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Research article

Indirect stability of a 2D wave-plate coupling system with memory viscoelastic damping

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Abstract: We performed a stability analysis of a 2D wave-plate coupling system equipped with memory viscoelastic damping. The study highlights the unique functionality of the damping mechanism, which operates indirectly and exclusively within either the wave or plate subsystem. The opposing subsystem receives dissipative signals indirectly through the coupling component. The primary objective of this study was to determine whether the indirect memory damping is sufficient to ensure the overall stability of the coupled system. To address this question, a frequency domain analysis was employed to establish explicit decay rates of the coupled system. Notably, a polynomial decay rate is observed when the memory damping is applied solely to either the plate or wave subsystem, which provides a conclusive answer to the posed question.

Keywords: wave-plate coupled system; infinite memory; frequency domain analysis; multiplier method
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1. Introduction

The issues of stability and control pertaining to coupled systems have garnered significant interest from researchers due to their widespread practical applications in modern control engineering. These applications include satellite antennas, fluid-structure interactions, structural-acoustic systems, and numerous others (see [9, 16, 17, 23, 31]). An intriguing question arises regarding the sufficiency of indirect dissipative mechanisms in stabilizing an entire system. Specifically, these dissipative mechanisms act exclusively on one subsystem, while the remaining subsystems only receive dissipative signals indirectly via the coupling part. The question then becomes whether these mechanisms, operating independently on one system, are adequate for ensuring overall system stability.

When the dissipative mechanism comes from a heat effect, Zhang and Zuazua [33] conducted a rigorous investigation into heat-wave systems, demonstrating that dissipation solely originating from heat ensures polynomial stability of the entire system. This same question was also addressed by Zhang and Zuazua [34] as well as Batty et al. [5], who examined various forms of boundary coupled conditions. Lebeau et al. [18] coupled an elastic structure with a thermal system to achieve stability. Zhang et al. [37] further delved into the exponential stability of a boundary-coupled heat-beam system both in one-dimensional space and two-dimensional space, while incorporating extra dissipation at the plate. Wang et al. [30] explored a heat-Schrödinger coupled system, aiming to establish exponential stability. Liu and Zhang [22] investigated the exponential stability of a heat-plate transmission system with memory using frequency domain methods. Kim [15] contributed to the field by studying the exponential stability of a linear thermoelastic bar-plate coupled system. Collectively, these studies provide valuable insights into the stability properties of coupled elastic-thermal systems, employing diverse methodologies to achieve exponential stability.

When the dissipative mechanism arises from frictional damping, Liu and Williams [21] conducted a thorough study on exponential stability in a wave transmission system under the influence of linear frictional feedback applied to the outer boundary. Subsequently, this conclusion was generalized to systems with variable coefficients by Chai [7]. Utilizing frequency domain analysis and the multiplier technique, Ammari et al. [2] studied a one-dimensional string-beam coupled model with boundary frictional damping feedback. Ammari [3] further extended the findings of the string-beam coupled model [2] to encompass a two-dimensional Euclidean space. Guo et al. [12] provided evidence that the system exhibits exponential decay when frictional damping acts on the wave only and polynomial decay when it acts on the plate only.

When the dissipative mechanism arises from memory damping, the exponential stability of two-dimensional plate equations with memory-type viscoelasticity was previously analyzed by Munoz-Rivera [25]. Zhang [36] further explored the exponential stability of a wave-heat coupled system with memory, where the heat conduction law falls into two categories: the Gurtin-Pipkin type and the Coleman-Gurtin type. The investigation employed frequency domain analysis under suitable assumptions. Zhang [35] also studied the polynomial decay of a wave transmission system with non-integral viscoelastic damping, using a frequency domain analysis technique. Han et al. [13] investigated the exponential stability of a one-dimensional beam-disk coupled system with memory-type feedback control. Their approach aligned with those used in [35, 36]. Zhang et al. [39] expanded their research to study the exponential decay of a piezoelectric beam with viscoelastic infinite memory, leveraging a semigroup approach and frequency domain method. Tyszka et al. [29] used a semigroup method to explore the stabilization of Kirchhoff and Euler–Bernoulli plates with memory damping. Their findings revealed that the system exhibits exponential decay when the dissipative mechanism acts on both equations and polynomial decay when it acts on only one equation. Li and Zhang [19] presented the polynomial stability of a high-dimensional viscoelastic wave-plate transmission system, leveraging frequency domain characterization and a geometrical multiplier approach. Liu, Özer, and Wang [20] presented the longtime dynamics of a new piezoelectric beam system with viscoelastic damping by a semigroup method. Feng and Özer [10] obtained the exponential stability of a piezoelectric beam with memory terms by a multiplier technique. The kernels considered above are all of exponential type. Zhang, Xu, and Han [38] proved the polynomial stability of a piezoelectric system with friction-type infinite memory term using frequency domain methods in high-dimensions. Messaoudi and Al-Gharabli [24] studied the general stability of wave systems with infinite memory terms, which depends on the decay rate of the kernels. Their research allows a wide class of kernels. Al-Mahdi, Al-Gharabli, and Messaoudi [1] improved the kernel's conditions.

The system under investigation in this study can be regarded as a coupled model for the stacked configuration of a plate and a membrane. It should be noted that, as far as we know, there are few studies related to the well-posedness and stability of wave-plate coupled systems with indirect memory-type viscoelastic damping. We aimed to conduct a rigorous investigation into the decay rate of energy within such a system, particularly when the dissipative mechanism, formulated by a memory term, occurs exclusively within either the wave subsystem or plate subsystem. The inherent dynamic damping characteristics of viscoelastic materials and their indirect influence pose significant challenges in constructing energy multipliers in the time domain. To address these complexities, we employed a frequency-domain approach in our rigorous analysis. We establish that, provided the kernel function of memory exhibits exponential decay, the wave-plate coupled system exhibits exponential decay when both the wave and plate are influenced by the viscoelastic term. The system demonstrates polynomial decay when the viscoelastic term solely acts on either the wave or the plate.

In the following, we describe the detailed wave-plate system studied in this paper. Let $\Omega \subset \mathbb{R}^2$ be a bounded domain with smooth boundary $\Gamma = \Gamma_1 \cup \Gamma_2$. Here, Γ_1 and Γ_2 are open sets satisfying $\Gamma_1 \cap \Gamma_2 = \emptyset$ and meas $(\Gamma_i) > 0$, $i = 1, 2$. We consider the coupled system which is modeled by

$$\begin{cases} \partial_{tt}u_1 - \Delta u_1 + \alpha_1 \displaystyle\int_0^\infty g(s)\Delta u_1(t-s)\mathrm{d}s + k(u_1 - u_2) = 0 & \text{in } \Omega \times \mathbb{R}^+, \\ \partial_{tt}u_2 + \Delta^2 u_2 - \alpha_2 \displaystyle\int_0^\infty g(s)\Delta^2 u_2(t-s)\mathrm{d}s - k(u_1 - u_2) = 0 & \text{in } \Omega \times \mathbb{R}^+, \end{cases} \tag{1.1}$$

supplemented by boundary conditions

$$\begin{cases} u_1 = u_2 = \dfrac{\partial u_2}{\partial \nu} = 0 & \text{on } \Gamma_1 \times \mathbb{R}^+, \\ \dfrac{\partial u_1}{\partial \nu} = \Delta u_2 = \dfrac{\partial \Delta u_2}{\partial \nu} = 0 & \text{on } \Gamma_2 \times \mathbb{R}^+, \end{cases} \tag{1.2}$$

and the initial conditions for $i = 1, 2$

$$\begin{cases} u_i(x,0) = u_i^0(x),\ \partial_t u_i(x,0) = u_i^1(x) & \text{in } \Omega, \\ u_i(x,-s) = \phi_i(x,s) & \text{in } \Omega, \quad s > 0, \end{cases} \tag{1.3}$$

where ν is the unit outer normal vector of $\partial\Omega$, τ is the unit tangent vector of $\partial\Omega$, $\mathbb{R}^+$ is the real interval $(0, \infty)$, and k is a positive real constant. The real constants $\alpha_1,\ \alpha_2 > 0$. The functions $u_i^0,\ u_i^1$ are the initial states, and ϕ_i is the memory history value, $i = 1, 2$. The memory kernel $g : \mathbb{R}_+ \to \mathbb{R}_+$ is a non-increasing C^1 function satisfying $g(s) > 0$ for $s \in \mathbb{R}_+$, and

$$0 < \beta_i := 1 - \alpha_i \int_0^\infty g(s)\mathrm{d}s < \infty,\ i = 1, 2. \tag{1.4}$$

The boundary condition in $(1.2)_1$ shows that both wave and plate are clamped at Γ. The boundary condition in $(1.2)_2$ shows that both wave and plate are free at Γ_2. For the free end, $\Delta u_2 = 0$ describes that the bending moment of the plate is zero, and $\frac{\partial \Delta u_2}{\partial \nu} = 0$ states that the shear force is zero (see [4]).

