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Research article

Almost periodic solutions for Clifford-valued stochastic shunting inhibitory cellular neural networks with mixed delays

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Abstract: We adopted a non decomposition method to study the existence and stability of Stepanov almost periodic solutions in the distribution sense of stochastic shunting inhibitory cellular neural networks (SICNNs) with mixed time delays. Due to the lack of linear structure in the set composed of Stepanov almost periodic stochastic processes in the distribution sense. Due to the lack of linear structure in the set composed of distributed Stepanov periodic stochastic processes, it poses difficulties for the existence of Stepanov almost periodic solutions in the distribution sense of SICNNs. To overcome this difficulty, we first proved that the network under consideration has a unique solution in a space composed of $\mathcal{L}^p$ bounded and $\mathcal{L}^p$ uniformly continuous stochastic processes. Then, using stochastic analysis, inequality techniques, and the definition of Stepanov almost periodic stochastic processes in the distribution sense, we proved that this solution is also a Stepanov almost periodic solution in the distribution sense. Moreover, the result of the global exponential stability of this almost periodic solution is given. It is worth noting that even if the network under consideration degenerated into a real-valued network, our results are novel. Finally, we provided a numerical example to validate our theoretical findings.

Keywords: stochastic neural networks; shunting inhibitory cellular; Clifford-valued neural network; almost periodic solution in the distribution sense; fixed point theorem
Mathematics Subject Classification: 34K50, 34K14, 92B20

1. Introduction

As stated in [1], a nervous system in the real world, synaptic transmission is a noisy process caused by random fluctuations in neurotransmitter release and other probabilistic factors. Therefore, it is necessary to consider stochastic neural networks (NNs) because random inputs may change the dynamics of the (NN) [2–5].

SICNNs, which were proposed in [6], have attracted the interest of many scholars since their introduction due to their special roles in psychophysics, robotics, adaptive pattern recognition, vision, and image processing. In the above applications, their dynamics play an important role. Thereupon, their various dynamics have been extensively studied (see [7–13] and references therein). However, there is limited research on the dynamics of stochastic SICNNs. Therefore, it is necessary to further study the dynamics of such NNs.

On the one hand, research on the dynamics of NNs that take values from a non commutative algebra, such as quaternion-valued NNs [14–16], octonion-valued NNs [17–20], and Clifford-valued NNs [21–23], has gained the interest of many researchers because such neural networks can include typical real-valued NNs as their special cases, and they have superior multi-dimensional signal processing and data storage capabilities compared to real-valued NNs. It is worth mentioning that in recent years, many authors have conducted extensive research on various dynamics of Clifford-valued NNs, such as the existence, multiplicity and stability of equilibrium points, and the existence, multiplicity and stability of almost periodic solutions as well as the synchronization problems [22–30]. However, most of the existing results for the dynamics of Clifford-valued NNs has been obtained through decomposition methods [24–27]. However, the results obtained by decomposition methods are generally not convenient for direct application, and there is little research on Clifford-valued NNs using non decomposition methods [28–30]. Therefore, further exploration of using non decomposition methods to study the dynamics of Clifford-valued NNs has important theoretical significance and application value.

On the other hand, Bohr's almost periodicity is a special case of Stepanov's almost periodicity, but there is little research on the Stepanov periodic oscillations of NNs [19, 31–33], especially the results of Stepanov's almost periodic solutions of stochastic SICNNs with discrete and infinitely distributed delays have not been published yet.

Motivated by the discussion above, our purpose of this article is to establish the existence and global exponential stability of Stepanov almost periodic solutions in the distribution sense for a stochastic Clifford-valued SICNN with mixed delays via non decomposition methods.

The subsequent sections of this article are organized as follows. Section 2 introduces some concepts, notations, and basic lemmas and gives a model description. Section 3 discusses the existence and stability of Stepanov almost periodic solutions in the distribution sense of the NN under consideration. An example is provided in Section 4. Finally, Section 5 provides a brief conclusion.

2. Preliminaries and model description

Let $\mathscr{A} = \{\sum_{\vartheta\in\mathbb{P}} x^{\vartheta}e_{\vartheta}, x^{\vartheta} \in \mathbb{R}\}$ be a real Clifford-algebra with N generators $e_{\emptyset} = e_0 = 1$, and $e_h, h = 1, 2, \cdots, N$, where $\mathbb{P} = \{\emptyset, 0, 1, 2, \cdots, \vartheta, \cdots, 12\cdots N\}$, $e_i^2 = 1, i = 1, 2, \cdots, r, e_i^2 = -1, i = r+1, r+2, \cdots, m, e_ie_j+e_je_i = 0, i \neq j$ and $i, j = 1, 2, \cdots, N$. For $x = \sum_{\vartheta\in\mathbb{P}} x^{\vartheta}e_{\vartheta} \in \mathscr{A}$, we indicate $\|x\|_\flat = \max_{\vartheta\in\mathbb{P}}\{|x^{\vartheta}|\}, x^c = \sum_{\vartheta\neq\emptyset} x^{\vartheta}e_{\vartheta}, x^0 = x - x^c$, and for $x = (x_{11}, x_{12}, \ldots, x_{1n}, x_{21}, x_{22}, \ldots, x_{2n}, \ldots, x_{mn})^T \in \mathscr{A}^{m\times n}$, we denote $\|x\|_0 = \max\{\|x_{ij}\|_\flat, 1 \leq i \leq m, 1 \leq j \leq n\}$. The derivative of $x(t) = \sum_{\vartheta\in\mathbb{P}} x^{\vartheta}(t)e_{\vartheta}$ is defined by $\dot{x}(t) = \sum_{\vartheta\in\mathbb{P}} \dot{x}^{\vartheta}(t)e_{\vartheta}$ and the integral of $x(t) = \sum_{\vartheta\in\mathbb{P}} x^{\vartheta}(t)e_{\vartheta}$ over the interval $[a, b]$ is defined by $\int_a^b x(t)dt = \sum_{\vartheta\in\mathbb{P}} \big(\int_a^b x^{\vartheta}(t)dt\big)e_{\vartheta}$.

