AIMS Mathematics

AIMS Mathematics, 9(6): 14997–15007.
DOI: 10.3934/math.2024726
Received: 02 January 2024
Revised: 13 April 2024
Accepted: 19 April 2024
Published: 25 April 2024

http://www.aimspress.com/journal/Math

Research article

Generalization of Snell's Law for the propagation of acoustic waves in elliptically anisotropic media

Luis M. Pedruelo-González* and Juan L. Fernández-Martínez

Group of Inverse Problems, Optimization and Machine Learning, Department of Mathematics, Universidad de Oviedo, Oviedo, Spain

* **Correspondence:** Email: pedrueloluis@uniovi.es.

Abstract: In seismic data processing, both in inversion (Inverse Processing) and modeling (Direct Processing), it is essential to consider anisotropy to unravel the geological structure of the subsoil. Besides, in most cases, the macroscopic model of anisotropy in 2D seismic surveys is elliptical and weak, with ratios of anisotropy close to one. Therefore, it is crucial to have at disposal the analytical formulas for acoustic wave propagation in elliptical anisotropic media. We presented the generalization of the Snell's Law for the case of acoustic wave propagation in elliptically anisotropic media. The generalization of the Snell's Law for acoustic anisotropic media had different applications in digital processing, raytracing, and acoustic inversion to properly consider elliptical anisotropy.

Keywords: transmission tomography; elliptical anisotropic media; inverse problems
Mathematics Subject Classification: 86-XX

1. Introduction

Seismic anisotropy is a topic treated in many research articles [1–5]. During the last decades, the separation between sources and receivers of seismic data has increased, and the quality and processing capacity of such seismic data has improved considerably. Thus, it is necessary to rethink its treatment. Although the subject is extensive and the mathematics describing its behavior is, in general, complex, this work intends to advance in facilitating the incorporation of anisotropy in the representation, processing, and inversion of seismic data, improving in this way the efficiency of exploration techniques.

The generalization of the Snell's Law to acoustic anisotropic media has different applications in digital processing, raytracing, and inversion. For instance, the transmission tomographic inverse problem could be solved in a discretized medium where the velocity field has a local anisotropic behavior. Using this formula, it is possible to solve both the forward and inverse problems to infer the major acoustic elliptical anisotropic parameters of the geological medium.

In exploration seismology, a medium is defined as showing seismic anisotropy if the seismic velocity varies when the direction of propagation varies. When the velocity does not vary as a function of direction, the medium is called isotropic, thus being a particular case of anisotropy. Heterogeneity describes the variations in physical properties between two points. Although heterogeneity and anisotropy are not the same thing, they generally coexist by mimicking their dynamic effects. An anisotropic and homogeneous model is valid for seismic data coming from environments with great structural geological complexity as well as for long distances between sources and receivers [6].

Nevertheless, it has been shown that most of the geological media show a macroscopic elliptical anisotropy, and in most of the cases, this anisotropy is weak, that is, the anisotropy ratio, which is the quotient between the minimum and the maximum acoustic velocity of the medium, is close to 1 (0.95–0.99). This is because of fine stratifications, fluid content, and organic matter [7–8]. This fact outlines the importance of the elliptical anisotropic model in geophysical exploration.

Besides, this research is circumscribed to the field of acoustic media. We are aware of the importance of wave propagation in elastic media, but there is no free lunch in modelling, that is, every mathematical model is suited for a particular purpose. Also, due to the importance of Snell's law and eikonal equation in raytracing, digital processing, and in inversion, we have adopted the Fermat's principle as the simplest method to deduce this law as shown in Berryman [9].

Snell's Law models the refraction that occurs in the path followed by an acoustic beam between two contiguous regions with homogeneous isotropic slowness s_1 and s_2. This law states that this is the transition between these two media, so we have:

$$s_1 \cdot \sin\theta_1 = s_2 \cdot \sin\theta_2 \tag{1}$$

where θ_1 and θ_2 are the respective angles of incidence and refraction (see Figure 1).

Snell's Law can be deduced as a corollary of the Fermat's principle, Berryman [9], that states: The ray between points A and B will follow the path of minimum travel time, that is, acoustic propagation follows variational principles. Snell's Law can be also deduced in elastic media via continuity principles [10].

