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TOPSIS method based on q-rung orthopair picture fuzzy soft environment and its application in the context of green supply chain management

Sumbal Ali[1], **Asad Ali**[1,*], **Ahmad Bin Azim**[1], **Abdul Samad Khan**[2], **Fuad A. Awwad**[3] and **Emad A. A. Ismail**[3]

[1] Department of Mathematics & Statistics, Hazara University, Mansehra, 21300, Pakistan
[2] Research Centre for Computational Science, School of Mathematics and Statistics, Northwestern Polytechnical University, Xi'an 710129, China
[3] Department of Quantitative analysis, College of Business Administration, King Saud University, P. O. Box 71115, Riyadh 11587, Saudi Arabia

* **Correspondence:** Email: asad_maths@hu.edu.pk; Tel: +923339508166.

Abstract: Green supplier selection has been an important technique for environmental sustainability and reducing the harm of ecosystems. In the current climate, green supply chain management (GSCM) is imperative for maintaining environmental compliance and commercial growth. To handle the change related to environmental concern and how the company manages and operates, they are integrated the GSCM into traditional supplier selection process. The main aims of this study were to outline both traditional and environmental criteria for selecting suppliers, providing a comprehensive framework to assist decision-maker in prioritizing green supplier effectively. In order to address issue to simulate decision-making problems and manage inaccurate data. A useful technique of fuzzy set was proposed to handle uncertainty in various real-life problems, but all types of data could not be handled such as incomplete and indeterminate. However, several extensions of fuzzy set were considered, such as intuitionistic fuzzy set, Pythagorean fuzzy set, q-rung orthopair fuzzy set, and q-rung orthopair fuzzy soft set considering membership and nonmember ship grade to handle the uncertainty problem. However, there was a lack of information about the neutral degree and parameterization axioms lifted by existing approaches, so to fill this gap and overcome the difficulties Ali et al. proposed a generalized structure by combining the structure of picture fuzzy set and q-rung orthopair fuzzy soft set, known as q-rung orthopair picture fuzzy soft sets, characterized by positive, neutral and negative membership degree with parameterization tools and aggregation operator to solve the multi criteria group decision-making problem. Additionally, the TOPSIS method is a widely utilized to assist individuals and organizations in selecting the most appropriate option from a range of choices, taking into account various criteria. Finally, we demonstrate an illustrative example related to GSCM to enhance competitiveness, based on criteria both in general and with a focus on environmental consideration, accompanied by an algorithm and flow chart.

Keywords: q-rung orthopair picture fuzzy soft numbers; TOPSIS technique; green supplier selection; MADM problem
Mathematics Subject Classification: 60L70, 68N17

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1. Introduction

The technique of order preferences similarity and ideal solution method is a widely utilized decision-making method in the field of multi-criteria decision analysis. It assists individuals and organizations in selecting the most appropriate option from a range of choices, taking into account various criteria. In 1981 Hwang and Yoon [1] initiated a useful method for solving the real world multi criteria group decision-making problem, called TOPSIS. This is a powerful technique to finding the ranking by using the distance measures. The fuzzy language framework has benefits in a number of areas, which are mostly described in a qualitative way. The rationale for utilizing words instead of solely relying on numbers is that linguistic descriptions or categorizations are typically even less precise compared to those found in algebra or mathematics. Risky situations often give rise to questions when making decisions. These inquiries also create stress regarding the intricate issue of models and management that arises due to these uncertainties.

The crisp set, which is also known as the Boolean set or the classical set, is the basic concept of set theory and mathematical logic, defined by the well-defined criteria of membership. In the crisp set, the element either belongs or does not belong to the set, and there is no uncertainty about its member status. If an element of the set possesses specific property its membership value is 1 or 0. Most of the time, it's hard to deal with the specific property, not entirely, but to some extent, of what value will be assigned to the element. So, this was the main reason behind introducing the concept of fuzzy set to handle these hindrances. To model fuzzy data as mathematically, Zadeh [2] initiated the fuzzy set (FS) in 1965. FS described by membership values belongs to [0,1], but there is no information about the nonmember ship function of an element. So, to tackle this situation in 1986 Atanassov [3] add non membership in FS, to deal the positive as well as negative aspect of the object, with the condition that $0 \leq (\mathrm{MD}) + (\mathrm{NMD}) \leq 1$. Under the environment of intuitionistic FS Born et al. [4] in 2009 proposed the TOPSIS method to solve the multi criteria group decision-making problem. FS and IFS deal with the linear relation of membership and nonmember ship degree, by following the condition $0 \leq (\mathrm{MD}) + (\mathrm{NMD}) \leq 1$, but there a problem arises. If we take $\mathrm{MD} = 0.6$ and $\mathrm{NMD} = 0.7$, then $(\mathrm{MD}) + (\mathrm{NMD}) > 1$, so FS and IFS cannot be handling this type of issue. So, to handle this problem in 2013, Yager [5] replaced the condition of IFS $0 \leq (\mathrm{MD}) + (\mathrm{NMD}) \leq 1$ to $0 \leq (\mathrm{MD})^2 + (\mathrm{NMD})^2 \leq 1$, and proposed the Pythagorean fuzzy set (PyFS). PyFS having the ability to handle such an uncertainty is stronger than IFS. In 2016, Yager [6], relaxed the condition of PyFS with the condition that $0 \leq (\mathrm{MD})^{\mathrm{q}} + (\mathrm{NMD})^{\mathrm{q}} \leq 1$. The only restriction on q is that $q \geq 1$ and the offered the generalized concept of IFS and PFS called q-rung orthopair fuzzy set (q-ROFS). In FS, IFS, PFS and q-ROFS, we solve the uncertainties the in MCDM problem with membership and nonmember ship, but in all these theories there is no information about the parametrization tool. So, in 1999, Molodtsov [7] proposed a novel idea called soft set theory, free from all this difficulty. In some senses, FS can be considered a special case of the soft set theory. Molodtsov's soft set merged with different structures and developed a new idea such as in 2001 when Maji et al. [8] combined soft set and FS, introducing fuzzy soft set and proposing a hybrid structure of IFS and soft set termed intuitionistic fuzzy soft set offered by Maji et al. [9]. In 2015, Peng et al. [10] initiated Pythagorean fuzzy soft set with some basic operation and significant results in the environment of PyFSS. In 2019, Naeem et al. [11] initiated TOPSIS and VIKOR methods based on PyFStS and Pythagorean m-polar fuzzy soft set with the TOPSIS method proposed by Riaz et al. [12] in 2020. Hussain et al. [13] proposed the q-rung orthopair fuzzy soft set and their application in MCGDM by combining the idea of soft set and q-rung orthopair fuzzy sets in 2020, which deal with uncertainty with membership, nonmember ship degree with parameterization axioms. Riaz et al. [14] developed TOPSIS, VIKOR and aggregation operators to solve the MCDM problem based on q-ROFS$_t$S in 2020. In 2021, Chinram et al. [15] introduced geometric aggregation operators based on q-ROFS$_t$S and their application in medical diagnoses to solve MCGDM. In 2022, Zulqarnain et al. [16] proposed a novel MCDM approach for interactive aggregation operators in the environment of q-rung orthopair fuzzy soft set. In the above literature, we analyze that there is a lack of information about the neutral degree, and Coung in 2014 fills this gap by introducing the picture fuzzy set [17] with the condition that $0 \leqslant$ (positive membership)+(neutral membership)+(negative membership)$\leqslant 1$. In 2023, Ali et al. [18] merged the concept of picture fuzzy set and q-ROFS$_t$S and initiated a generalized idea of q-rung orthopair picture fuzzy soft sets and their application in MCGDM problems. q-ROPFS$_t$S provides a more flexible and generalized structure, with the constrain condition $0 \leqslant$ (positive membership)^q+(neutral membership)^q+(negative membership)$^q \leqslant 1$ ($q \geqslant 1$), to overcome the gap of neutral degree in the structure of q-ROFS$_t$S. In the other words we say that q-ROPFS$_t$S is a special case of FS, Interval-valued fuzzy set, IFS, PyFS, q-ROFS, PFS, rough set, and q-ROFS$_t$S.