Let $(\mathbb{Y}, \rho)$ be a separable metric space and $\mathcal{P}(\mathbb{Y})$ the collection of all probability measures defined on Borel σ-algebra of $\mathbb{Y}$. Denote by $C_b(\mathbb{Y})$ the set of continuous functions $f : \mathbb{Y} \to \mathbb{R}$ with $\|g\|_\infty := \sup_{x\in\mathbb{Y}}\{|g(x)|\} < \infty$.

For $g \in C_b(\mathbb{Y})$, $\mu, \nu \in \mathcal{P}(\mathbb{Y})$, let us define

$$\|g\|_L = \sup_{x \neq y} \frac{|g(x) - g(y)|}{\rho(x,y)}, \quad \|g\|_{BL} = \max\{\|g\|_\infty, \|g\|_L\},$$

$$\rho_{BL}(\mu, \nu) := \sup_{\|g\|_{BL} \leq 1} \left| \int_{\mathbb{Y}} g d(\mu - \nu) \right|.$$

According to [34], $(\mathbb{Y}, \rho_{BL}(\cdot, \cdot))$ is a Polish space.

Definition 2.1. *[35] A continuous function $g : \mathbb{R} \to \mathbb{Y}$ is called almost periodic if for every $\varepsilon > 0$, there is an $\ell(\varepsilon) > 0$ such that each interval with length ℓ has a point τ meeting*

$$\rho(g(t+\tau), g(t)) < \varepsilon, \quad \text{for all } t \in \mathbb{R}.$$

We indicate by $AP(\mathbb{R}, \mathbb{Y})$ the set of all such functions.

Let $(\mathbb{X}, \|\cdot\|)$ signify a separable Banach space. Denote by $\mu(X) := P \circ X^{-1}$ and $E(X)$ the distribution and the expectation of $X : (\Omega, \mathcal{F}, P) \to \mathbb{X}$, respectively.

Let $\mathcal{L}^p(\Omega, \mathbb{X})$ indicate the family of all $\mathbb{X}$-valued random variables satisfying $E(\|X\|^p) = \int_\Omega \|X\|^p dP < \infty$.

Definition 2.2. *[21] A process $Z : \mathbb{R} \to \mathcal{L}^p(\Omega, \mathbb{X})$ is called $\mathcal{L}^p$-continuous if for any $t_0 \in \mathbb{R}$,*

$$\lim_{t \to t_0} E\|Z(t) - Z(t_0)\|^p = 0.$$

It is $\mathcal{L}^p$-bounded if $\sup_{t \in \mathbb{R}} E\|Z(t)\|^p < \infty$.

For $1 < p < \infty$, we denote by $\mathcal{L}^p_{loc}(\mathbb{R}, \mathbb{X})$ the space of all functions from $\mathbb{R}$ to $\mathbb{X}$ which are locally p-integrable. For $g \in \mathcal{L}^p_{loc}(\mathbb{R}, \mathbb{X})$, we consider the following Stepanov norm:

$$\|g\|_{S^p} = \sup_{t \in \mathbb{R}} \left(\int_t^{t+1} \|g(s)\|^p ds \right)^{\frac{1}{p}}.$$

Definition 2.3. *[35] A function $g \in \mathcal{L}^p_{loc}(\mathbb{R}, \mathbb{X})$ is called p-th Stepanov almost periodic if for any $\varepsilon > 0$, it is possible to find a number $\ell > 0$ such that every interval with length ℓ has a number τ such that*

$$\|g(t+\tau) - g(t)\|_{S^p} < \varepsilon.$$

Definition 2.4. *[9] A stochastic process $Z \in \mathcal{L}^p_{loc}(\mathbb{R}, \mathcal{L}^p(\Omega, \mathbb{X}))$ is said to be S^p-bounded if*

$$\|Z\|_{S^p_s} := \sup_{t \in \mathbb{R}} \left(\int_t^{t+1} E\|Z(s)\|^p ds \right)^{\frac{1}{p}} < \infty.$$

Definition 2.5. *[9] A stochastic process $Z \in \mathcal{L}_{loc}(\mathbb{R}, \mathcal{L}^p(\Omega, \mathbb{H}))$ is called Stepanov almost periodic in p-th mean if for any $\varepsilon > 0$, it is possible to find a number $\ell > 0$ such that every interval with length ℓ has a number τ such that*

$$\|Z(t+\tau) - Z(t)\|_{S^p_s} < \varepsilon.$$

Definition 2.6. *[9] A stochastic process $Z : \mathbb{R} \to \mathcal{L}^p(\Omega, \mathbb{X}))$ is said to be p-th Stepanov almost periodic in the distribution sense if for each $\varepsilon > 0$, it is possible to find a number $\ell > 0$ such that any interval with length ℓ has a number τ such that*

$$\sup_{a\in\mathbb{R}}\Big(\int_a^{a+1} d_{BL}^p(P\circ[Z(t+\tau)]^{-1}, P\circ[Z(t)]^{-1})dt\Big)^{\frac{1}{p}} < \varepsilon.$$

