Mathematics

Research article

Event-triggered impulsive control for second-order nonlinear multi-agent systems under DoS attacks

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Abstract: We investigated impulsive consensus in second-order nonlinear multi-agent systems (MASs) under Denial-of-Service (DoS) attacks. We consided scenarios where the communication network is subjected to DoS attacks, disrupting communication links and causing changes in the communication topology. An event-triggered impulsive control(ETIC) approach is proposed to flexibly address these issues. Additionally, an upper bound on the DoS attack period is introduced. Finally, a numerical example is given to verify the validity of the major results.

1. Introduction

In recent years, the issue of multi-agent consensus has garnered considerable attention and research across various disciplines, including computer science, control theory, communication, and related fields. Multi-agent systems comprise numerous interacting intelligent agents collaborating to accomplish predefined objectives. Many researchers have studied the issue of achieving consensus in multi-agent systems under different conditions. There are several effective methods for achieving consensus, such as distributed control [1,2], sliding mode control [3], fault-tolerant control [4,5], and impulsive control [6–9].

Compared to the continuous control methods mentioned above, the use of impulsive control protocols significantly reduces information transmission. Impulsive control involves only information transfer at discrete impulsive moments. This helps to reduce the risk of communication losses. In recent decades, impulsive control has widespread applications in fields such as signal processing,

communication, and medical devices. In [10], researchers investigated bounded consensus in heterogeneous multi-agent systems using impulsive control. Additionally, some multi-agent systems with heterogeneous delays were discussed and studied in [11,12]. In [13], two different impulsive control were proposed: Partial state constraint and full state constraint. They delved into the consensus problem of nonlinear MASs featuring grouping through state constraint impulsive protocols. They examined two scenarios: One involving a leader agent and another without, each analyzed using distinct two kinds of protocol. Considering that the network communication topology is not always constant, in [14], scholars employed algebraic graph theory and impulsive stability theory to explore the consensus problem in fixed-topology systems using impulsive control. In [15], the flexible handling of communication interruptions and topology switches was achieved using the T-S fuzzy model and impulsive control. It is important to note that the above-mentioned impulsive control methods are all time-triggered, which may result in additional communication and control overhead during information transmission. Therefore, it is necessary to study a new impulsive control method to address this issue. In [16], a distributed ETIC strategy was proposed to achieve consensus of MASs. Considering the evident benefits of ETIC, in recent years, numerous scholars have conducted research on ETIC and reported their corresponding results [17]. In [18], a distributed ETIC protocol was proposed to achieve consensus in MASs. Furthermore, in [19], an innovative dynamic event-triggered impulsive control protocol has been introduced, offering a more efficient means to reduce controller updates and minimize the waste of communication resources.

In practice, complex network systems or MAS tend to be more complex and vulnerable to cyber attacks by malicious actors. In recent years, scholars have gradually turned their attention to nonlinear multi-agent systems. In [20] scholars proposed an improved prescribed performance approach for human-in-the-loop consensus tracking control in unmanned aerial vehicle (UAV) systems. Their research underscores the importance of establishing effective interactions between humans and machines to achieve efficient control of UAV systems. In [21], scholars explored a privacy preservation mechanism for multi-agent systems and introduced a gain iterative disturbance observer. Their research focuses on safeguarding information privacy within systems while maintaining stability and reliability of system performance. In [22], scholars proposed an improved predefined-time adaptive neural control approach for addressing control issues in nonlinear multi-agent systems. Their research aims to overcome limitations of traditional control methods in complex systems, presenting a control solution with enhanced adaptability and robustness. In [23], scholars investigated quantized guaranteed cost memory consensus for nonlinear multi-agent systems with switching topology and actuator faults. Their study addresses uncertainties and faults within the system, introducing a novel guaranteed cost memory consensus method. Furthermore, cyber attacks have also been a hot research direction in recent years. Cyber attacks are typically categorized into two types: deception attacks [24,25] and DoS attacks [26–33]. Deception attacks focus on tampering with the data in the communication network, while DoS attacks typically overload the resources of the target system, rendering it unable to provide services and thus disrupting the information transmission channel. In [24], the impulsive consensus of nonlinear leader-follower MASs under DoS attacks was studied. The authors in [25] considered cases where sensor data transmission involves packet loss, with the objective of maximizing the terminal state covariance as the attack target, and presents optimal energy-constrained DoS attack scheduling schemes. In [26], the scholars developed a resilient cooperative ETIC scheme of linear MASs with DoS attacks. In [27], the guided bounded consensus of MASs with ETIC was studied in the presence