Wave-plate coupled vibration systems such as (1.1)–(1.3) can simulate the influence of a coal mine ventilator and its pipeline vibration or the vibration of aircraft wings in air, which have attracted increasing attention. Previous studies usually consider differential viscoelastic damping, frictional damping, or viscous damping, as the damping devices act on wave-plate coupled systems (see [3, 12, 19]). The damping considered in this paper is memory viscoelastic damping, with the integral terms in (1.1), which possesses properties of both viscosity and elasticity. Due to the special properties of the damping and the effect of the past history on the present state, the influence of such damping in wave-plate coupled systems is a problem worth studying.

This paper organized as follows. In Section 2, we introduce some notations and preliminaries to establish the foundation for our subsequent discussions. In Section 3, the generation of semigroup $e^{t\mathcal{A}}$ is argued by the semigroup approach, and the well-posedness of the system is further discussed. In Section 4, we dedicate our attention to analyze the long-time behavior of $e^{t\mathcal{A}}$ across three distinct scenarios:

1) memory damping acts both on wave and plate: $\alpha_1 > 0$ and $\alpha_2 > 0$;
2) memory damping acts only on wave: $\alpha_1 > 0$ and $\alpha_2 = 0$;
3) memory damping acts only on plate: $\alpha_1 = 0$ and $\alpha_2 > 0$.

In Section 5, we give a conclusion of this paper.

2. Preliminaries

In this section, we present crucial notations that are fundamental to our discussion. Drawing inspiration from the work of Dafermos [8], we introduce two new variables, for $t, s > 0$,

$$\eta_i^t(x, s) = u_i(x, t) - u_i(x, t-s) \ \text{ in } \Omega, \ i = 1, 2.$$

Thus, system (1.1)–(1.3) can be rewritten as

$$\begin{cases} \partial_{tt}u_1 - \beta_1\Delta u_1 - \alpha_1 \displaystyle\int_0^\infty g(s)\Delta\eta_1^t(s)\mathrm{d}s + k(u_1 - u_2) = 0 & \text{in } \Omega \times \mathbb{R}^+, \\ \partial_{tt}u_2 + \beta_2\Delta^2 u_2 + \alpha_2 \displaystyle\int_0^\infty g(s)\Delta^2\eta_2^t(s)\mathrm{d}s - k(u_1 - u_2) = 0 & \text{in } \Omega \times \mathbb{R}^+, \\ \partial_t\eta_i^t + \partial_s\eta_i^t = \partial_t u_i, \ i = 1, 2 & \text{in } \Omega \times \mathbb{R}^+ \times \mathbb{R}^+ \end{cases} \tag{2.1}$$

supplemented by boundary conditions

$$\begin{cases} u_1 = \eta_1^t = u_2 = \eta_2^t = \frac{\partial u_2}{\partial \nu} = \frac{\partial \eta_2^t}{\partial \nu} = 0, & \text{on } \Gamma_1 \times \mathbb{R}^+, \\ \frac{\partial u_1}{\partial \nu} = \frac{\partial \eta_1^t}{\partial \nu} = \Delta u_2 = \Delta\eta_2^t = \frac{\partial \Delta u_2}{\partial \nu} = \frac{\partial \Delta\eta_2^t}{\partial \nu} = 0, & \text{on } \Gamma_2 \times \mathbb{R}^+, \end{cases} \tag{2.2}$$

and the initial conditions for $i = 1, 2$

$$\begin{cases} u_i(x, 0) = u_i^0(x), \ \partial_t u_i(x, 0) = u_i^1(x) & \text{in } \Omega, \\ \eta_i^0(x, s) = u_i^0(x) - \phi_i(x, s) & \text{in } \Omega \times \mathbb{R}^+. \end{cases} \tag{2.3}$$

Let L^2 denote the space $L^2(\Omega)$ equipped with the inner product $\langle \cdot, \cdot \rangle = \langle \cdot, \cdot \rangle_{L^2(\Omega)}$ and the associated norm $\|\cdot\| = \|\cdot\|_{L^2(\Omega)}$. Furthermore, we introduce the space $H^1_{\Gamma_1} = \left\{ f \in H^1(\Omega) \mid f = 0 \text{ on } \Gamma_1 \right\}$ endowed with inner product

$$\langle f, g \rangle_{H^1_{\Gamma_1}} = \langle \nabla f, \nabla g \rangle, \quad \forall f, g \in H^1_{\Gamma_1},$$

and norm $\|\cdot\|_{H^1_{\Gamma_1}} = \langle\cdot,\cdot\rangle^{1/2}_{H^1_{\Gamma_1}}$; the space $H^2_{\Gamma_1} = \left\{f \in H^2(\Omega) \mid f = \frac{\partial f}{\partial \nu} = 0 \text{ on } \Gamma_1\right\}$ endowed with inner product

$$\langle f, g\rangle_{H^2_{\Gamma_1}} = \langle \Delta f, \Delta g\rangle, \quad \forall f, g \in H^2_{\Gamma_1}$$

and norm $\|\cdot\|_{H^2_{\Gamma_1}} = \langle\cdot,\cdot\rangle^{1/2}_{H^2_{\Gamma_1}}$; and the space

$$\mathcal{M}_i = \left\{\eta(x,s) \in H^i_{\Gamma_1} \mid \int_0^\infty g(s)\|\eta(x,s)\|_{H^i_{\Gamma_1}} \mathrm{d}s < \infty\right\}$$

endowed with the inner product

$$\langle \eta, \xi\rangle_{\mathcal{M}_i} = \int_0^\infty g(s)\langle \eta(\cdot,s), \xi(\cdot,s)\rangle_{H^i_{\Gamma_1}} \mathrm{d}s, \quad \forall \eta, \xi \in \mathcal{M}_i$$

and norm $\|\cdot\|_{\mathcal{M}_i} = \langle\cdot,\cdot\rangle^{1/2}_{\mathcal{M}_i}$, $i = 1, 2$.

We study system (1.1)–(1.3) within the context of the energy domain

$$\mathcal{H} = H^1_{\Gamma_1} \times L^2 \times H^2_{\Gamma_1} \times L^2 \times \mathcal{M}_1 \times \mathcal{M}_2$$

with inner product

$$\langle X, \tilde{X}\rangle_{\mathcal{H}} = \sum_{i=1}^{2}\left[\beta_i\langle u_i, \tilde{u}_i\rangle_{H^i_{\Gamma_1}} + \langle v_i, \tilde{v}_i\rangle + \alpha_i\langle \eta_i, \tilde{\eta}_i\rangle_{\mathcal{M}_i}\right] + k\langle u_1 - u_2, \tilde{u}_1 - \tilde{u}_2\rangle$$

for all $X = (u_1, v_1, u_2, v_2, \eta_1, \eta_2)^\top \in \mathcal{H}$ and $\tilde{X} = (\tilde{u}_1, \tilde{v}_1, \tilde{u}_2, \tilde{v}_2, \tilde{\eta}_1, \tilde{\eta}_2)^\top \in \mathcal{H}$, and the induced norm $\|\cdot\|_{\mathcal{H}} = \langle\cdot,\cdot\rangle^{\frac{1}{2}}_{\mathcal{H}}$ which is equivalent to the normal inner product in $\mathcal{H}$.

3. Well-posedness of solution

This section is devoted to establish the well-posedness of the solution to the system (2.1)–(2.3) by a semigroup method. Initially, we convert the coupled system into an abstract problem. Toward this end, we introduce the system operator $\mathcal{A}$ acting on $\mathcal{H}$ as follows:

$$\mathcal{A}X = \left(v_1, \Delta\zeta_1 - k(u_1 - u_2), v_2, -\Delta^2\zeta_2 + k(u_1 - u_2), v_1 - \partial_s\eta_1, v_2 - \partial_s\eta_2\right)^\top \tag{3.1}$$

for all

$$\begin{aligned} X &:= (u_1, v_1, u_2, v_2, \eta_1, \eta_2)^\top \in D(\mathcal{A}) \\ &= \left\{ X \in \mathcal{H} \;\middle|\; \begin{array}{c} v_1 \in H^1_{\Gamma_1},\ v_2 \in H^2_{\Gamma_1},\ \zeta_i \in H^{2i}(\Omega), \partial_s\eta_i \in \mathcal{M}_i, \eta_i(s=0) = 0, \quad i = 1, 2, \\ \dfrac{\partial u_1}{\partial \nu} = \dfrac{\partial \eta_1^t}{\partial \nu} = \Delta u_2 = \Delta\eta_2 = \dfrac{\partial \Delta u_2}{\partial \nu} = \dfrac{\partial \Delta\eta_2}{\partial \nu} = 0 \text{ on } \Gamma_2 \end{array} \right\}. \end{aligned}$$

Here,

$$\zeta_i = \beta_i u_i + \alpha_i \int_0^\infty g(s)\eta_i(s)\mathrm{d}s, \quad i = 1, 2.$$

Then, the system can be reformulated as

$$\begin{cases} \dfrac{\mathrm{d}X(t)}{\mathrm{d}t} = \mathcal{A}X(t), \quad t > 0, \\ X(0) = X_0, \end{cases} \tag{3.2}$$

where $X(t) = (u_1, \partial_t u_1, u_2, \partial_t u_2, \eta_1^t, \eta_2^t)^\top$ and $X_0 = (u_1^0, u_1^1, u_2^0, u_2^1, u_1^0 - \phi_1, u_2^0 - \phi_2)^\top$. Hence, the ensuing theorem established below attests to the well-posedness of the system (1.1)–(1.3).

