Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 9(8): 21952–21971.
DOI: 10.3934/math.20241067
Received: 28 March 2024
Revised: 06 June 2024
Accepted: 11 June 2024
Published: 11 July 2024

Research article

Analysis and forecasting of electricity prices using an improved time series ensemble approach: an application to the Peruvian electricity market

Salvatore Mancha Gonzales[1], **Hasnain Iftikhar**[1,2,*] **and Javier Linkolk López-Gonzales**[1]

[1] Escuela de Posgrado, Universidad Peruana Unión, Lima, Peru
[2] Department of Statistics, Quaid-i-Azam University, Islamabad 45320, Pakistan

* **Correspondence:** Email: hasnain@stat.qau.edu.pk.

Abstract: In today's electricity markets, accurate electricity price forecasting provides valuable insights for decision-making among participants, ensuring reliable operation of the power system. However, the complex characteristics of electricity price time series hinder accessibility to accurate price forecasting. This study addressed this challenge by introducing a novel approach to predicting prices in the Peruvian electricity market. This approach involved preprocessing the monthly electricity price time series by addressing missing values, stabilizing variance, normalizing data, achieving stationarity, and addressing seasonality issues. After this, six standard base models were employed to model the time series, followed by applying three ensemble models to forecast the filtered electricity price time series. Comparisons were conducted between the predicted and observed electricity prices using mean error accuracy measures, graphical evaluation, and an equal forecasting accuracy statistical test. The results showed that the proposed novel ensemble forecasting approach was an efficient and accurate tool for forecasting monthly electricity prices in the Peruvian electricity market. Moreover, the ensemble models outperformed the results of earlier studies. Finally, while numerous global studies have been conducted from various perspectives, no analysis has been undertaken using an ensemble learning approach to forecast electricity prices for the Peruvian electricity market.

Keywords: Peruvian electricity market; analysis and forecasting electricity prices; time series models; weighting ensemble modeling; a novel time series ensemble approach
Mathematics Subject Classification: 62E15, 62G30

1. Introduction

The Peruvian electricity market is collaborative, with key players working together to ensure efficiency and reliability. The Ministry of Energy and Mines formulates and evaluates national policies related to energy development, promoting sustainability while minimizing environmental impact. The Supervisory Agency of Investment in Energy and Mining oversees and regulates the Energy and Mining sectors, setting regulated electricity prices and ensuring compliance with regulations. The Committee for the Economic Operation of the National Interconnected System manages the balance between electricity generation and demand, determining the optimal dispatch of resources and coordinating the expansion of the transmission infrastructure [1].

The National Institute for the Defense of Free Competition and the Protection of Intellectual Property ensures fair competition, protects consumer rights, and investigates potential violations in the electricity market—Peru's electricity market functions as a regulated system. The regulatory authority establishes a comprehensive tariff structure that includes costs associated with generation, transmission, distribution, and a reasonable profit margin for distribution companies. This regulated price aims to provide stability and predictability for consumers, and any changes require approval from the Supervisory Agency of Investment in Energy and Mining [2].

Grasping electric power price is critical for all system users. This insight allows for educated decisions on energy infrastructure investment, pricing strategies, and overall system dependability (see [3, 4]). Forecasting electric power prices is valuable, especially when studying historical electrical power price time series [5–7]. These studies give valuable insights by detecting trends, seasonality, patterns, and peak-price times linked with high system load. The inherent unpredictability of the energy price series has prompted substantial studies into electricity price forecasting, employing various modeling tools and approaches [8–10]. Initial studies on electricity price forecasting typically employed statistical techniques and decomposition-combination approaches [11–13]. More advanced methods, such as machine learning models, hybrid learning techniques, and ensemble learning algorithms, have also been employed [14, 15]. The statistical methods included linear and nonlinear autoregressive models and other variants, such as integrated moving averages. Additionally, uncomplicated, double, and triple exponential smoothing models were applied, and they are known for their relatively straightforward mathematical structures compared to more computationally intensive models [16]. For example, the study in [17] proposed a model decomposed into a deterministic component encompassing trends and periodicities and a stochastic component. The proposed forecasting technique was applied to the Italian electricity price time series, which generated one-day-ahead forecasts for a year, demonstrating the methodology's effectiveness in forecasting electricity prices.

Unlike traditional methods and models reliant on standard regression and time series approaches, machine learning models excel at addressing the nonlinearities inherent in electricity price forecasting problems. For example, the authors in [18] introduced machine learning models that predict electricity prices, including long short-term memory, convolutional neural networks, gated recurrent units, and deep neural networks. The study then conducted a comprehensive benchmark to assess the models' accuracy. All four models achieved significant results, with deep neural networks demonstrating the best overall performance. On the other hand, the researchers have also investigated hybrid models combining features from various existing models [19–21]. For instance, the work in [22] utilized wavelet transforms to enhance accuracy and incorporated machine learning models like kernel extreme learning machine and Self-adaptive particle swarm optimization for optimization. The results demonstrated improved prediction accuracy and greater applicability than individual or hybrid methods. However, many authors used ensemble learning to improve forecasting accuracy, where multiple learners are trained and combined for better accuracy, gaining traction in academic literature [23, 24]. For instance, a study in [25] proposed an ensemble method for the Italian market that involved filtering the price series and utilizing machine learning models for the residual segment. The final prediction combined estimates from both components and outperformed other individual models, highlighting the ensemble approach's potential.

This study proposes a new approach to forecasting monthly prices in the Peruvian electricity market. The strategy involves training multiple learners to combine their predictions later to improve accuracy, a technique known as ensemble learning. The goal is to leverage individual learners' diversity and strengths, often generated using different algorithms or subsets of data, to create a more robust, accurate, and efficient forecasting model. The proposed time series ensemble approach treats the first preprocessed electricity prices time series for missing values, variance stabilization, normalization, stationarity, and seasonality issues to achieve this. It then uses six standard time series models and three proposed ensemble models to anticipate the clean prices time series free from initial treatments. The proposed ensemble models are based on the weighting technique, such as equal weight to single models, in-sample-based weighing (training), and out-of-sample (validation). The proposed improved time series ensemble forecasting approach anticipates monthly electric power prices using Peruvian electricity market data spanning January 2001 to December 2022. Therefore, the primary contributions to this work are as follows:

i) An improved approach to forecasting month-ahead electric power prices is suggested, which uses a time series ensemble method for better accuracy and efficiency.

ii) The efficiency of the proposed ensemble models is compared to various single linear and nonlinear time-series models, which are also used within the proposed improved ensemble forecasting approach.

iii) To evaluate the accuracy and efficiency of the proposed approach, six different accuracy average errors are determined, and a statistical equal forecast test, the Diebold-Marino test, and graphical evaluation using line plots, bar plots, autocorrelation, and partial correlation plots are performed.

iv) The outcomes of the best ensemble model are highly accurate and efficient compared to the best models reported in the literature for month-ahead electric power price forecasting.

v) The proposed improved time series ensemble forecasting approach can be extended and applied to other energy markets to assess its efficacy.

vi) This study is the first to propose a forecasting strategy for the Peruvian electricity market.

The rest of the manuscript is organized into distinct sections: Section 2 details the proposed approach, Section 3 applies it to the Peruvian dataset and compares it to existing models (Section 4), and Section 5 concludes the study with limitations and future research directions.