Then, the state error is defined as

$$\begin{cases} e_{pi}(t) = x_i(t) - x_0(t) \\ e_{vi}(t) = v_i(t) - v_0(t). \end{cases} \tag{3}$$

In this paper, we consider a second-order leader system under DoS attacks, so the impulsive control input can be designed as

$$u_i(t) = c \sum_{k=1}^{\infty} [\sum_{j \in N_i} a_{ij}(m)(x_j(t) - x_i(t)) + b_i(m)(x_0 - x_i) +$$

$$\sum_{j \in N_i} a_{ij}(m)(v_j(t) - v_i(t)) + b_i(m)(v_0 - v_i)]\delta(t - t_k), \tag{4}$$

where c denotes the coupling strength, m defines the topology under DoS attacks, the local neighborhood consensus error is $p_i(t) = \sum_{j \in N_i} a_{ij}(x_j(t) - x_i(t)) + b_i(x_0 - x_i) + \sum_{j \in N_i} a_{ij}(v_j(t) - v_i(t)) + b_i(v_0 - v_i)$, $\delta(\cdot)$ is the Dirac function, $\{t_k\}$ is the impulse sequence, it satisfies $\lim_{k \to \infty} t_k = \infty, k \in \mathbb{N}_+$.

From (1) and (2), the state error system are as follows

$$\begin{cases} \dot{e}_{pi}(t) = e_{vi}(t) \\ \dot{e}_{vi}(t) = Ae_{pi} + f(e_{pi}, e_{vi}) + u_i(t), \end{cases} \tag{5}$$

where $f(e_p, e_v) = f(x_i(t), v_i(t)) - f(x_0(t), v_0(t))$.

From the (3)–(5), we can get

$$\begin{cases} \dot{e}_{pi}(t) = e_{vi}(t) \\ \dot{e}_{vi}(t) = Ae_{pi} + f(e_{pi}, e_{vi}), t \neq t_k \\ \Delta e_{vi}(t_k) = \sum_{j=1}^{N} c[-p_i(t_k)(m)], t = t_k, \end{cases} \tag{6}$$

where $p_i(t_k)(m) = \sum_{j \in N_i} a_{ij}(m)(x_j(t) - x_i(t)) + b_i(m)(x_0 - x_i) + \sum_{j \in N_i} a_{ij}(m)(v_j(t) - v_i(t)) + b_i(m)(v_0 - v_i)$, $\Delta e_{vi}(t_k) = e_{vi}(t_k) - e_{vi}(t_k^-), e_{vi}(t_k) = e_{vi}(t_k^+) = \lim_{h \to 0^+} e_{vi}(t_k + h), e_{vi}(t_k^-) = \lim_{h \to 0^-} e_{vi}(t_k + h)$, and $e(t)$ is right-hand continuous at $t = t_k$.

The system (6) can be rewritten as:

$$\begin{cases} \dot{e}_p(t) = e_v(t) \\ \dot{e}_v(t) = (I_N \otimes A)e_p + F(e_p, e_v), t \neq t_k \\ \Delta e_v(t_k) = -c[(L(m) + B(m)) \otimes I_N](e_p(t_k^-) + e_v(t_k^-)), t = t_k, \end{cases} \tag{7}$$

where $e_p = [e_{p1}, e_{p2}, \cdots, e_{pN}]^T, e_v = [e_{v1}, e_{v2}, \cdots, e_{vN}]^T, F(e_p, e_v) = [f(e_{p1}, e_{v1}), f(e_{p2}, e_{v2}), \cdots, f(e_{pN}, e_{vN})]^T, B = diag\{b_i\} \in \mathbb{R}^{N \times N}$.