Theorem 3.1. *The operator $\mathcal{A}$ generates a C_0-semigroup of contractions $\left\{e^{t\mathcal{A}}\right\}_{t\geq 0}$ on $\mathcal{H}$.*

Proof. Initially, we note that $\mathcal{A}$ exhibits dissipative behavior. Indeed, for any $X = (u_1, v_1, u_2, v_2, \eta_1, \eta_2)^\top \in D(\mathcal{A})$,

$$\begin{aligned} \mathrm{Re}\,\langle \mathcal{A}X, X\rangle_{\mathcal{H}} &= \mathrm{Re}\Big[\langle v_1, u_1\rangle_{H^1_{\Gamma_1}} + \langle \Delta\zeta_1, v_1\rangle + \left\langle -\Delta^2\zeta_2, v_2\right\rangle + \langle v_2, u_2\rangle_{H^2_{\Gamma_1}} \\ &\quad + \langle v_1 - \partial_s\eta_1, \eta_1\rangle_{\mathcal{M}_1} + \langle v_2 - \partial_s\eta_2, \eta_2\rangle_{\mathcal{M}_2}\Big] \\ &= \int_0^\infty g'(s)\left(\alpha_1\|\eta_1(s)\|^2_{H^1_{\Gamma_1}} + \alpha_2\|\eta_2(s)\|^2_{H^2_{\Gamma_1}}\right)\mathrm{d}s \leq 0, \end{aligned} \tag{3.3}$$

which shows the dissipativeness of $\mathcal{A}$.

Next, we demonstrate that $0 \in \rho(\mathcal{A})$, the resolvent set of $\mathcal{A}$. In other words, given any $Y = (w_1, h_1, w_2, h_2, \xi_1, \xi_2)^\top \in \mathcal{H}$, it is necessary to find a solution $X = (u_1, v_1, u_2, v_2, \eta_1, \eta_2)^\top \in D(\mathcal{A})$ such that the equation $\mathcal{A}X = Y$ holds. Pursuant to the definition of the operator $\mathcal{A}$, the equality $\mathcal{A}X = Y$ is equivalent to

$$\begin{cases} v_1 = w_1, \\ \beta_1\Delta u_1 + \alpha_1\displaystyle\int_0^\infty g(s)\Delta\eta_1(s)\mathrm{d}s - k(u_1 - u_2) = h_1, \\ v_2 = w_2, \\ -\beta_2\Delta^2 u_2 - \alpha_2\displaystyle\int_0^\infty g(s)\Delta^2\eta_2(s)\mathrm{d}s + k(u_1 - u_2) = h_2, \\ v_1 - \partial_s\eta_1 = \xi_1, \\ v_2 - \partial_s\eta_2 = \xi_2. \end{cases} \tag{3.4}$$

We can see from $(3.4)_{2,4}$ that $\zeta_i \in H^{2i}(\Omega)$ because $h_i \in L^2$, $i = 1, 2$. Integrating $(3.4)_5$ and $(3.4)_6$, we have

$$\eta_1 = \int_0^s (w_1 - \xi_1(r))\,\mathrm{d}r \quad \text{and} \quad \eta_2 = \int_0^s (w_2 - \xi_2(r))\,\mathrm{d}r. \tag{3.5}$$

Then, transform $(3.4)_2$ and $(3.4)_4$ into

$$\beta_1\Delta u_1 - k(u_1 - u_2) = h_1 - \alpha_1\int_0^\infty g(s)\Delta\eta_1(s)\mathrm{d}s, \tag{3.6}$$

and

$$-\beta_2\Delta^2 u_2 + k(u_1 - u_2) = h_2 + \alpha_2\int_0^\infty g(s)\Delta^2\eta_2(s)\mathrm{d}s. \tag{3.7}$$

Multiplying (3.6) by $\overline{\varphi}$ (resp., (3.7) by $\overline{\psi}$), integrating in Ω, we have the variational formulation, for any pair $(\varphi, \psi) \in \mathbb{H} := H^1_{\Gamma_1} \times H^2_{\Gamma_1}$,

$$B(u_1, u_2) = F(\varphi, \psi), \tag{3.8}$$

where

$$B(u_1,u_2)=\beta_1\langle u_1,\varphi\rangle_{H_0^1}+\beta_2\langle u_2,\psi\rangle_{H^2_{\Gamma_1}}+k\langle u_1-u_2,\varphi-\psi\rangle, \tag{3.9}$$

and

$$F(\varphi,\psi)=\left\langle\left[\alpha_1\int_0^\infty g(s)\Delta\eta_1(s)\mathrm{d}s-h_1\right],\varphi\right\rangle-\left\langle\left[h_2+\alpha_2\int_0^\infty g(s)\Delta^2\eta_2(s)\mathrm{d}s\right],\psi\right\rangle.$$

Hölder's and Poincaré's inequalities, when applied to the given context, establish the boundedness of the functional F in $\mathbb{H}$. From $0\in\rho(\mathcal{A})$, we have $\overline{D(\mathcal{A})}=\mathcal{H}$. In fact, $0\in\rho(\mathcal{A})\neq\emptyset$ implies that $\mathcal{A}$ is closed. Then, we conclude that $\rho(\mathcal{A})$ is an open set, so we may find some positive number $\lambda_0\in\rho(\mathcal{A})$. Let $Y\in X$ satisfying $\langle Y,X\rangle_{\mathcal{H}}=0$ for all $X\in D(\mathcal{A})$. Since $\lambda_0\in\rho(\mathcal{A})$, there exists $X_0\in D(\mathcal{A})$ such that $sX_0-\mathcal{A}X_0=Y$, so

$$0=\mathrm{Re}\,\langle Y,X_0\rangle_{\mathcal{H}}=(\mathrm{Re}\,\lambda_0)\|X_0\|^2_{\mathcal{H}}-\mathrm{Re}\,\langle\mathcal{A}X_0,X_0\rangle_{\mathcal{H}}\geq(\mathrm{Re}\,\lambda_0)\|X_0\|^2_{\mathcal{H}}.$$

Thus, $X_0=0$, so $Y=0$, so that $D(\lambda_0\mathcal{E}-\mathcal{A})$ is dense, and so is $D(\mathcal{A})$.

Furthermore, it is evident that the bilinear form B exhibits both boundedness and coercivity in $\mathbb{H}$. Using the Lax–Milgram theorem, we can conclude that there exists a unique solution (u_1,u_2) satisfying (3.8). In conclusion, the inverse of the operator $\mathcal{A}$, denoted as $\mathcal{A}^{-1}$, exists and is bounded in the Hilbert space $\mathcal{H}$. Consequently, the Lumer–Phillips theorem, as detailed in [26], validates the desired conclusion. □

Theorem 3.2. *For any initial and history data $X_0\in\mathcal{H}$, the abstract problem* (3.2) *has a unique mild solution $e^{\mathcal{A}t}X_0$. Moreover, if $X_0\in D(\mathcal{A})$, the abstract problem* (3.2) *has a unique classical solution.*

4. Stability analysis of system

In this section, we delve into the stability analysis of the C_0-semigroup $e^{\mathcal{A}t}$ associated with the system (1.1)–(1.3). To accomplish this, we give a hypothesis regarding the memory kernel g:

(A1) There exist real constants $\mu_1,\mu_2>0$ such that $-\mu_1 g(t)\leq g'(t)\leq-\mu_2 g(t)$.