2. The proposed ensemble approach for time series

This section exposes a recently introduced time series ensemble technique tailored explicitly for predicting electricity prices within Peru's electricity market framework. The temporal scope under scrutiny pertains to the forecasting horizon of one month in advance. In the proposed ensemble forecasting approach, the first step involves preprocessing the monthly electricity price data to address missing values, variance stabilization, normalization, stationarity, and seasonality. Subsequently, a set of six standard time series models is systematically employed. These models include the autoregressive, autoregressive integrated moving averages, exponential smoothing model, nonlinear autoregressive integrated moving averages, neural network autoregressive, and the theta model. In addition, three ensemble models are introduced to predict future electricity prices without the issues mentioned, such as missing values and inconsistencies in variance, normalization, stationarity, and seasonality.

The ensemble models utilized a weighting technique incorporating equal weights for individual models, in-sample-based weighing during the training phase, and out-of-sample weighing during validation. An analysis of monthly electricity price data spanning from January 2001 to December 2022 was undertaken to complete the current study. Six distinct accuracy measures were employed, encompassing mean absolute error, mean absolute percent error, mean absolute scaled error, root relative squared error, root mean squared error, and root mean squared log error.

The proposed novel time series ensemble technique was evaluated using line charts, bar charts, autocorrelation, and partial correlation charts. Additionally, hypothesis testing was incorporated, utilizing the Diebold-Mariano test. Furthermore, a comparative analysis was conducted by pitting the forecasting results of the best ensemble model against the most robust evidence presented by individual models.

2.1. Preparation of raw data

The main goal of this study is to achieve an accurate and efficient one-month-ahead forecast for Peruvian electricity prices. To achieve this objective, we implemented a sequential methodology to model and forecast the electricity price time series in the Peruvian market. We aim to understand the complex characteristics of electricity price dynamics over time. These characteristics are expected to include missing values, a nonlinear long-run trend, pronounced seasonality, high volatility, non-normality, and non-stationarity.

See Figure 1a for the monthly electricity price time series, Figure 1b for the mean of twenty years' monthly variations in electricity prices, Figure 1c for the autocorrelation plot of the original electricity prices at sixty lags, and Figure 1d for the partial autocorrelation plot of original electricity prices at sixty lags. The figure clearly illustrates a discernible nonlinear long-run trend and an annual seasonality. Furthermore, non-normality and non-stationarity are also evident from these visual representations.

The proposed approach addresses these irregularities before modeling and forecasting to achieve high accuracy and efficient forecasts. First, it replaces a few missing observations with the mean of their five surrounding values. Second, the natural logarithm is applied to stabilize variance and standard deviation. Third, the regression splines method captures the nonlinear trend, and dummy variables address monthly seasonality. For instance, model these components using the following procedure: Let the time series of the electric power price time series be denoted by $log(Y_m)$; m shows the mth month data point. Thus, the dynamics of the log monthly electric prices times series, $log(Y_m)$, may be described as:

$$log(Y_m) = \tau_m + a_m + y_m. \tag{2.1}$$

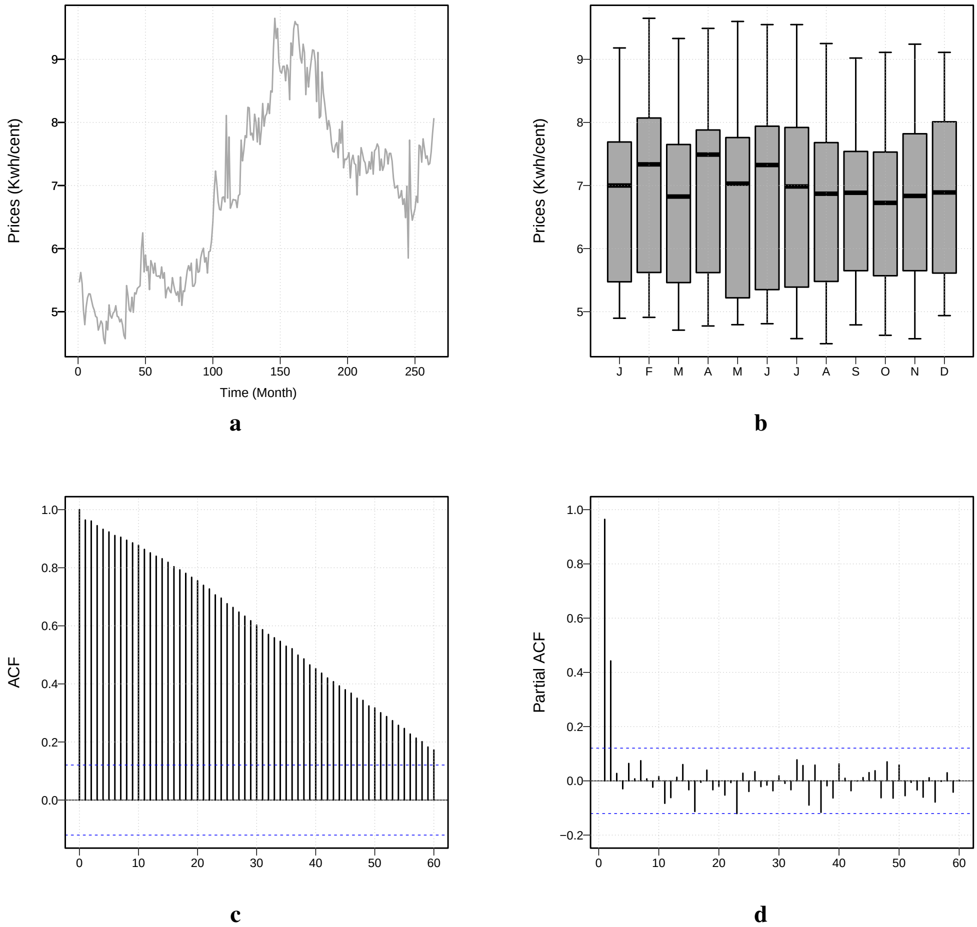


Figure 1. Characterization of Peruvian electricity prices (2001–2022): time series plot (a): visualizes the price movements over time; mean monthly variation (b): displays the average monthly change in prices; autocorrelation function plot (c): analyzes the correlation of price changes at different time lags; partial autocorrelation function plot (d): examines the correlation between price changes after accounting for the influence of past lags.

That is, the $\log(Y_m)$ is divided into these components: a long-run nonlinear trend component (τ_m), a yearly seasonality component (a_m), and a residual component (y_m). The (τ_m) component is a function of the series $(1, 2, 3, \ldots, m)$ and is estimated by the regression splines method, and dummies capture the annual periodicity

$$a_{m,j} = \sum_{\mathrm{j}=1}^{12} \zeta_j I_{m,j}.$$

The variable $I_{m,j}$ is assigned a value of 1 when j refers to the ith month and 0 otherwise. The regression coefficients (ζ_j) associated with these components are determined using the ordinary least square method. Once the estimated deterministic component (long-run trend and annual periodicity) is obtained, the residual or stochastic component can be derived as

$$y_m = log(Y_m) - (\hat{\tau}_m + \hat{a}_m). \tag{2.2}$$

Thus, once the electric power price time series (Y_m) is preprocessed (to address the issue of missing values and their imputation, stabilize the variance and standard deviation, and remove the deterministic properties), the next step is to model the remaining residual y_m series; the current work considers six single-time series models autoregressive, exponential smoothing, autoregressive integrated moving averages, nonlinear autoregressive, Theta, and neural network autoregressive, and three of their proposed ensemble models. The ensemble models utilize a weighting technique, assigning equal weights, in-sample-based weighting during training, and out-of-sample weighting during validation. Dedicated subsections provide detailed descriptions of these models, ensuring a methodical and structured presentation.

