
(2) If $\mathcal{S}(\beta) > \mathcal{S}(\gamma)$, then $\beta > \gamma$;
(3) If $\mathcal{S}(\beta) = \mathcal{S}(\gamma)$, then
(i) If $\mathcal{H}(\beta) < \mathcal{H}(\gamma)$, then $\beta < \gamma$;
(ii) If $\mathcal{H}(\beta) > \mathcal{H}(\gamma)$, then $\beta > \gamma$;
(iii) If $\mathcal{H}(\beta) = \mathcal{H}(\gamma)$, then $\beta = \gamma$.

Example 4.1. Suppose that $IQ_1 = ([0.1, 0.3], [0.3, 0.4], [0.5, 0.6], [0.1, 0.2])$ and $IQ_2 = ([0.2, 0.4], [0.2, 0.3], [0.4, 0.5], [0.2, 0.3])$ are two CIVIFSs-IQ. Using the above methods, we have $\mathcal{S}(IQ_1) = \mathcal{S}(IQ_2) = -0.15$, $\mathcal{H}(IQ_1) = \mathcal{H}(IQ_2) = 1.25$, but $IQ_1 \neq IQ_2$. Thus, the above method cannot rank these two CIVIFSs-IQ.

After the above analysis, we find that the existing ranking methods have defects in the ranking of CIVIFSs-IQ. In order to overcome the shortcomings of existing CIVIFS scoring methods and achieve better CIVIFS-IQ scoring, we set up a new scoring function.

In order to create a new score function with complementary advantages, Gao et al. [35] combined the accuracy function provided by Hong [36] with the scoring function defined by Zhang [37]. The idea of [35] is extended to CIVIFs-IQ in this study. The degree intervals of real membership, imaginary membership, real nonmembership and imaginary nonmembership, as well as the absolute difference and useful information about these are examined, along with the impact of abstention information on decision-making. A CIVIFS-IQ has the following score function, which is given.

Give a CIVIFS $IQ = ([\ddot{A}^-, \ddot{A}^+], [\ddot{B}^-, \ddot{B}^+], [\ddot{C}^-, \ddot{C}^+], [\ddot{D}^-, \ddot{D}^+]) \in IQ^*$, and we define IQ score function as follows:

$$\mathcal{G}(IQ) = \frac{1}{4}(\ddot{A}^- + \ddot{A}^+ + \ddot{B}^- + \ddot{B}^+ - \ddot{C}^- - \ddot{C}^+ - \ddot{D}^- - \ddot{D}^+)(1 + \frac{1}{\ddot{A}^- + \ddot{A}^+ + \ddot{B}^- + \ddot{B}^+ - (\ddot{A}^-\ddot{C}^- + \ddot{B}^-\ddot{D}^-) + (\ddot{A}^+\ddot{C}^+ + \ddot{B}^+\ddot{D}^+)}).$$

For two CIVIFSs IQ_1 and IQ_2,
(1) If $\mathcal{G}(IQ_1) < \mathcal{G}(IQ_2)$, then $IQ_1 < IQ_2$;
(2) If $\mathcal{G}(IQ_1) > \mathcal{G}(IQ_2)$, then $IQ_1 > IQ_2$;
(3) If $\mathcal{G}(IQ_1) = \mathcal{G}(IQ_2)$, then
(i) If $\mathcal{H}(IQ_1) < \mathcal{H}(IQ_2)$, then $IQ_1 < IQ_2$;
(ii) If $\mathcal{H}(IQ_1) > \mathcal{H}(IQ_2)$, then $IQ_1 > IQ_2$;
(iii) If $\mathcal{H}(IQ_1) = \mathcal{H}(IQ_2)$, then $IQ_1 = IQ_2$.

The following uses the above newly defined score function to calculate

$$\mathcal{G}(IQ_1) = -0.1375, \mathcal{G}(IQ_2) = -0.1340, \mathcal{G}(IQ_1) < \mathcal{G}(IQ_2),$$

then

$$IQ_1 < IQ_2.$$

During the decision-making process, one of the most crucial methods to deal with the information value of imprecise and uncertain data is to quantify the dependence of two variables using the correlation coefficient approach. Gerstenkorn and Manko [38] first proposed the concept of the correlation coefficient of IFSs in 1991. The concept of correlation coefficients was extended in 1995 by Bustince and Burillo [39] from IFS to an IVIFS environment. Garg introduced correlation coefficients for Pythagorean fuzzy sets [40, 41]. The researchers also looked at different kinds of correlation coefficients [42–45] and used them in MCDM. Although a great deal of research has been done on correlation coefficients, little is known about the correlation coefficient of CIVIFS. So, next, we define the correlation coefficient in the CIVIFS-IQ environment.

Let $X = \{x_1, x_2, x_3, ..., x_n\}$ be a finite universal set and $A, B \in CIVIFS - IQ(X)$ be given by $A = \{\langle x_i, [\ddot{A}_A^-(x_i), \ddot{A}_A^+(x_i)], [\ddot{B}_A^-(x_i), \ddot{B}_A^+(x_i)], [\ddot{C}_A^-(x_i), \ddot{C}_A^+(x_i)], [\ddot{D}_A^-(x_i), \ddot{D}_A^+(x_i)] >: x_i \in X\}$ and $B = \{\langle x_i, [\ddot{A}_B^-(x_i), \ddot{A}_B^+(x_i)], [\ddot{B}_B^-(x_i), \ddot{B}_B^+(x_i)], [\ddot{C}_B^-(x_i), \ddot{C}_B^+(x_i)], [\ddot{D}_B^-(x_i), \ddot{D}_B^+(x_i)] >: x_i \in X\}$.

The informational energies of two CIVIFSs-IQ A and B are expressed as

$$\mathcal{T}(A) = \sum_{i=1}^{n} (\ddot{A}_A^-(x_i)^2 + \ddot{A}_A^+(x_i)^2 + \ddot{B}_A^-(x_i)^2 + \ddot{B}_A^+(x_i)^2 + \ddot{C}_A^-(x_i)^2 + \ddot{C}_A^+(x_i)^2 + \ddot{D}_A^-(x_i)^2 + \ddot{D}_A^+(x_i)^2). \tag{4.1}$$

$$\mathcal{T}(B) = \sum_{i=1}^{n} (\ddot{A}_B^-(x_i)^2 + \ddot{A}_B^+(x_i)^2 + \ddot{B}_B^-(x_i)^2 + \ddot{B}_B^+(x_i)^2 + \ddot{C}_B^-(x_i)^2 + \ddot{C}_B^+(x_i)^2 + \ddot{D}_B^-(x_i)^2 + \ddot{D}_B^+(x_i)^2). \tag{4.2}$$

$$\begin{aligned} C(A, B) = \sum_{i=1}^{n} (&\ddot{A}_A^-(x_i)\ddot{A}_B^-(x_i) + \ddot{A}_A^+(x_i)\ddot{A}_B^+(x_i) + \ddot{B}_A^-(x_i)\ddot{B}_B^-(x_i) + \ddot{B}_A^+(x_i)\ddot{B}_B^+(x_i) + \ddot{C}_A^-(x_i)\ddot{C}_B^-(x_i) \\ &+ \ddot{C}_A^+(x_i)\ddot{C}_B^+(x_i) + \ddot{D}_A^-(x_i)\ddot{D}_B^-(x_i) + \ddot{D}_A^+(x_i)\ddot{D}_B^+(x_i)). \end{aligned} \tag{4.3}$$

From Eq (4.3), it is clearly seen that correlation of CIVIFNs-IQ satisfies the following properties.
(1) $C(A, B) = C(B, A)$.
(2) $C(A, A) = \mathcal{T}(A)$.

Next, we determined the correlation coefficient between A and B using these as our basis.