Remark 4.1. *Assumption (A1), which only exhibits exponential decay of g, imposes strong constraints on the relaxation function g. In fact, there are many other types of g satisfying condition* (1.4)*, excluding the typical case of exponential decay. Al-Mahdi, Al-Gharabli, and Messaoudi [1, 24] provided more general assumptions about g, allowing a wide class of kernels: for example,*

$$g(t)=\left(\frac{a}{1+t}\right)^m,\ \ m>1,$$

$$g(t)=\left(\frac{a}{(t+2)\ln(t+2)}\right)^m,\ \ m>1,$$

and

$$g(t)=\left(\frac{a}{te^t}\right)^m,\ \ m>0,$$

where a is some positive constant. We select (A1) to explore the transmission of the effect of indirect memory-type damping which is of exponential-decay type in a wave-plate coupled system.

Then, we present the frequency-domain theories, which are essential to the establishment of our primary stability outcomes.

Lemma 4.2. *[28] Suppose $\left\{e^{\mathcal{A}t}\right\}_{t\geq 0}$ is a C_0-semigroup of contractions on $\mathcal{H}$ such that* $\mathrm{i}\mathbb{R} \subset \rho(\mathcal{A})$ *. Then, $\left\{e^{\mathcal{A}t}\right\}_{t\geq 0}$ is exponentially stable if and only if*

$$\lim_{|\lambda|\to\infty} \sup \|(\mathrm{i}\lambda\mathcal{E} - \mathcal{A})^{-1}\|_{\mathcal{H}} < \infty, \tag{4.1}$$

where $\mathcal{E}$ is an identical transformation in $\mathcal{H}$.

Lemma 4.3. *[6, 11, 14, 27] Suppose $\left\{e^{\mathcal{A}t}\right\}_{t\geq 0}$ is a C_0-semigroup of contractions on $\mathcal{H}$ such that* $\mathrm{i}\mathbb{R} \subset$ *$\rho(\mathcal{A})$. Then,*

$$\left\|e^{\mathcal{A}t}X_0\right\| \leq Ct^{-1/\theta}\,\|X_0\|_{D(\mathcal{A})}\,,\quad \forall t > 0, X_0 \in D(\mathcal{A}), \tag{4.2}$$

if and only if

$$\lim_{|\lambda|\to\infty} \sup |\lambda|^{-\theta}\|(i\lambda\mathcal{E} - \mathcal{A})^{-1}\|_{\mathcal{H}} < \infty. \tag{4.3}$$

4.1. Exponential stability of $e^{t\mathcal{A}}$ with $\alpha_1 > 0$ and $\alpha_2 > 0$

In this section, we delve into the scenario where viscoelastic damping simultaneously impacts both wave and plate ($\alpha_1 > 0$ and $\alpha_2 > 0$). We provide a rigorous analysis of the exponential stability of $e^{t\mathcal{A}}$ when memory effects are present in both wave and plate.

Theorem 4.4. *If the memory operates concurrently on both wave and plate, namely, $\alpha_1 > 0$ and $\alpha_2 > 0$, and the hypothesis (A1) holds, then the semigroup $e^{\mathcal{A}t}$ exhibits exponential decay on $\mathcal{H}$. In other words, there exist $C,\ \omega > 0$ such that*

$$\|e^{t\mathcal{A}}X_0\|_{\mathcal{H}} \leq Ce^{-\omega t}\|X_0\|_{\mathcal{H}},\ \ \forall X_0 \in \mathcal{H}. \tag{4.4}$$

Proof. We prove the theorem above by rigorously examining the conditions of Lemma 4.2. First, we prove (4.1) through the method of contradiction. If it is incorrect, then there exist $X_n = (u_1^n, v_1^n, u_2^n, v_2^n, \eta_1^n, \eta_2^n)^\top \in D(\mathcal{A})$ with $\|X_n\|_{\mathcal{H}} = 1$ and a sequence $\{\lambda_n\}_{n=1}^\infty \subset \mathbb{R}$ such that

$$(\mathrm{i}\lambda_n\mathcal{E} - \mathcal{A})\,X_n = Y_n = o(1) \text{ in } \mathcal{H} \tag{4.5}$$

when $\lim\limits_{n\to\infty} \lambda_n = \infty$. Here, $Y_n = (w_1^n, h_1^n, w_2^n, h_2^n, \xi_1^n, \xi_2^n)^\top \in \mathcal{H}$. Equation (4.5) implies that

$$\mathrm{i}\lambda_n u_1^n - v_1^n = w_1^n = o(1) \text{ in } H_0^1, \tag{4.6}$$

$$\mathrm{i}\lambda_n v_1^n - \beta_1\Delta u_1^n - \alpha_1\int_0^\infty g(s)\Delta\eta_1^n(s)\mathrm{d}s + k(u_1^n - u_2^n) = h_1^n = o(1) \text{ in } L^2, \tag{4.7}$$

$$\mathrm{i}\lambda_n u_2^n - v_2^n = w_2^n = o(1) \text{ in } H_{\Gamma_1}^2, \tag{4.8}$$

$$\mathrm{i}\lambda_n v_2^n + \beta_2\Delta^2 u_2^n + \alpha_2\int_0^\infty g(s)\Delta^2\eta_2^n(s)\mathrm{d}s - k(u_1^n - u_2^n) = h_2^n = o(1) \text{ in } L^2, \tag{4.9}$$

$$\mathrm{i}\lambda_n\eta_1^n - v_1^n + \partial_s\eta_1^n = \xi_1^n = o(1) \text{ in } \mathcal{M}_1, \tag{4.10}$$

$$\mathrm{i}\lambda_n\eta_2^n - v_2^n + \partial_s\eta_2^n = \xi_2^n = o(1) \text{ in } \mathcal{M}_2. \tag{4.11}$$

We get from boundary conditions that

$$\mathrm{i}\lambda_n \eta_i^n - v_i^n + \partial_s \eta_i^n = \xi_i^n = o(1) \text{ in } L_g^2, \quad i = 1, 2. \tag{4.12}$$

Here,

$$L_g^2 = L_g^2(\mathbb{R}_+; L^2(\Omega)) = \left\{ \eta(x,s) \in L^2 \mid \int_0^\infty g(s) \|\eta(x,s)\| \mathrm{d}s < \infty \right\}$$

with inner product

$$\langle \eta, \xi \rangle_g = \int_0^\infty g(s) \langle \eta(\cdot, s), \xi(\cdot, s) \rangle_{L^2} \mathrm{d}s, \quad \forall \eta, \xi \in L_g^2,$$

and norm $\|\cdot\|_g = \langle \cdot, \cdot \rangle_g^{1/2}$. To establish a direct contradiction to $\|X_n\|_{\mathcal{H}} = 1$, it is imperative to focus on the objective of

$$\lim_{n\to\infty} \|u_1^n\|_{H^1_{\Gamma_1}} = \lim_{n\to\infty} \|u_2^n\|_{H^2_{\Gamma_1}} = \lim_{n\to\infty} \|v_i^n\| = \lim_{n\to\infty} \|\eta_i^n\|_{\mathcal{M}_i} = 0, \quad i = 1, 2.$$

To ensure clarity and rigor, it will be structured into three distinct steps.

Step 1. Prove $\|\eta_i^n\|_{\mathcal{M}_i} = o(1), \ i = 1, 2.$

By (3.3) and (4.5), we conclude that

$$\begin{aligned} &\lim_{n\to\infty} \int_0^\infty (-g'(s)) \left(\alpha_1 \|\eta_1^n(s)\|^2_{H^1_{\Gamma_1}} + \alpha_2 \|\eta_2^n(s)\|^2_{H^2_{\Gamma_1}} \right) \mathrm{d}s \\ = &-\lim_{n\to\infty} \operatorname{Re} \langle \mathcal{A} X_n, X_n \rangle_{\mathcal{H}} \\ = &\lim_{n\to\infty} \operatorname{Re} \langle (i\lambda_n \mathcal{E} - \mathcal{A}) X_n, X_n \rangle_{\mathcal{H}} = 0. \end{aligned} \tag{4.13}$$

Due to the hypothesis (A1), we have

$$\|\eta_1^n\|^2_{\mathcal{M}_1} \le \frac{1}{\mu_2} \int_0^\infty (-g'(s)) \|\eta_1^n(s)\|^2_{H^1_{\Gamma_1}} \mathrm{d}s \to 0, \quad n \to \infty. \tag{4.14}$$

$$\|\eta_2^n\|^2_{\mathcal{M}_2} \le \frac{1}{\mu_2} \int_0^\infty (-g'(s)) \|\eta_2^n(s)\|^2_{H^2_{\Gamma_1}} \mathrm{d}s \to 0, \quad n \to \infty. \tag{4.15}$$

Step 2. Prove $\|v_i^n\| = o(1), \ i = 1, 2.$

Using multiplier $v_i^n(x)$ in (4.12), we have, for $i = 1, 2$,

$$-\mathrm{i}\lambda_n \langle v_i^n, \eta_i^n \rangle_g - \|v_i^n\|_g^2 + \langle v_i^n, \partial_s \eta_i^n \rangle_g = \langle v_i^n, \xi_i^n \rangle_g \to 0. \tag{4.16}$$