Lemma 1. ([35] Differential Mean Value Theorem) Let $f(\cdot)$ is a continuously differentiable nonlinear function, for any $x_1, x_2 \in \mathbb{R}$, the following formula is satisfied:

$$f(x_i(t), v_i(t)) - f(x_i(t_0), v_i(t_0)) = \frac{\partial f(z_i)}{\partial x_i}(x_i(t) - x_i(t_0)) + \frac{\partial f(z_i)}{\partial v_i}(v_i(t) - v_i(t_0)),$$

where $f(z_i) = f(x_i(t_0) + \Delta(x_i(t) - x_i(t_0)), v_i(t_0 + \Delta(v_i(t) - v_i(t_0))), 0 < \Delta < 1$.

Definition 1. [28] The second-order MASs (1) and (2) with an impulsive controller $u_i(t)$ is said to be global exponential consensus, if there exist a scalar θ and a decay rate ε such that for any $x_1(t_0), x_0(t_0) \in \mathbb{R}$.

$$\|x_i(t) - x_0(t)\| \leq \theta \|x_i(t_0) - x_0(t_0))\| e^{-\varepsilon(t - t_0)}.$$

2.3. DoS attack model

When MASs are targeted by DoS attacks, the attacks focus on disrupting the communication network, causing varying degrees of damage to the communication connections among system nodes. Consequently, leading to a disconnection of the communication channel, the network communication topology undergoes changes. It is noted that the MASs incorporates a recovery mechanism that allows the communication topology to be restored once the attack concludes. We assume that DoS attacks are not continuous and that the attacks needs time to recover energy after one attack. Thus, there is a minimum interval between attacks.

We assume that the time interval of an attack cycle is divided into three periods.

(1) Normal control period(N_n): The multi-agent system is normal working. The DoS attack has not arrived yet.

(2) Attack period(N_a): DoS attacks and switching topologies occur in this interval.

(3) Recovery period(N_r): The period during which the system recovers from a DoS attack. (minimum interval between attacks)

Definition 2. [28] Define the average impulsive interval as $\phi > 0$ in the time interval $[t_0, t]$. If there exist positive integer N_0, such that

$$\frac{t - t_0}{\phi} - N_0 \leq N(t_0, t) \leq \frac{t - t_0}{\phi} + N_0, \tag{8}$$

where $N(t_0, t)$ is the number of impulsive times in $[t_0, t]$.

Definition 3. [28] According to the DoS attack model, suppose that $N_n(t_0, t)$, $N_a(t_0, t)$ and $N_r(t_0, t)$ denote the number of impulse instants in the normal control period, attack period and recovery period. The total time of an attack cycle is $N(T_0, T) = N_n(t_0, t) + N_a(t_0, t) + N_r(t_0, t)$. Then the impulsive attack ratio η_a can be defined as

$$\eta_a = \frac{N_a(t_0, t)}{N(t_0, t)}. \tag{9}$$

Assumption 1. The graph $\mathcal{G}$ contains a spanning tree which the root node is the leader.

3. Main results

In this section, by designing an appropriate Lyapunov function and utilizing linear matrix inequalities, the system errors can be driven towards a bounded set, resulting in successful synchronization. Next, we designed the event trigger condition to be

$$t_{k+1} = \inf \left\{ t \geq t_k : V(t) \geq e^{a + \xi(t_k - t)} V(t_k) \right\},$$

where $a, \xi > 0$ are event trigger parameters.

Theorem 1. Under that Assumption 1, the MASs (1) and (2) are said to be globally exponentially synchronized, if there exist a symmetric matrix $P > 0$, system parameters $\alpha, \phi, \xi > 0$, $0 < \mu_1, \mu_m < 1$, such that

$$P\bar{A} + \bar{A}^T P - \alpha P \leq 0, \tag{10}$$

$$-\frac{\xi + \alpha}{\phi} < \ln \mu_1 < -\frac{ka}{N_0 + k}, \tag{11}$$

$$\eta_a < -\frac{\ln \mu_1 + \alpha\phi}{\ln \mu_m - \ln \mu_1}, \tag{12}$$

$$\phi < -\frac{\ln \mu_1}{\alpha}, \tag{13}$$

where we denote

$$\mu(t_k) = \lambda_{max}((I_{2N \times 2N} - R(m))^T P(I_{2N \times 2N} - R(m))),$$

$$\mu_1 = \lambda_{max}((I_{2N \times 2N} - R)^T P(I_{2N \times 2N} - R)),$$

$$\mu_m = \max_{\mu(t_k) \in \{2,...,m\}} \lambda_{max}((I_{2N \times 2N} - R(m))^T P(I_{2N \times 2N} - R(m))).$$

The definition of R and $R(m)$ are given later.