1.1. Literature review

In advance operation management scientific research, choosing the environmentally-friendly suppliers in the medical industry while considering economic and environmental criteria, to highlights the significance and the important of keeping harmful materials away from the environments [19]. To understand the evolution and effect of GSCM in China, a systemic literature review, analysis and comparison approach is utilized in the manufacture sector [20]. The authors concluded to improve the existing legal framework-regarding tax subsidies, industry coverage and environmental data disclosure. They suggested to address this issue-by introducing new group decision-making model based on plan-do-check-act to assess the GSCM in manufacture firms, and use this tool to integrate fuzzy MCDM approach, fuzzy analytical hierarchy process method to determine criteria weights, and the fuzzy technique to classify organizations [21]. In addition, Libyan Iron and steel company (LISCO) implemented a hybrid gray theory method regarding the selection of supplier for decision-making [22]. Further, some researchers, used various techniques regarding the selection of supplier by social, economic, and environmental practices, see [23,24]. Companies are recognizing the importance of environmental measure in their operations due to increasing awareness. To enhance environmental performance, GSCM is a proactive strategy, but unfortunately there's a lack of research, especially in administrative organizations. Integrating environmental aspects into supply chain research is urgently needed. Many corporations adopt GSCM for financial support, including selecting green suppliers. Decision-making process in GSCM can be affected by neglecting uncertainties in relationships. In the literature, various mathematical and analytical methods have been detailed about the GSCM environment. Qu et al. [25] identified suitable environmentally-friendly suppliers for a Chinese internet company by utilizing the methodologies of fuzzy TOPSIS and ELECTRE. Zhou and Chen [26] initiated integrated method in the context of combining the AHP-VIKOR-MRM tool for solving the group green supplier selection problem. Sahoo et al. [27] used the concept of binary code genetic algorithm the select the level of supplier in supply chain management with the help of pareto optimality. Adegbola [28] used simulation optimization to address the long-standing stochastic single-vendor, multi-manufacture inventory control program. The idea of sustainable development initiatives and company economic performance was given by Malys [29]. Modarres et al. [30] examined a European environmental sustainability path toward a green commercial aviation supply chain. Venkataraman, A., and Rajkumar [31] proposed a systematic evaluation and bibliometric analysis of emerging technologies related to the sustainability of the supply chain. Ogutu et al. [32] discussed the bibliometric meta-data analysis to present an analysis of current trends in sustainable organizational management. Cannas et al. [33] discussed that how Artificial Intelligence (AI) methods in OSCM can increase the companies' competitiveness by reducing costs and lead times and improving service levels, quality, safety, and sustainability. Furthermore, for application of green supply chain management in various direction see (Nascimento and Loureiro [34], Jain et al. [35], Slabe-Erker et al. [36]). Using the green supply selection technique various criteria are used by the company, and these criteria are linked to environmental and economic consideration. In the context of selecting green suppliers, MCDM are commonly employed. q-rung orthopair picture fuzzy soft sets find practical application in a scenario where a mathematical description is unavailable, so to overcome these difficulties, a novel model of q-rung orthopair picture fuzzy soft sets are proposed, which can manage the diverse interactions among input arguments.

1.2. Research gap

After reviewing the literature study, Ali et at. [18] in 2023 introduced a generalized structure of q-ROFS$_t$Ss to address the limitations of abstinence degree, proposed q-rung orthopair picture fuzzy soft sets with fundamental operational laws and some aggregation operations, having the ability to handle the MCDM problem with triplet membership degree with parameters. There is no more work in the literature on the q- rung orthopair picture fuzzy soft TOPSIS method, so our main aim is to improve the TOPSIS method based on q-ROPFS$_t$S. Finally, we develop a framework that can help decision-maker to select and prioritize the right green supplier, based on criteria both in general and with a focus on environmental consideration, utilizing the q-rung orthopair picture fuzzy soft TOPSIS method.

Motivation of the article:

The utilization of the existing concept of the q-rung orthopair fuzzy soft set and their aggregation operators generates a significant interest in various researchers across diverse fields. Despite their widespread applications, these concepts have the lack of information of neutral degree to handle the DM problem. To addressing these limitation Ali et al. [18] proposed more flexible and novel framework, namely, the q-rung orthopair picture fuzzy soft sets with it constrain condition $0 \leq (\alpha)^q + (\beta)^q + (\gamma)^q \leq 1$ ($q \geq 1$). This innovative structure incorporates with triplet membership degree: positive, neutral, and negative degree with parameterization tools. Our goal with merging q-spherical picture fuzzy soft sets with TOPSIS is to overcome the drawbacks of conventional decision models in handling rough and uncertain data.

Contribution of the article:

- To develop TOPSIS technique based on q-ROPFS$_t$ environment.
- To enhance the application of TOPSIS method in q-ROPFS$_t$Ns, we initiated a more reasonable way for determining positive ideal, negative ideal solution and distance between q-ROPFS$_t$Ns.
- To ensure the reliability and flexibility of the proposed model, a GSCM problem has been discussed.
- To verify the stability of proposed model, we make a parameter analysis test, by increasing the values of "q".
- To signify the validity, we make comparative analysis of the proposed model with existing approaches.

The article is organized by the following section: Section 1 contains the introduction. Section 2 offers the fundamental preliminaries. Section 3 represents the q-rung orthopair picture fuzzy soft TOPSIS method accompanied by an algorithm and flow chart. Section 4 consists of a decision model and case study. In Section 5, managerial implications are discussed with a parameters analysis test, comparison analysis, and advantages of the proposed model. In Section 6, the conclusion serves as a summary of the major findings, emphasizing the overall significance of the research, limitations, and future recommendations. The comprehensive structure of the paper is shown in Figure 1.

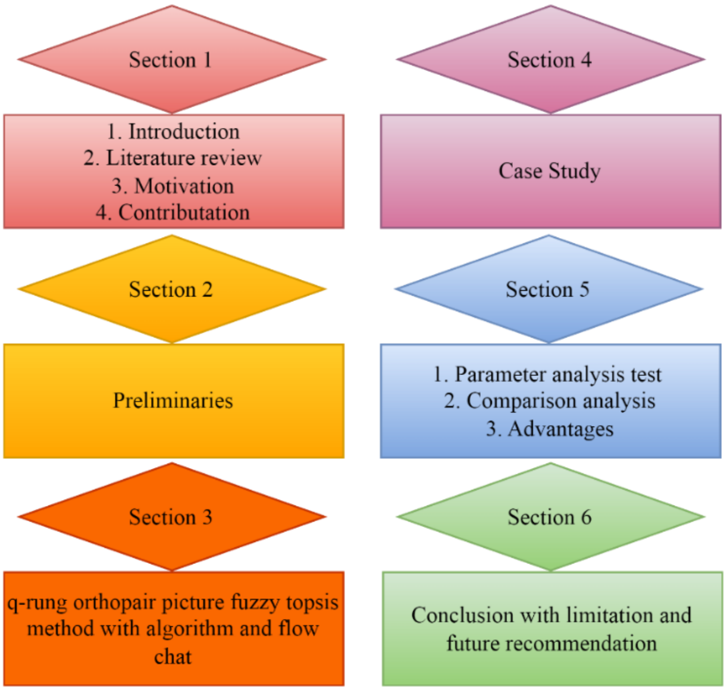

Figure 1. Layout of the article.

2. Preliminaries

Definition 2.1. [2] The expression of FS $\mathfrak{A}$ on a nonempty set Ɓ is written as

$$\mathfrak{A} = \{\langle ɓ, ƥ_{Ɓ}(ɓ)\rangle : ɓ \in Ɓ\}, \tag{1}$$

where $ƥ_{Ɓ}(\mathscr{z})$ expressed the membership degree belonging to [0,1], with the condition $0 \leq ƥ_{Ɓ}(ɓ) \leq 1$. The graphical representation of fuzzy spaces as shown in Figure 2.

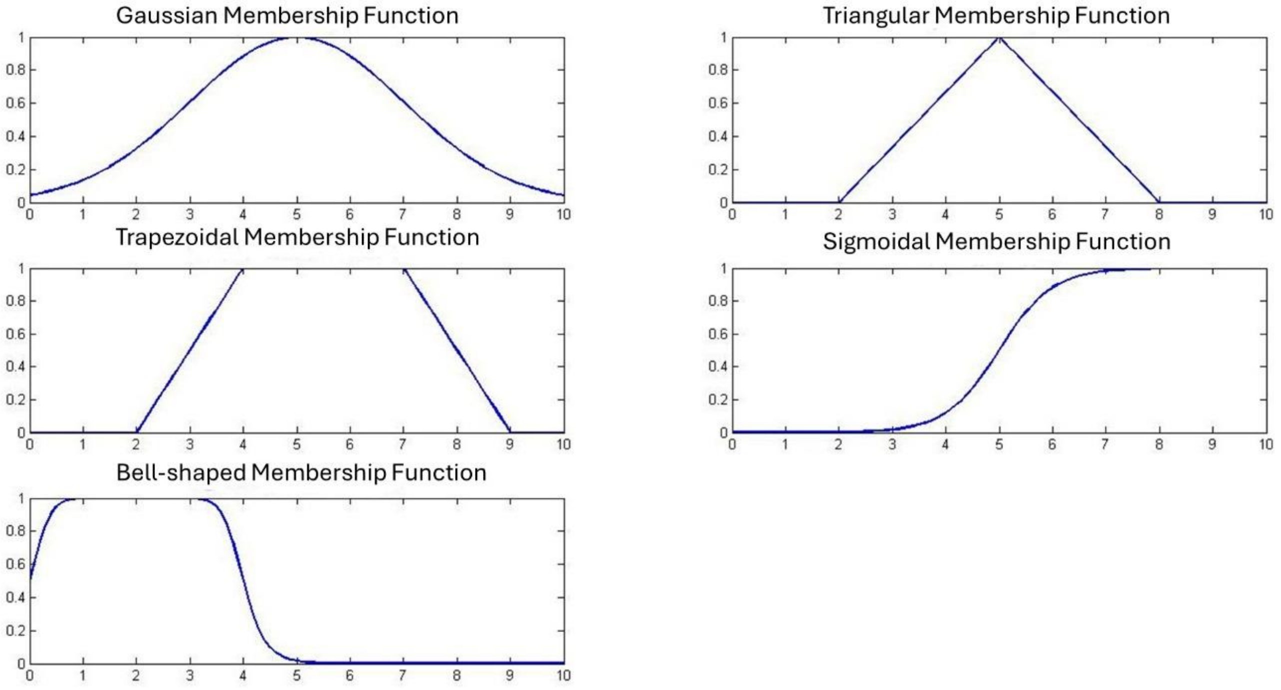

Figure 2. Some graphical representations of fuzzy spaces.