Next, we proceed to the estimation of each individual term above. Leveraging Poincaré's inequality and Hölder's inequality, in conjunction with (4.7), we arrive at

$$\begin{aligned} \left| \mathrm{i}\lambda_n \langle v_1^n, \eta_1^n \rangle_g \right| &= \left| \beta_1 \langle u_1^n, \eta_1^n \rangle_{\mathcal{M}_1} + \alpha_1 \left\| \int_0^\infty g(s) \eta_1^n(s) \mathrm{d}s \right\|^2_{H^1_{\Gamma_1}} + k \langle u_1^n - u_2^n, \eta_1^n \rangle_g - \langle h_1^n, \eta_1^n \rangle_g \right| \\ &\le C \|\eta_1^n\|_{\mathcal{M}_1} \left(\|u_1^n\|_{H^1_{\Gamma_1}} + \|\eta_1^n\|_{\mathcal{M}_1} + \|u_1^n - u_2^n\| + \|h_1^n\| \right) \to 0. \end{aligned} \tag{4.17}$$

Similarly, it can be inferred that

$$\left| \mathrm{i}\lambda_n \langle v_2^n, \eta_2^n \rangle_g \right| \le C \|\eta_2^n\|_{\mathcal{M}_2} \left(\|u_2^n\|_{H^2_{\Gamma_1}} + \|\eta_2^n\|_{\mathcal{M}_2} + \|u_1^n - u_2^n\| + \|h_2^n\| \right) \to 0. \tag{4.18}$$

Using the integration-by-parts formula and Cauchy's inequality, and thanks to hypothesis (A1), we deduce that, for $i = 1, 2$,

$$\left|\langle v_i^n, \partial_s \eta_i^n \rangle_g\right| = \left|\int_0^\infty g'(s)\langle v_i^n, \eta_i^n \rangle \mathrm{d}s\right| \leq C \left|\int_0^\infty g(s)\langle v_i^n, \eta_i^n \rangle \mathrm{d}s\right| \leq C\|v_i^n\|\|\eta_i^n\|_g \to 0. \tag{4.19}$$

Then, (4.16)–(4.19) show that

$$\int_0^\infty g(s)\mathrm{d}s\|v_i^n\|^2 = \|v_i^n\|_g^2 \to 0. \tag{4.20}$$

Step 3. Prove $\|u_1^n\|_{H^1_{\Gamma_1}} = o(1)$ and $\|u_2^n\|_{H^2_{\Gamma_1}} = o(1)$.

From (4.6)–(4.9), we have

$$\|\lambda_n u_i^n\|^2 = \|v_i^n + w_i^n\|^2 \leq (\|v_i^n\| + \|w_i^n\|)^2 \to 0, \quad i = 1, 2, \tag{4.21}$$

$$-\lambda_n^2 u_1^n - \beta_1 \Delta u_1^n - \alpha_1 \int_0^\infty g(s)\Delta\eta_1^n(s)\mathrm{d}s + k(u_1^n - u_2^n) - \mathrm{i}\lambda_n w_1^n = h_1^n, \tag{4.22}$$

and

$$-\lambda_n^2 u_2^n + \beta_2 \Delta^2 u_2^n + \alpha_2 \int_0^\infty g(s)\Delta^2\eta_2^n(s)\mathrm{d}s - k(u_1^n - u_2^n) - \mathrm{i}\lambda_n w_2^n = h_2^n. \tag{4.23}$$

Then, using multiplier u_1^n in (4.22) and combining with (4.21), we have

$$\begin{aligned} \beta_1\|u_1^n\|^2_{H^1_{\Gamma_1}} &= \Big|\|\lambda_n u_1^n\|^2 - \alpha_1\langle \eta_1^n, u_1^n\rangle_{\mathcal{M}_1} - k\langle u_1^n - u_2^n, u_1^n\rangle \\ &\quad +\mathrm{i}\langle w_1^n, \lambda_n u_1^n\rangle + \langle h_1^n, u_1^n\rangle\Big| \\ &\leq \|\lambda_n u_1^n\|^2 + \alpha_1\|u_1^n\|_{H^1_{\Gamma_1}}\|\eta_1^n\|_{\mathcal{M}_1} + k\|u_1^n - u_2^n\|\|u_1^n\| \\ &\quad +\|w_1^n\|\|\lambda_n u_1^n\| + \|h_1^n\|\|u_1^n\| \to 0. \end{aligned} \tag{4.24}$$

Analogous to the aforementioned inequality, we obtain $\|u_2^n\|^2_{H^2_{\Gamma_1}} \to 0$.

In conclusion, it is evident that the limit $\lim\limits_{n\to 0}\|X_n\|_{\mathcal{H}} = 0$ holds, which presents a contradiction. Leveraging Lemma 4.2, the exponential stability of $e^{t\mathcal{A}}$ with memory acting concurrently on both wave and plate can be established by demonstrating that $\mathrm{i}\mathbb{R} \subset \rho(\mathcal{A})$. In fact, if this condition is not met, it follows from the openness of $\rho(\mathcal{A})$ and $0 \in \rho(\mathcal{A})$ that

$$0 < \tilde{\lambda}\Delta q \sup\left\{\bar{\lambda} > 0 \mid [-\mathrm{i}\bar{\lambda}, \mathrm{i}\bar{\lambda}] \subset \rho(\mathcal{A})\right\} < \infty.$$

Based on the Banach-Steinhaus theorem, there exist sequences $\{\lambda_n\}$ converging to $\tilde{\lambda}$ and $\{X_n\} = \{(u_1^n, v_1^n, u_2^n, v_2^n, \eta_1^n, \eta_2^n)^\top\}$ belonging to the domain $D(\mathcal{A})$ with $\|X_n\|_{\mathcal{H}} = 1$ such that $(\mathrm{i}\lambda_n - \mathcal{A})X_n \to 0$ as $n \to \infty$ in the Hilbert space $\mathcal{H}$. Consequently, (4.6)–(4.11) hold for $\lambda_n \to \tilde{\lambda}$. By repeating the previous steps, we obtain the contradiction $\lim_{n\to\infty}\|X_n\|_{\mathcal{H}} = 0$, indicating that $\mathrm{i}\mathbb{R} \subset \rho(\mathcal{A})$. This completes the proof. □

4.2. *Polynomial stability of $e^{t\mathcal{A}}$ with $\alpha_1 > 0$ and $\alpha_2 = 0$*

The polynomial stability of $e^{t\mathcal{A}}$ with memory solely influencing the wave motion is presented hereinafter.

Theorem 4.5. *If the memory only operates within the wave equation, namely, $\alpha_1 > 0$ and $\alpha_2 = 0$, and the hypothesis (A1) is satisfied, then the semigroup $e^{\mathcal{A}t}$ exhibits polynomial decay on the space $\mathcal{H}$, i.e., there exists $C > 0$ such that*

$$\|e^{t\mathcal{A}}X_0\|_{\mathcal{H}} \leq Ct^{-1/8}\|X_0\|_{D(\mathcal{A})}, \quad \forall X_0 \in D(\mathcal{A}). \tag{4.25}$$

Proof. The theorem can be established by verifying the conditions outlined in Lemma 4.3. By examining the proof of Theorem 4.4, it suffices to establish the validity of (4.3). Furthermore, the condition $\mathrm{i}\mathbb{R} \subset \rho(\mathcal{A})$ can be readily confirmed. In the event that (4.3) is incorrect, there exist sequences of the form $X_n = (u_1^n, v_1^n, u_2^n, v_2^n, \eta_1^n, 0)^\top \in D(\mathcal{A})$ with $\|X_n\|_{\mathcal{H}} = 1$ and a corresponding sequence $\{\lambda_n\}_{n=1}^\infty \subset \mathbb{R}$ such that

$$\lambda_n^8\,(\mathrm{i}\lambda_n\mathcal{E} - \mathcal{A})\,X_n = Y_n = o(1) \text{ in } \mathcal{H}. \tag{4.26}$$