Proof. Let $e(t) = [e_p(t), e_v(t)]^T$, according to Lemma 1 and Assumption 1, we can get

$$\dot{e}(t) = \bar{A}e(t), \tag{14}$$

where $\bar{A} = \begin{bmatrix} 0 & I_N \\ I_N + \frac{\partial f(z_i)}{\partial x} & \frac{\partial f(z_i)}{\partial v} \end{bmatrix}, \frac{\partial f(z_i)}{\partial x} = diag_{i=1}^N[\frac{\partial f(z_i)}{\partial x_i}], \frac{\partial f(z_i)}{\partial v} = diag_{i=1}^N[\frac{\partial f(z_i)}{\partial v_i}].$

Then, choose a Lyapunov function as

$$V(t) = e(t)^T P e(t). \tag{15}$$

For $t \in [t_{k-1}, t_k)$, differentiating $V(t)$ with respect to time t,

$$\begin{aligned} \dot{V}(t) &= 2e(t)^T P\dot{e}(t) \\ &= 2e(t)^T P\bar{A}e(t) = e(t)^T(P\bar{A} + \bar{A}^T P)e(t). \end{aligned} \tag{16}$$

Based on (10), one has

$$\dot{V}(t) \le \alpha V(t), \tag{17}$$

which leads to

$$V(t) \le e^{\alpha(t-t_{k-1})} V(t_{k-1}), t \in [t_{k-1}, t_k). \tag{18}$$

On the another hand, when $t = t_k, k = 1, 2, \cdots, n,$

$$\begin{aligned} V(t_k) &= e(t_k)^T P e(t_k) \\ &= (e(t_k^-) - c[(L(m) + B(m)) \otimes I_N](e_p(t_k^-) + e_v(t_k^-)))^T P \\ &\quad (e(t_k^-) - c[(L(m) + B(m)) \otimes I_N](e_p(t_k^-) + e_v(t_k^-))) \\ &= e(t_k^-)^T (I_{2N \times 2N} - R(m))^T P e(t_k^-)(I_{2N \times 2N} - R(m)) \\ &\le \lambda_{max}((I_{2N \times 2N} - R(m))^T P(I_{2N \times 2N} - R(m))V(t_k) = \mu(t_k)V(t_k^-), \end{aligned} \tag{19}$$

where

$$R = \begin{bmatrix} -c[(L+B) \otimes I_N] & 0 \\ 0 & -c[(L+B) \otimes I_N] \end{bmatrix},$$

$$R(m) = \begin{bmatrix} -c[(L(m)+B(m)) \otimes I_N] & 0 \\ 0 & -c[(L(m)+B(m)) \otimes I_N] \end{bmatrix}.$$

Let $t \in [t_0, t_1)$, from (17),

$$V(t_1) \leq \mu(t_1)V(t_1^-) \leq \mu(t_1)e^{\alpha(t_1-t_0)}V(t_0). \tag{20}$$

For $t \in [t_1, t_2)$, from (17), (19) and (20)

$$\begin{aligned} V(t_2) &\leq \mu(t_2)V(t_2^-) \\ &\leq \mu(t_2)e^{\alpha(t_2-t_1)}V(t_1) \\ &\leq \mu(t_2)\mu(t_1)e^{\alpha(t_2-t_1)}e^{\alpha(t_1-t_0)}V(t_0) \\ &= \mu(t_2)\mu(t_1)e^{\alpha(t_2-t_0)}V(t_0). \end{aligned} \tag{21}$$

By induction, for $t \in [t_k, t_{k+1})$,

$$\begin{aligned} V(t) &\leq e^{\alpha(t-t_k)}V(t_k) \\ &\leq \prod_{m=1}^{k}\mu(t_m^-)e^{\alpha(t-t_0)}V(t_0). \end{aligned} \tag{22}$$

According to the Definitions 2 and 3, it is easy to get

$$\frac{N_a(t_0,t)}{N(t_0,t)}(\frac{t-t_0}{\phi} - N_0) \leq N_a(t_0,t) \leq \frac{N_a(t_0,t)}{N(t_0,t)}(\frac{t-t_0}{\phi} + N_0).$$

In a similar way, N_n and N_r is similar to N_a.