Definition 2.2. [3] The expression of IFS $\mathfrak{A}$ on a nonempty set Ɓ is written as

$$\mathfrak{A} = \{\langle ɓ, ƥ_{Ɓ}(ɓ), ƕ_{Ɓ}(ɓ)\rangle : ɓ \in Ɓ\}, \tag{2}$$

where $ƥ_{Ɓ}(ɓ) \in [0,1]$ and $ƕ_{Ɓ}(ɓ) \in [0,1]$ express membership degree and nonmember ship degree, with the restriction

$$0 \leq ƥ_{Ɓ}(ɓ) + ƕ_{Ɓ}(ɓ) \leq 1.$$

Definition 2.3. [5] The expression of PyFS $\mathfrak{A}$ on nonempty set Ɓ is written as

$$\mathfrak{A} = \{\langle ɓ, ƥ_{Ɓ}(ɓ), ƕ_{Ɓ}(ɓ)\rangle : ɓ \in Ɓ\}, \tag{3}$$

where $ƥ_{Ɓ}(ɓ) \in [0,1]$ express membership degree and $ƕ_{Ɓ}(ɓ) \in [0,1]$ express nonmember ship degree, with the domain

$$0 \leq \left(ƥ_{Ɓ}(ɓ)\right)^2 + \left(ƕ_{Ɓ}(ɓ)\right)^2 \leq 1.$$

Definition 2.4. [6] The expression of q-rung orthopair fuzzy set $\mathfrak{A}$ on a nonempty set Ɓ is expressed as

$$\mathfrak{A} = \{\langle ƀ, ƥ_{Ɓ}(ƀ), ƕ_{Ɓ}(ƀ)\rangle : ƀ \in Ɓ\}, \tag{4}$$

where $ƥ_{Ɓ}(ƀ) \in [0,1]$ and $ƕ_{Ɓ}(ƀ) \in [0,1]$ represent membership and nonmember ship function, with the domain $0 \le \left(ƥ_{Ɓ}(ƀ)\right)^q + \left(ƕ_{Ɓ}(ƀ)\right)^q \le 1 (q \ge 1)$. The comparison between intuitionistic, Pythagorean and q-rung orthopair fuzzy spaces as shown in Figure 3.

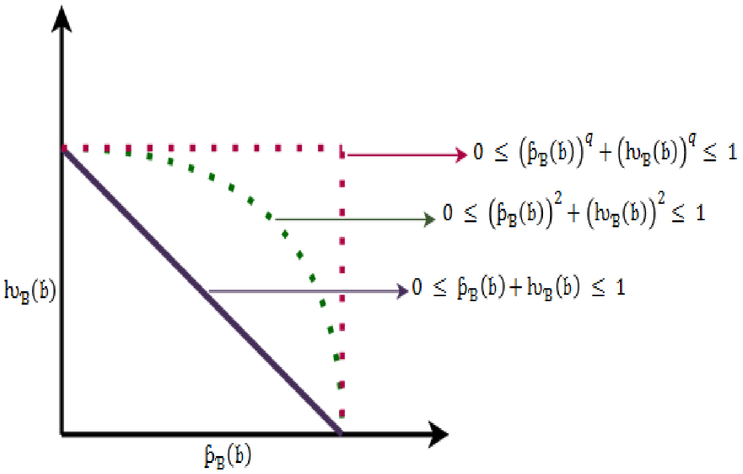

Figure 3. A comparison of the differences between Pythagorean, intuitionistic and q-rung orthopair fuzzy spaces.

Definition 2.4. [17] The expression of picture fuzzy set $\mathfrak{A}$ on a nonempty set $Ɓ$ is expressed as

$$\mathfrak{A}=\{\langle ƀ, ƥ_{Ɓ}(ƀ), ƕ_{Ɓ}(ƀ), ƙ_{Ɓ}(ƀ)\rangle : ƀ \in Ɓ\}, \tag{5}$$

with the domain $0 \le ƥ_{Ɓ}(ƀ) + ƕ_{Ɓ}(ƀ) + ƙ_{Ɓ}(ƀ) \le 1$. The visual representation of picture fuzzy set as shown in Figure 4.

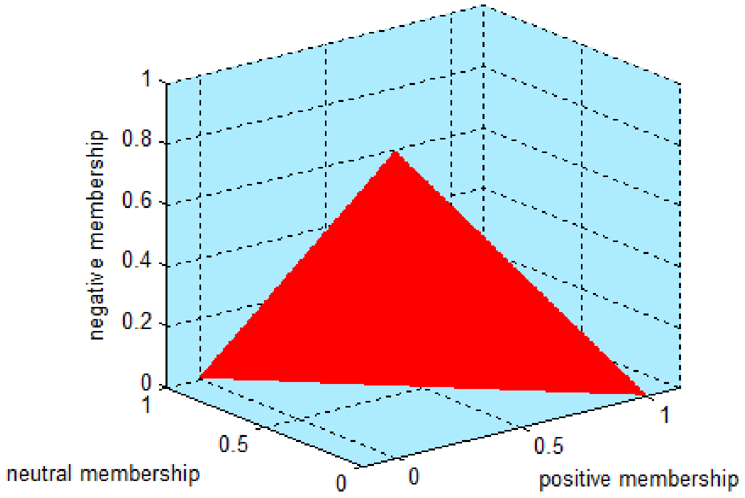

Figure 4. A visual representation of picture fuzzy spaces.

Definition 2.5. [7] A pair $(Ɯ, \dot{\mathrm{A}})$ is often referred to as a soft set over $Ɓ$, where $Ɯ: \dot{\mathrm{A}} \longrightarrow Ƥ(Ɓ)$ and $\dot{\mathrm{A}} \subseteq \ddot{\mathrm{E}}$, $\ddot{\mathrm{E}}$ expresses a set of parameters and $Ƥ(Ɓ)$ represents a power set of $Ɓ$.

Definition 2.6. [14] Consider a soft set $(Ɯ, \dot{\mathrm{A}})$ and $\mathrm{T} \subseteq \dot{\mathrm{A}}$, then q-ROFS$_t$S over $Ɓ$ is defined as

$$Ꞧ_{e_j}(ƀ_i) = \left\{\langle ƀ, ƥ_\mathrm{j}(ƀ_i), ƕ_j(ƀ_i)\rangle : ƀ_i \in Ɓ, e_j \in \dot{\mathrm{A}}\right\}, \tag{6}$$

where $Ꞧ: \dot{\mathrm{A}} \longrightarrow \mathrm{q-ROFS}^{Ɓ}$ and $ƥ_\mathrm{j}(ƀ_i) \in [0,1]$ and $ƕ_j(ƀ_i) \in [0,1]$ express membership and nonmember ship function, with the condition $0 \leq \left(ƥ_j(ƀ_i)\right)^q + \left(ƕ_j(ƀ_i)\right)^q \leq 1\ (q \geq 1)$. The comparison between intuitionistic fuzzy soft space, Pythagorean fuzzy soft space and q-rung orthopair fuzzy soft spaces as shown in Figure 5.