2.1.1. Autoregressive model

The autoregressive (AR) model is a time-series framework that uses past observations to predict future values. It assumes that a variable's future value depends linearly on past values, like the previous month's data influencing the next. The model uses coefficients to determine the weight of each past observation in the prediction. The order of an AR model is denoted as 'p', which specifies the number of past values considered [26]. For example, an AR(1) model uses only the previous value, while an AR(2) model uses the previous two values

$$v_m = \alpha_0 + \alpha_1 v_{m-1} + \alpha_2 v_{m-2} + \ldots + \alpha_p v_{m-p} + e_m. \tag{2.3}$$

In Eq (2.3), the α_0, denotes the intercept, and $\alpha_1, \alpha_1, \ldots, \alpha_p$, are the coefficient of $v_{m-1}, v_{m-2}, \ldots, v_{m-p}$, respectively.

2.1.2. Exponential smoothing (ESM) model

The ESM is a forecasting model used for time series data. It assigns decreasing weights to past observations as they get older, giving more importance to recent data. This reflects the idea that future values are more likely to be similar to recent observations than those from the distant past [27]. The ESM can be expressed mathematically as follows:

$$v_{m+1} = \alpha . v_m + (1 - \alpha_{)} \cdot v_{m-1}. \tag{2.4}$$

In Eq (2.4), α denotes the smoothing parameter that determines the weight of the most recent observation. However, the v_{m+1} denotes the predicted value of the variable at a time (m+1), v_m, the observed value of the variable at a time (m), and v_{m-1} observed the value of the variable at a time (m-1).

2.1.3. Theta model

The theta model is a different forecasting approach. It focuses on modifying the local curvature of the time series data through a coefficient theta (η), not directly calculating an average, while the provided formula represents a simple moving average [27]

$$v_{m,\eta} = a_\eta + b_\eta (t - 1) + \eta v_m. \tag{2.5}$$

In Eq (2.5),$v_{m,\eta}$ shows the forecasted value at a time (t) for a specific η value, and a_η, b_η constants are estimated through a minimization process. v_m denotes the actual data point at a time (t). η is the parameter influencing the forecast.

The theta model, characterized by its simplicity and user-friendliness, is an appropriate methodology for short-term forecasting within the domain of stationary time series. However, its performance may be limited by strong trends or seasonality.

2.1.4. Autoregressive moving average (ARMA) model

The ARMA model combines the strengths of both AR and moving average (MA) models to predict future values. It considers past observations, similar to AR models, and past forecast errors, like MA models, to make predictions. This model is defined by two parameters: p and q. The p represents the number of past observations used (AR order), and the q represents the number of past forecast errors included (MA order). The AR component captures the influence of past data points on the current value, while the MA component accounts for the impact of past forecasting errors [28]. Mathematically, an ARMA model can be expressed as follows:

$$v_m = \alpha_0 + \alpha_1 v_{m-1} + \alpha_2 v_{m-2} + \ldots + \alpha_p v_{m-p} + e_m + \theta_1 e_{m-1} + \theta_2 e_{m-2} + \ldots + \theta_q e_{m-q}. \tag{2.6}$$

In Eq (2.6), v_m shows the current value of the response variable; α_0 denotes the intercept, α_1, α_2, and α_p are the AR part coefficients; the e_m is an error term at time t, and $\theta_1, \ldots, \theta_q$ are the MA part coefficients.

2.1.5. Nonparametric autoregressive (NAR) model

NAR offers an alternative to traditional parametric models by utilizing flexible and adaptive techniques like kernel regression or spline functions. Unlike parametric models requiring mathematical equations, NAR eliminates the need to estimate specific parameters. Critical features of NAR include flexibility, the absence of predefined parameters, a focus on local relationships, and reliance on data-driven techniques to capture complex and nonlinear dependencies within time series data. Specifically, NAR does not assume a predetermined parametric form for the relationship between a data point v_m and its preceding terms, allowing them to accommodate potential nonlinearities [29]. This relationship is expressed as nonparametric, deviating from predefined mathematical structures. The relationship is expressed as

$$v_m = x_1(v_{m-1}) + x_2(v_{m-2}) + \ldots + x_p(v_{m-p}) + e_m. \tag{2.7}$$

In this context, x_j $(j = 1, 2, \ldots, p)$ denotes smoothing functions that capture the relationship between a data point v_m and its past values. For example, cubic regression splines are employed as the functions x_j, and lags 1, 2, and 3 are incorporated into the NAR modeling procedure.

2.1.6. Neural network autoregressive (NNA) model

The NNA model is a machine learning technique used in time series analysis to predict future values based on past observations. It considers past values denoted as $v_{m-1}, v_{m-2}, \ldots, v_{m-p}$, where p is the time lag parameter, to learn a function that maps these past observations to future values. The NNA model is trained using backpropagation and gradient descent to minimize the difference between its predictions and observed values. The forecasting process first involves determining the AR order, which specifies the number of past values needed to predict the current value. The NNA is then trained on a dataset that reflects this order. The number of input nodes in the NNA corresponds to the autoregressive order, with each node representing a past-lag observation in the univariate time series. The NNA's output node then provides the predicted value. Determining the optimal number of hidden nodes in an NNA often requires experimentation [30]. Therefore, careful consideration is necessary when selecting the number of training iterations to avoid overfitting the model. This study uses a simple NNA architecture with a single hidden layer containing three nodes. This can be expressed as

$$v_m = g(v_{m-1}, v_{m-2}, v_{m-3}),$$

where v_m represents the predicted value (e.g., preprocessed monthly electricity price). g represents the neural network function with three hidden nodes in a single layer. v_{m-1}, v_{m-2}, and v_{m-3} represent the past values used for prediction lagged by 1, 2, and 3 periods, respectively.

2.1.7. The proposed ensemble models

An ensemble technique, at its core, combines predictions from multiple models, each carefully calibrated before being merged. This approach leverages the strengths of individual models while mitigating their weaknesses. In this study, ensemble techniques were initially used to calculate weights for the outputs of individual models. Consequently, the proposed ensemble incorporates three distinct weighting strategies:

a) Equal weight: all individual models contribute equally to the ensemble, denoted as ensemble (E).

b) Weighting by training accuracy: weights are assigned based on the mean accuracy measures (the mean absolute error (MAE), the mean scaled absolute error (MASE), the mean absolute percent error (MAPE), the root mean squared error (RMSE), the root relative squared error (RRSE), and the root mean log squared error (RMSLE)), during training, denoted as ensemble (I). Models with higher training accuracy receive higher weights.

c) Weighting by validation accuracy: weights are assigned based on the mean accuracy measures on the validation set denoted as ensemble (O).

Models with higher validation accuracy receive higher weights. The model assigns greater weight to the ensemble model for datasets with a lower mean error (not mean accuracy errors). Models with higher mean errors contribute less weight to the ensemble. Notably, the model weights are small positive values that sum to one, representing the relative contribution of each model to the overall ensemble performance.