According to Assumption 2, the multi-agent system is not attacked ($N(t_0,t) = N_n(t_0,t) + N_r(t_0,t)$), one has

$$\begin{aligned} V(t) &< \mu_1^{N(t_0,t)}e^{\alpha(t-t_0)}V(t_0) \\ &< \mu_1^{-N_0}e^{(\frac{\ln \mu_1}{\phi}+\alpha)(t-t_0)}V(t_0). \end{aligned} \tag{23}$$

Based on trigger condition, for any $t \in [t_k, t_{k+1})$, we have

$$V(t) \leq \mu_1^k e^{ka+\xi(t_0-t_{k+1})}V(t_0). \tag{24}$$

During the second pulse control time, there are three possible event triggering conditions: Event triggering zero times, event triggering one time, and event triggering multiple times.
(1) Event triggering zero times: $V(t) \leq e^{a+\xi(t_0-t_1)}V(t_0)$.
(2) Event triggering one times: $V(t) \leq \mu_1 e^{2a+\xi(t_0-t_2)}V(t_0)$.
(3) Event triggering k times: $V(t) \leq \mu_1^k e^{ka+\xi(t_0-t_{k+1})}V(t_0)$.
In general, we have $V(t) \leq e^{k(\ln \mu_1+a)}e^{-\xi(t-t_0)}V(t_0)$, when $t \in [t_0, t]$, $k = 0, 1, 2, \cdots, n$

According to the condition,

$$V(t) \leq e^{k(\ln \mu_1+a)}e^{-\xi(t-t_0)}V(t_0) \leq \mu_1^{-N_0}e^{(\frac{\ln \mu_1}{\phi}+\alpha)(t-t_0)}V(t_0). \tag{25}$$

On the another hand, when there are attacks($N(t_0, t) = N_n(t_0, t) + N_a(t_0, t) + N_r(t_0, t)$).

$$
\begin{aligned}
V(t) &\leq \mu_1^{\frac{N_n(t_0,t)}{N(t_0,t)}(\frac{t-t_0}{\phi}-N_0)} \mu_m^{\frac{N_a(t_0,t)}{N(t_0,t)}(\frac{t-t_0}{\phi}-N_0)} \mu_1^{\frac{N_r(t_0,t)}{N(t_0,t)}(\frac{t-t_0}{\phi}-N_0)} e^{\alpha(t-t_0)} V(t_0) \\
&\leq \mu_1^{-N_0 \frac{N_n(t_0,t)+N_r(t_0,t)}{N(t_0,t)}} \mu_m^{-N_0 \frac{N_a(t_0,t)}{N(t_0,t)}} e^{(\frac{\ln\mu_1}{\phi} \frac{N_n(t_0,t)+N_r(t_0,t)}{N(t_0,t)} + \frac{\ln\mu_m}{\phi} \frac{N_a(t_0,t)}{N(t_0,t)} +\alpha)(t-t_0)} V(t_0) \\
&= \theta e^{-\varepsilon(t-t_0)} V(t_0),
\end{aligned} \tag{26}
$$

where $\theta = \mu_1^{-N_0 \frac{N_n(t_0,t)+N_r(t_0,t)}{N(t_0,t)}} \mu_m^{-N_0 \frac{N_a(t_0,t)}{N(t_0,t)}}$, $\varepsilon = -(\frac{\ln\mu_1}{\phi} \frac{N_n(t_0,t)+N_r(t_0,t)}{N(t_0,t)} + \frac{\ln\mu_m}{\phi} \frac{N_a(t_0,t)}{N(t_0,t)} +\alpha) = -(\frac{\ln\mu_1}{\phi}(1-\eta_a) + \frac{\ln\mu_m}{\phi}\eta_a + \alpha)$. According to the condition and Definition 1, the proof is completed.