Here, $Y_n = (w_1^n, h_1^n, w_2^n, h_2^n, \xi_1^n, 0)^\top \in \mathcal{H}$. Equation (4.26) gives

$$\lambda_n^8\,(\mathrm{i}\lambda_n u_1^n - v_1^n) = w_1^n = o(1) \text{ in } H_0^1, \tag{4.27}$$

$$\lambda_n^8\left(\mathrm{i}\lambda_n v_1^n - \beta_1\Delta u_1^n - \alpha_1\int_0^\infty g(s)\Delta\eta_1^n(s)\mathrm{d}s + k(u_1^n - u_2^n)\right) = h_1^n = o(1) \text{ in } L^2, \tag{4.28}$$

$$\lambda_n^8\,(\mathrm{i}\lambda_n u_2^n - v_2^n) = w_2^n = o(1) \text{ in } H_{\Gamma_1}^2, \tag{4.29}$$

$$\lambda_n^8\left(\mathrm{i}\lambda_n v_2^n + \Delta^2 u_2^n - k(u_1^n - u_2^n)\right) = h_2^n = o(1) \text{ in } L^2, \tag{4.30}$$

$$\lambda_n^8\,(\mathrm{i}\lambda_n\eta_1^n - v_1^n + \partial_s\eta_1^n) = \xi_1^n = o(1) \text{ in } \mathcal{M}_1. \tag{4.31}$$

We get from (4.31) and the boundary condition that

$$\lambda_n^8\,(\mathrm{i}\lambda_n\eta_1^n - v_1^n + \partial_s\eta_1^n) = \xi_1^n = o(1) \text{ in } L_g^2. \tag{4.32}$$

Next, we prove

$$\lim_{n\to\infty}\|u_1^n\|_{H_{\Gamma_1}^1} = \lim_{n\to\infty}\|u_2^n\|_{H_{\Gamma_1}^2} = \lim_{n\to\infty}\|v_i^n\| = \lim_{n\to\infty}\|\eta_1^n\|_{\mathcal{M}_1} = 0, \quad i = 1, 2.$$

Step 1. Prove $\|\eta_1^n\|_{\mathcal{M}_1} = \lambda_n^{-4}o(1)$.
Similar to (4.13), we get

$$\lim_{n\to\infty}\int_0^\infty(-g'(s))\,\|\lambda_n^4\eta_1^n(s)\|_{H_{\Gamma_1}^1}^2\,\mathrm{d}s = \frac{1}{\alpha_1}\lim_{n\to\infty}\mathrm{Re}\left\langle\lambda_n^8(i\lambda_n\mathcal{E} - \mathcal{A})X_n, X_n\right\rangle_{\mathcal{H}} = 0. \tag{4.33}$$

Thus, from the hypothesis (A1), we have

$$\|\lambda_n^4\eta_1^n\|_{\mathcal{M}_1}^2 \leq \frac{1}{\mu_2}\int_0^\infty(-g'(s))\,\|\lambda_n^4\eta_1^n(s)\|_{H_{\Gamma_1}^1}^2\,\mathrm{d}s = o(1). \tag{4.34}$$

Step 2. Prove $\|v_1^n\| = \lambda_n^{-2}o(1)$.
Taking the inner product of (4.32) with $v_1^n(x)$ in L_g^2, we have

$$-\mathrm{i}\lambda_n\langle v_1^n, \eta_1^n\rangle_g - \|v_1^n\|_g^2 + \langle v_1^n, \partial_s\eta_1^n\rangle_g = \langle v_1^n, \lambda_n^{-8}\xi_1^n\rangle_g. \tag{4.35}$$

Consult Eq (4.17)–(4.19) and combine with $\|X_n\|_{\mathcal{H}} = 1$ and $\|\eta_1^n\|_{\mathcal{M}_1} = \lambda_n^{-4}o(1)$ to derive

$$\left|\mathrm{i}\lambda_n\langle v_1^n, \eta_1^n\rangle_g\right| \le C\|\eta_1^n\|_{\mathcal{M}_1}\left(\|u_1^n\|_{H^1_{\Gamma_1}} + \|\eta_1^n\|_{\mathcal{M}_1} + \|u_1^n - u_2^n\| + \|\lambda_n^{-8}h_1^n\|\right) = \lambda_n^{-4}o(1), \tag{4.36}$$

and $\left|\langle v_1^n, \partial_s\eta_1^n\rangle_g\right| \le C\|v_1^n\|\|\eta_1^n\|_g = \lambda_n^{-4}o(1)$. This implies that

$$\int_0^\infty g(s)\mathrm{d}s\|v_1^n\|^2 = \|v_1^n\|_g^2 = \lambda_n^{-4}o(1). \tag{4.37}$$

Step 3. Prove $\|u_1^n\|_{H^1_{\Gamma_1}} = \lambda_n^{-2}o(1)$.

From (4.27) and (4.28), we have

$$\|\lambda_n u_1^n\| \le \left(\|v_1^n\| + \|\lambda_n^{-8}w_1^n\|\right) = \lambda_n^{-2}o(1), \tag{4.38}$$

and

$$-\lambda_n^2 u_1^n - \beta_1\Delta u_1^n - \alpha_1\int_0^\infty g(s)\Delta\eta_1^n(s)\mathrm{d}s + k(u_1^n - u_2^n) - \mathrm{i}\lambda_n^{-7}w_1^n = \lambda_n^{-8}h_1^n. \tag{4.39}$$

Due to (4.38), we have

$$\|\lambda_n^{-1}u_1^n\| \le \lambda_n^{-4}o(1). \tag{4.40}$$

Also, from (4.29) and $\|X_n\|_{\mathcal{H}} = 1$, we have

$$\left\|\lambda_n u_2^n\right\| = \left\|v_2^n + \lambda_n^{-8}w_2^n\right\| \le \left\|v_2^n\right\| + \left\|\lambda_n^{-8}w_2^n\right\| = O(1),$$

which implies that

$$\left\|\lambda_n\left(u_1^n - u_2^n\right)\right\| \le \left\|\lambda_n u_1^n\right\| + \left\|\lambda_n u_2^n\right\| = O(1). \tag{4.41}$$

Using multiplier u_1^n in (4.39) and combining with (4.38), (4.40), and (4.41), we have

$$\begin{aligned}\beta_1\|u_1^n\|^2_{H^1_{\Gamma_1}} &\le \|\lambda_n u_1^n\|^2 + \alpha_1\|u_1^n\|_{H^1_{\Gamma_1}}\|\eta_1^n\|_{\mathcal{M}_1} + k\left\|\lambda_n\left(u_1^n - u_2^n\right)\right\|\|\lambda_n^{-1}u_1^n\| \\ &+\|\lambda_n^{-7}w_1^n\|\|u_1^n\| + \left\|\lambda_n^{-8}h_1^n\right\|\|u_1^n\| = \lambda_n^{-4}o(1).\end{aligned} \tag{4.42}$$

Step 4. Prove $\lim\limits_{n\to\infty}\|v_2^n\| = \lim\limits_{n\to\infty}\|u_2^n\|_{H^2_{\Gamma_1}} = \lim\limits_{n\to\infty}\|\lambda_n u_2^n\|$.

From (4.29), it is obvious that $\lim\limits_{n\to\infty}\|v_2^n\| = \lim\limits_{n\to\infty}\|\lambda_n u_2^n\|$. Adding (4.28) and (4.30) to eliminate the coupled term $k(u_1^n - u_2^n)$, then taking the L^2-inner product with u_2^n, we get

$$\begin{aligned}&\|\lambda_n u_2^n\|^2 - \|u_2^n\|^2_{H^2_{\Gamma_1}} \\ &= \langle\mathrm{i}\lambda_n v_1^n, u_2^n\rangle + \beta_1\langle\nabla u_1^n, \nabla u_2^n\rangle + \alpha_1\int_0^\infty g(s)\langle\nabla\eta_1^n(s), \nabla u_2^n\rangle\mathrm{d}s \\ &\quad -\langle\mathrm{i}\lambda_n^{-7}w_2^n + \lambda_n^{-8}(h_1^n + h_2^n), u_2^n\rangle.\end{aligned} \tag{4.43}$$

By Hölder's and Poincaré's inequalities and the conclusions above, we know $\|\lambda_n u_2^n\|^2 - \|u_2^n\|^2_{H^2_{\Gamma_1}} = \lambda_n^{-1}o(1)$.

Step 5. Prove $\|\lambda_n u_2^n\| = o(1)$.

From (4.29), we know that

$$\|\lambda_n u_2^n\| \le C\left(\|v_2^n\| + \|\lambda_n^{-8}w_2^n\|\right) \le C\left(\|X_n\|_{\mathcal{H}} + \|\lambda_n^{-8}Y_n\|_{\mathcal{H}}\right),$$

which implies that $\|\lambda_n u_2^n\|$ is bounded. Using multiplier u_2^n in (4.28) and Cauchy's inequality, we have

$$\begin{aligned}\|\lambda_n u_2^n\|^2 \leq\ & C\|\lambda_n u_2^n\|\left(\|\lambda_n^2 v_1^n\| + \|\lambda_n u_1^n\| + \|\lambda^{-7} h_1^n\|\right)\\ &+C\|u_2^n\|_{H^2_{\Gamma_1}}\left(\|\lambda_n^2 u_1^n\|_{H^1_{\Gamma_1}} + \|\lambda_n^2 \eta_1^n\|_{\mathcal{M}_1}\right),\end{aligned} \tag{4.44}$$

which implies $\|\lambda_n u_2^n\| = o(1)$.