Thus, after estimating the long-run trend component and annual periodicity using the spline regression method and dummy variables discussed above, the next step is forecasting the remaining part (y_m) using six single and three proposed ensemble models as discussed above. Thus, this work can obtain the monthly electric power demand for the next month's forecast as follows:

$$\hat{Y}_{m+1} = \exp(\hat{\tau}_m + \hat{a}_m + \hat{y}_m). \tag{2.8}$$

2.2. Evaluation criteria

This study examines three distinct evaluation criteria applied to the proposed time series ensemble forecasting technique: accuracy metrics, a test of equal forecast accuracy, and graphical assessment (see [31]). Primarily, the accuracy metrics include the MAE, the MASE, the MAPE, the RMSE, the RMSLE, and the RRSE. Thus, the mathematical form of these accuracy metrics is presented in the following way:

$$\mathbf{MAE} = \frac{1}{\mathrm{M}} \sum_{\mathrm{m=1}}^{\mathrm{M}} |\mathrm{v_m} - \hat{v}_\mathrm{m}|,$$

$$\mathbf{MASE} = \frac{1}{\mathrm{M}} \left[\frac{|\mathrm{v_m} - \hat{v}_\mathrm{m}|}{\frac{1}{\mathrm{M\text{-}1}} \sum_{\mathrm{m=2}}^{\mathrm{M}} |\mathrm{v_m} - v_{\mathrm{m\text{-}1}}|} \right],$$

$$\mathbf{MAPE} = \frac{1}{\mathrm{M}} \sum_{\mathrm{m=1}}^{\mathrm{M}} \left| \frac{\mathrm{v_m} - \hat{v}_\mathrm{m}}{\mathrm{v_m}} \right|,$$

$$\mathbf{RMSE} = \left[\sum_{m=1}^{M} \left[\frac{(v_m - \hat{v}_m)^2}{M} \right] \right]^{0.5},$$

$$\mathbf{RMSLE} = \left[\frac{1}{M} \sum_{m=1}^{M} [\log(v_m + 1) - \log(\hat{v}_m + 1)]^2 \right]^{0.5},$$

$$\mathbf{RRSE} = \left[\frac{\sum_{m=1}^{M} (v_m - \hat{v}_m)^2}{\sum_{m=1}^{M} (v_m - \overline{v}_m)^2} \right]^{0.5}.$$

In the given formulations, v_m denotes observed values, while $\hat{v}_m$ represents forecasted electricity prices for the m-th observation ($m = 1, 2, ..., 48$). Generally, lower values for all the listed metrics indicate better predictive accuracy of the model.

Second, the Diebold-Mariano (DM) test [32] is employed to evaluate the accuracy of the proposed ensemble forecasting approach. This statistical test is commonly used in time series analysis to compare the accuracy of two forecasting models [33]. It evaluates whether the errors generated by one model are statistically different from the errors of another. To perform the DM test, the forecast errors of each model are calculated using a loss function. These errors are the differences between the observed values (v_m) and the forecasted values ($\hat{v}_m$). The test statistic is then calculated by comparing the mean squared errors of the two models. Suppose the test statistic is greater than a certain threshold and the p-value is lower than a predetermined significance level (e.g., $\alpha = 0.05$). In that case, we conclude that the forecasts from one model are statistically significantly better than the other.

The null and alternative hypotheses of the DM test are:

H_0: There is no difference in forecast accuracy between the two models ($H_0 : \bar{d} = 0$).

H_a: The two models differ in the forecast accuracy ($H_a : \bar{d} \neq 0$).

Therefore, the null hypothesis implies no statistically significant difference in forecast accuracy between the models, while the alternative hypothesis suggests a significant difference exists.

In the third phase of the analysis, a graphical evaluation is performed on the residuals derived from the forecasted and observed electricity prices generated by the final optimal model. This involves creating and analyzing various diagnostic plots, including histograms, line plots, box plots, and autocorrelation/partial autocorrelation plots. To summarize this section, the primary procedural steps of the proposed time series ensemble forecasting techniques are listed in bullet points below. Furthermore, a visual representation of the procedural flow is provided in Figure 2.

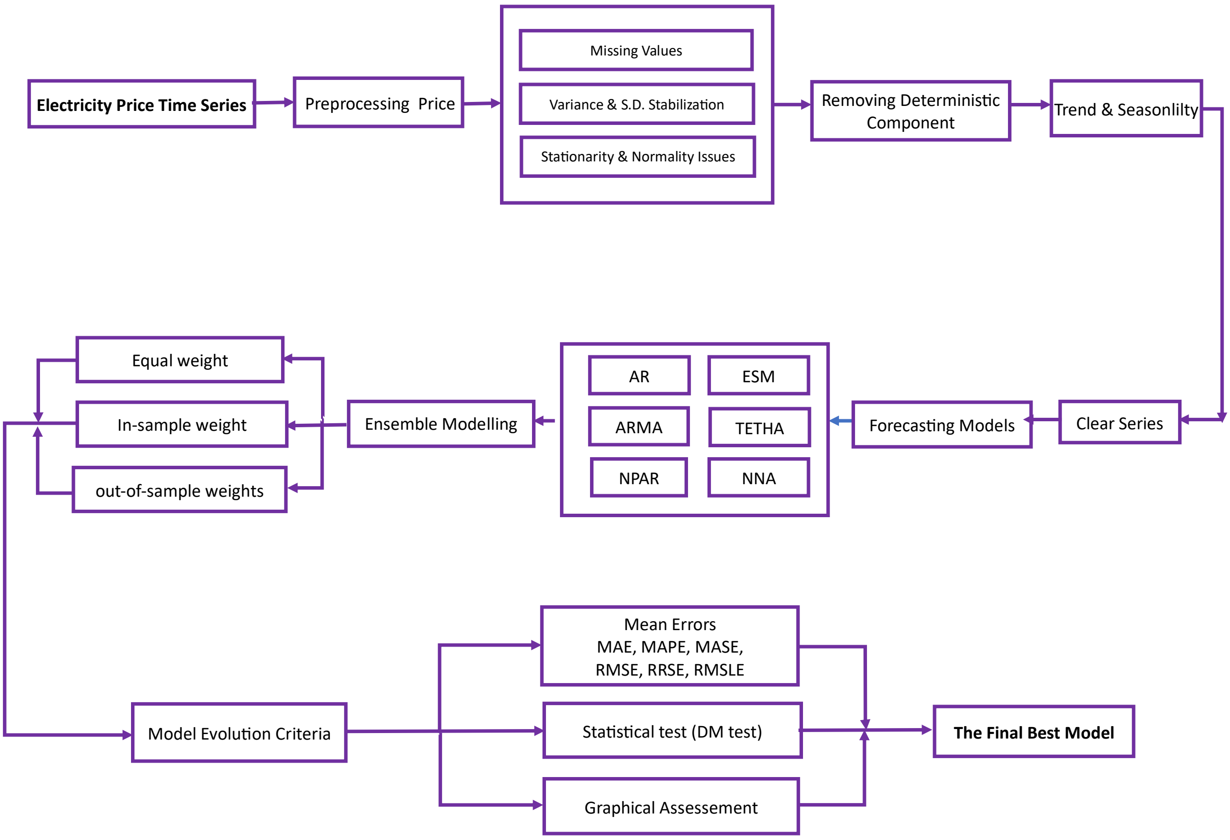


Figure 2. A proposed time series ensemble approach framework.