Definition 4.1. If $A = \{\langle x_i, [\ddot{A}_A^-(x_i), \ddot{A}_A^+(x_i)], [\ddot{B}_A^-(x_i), \ddot{B}_A^+(x_i)], [\ddot{C}_A^-(x_i), \ddot{C}_A^+(x_i)], [\ddot{D}_A^-(x_i), \ddot{D}_A^+(x_i)] >: x_i \in X\}$ and $B = \{\langle x_i, [\ddot{A}_B^-(x_i), \ddot{A}_B^+(x_i)], [\ddot{B}_B^-(x_i), \ddot{B}_B^+(x_i)], [\ddot{C}_B^-(x_i), \ddot{C}_B^+(x_i)], [\ddot{D}_B^-(x_i), \ddot{D}_B^+(x_i)] >: x_i \in X\}$ are two CIVIFSs-IQ on X, then the correlation coefficient between them is:

$$\begin{aligned} \mathcal{K}(A, B) &= \frac{C(A, B)}{\sqrt{\mathcal{T}(A) \times \mathcal{T}(B)}} \\ &= \frac{\sum_{i=1}^{n} \begin{pmatrix} \ddot{A}_A^-(x_i)\ddot{A}_B^-(x_i) + \ddot{A}_A^+(x_i)\ddot{A}_B^+(x_i) + \ddot{B}_A^-(x_i)\ddot{B}_B^-(x_i) + \ddot{B}_A^+(x_i)\ddot{B}_B^+(x_i) \\ + \ddot{C}_A^-(x_i)\ddot{C}_B^-(x_i) + \ddot{C}_A^+(x_i)\ddot{C}_B^+(x_i) + \ddot{D}_A^-(x_i)\ddot{D}_B^-(x_i) + \ddot{D}_A^+(x_i)\ddot{D}_B^+(x_i) \end{pmatrix}}{\left\{ \begin{array}{l} \sqrt{\sum_{i=1}^{n} \begin{pmatrix} \ddot{A}_A^-(x_i)^2 + \ddot{A}_A^+(x_i)^2 + \ddot{B}_A^-(x_i)^2 + \ddot{B}_A^+(x_i)^2 \\ + \ddot{C}_A^-(x_i)^2 + \ddot{C}_A^+(x_i)^2 + \ddot{D}_A^-(x_i)^2 + \ddot{D}_A^+(x_i)^2 \end{pmatrix}} \\ \times \sqrt{\sum_{i=1}^{n} \begin{pmatrix} \ddot{A}_B^-(x_i)^2 + \ddot{A}_B^+(x_i)^2 + \ddot{B}_B^-(x_i)^2 + \ddot{B}_B^+(x_i)^2 \\ + \ddot{C}_B^-(x_i)^2 + \ddot{C}_B^+(x_i)^2 + \ddot{D}_B^-(x_i)^2 + \ddot{D}_B^+(x_i)^2 \end{pmatrix}} \end{array} \right\}}. \end{aligned} \tag{4.4}$$

Theorem 4.1. The correlation coefficient $\mathcal{K}$ between two CIVIFSs-IQ A and B defined on X satisfies the following properties:
(1) $0 \leq \mathcal{K}(A, B) \leq 1$.

(2) $\mathcal{K}(A,B)=\mathcal{K}(B,A)$.
(3) $\mathcal{K}(A,B)=1$ if $A=B$.

Proof. $A=\{\langle x_i,[\ddot{A}_A^-(x_i),\ddot{A}_A^+(x_i)],[\ddot{B}_A^-(x_i),\ddot{B}_A^+(x_i)],[\ddot{C}_A^-(x_i),\ddot{C}_A^+(x_i)],[\ddot{D}_A^-(x_i),\ddot{D}_A^+(x_i)]>: x_i\in X\}$ and $B=\{\langle x_i,[\ddot{A}_B^-(x_i),\ddot{A}_B^+(x_i)],[\ddot{B}_B^-(x_i),\ddot{B}_B^+(x_i)],[\ddot{C}_B^-(x_i),\ddot{C}_B^+(x_i)],[\ddot{D}_B^-(x_i),\ddot{D}_B^+(x_i)]>: x_i\in X\}$ are two CIVIFSs-IQ defined on X. Then, we have:

(1) The inequality $\mathcal{K}(A,B)\geq 0$ is obvious due to $C(A,B)\geq 0$ being obtained from the Eq (4.4). Now, we shall prove $\mathcal{K}(A,B)\leq 1$. For it, based on the Definition 4.1, we get

$$\begin{aligned}\mathcal{K}(A,B)&=\frac{C(A,B)}{\sqrt{\mathcal{T}(A)\times\mathcal{T}(B)}}\\&=\frac{\sum_{i=1}^{n}\begin{pmatrix}\ddot{A}_A^-(x_i)\ddot{A}_B^-(x_i)+\ddot{A}_A^+(x_i)\ddot{A}_B^+(x_i)+\ddot{B}_A^-(x_i)\ddot{B}_B^-(x_i)+\ddot{B}_A^+(x_i)\ddot{B}_B^+(x_i)\\+\ddot{C}_A^-(x_i)\ddot{C}_B^-(x_i)+\ddot{C}_A^+(x_i)\ddot{C}_B^+(x_i)+\ddot{D}_A^-(x_i)\ddot{D}_B^-(x_i)+\ddot{D}_A^+(x_i)\ddot{D}_B^+(x_i)\end{pmatrix}}{\left\{\begin{array}{l}\sqrt{\sum_{i=1}^{n}\begin{pmatrix}\ddot{A}_A^-(x_i)^2+\ddot{A}_A^+(x_i)^2+\ddot{B}_A^-(x_i)^2+\ddot{B}_A^+(x_i)^2\\+\ddot{C}_A^-(x_i)^2+\ddot{C}_A^+(x_i)^2+\ddot{D}_A^-(x_i)^2+\ddot{D}_A^+(x_i)^2\end{pmatrix}}\\\times\sqrt{\sum_{i=1}^{n}\begin{pmatrix}\ddot{A}_B^-(x_i)^2+\ddot{A}_B^+(x_i)^2+\ddot{B}_B^-(x_i)^2+\ddot{B}_B^+(x_i)^2\\+\ddot{C}_B^-(x_i)^2+\ddot{C}_B^+(x_i)^2+\ddot{D}_B^-(x_i)^2+\ddot{D}_B^+(x_i)^2\end{pmatrix}}\end{array}\right\}}\\&=\frac{\begin{pmatrix}\sum_{i=1}^{n}\ddot{A}_A^-(x_i)\ddot{A}_B^-(x_i)+\sum_{i=1}^{n}\ddot{A}_A^+(x_i)\ddot{A}_B^+(x_i)+\sum_{i=1}^{n}\ddot{B}_A^-(x_i)\ddot{B}_B^-(x_i)+\sum_{i=1}^{n}\ddot{B}_A^+(x_i)\ddot{B}_B^+(x_i)\\+\sum_{i=1}^{n}\ddot{C}_A^-(x_i)\ddot{C}_B^-(x_i)+\sum_{i=1}^{n}\ddot{C}_A^+(x_i)\ddot{C}_B^+(x_i)+\sum_{i=1}^{n}\ddot{D}_A^-(x_i)\ddot{D}_B^-(x_i)+\sum_{i=1}^{n}\ddot{D}_A^+(x_i)\ddot{D}_B^+(x_i)\end{pmatrix}}{\left\{\begin{array}{l}\sqrt{\sum_{i=1}^{n}\begin{pmatrix}\ddot{A}_A^-(x_i)^2+\ddot{A}_A^+(x_i)^2+\ddot{B}_A^-(x_i)^2+\ddot{B}_A^+(x_i)^2\\+\ddot{C}_A^-(x_i)^2+\ddot{C}_A^+(x_i)^2+\ddot{D}_A^-(x_i)^2+\ddot{D}_A^+(x_i)^2\end{pmatrix}}\\\times\sqrt{\sum_{i=1}^{n}\begin{pmatrix}\ddot{A}_B^-(x_i)^2+\ddot{A}_B^+(x_i)^2+\ddot{B}_B^-(x_i)^2+\ddot{B}_B^+(x_i)^2\\+\ddot{C}_B^-(x_i)^2+\ddot{C}_B^+(x_i)^2+\ddot{D}_B^-(x_i)^2+\ddot{D}_B^+(x_i)^2\end{pmatrix}}\end{array}\right\}}.\end{aligned}$$

By using the Cauchy-Schwarz inequality,

$$\left(\sum_{i=1}^{n}a_ib_i\right)^2\leq\left(\sum_{i=1}^{n}a_i\right)\left(\sum_{i=1}^{n}b_i\right).$$