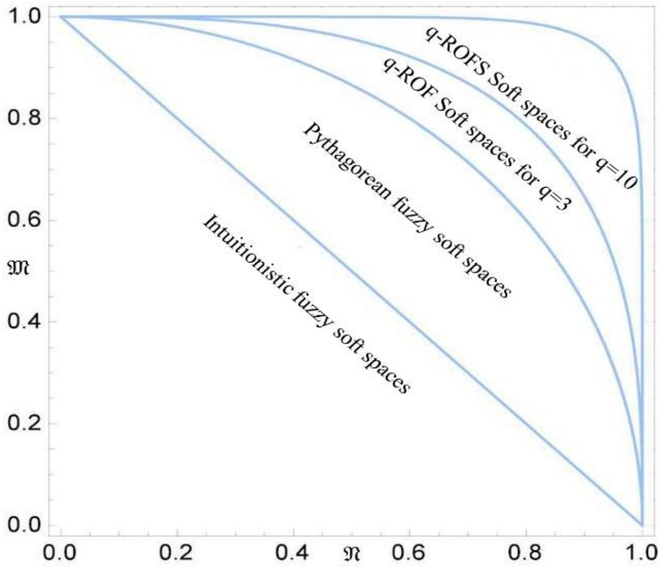

Figure 5. A comparison of the differences between intuitionistic, Pythagorean and q-rung orthopair fuzzy soft spaces.

Definition 2.7. [18] Consider a soft set $(Ɯ, \dot{\mathrm{A}})$ and $\mathrm{T} \subseteq \dot{\mathrm{A}}$, then q-ROPFS$_f$S over $Ɓ$ is a pair $(Ꞧ, \dot{\mathrm{A}})$ defined as

$$Ꞧ_{e_j}(ƀ_i) = \left\{\langle ƀ_i, ƥ_\mathrm{j}(ƀ_i), ƕ_j(ƀ_i), ꝁ_j(ƀ_i)\rangle : ƀ_i \in Ɓ, e_j \in \dot{\mathrm{A}}\right\},$$

where $Ꞧ: \dot{\mathrm{A}} \longrightarrow \mathrm{q-ROPFS}^{Ɓ}$ and $ƥ_\mathrm{j}(ƀ_i), ƕ_j(ƀ_i), ꝁ_j(ƀ_i) \in [0,1]$, express positive, neutral and negative membership degree, with the restriction

$$0 \leq \left(ƥ_j(ƀ_i)\right)^q + \left(ƕ_j(ƀ_i)\right)^q + \left(ꝁ_j(ƀ_i)\right)^q \leq 1 (q \geq 1).$$

For simplicity we use $Ꞧ_{e_j}(ƀ_i) = \langle ƀ_i, ƥ_\mathrm{j}(ƀ_i), ƕ_j(ƀ_i), ꝁ_j(ƀ_i)\rangle_q$ and q-ROPFS$_f$N is represented by

$Ɍ_{e_{ij}} = \langle ƥ_{\mathrm{ij}}, ƕ_{ij}, ꝁ_{ij} \rangle$.

Definition 2.8. [18] The basic operations of three $\mathrm{q-ROPFS_fN_s}$ $Ɍ = \langle ƥ, ƕ, ꝁ \rangle$ and $Ɍ_{e_{1j}} = \langle ƥ_{\mathrm{1j}}, ƕ_{1j}, ꝁ_{1j} \rangle$ for $\lambda, \lambda_1, \lambda_2 \succ 0$ and $(j = 1{,}2)$ are defined as

1) $Ɍ_{e_{11}} \cup Ɍ_{e_{12}} = \{\max(ƥ_{11}, ƥ_{12}), \min(ƕ_{11}, ƕ_{12}), \min(ꝁ_{11}, ꝁ_{12})\}$;
2) $Ɍ_{\mathrm{e_{11}}} \cap Ɍ_{\mathrm{e_{12}}} = \{\min(ƥ_{11}, ƥ_{12}), \min(ƕ_{11}, ƕ_{12}), \max(ꝁ_{11}, ꝁ_{12})\}$;
3) $Ɍ^{co} = (ꝁ, ƕ, ƥ)$;
4) $Ɍ_{\mathrm{e_{11}}} \leq Ɍ_{e_{12}}$ if and only if $\{ƥ_{11} \leq ƥ_{12}, ƕ_{11} \leq ƕ_{12}, ꝁ_{11} \geq ꝁ_{12}\}$;
5) $Ɍ_{e_{11}} \oplus Ɍ_{e_{12}} = \left(\sqrt[q]{(ƥ_{11})^q + (ƥ_{12})^q - (ƥ_{11})^q(ƥ_{12})^q}\,, ƕ_{11}ƕ_{12},\ ꝁ_{11}ꝁ_{12}\right)$;
6) $Ɍ_{e_{11}} \otimes Ɍ_{e_{12}} = \begin{pmatrix} ƥ_{11}ƥ_{12}\,, \\ \sqrt[q]{(ƕ_{11})^q + (ƕ_{12})^q - (ƕ_{11})^q(ƕ_{12})^q}, \\ \sqrt[q]{(ꝁ_{11})^q + (ꝁ_{12})^q - (ꝁ_{11})^q(ꝁ_{12})^q} \end{pmatrix}$;
7) $\lambda\, Ɍ = \left(\sqrt[q]{1 - (1 - ƥ)^{\lambda}}, ƕ^{\lambda}, ꝁ^{\lambda}\right)$;
8) $Ɍ^{\lambda} = \left(ƥ^{\lambda}, \sqrt[q]{1 - (1 - ƕ)^{\lambda}}, \sqrt[q]{1 - (1 - ꝁ)^{\lambda}}\right)$.

Definition 2.8. [18] The score function $q - ROPFS_fN$ $Ɍ_{e_{ij}} = \langle ƥ_{\mathrm{ij}}, ƕ_{ij}, ꝁ_{ij} \rangle$ is defined as

$$S\left(Ɍ_{e_{ij}}\right) = ƥ_{ij}^q - ƕ_{ij}^q - ꝁ_{ij}^q + \left(\frac{e^{ƥ_{ij}^q - ƕ_{ij}^q - ꝁ_{ij}^q}}{e^{ƥ_{ij}^q - ƕ_{ij}^q - ꝁ_{ij}^q} + 1} - \frac{1}{2}\right)\pi_{Ɍ_{e_{ij}}}^q,$$

where $\mathrm{S}\left(Ɍ_{e_{ij}}\right) \in [-1{,}1]$, the accuracy function is defined as $\mathrm{Acc}\left(Ɍ_{e_{ij}}\right) = ƥ_{ij}^q + ƕ_{ij}^q + ꝁ_{ij}^q$, where $\mathrm{Acc}\left(Ɍ_{e_{ij}}\right) \in [0{,}1]$.

3. q-Rung orthopair picture fuzzy soft topsis

In this section we study how we solve multi-criteria group decision-making problem, by using q-rung orthopair picture fuzzy soft set with TOPSIS method. The following steps are involved, the for calculating the $\mathrm{q\text{-}ROPF\,S_t}$ TOPSIS method. Throughout the article we use WDM for weighted decision matrix.

Algorithm

Step 1. Define the problem. Consider the set of experts, based on the collection of suppliers (Alternative) $\mathrm{Alt} = \{\mathrm{Alt_i}: \mathrm{i} = 1{,}2{,}3{,}4\}$ and family of criteria $\mathrm{Cri} = \{\mathrm{Cri_i}: \mathrm{i} = 1{,}2{,}3{,}4\}$.

Step 2. Using the linguistic term represented in Table 1. construct WDM, then the transformed WDM is into a weighted q-ROPFS$_\mathrm{f}$ decision matrix using the q-ROPFS$_\mathrm{f}$ values given in Table 1.

Table 1. Show the linguistic term of alternatives.

Linguistic term	$(\mathfrak{B}, \mathfrak{C}, \mathfrak{D})$
(VHI) Very High Important	$(0.95, 0.11, 0.20)$
(HI) High Important	$(0.85, 0.22, 0.30)$
(SI) Slightly Important	$(0.75, 0.33, 0.40)$
(LI) Less Important	$(0.65, 0.44, 0.50)$

Step 3. To evaluate normalized WDM, multiply the weights of the criteria by the evaluations of decision matrix.

Step 4. For the defuzzification of the WDM, calculate the score function using the formula as given below:

$$\mathrm{S}\left(\text{Ꞧ}_{e_{ij}}\right) = (\text{ƥ}_{ij})^q - (\text{ƕ}_{ij})^q - (\text{ꝁ}_{ij})^q + \left(\frac{\mathrm{e}^{(\text{ƥ}_{ij})^q - (\text{ƕ}_{ij})^q - (\text{ꝁ}_{ij})^q}}{e^{(\text{ƥ}_{ij})^q - (\text{ƕ}_{ij})^q - (\text{ꝁ}_{ij})^q}} - \frac{1}{2}\right)\pi^{\mathrm{q}}_{\text{Ꞧ}_{e_{ij}}}, (\mathrm{q} \geq 1),$$

where $\mathrm{S}\left(\text{Ꞧ}_{e_{ij}}\right) \epsilon[-1,1]$.

Step 5. Construct the q-ROPFS$_\mathrm{f}$ positive ideal solution and q-ROPFS$_\mathrm{f}$ negative ideal solution, on the basis of score function.