Our purpose is to generalize the expression (1) for the case of acoustic wave propagation between two elliptically anisotropic media and to find the analytical expression that governs this situation.

The velocity models of each of the media exhibit in this case a weak elliptical anisotropy (see Figure 2) described by the following parameters:

$V_{\max}$: The maximum velocity,

α: The direction of anisotropy,

$\lambda = \dfrac{V_{\min}}{V_{\max}}$: The anisotropy ratio.

Let us point out that this elliptic anisotropic velocity model includes the isotropic case (for $\lambda = 1$).

Most cases of geophysical media show a weak elliptical anisotropy with an anisotropy ratio in the interval (0.95, 1) [11,12].

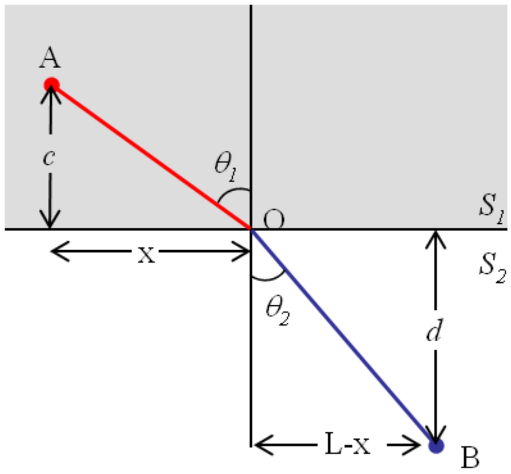

Figure 1. Snell's Law. A consequence of Fermat's principle stating that the path followed by a beam between a source (A) located in a medium and a receiver and (B) located in another medium is that one taking the minimum traveling time.

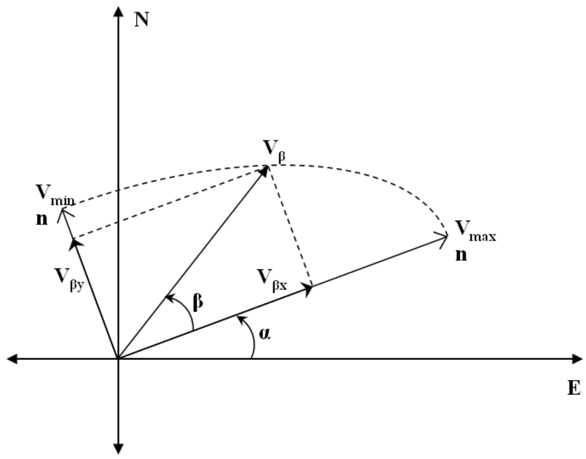

Figure 2. Elliptic anisotropic velocity model, involved variables: the anisotropy direction, anisotropy ratio, and the maximum velocity.

2. Deduction of Snell's Law in anisotropic media

Without any loss of generality, we can consider an elliptically anisotropic medium, consisting of two regions separated by a plane boundary, as well as a beam connecting two points A and B located on either side of the boundary (Figure 1). Each of the media, homogeneous and elliptically anisotropic respectively, will be characterized by a set of three parameters $(V_{\max}, \alpha, \lambda)$. If we call $v_\beta = (v_{\beta x}, v_{\beta y})$ the propagation velocity in the direction forming an angle β with respect to the direction of maximum velocity α we obtain the following relation:

$$\frac{v_{\beta x}^2}{V_{\max}^2}+\frac{v_{\beta y}^2}{V_{\min}^2}=1 \tag{2}$$

and since

$$\lambda=\frac{V_{\min}}{V_{\max}}. \tag{3}$$

Then, we can express v_β in terms of anisotropy parameters:

$$v_\beta = a_{\lambda\beta} V_{\max} = \left|\sqrt{\frac{1+\tan^2\beta}{1+\frac{\tan^2\beta}{\lambda^2}}}\right| V_{\max}. \tag{4}$$

The time a beam takes to go from source A to receiver B is:

$$t(x)=\left\|\overline{AO}\right\|\cdot s_1+\left\|\overline{OB}\right\|\cdot s_2=\frac{\sqrt{c^2+x^2}}{V_{\theta_1}}+\frac{\sqrt{d^2+(L-x)^2}}{V_{\theta_2}}. \tag{5}$$