The equality holds as valid if at least one of $a_i,b_i,i=1,2,...,n$ is all zero or if $\frac{b_1}{a_1}=\frac{b_1}{a_1}=\cdots=\frac{b_n}{a_n}$. Therefore,

$$\mathcal{K}(A,B) \leq \frac{\left(\begin{array}{l} \sqrt{\sum_{i=1}^{n} \ddot{A}_A^-(x_i)^2}\sqrt{\sum_{i=1}^{n} \ddot{A}_B^-(x_i)^2} + \sqrt{\sum_{i=1}^{n} \ddot{A}_A^+(x_i)^2}\sqrt{\sum_{i=1}^{n} \ddot{A}_B^+(x_i)^2} + \sqrt{\sum_{i=1}^{n} \ddot{B}_A^-(x_i)^2}\sqrt{\sum_{i=1}^{n} \ddot{B}_B^-(x_i)} \\ + \sqrt{\sum_{i=1}^{n} \ddot{B}_A^+(x_i)^2}\sqrt{\sum_{i=1}^{n} \ddot{B}_B^+(x_i)^2} + \sqrt{\sum_{i=1}^{n} \ddot{C}_A^-(x_i)^2}\sqrt{\sum_{i=1}^{n} \ddot{C}_B^-(x_i)^2} + \sqrt{\sum_{i=1}^{n} \ddot{C}_A^+(x_i)^2}\sqrt{\sum_{i=1}^{n} \ddot{C}_B^+(x_i)} \\ + \sqrt{\sum_{i=1}^{n} \ddot{D}_A^-(x_i)}\sqrt{\sum_{i=1}^{n} \ddot{D}_B^-(x_i)} + \sqrt{\sum_{i=1}^{n} \ddot{D}_A^+(x_i)}\sqrt{\sum_{i=1}^{n} \ddot{D}_B^+(x_i))} \end{array}\right)}{\left\{\begin{array}{l} \sqrt{\sum_{i=1}^{n}(\ddot{A}_A^-(x_i)^2 + \ddot{A}_A^+(x_i)^2 + \ddot{B}_A^-(x_i)^2 + \ddot{B}_A^+(x_i)^2 + \ddot{C}_A^-(x_i)^2 + \ddot{C}_A^+(x_i)^2 + \ddot{D}_A^-(x_i)^2 + \ddot{D}_A^+(x_i)^2)} \\ \times \sqrt{\sum_{i=1}^{n}(\ddot{A}_B^-(x_i)^2 + \ddot{A}_B^+(x_i)^2 + \ddot{B}_B^-(x_i)^2 + \ddot{B}_B^+(x_i)^2 + \ddot{C}_B^-(x_i)^2 + \ddot{C}_B^+(x_i)^2 + \ddot{D}_B^-(x_i)^2 + \ddot{D}_B^+(x_i)^2)} \end{array}\right\}}.$$

Let us adopt the following notations:

$$\begin{array}{llll} \sum_{i=1}^{n} \ddot{A}_A^-(x_i)^2 = a, & \sum_{i=1}^{n} \ddot{A}_B^-(x_i)^2 = b, & \sum_{i=1}^{n} \ddot{A}_A^+(x_i)^2 = c, & \sum_{i=1}^{n} \ddot{A}_B^+(x_i)^2 = d, \\ \sum_{i=1}^{n} \ddot{B}_A^-(x_i)^2 = e, & \sum_{i=1}^{n} \ddot{B}_B^-(x_i)^2 = f, & \sum_{i=1}^{n} \ddot{B}_A^+(x_i)^2 = g, & \sum_{i=1}^{n} \ddot{B}_B^+(x_i)^2 = h, \\ \sum_{i=1}^{n} \ddot{C}_A^-(x_i)^2 = k, & \sum_{i=1}^{n} \ddot{C}_B^-(x_i)^2 = l, & \sum_{i=1}^{n} \ddot{C}_A^+(x_i)^2 = m, & \sum_{i=1}^{n} \ddot{C}_B^+(x_i)^2 = n, \\ \sum_{i=1}^{n} \ddot{D}_A^-(x_i)^2 = p, & \sum_{i=1}^{n} \ddot{D}_B^-(x_i)^2 = q, & \sum_{i=1}^{n} \ddot{D}_A^+(x_i)^2 = r, & \sum_{i=1}^{n} \ddot{D}_B^+(x_i)^2 = t, \end{array}$$

and the above inequality is equivalent to

$$\mathcal{K}(A,B) \leq \frac{\sqrt{ab} + \sqrt{cd} + \sqrt{ef} + \sqrt{gh} + \sqrt{kl} + \sqrt{mn} + \sqrt{pq} + \sqrt{rt}}{\sqrt{(a+c+d+e+g+k+l+m+p+r)\cdot(b+d+f+h+l+n+q+t)}}$$

$$\mathcal{K}(A,B)^2 - 1 \leq \frac{(\sqrt{ab} + \sqrt{cd} + \sqrt{ef} + \sqrt{gh} + \sqrt{kl} + \sqrt{mn} + \sqrt{pq} + \sqrt{rt})^2}{(a+c+d+e+g+k+l+m+p+r)\cdot(b+d+f+h+l+n+q+t)} - 1$$

$$\mathcal{K}(A,B)^2 - 1 \leq -\frac{\left(\begin{array}{l} (\sqrt{ab} - \sqrt{cd})^2 + (\sqrt{af} - \sqrt{be})^2 + (\sqrt{ah} - \sqrt{bg})^2 + (\sqrt{cf} - \sqrt{de})^2 + (\sqrt{ch} - \sqrt{dg})^2 \\ + (\sqrt{eh} - \sqrt{gf})^2 + (\sqrt{al} - \sqrt{bk})^2 + (\sqrt{an} - \sqrt{bm})^2 + (\sqrt{aq} - \sqrt{bq})^2 + (\sqrt{at} - \sqrt{br})^2 \\ + (\sqrt{cl} - \sqrt{dk})^2 + (\sqrt{cn} - \sqrt{dm})^2 + (\sqrt{cq} - \sqrt{dq})^2 + (\sqrt{ct} - \sqrt{dr})^2 + (\sqrt{el} - \sqrt{fk})^2 \\ + (\sqrt{et} - \sqrt{fr})^2 + (\sqrt{en} - \sqrt{fm})^2 + (\sqrt{eq} - \sqrt{fp})^2 + (\sqrt{gl} - \sqrt{hk})^2 + (\sqrt{gn} - \sqrt{hm})^2 \\ + (\sqrt{gq} - \sqrt{hp})^2 + (\sqrt{gt} - \sqrt{hr})^2 + (\sqrt{kn} - \sqrt{km})^2 + (\sqrt{kq} - \sqrt{lp})^2 + (\sqrt{kt} - \sqrt{lr})^2 \\ + (\sqrt{mq} - \sqrt{np})^2 + (\sqrt{mt} - \sqrt{nr})^2 + (\sqrt{pt} - \sqrt{qr})^2 \end{array}\right)}{(a+c+d+e+g+k+l+m+p+r)\cdot(b+d+f+h+l+n+q+t)}$$

$$\mathcal{K}(A,B)^2 - 1 \leq 0$$

$$\mathcal{K}(A,B)^2 \leq 1.$$

Hence, $\mathcal{K}(A,B)^2 \leq 1$, which implies $\mathcal{K}(A,B) \leq 1$. So, $0 \leq \mathcal{K}(A,B) \leq 1$.

(2) For any two CIVIFSs-IQ A and B, the proof of Theorem 4.1 (2) is straightforward.

(3) If $A = B$, this implies that $\ddot{A}_A^-(x_i) = \ddot{A}_B^-(x_i), \ddot{A}_A^+(x_i) = \ddot{A}_B^+(x_i), \ddot{B}_A^-(x_i) = \ddot{B}_B^-(x_i), \ddot{B}_A^+(x_i) = \ddot{B}_B^+(x_i), \ddot{C}_A^-(x_i) = \ddot{C}_B^-(x_i), \ddot{C}_A^+(x_i) = \ddot{C}_B^+(x_i), \ddot{D}_A^-(x_i) = \ddot{D}_B^-(x_i), \ddot{D}_A^+(x_i) = \ddot{D}_B^+(x_i)$, for all i, and, thus, from Eq (4.4), it follows that $\mathcal{K}(A, B) = 1$.

Example 4.2. Let $A = \{(x_1, [0.3, 0.4], [0.1, 0.3], [0.1, 0.3], [0.3, 0.4]), (x_2, [0.1, 0.3], [0.3, 0.4], [0.3, 0.4], [0.1, 0.3]), (x_3, [0.3, 0.4], [0.1, 0.3], [0.1, 0.2], [0.3, 0.5]), (x_4, [0.1, 0.3], [0.3, 0.4], [0.3, 0.5], [0.1, 0.2])\}$, and $B = \{(x_1, [0.2, 0.5], [0.3, 0.4], [0.2, 0.3], [0.1, 0.4]), (x_2, [0.3, 0.4], [0.2, 0.5], [0.1, 0.4], [0.2, 0.3]), (x_3, [0.2, 0.4], [0.4, 0.5], [0.1, 0.2], [0.3, 0.5]), (x_4, [0.4, 0.5], [0.2, 0.4], [0.3, 0.5], [0.1, 0.2])\}$ be two CIVIFSs-IQ defined on the universal set X. By Eq(4.1), we obtain the informational energy of A as

$$\begin{aligned}
\mathcal{T}(A) &= \sum_{i=1}^{n}(\ddot{A}_A^-(x_i)^2 + \ddot{A}_A^+(x_i)^2 + \ddot{B}_A^-(x_i)^2 + \ddot{B}_A^+(x_i)^2 + \ddot{C}_A^-(x_i)^2 + \ddot{C}_A^+(x_i)^2 + \ddot{D}_A^-(x_i)^2 + \ddot{D}_A^+(x_i)^2) \\
&= (0.3)^2 + (0.4)^2 + (0.1)^2 + (0.3)^2 + (0.1)^2 + (0.3)^2 + (0.3)^2 + (0.4)^2 + (0.1)^2 + (0.3)^2 + (0.3)^2 \\
&\quad + (0.4)^2 + (0.3)^2 + (0.4)^2 + (0.1)^2 + (0.3)^2 + (0.3)^2 + (0.4)^2 + (0.1)^2 + (0.3)^2 + (0.1)^2 + (0.2)^2 \\
&\quad + (0.3)^2 + (0.5)^2 + (0.1)^2 + (0.3)^2 + (0.3)^2 + (0.4)^2 + (0.3)^2 + (0.5)^2 + (0.1)^2 + (0.2)^2 \\
&= 2.88.
\end{aligned}$$