In summary, it has been established that $\lim_{n\to 0}\|X_n\|_{\mathcal{H}} = 0$, which is in direct contradiction with $\lim_{n\to 0}\|X_n\|_{\mathcal{H}} = 1$. Consequently, the derivation of this theorem relies crucially on Lemma 4.3. The proof is hereby concluded. □

4.3. Polynomial stability of $e^{t\mathcal{A}}$ with $\alpha_1 = 0$ and $\alpha_2 > 0$

We initially introduce the subsequent hypothesis and lemma, which are vital in establishing the stability of $e^{t\mathcal{A}}$ with memory solely acting on the plate.

(A2) Assume that there exist some spatial point $x_0 \in \mathbb{R}^2$ and constant $\rho > 0$ such that, for the spatial vector field defined as $h = x - x_0$,

$$h\cdot\nu \leq 0 \text{ on } \Gamma_1 \quad \text{and} \quad h\cdot\nu > \rho \text{ on } \Gamma_2.$$

Lemma 4.6. *Let $u \in H^2(\Omega)\cap H^1_{\Gamma_1}$, and the following equality holds:*

$$\langle -\Delta u, h\cdot\nabla u\rangle = -\frac{1}{2}\int_{\Gamma_1}\left(\frac{\partial u}{\partial\nu}\right)^2 h\cdot\nu\mathrm{d}S + \frac{1}{2}\int_{\Gamma_2}\left(\frac{\partial u}{\partial\tau}\right)^2 h\cdot\nu\mathrm{d}S. \tag{4.45}$$

Proof. Utilize Green's formula to derive

$$\langle -\Delta u, h\cdot\nabla u\rangle = \langle \nabla u, \nabla(h\cdot\nabla u)\rangle - \int_\Gamma \frac{\partial u}{\partial\nu} h\cdot\nabla u\mathrm{d}S. \tag{4.46}$$

The first item on the right side of the formula (4.46) can be processed as follows

$$\begin{aligned}\langle \nabla u, \nabla(h\cdot\nabla u)\rangle &= \|\nabla u\|^2 + \frac{1}{2}\int_\Omega h\cdot\nabla\left(|\nabla u|^2\right)\mathrm{d}x\\ &= \|\nabla u\|^2 + \frac{1}{2}\int_\Gamma |\nabla u|^2 h\cdot\nu\mathrm{d}S - \frac{1}{2}\int_\Omega |\nabla u|^2 \mathrm{div}\, h\mathrm{d}x\\ &= \frac{1}{2}\int_\Gamma |\nabla u|^2 h\cdot\nu\mathrm{d}S.\end{aligned} \tag{4.47}$$

From the boundary conditions, we have $\nabla u = \frac{\partial u}{\partial\nu}\nu$ on Γ_1, and $\nabla u = \frac{\partial u}{\partial\tau}\tau$ on Γ_2. Combining these with (4.46) and (4.47), we deduce (4.45). Thus, we complete the proof. □

Theorem 4.7. *If the memory acts only on the plate, namely, $\alpha_1 = 0$ and $\alpha_2 > 0$, and the hypotheses (A1), (A2) hold, then the semigroup $e^{t\mathcal{A}}$ exhibits polynomial decay on the space $\mathcal{H}$, i.e., there exists $C > 0$ such that*

$$\|e^{t\mathcal{A}}X_0\|_{\mathcal{H}} \leq Ct^{-1/2}\|X_0\|_{D(\mathcal{A})},\quad \forall X_0\in D(\mathcal{A}). \tag{4.48}$$

Proof. The assertion $\mathrm{i}\mathbb{R} \subset \rho(\mathcal{A})$ can be rigorously established through a proof analogous to that of Theorem 4.4. We proceed to demonstrate the validity of (4.3). To accomplish this, we adopt a proof by contradiction. Suppose, for the sake of contradiction, that the assertion is false. Then, invoking the Banach–Steinhaus theorem, we can construct sequences $X_n = (u_1^n, v_1^n, u_2^n, v_2^n, 0, \eta_2^n)^\top \in D(\mathcal{A})$ satisfying $\|X_n\|_{\mathcal{H}} = 1$. Furthermore, there exists a sequence $\lambda_n \to +\infty$, such that the sequences satisfy the following condition:

$$\lambda_n^2\,(\mathrm{i}\lambda_n\mathcal{E} - \mathcal{A})\,X_n = Y_n \to 0 \text{ in } \mathcal{H},$$

in which $Y_n = (w_1^n, h_1^n, w_2^n, h_2^n, 0, \xi_2^n)^\top \in \mathcal{H}$. That is,

$$\lambda_n^2\,(\mathrm{i}\lambda_n u_1^n - v_1^n) = w_1^n = o(1) \text{ in } H_0^1, \tag{4.49}$$

$$\lambda_n^2\,(\mathrm{i}\lambda_n v_1^n - \Delta u_1^n + k(u_1^n - u_2^n)) = h_1^n = o(1) \text{ in } L^2, \tag{4.50}$$

$$\lambda_n^2\,(\mathrm{i}\lambda_n u_2^n - v_2^n) = w_2^n = o(1) \text{ in } H_{\Gamma_1}^2, \tag{4.51}$$

$$\lambda_n^2\left(\mathrm{i}\lambda_n v_2^n + \beta_2\Delta^2 u_2^n + \alpha_2\int_0^\infty g(s)\Delta^2\eta_2^n(s)\mathrm{d}s - k(u_1^n - u_2^n)\right) = h_2^n = o(1) \text{ in } L^2, \tag{4.52}$$

$$\lambda_n^2\,(\mathrm{i}\lambda_n\eta_2^n - v_2^n + \partial_s\eta_2^n) = \xi_2^n = o(1) \text{ in } \mathcal{M}_2. \tag{4.53}$$

We get from (4.53) and the boundary condition that

$$\lambda_n^2\,(\mathrm{i}\lambda_n\eta_2^n - v_2^n + \partial_s\eta_2^n) = \xi_2^n \to 0 \text{ in } L_g^2. \tag{4.54}$$

Next, the process is divided into five steps.

Step 1. Prove $\|\eta_2^n\|_{\mathcal{M}_2} = \lambda_n^{-1}o(1)$.

$$\begin{aligned}\lim_{n\to\infty}\int_0^\infty (-g'(s))\,\|\lambda_n\eta_2^n(s)\|_{H_{\Gamma_1}^2}^2\,\mathrm{d}s &= -\frac{1}{\alpha_2}\lim_{n\to\infty}\mathrm{Re}\left\langle\lambda_n^2\mathcal{A}X_n, X_n\right\rangle_{\mathcal{H}} \\ &= \lim_{n\to\infty}\mathrm{Re}\left\langle\lambda_n^2(\mathrm{i}\lambda_n\mathcal{E} - \mathcal{A})X_n, X_n\right\rangle_{\mathcal{H}} = 0.\end{aligned} \tag{4.55}$$

Due to the assumption (A1), we have

$$\|\lambda_n\eta_2^n\|_{\mathcal{M}_2}^2 \le \frac{1}{\mu_2}\int_0^\infty (-g'(s))\,\|\lambda_n\eta_2^n(s)\|_{H_{\Gamma_1}^2}^2\,\mathrm{d}s \to 0, \tag{4.56}$$

which implies that $\|\lambda_n\eta_2^n\|_{\mathcal{M}_2} = o(1)$.

Step 2. Prove $\|v_2^n\|_{L^2} = \lambda_n^{-1/2}o(1)$.

Using multiplier $g(s)v_2^n(x)$ in (4.54) and subsequently integrating the resulting expression with respect to s, we arrive at

$$\langle\lambda_n v_2^n, \mathrm{i}\lambda_n\eta_2^n\rangle_g - \lambda_n\|v_2^n\|_g^2 + \langle v_2^n, \lambda_n\partial_s\eta_2^n\rangle_g = \langle v_2^n, \lambda_n^{-1}\xi_2^n\rangle_g. \tag{4.57}$$

Next, we estimate each term above. Similarly to (4.36), we have

$$\left|\langle\mathrm{i}\lambda_n v_2^n, \lambda_n\eta_2^n\rangle_g\right| \le C\|\lambda_n\eta_2^n\|_{\mathcal{M}_2}\left(\|u_2^n\|_{H_{\Gamma_1}^2} + \|\eta_2^n\|_{\mathcal{M}_2} + \|u_1^n - u_2^n\| + \|\lambda_n^{-2}h_2^n\|\right) \to 0, \tag{4.58}$$

and

$$\left|\langle v_2^n, \lambda_n\partial_s\eta_2^n\rangle_g\right| \le \mu_1\|v_2^n\|\|\lambda_n\eta_2^n\|_g \to 0. \tag{4.59}$$

From (4.58) and (4.59), we deduce that

$$\lambda_n \|v_2^n\|_g^2 = \lambda_n \int_0^\infty g(s)\mathrm{d}s \|v_2^n\|^2 \to 0. \tag{4.60}$$

Step 3. Prove $\|u_2^n\|_{H^2_{\Gamma_1}} = \lambda_n^{-1} o(1)$.