Lemma 2.1. *[36] (Burkholder-Davis-Gundy inequality) If $f \in \mathcal{L}^2(J, \mathbb{R})$, $p > 2$, $B(t)$ is Brownian motion, then*

$$E\Big[\sup_{t\in J}\Big|\int_{t_0}^t f(s)dB(s)\Big|^p\Big] \leq C_p E\Big[\int_{t_0}^T |f(s)|^2 ds\Big]^{\frac{p}{2}},$$

where $c_p = \Big(\frac{p^{p+1}}{2(p-1)^{p-1}}\Big)^{\frac{p}{2}}$.

The model that we consider in this paper is the following stochastic Clifford-valued SICNN with mixed delays:

$$\begin{aligned} dx_{ij}(t) =& \Big[-a_{ij}(t)x_{ij}(t) + \sum_{C_{kl}\in N_{h_1}(i,j)} C_{ij}^{kl}(t)f(x_{kl}(t-\tau_{kl}(t)))x_{ij}(t) \\ &+ \sum_{C_{kl}\in N_{h_2}(i,j)} B_{ij}^{kl}(t)\int_0^\infty K_{ij}(u)g(x_{kl}(t-u))du x_{ij}(t) + L_{ij}(t)\Big]dt \\ &+ \sum_{C_{kl}\in N_{h_3}(i,j)} E_{ij}^{kl}(t)\delta_{ij}(x_{ij}(t-\sigma_{ij}(t)))d\omega_{ij}(t), \end{aligned} \tag{2.1}$$

where $i = 1, 2, \cdots, m, j = 1, 2, \cdots, n$, $C_{ij}(t)$ represents the cell at the (i, j) position, the h_1-neighborhood $N_{h_1}(i, j)$ of C_{ij} is given as:

$$N_{h_1}(i, j) = \{C_{kl} : \max(|k-i|, |l-j|) \leq h_1, \quad 1 \leq k \leq m, 1 \leq l \leq n\},$$

$N_{h_2}(i, j), N_{h_3}(i, j)$ are similarly defined, x_{ij} denotes the activity of the cell C_{ij}, $L_{ij}(t) : \mathbb{R} \to \mathscr{A}$ corresponds to the external input to C_{ij}, the function $a_{ij}(t) : \mathbb{R} \to \mathscr{A}$ represents the decay rate of the cell activity, $C_{ij}^{kl}(t) : \mathbb{R} \to \mathscr{A}, B_{ij}^{kl}(t) : \mathbb{R} \to \mathscr{A}$ and $E_{ij}^{kl}(t) : \mathbb{R} \to \mathscr{A}$ signify the connection or coupling strength of postsynaptic activity of the cell transmitted to the cell C_{ij}, and the activity functions $f(\cdot) : \mathscr{A} \to \mathscr{A}$, and $g(\cdot) : \mathscr{A} \to \mathscr{A}$ are continuous functions representing the output or firing rate of the cell C_{kl}, and $\tau_{kl}(t), \sigma_{ij}(t) : \mathbb{R} \to \mathbb{R}^+$ are the transmission delay, the kernel $K_{ij}(t) : \mathbb{R} \to \mathbb{R}$ is an integrable function, $\omega_{ij}(t)$ represents the Brownian motion defined on a complete probability space, $\delta_{ij}(\cdot) : \mathscr{A} \to \mathscr{A}$ is a Borel measurable function.

Let $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geqslant 0}, P)$ be a complete probability space in which $\{\mathscr{F}_t\}_{t\geqslant 0}$ is a natural filtration meeting the usual conditions. Denote by $B_{\mathscr{F}_0}([-\theta, 0], \mathscr{A}^n)$ the family of bounded, $\mathscr{F}_0$-measurable and $\mathscr{A}^n$-valued random variables from $[-\theta, 0] \to \mathscr{A}^n$. The initial values of system (2.1) are depicted as

$$x_i(s) = \phi_i(s), \quad s \in [-\theta, 0],$$

where $\phi_i \in B_{\mathscr{F}_0}([-\theta, 0], \mathscr{A}), \theta = \max_{1\leq i,j\leq n}\{\sup_{t\in\mathbb{R}} \tau_{ij}(t), \sup_{t\in\mathbb{R}} \sigma_{ij}(t)\}$.