Similarly, the informational energy of CIVIFS-IQ B is

$$\begin{aligned}
\mathcal{T}(B) &= \sum_{i=1}^{n}(\ddot{A}_B^-(x_i)^2 + \ddot{A}_B^+(x_i)^2 + \ddot{B}_B^-(x_i)^2 + \ddot{B}_B^+(x_i)^2 + \ddot{C}_B^-(x_i)^2 + \ddot{C}_B^+(x_i)^2 + \ddot{D}_B^-(x_i)^2 + \ddot{D}_B^+(x_i)^2) \\
&= (0.2)^2 + (0.5)^2 + (0.3)^2 + (0.4)^2 + (0.2)^2 + (0.3)^2 + (0.1)^2 + (0.4)^2 + (0.3)^2 + (0.4)^2 + (0.2)^2 \\
&\quad + (0.5)^2 + (0.1)^2 + (0.4)^2 + (0.2)^2 + (0.3)^2 + (0.2)^2 + (0.4)^2 + (0.4)^2 + (0.5)^2 + (0.1)^2 + (0.2)^2 \\
&\quad + (0.3)^2 + (0.5)^2 + (0.4)^2 + (0.5)^2 + (0.2)^2 + (0.4)^2 + (0.3)^2 + (0.5)^2 + (0.1)^2 + (0.2)^2 \\
&= 3.68.
\end{aligned}$$

By using Eq (4.3), the correlation between CIVIFSs-IQ A and B is computed as

$$\begin{aligned}
C(A, B) &= \sum_{i=1}^{n}(\ddot{A}_A^-(x_i)\ddot{A}_B^-(x_i) + \ddot{A}_A^+(x_i)\ddot{A}_B^+(x_i) + \ddot{B}_A^-(x_i)\ddot{B}_B^-(x_i) + \ddot{B}_A^+(x_i)\ddot{B}_B^+(x_i) + \ddot{C}_A^-(x_i)\ddot{C}_B^-(x_i) + \ddot{C}_A^+(x_i)\ddot{C}_B^+(x_i) \\
&\quad + \ddot{D}_A^-(x_i)\ddot{D}_B^-(x_i) + \ddot{D}_A^+(x_i)\ddot{D}_B^+(x_i)) \\
&= 0.3 \times 0.2 + 0.4 \times 0.5 + 0.1 \times 0.3 + 0.3 \times 0.4 + 0.1 \times 0.2 + 0.3 \times 0.3 + 0.3 \times 0.1 + 0.4 \times 0.4 \\
&\quad + 0.1 \times 0.3 + 0.3 \times 0.4 + 0.3 \times 0.2 + 0.4 \times 0.5 + 0.3 \times 0.1 + 0.4 \times 0.4 + 0.1 \times 0.2 + 0.3 \times 0.3 \\
&\quad + 0.3 \times 0.2 + 0.4 \times 0.4 + 0.1 \times 0.4 + 0.3 \times 0.5 + 0.1 \times 0.1 + 0.2 \times 0.2 + 0.3 \times 0.3 + 0.5 \times 0.5 \\
&\quad + 0.1 \times 0.4 + 0.3 \times 0.5 + 0.3 \times 0.2 + 0.4 \times 0.4 + 0.3 \times 0.3 + 0.5 \times 0.5 + 0.1 \times 0.1 + 0.2 \times 0.2 \\
&= 3.02.
\end{aligned}$$

Thus, the correlation coefficient between A and B is given by Eq (4.4) as

$$\mathcal{K}(A, B) = \frac{C(A, B)}{\sqrt{\mathcal{T}(A) \times \mathcal{T}(B)}} = 0.9277.$$

Each element of the universal set is given equal weight in the formulas for computing the coefficient correlation that is defined above. This might not always be feasible, though, in real-world circumstances. There are components of the universal set that are more significant than others. Therefore, we need to consider the appropriate weights assigned to each component of the universal set.

We provide a weighted correlation coefficient between CIVIFSs in the section that follows. Assume that the weight vector $\xi = (\xi_1, \xi_2, ..., \xi_n)^T$ corresponds to the elements $x_i (i = 1, 2, ..., n)$ where $\xi_i > 0$, $\sum_{i=1}^{n} \xi_i = 1$. Then, we define the weight correlation coefficient as follows:

$$\begin{aligned}\mathcal{K}(A,B) &= \frac{C_w(A,B)}{\sqrt{\mathcal{T}_w(A)\times\mathcal{T}_w(B)}}\\ &= \frac{\sum_{i=1}^{n}\left(\xi_i\begin{bmatrix}\ddot{A}_A^-(x_i)\ddot{A}_B^-(x_i)+\ddot{A}_A^+(x_i)\ddot{A}_B^+(x_i)+\ddot{B}_A^-(x_i)\ddot{B}_B^-(x_i)+\ddot{B}_A^+(x_i)\ddot{B}_B^+(x_i)\\ +\ddot{C}_A^-(x_i)\ddot{C}_B^-(x_i)+\ddot{C}_A^+(x_i)\ddot{C}_B^+(x_i)+\ddot{D}_A^-(x_i)\ddot{D}_B^-(x_i)+\ddot{D}_A^+(x_i)\ddot{D}_B^+(x_i)\end{bmatrix}\right)}{\left\{\begin{array}{l}\sqrt{\sum_{i=1}^{n}\left(\xi_i\begin{bmatrix}\ddot{A}_A^-(x_i)^2+\ddot{A}_A^+(x_i)^2+\ddot{B}_A^-(x_i)^2+\ddot{B}_A^+(x_i)^2\\ +\ddot{C}_A^-(x_i)^2+\ddot{C}_A^+(x_i)^2+\ddot{D}_A^-(x_i)^2+\ddot{D}_A^+(x_i)^2\end{bmatrix}\right)}\\ \times\sqrt{\sum_{i=1}^{n}\left(\xi_i\begin{bmatrix}\ddot{A}_B^-(x_i)^2+\ddot{A}_B^+(x_i)^2+\ddot{B}_B^-(x_i)^2+\ddot{B}_B^+(x_i)^2\\ +\ddot{C}_B^-(x_i)^2+\ddot{C}_B^+(x_i)^2+\ddot{D}_B^-(x_i)^2+\ddot{D}_B^+(x_i)^2\end{bmatrix}\right)}\end{array}\right\}}.\end{aligned} \tag{4.5}$$

5. MCDM approach based on the proposed correlation coefficient

In this section, we use the correlation coefficients of CIVIFS-IQ to propose the method of MCDM, and give concrete examples.

5.1. A model for attribute weight

For an IVIFS $A = \{< x, [u_A^-(x), u_A^+(x)], [\upsilon_A^-(x), \upsilon_A^+(x)] > | x \in X\}$, Wei et al. [46] defined the entropy formula by

$$E(A) = \frac{1}{n}\sum_{i=1}^{n}\left\{\frac{\min\{u_A^-(x), \upsilon_A^-(x)\} + \min\{u_A^+(x), \upsilon_A^+(x)\}}{\max\{u_A^-(x), \upsilon_A^-(x)\} + \max\{u_A^+(x), \upsilon_A^+(x)\}}\right\}. \tag{5.1}$$

For a CIFS $P = \{< x, r_P(x)e^{i\omega_{r_P}(x)}, k_P(x)e^{i\omega_{k_P}(x)} >: x \in U\}$, Garg and Rani [47] defined the entropy formula by

$$E(P) = \frac{1}{n}\sum_{i=1}^{n}\left\{\frac{\min\{r_P(x_i), k_P(x_i)\} + \frac{1}{2\pi}\min\{\omega_{r_P}(x_i), \omega_{k_p}(x_i)\}}{\max\{r_P(x_i), k_P(x_i)\} + \frac{1}{2\pi}\max\{\omega_{r_P}(x_i), \omega_{k_p}(x_i)\}}\right\}. \tag{5.2}$$

Definition 5.1. For any set $A \in CIVIFS(X)$, a real-valued function E: $CIVIFS(X) \longrightarrow [0, 1]$ is called the entropy of $CIVIFS(X)$ if it satisfies the following properties:
(1) $0 \le E(A) \le 1$.
(2) $E(A) = 0$ if, and only if, A is a crisp set.
(3) $E(A) = 1$ if, and only if, $[r_A^-(x_i), r_A^+(x_i)] = [k_A^-(x_i), k_A^+(x_i)]$ and $[\omega_{r_A}^-(x_i), \omega_{r_A}^+(x_i)] = [\omega_{k_A}^-(x_i), \omega_{k_A}^+(x_i)]$.
(4) $E(A) = E(A^c)$.
(5) $E(A) \le E(B)$ if either $A \subseteq B$ with $r_B^-(x_i) \le k_B^-(x_i)$, $r_B^+(x_i) \le k_B^+(x_i)$, $\omega_{r_B}^-(x_i) \le \omega_{k_B}^-(x_i)$, $\omega_{r_B}^+(x_i) \le \omega_{k_B}^+(x_i)$ for each $x \in X$, or $A \supseteq B$ with $r_B^-(x_i) \ge k_B^-(x_i)$, $r_B^+(x_i) \ge k_B^+(x_i)$, $\omega_{r_B}^-(x_i) \ge \omega_{k_B}^-(x_i)$, $\omega_{r_B}^+(x_i) \ge \omega_{k_B}^+(x_i)$ for each $x \in X$.