For the q-ROPFS$_\mathrm{f}$-PIS:

$$\mathrm{X}^{+} = \left\{\mathrm{C_j}, \mathrm{V_i} < \mathrm{Score}\left(\mathrm{C_j}(\mathrm{X_{iw}})\right) > \mathrm{j} = 1,2, \dots, \mathrm{n}\right\}.$$

For the q-ROPFS$_\mathrm{f}$-NIS:

$$\mathrm{X}^{-} = \left\{\mathrm{C_j}, \Lambda_\mathrm{i} < \mathrm{Score}\left(\mathrm{C_j}(\mathrm{X_{iw}})\right) > \mathrm{j} = 1,2, \dots, \mathrm{n}\right\}.$$

Step 6. Evaluate q-ROPFS$_\mathrm{f}$ Euclidean distance from q-ROPFS$_\mathrm{f}$-NIS and q-ROPFS$_\mathrm{f}$-PIS for alternatives, by the equations:

For the q-ROPFS$_\mathrm{f}$-NIS:

$$\mathrm{D}(\mathrm{X_i}, \mathrm{X}^{-}) = \sqrt[q]{\frac{1}{2n}\sum_{i=1}^{n}((\text{ƥ}_x{}^q - \text{ƥ}_{x^-}{}^q)^2 + (\text{ƕ}_x{}^q - \text{ƕ}_{x^-}{}^q)^2 + (\text{ꝁ}_x{}^q - \text{ꝁ}_{x^-}{}^q)^2)}.$$

For the q-ROPFS$_\mathrm{f}$-PIS:

$$D(X_i, X^{+}) = \sqrt[q]{\frac{1}{2n}\sum_{i=1}^{n}((\text{ƥ}_x{}^q - \text{ƥ}_{x^+}{}^q)^2 + (\text{ƕ}_x{}^q - \text{ƕ}_{x^+}{}^q)^2 + (\text{ꝁ}_x{}^q - \text{ꝁ}_{x^+}{}^q)^2)}.$$

Step 7. Evaluate the relative coefficient by the formula:

$$\text{Relative coefficient } = \mathrm{R}(\mathrm{X_i}) = \frac{\mathrm{D}(\mathrm{X_i}, \mathrm{X}^{-})}{\mathrm{D}(\mathrm{X_i}, \mathrm{X}^{+}) + \mathrm{D}(\mathrm{X_i}, \mathrm{X}^{-})} \in [0,1].$$

Step 8. Ranking each alternative in descending order to get the best one.

The procedure of decision-making problem through topsis method is represented by the flow chat as shown in Figure 6.

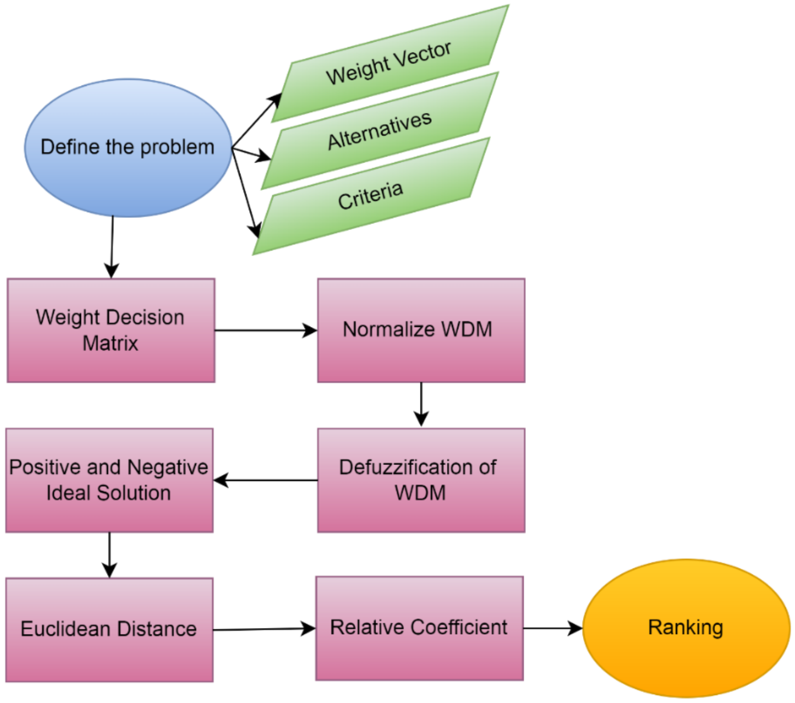

Figure 6. Flow chart represents the procedure of decision-making problem through TOPSIS method.

4. Decision model and case study

An attempt will be made in the following section to construct an illusory application for a TOPSIS approach using the theory of q-ROPFS$_t$S. Using the step-by-step algorithm of the proposed method tackle the following problem using the following linguistic term.

Case study:

A green supply chain, which is also called a sustainable or eco-friendly supply chain, was designed strategically to protect the environment by conducting business and sustainability throughout the entire supply chain process. To address the change related to environmental concern that how company manage and operate, they are integrated GSCM into the traditional supplier selection process. The main aims of this study are to outline both traditional and environmental criteria for supplier's selection offering a comprehensive framework to assist the decision-maker in prioritizing green supplier effectively. The critical stages will be addressed by this approach, including sourcing raw material, distribution, production and end-of-life product disposal. To protect the environment and reduce harm of ecosystems companies participate in green supply chain activity.

For this purpose, we consider the panel of experts to assess the best supplier chain involving integrating environmental consideration into the traditional supply chain process. From the collection of suppliers $\mathrm{Alt}=\{\mathrm{Alt}_i: i=1,2,3,4\}$ we select the best supplier to address the change related to environmental concern on the basis of criteria $\mathrm{Cri}=\{\mathrm{Cri}_i: i=1,2,3,4\}$ represented in Table 2. Note that here suppliers expressed the alternatives.

Collection of Alternatives:

$\mathrm{Alt}=\{\mathrm{Alt}_i: i=1,2,3,4\}$.

Alt_1: Sustainable Sourcing Initiative (SSI).

Alt_2: Renewable Energy Integration (REI).
Alt_3: Green Logistic Initiative (GLI).
Alt_4: Circular Economy Approach (CEA).

Advantages of alternatives:

(SSI): SSI promote suspensible sources of raw material.
(REI): Improve the reliability of energy resources and enhances the company reputation.
(GLI): Improves efficiency and reduces emissions in logistic.
(CEA): Promotes resource efficiency and cost savings.

Disadvantages of alternatives:

(SSI): Initial implementation costs might be high.
(REI): Technological limitations and intermittency of some renewable sources.
(GLI): Requires infrastructure investment and coordination efforts.
(CEA): Requires organizational changes and technological investment.

Step 1. Analyzing the problem, expressed as above based on the criteria and explanation shown in Table 2 and with linguistic term of alternatives related to green supply chain shown in Table 1.

Step 2. On the basis of Table 1, we first construct weighted decision matrix, as shown in Table 3, and then transformed into a weighted q-rung orthopair picture fuzzy soft decision matrix as shown in Table 4, by using the linguistic term of alternatives of Table 1.

Step 3. In this step we design a normalized decision matrix represented by Table 6, by multiplying the weight of criteria as shown in Table 5, to the evaluation of q-ROPFS$_f$ decision matrix of Table 4.

For Alt_1 and Cri_1,

$$\mathrm{W}_1 = 0.3 \text{ and } \mathfrak{p}_{11} = 0.95,\ \mathfrak{p}_{11} \times \mathrm{W}_1 = 0.95 \times 0.3 = 0.285.$$

Same procedure for the remaining values.

Table 2. Criteria and explanations.

Criteria	Explanation	Benefit
Cri_1	Sustainable resources of raw material	Promoting biodiversity and reducing deforestation.
Cri_2	Energy efficient manufacturing processes	Reducing less resources of renewable energy
Cri_3	Reduced packages and waste	Promoting the system of recycle, to reduce waste.
Cri_4	Logistics and transportation optimization	Plan the efficient routes to decrease the pollution from moving goods

Table 3. Represent weighted decision matrix.

Alternatives	Cri_1	Cri_2	Cri_3	Cri_4
Alt_1	VHI	HI	SI	LI
Alt_2	HI	LI	SI	VHI
Alt_3	SI	VHI	LI	HI
Alt_4	LI	SI	HI	VHI

Table 4. Evaluation of q-ROPFS$_f$ decision matrix.

Alternatives	Cri_1	Cri_2	Cri_3	Cri_4
Alt_1	(0.95, 0.11, 0.20)	(0.85, 0.22, 0.30)	(0.75, 0.33, 0.40)	(0.65, 0.44, 0.50)
Alt_2	(0.85, 0.22, 0.30)	(0.65,0.44,0.50)	(0.75, 0.33, 0.40)	(0.95,0.11,0.20)
Alt_3	(0.75, 0.33, 0.40)	(0.95, 0.11, 0.20)	(0.65, 0.44,0.50)	(0.85,0.22,0.30)
Alt_4	(0.65, 0.44, 0.50)	(0.75, 0.33, 0.40)	(0.85,0.22,0.30)	(0.95,0.11,0.20)

Table 5. Represent weight of criteria.