- In the first step, the electric power prices time series (Y_m) is preprocessed (to address the issue of missing values and their imputation, stabilize the variance and standard deviation, and remove the deterministic properties), discussed in detail in Section 2.1.
- In the second step, we divide the stochastic (short-run dynamic) electric power time (y_m) into three parts: training, validation, and testing datasets. Let

$$y_n, n = 1, 2, ..., N(264)$$

represent the electricity time series of electricity prices. Then, training dataset:

$$y_t, t = 1, 2, ..., T(180);$$

validation dataset:

$$y_u, u = 1, 2, ..., U(48);$$

testing dataset:

$$y_m, m = 1, 2, ..., M(48);$$

where

$$N(N = T + U + M)$$

is the total number of data points.

- In the third step, model the train data using single models, i.e., AR, ARMA, ESM, NAR, NNA, and theta models.
- In the fourth step, calculate the one-month-ahead forecast using the expanding window technique. The forecast values, $\hat{y}^j_{N-(T+U+m)}$ for j = 1, 2, 3, 4, 5, 6, are obtained by the models listed in step 3.
- In the fifth step, the output of a basic ensemble method is mathematically described by Eq (2.9).

$$\hat{y}^j_{N-(T+V)+m} = \sum_{j=1}^{6} \omega_j \hat{y}^j_{N-(T+V)+m}, \tag{2.9}$$

where ω_j, $j = 1, 2, \ldots, 6$ are obtained by three weighting strategies:
a) Equal weight to all single models and denoted by ensemble (E);

b) Weight assigned based on training, mean accuracy measures (MAE, MASE, MAPE, RMSE, RRSE, and RMSLE), and denoted by (ensemble (I));

c) Weight assigned based on validation mean accuracy measures and denoted by (ensemble (O)).

The lower accuracy means the error model assigns more weight to the ensemble model in training and validation datasets. In contrast, the highest accuracy mean error model has less weight than the ensemble model. However, the model weights are small, positive values, and the sum of all weights equals one, indicating the percentage of trust or expected performance from each model.
- In the sixth step, obtain the one-month-ahead forecast values using equations for the E, I, and O models.
- In the last step, evaluate the model based on accuracy and average errors (MAPE, MASE, MAE, RMSE, RMSLE, and RRSE).

3. Case study results

This study investigates monthly electricity prices in the Peruvian market from January 2001 to December 2022 (22 years, 264 data points). The data is divided into three sets: training (2001–2014), validation (2015–2018), and testing (2019–2022). Exploratory data analysis (Figure 1) reveals the presence of a nonlinear trend, seasonality, non-normality, and non-stationarity. Table 1 summarizes the characteristics of the data before and after various transformations, like taking the natural logarithm (addressing variance stabilization) and differencing (exploring stationarity). While a single statistical value suggests non-stationarity, further investigation using multiple tests is crucial for confirmation. The first-order difference may be applied to achieve stationarity, but its effectiveness needs to be statistically evaluated. If stationarity is confirmed, the transformed data can be used for modeling and forecasting. This study employs nine forecasting models (six individual and three ensembles). It compares their performance using various error metrics (MAE, MASE, MAPE, RMSE, RRSE, and RMSLE) presented in Section 2.2.

Table 1. Peruvian electricity market: analyzing stationarity, seasonality, linearity, and normality in natural logarithm-transformed electricity prices.

Statistic	Original prices	log(prices)	diff(log(prices)	R(diff(log(prices))
Min	4.493000	1.502500	-0.178000	-0.318400
Q1	5.547500	1.713300	-0.023100	-0.038900
Q2	6.980000	1.943000	-0.000900	0.013000
Mean	6.847100	1.903700	0.001500	0.000000
Q3	7.762500	2.049300	0.023700	0.052200
Mode	7.530000	2.018900	0.000000	-0.031100
Var	1.849300	0.041100	0.002500	0.003600
SD	1.359900	0.202600	0.049600	0.059700
Max	9.650000	2.267000	0.277400	0.097100
Stationary	-1.210800	-1.185300	-7.304700	-0.909800
Seasonality	4.706700	4.779000	5.435100	0.020000
Normality	13.165900	16.131600	340.316200	339.129400
Linearity	8.359543	6.913214	-	-

Based on the results presented in Table 2, the proposed ensemble models generally outperform the single time series models regarding one-month-ahead electricity price prediction accuracy. Specifically, the ESM exhibits the lowest error values among the single models across most metrics (MAPE, MAE, MASE, RMSE, RRSE, and RMLSE), indicating its strong performance. Among the ensemble models: The ensemble (O) demonstrates superior performance, achieving the lowest error values for all considered metrics (MAPE, MAE, MASE, RMSE, RRSE, and RMLSE). Notably, its MAPE value of 0.025278 suggests a high level of forecasting accuracy. Therefore, based on the provided metrics, ensemble (O) emerges as the most accurate and efficient model for one-month-ahead electricity price forecasting compared to all nine models considered in this study. Ensemble (I) is the second-best performing model within the ensemble category.

Table 2. One-month-ahead electricity price forecasting: average forecast errors.

Model	MAPE	MAE	MASE	RMSE	RRSE	RMLSE
AR	0.031887	0.226967	0.906638	0.323678	0.806174	0.040367
ARIMA	0.032208	0.229174	0.915454	0.324077	0.807169	0.040419
ESM	0.031869	0.226952	0.906580	0.323457	0.805624	0.040327
NNA	0.038966	0.275971	1.102387	0.364993	0.909076	0.045213
NAR	0.042357	0.300534	1.200509	0.426857	1.063159	0.053710
Theta	0.032195	0.229349	0.916155	0.324072	0.807155	0.040381
ensemble (E)	0.034287	0.243722	0.973568	0.342232	0.852387	0.042702
ensemble (I)	0.026900	0.191328	0.764274	0.255640	0.636716	0.031939
ensemble (O)	0.025278	0.179678	0.717738	0.276624	0.688979	0.034370

After calculating the performance metrics, we used the DM test to statistically assess the superiority of models within the proposed ensemble technique (see Table 3 for p-values). Our analysis indicates a 5% significance level. The performance comprises nine forecasting models, including six base models and three proposed ensemble models. Statistical analysis (DM test) revealed that the ensemble (O) model achieved statistically superior performance across all models. Notably, the ensemble (I) model also showed strong results, outperforming seven other models. These findings confirm the ensemble (O)'s accuracy as the most reliable model for one-month-ahead electricity price forecasting within the scope of this study. These results support the conclusion that the ensemble (O) model offers the most accurate and reliable one-month-ahead electricity price forecasts among the models considered.

Table 3. The DM test: comparing one model to others.

Models	AR	ARIMA	ESM	NNA	NAR	Theta	ensemble (E)	ensemble (I)	ensemble (O)
AR	0.000000	0.106350	0.125330	0.891280	0.847020	0.770150	0.885110	0.117180	0.009630
ARIMA	0.893650	0.000000	0.896050	0.891410	0.849260	0.886300	0.889560	0.125850	0.011190
ESM	0.874670	0.103950	0.000000	0.891170	0.847190	0.834170	0.883950	0.116730	0.009040
NNA	0.108720	0.108590	0.108830	0.000000	0.696380	0.108290	0.108370	0.109670	0.008860
NAR	0.152980	0.150740	0.152810	0.303620	0.000000	0.152690	0.154870	0.149270	0.047020
Theta	0.229850	0.113700	0.165830	0.891710	0.847310	0.000000	0.893220	0.120390	0.012010
ensemble (E)	0.114890	0.110440	0.116050	0.891630	0.845130	0.106780	0.000000	0.116510	0.010830
ensemble (I)	0.882820	0.874150	0.883270	0.890330	0.850730	0.879610	0.883490	0.000000	0.092750
ensemble (O)	0.890370	0.888810	0.890960	0.891140	0.852980	0.887990	0.889170	0.907250	0.000000

We performed a graphical analysis to further validate our proposed ensemble (O) model's superiority. Figure 3 visually compares the actual and forecasted electricity prices for the top three models: ensemble (O), ensemble (I), and ESM. The ensemble (O) model's forecasts closely track the actual prices, demonstrating its exceptional accuracy. Additionally, we examined the correlogram plots (autocorrelation and partial autocorrelation) of the residuals for these three models (Figure 4). The absence of significant autocorrelation in the residuals of all models indicates that they have been sufficiently whitened, signaling satisfactory model performance.