Definition 5.2. The entropy measure of CIVIFSs $A = \{< x, [r_A^-(x), r_A^+(x)]e^{i[\omega_{r_A}^-(x), \omega_{r_A}^+(x)]}, [k_A^-(x), k_A^+(x)]$ $e^{i[\omega_{k_A}^-(x), \omega_{k_A}^+(x)]} > | x \in X\}$ is defined as follows:

$$E(A) = \frac{1}{n}\sum_{i=1}^{n}\left\{\frac{\left(\begin{array}{l}\min\{r_A^-(x_i), k_A^-(x_i)\} + \frac{1}{2\pi}\min\{\omega_{r_A}^-(x_i), \omega_{k_A}^-(x_i)\} \\ + \min\{r_A^+(x_i), k_A^+(x_i)\} + \frac{1}{2\pi}\min\{\omega_{r_A}^+(x_i), \omega_{k_A}^+(x_i)\}\end{array}\right)}{\left(\begin{array}{l}\max\{r_A^-(x_i), k_A^-(x_i)\} + \frac{1}{2\pi}\max\{\omega_{r_A}^-(x_i), \omega_{k_A}^-(x_i)\} \\ + \max\{r_A^+(x_i), k_A^+(x_i)\} + \frac{1}{2\pi}\max\{\omega_{r_A}^+(x_i), \omega_{k_A}^+(x_i)\}\end{array}\right)}\right\}. \tag{5.3}$$

Theorem 5.1. The CIVIFS entropy measure E satisfies the following properties:
(1) $0 \le E(A) \le 1$.
(2) $E(A) = 0$ if, and only if, A is a crisp set.
(3) $E(A) = 1$ if, and only if, $[r_A^-(x_i), r_A^+(x_i)] = [k_A^-(x_i), k_A^+(x_i)]$ and $[\omega_{r_A}^-(x_i), \omega_{r_A}^+(x_i)] = [\omega_{k_A}^-(x_i), \omega_{k_A}^+(x_i)]$.
(4) $E(A) = E(A^c)$.
(5) $E(A) \le E(B)$ if either $A \subseteq B$ with $r_B^-(x_i) \le k_B^-(x_i)$, $r_B^+(x_i) \le k_B^+(x_i)$, $\omega_{r_B}^-(x_i) \le \omega_{k_B}^-(x_i)$, $\omega_{r_B}^+(x_i) \le \omega_{k_B}^+(x_i)$ for each $x \in X$, or $A \supseteq B$ with $r_B^-(x_i) \ge k_B^-(x_i)$, $r_B^+(x_i) \ge k_B^+(x_i)$, $\omega_{r_B}^-(x_i) \ge \omega_{k_B}^-(x_i)$, $\omega_{r_B}^+(x_i) \ge \omega_{k_B}^+(x_i)$ for each $x \in X$.

Proof. (1) For any CIVIFS A, the proof property (1) is straightforward.

(2) From $A = \{[1,1]e^{i[0,0]}, [0,0]e^{i[0,0]}\}$ or $A = \{[0,0]e^{i[0,0]}, [1,1]e^{i[0,0]}\}$, we have

$$\min\{r_A^-(x_i), k_A^-(x_i)\} = \min\{r_A^+(x_i), k_A^+(x_i)\} = \min\{\omega_{r_A}^-(x_i), \omega_{k_A}^-(x_i)\}\} = \min\{\omega_{r_A}^+(x_i), \omega_{k_A}^+(x_i)\} = 0,$$

so $E(A) = 0$.

(3) If $[r_A^-(x_i), r_A^+(x_i)] = [k_A^-(x_i), k_A^+(x_i)]$ and $[\omega_{r_A}^-(x_i), \omega_{r_A}^+(x_i)] = [\omega_{k_A}^-(x_i), \omega_{k_A}^+(x_i)]$, we have $r_A^-(x_i) = k_A^-(x_i)$, $r_A^+(x_i) = k_A^+(x_i)$, $\omega_{r_A}^-(x_i) = \omega_{k_A}^-(x_i)$, and $\omega_{r_A}^+(x_i) = \omega_{k_A}^+(x_i)$. Thus,

$$\min\{r_A^-(x_i), k_A^-(x_i)\} = \max\{r_A^-(x_i), k_A^-(x_i)\}, \min\{r_A^+(x_i), k_A^+(x_i)\} = \max\{r_A^+(x_i), k_A^+(x_i)\},$$

$$\min\{\omega_{r_A}^-(x_i), \omega_{k_A}^-(x_i)\} = \max\{\omega_{r_A}^-(x_i), \omega_{k_A}^-(x_i)\}, \min\{\omega_{r_A}^+(x_i), \omega_{k_A}^+(x_i)\} = \max\{\omega_{r_A}^+(x_i), \omega_{k_A}^+(x_i)\},$$

so $E(A) = 1$.

(4) It is clear that $A^c = \{[k_A^-(x_i), k_A^+(x_i)]e^{i[\omega_{k_A}^-(x_i), \omega_{k_A}^+(x_i)]}, [r_A^-(x_i), r_A^+(x_i)]e^{i[\omega_{r_A}^-(x_i), \omega_{r_A}^+(x_i)]}]\}$ for $x \in X$. By applying Eq (5.3), we have $E(A) = E(A^c)$.

(5) When $A \subseteq B$, with $r_B^-(x_i) \le k_B^-(x_i)$, $r_B^+(x_i) \le k_B^+(x_i)$, $\omega_{r_B}^-(x_i) \le \omega_{k_B}^-(x_i)$, $\omega_{r_B}^+(x_i) \le \omega_{k_B}^+(x_i)$ for each $x \in X$, we have

$$r_A^-(x_i) \le r_B^-(x_i) \le k_B^-(x_i) \le k_A^-(x_i),$$

$$r_A^+(x_i) \le r_B^+(x_i) \le k_B^+(x_i) \le k_A^+(x_i),$$

$$\omega_{r_A}^-(x_i) \le \omega_{r_B}^-(x_i) \le \omega_{k_B}^-(x_i) \le \omega_{k_A}^-(x_i),$$

$$\omega_{r_A}^+(x_i) \le \omega_{r_B}^+(x_i) \le \omega_{k_B}^+(x_i) \le \omega_{k_A}^+(x_i).$$

Thus,

$$\min\{r_A^-(x_i), k_A^-(x_i)\} \le \min\{r_B^-(x_i), k_B^-(x_i)\}, \min\{r_A^+(x_i), k_A^+(x_i)\} \le \min\{r_B^+(x_i), k_B^+(x_i)\},$$

$$\max\{r_A^-(x_i), k_A^-(x_i)\} \ge \max\{r_B^-(x_i), k_B^-(x_i)\}, \max\{r_A^+(x_i), k_A^+(x_i)\} \ge \max\{r_B^+(x_i), k_B^+(x_i)\},$$

$$\min\{\omega^-_{r_A}(x_i), \omega^-_{k_A}(x_i)\} \le \min\{\omega^-_{r_B}(x_i), \omega^-_{k_B}(x_i)\}, \min\{\omega^+_{r_A}(x_i), \omega^+_{k_A}(x_i)\} \le \min\{\omega^+_{r_B}(x_i), \omega^+_{k_B}(x_i)\},$$

$$\max\{\omega^-_{r_A}(x_i), \omega^-_{k_A}(x_i)\} \ge \max\{\omega^-_{r_B}(x_i), \omega^-_{k_B}(x_i)\}, \max\{\omega^+_{r_A}(x_i), \omega^+_{k_A}(x_i)\} \ge \max\{\omega^+_{r_B}(x_i), \omega^+_{k_B}(x_i)\}.$$

By Eq (5.3), we have $E(A) \le E(B)$.