For convenience, we introduce the following notations:

$$
\begin{aligned}
&\underline{a}^0=\min_{ij\in\Lambda}\underline{a}_{ij}^0=\min_{ij\in\Lambda}\inf_{t\in\mathbb{R}}a_{ij}^0(t),\ \ \bar{a}^0=\max_{ij\in\Lambda}\bar{a}_{ij}^0=\max_{ij\in\Lambda}\sup_{t\in\mathbb{R}}a_{ij}^0(t),\ \ C_{ij}^{kl^+}=\sup_{t\in\mathbb{R}}\|C_{ij}^{kl}(t)\|_\flat,\\
&\bar{a}^c=\max_{ij\in\Lambda}\bar{a}_{ij}^c=\max_{ij\in\Lambda}\sup_{t\in\mathbb{R}}\|a_{ij}^c(t)\|_\flat,\ \ B_{ij}^{kl^+}=\sup_{t\in\mathbb{R}}\|B_{ij}^{kl}(t)\|_\flat,\ \ E_{ij}^{kl^+}=\sup_{t\in\mathbb{R}}\|E_{ij}^{kl}(t)\|_\flat,\\
&K_{ij}^+=\sup_{t\in\mathbb{R}}K_{ij}(t),\ \ \tau_{kl}^+=\sup_{t\in\mathbb{R}}\tau_{kl}(t),\ \ \dot{\tau}_{kl}^+=\sup_{t\in\mathbb{R}}\dot{\tau}_{kl}(t),\sigma_{ij}^+=\sup_{t\in\mathbb{R}}\sigma_{ij}(t),\ \ \dot{\sigma}_{ij}^+=\sup_{t\in\mathbb{R}}\dot{\sigma}_{ij}(t),\\
&M_L=\max_{ij\in\Lambda}L_{ij}^+=\max_{ij\in\Lambda}\sup_{t\in\mathbb{R}}\|L_{ij}(t)\|_\flat,\ \ \theta=\max_{ij\in\Lambda}\{\tau_{ij}^+,\sigma_{ij}^+\},\ \ \Lambda=\{11,12,\cdots,1n,\cdots,mn\}.
\end{aligned}
$$

Throughout this paper, we make the following assumptions:

$(A1)$ For $ij\in\Lambda$, $f,g,\delta_{ij}\in C(\mathscr{A},\mathscr{A})$ satisfy the Lipschitz condition, and f,g are bounded, that is, there exist constants $L_f>0, L_g>0, L_{ij}^\delta>0, M_f>0, M_g>0$ such that for all $x,y\in\mathscr{A}$,

$$
\begin{aligned}
\|f(x)-f(y)\|_\flat\le L_f\|x-y\|_\flat,\ \ \|g(x)-g(y)\|_\flat\le L_g\|x-y\|_\flat,\\
\|\delta_{ij}(x)-\delta_{ij}(y)\|_\flat\le L_{ij}^\delta\|x-y\|_\flat,\|f(x)\|_\flat\le M_f,\ \ \|g(x)\|_\flat\le M_g;
\end{aligned}
$$

furthermore, $f(0)=g(0)=\delta_{ij}(0)=0$.

$(A2)$ For $ij\in\Lambda$, $a_{ij}^0\in AP(\mathbb{R},\mathbb{R}^+), a_{ij}^c\in AP(\mathbb{R},\mathscr{A}), \tau_{ij},\sigma_{ij}\in AP(\mathbb{R},\mathbb{R}^+)\cap C^1(\mathbb{R},\mathbb{R})$ satisfying $1-\dot{\tau}_{ij}^+, 1-\dot{\sigma}_{ij}^+>0$, $C_{ij}^{kl}, B_{ij}^{kl}, E_{ij}^{kl}\in AP(\mathbb{R},\mathscr{A})$, $L=(L_{11},L_{12},\cdots,L_{mn})\in\mathscr{L}_{loc}^p(\mathbb{R},L^p(\Omega,\mathscr{A}^{m\times n}))$ is almost periodic in the sense of Stepanov.

$(A3)$ For $p>2$, $\frac{1}{p}+\frac{1}{q}=1$,

$$
\begin{aligned}
0<r^1:=&\frac{8^p}{4}\max_{ij\in\Lambda}\bigg\{\Big(\frac{p}{q\underline{a}_{ij}^0}\Big)^{\frac{p}{q}}\frac{q}{p\underline{a}_{ij}^0}\Big[(\bar{a}_{ij}^c)^p+\Big(\sum_{C_{kl}\in N_{h_1}(i,j)}(C_{ij}^{kl^+})^q\Big)^{\frac{p}{q}}(2\kappa L_f+M_f)^p\\
&+\Big(\sum_{C_{kl}\in N_{h_2}(i,j)}(B_{ij}^{kl^+})^q\Big)^{\frac{p}{q}}\Big((2\kappa L_g+M_g)\int_0^\infty|K_{ij}(u)|du\Big)^p\Big]\\
&+C_p\Big(\frac{p-2}{2\underline{a}_{ij}^0}\Big)^{\frac{p-2}{2}}\frac{q}{p\underline{a}_{ij}^0}\Big(\sum_{C_{kl}\in N_{h_3}(i,j)}(E_{ij}^{kl^+})^q\Big)^{\frac{p}{q}}(L_{ij}^\delta)^p\bigg\}<1,
\end{aligned}
$$

and for $p=2$,

$$
\begin{aligned}
0<r^2:=16&\max_{ij\in\Lambda}\bigg\{\frac{1}{(\underline{a}_{ij}^0)^2}\Big[(\bar{a}_{ij}^c)^2+\sum_{C_{kl}\in N_{h_1}(i,j)}(C_{ij}^{kl^+})^2(2\kappa L_f+M_f)^2+\sum_{C_{kl}\in N_{h_2}(i,j)}(B_{ij}^{kl^+})^2\\
&\times\Big((2\kappa L_g+M_g)\int_0^\infty|K_{ij}(u)|du\Big)^2\Big]+\frac{1}{2\underline{a}_{ij}^0}\sum_{C_{kl}\in N_{h_3}(i,j)}(E_{ij}^{kl^+})^2(L_{ij}^\delta)^2\bigg\}<1.
\end{aligned}
$$