From (4.51) and (4.52), we have

$$\|\lambda_n u_2^n\| = \|v_2^n + \lambda_n^{-2} w_2^n\| \leq \|v_2^n\| + \|\lambda_n^{-2} w_2^n\| = \lambda_n^{-1/2} o\left(1\right), \tag{4.61}$$

and

$$-\lambda_n^2 u_2^n + \beta_2 \Delta^2 u_2^n + \alpha_2 \int_0^\infty g(s) \Delta^2 \eta_2^n(s)\mathrm{d}s - k(u_1^n - u_2^n) - \mathrm{i}\lambda_n^{-1} w_2^n = \lambda_n^{-2} h_2^n. \tag{4.62}$$

Then, using multiplier u_2^n in (4.62) and combining with (4.61), we have

$$\begin{aligned} \beta_2 \|u_2^n\|^2_{H^2_{\Gamma_1}} &= \|\lambda_n u_2^n\|^2 - \alpha_2 \langle \eta_2^n(s), u_2^n \rangle_{\mathcal{M}_2} + k\langle u_1^n - u_2^n, u_2^n \rangle \\ &\quad + \mathrm{i}\langle \lambda_n^{-1} w_2^n, u_2^n \rangle + \langle \lambda_n^{-2} h_2^n, u_2^n \rangle \\ &\leq \|\lambda_n u_2^n\|^2 + \alpha_2 \|u_2^n\|_{H^2_{\Gamma_1}} \|\eta_2^n\|_{\mathcal{M}_2} + k\|u_1^n - u_2^n\| \|u_2^n\| \\ &\quad + \|\lambda_n^{-1} w_2^n\| \|\lambda_n u_2^n\| + \|\lambda_n^{-2} h_2^n\| \|u_2^n\| = o\left(\lambda_n^{-1}\right). \end{aligned} \tag{4.63}$$

Step 4. Prove $\lim\limits_{n\to\infty} \|v_1^n\| = \lim\limits_{n\to\infty} \|u_1^n\|_{H^1_{\Gamma_1}} = \lim\limits_{n\to\infty} \|\lambda_n u_1^n\|$.

From (4.49), we know that

$$\|\lambda_n u_1^n\|^2 = \|\mathrm{i}\lambda_n u_1^n\|^2 = \|\lambda_n^{-2} w_1^n + v_1^n\|^2 = \|v_1^n\|^2 + \|\lambda_n^{-2} w_1^n\|^2 + 2\mathrm{Re}\langle v_1^n, \lambda_n^{-2} w_1^n \rangle, \tag{4.64}$$

and

$$\|\lambda_n u_1^n\| \leq C\left(\|v_1^n\| + \|\lambda_n^{-2} w_1^n\|\right) \leq C\left(\|X_n\|^2_{\mathcal{H}} + \|Y_n\|^2_{\mathcal{H}}\right). \tag{4.65}$$

(4.64) implies that $\lim\limits_{n\to\infty} \|v_1^n\|^2 = \lim\limits_{n\to\infty} \|\lambda_n u_1^n\|^2$, and (4.65) implies that $\|\lambda_n u_1^n\|$ is bounded, which also implies that $\lim\limits_{n\to\infty} \|u_1^n\| = 0$. Moreover, we deduce from (4.52) that

$$\|\lambda_n^{-1} \Delta^2 \zeta_2^n\| \leq C\left(\|\lambda_n^{-3} h_2^n\| + \|v_2^n\| + \|\lambda_n^{-1}\left(u_1^n - u_2^n\right)\|\right) \to 0. \tag{4.66}$$

Based on (4.49) to (4.52), we have

$$\lambda_n^2\left(u_1^n + u_2^n\right) + \Delta u_1^n - \Delta^2 \zeta_2^n + \mathrm{i}\lambda_n^{-1}\left(w_1^n + w_2^n\right) + \lambda_n^{-2}\left(h_1^n + h_2^n\right) = 0. \tag{4.67}$$

Using multiplier u_1^n in (4.67), we have

$$\begin{aligned} \|u_1^n\|^2_{H^1_0} - \|\lambda_n u_1^n\|^2 &= \langle \lambda_n u_2^n, \lambda_n u_1^n \rangle + \mathrm{i}\langle \lambda_n^{-1}\left(w_1^n + w_2^n\right), u_1^n \rangle \\ &\quad - \langle \lambda_n^{-1} \Delta^2 \zeta_2^n, \lambda_n u_1^n \rangle + \langle \lambda_n^{-2}\left(h_1^n + h_2^n\right), u_1^n \rangle, \end{aligned} \tag{4.68}$$

which, combining with (4.61) and (4.66), implies that $\lim\limits_{n\to\infty} \|u_1^n\|^2_{H^1_{\Gamma_1}} = \lim\limits_{n\to\infty} \|\lambda_n u_1^n\|^2$.

Step 5. Prove $\|\lambda_n u_1^n\|^2 = o(1)$.

Calculating the gradient of Eq (4.49) on both sides and then taking the dot product with the vector field h, we have

$$\mathrm{i}\lambda_n h \cdot \nabla u_1^n - h \cdot \nabla v_1^n = \lambda_n^{-2} h \cdot \nabla w_1^n,$$

Using multiplier v_1^n in the above equation, we have

$$\langle v_1^n, \mathrm{i}\lambda_n h \cdot \nabla u_1^n \rangle - \langle v_1^n, h \cdot \nabla v_1^n \rangle \to 0. \tag{4.69}$$

On the other hand, using multiplier $h \cdot \nabla u_1^n$ in (4.50), we have

$$\langle \mathrm{i}\lambda_n v_1^n, h \cdot \nabla u_1^n \rangle - \langle \Delta u_1^n, h \cdot \nabla u_1^n \rangle + k \langle u_1^n - u_2^n, h \cdot \nabla u_1^n \rangle \to 0. \tag{4.70}$$

Based on the facts that $\|u_i^n\| = o(1),\ i = 1, 2$, we know $\|u_1^n - u_2^n\| = o(1)$. Then, according to Lemma 4.6, the combination of (4.69) and (4.70) results in

$$\|v_1^n\|^2 - \frac{1}{2} \int_{\Gamma_1} \left(\frac{\partial u_1^n}{\partial \nu} \right)^2 h \cdot \nu \mathrm{d}S + \frac{1}{2} \int_{\Gamma_2} \left(\frac{\partial u_1^n}{\partial \tau} \right)^2 h \cdot \nu \mathrm{d}S \to 0. \tag{4.71}$$

From assumption (A2), we directly deduce that $\|v_1^n\|^2 = o(1)$. In summary, we have derived the limit $\lim_{n\to 0} \|X_n\|_{\mathcal{H}} = 0$, which contradicts the stated condition $\lim_{n\to 0} \|X_n\|_{\mathcal{H}} = 1$. Therefore, the validity of this theorem relies crucially on Lemma 4.3. The proof is hereby concluded. □

Remark 4.8. *The damping term in system* (1.1)–(1.3) *is infinite history memory. We can also deal with finite history memory in wave-plate systems using the method of Xu [32], which will be discussed in future work.*

5. Conclusions

We studied the stability of a wave-plate system with memory viscoelastic damping. First, the well-posedness of the solution was proved by the semigroup method under suitable conditions. Next, using frequency domain theories, we showed that the system decays exponentially when the viscoelastic damping acts on both the wave and plate, decays polynomially with order $t^{-1/8}$ when the viscoelastic damping acts only on the wave, and decays polynomially with order $t^{-1/2}$ when the viscoelastic damping acts on the plate only. The given polynomial decay rates are not proved optimal, which will be part of future work. Furthermore, we will also consider the other types of relaxation functions g in system (1.1)–(1.3), such as the polynomial-decay type, and long-time behavior of the wave-plate transmission systems with boundary-coupling in future work.

Author contributions

Peipei Wang: Conceptualization, Methodology, Writing-original draft, Formal analysis, Investigation; Yanting Wang: Methodology, Writing-review & editing; Fei Wang: Conceptualization, Methodology. All authors have read and approved the final version of the manuscript for publication.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflicts of interest.

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