Weight criteria	Wei_1	Wei_2	Wei_3	Wei_4
$(\mathfrak{B}, \mathfrak{C}, \mathfrak{D})$	(0.3,0.1,0.1)	(0.3,0.2,0.2)	(0.2,0.3,0.3)	(0.2,0.4,0.4)

Table 6. Evaluation of weighted q-ROPFS$_f$ decision matrix.

Alternatives	Cri_1	Cri_2	Cri_3	Cri_4
Alt_1	(0.285, 0.2326, 0.2079)	(0.255, 0.2648, 0.3264)	(0.15, 0.3957, 0.4469)	(0.13, 0.5238, 0.5657)
Alt_2	(0.255, 0.s2266, 0.3036)	(0.195, 0.4523, 0.5092)	(0.15, 0.3957, 0.4469)	(0.19, 0.4026, 0.4150)
Alt_3	(0.225, 0.3329, 0.4019)	(0.285, 0.2104, 0.2516)	(0.13, 0.4790, 0.5297)	(0.17, 0.4198, 0.4469)
Alt_4	(0.195, 0.4416, 0.5012)	(0.225, 0.3521, 0.4150)	(0.17, 0.3343, 0.3763)	(0.19, 0.4026, 0.4150)

Step 4. Calculated the score values by using the value of Table 6, for defuzzification of the q-ROPF$_f$ weighted decision matrix, by using the score function. The result is shown in Table 7.

Score function:

$$\mathrm{S}\left(\text{Ɍ}_{e_{ij}}\right) = \left(\text{ƥ}_{ij}\right)^q - \left(\text{ƕ}_{ij}\right)^q - \left(\text{ꝁ}_{ij}\right)^q + \left(\frac{\mathrm{e}^{\left(\text{ƥ}_{ij}\right)^q - \left(\text{ƕ}_{ij}\right)^q - \left(\text{ꝁ}_{ij}\right)^q}}{e^{\left(\text{ƥ}_{ij}\right)^q - \left(\text{ƕ}_{ij}\right)^q - \left(\text{ꝁ}_{ij}\right)^q}} - \frac{1}{2}\right) \pi^{\mathrm{q}}_{\text{Ɍ}_{e_{ij}}}, (\mathrm{q} \geq 1).$$

For Alt_1 and Cri_1,

$\text{ƥ}_{11} = 0.285$; $\text{ƕ}_{11} = 0.2326$; $\text{ꝁ}_{11} = 0.2079$; and q = 3;

$$\mathrm{S}\left(\text{Ɍ}_{e_{11}}\right) = (0.285)^q - (0.2326)^q - (0.2079)^q + \left(\frac{\mathrm{e}^{(0.285)^3 - (0.2326)^3 - (0.2079)^3}}{e^{(0.285)^3 - (0.2326)^3 - (0.2079)^3}} - \frac{1}{2}\right) \pi^3_{\text{Ɍ}_{e_{11}}};$$

$$\mathrm{S}\left(\text{Ɍ}_{e_{11}}\right) = 0.0020.$$

Same for the remaining score values.

Table 7. Score values.

Alternatives	Cri_1	Cri_2	Cri_3	Cri_4
Alt_1	0.0020	−0.0461	−0.1864	−0.4102
Alt_2	−0.0288	−0.2748	−0.1864	−0.1636
Alt_3	−0.1137	−0.0026	−0.3250	−0.1998
Alt_4	−0.2588	−0.1305	−0.1077	−0.1636

Step 5. Using the information of Table 7 to calculate the q-ROPFS$_f$ positive ideal solution and q-ROPFS$_f$ negative ideal solution, which are displayed in Table 8.

Table 8. Show q-ROPFS$_f$-PIS and q-ROPFS$_f$-NIS.

	Cri_1	Cri_2	Cri_3	Cri_4
PSI	$\begin{pmatrix}0.285,\\0.2326,\\0.2079\end{pmatrix}$	$\begin{pmatrix}0.285,\\0.2104,\\0.2516\end{pmatrix}$	$\begin{pmatrix}0.17,\\0.3343,\\0.3763\end{pmatrix}$	$\begin{pmatrix}0.19,\\0.4026,\\0.4150\end{pmatrix}$
NSI	$\begin{pmatrix}0.195,\\0.4416,\\0.5012\end{pmatrix}$	$\begin{pmatrix}0.195,\\0.4523,\\0.5092\end{pmatrix}$	$\begin{pmatrix}0.13,\\0.4790,\\0.5297\end{pmatrix}$	$\begin{pmatrix}0.13,\\0.5238,\\0.5657\end{pmatrix}$

For the q-ROPFS$_f$-PIS:

$$X^+ = \left\{C_j, \vee_i < \mathrm{Score}\left(C_j(X_{iw})\right) > j = 1,2,\dots,n\right\},$$

$$X^+ = \left\{C_j, \vee_i < \mathrm{Score}(0.0020, -0.0288, -0.1137, -0.2588\,) > j = 1,2,\dots,n\right\},$$

$$X^+ = 0.0020.$$

Now using the information of Table 6. $X^+ = 0.0020 = (0.285, 0.2326, 0.2079)$. Same for the remaining criteria.

For the q-ROPFS$_f$-NIS:

$$X^- = \left\{C_j, \wedge_i < \mathrm{Score}\left(C_j(X_{iw})\right) > j = 1,2,\dots,n\right\},$$

$$X^- = \left\{C_j, \wedge_i < \mathrm{Score}(0.0020, -0.0288, -0.1137, -0.2588) > j = 1,2,\dots,n\right\},$$

$$X^- = \left\{C_j, \wedge_i < \mathrm{Score}(0.0020, -0.0288, -0.1137, -0.2588) > j = 1,2,\dots,n\right\},$$

$$X^- = -0.2588.$$

Now using the information of Table 6. $X^- = -0.2588 {=} (0.195, 0.4416, 0.5012)$. The same is for the remaining criteria.

Step 6. Evaluate q-ROPFS$_f$ Euclidean distance from q-ROPFS$_f$-PIS and q-ROPFS$_f$-NIS for alternatives.

For the q-ROPFS$_f$-PIS:

$$D(X_i, X^+) = \sqrt[q]{\frac{1}{2n}\sum_{i=1}^{n}((\text{ƥ}_x{}^q - \text{ƥ}_{x^+}{}^q)^2 + (\text{ƕ}_x{}^q - \text{ƕ}_{x^+}{}^q)^2 + (\text{ꝁ}_x{}^q - \text{ꝁ}_{x^+}{}^q)^2)}.$$

For Alt_1,

$$\begin{pmatrix} a_1 = 0.285;\ \mathrm{b}_1 = 0.2326;\ \mathrm{c}_1 = 0.2079; \\ \mathrm{d}_1 = 0.255;\ \mathrm{e}_1 = 0.2648;\ \mathrm{f}_1 = 0.3264; \\ \mathrm{g}_1 = 0.15;\ \mathrm{h}_1 = 0.3957;\ \mathrm{i}_1 = 0.4469; \\ \mathrm{j}_1 = 0.13;\ \mathrm{k}_1 = 0.5238;\ \mathrm{h}_1 = 0.5657; \end{pmatrix} \text{ and } \begin{pmatrix} a_2 = 0.285;\ \mathrm{b}_2 = 0.2326;\ \mathrm{c}_2 = 0.2079; \\ \mathrm{d}_2 = 0.285;\ \mathrm{e}_2 = 0.2104;\ \mathrm{f}_2 = 0.2516; \\ \mathrm{g}_2 = 0.17;\ \mathrm{h}_2 = 0.3343;\ \mathrm{i}_2 = 0.3763; \\ \mathrm{j}_2 = 0.19;\ \mathrm{k}_2 = 0.4026;\ \mathrm{h}_2 = 0.4150; \end{pmatrix},$$

$$D(X_4, X^+) = \sqrt[q]{\frac{1}{2(4)}\begin{pmatrix} (a_1{}^q - a_{2^+}{}^q)^2 + (b_1{}^q - b_{2^+}{}^q)^2 + (c_1{}^q - c_{2^+}{}^q)^2 \\ (d_1{}^q - d_{2^+}{}^q)^2 + (e_1{}^q - e_{2^+}{}^q)^2 + (\mathrm{f}_1{}^q - \mathrm{f}_{2^+}{}^q)^2 \\ (\mathrm{g}_1{}^q - \mathrm{g}_{2^+}{}^q)^2 + (\mathrm{h}_1{}^q - \mathrm{h}_{2^+}{}^q)^2 + (\mathrm{i}_1{}^q - \mathrm{i}_{2^+}{}^q)^2 \\ (\mathrm{j}_1{}^q - \mathrm{j}_{2^+}{}^q)^2 + (\mathrm{k}_1{}^q - \mathrm{k}_{2^+}{}^q)^2 + (\mathrm{l}_1{}^q - \mathrm{l}_{2^+}{}^q)^2 \end{pmatrix}},$$

$$D(X_4, X^+) = \sqrt[3]{\frac{1}{8}\begin{pmatrix} (0.285^3 - 0.285^3)^2 + (0.2326^3 - 0.2326^3)^2 + (0.2079^3 - 0.2079^3)^2 \\ (0.255^3 - 0.285^3)^2 + (0.2648^3 - 0.2104^3)^2 + (0.3264^3 - 0.2516^3)^2 \\ (0.15^3 - 0.17^3)^2 + (0.3957^3 - 0.3343^3)^2 + (0.4469^3 - 0.3763^3)^2 \\ (0.13^3 - 0.19^3)^2 + (0.5238^3 - 0.4026^3)^2 + (0.5657^3 - 0.4150^3)^2 \end{pmatrix}},$$

$$D(X_4, X^+) = \mathrm{Alt}_1 = 0.1370.$$

Same for the remaining Alt_2, Alt_3, Alt_4, and for the negative ideal solution. The distance measure of positive ideal solution and negative ideal solution of alternative as shown in Table 9.