$(A4)$ For $\frac{1}{p}+\frac{1}{q}=1$,

$$
0<\frac{q}{p\underline{a}^0}\rho^1:=16^{p-1}\frac{q}{p\underline{a}^0}\max_{ij\in\Lambda}\bigg\{\Big(\frac{p}{q\underline{a}_{ij}^0}\Big)^{\frac{p}{q}}\Big[(\bar{a}_{ij}^c)^p+\Big(\sum_{C_{kl}\in N_{h_1}(i,j)}(C_{ij}^{kl^+})^q\Big)^{\frac{p}{q}}\Big[2^{p-1}(L_f)^p
$$

$$\times \sum_{C_{kl}\in N_{h_1}(i,j)} \frac{e^{\frac{p}{q}\underline{a}_{ij}^0\tau_{kl}^+}(2\kappa)^p}{1-\dot{\tau}_{kl}^+} + (M_f)^p\Big] + \Big(\sum_{C_{kl}\in N_{h_2}(i,j)} (B_{ij}^{kl^+})^q\Big)^{\frac{p}{q}}\Big[\Big(2\kappa L_g$$

$$\times \int_0^\infty |K_{ij}(u)|du\Big)^p + \Big(M_g\int_0^\infty |K_{ij}(u)|du\Big)^p\Big]\Big] + 2^{p-1}C_p(\frac{p-2}{2\underline{a}_{ij}^0})^{\frac{p-2}{2}}$$

$$\times\Big(\sum_{C_{kl}\in N_{h_3}(i,j)} (E_{ij}^{kl^+})^q\Big)^{\frac{p}{q}}(L_{ij}^\delta)^p\frac{e^{\frac{p}{q}\underline{a}_{ij}^0\sigma_{ij}^+}}{1-\dot{\sigma}_{ij}^+}\Big\} < 1,\ (p>2),$$

$$0<\frac{\rho^2}{\underline{a}^0} := \frac{32}{\underline{a}^0}\max_{ij\in\Lambda}\Big\{\Big(\frac{1}{\underline{a}_{ij}^0}\Big)\sum_{C_{kl}\in N_{h_1}(i,j)}(C_{ij}^{kl^+})^2\Big[(L_f)^2\sum_{C_{kl}\in N_{h_1}(i,j)}\frac{e^{\underline{a}_{ij}^0\tau_{kl}^+}(2\kappa)^2}{1-\dot{\tau}_{kl}^+}+\frac{(M_f)^2}{2}\Big]$$

$$+\sum_{C_{kl}\in N_{h_3}(i,j)}(E_{ij}^{kl^+})^2(L_{ij}^\delta)^2\frac{e^{2\underline{a}_{ij}^0\sigma_{ij}^+}}{1-\dot{\sigma}_{ij}^+}+\frac{1}{2\underline{a}_{ij}^0}\sum_{C_{kl}\in N_{h_2}(i,j)}(B_{ij}^{kl^+})^2(4\kappa^2L_g^2+M_g^2)$$

$$\times\Big(\int_0^\infty |K_{ij}(u)|du\Big)^2+\frac{(\bar{a}_{ij}^c)^2}{2\underline{a}_{ij}^0}\Big\} < 1,\ (p=2).$$

($A5$) The kernel K_{ij} is almost periodic and there exist constants $M>0$ and $u>0$ such that $|K_{ij}(t)|\le Me^{-ut}$ for all $t\in\mathbb{R}$.

3. Main results

Let $\mathbb{X}$ indicate the space of all $\mathcal{L}^p$-bounded and $\mathcal{L}^p$-uniformly continuous stochastic processes from $\mathbb{R}$ to $\mathcal{L}^p(\Omega,\mathscr{A}^{m\times n})$, then with the norm $\|\phi\|_{\mathbb{X}}=\sup_{t\in\mathbb{R}}\{E\|\phi(t)\|_0^p\}^{\frac{1}{p}}$, where $\phi=(\phi_{11},\phi_{12},\dots,\phi_{mn})\in\mathbb{X}$, it is a Banach space.

Set $\phi^0=(\phi_{11}^0,\phi_{12}^0,\dots,\phi_{mn}^0)^T$, where $\phi_{ij}^0(t)=\int_{-\infty}^t e^{-\int_s^t a_{ij}^0(u)du}L_{ij}(s)ds, t\in\mathbb{R}, ij\in\Lambda$. Then, ϕ^0 is well defined under assumption ($A2$). Consequently, we can take a constant κ such that $\kappa\ge\|\phi^0\|_{\mathbb{X}}$.