Since Fermat's path minimizes travel time, the following conditions must be fulfilled:

$$\frac{dt}{dx}=0. \tag{6}$$

Notice that this condition in isotropic media leads us to the well-known Snell's Law. Let us see what this condition leads us to if applied to Eq (5). If we call:

$$t_1(x)=\frac{\left(\sqrt{c^2+x^2}\right)}{V_{\theta_1}}, \tag{7}$$

$$t_2(x)=\frac{\left(\sqrt{d^2+\left(L-x\right)^2}\right)}{V_{\theta_2}}, \tag{8}$$

then

$$t(x)=t_1(x)+t_2(x) \quad\Rightarrow\quad \frac{dt}{dx}=\frac{dt_1}{dx}+\frac{dt_2}{dx}. \tag{9}$$

Let us start calculating $\frac{dt_1}{dx}$. From expression (7):

$$\frac{dt_1}{dx} = \frac{V_{\theta_1}\frac{x}{\sqrt{c^2+x^2}} - \sqrt{c^2+x^2}\ \frac{dV_{\theta_1}}{dx}}{V_{\theta_1}^2} = \frac{V_{\theta_1} sin\theta_1 - \sqrt{c^2+x^2}\ \frac{dV_{\theta_1}}{dx}}{V_{\theta_1}^2}, \tag{10}$$

and calling

$$V_{\theta_1} = a_1 V_{1\max}, \tag{11}$$

where

$$a_1 = \sqrt{\frac{1+\tan^2\beta_1}{1+\frac{\tan^2\beta_1}{\lambda_1^2}}}, \tag{12}$$

then we obtain:

$$\frac{dV_{\theta_1}}{dx} = \frac{da_1}{dx} V_{1\max}. \tag{13}$$

Besides, $\frac{da_1}{dx}$ is given by:

$$\frac{da_1}{dx} = \frac{1}{2\sqrt{\frac{1+tan^2\beta_1}{1+\frac{tan^2\beta_1}{\lambda_1^2}}}} \cdot \frac{2\,tan\,\beta_1\cdot(1+tan^2\beta_1)\cdot\left(1+\frac{tan^2\beta_1}{\lambda_1^2}\right) - (1+tan^2\beta_1)\cdot\frac{tan\,\beta_1}{\lambda_1^2}(1+tan^2\beta_1)}{\left(1+\frac{tan^2\beta_1}{\lambda_1^2}\right)^2} \cdot \frac{d\beta_1}{dx}, \tag{14}$$

which can also be expressed as follows:

$$\frac{da_1}{dx} = \left(\tan\beta_1\cdot a_1 - a_1^3\cdot\frac{\tan\beta_1}{\lambda_1^2}\right)\cdot\frac{d\beta_1}{dx} = \left(\tan\beta_1\cdot a_1\cdot\left(1-\frac{a_1^2}{\lambda_1^2}\right)\right)\cdot\frac{d\beta_1}{dx}. \tag{15}$$

Moreover (see Figure 3):

$$\beta_1 = \theta_1 + 90^o - \alpha, \tag{16}$$

$$\frac{d\beta_1}{dx} = \frac{d\theta_1}{dx} = \frac{d}{dx}\left(arcsin\frac{x}{\sqrt{x^2+c^2}}\right) = \frac{1}{\sqrt{1-\frac{x^2}{\sqrt{x^2+c^2}}}}\cdot\frac{\sqrt{x^2+c^2} - \frac{x^2}{\sqrt{x^2+c^2}}}{x^2+c^2}, \tag{17}$$

that can also be expressed in reduced form, as:

$$\frac{d\beta_1}{dx} = \frac{d\theta_1}{dx} = \frac{c}{x^2+c^2}. \tag{18}$$

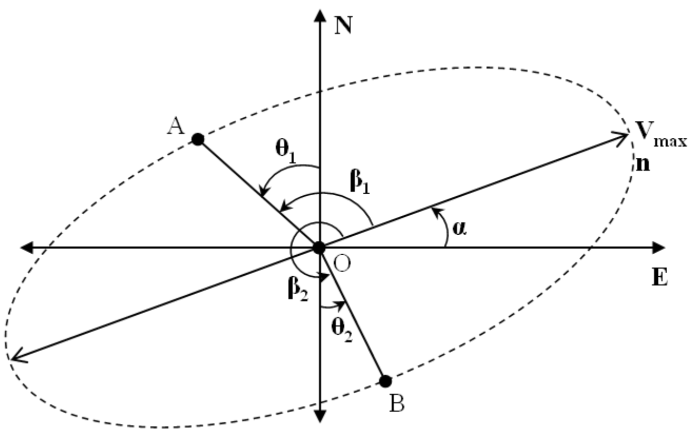

Figure 3. Angles definitions.