Next, it is proved that there is no Zeno behavior. Assume that $t_1 < t_2 < ... < t_k$ are the trigger instants, and based on (26), it holds that

$$
V(t_1) = e^{a+\varepsilon(t_0-t_1)} V(t_0) \leq \mu^{-N_0} e^{(\frac{\ln\mu_1}{\phi}+\alpha)(t_1-t_0)} V(t_0),
$$

which implies that $t_1 - t_0 \geq \frac{\phi(a+N_0\ln\mu_1)}{\phi\alpha+\phi\varepsilon+\ln\mu_1}$. It means that event-triggered time interval $\left\{t_{k+1} - t_k\right\}_{k\in\mathbb{N}}$ are lower bounded by τ ($\tau = \frac{\phi(a+N_0\ln\mu_1)}{\phi\alpha+\phi\varepsilon+\ln\mu_1}$). The conditions in Theorem 1 prove that $\tau > 0$. Therefore, the system does not exhibit Zeno behavior.

Remark 1. We analyze the time upper bound and attack intensity that can sustain the DoS attack cycle of the system in Theorem 1. In this paper, an attack cycle is divided into three periods to be analyzed, and it is worth noting that the system will not be attacked during the adjustment period. In addition, different from the general event-triggered control strategy, in the control protocol proposed in this paper, the controller works only when the system state meets the trigger condition, and the controller does not perform pulse control in the rest time. This effectively reduces the number of information transmission and avoids the congestion of information transmission channels.

Remark 2. Compared with literature [36], literature [36] introduced an event trigger interval parameter and a mandatory immediate sequence. The event trigger mechanism in this paper only needs the system information of the previous event trigger time to determine the departure time of the next event, which makes the proposed algorithm easy to implement.

4. Simulation examples

Consider MASs with nonlinear dynamics (1), where the network topology is shown in Figure 1. 0 is the leader, and the rest are the followers. Control input matrix A, and the Laplace matrix of the switching topology are described as
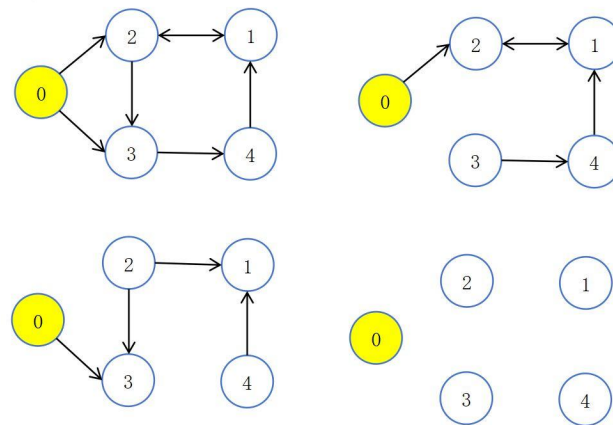
Figure 1. Graph $\mathcal{G}$ in the example.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, L_1 = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}, L_2 = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} 2 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, L_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Take the initial values of the leader as $X_0 = [0.2356\,0.5688\,0.6325]$, nonlinear function $f(x_{ij}(t)) = 0.5 \times (|x_{ij} + 1| - |x_{ij} - 1|)$, and other system parameters $c = 0.4$, $a = 0.05$, $\xi = 0.1$.

Figures 2 and 3 describe the position error and velocity error images. It is evident from these graphs that the system error gradually diminishes over time. Figure 4 describes the motion position trajectory of multi-agent i (0, 1, 2, 3, 4). It is easy to see that the followers' states converge towards those of the leader as time progresses. In addition, the system was attacked in 8–10 seconds, 15–18 seconds, 23–29 seconds. The network topology is changed due to a DoS attack. The trigger time of each agent is shown in Figure 5. In the initial 10 seconds, the system state error is large and the trigger is more frequent. When t=10s, the position of multiple agents gradually tends to be consistent, the trigger times decrease, and the event trigger pulse control does not trigger after reaching an agreement, which indicates that the event trigger pulse control can effectively solve the problem of limited bandwidth. Moreover, comparing the proposed event-triggered mechanism in this paper with the one in [35] under similar consensus performance, one can conclude from Table 1 that the communication burden can be reduced greatly by adopting the mechanism in this paper. The design of event triggering function in literature [35] is as follows