Definition 3.1. *[37] An $\mathscr{F}_t$-progressively measurable stochastic process $x(t)=(x_{11}(t),x_{12}(t),\dots,x_{mn}(t))^T$ is called a solution of system* (2.1)*, if $x(t)$ solves the following integral equation:*

$$\begin{aligned}x_{ij}(t)=&x_{ij}(t_0)e^{-\int_{t_0}^t a_{ij}^0(u)du}+\int_{t_0}^t e^{-\int_s^t a_{ij}^0(u)du}\Big[-a_{ij}^c(s)x_{ij}(s)+\sum_{C_{kl}\in N_{h_1}(i,j)}C_{ij}^{kl}(s)\\&\times f(x_{kl}(s-\tau_{kl}(s)))x_{ij}(s)+\sum_{C_{kl}\in N_{h_2}(i,j)}B_{ij}^{kl}(s)\int_0^\infty K_{ij}(u)g(x(s-u))dux_{ij}(s)\\&+L_{ij}(s)\Big]ds+\int_{t_0}^t e^{-\int_s^t a_{ij}^0(u)du}\sum_{C_{kl}\in N_{h_3}(i,j)}E_{ij}^{kl}(s)\delta_{ij}(x_{ij}(s-\sigma_{ij}(s)))dw_{ij}(s).\end{aligned}\tag{3.1}$$

In (3.1), let $t_0\to-\infty$, then one gets

$$x_{ij}(t)=\int_{-\infty}^t e^{-\int_s^t a_{ij}^0(u)du}\Big[-a_{ij}^c(s)x_{ij}(s)+\sum_{C_{kl}\in N_{h_1}(i,j)}C_{ij}^{kl}(s)f(x_{kl}(s-\tau_{kl}(s)))x_{ij}(s)$$

$$+\sum_{C_{kl}\in N_{h_2}(i,j)} B_{ij}^{kl}(s)\int_0^\infty K_{ij}(u)g(x(s-u))dux_{ij}(s)+L_{ij}(s)\Big]ds+\int_{-\infty}^t e^{-\int_s^t a_{ij}^0(u)du}$$
$$\times\sum_{C_{kl}\in N_{h_3}(i,j)} E_{ij}^{kl}(s)\delta_{ij}(x_{ij}(s-\sigma_{ij}(s)))dw_{ij}(s), t\geq t_0, ij\in\Lambda. \tag{3.2}$$

It is easy to see that if $x(t)$ solves (3.2), then it also solves (2.1).

Theorem 3.1. *Assume that* (A1)–(A4) *hold. Then the system* (2.1) *has a unique $\mathcal{L}^p$-bounded and $\mathcal{L}^p$-uniformly continuous solution in $\mathbb{X}^* = \{\phi\in\mathbb{X} : \|\phi-\phi^0\|_{\mathbb{X}}\leq\kappa\}$, where κ is a constant satisfying $\kappa\geq\|\phi^0\|_{\mathbb{X}}$.*

Proof. Define an operator $\phi:\mathbb{X}^*\to\mathbb{X}$ as follows:

$$(\Psi\phi)(t) = ((\Psi_{11}\phi)(t),(\Psi_{12}\phi)(t),\ldots,(\Psi_{mn}\phi)(t))^T,$$

where $(\phi_{11},\phi_{12},\ldots,\phi_{mn})^T\in\mathbb{X}$, $t\in\mathbb{R}$ and

$$(\Psi_{ij}\phi)(t)=\int_{-\infty}^t e^{-\int_s^t a_{ij}^0(u)du}\Big[-a_{ij}^c(s)\phi_{ij}(s)+\sum_{C_{kl}\in N_{h_1}(i,j)} C_{ij}^{kl}(s)f(\phi_{kl}(s-\tau_{kl}(s)))\phi_{ij}(s)$$
$$+\sum_{C_{kl}\in N_{h_2}(i,j)} B_{ij}^{kl}(s)\int_0^\infty K_{ij}(u)g(\phi_{kl}(s-u))du\phi_{ij}(s)+L_{ij}(s)\Big]ds$$
$$+\int_{-\infty}^t e^{-\int_s^t a_{ij}^0(u)du}\sum_{C_{kl}\in N_{h_3}(i,j)} E_{ij}^{kl}(s)\delta_{ij}(\phi_{ij}(s-\sigma_{ij}(s)))d\omega_{ij}(s), ij\in\Lambda. \tag{3.3}$$

First of all, let us show that $E\|\Psi\phi(t)-\phi^0(t)\|_0^p\leq\kappa$ for all $\phi\in\mathbb{X}^*$.
Noticing that for any $\phi\in\mathbb{X}^*$, it holds

$$\|\phi\|_{\mathbb{X}}\leq\|\phi^0\|_{\mathbb{X}}+\|\phi-\phi^0\|_{\mathbb{X}}\leq 2\kappa.$$

Then, we deduce that

$$\begin{aligned}
&E\|\Psi\phi(t)-\phi^0(t)\|_0^p\\
\leq&4^{p-1}\max_{ij\in\Lambda}\Big\{E\Big\|\int_{-\infty}^t -e^{-\int_s^t a_{ij}^0(u)du}a_{ij}^c(s)\phi_{ij}(s)\Big\|_\flat^p\Big\}+4^{p-1}\max_{ij\in\Lambda}\Big\{E\Big\|\int_{-\infty}^t e^{-\int_s^t a_{ij}^0(u)du}\\
&\times\sum_{C_{kl}\in N_{h_1}(i,j)} C_{ij}^{kl}(s)f(\phi_{kl}(s-\tau_{kl}(s)))\phi_{ij}(s)ds\Big\|_\flat^p\Big\}+4^{p-1}\max_{ij\in\Lambda}\Big\{E\Big\|\int_{-\infty}^t e^{-\int_s^t a_{ij}^0(u)du}\\
&\times\sum_{C_{kl}\in N_{h_2}(i,j)} B_{ij}^{kl}(s)\int_0^\infty K_{ij}(u)g(\phi_{kl}(s-u))du\phi_{ij}(s)ds\Big\|_\flat^p\Big\}\\
&+4^{p-1}\max_{ij\in\Lambda}\Big\{E\Big\|\int_{-\infty}^t e^{-\int_s^t a_{ij}^0(u)du}\sum_{C_{kl}\in N_{h_3}(i,j)} E_{ij}^{kl}(s)\delta_{ij}(\phi_{ij}(s-\sigma_{ij}(s)))d\omega_{ij}(s)\Big\|_\flat^p\Big\}\\
:=&F_1+F_2+F_3+F_4.
\end{aligned}\tag{3.4}$$