Consequently, merging the above, we could come back to expression (10), incorporating the result of (13), obtaining:

$$\frac{dt_1}{dx}=\frac{V_{\theta_1}sin\theta_1-\sqrt{c^2+x^2}\ \frac{dV_{\theta_1}}{dx}}{V_{\theta_1}^2}=\frac{V_{\theta_1}sin\theta_1-\sqrt{c^2+x^2}\cdot\frac{da_1}{dx}\cdot V_{1\max}}{V_{\theta_1}^2}, \tag{19}$$

Considering (15), we have:

$$\frac{dt_1}{dx}=\frac{V_{\theta_1}sin\theta_1-\sqrt{c^2+x^2}\cdot\left(\tan\beta_1\cdot a_1\cdot\left(1-\frac{a_1^2}{\lambda_1^2}\right)\right)\cdot\frac{d\beta_1}{dx}\cdot V_{1\max}}{V_{\theta_1}^2}. \tag{20}$$

Finally, including the result of (18), we arrive at the expression:

$$\frac{dt_1}{dx}=\frac{sin\theta_1}{V_{\theta_1}}-\frac{cos\theta_1}{V_{\theta_1}}\cdot\tan\beta_1\cdot\left(1-\frac{a_1^2}{\lambda_1^2}\right). \tag{21}$$

Let us now find the derivative of expression (8), $\frac{dt_2}{dx}$:

$$\frac{dt_2}{dx}=\frac{V_{\theta_2}\frac{-2(L-x)}{2\sqrt{(L-x)^2+d^2}}-\sqrt{(L-x)^2+d^2}\ \frac{dV_{\theta_2}}{dx}}{V_{\theta_2}^2}. \tag{22}$$

Let us now call similar to (11):

$$V_{\theta_2}=a_2V_{2\max}, \tag{23}$$

where

$$a_2 = \sqrt{\frac{1+\tan^2\beta_2}{1+\frac{\tan^2\beta_2}{\lambda_2^2}}}\,. \tag{24}$$

Then, we have

$$\frac{dV_{\theta_2}}{dx} = \frac{da_2}{dx}V_{2\max}\,, \tag{25}$$

where $\frac{da_2}{dx}$ is given by the expression:

$$\frac{da_2}{dx} = \left(\tan\beta_2 \cdot a_2 - a_2^3 \cdot \frac{\tan\beta_2}{\lambda_2^2}\right) \cdot \frac{d\beta_2}{dx} = \left(\tan\beta_2 \cdot a_2 \cdot \left(1 - \frac{a_2^2}{\lambda_2^2}\right)\right) \cdot \frac{d\beta_2}{dx}\,. \tag{26}$$

Therefore

$$\frac{dV_{\theta_2}}{dx} = \frac{da_2}{dx} V \left(tan\,\beta_2 \cdot a_2 \cdot \left(1 - \frac{a_2^2}{\lambda_2^2}\right)\right) \frac{d\beta_2}{dx}{}_{2max}{}_{2max}\,. \tag{27}$$

Since

$$\beta_2 = (\theta_2 + 270^{\,o} - \alpha), \tag{28}$$

then

$$\frac{d\beta_2}{dx} = \frac{d\theta_2}{dx} = \frac{d}{dx}\left(arcsin\frac{L-x}{\sqrt{(L-x)^2+d^2}}\right) = \frac{1}{\sqrt{1-\frac{(L-x)^2}{(L-x)^2+d^2}}} \cdot \frac{-\sqrt{(L-x)^2+d^2} - (L-x)\frac{-2(L-x)}{2\sqrt{(L-x)^2+d^2}}}{(L-x)^2+d^2}, \tag{29}$$

that can be expressed in reduced form as:

$$\frac{d\beta_2}{dx} = \frac{d\theta_2}{dx} = \frac{-d}{(L-x)^2+d^2}\,. \tag{30}$$