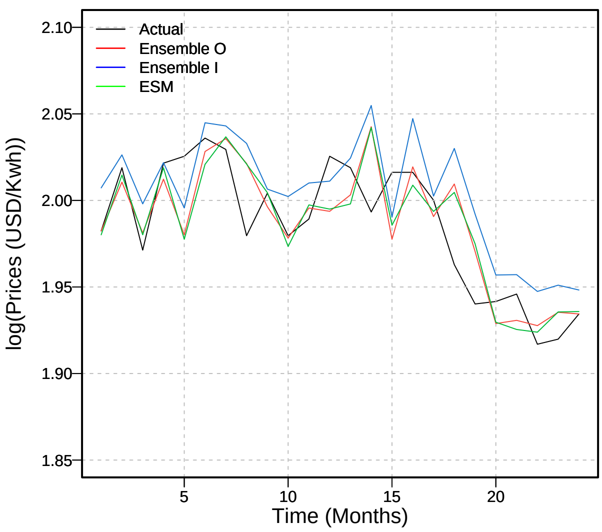
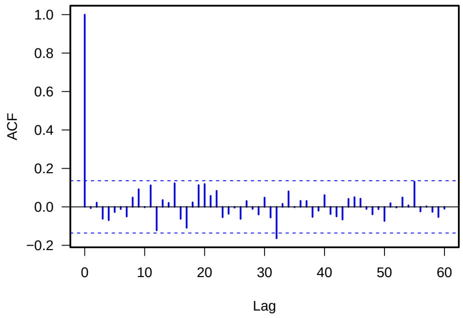
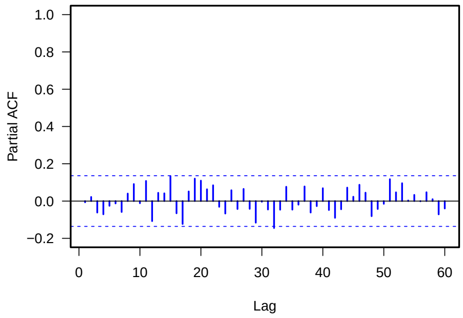


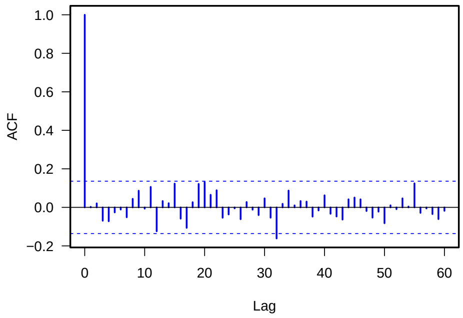
Figure 3. Comparison of original and forecasted Peruvian electricity prices: ensemble (O), ensemble (I), and ESM models (24 months).



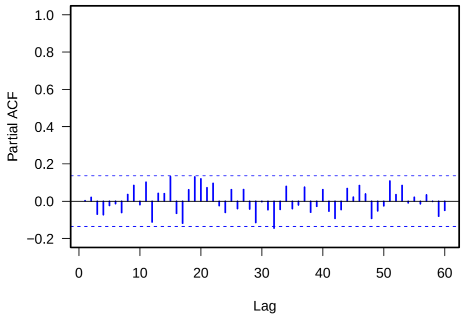
a



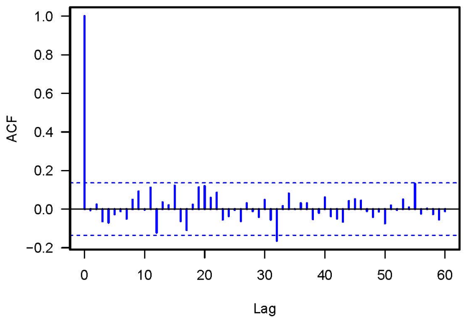
b



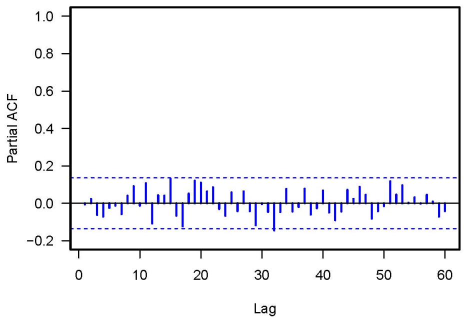
c



d



e



f

Figure 4. Autocorrelation function and partial autocorrelation plots for the three best models among all nine considered models: ensemble (O) model (a,b); ensemble (I) model (c,d); and ESM model (e,f).

In conclusion, the combination of accuracy metrics (MAE, MASE, MAPE, RMSE, RRSE, and RMLSE), statistical testing (DM test), and graphical analysis provides compelling evidence for the superiority of our proposed ensemble forecasting approach in one-month-ahead Peruvian electricity price prediction. Specifically, the ensemble (O) model consistently generates the most precise forecasts compared to this study's other single and ensemble models.

4. Discussion

Having identified ensemble (O) as the best model based on accuracy metrics (MAPE, MAE, MASE, RMSE, RRSE, and RMLSE), statistical testing (DM test), and graphical analysis, we compared it with the existing literature. We found our model highly competitive based on the performance metrics achieved by other proposed methods. Figure 5 and Table 4 compare ensemble (O) with the best models from four previous studies. We applied several models from these studies to our data, including the coyote optimization algorithm-complementary ensemble empirical mode decomposition (COA-CEEMD) model (see [24]): this model produced consistently higher error values than ensemble (O) across all metrics. For example, its MAPE was 0.029868, exceeding ensemble (O)'s 0.025278. Similarly, MAE, MASE, RMSE, RRSE, and RMSLE values were also higher for this model. The deep feedforward neural network (DFNN) model [34] This model also had higher prediction errors than ensemble (O), with a MAPE of 0.031578 and higher values for other metrics.

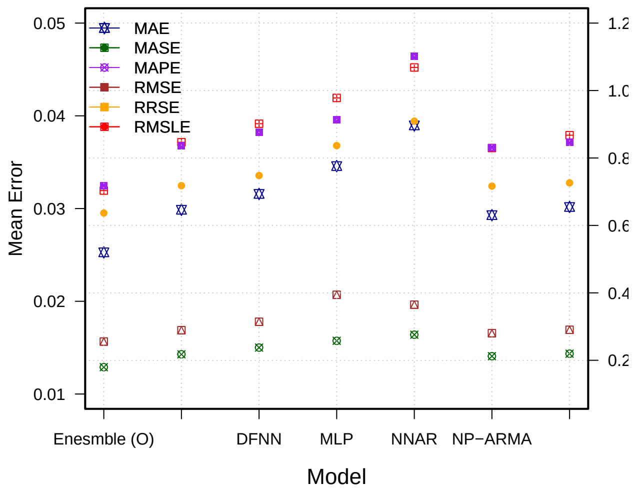


Figure 5. Performance of the proposed approach compared to benchmarks.