Similarly, when $A \supseteq B$ with $r^-_B(x_i) \ge k^-_B(x_i)$, $r^+_B(x_i) \ge k^+_B(x_i)$, $\omega^-_{r_B}(x_i) \ge \omega^-_{k_B}(x_i)$, $\omega^+_{r_B}(x_i) \ge \omega^+_{k_B}(x_i)$ for each $x \in X$, one can also prove $E(A) \le E(B)$.

Remark 5.1. The entropy measure provided by (5.3) reduces to the CIFSs entropy measure defined by (5.2) if a CIVIFS A reduces to being a CIFS. Our entropy formula for CIVIFS is therefore, in a way, a generalization of those for CIFSs.

Remark 5.2. If $A = \{\langle x_i, [\ddot{A}^-_A(x_i), \ddot{A}^+_A(x_i)], [\ddot{B}^-_A(x_i), \ddot{B}^+_A(x_i)], [\ddot{C}^-_A(x_i), \ddot{C}^+_A(x_i)], [\ddot{D}^-_A(x_i), \ddot{D}^+_A(x_i)] >: x_i \in X\}$ is a CIVIFS-IQ defined on X, $X = \{x_1, x_2, x_3, ..., x_n\}$ is a finite universal set. Then, formula (5.3) can be written as follows:

$$E(A) = \frac{1}{n}\sum_{i=1}^{n}\left\{\frac{\begin{pmatrix}\min\{\ddot{A}^-_A(x_i), \ddot{C}^-_A(x_i)\} + \min\{\ddot{A}^+_A(x_i), \ddot{C}^+_A(x_i)\} \\ + \min\{\ddot{B}^-_A(x_i), \ddot{D}^-_A(x_i)\} + \min\{\ddot{B}^+_A(x_i), \ddot{D}^+_A(x_i)\}\end{pmatrix}}{\begin{pmatrix}\max\{\ddot{A}^-_A(x_i), \ddot{C}^-_A(x_i)\} + \max\{\ddot{A}^+_A(x_i), \ddot{C}^+_A(x_i)\} \\ + \max\{\ddot{B}^-_A(x_i), \ddot{D}^-_A(x_i)\} + \max\{\ddot{B}^+_A(x_i), \ddot{D}^+_A(x_i)\}\end{pmatrix}}\right\}. \tag{5.4}$$

Definition 5.3. In MCDM with CIVIF information, if the value of the q th attribute of the p th alternative is $a_{pq} = \{[\ddot{A}^-_{pq}(x_i), \ddot{A}^+_{pq}(x_i)], [\ddot{B}^-_{pq}(x_i), \ddot{B}^+_{pq}(x_i)], [\ddot{C}^-_{pq}(x_i), \ddot{C}^+_{pq}(x_i)], [\ddot{D}^-_{pq}(x_i), \ddot{D}^+_{pq}(x_i)]\}(p = 1, 2, ..., m, q = 1, 2, ..., n.)$, then it is assumed that there are m different alternatives that will be evaluated under the set of the n criteria. Then, the CIVIF entropy of the q th attribute is

$$E_q = \frac{1}{n}\sum_{i=1}^{n}\left\{\frac{\begin{pmatrix}\min\{\ddot{A}^-_{pq}(x_i), \ddot{C}^-_{pq}(x_i)\} + \min\{\ddot{A}^+_{pq}(x_i), \ddot{C}^+_{pq}(x_i)\} \\ + \min\{\ddot{B}^-_{pq}(x_i), \ddot{D}^-_{pq}(x_i)\} + \min\{\ddot{B}^+_{pq}(x_i), \ddot{D}^+_{pq}(x_i)\}\end{pmatrix}}{\begin{pmatrix}\max\{\ddot{A}^-_{pq}(x_i), \ddot{C}^-_{pq}(x_i)\} + \max\{\ddot{A}^+_{pq}(x_i), \ddot{C}^+_{pq}(x_i)\} \\ + \max\{\ddot{B}^-_{pq}(x_i), \ddot{D}^-_{pq}(x_i)\} + \max\{\ddot{B}^+_{pq}(x_i), \ddot{D}^+_{pq}(x_i)\}\end{pmatrix}}\right\}.$$

The weight of the q th attribute can be expressed as

$$W_q = \frac{nc - E_q}{n \times nc - \sum\limits_{q=1}^{n} E_q}$$

where nc is a constant, usually set to 1, and can also be adjusted appropriately according to the weight relationship. For objective weights, in principle, the maximum weight and minimum weight of the indicator should be controlled within one time; if the gap is too large, the value of nc can be adjusted appropriately.

When $nc = 1$, the weight is

$$W_q = \frac{1 - E_q}{n - \sum\limits_{q=1}^{n} E_q}.$$

According to entropy theory, if the entropy value of a criterion or feature is smaller in different alternatives, it can provide more useful information for decision makers. Therefore, criteria or characteristics should be given greater weight; otherwise, such criteria or characteristics will be considered unimportant by most decision makers. In other words, such criteria or characteristics should be evaluated with less weight.

When the attribute information data of each alternative is a CIVIFS-IQ, from the perspective of reflecting the original decision information, then the more fuzzy and uncertain the attribute information is, the less information of the attribute that is available for the scheme, the greater the entropy value, and the smaller the weight that should be assigned, and vice versa. Therefore, using CIVIF entropy to determine the weight of the attribute index can not only reduce the loss of evaluation information but also reflect the will of decision-makers.

5.2. MCDM approach based on the proposed correlation coefficient

It is assumed that there are m diverse alternatives, designated by $O_1, O_2, \ldots, O_m$, for MCDM with CIVIF information. These alternatives will be assessed under a set of n criteria, denoted by $Q_1, Q_2, \ldots, Q_n$. Let's say a specialist is asked to assess these alternatives based on each set of criteria. Consider the values offered by CIVIFS-IQ environment experts. These values can be viewed as CIVIFS-IQ. The following is a representation of the rating values for each alternative as CIVIFSs-IQ $O_p(p = 1, 2, \ldots, m)$.

$$O_p = \{(Q_q, \ddot{A}_{pq}(Q_q), \ddot{B}_{pq}(Q_q), \ddot{C}_{pq}(Q_q), \ddot{D}_{pq}(Q_q) \mid q = 1, 2, \ldots n; p = 1, 2, \ldots m\},$$

where $\ddot{A}_{pq}(Q_q) \subseteq [0, 1]$ represents the satisfaction degree interval of the alternative O_p toward the criteria Q_q and $\ddot{C}_{pq}(Q_q)$ represents the possible degree interval of the rejection for the alternative O_p under the criteria Q_q. For convenience, we denote this CIVIFSs-IQ by $a_{pq} = (\ddot{A}_{pq}, \ddot{B}_{pq}, \ddot{C}_{pq}, \ddot{D}_{pq})$, where $\ddot{A}_{pq}, \ddot{B}_{pq}, \ddot{C}_{pq}, \ddot{D}_{pq} \subseteq [0, 1]$ and $\ddot{A}_{pq} + \ddot{C}_{pq} \subseteq [0.1], \ddot{B}_{pq} + \ddot{D}_{pq} \subseteq [0, 1]$ for $p = 1, 2, \ldots, m; q = 1, 2, \ldots, n$, and call it a complex interval-value intuitionistic fuzzy number by interval quaternion number (CIVIFN-IQN). Then, we utilize the following steps based on the proposed correlation coefficients for solving the MCDM problems in the CIVIFS-IQ environment.

Step 1. Construct a decision matrix based on the collective information of the alternatives $O_p(p = 1, 2, \ldots, m)$ as provided by an expert in terms of CIVIFSs-IQ $a_{pq} = (\ddot{A}_{pq}, \ddot{B}_{pq}, \ddot{C}_{pq}, \ddot{D}_{pq}) = ([\ddot{A}^-_{pq}, \ddot{A}^+_{pq}], [\ddot{B}^-_{pq}, \ddot{B}^+_{pq}], [\ddot{C}^-_{pq}, \ddot{C}^+_{pq}], [\ddot{D}^-_{pq}, \ddot{D}^+_{pq}])$. Such a matrix was designated as $D = (a_{pq})_{m\times n}$, which is expressed as

$$D = \begin{array}{c} \\ O_1 \\ O_2 \\ \vdots \\ O_m \end{array} \begin{array}{c} \begin{array}{cccc} Q_1 & Q_2 & \cdots & Q_n \end{array} \\ \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \end{array}.$$

Step 2. To create the ideal set. The CIVIFS-IQ ideal set is $B^+ = (b_1^+, b_2^+, ..., b_n^+)$, where $b_q^+ =$