$$t_{k+1}^i = \inf \left\{ t > t_k^i | \, \|q_i(t)\|^2 - \rho \|e_i(t)\|^2 \geq 0 \right\},$$
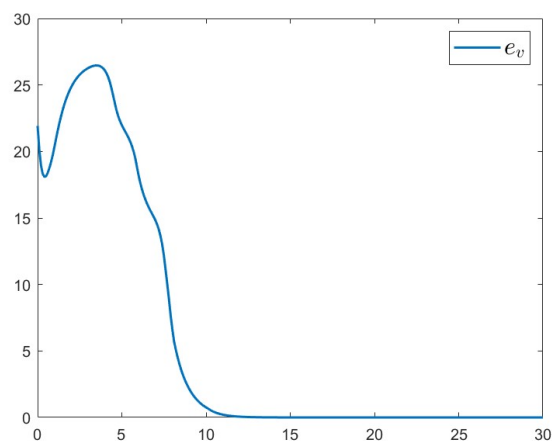
where $\rho = 1.2$.
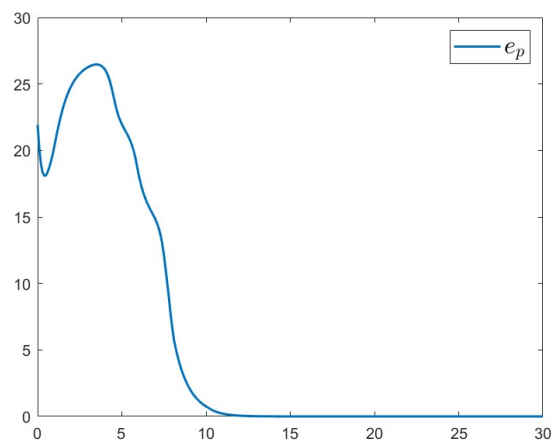
Figure 2. The Velocity errors' response curves.

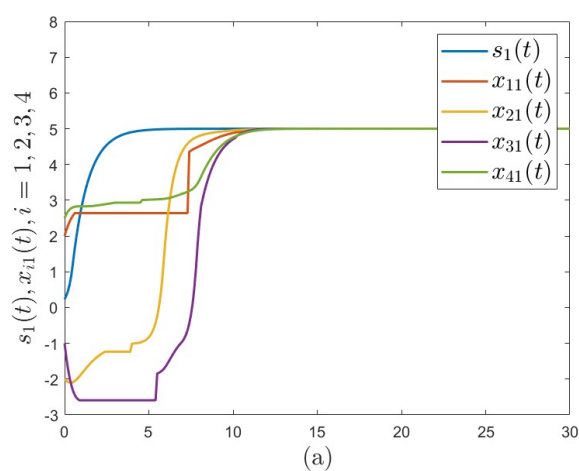
Figure 3. The position errors' response curves.
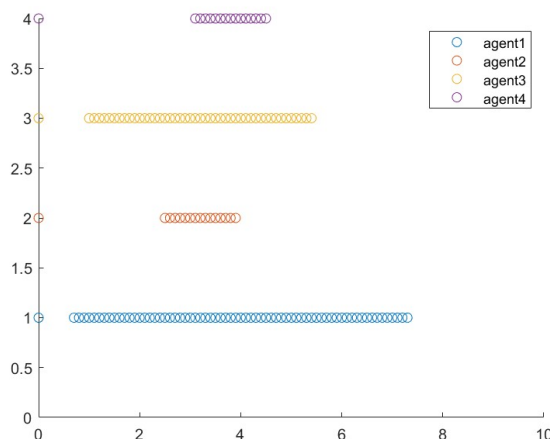

Figure 4. Position's response curves.

Figure 5. Event trigger time instant for all agents.

Table 1. Compared with the traditional triggering protocols in [0,10 s].

Agent i	1	2	3	4
[35]	288	295	294	297
Our ETC	67	15	45	15

5. Conclusions

We investigate event-triggered impulsive consensus in second-order nonlinear MASs under DoS attacks. By employing Lyapunov functions, matrix graph theory, and inequality techniques, the conditions for asymptotic convergence of the global synchronization error system are studied. The influence of DoS attacks is solved by the design of a appropriate event-triggered impulsive controller. An example simulation substantiates the efficiency of the control schema. Furthermore, in future research, we will consider other control strategies to alleviate the impact of diverse network attacks.

Use of AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The author declares no conflicts of interest in this paper.

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