Table 4. One-month-ahead accuracy comparison of proposed ensemble models with existing literature in Peruvian electricity price forecasting.

Model	MAPE	MAE	MASE	RMSE	RRSE	RMLSE
ensemble (O)	0.025278	0.179678	0.717738	0.255640	0.636716	0.031939
The COA–CEEMD–Proposed [24]	0.029868	0.217778	0.836838	0.289300	0.717916	0.037149
The DFNN model [34]	0.031578	0.237778	0.876648	0.314300	0.747916	0.039149
The MLP model [35]	0.034578	0.257778	0.913028	0.394300	0.836616	0.041929
The NNAR model [36]	0.038966	0.275971	1.102387	0.364993	0.909076	0.045213
The NP-ARMA Model [37]	0.029288	0.212178	0.829638	0.280500	0.716616	0.036529
The DR-SFGM model [38]	0.030168	0.219578	0.846838	0.290500	0.726216	0.037909

Multilayer perceptron (MLP) model [35] The average accuracy measures obtained using this model on our data were significantly higher compared to ensemble (O). Neural network autoregressive (NNAR) model [36]: The performance obtained with this model was worse than ensemble (O) on our dataset. Nonlinear autoregressive moving average (NP-ARMA) model [37]: This model also yielded higher error metrics compared to our ensemble (O) model. Dynamic regression with stochastic gradient matching (DR-SFGM) model [38]: This model achieved lower accuracy metrics than our ensemble (O) model when applied to our dataset. In conclusion, these comparisons suggest that ensemble (O) offers competitive performance for one-month-ahead electricity price forecasting in the Peruvian market, even when compared to existing methods.

However, this work only focuses on the Peruvian electricity market; in the future, the proposal of the current research study should be extended to other electricity markets—for instance, the European electricity markets, the United American electricity market, the United Kingdom, the Chinese electricity markets, and so on. On the other hand, it may also be expanded and used for other datasets, such as electrical power systems [39, 40], air pollution [41], and railway signaling [42].

The current work proposal relies on only single linear and nonlinear time-series models; it might use other models like machine learning, deep learning, patch time series transformer, and decomposition linear model in future projects within the current proposal. Furthermore, exogenous variables (consumer load, fuel, carbon dioxide emission prices, average solar radiation, and wind speed) should be incorporated within the proposed ensemble time series forecasting technique in the future to enhance the forecasting accuracy of electricity prices.

5. Conclusions

In today's dynamic electricity markets, accurate price forecasting enables market players to make informed decisions that optimize costs and ensure system stability. However, the complex nature of electricity prices often presents significant challenges to forecasting accuracy. This study proposes a novel time series ensemble technique specifically tailored for electricity price forecasting in the Peruvian market. Preprocessing plays a crucial role in achieving accurate forecasts. It addresses missing values, variance fluctuations, non-normalization, and non-stationarity. Combining base forecasting models (both linear and nonlinear) with ensemble techniques can potentially enhance forecasting accuracy. The evaluation process employed a combination of accuracy metrics, graphical analysis, and statistical testing to demonstrate the effectiveness of the proposed methodology. The study's findings suggest that the proposed time series ensemble technique offers a highly efficient and accurate approach for predicting monthly electricity prices in the Peruvian market. Compared to other models, the proposed ensemble models exhibited the lowest mean errors, highlighting the effectiveness of the ensemble approach in capturing the complex dynamics of electricity prices.

Author contributions

Salvatore Mancha Gonzales: validation, formal analysis, investigation, resources, data curation, writing original draft preparation, writing review and editing, visualization; Hasnain Iftikhar: conceptualization, methodology, software, validation, formal analysis, investigation, resources, data curation, writing original draft preparation, writing review and editing, visualization, supervision, project administration, funding acquisition; Javier Linkolk López-Gonzales: resources, data curation, writing original draft preparation, writing review and editing, supervision, project administration, funding acquisition. All authors have read and agreed to the published version of the manuscript.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare no conflicts of interest.

References

1. World Bank Group, International experience with private sector participation in power grids: Peru case study, 2012. Available from: http://hdl.handle.net/10986/23616.
2. E. V. Guevara, Competition and wholesale electricity market: the monitoring task assigned to the Peruvian electricity coordinator (COES), *IUS ET Veritas*, **61** (2020), 94–112. https://doi.org/10.18800/iusetveritas.202002.006
3. M. Pinhão, M. Fonseca, R. Covas, Electricity spot price forecast by modelling supply and demand curve, *Mathematics*, **10** (2022), 2012. https://doi.org/10.3390/math10122012
4. B. Li, J. Wang, A. A. Nassani, R. H. Binsaeed, Z. Li, The future of Green energy: a panel study on the role of renewable resources in the transition to a Green economy, *Energy Econ.*, **127** (2023), 107026. https://doi.org/10.1016/j.eneco.2023.107026
5. K. G. Olivares, C. Challu, G. Marcjasz, R. Weron, A. Dubrawski, Neural basis expansion analysis with exogenous variables: forecasting electricity prices with NBEATSx, *Int. J. Forecast.*, **39** (2023), 884–900. https://doi.org/10.1016/j.ijforecast.2022.03.001
6. Y. Duan, Y. Zhao, J. Hu, An initialization-free distributed algorithm for dynamic economic dispatch problems in microgrid: modeling, optimization, and analysis, *Sustain. Energy Grids Networks*, **34** (2023), 101004. https://doi.org/10.1016/j.segan.2023.101004
7. X. Li, Y. Jiang, X. Xin, A. A. Nassani, C. Yang, The asymmetric role of natural resources, fintech, and green innovations in the Chinese economy. Evidence from QARDL approach, *Resour. Policy*, **90** (2024), 104731. https://doi.org/10.1016/j.resourpol.2024.104731
8. M. Alrashidi, Ultra-short-term solar forecasting with reduced pre-acquired data considering optimal heuristic configurations of deep neural networks. *AIMS Math.*, **9** (2024), 12323–12356. https://doi.org/10.3934/math.2024603
9. R. A. de Marcos, A. Bello, J. Reneses, Electricity price forecasting in the short term hybridising fundamental and econometric modelling, *Electr. Power Syst. Res.*, **167** (2019), 240–251. https://doi.org/10.1016/j.epsr.2018.10.034
10. R. Wang, R. Zhang, Techno-economic analysis and optimization of hybrid energy systems based on hydrogen storage for sustainable energy utilization by a biological-inspired optimization algorithm, *J. Energy Storage*, **66** (2023), 107469. https://doi.org/10.1016/j.est.2023.107469