$\{[\bar{r}_q^-, \bar{r}_q^+], [\bar{\omega}_{r_q}^-, \bar{\omega}_{r_q}^+], [\bar{k}_q^-, \bar{k}_q^+], [\bar{\omega}_{k_q}^-, \bar{\omega}_{k_q}^+]\}, q = 1, 2, ..., n$, and where

$$b_q^+ = \begin{cases} \bar{r}_q^- = \max_P \ddot{A}_{pq}^-, & \bar{r}_q^+ = \max_p \ddot{A}_{pq}^+, \\ \bar{\omega}_{r_q}^- = \max_p \ddot{B}_{pq}^-, & \bar{\omega}_{r_q}^+ = \max_p \ddot{B}_{pq}^+, \\ \bar{k}_q^- = \min_p \ddot{C}_{pq}^-, & \bar{k}_q^+ = \min_p \ddot{C}_{pq}^+, \\ \bar{\omega}_{k_q}^- = \min_p \ddot{D}_{pq}^-, & \bar{\omega}_{k_q}^+ = \min_p \ddot{D}_{pq}^+. \end{cases}$$

Step 3. Use the following formula to calculate the weight value of each criterion.

$$W_q = \frac{1 - E_q}{n - \sum_{q=1}^{n} E_q} \tag{5.5}$$

where

$$E_q = \frac{1}{m} \sum_{p=1}^{m} \left\{ \frac{\min\{\ddot{A}_{pq}^-, \ddot{C}_{pq}^-\} + \min\{\ddot{A}_{pq}^+, \ddot{C}_{pq}^+\} + \min\{\ddot{B}_{pq}^-, \ddot{D}_{pq}^-\} + \min\{\ddot{B}_{pq}^+, \ddot{D}_{pq}^+\}}{\max\{\ddot{A}_{pq}^-, \ddot{C}_{pq}^-\} + \max\{\ddot{A}_{pq}^+, \ddot{C}_{pq}^+\} + \max\{\ddot{B}_{pq}^-, \ddot{D}_{pq}^-\} + \max\{\ddot{B}_{pq}^+, \ddot{D}_{pq}^+\}} \right\}. \tag{5.6}$$

Step 4. Calculate the correlation coefficient $\mathcal{K}(B^+, O_p)$ between the alternative $O_p(p = 1, 2, \ldots, m)$ and the ideal set B^+ by the following formulas.

$$\begin{aligned} \mathcal{K}(B^+, O_p) &= \frac{C_w(B^+, O_p)}{\sqrt{\mathcal{T}_w(B^+) \times \mathcal{T}_w(O_p)}} \\ &= \frac{\sum_{q=1}^{n} \left(w_q \begin{bmatrix} \ddot{A}_{pq}^- \bar{r}_q^- + \ddot{A}_{pq}^+ \bar{r}_q^+ + \ddot{B}_{pq}^- \bar{\omega}_{r_q}^- + \ddot{B}_{pq}^+ \bar{\omega}_{r_q}^+ \\ + \ddot{C}_{pq}^- \bar{k}_q^- + \ddot{C}_{pq}^+ \bar{k}_q^+ + \ddot{D}_{pq}^- \bar{\omega}_{k_q}^- + \ddot{D}_{pq}^+ \bar{\omega}_{k_q}^+ \end{bmatrix} \right)}{\left\{ \begin{array}{l} \sqrt{\sum_{q=1}^{n} \left(w_q \begin{bmatrix} (\ddot{A}_{pq}^-)^2 + (\ddot{A}_{pq}^+)^2 + (\ddot{B}_{pq}^-)^2 + (\ddot{B}_{pq}^+)^2 \\ + (\ddot{C}_{pq}^-)^2 + (\ddot{C}_{pq}^+)^2 + (\ddot{D}_{pq}^-)^2 + (\ddot{D}_{pq}^+)^2 \end{bmatrix} \right)} \\ \times \sqrt{\sum_{q=1}^{n} \left(w_q \begin{bmatrix} (\bar{r}_q^-)^2 + (\bar{r}_q^+)^2 + (\bar{\omega}_{r_q}^-)^2 + (\bar{\omega}_{r_q}^+)^2 \\ + (\bar{k}_q^-)^2 + (\bar{k}_q^+)^2 + (\bar{\omega}_{k_q}^-)^2 + (\bar{\omega}_{k_q}^+)^2 \end{bmatrix} \right)} \end{array} \right\}}. \end{aligned} \tag{5.7}$$

Step 5. The alternatives are ranked according to the value of the correlation coefficient calculated in Step 4. The better the alternatives, the higher the correlation coefficient value.

5.3. Illustrative example

We present an example to highlight the potential uses of the developed methodology. The source of the example is Garg and Rani [27]. Let's say a business owner chooses to purchase new equipment for his enterprise, and the manufacturer offers details on four models, each with a distinct manufacturing date. The business owner made the decision to choose machinery based on four criteria: Q_1: Reliability; Q_2: Safety; Q_3: Flexibility; and Q_4: Productivity. The same sort of machine's manufacture date modification could likewise affect these criteria. The entrepreneur wants to select the best model from the range of machines that are offered. Assume the person making the decisions expresses his preferences using CIVIFS-IQ. Consider the following scenario: From 50% to 60%, the decision-makers agree that O_1 is appropriate at Q_1, while from 10% to 30%, they disagree. The phase term that denotes the production date is provided as the decision-maker agrees, where O_1 has a production date being appropriately at Q_1 from 30% to 50% and disagrees from 20% to 40%. Therefore, the information that the decision-maker has about O_1 at Q_1 can be written as follows: ([0.5,0.6],[0.3,0.5],[0.1,0.3],[0.2,0.4]). Likewise, all information is acquired in terms of CIVIFN-IQN. We rate the alternatives using the following steps and apply the defined method to determine which mechanical device is best.

Step 1. The rating values of each alternative $O_P(p = 1, 2, 3, 4)$ are expressed by an expert under the set of criteria $Q_q(q = 1, 2, 3, 4)$ and is represented by the following matrix D as

$$D = \begin{array}{c} \\ O_1 \\ O_2 \\ O_3 \\ O_4 \end{array} \begin{array}{c} \begin{array}{cccc} \quad Q_1 \quad & \quad Q_2 \quad & \quad Q_3 \quad & \quad Q_4 \quad \end{array} \\ \left(\begin{array}{cccc} \begin{pmatrix} [0.5, 0.6], [0.3, 0.5], \\ [0.1, 0.3], [0.2, 0.4] \end{pmatrix} & \begin{pmatrix} [0.4, 0.7], [0.4, 0.5], \\ [0.2, 0.3], [0.1, 0.3] \end{pmatrix} & \begin{pmatrix} [0.3, 0.4], [0.1, 0.4], \\ [0.2, 0.3], [0.3, 0.5] \end{pmatrix} & \begin{pmatrix} [0.2, 0.5], [0.3, 0.4], \\ [0.1, 0.3], [0.1, 0.3] \end{pmatrix} \\ \begin{pmatrix} [0.2, 0.4], [0.1, 0.3], \\ [0.3, 0.4], [0.2, 0.6] \end{pmatrix} & \begin{pmatrix} [0.4, 0.6], [0.4, 0.5], \\ [0.1, 0.2], [0.3, 0.4] \end{pmatrix} & \begin{pmatrix} [0.2, 0.5], [0.4, 0.6], \\ [0.2, 0.5], [0.3, 0.5] \end{pmatrix} & \begin{pmatrix} [0.3, 0.5], [0.2, 0.5], \\ [0.2, 0.3], [0.2, 0.5] \end{pmatrix} \\ \begin{pmatrix} [0.1, 0.5], [0.3, 0.5], \\ [0.2, 0.4], [0.3, 0.4] \end{pmatrix} & \begin{pmatrix} [0.5, 0.6], [0.2, 0.5], \\ [0.1, 0.2], [0.1, 0.3] \end{pmatrix} & \begin{pmatrix} [0.2, 0.4], [0.3, 0.6], \\ [0.1, 0.3], [0.1, 0.2] \end{pmatrix} & \begin{pmatrix} [0.1, 0.5], [0.1, 0.5], \\ [0.2, 0.4], [0.1, 0.2] \end{pmatrix} \\ \begin{pmatrix} [0.2, 0.4], [0.3, 0.4], \\ [0.1, 0.5], [0.1, 0.2] \end{pmatrix} & \begin{pmatrix} [0.3, 0.4], [0.2, 0.5], \\ [0.1, 0.2], [0.1, 0.5] \end{pmatrix} & \begin{pmatrix} [0.5, 0.6], [0.3, 0.5], \\ [0.2, 0.3], [0.1, 0.4] \end{pmatrix} & \begin{pmatrix} [0.3, 0.4], [0.3, 0.5], \\ [0.1, 0.4], [0.2, 0.4] \end{pmatrix} \end{array} \right) \end{array}.$$

Step 2. Set B^+ is utilized as an ideal set, and an expert gave their preferences with respect to all the criteria in terms of CIVIFS-IQ. These can be summed up as follows.

$$\begin{aligned} B^+ = \{ & (Q_1, [0.5, 0.6], [0.3, 0.5], [0.1, 0.3], [0.1, 0.2]), (Q_2, [0.5, 0.7], [0.4, 0.5], [0.1, 0.2], [0.1, 0.3]), \\ & (Q_3, [0.5, 0.6], [0.4, 0.6], [0.1, 0.3], [0.1, 0.2]), (Q_4, [0.3, 0.5], [0.3, 0.5], [0.1, 0.3], [0.1, 0.2])\}. \end{aligned}$$

Step 3. We use Eqs (5.6) and (5.5) to calculate the entropy measure E_q and weight value W_q of each criteria, respectively.