Table 9. Show the distance measure of PIS and NIS.

Distance measures	Alt_1	Alt_2	Alt_3	Alt_4
PSI	0.1370	0.1421	0.1325	0.1437
NSI	0.1712	0.1699	0.1711	0.1690

The graphical representation of positive ideal solution and negative ideal solution as shown in Figure 7.

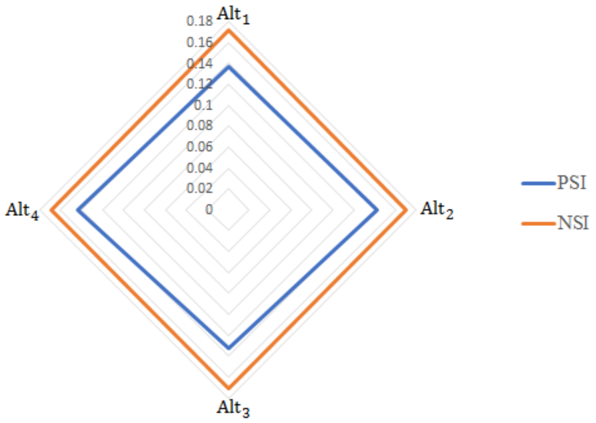

Figure 7. Show graphical representation of PIS and NIS.

Step 7. Evaluate the closeness coefficient-by using the formula

$$\text{Relative coefficient} = R(X_i) = \frac{D(X_i,X^-)}{D(X_i,X^+)+D(X_i,X^-)},$$

$$R(X_1)= \frac{0.1712}{0.1370+0.1712} = \frac{0.1712}{0.3082} = 0.5555,\ R(X_2)= \frac{0.1699}{0.1421+0.1699} = \frac{0.1699}{0.312} = 0.5445,$$

$$R(X_3)= \frac{0.1711}{0.1325+0.1711} = \frac{0.1711}{0.3036} = 0.5636,\ R(X_4)= \frac{0.1690}{0.1437+0.1690} = \frac{0.1690}{0.3127} = 0.5404.$$

The closeness coefficient of alternative as shown in Table 10.

Table 10. Show the closeness coefficient.

	Alt_1	Alt_2	Alt_3	Alt_4
Closeness coefficient	0.5555	0.5445	0.5636	0.5404

Step 8. Rank the alternative in descending order, and get the best one.

$$0.5636 > 0.5555 > 0.5445 > 0.5404,$$
$$Alt_3 > Alt_1 > Alt_2 > Alt_4.$$

Alt_3 is the best supplier to reduce harm of ecosystem companies participating in green supply chain activity. The graphical representation of ranking of alternatives as shown in Figure 8.

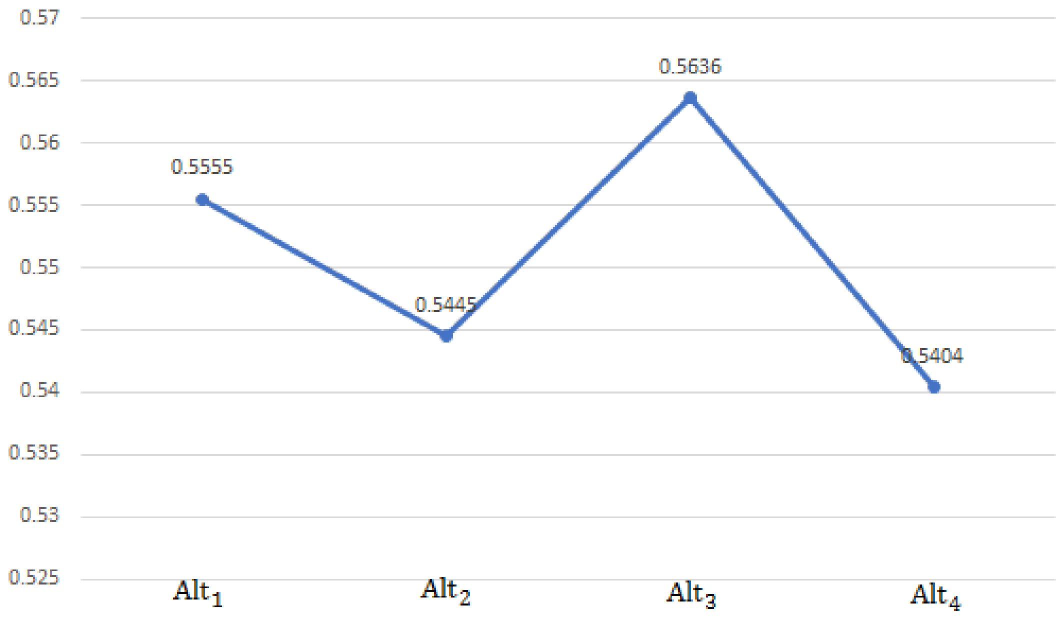

Figure 8. Show the ranking of alternatives.

Parameter analysis test:

In this section, we check the stability and effectiveness of our proposed model. Initially, our focus is directed toward examining how the various values of q effect the score values of alternatives. For this analysis, we performed a parameters analysis test to find out the effectiveness of various values of parameter (q = 2 to 10) on our proposed model using the case study. The result illustrated in Table 11 and Figure 9 demonstrate the impact of various values of "q" on the outcomes of best alternative ranking and the closeness coefficient. This indicates that there is no negative effect on the stability of best alternative by increase in the q values, which shows a strong consistency in decision outcomes. Through this investigation, we demonstrate a robust stability in decision-making to get educated judgments based on the result. The behavior of alternatives for different values of "q" is presented in Figure 9.

Table 11. Show the parameter analysis test of various values of q.

Closeness coefficient	q	Alt_1	Alt_2	Alt_3	Alt_4	Best alternative
	2	0.6244	0.5289	0.5863	0.4975	Alt_3
	3	0.5555	0.5445	0.5636	0.5404	
	4	0.5220	0.5517	0.5579	0.5519	
	5	0.5036	0.5553	0.5653	0.5651	
	6	0.4928	0.5571	0.5683	0.5413	
	7	0.4861	0.5577	0.5693	0.5388	
	8	0.4817	0.5579	0.5691	0.5373	
	9	0.4787	0.5577	0.5684	0.5362	
	10	0.4766	0.5573	0.5676	0.5355	

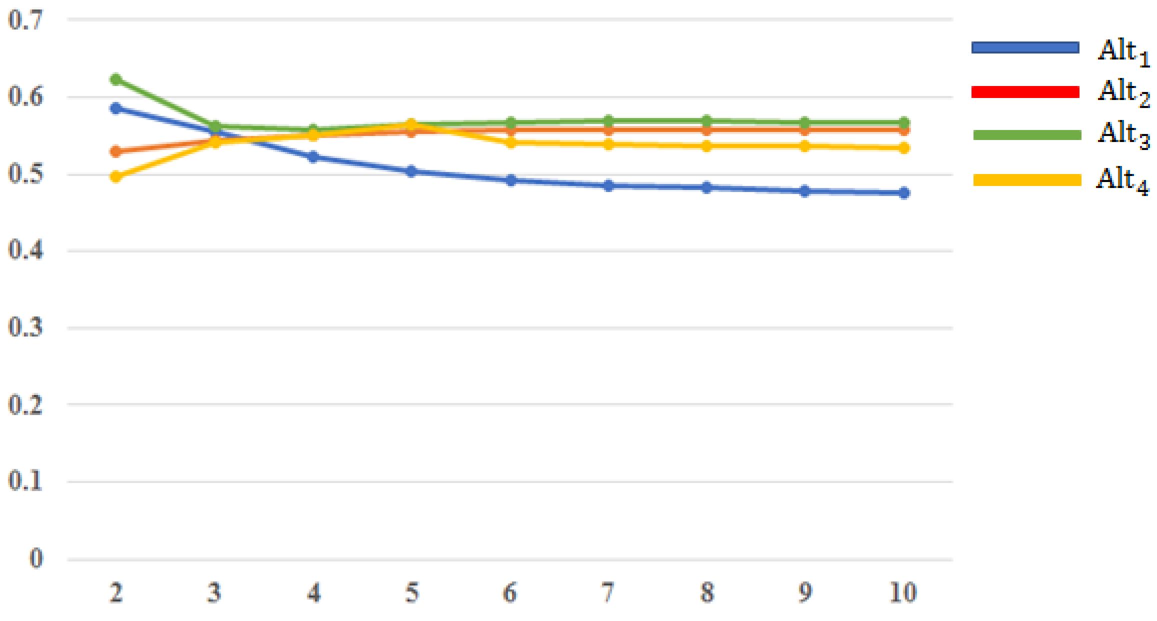

Figure 9. Show behavior of alternatives for different values of "q".