Now, merging what has been shown above we can re-formulate expression (22) as follows:

$$\frac{dt_2}{dx} = \frac{V_{\theta_2}\cdot(-sin\theta_2) - \sqrt{(L-x)^2+d^2}\cdot\frac{dV_{\theta_2}}{dx}}{V_{\theta_2}^2} = \frac{-V_{\theta_2}sin\theta_1 - \sqrt{(L-x)^2+d^2}\cdot\frac{da_2}{dx}\cdot V_{2max}}{V_{\theta_2}^2}, \tag{31}$$

that is:

$$\frac{dt_2}{dx}=\frac{-V_{\theta_2}sin\theta_2+\sqrt{(L-x)^2+d^2}\cdot\left(\tan\beta_2\cdot a_2\cdot\left(1-\frac{a_2^2}{\lambda_2^2}\right)\right)\cdot\frac{d\beta_2}{dx}\cdot V_{2\max}}{V_{\theta_2}^2}. \tag{32}$$

Finally, we conclude:

$$\frac{dt_2}{dx}=\frac{-sin\theta_2}{V_{\theta_2}}+\frac{cos\theta_2}{V_{\theta_2}}\cdot\tan\beta_2\cdot\left(1-\frac{a_2^2}{\lambda_2^2}\right). \tag{33}$$

Considering expressions (9), (21), and (33), we have:

$$\frac{dt}{dx}=\left(\frac{sin\theta_1}{V_{\theta_1}}-\frac{cos\theta_1}{V_{\theta_1}}\cdot\tan\beta_1\cdot\left(1-\frac{a_1^2}{\lambda_1^2}\right)\right)+\left(\frac{-sin\theta_2}{V_{\theta_2}}+\frac{cos\theta_2}{V_{\theta_2}}\cdot\tan\beta_2\cdot\left(1-\frac{a_2^2}{\lambda_2^2}\right)\right). \tag{34}$$

Finally, imposing the stationary condition, that the Fermat's ray paths should correspond to the minimum travel time:

$$\frac{dt}{dx}=0 \quad \Leftrightarrow$$

$$\Leftrightarrow \quad \left(\frac{sin\theta_1}{V_{\theta_1}}-\frac{cos\theta_1}{V_{\theta_1}}\cdot\tan\beta_1\cdot\left(1-\frac{a_1^2}{\lambda_1^2}\right)\right)=\left(\frac{sin\theta_2}{V_{\theta_2}}-\frac{cos\theta_2}{V_{\theta_2}}\cdot\tan\beta_2\cdot\left(1-\frac{a_2^2}{\lambda_2^2}\right)\right), \tag{35}$$

we arrive at:

$$\frac{V_{\theta_1}}{V_{\theta_2}}=\frac{sin\theta_1-cos\theta_1\tan\beta_1\cdot\left(1-\frac{a_1^2}{\lambda_1^2}\right)}{sin\theta_2-cos\theta_2\tan\beta_2\cdot\left(1-\frac{a_2^2}{\lambda_2^2}\right)}. \tag{36}$$

In the case of isotropic media, this expression simplifies to the well-known Snell's Law. Therefore, Eq (36) shall be called Generalized Snell's Law for elliptically anisotropic media.

3. Applications

3.1. Application in raytracing

When this formula is used in tracing programs is used, the determination of angle θ_2 should be solved numerically according to:

$$V_{\theta_2}=\frac{V_{\theta_1}\cdot\left(sin\theta_2-cos\theta_2\tan\beta_2\cdot\left(1-\frac{a_2^2}{\lambda_2^2}\right)\right)}{sin\theta_1-cos\theta_1\tan\beta_1\cdot\left(1-\frac{a_1^2}{\lambda_1^2}\right)}. \tag{37}$$

Considering that K is a constant:

$$K = \frac{V_{\theta_1}}{sin\theta_1 - cos\theta_1 \, tan\, \beta_1 \cdot \left(1 - \frac{a_1^2}{\lambda_1^2}\right)}, \tag{38}$$

we can express (37) as:

$$V_{\theta_2} = a_2 V_{2\max} = K \cdot \left(sin\theta_2 - \cos\theta_2 \tan\beta_2 \cdot \left(1 - \frac{a_2^2}{\lambda_2^2}\right)\right), \tag{39}$$

or:

$$\mathrm{a}_2 = \frac{K}{V_{2\max}} \cdot \left(sin\theta_2 - cos\theta_2 \tan\beta_2 \cdot \left(1 - \frac{a_2^2}{\lambda_2^2}\right)\right), \tag{40}$$

that can be solved numerically.