$E_1 = 0.6411$, $E_2 = 0.5020$, $E_3 = 0.6355$, $E_4 = 0.6800$.

$W_1 = 0.2328$, $W_2 = 0.3230$, $W_3 = 0.2365$, $W_4 = 0.2077$.

Step 4. By applying the correlation coefficient $\mathcal{K}$ as given in Eq (5.7) between the set $O_P(p = 1, 2, 3, 4)$ and the ideal set B^+, we get their measurement values as

$\mathcal{K}(O_1, B^+) = 0.9468, \mathcal{K}(O_2, B^+) = 0.9856, \mathcal{K}(O_3, B^+) = 0.9414, \mathcal{K}(O_4, B^+) = 0.9373.$

Step 5. Based on the optimal values of the alternative, we conclude that its ranking order is $O_2 \succ O_1 \succ O_3 \succ O_4$. Therefore, the decision maker can choose alternative O_2.

5.4. Comparative analysis

This section provides a comparison between the proposed measure performance and some of the existing methods used in the CIVIFS as well as IVIFS environments.

5.4.1. Comparison with different CIVIFS methods

An analysis of the taken into consideration data has been done in order to compare the final results of the proposed method with some of the existing methods [27, 32] within the CIVIFS environment.

Since the examples used in [27,32] are also used in this article, the analysis that follows is done directly. The following is an overview of the outcomes that correlate to these approaches:
(1) If we use the method of [27], the ranking of the alternatives is $O_2 > O_3 > O_1 > O_4$, and, hence, we conclude that the best alternative is O_2.
(2) If we use the method of [32], the ranking of the alternatives is $O_2 > O_3 > O_4 > O_1$, and, hence, we conclude that the best alternative is O_2.
The best alternative produced from the proposed method coincides with the existing method, according to these comparison studies, validating the approach's feasibility in the CIVIFS-IQ environment.

The method used in [27] is the aggregate operator, and the method used in [32] is TODIM. The attribute weights of both methods are given directly, but in the method proposed in this paper, the attribute weights are calculated using the entropy weight method.

The entropy weight method is an objective weighting method based on the principle of information entropy, which is used for the weight of each index in a multi-index comprehensive evaluation. The advantage of the entropy weight method is that it has strong objectivity, is not affected by subjective factors, and can better reflect the difference and importance of indicators. However, it also has some disadvantages, such as being more sensitive to abnormal values, which can lead to improper weight allocation. In addition, the entropy weight method has high requirements for data integrity, and missing data may affect the accuracy of evaluation results.

5.4.2. Comparison with different IVIFS methods

We transform the considered data into the IVIFNs since the IVIFS is a special case of the CIVIFS. This allows us to compare the performance of the proposed approach in this context. To do this, we set each CIVIFN's phase term to zero; this is equivalent to $\ddot{B}_{pq} = 0, \ddot{D}_{pq} = 0$.
(1) Utilizing the weighted correlation coefficient (W_1) as suggested by [48], the measurement values for each alternative are summed up as follows: $W_1(O_1, P) = 0.7929$, $W_1(O_2, P) = 0.8480$, $W_1(O_3, P) = 0.7717$, and $W_1(O_4, P) = 0.7986$. Then, the ranking of the alternatives is $O_2 > O_4 > O_1 > O_3$ and, hence, we conclude that the best alternative is O_2.
(2) Upon applying the relative similarity measure (W_2), as described by [46], to the considered data, we obtain $W_2(O_1) = 0.4873$, $W_2(O_2) = 0.5324$, $W_2(O_3) = 0.5029$, and $W_2(O_4) = 0.4930$. Then, the ranking of the alternatives is $O_2 > O_3 > O_4 > O_1$ and, hence, we conclude that the best alternative is O_2.
(3) If we use the method of [49], we obtain $W_3(O_1) = 0.2235$, $W_3(O_2) = 0.2283$, $W_3(O_3) = 0.1557$, and $W_3(O_4) = 0.1432$. The ranking of the alternatives is $O_2 > O_1 > O_3 > O_4$, and, hence, we conclude that the best alternative is O_2.
(4) If we use the method of [13], we obtain $W_4(O_1) = 0.0162$, $W_4(O_2) = 0.0212$, $W_4(O_3) = -0.0307$, and $W_4(O_4) = -0.0292$. The ranking of the alternatives is $O_2 > O_1 > O_4 > O_3$, and, hence, we conclude that the best alternative is O_2.
(5) If we use the method of [9], we obtain $W_5(O_1) = 0.2150$, $W_5(O_2) = 0.2205$, $W_5(O_3) = 0.1495$, and $W_5(O_4) = 0.1400$. The ranking of the alternatives is $O_2 > O_1 > O_3 > O_4$, and, hence, we conclude that the best alternative is O_2.

This analysis makes it evident that the results produced by the existing approaches agree with the proposed one, hence supporting the proposed method.

Table 1 presents an analysis that compares the proposed method with other similar approaches that have been described in different literature sources. Compared to other techniques, the CIVIFS-IQ-MCDM methodology is found to reflect greater levels of overall dominance scores. This is important because it suggests that the CIVIFS-IQ-MCDM technique could be able to keep the fuzzy information more in line with actual circumstances, making the rankings more trustworthy.

Table 1. Comparison analysis between the proposed CIVIF-MCDM approach and the existing fuzzy MCDM approaches.

Method	Uncertainty	Hesitancy	Multi-dimensional data	Periodicity	Whether describe information by interval-valued numbers
FS-MCDM	✓	×	×	×	×
IFS-MCDM	✓	✓	×	×	×
IVIFS-MCDM	✓	✓	×	×	✓
CIFS-MCDM	✓	✓	✓	✓	×
Proposed model	✓	✓	✓	✓	✓

5.4.3. Advantages of the proposed approach

From the existing research and the proposed method, the advantages of using the correlation coefficient to solve the decision problem in the CIVIFSs-IQ environment are as follows.

(1) A CIVIFS is a generalization of the existing studies such as CIFS, CFS, and IVIFS, IFS, FS. It does this by centralizing the processing of four-dimensional information into a single set and accounting for more object information. Experts' processing freedom can be increased by the use of interval values, which lowers information loss. As a result, compared to the current correlation coefficient, the correlation coefficient proposed in the CIVIFS-IQ environment is more general.

(2) The proposed decision-making approach's main benefits are that it takes into account a lot more data to access alternatives and lessen information loss. Furthermore, the correlation coefficients based on the with or without weighting factor will enable the decision-maker to select the best alternatives with increased accuracy.

(3) In the method proposed in this paper, we use the entropy weight method to calculate the criteria weight. The entropy weight method has higher objectivity, is free from the interference of subjective factors, and can reflect the difference and importance of indicators more accurately. Therefore, the method proposed in this paper can deal with the MCDM problem with completely unknown criteria weights and provide effective help for decision-makers.

6. Conclusions

CIVIFSs are helpful in simulating real-world scenarios that take uncertainty and periodic data into account simultaneously. Because in the process of data collection, due to the limitations of many factors, it is not always possible to expect accurate statements to be collected, and one must agree to minimize errors as much as possible. As a result, selecting an interval value will better reflect the real world when conveying information value. The CIVIFS is a more extensive extension of the existing FS theory. In this paper, the interval quaternion is used to represent the CIVIFS, the order relation is analyzed, and a new operation based on the interval quaternion is introduced. Furthermore, the proposed method has the ability to transmit multidimensional fuzzy information and capture complex features. A new scoring function is proposed in the CIVIFS-IQ environmental. The score function proposed in this paper overcomes the shortcomings of other score functions to some extent, is more faithful to decision information, and more truly reflects the will of decision makers. In FS theory, the degree of reliance between two FSs is determined by the correlation coefficient, which provides a measure of the correlation between two variables. In this paper, the correlation coefficient in the CIVIFS-IQ environment is presented, and the interval quaternion representation and correlation coefficient are applied to the MCDM model, and the model is applied to the enterprise decision problem.

Author contributions

Zengtai Gong: Conceptualization, formal analysis, investigation, methodology, and supervision; Yanhong Su: Conceptualization, methodology, writing-original; Na Qin: Draft resources and editing. All authors have read and approved the final version of the manuscript for publication.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that they have no conflict of interest.

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