11. I. Shah, H. Iftikhar, S. Ali, Modeling and forecasting electricity demand and prices: a comparison of alternative approaches, *J. Math.*, **2022** (2022), 3581037. https://doi.org/10.1155/2022/3581037
12. P. Li, J. Hu, L. Qiu, Y. Zhao, B. K. Ghosh, A distributed economic dispatch strategy for power-water networks, *IEEE Trans. Control Network Syst.*, **9** (2022), 356–366. https://doi.org/10.1109/TCNS.2021.3104103
13. J. Janczura, Expectile regression averaging method for probabilistic forecasting of electricity prices, *Comput. Stat.*, **18** (2024), 1613–9658. https://doi.org/10.1007/s00180-024-01508-y
14. F. Abid, M. Alam, F. S. Alamri, I. Siddique, Multi-directional gated recurrent unit and convolutional neural network for load and energy forecasting: a novel hybridization, *AIMS Math.*, **8** (2023), 19993–20017. https://doi.org/10.3934/math.20231019
15. M. Shirkhani, J. Tavoosi, S. Danyali, A. K. Sarvenoee, A. Abdali, A. Mohammadzadeh, et al., A review on microgrid decentralized energy/voltage control structures and methods, *Energy Rep.*, **10** (2023), 368–380. https://doi.org/10.1016/j.egyr.2023.06.022
16. A. L. de Rojas, M. A. Jaramillo-Morán, J. E. Sandubete, EMDFormer model for time series forecasting, *AIMS Math.*, **9** (2024), 9419–9434. https://doi.org/10.3934/math.2024459
17. H. Iftikhar, J. E. Turpo-Chaparro, P. C. Rodrigues, J. L. López-Gonzales, Forecasting day-ahead electricity prices for the Italian electricity market using a new decomposition-combination technique, *Energies*, **16** (2023), 6669. https://doi.org/10.3390/en16186669
18. Mustaqeem, M. Ishaq, S. Kwon, Short-term energy forecasting framework using an ensemble deep learning approach, *IEEE Access*, **9** (2021), 94262–94271. https://doi.org/10.1109/ACCESS.2021.3093053
19. I. B. Todorov, F. S. Lasheras, Forecasting applied to the electricity, energy, gas and oil industries: a systematic review, *Mathematics*, **10** (2022), 3930. https://doi.org/10.3390/math10213930
20. J. Hu, Y. Zou, N. Soltanov, A multilevel optimization approach for daily scheduling of combined heat and power units with integrated electrical and thermal storage, *Expert Syst. Appl.*, **250** (2024), 123729. https://doi.org/10.1016/j.eswa.2024.123729
21. A. L. de Rojas, Data augmentation in economic time series: behavior and improvements in predictions, *AIMS Math.*, **8** (2023), 24528–24544. https://doi.org/10.3934/math.20231251
22. Z. Yang, L, Ce, L. Lian, Electricity price forecasting by a hybrid model, combining wavelet transform, ARMA and kernel-based extreme learning machine methods, *Appl. Energy*, **190** (2017), 291–305. https://doi.org/10.1016/j.apenergy.2016.12.130
23. S. Khan, S. Aslam, I. Mustafa, Short-term electricity price forecasting by employing ensemble empirical mode decomposition and extreme learning machine, *Forecasting*, **3** (2021), 460–477. https://doi.org/10.3390/forecast3030028
24. M. H. D. M. Ribeiro, S. F. Stefenon, J. D. de Lima, A. Nied, V. C Mariani, L. D. S. Coelho, Electricity price forecasting based on self-adaptive decomposition and heterogeneous ensemble learning, *Energies*, **13** (2020), 5190. https://doi.org/10.3390/en13195190
25. N. Bibi, I. Shah, A. Alsubie, S. Ali, S. A. Lone, Electricity spot prices forecasting based on ensemble learning, *IEEE Access*, **9** (2012), 150984–15099. https://doi.org/10.1109/ACCESS.2021.3126545

26. P. J. Brockwell, R. A. Davis, *Introduction to time series and forecasting*, Springer, 2016. https://doi.org/10.1007/978-3-319-29854-2

27. R. J. Hyndman, G. Athanasopoulos, *Forecasting: principles and practice*, OTexts, 2018.

28. H. Iftikhar, A. Zafar, J. E. Turpo-Chaparro, P. C. Rodrigues, J. L. López-Gonzales, Forecasting day-ahead Brent crude oil prices using hybrid combinations of time series models, *Mathematics*, **11** (2023), 3548. https://doi.org/10.3390/math11163548

29. L. Wasserman, *All of nonparametric statistics*, Springer Science & Business Media, 2006. https://doi.org/10.1007/0-387-30623-4

30. H. Iftikhar, M. Khan, J. E. Turpo-Chaparro, P. C. Rodrigues, J. L. López-Gonzales, Forecasting stock prices using a novel filtering-combination technique: application to the Pakistan stock exchange, *AIMS Math.*, **9** (2024), 3264–3288. https://doi.org/10.3934/math.2024159

31. N. Carbo-Bustinza, H. Iftikhar, M. Belmonte, R. J. Cabello-Torres, A. R. H. De La Cruz, J. L. López-Gonzales, Short-term forecasting of ozone concentration in metropolitan Lima using hybrid combinations of time series models, *Appl. Sci.*, **13** (2023), 10514. https://doi.org/10.3390/app131810514

32. F. X. Diebold, R. S. Mariano, Comparing predictive accuracy, *J. Bus. Econ. Stat.*, **20** (2012), 134–144. http://doi.org/10.1198/073500102753410444

33. L. Inglada-Pérez, S. G. Gil, A study on the nature of complexity in the Spanish electricity market using a comprehensive methodological framework, *Mathematics*, **12** (2024), 893. http://doi.org/10.3390/math12060893

34. T. Windler, J. Busse, J. Rieck, One month-ahead electricity price forecasting in the context of production planning, *J. Clean. Prod.*, **238** (2019), 117910. https://doi.org/10.1016/j.jclepro.2019.117910

35. F. L. C. da Silva, K. da Costa, P. C. Rodrigues, R. Salas, J. L. López-Gonzales, Statistical and artificial neural networks models for electricity consumption forecasting in the Brazilian industrial sector, *Energies*, **15** (2022), 588. https://doi.org/10.3390/en15020588

36. S. Krstev, J. Forcan, D. Krneta, An overview of forecasting methods for monthly electricity consumption, *Tehnički Vjesnik*, **30** (2023), 993–1001. https://doi.org/10.17559/TV-20220430111309

37. I. Shah, H. Iftikhar, S. Ali, Modeling and forecasting medium-term electricity consumption using component estimation technique, *Forecasting*, **2** (2020), 163–179. https://doi.org/10.3390/forecast2020009

38. S. Ding, Z. Tao, R. Li, X. Qin, A novel seasonal adaptive grey model with the data-restacking technique for monthly renewable energy consumption forecasting, *Expert Syst. Appl.*, **208** (2022), 118115. https://doi.org/10.1016/j.eswa.2022.118115

39. X. Zhang, L. Gong, X. Zhao, R. Li, L. Yang, B. Wang, Voltage and frequency stabilization control strategy of virtual synchronous generator based on small signal model, *Energy Rep.*, **9** (2023), 583–590. https://doi.org/10.1016/j.egyr.2023.03.071

40. Y. Lei, Y. Chen, H. Hai, R. Gao, W. Wu, DGNet: an adaptive lightweight defect detection model for new energy vehicle battery current collector, *IEEE Sensors J.*, **23** (2023), 29815–29830. https://doi.org/10.1109/JSEN.2023.3324441

41. F. Quispe, E. Salcedo, H. Iftikhar, A. Zafar, M. Khan, J. E. Turpo-Chaparro, et al., Multi-step ahead ozone level forecasting using a component-based technique: a case study in Lima, Peru, *AIMS Environ. Sci.*, **11** (2024), 401–425. https://doi.org/10.3934/environsci.2024020

42. X. Hu, L. Tan, T. Tang, M^{2}BIST-SPNet: RUL prediction for railway signaling electromechanical devices, *J. Supercomput.*, 2024. https://doi.org/10.1007/s11227-024-06111-y