3.2. Application in transmission tomography

The simplest use of this model in inversion consists in finding the anisotropic parameters from transmision traveltime data in a medium composed by two homgeneous elliptical anisotropic suddomains separated by an interface. In this case, we have at disposal m traveltime data between m pairs of sources and receivers, and the dimension of the model space is 6. The solution has to be iterative to guess the direction of the different rays between sources and receivers. Besides, the inverse problem is nonlinear due to the dependence of Snell's Law on the anisotropic paramerters. This provides an example of a simple application in inversion.

4. Conclusions and discussion

Wave propagation in geophysical acoustic media is particularly important in several fields of geophysics:

- In seismic exploration of natural resources such as oil, gas, and minerals.
- In earthquake studies to provide valuable insights into the earth's internal structure and the dynamics of seismic events.
- In volcanic monitoring to study the acoustic waves induced by magma movement and volcanic eruptions.
- In subsurface imaging and reservoir characterization to optimize hydrocarbon exploration and recovery.
- In geothermal exploration and environmental monitoring, among others.

Besides, some degree of anisotropy is commonly observed in geological media. Anisotropy is a kind of geological heterogeneity, which is usually the target in geophysical exploration. Particularly, elliptical anisotropy refers to a type of anisotropy in which the velocities of propagation vary in different directions, forming the ellipse of anisotropy that in 2D is characterized by the direction of the anisotropy, the maximum velocity and the ratio of anisotropy, that is a real parameter between 0 and

1. Nevertheless, the manifestation of elliptical anisotropy in geological media tends to be relatively weak, that is, the anisotropy ratio is close to one (the isotropic case) due to the complex interplay of geological processes, material heterogeneity, and structural deformation.

Geological processes such as sedimentation, compaction, and metamorphism often lead to the alignment of geological features in preferred orientations. This preferred orientation of foliation or bedding (fine layering) can introduce some degree of anisotropy in the rock's properties, resulting in weaker elliptical anisotropy. Besides, geological media typically exhibit significant heterogeneity in their composition and structure. The presence of multiple mineral phases with different orientations can average out any directional variations, leading to weak overall anisotropy. Also, rocks experience a wide range of pressure and temperature conditions during their formation and deformation. These conditions can influence the mechanical and elastic properties of the rock, potentially reducing the degree of elliptical anisotropy. High pressures and temperatures may lead to recrystallization or mineral reorientation, which can diminish the effects of any pre-existing anisotropy. This includes the effect of organic matter. Besides rocks undergo various forms of deformation, including folding, faulting, and shearing, which can modify their internal structure and properties. These deformation processes can disrupt any pre-existing anisotropic fabric or alignment, resulting in a weakening of elliptical anisotropy. Finally, the measurement scale of the seismic surveys is as source of weak anisotropy due to the challenge of the detection and characterization of the elliptical anisotropy of the geological media, causing the macroscopic geophysical model to be weak. This fact provokes the elliptical anisotropy identification from travel time data to be challenging (ill-posed problem).

In this paper, we have deduced the analytical expressions of the Snell's Law for the propagation of acoustic waves in elliptically anisotropic media. These formulas will be of great use in the study of the geological structure of the subsoil by means of seismic data, either in inversion (Inverse Processing) or in Raytracing (Direct Processing), and will allow the estimation of the elliptic anisotropy model from experimental travel time data. This knowledge is very important since elliptic anisotropy is a common feature in real practice [13–15].

This article remains a theoretical deduction of this law using Fermat's principle of ray propagation. To avoid increasing its length and difficulty, possible practical cases have been omitted, since in the different fields each researcher develops specific software for solving inverse problems in acoustic media. The generalization of Snell's Law to weak anisotropic media allows its use in digital processing, forward propagation, and inversion. We outline some potential applications of this formula.

Use of AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

All authors declare no conflicts of interest in this paper.

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