By the Hölder inequality, we have

$$\begin{aligned}F_2 \leq & 4^{p-1}\max_{ij\in\Lambda}\Big\{E\Big\|\Big[\int_{-\infty}^{t} e^{-\frac{q}{p}\int_s^t a_{ij}^0(u)du}ds\Big]^{\frac{p}{q}}\Big[\int_{-\infty}^{t} e^{-\frac{p}{q}\int_s^t a_{ij}^0(u)du}\\ &\times\Big(\sum_{C_{kl}\in N_{h_1}(i,j)} C_{ij}^{kl}(s)f(\phi_{kl}(s-\tau_{kl}(s)))\phi_{ij}(s)\Big)^p ds\Big]\Big\|_{\flat}\Big\}\\ \leq & 4^{p-1}\max_{ij\in\Lambda}\Big\{\Big(\frac{p}{q\underline{a}_{ij}^0}\Big)^{\frac{p}{q}}E\Big[\int_{-\infty}^{t} e^{-\frac{p}{q}\int_s^t a_{ij}^0(u)du}\Big(\sum_{C_{kl}\in N_{h_1}(i,j)}(\|C_{ij}^{kl}(s)\|_{\flat})^q\Big)^{\frac{p}{q}}\\ &\times\sum_{ij\in\Lambda}(2\kappa L_f)^p\|\phi_{ij}(s)\|_{\flat}^p ds\Big]\Big\}\\ \leq & 4^{p-1}\max_{ij\in\Lambda}\Big\{\Big(\frac{p}{q\underline{a}_{ij}^0}\Big)^{\frac{p}{q}}\frac{q}{p\underline{a}_{ij}^0}\Big(\sum_{C_{kl}\in N_{h_1}(i,j)}(C_{ij}^{kl^+})^q\Big)^{\frac{p}{q}}(2\kappa L_f)^p\Big\}\|\phi\|_{\mathbb{X}}^p. \end{aligned} \tag{3.5}$$

Similarly, one has

$$F_1 \leq 4^{p-1}\max_{ij\in\Lambda}\Big\{\Big(\frac{p}{q\underline{a}_{ij}^0}\Big)^{\frac{p}{q}}\frac{q}{p\underline{a}_{ij}^0}(\bar{a}_{ij}^c)^p\Big\}\|\phi\|_{\mathbb{X}}^p, \tag{3.6}$$

$$F_3 \leq 4^{p-1}\max_{ij\in\Lambda}\Big\{\Big(\frac{p}{q\underline{a}_{ij}^0}\Big)^{\frac{p}{q}}\frac{q}{p\underline{a}_{ij}^0}\Big(\sum_{C_{kl}\in N_{h_2}(i,j)}(B_{ij}^{kl^+})^q\Big)^{\frac{p}{q}}\Big(2\kappa L_g\int_0^{\infty}|K_{ij}(u)|du\Big)^p\Big\}\|\phi\|_{\mathbb{X}}^p. \tag{3.7}$$

By the Burkolder-Davis-Gundy inequality and the Hölder inequality, when $p > 2$, we infer that

$$\begin{aligned}F_4 \leq & 4^{p-1}C_p\max_{ij\in\Lambda}\Big\{E\Big[\int_{-\infty}^{t}\Big\|e^{-\int_s^t a_{ij}^0(u)du}\sum_{C_{kl}\in N_{h_3}(i,j)}E_{ij}^{kl}(s)\delta_{ij}(\phi_{ij}(s-\sigma_{ij}(s)))\Big\|_{\flat}^2 ds\Big]^{\frac{p}{2}}\Big\}\\ \leq & 4^{p-1}C_p\max_{ij\in\Lambda}\Big\{E\Big[e^{-2\int_s^t a_{ij}^0(u)du}\Big\|\sum_{C_{kl}\in N_{h_3}(i,j)}E_{ij}^{kl}\delta_{ij}(\phi_{ij}(s-\sigma_{ij}(s)))\Big\|_{\flat}^2 ds\Big]^{\frac{p}{2}}\Big\}\\ \leq & 4^{p-1}C_p\max_{ij\in\Lambda}\Big\{E\Big[\int_{-\infty}^{t}(e^{-2\int_s^t a_{ij}^0(u)du})^{\frac{p}{p-2}\times\frac{1}{p}}ds\Big]^{\frac{p-2}{p}\times\frac{p}{2}}\\ &\times E\Big[\int_{-\infty}^{t}(e^{-2\int_s^t a_{ij}^0(u)du})^{\frac{1}{q}\times\frac{p}{2}}\Big(\Big\|\sum_{C_{kl}\in N_{h_3}(i,j)}E_{ij}^{kl}(s)\delta_{ij}\phi_{ij}(s-\sigma_{ij}(s))\Big\|_{\flat}^2\Big)^{\frac{p}{2}}ds\Big]\Big\}\\ \leq & 4^{p-1}C_p\max_{ij\in\Lambda}\Big\{\Big(\frac{p-2}{2\underline{a}_{ij}^0}\Big)^{\frac{p-2}{2}}\frac{q}{p\underline{a}_{ij}^0}E\Big\|\sum_{C_{kl}\in N_{h_3}(i,j)}E_{ij}^{kl}(s)\delta_{ij}(\phi_{ij}(s-\sigma_{ij}(s)))\Big\|_{\flat}^p\Big\}\\ \leq & 4^{p-1}C_p\max_{ij\in\Lambda}\Big\{\Big(\frac{p-2}{2\underline{a}_{ij}^0}\Big)^{\frac{p-2}{2}}\frac{q}{p\underline{a}_{ij}^0}\Big(\sum_{C_{kl}\in N_{h_3}(i,j)}(E_{ij}^{kl^+})^q\Big)^{\frac{p}{q}}(L_{ij}^{\delta})^p\Big\}\|\phi\|_{\mathbb{X}}^p. \end{aligned} \tag{3.8}$$