5. Comparison analysis and discussions

The main purpose of this section is to demonstrate the authenticity, reliability, and effectiveness of our proposed method. For this purpose, we evaluated our proposed model against to the existing approaches such as Born et al. [4], Zulqarnain et al. [37], Yang and Pang [38] and Vimala et al. [39], which are summarized in Table 12 and Figure 10. The sequence in which the ranking order of alternatives are ranked aligns with the proposed ranking order, and the same best alternative, indicating that our proposed technique is more flexible and authentic by three incorporating characteristics positive, neutral, and negative membership function with attributes which play a significant role throughout evaluation of alternatives as compared to the existing approaches shown in comparison analysis. Our proposed model is more flexible and has a free structure, enabling decision-maker to navigate the complication of the DM process in a more realistic manner, because existing approaches such as [2], [3], [5], [6], [13] and [17] are restricted by certain limitations and lack information about neutral degree, as shown in characteristic analysis. The main advantage of our proposed method is the superiority to handle real-life issues and prevent these limitations by utilizing the parameterization axioms, as shown in Table 13.

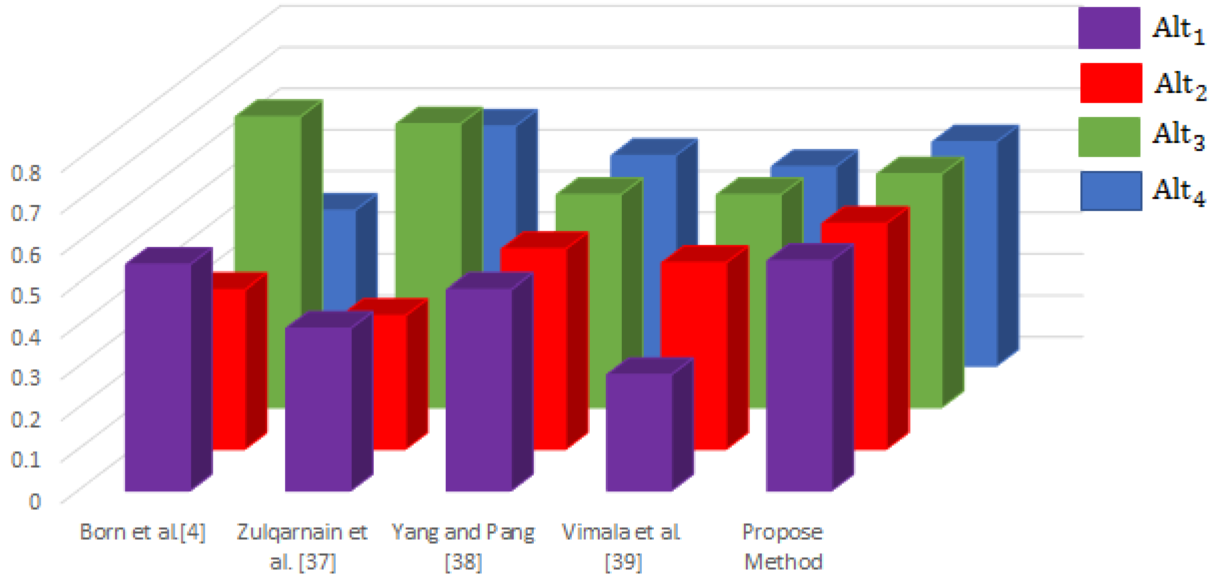

Figure 10. Show the graphical representation of comparison analysis.

Table 12. Comparative analysis with existing methods.

TOPSIS methods	Score values				Ranking order
	Alt_1	Alt_2	Alt_3	Alt_4	
Born et al. [4]	0.546	0.385	0.702	0.375	$\text{Alt}_3 > \text{Alt}_4 > \text{Alt}_2 > \text{Alt}_1$
Zulqarnain et al. [37]	0.39272	0.32281	0.68478	0.57876	$\text{Alt}_3 > \text{Alt}_4 > \text{Alt}_1 > \text{Alt}_2$
Yang and Pang [38]	0.4861	0.4833	0.5137	0.5076	$\text{Alt}_3 > \text{Alt}_4 > \text{Alt}_1 > \text{Alt}_2$
Vimala et al. [39]	0.28071	0.44978	0.51366	0.48063	$\text{Alt}_3 > \text{Alt}_4 > \text{Alt}_2 > \text{Alt}_1$
Propose Method	0.5555	0.5445	0.5636	0.5404	$\text{Alt}_3 > \text{Alt}_1 > \text{Alt}_2 > \text{Alt}_4$

Table 13. Characteristic analysis with existing theories.

Methods	Positive MD	Neutral MD	Negative MD	Parametrization tools	Domian
FS [2]	✓	×	×	×	$0 \leq (a) \leq 1$
IFS [3]	✓	×	✓	×	$0 \leq (a) + (b) \leq 1$
PyFS [5]	✓	×	✓	×	$0 \leq (a)^2 + (b)^2 \leq 1$
q-ROFS [6]	✓	×	✓	×	$0 \leq (a)^q + (b)^q \leq 1$ $q \geq 1$
q-ROFSS [13]	✓	×	✓	✓	$0 \leq (a)^q + (b)^q \leq 1$ $q \geq 1$
PFS [17]	✓	✓	✓	×	$0 \leq a + b + c \leq 1$
q-ROPFSS [18] (Proposed)	✓	✓	✓	✓	$0 \leq (\mathrm{a})^q + (\mathrm{b})^q + (\mathrm{c})^q \leq 1$ $q \geq 1$

Advantages:

- Our proposed model demonstrates high level of proficiency to address the challenges lifted by FS and its extended theories, such as inaccurate data and decision-making simulation, which provide a robust framework to manage uncertainty across various situations.
- In terms of the DM problem, our generalized structure of the q-rung orthopair picture fuzzy soft sets with the TOPSIS method have a unique characteristic to overcome the gap of neutral degree lifted by existing approaches.
- The aim of the TOPSIS method within the context of the proposed model, addresses the decision-making problem related to green supplier chain selection management to reduce the harm of ecosystems and maintain the environmental sustainability.

6. Conclusions

This article established a generalized concept in terms of q-ROPFS$_t$S, by merging the concept of picture fuzzy set and q-ROFS$_t$S. This framework handles the inherent vagueness information by applying the triplet of membership linked to specific attributes. Literature theories lacked information regarding neutral degree and faced constraint due to certain limitations, and there is no research work on the q-rung orthopair picture fuzzy soft TOPSIS method. So, to improve the TOPSIS method our main aims to base on q-ROPFS$_t$ environment, a novel MCGDM strategy created for GSCM. On the basis of this concept, we proposed an algorithm for MCDM problem named q-ROPFS$_t$ TOPSIS method. By using the proposed technique for application, we tackled the decision model and case study related to GSCM to choose the best alternative on the bases of criteria. To check the validity of proposed method we compare our proposed q-ROPFS$_t$ TOPSIS method with existing TOPSIS methods to get the better result, and to check the stability and effectiveness of our proposed model, we demonstrate the parameter analysis test by using the various values of parameter “q”. Through comparison analysis we check reliability and effectiveness of our proposed model. The proposed study has limitations. However, the shortcoming of this study related to eco-friendly supply chain, which designed a strategy to protect the environment and reduce harm of ecosystems companies, participates in green supply chain activity. However, the scope of validation may be limited in this specific scenario. It could be very beneficial to exploring the broader range of real-life decision-making challenges across various industries to make sure of the robustness of the q-ROPFS$_t$S framework. In the future, we will extend our work into different directions such as AHP, complex proportional assessment method (COPRAS), VIKOR method, CODAS method and various extensions of FS like cubic m-polar fuzzy sets, Pythagorean m-polar fuzzy sets and different methods for the selection of investment projects.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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