When $p = 2$, by the Itô isometry, it follows that

$$F_4 \leq 4\max_{ij\in\Lambda}\Big\{E\Big[\int_{-\infty}^{t} e^{-2\int_s^t a_{ij}^0(u)du}\Big\|\sum_{C_{kl}\in N_{h_3}(i,j)}E_{ij}^{kl}(s)\delta_{ij}(\phi_{ij}(s-\sigma_{ij}(s)))\Big\|_{\mathscr{A}}^2 ds\Big]\Big\}$$

$$\leq 4\max_{ij\in\Lambda}\left\{\frac{1}{2\underline{a}_{ij}^0}\sum_{C_{kl}\in N_{h_3}(i,j)}(E_{ij}^{kl^+})^2(L_{ij}^{\delta})^2\right\}\|\phi\|_{\mathbb{X}}^2. \tag{3.9}$$

Putting (3.5)–(3.9) into (3.4), we obtain that

$$\begin{aligned}\|\Psi\phi-\phi^0\|_{\mathbb{X}}^p \leq &4^{p-1}\max_{ij\in\Lambda}\left\{\left(\frac{p}{q\underline{a}_{ij}^0}\right)^{\frac{p}{q}}\frac{q}{p\underline{a}_{ij}^0}\left[(\bar{a}_{ij}^c)^p+\left(\sum_{C_{kl}\in N_{h_1}(i,j)}(C_{ij}^{kl^+})^q\right)^{\frac{p}{q}}(2\kappa L_f)^p\right.\right.\\&\left.+\left(\sum_{C_{kl}\in N_{h_2}(i,j)}(B_{ij}^{kl^+})^q\right)^{\frac{p}{q}}\left(2\kappa L_g\int_0^\infty|K_{ij}(u)|du\right)^p\right]\\&\left.+C_p\left(\frac{p-2}{2\underline{a}_{ij}^0}\right)^{\frac{p-2}{2}}\frac{q}{p\underline{a}_{ij}^0}\left(\sum_{C_{kl}\in N_{h_3}(i,j)}(E_{ij}^{kl^+})^q\right)^{\frac{p}{q}}(L_{ij}^{\delta})^p\right\}\|\phi\|_{\mathbb{X}}^p\leq\kappa^p,\ (p>2),\end{aligned} \tag{3.10}$$

and

$$\begin{aligned}\|\Psi\phi-\phi^0\|_{\mathbb{X}}^2 \leq &4\max_{ij\in\Lambda}\left\{\frac{1}{(a_{ij}^-)^2}\left[(\bar{a}_{ij}^c)^2+\sum_{C_{kl}\in N_{h_1}(i,j)}(C_{ij}^{kl^+})^2(2\kappa L_f)^2+\sum_{C_{kl}\in N_{h_2}(i,j)}(B_{ij}^{kl^+})^2\left(2\kappa L_g\right.\right.\right.\\&\left.\left.\left.\times\int_0^\infty|K_{ij}(u)|du\right)^2\right]+\frac{1}{2\underline{a}_{ij}^0}\sum_{C_{kl}\in N_{h_3}(i,j)}(E_{ij}^{kl^+})^2(L_{ij}^{\delta})^2\right\}\|\phi\|_{\mathbb{X}}^2\leq\kappa^2,\ (p=2).\end{aligned} \tag{3.11}$$

It follows from (3.10), (3.11) and $(A3)$ that $\|\Psi\phi-\phi^0\|_{\mathbb{X}}\leq\kappa$.

Then, using the same method as that in the proof of Theorem 3.2 in [21], we can show that $\Psi\phi$ is $\mathcal{L}^p$-uniformly continuous. Therefore, we have $\Psi(\mathbb{X}^*)\subset\mathbb{X}^*$.

Last, we will show that Ψ is a contraction mapping. Indeed, for any $\psi,\varphi\in\mathbb{X}^*$, when $p>2$, we have

$$\begin{aligned}&E\|(\Phi\varphi)(t)-(\Phi\psi)(t)\|_0^p\\\leq&4^{p-1}\max_{ij\in\Lambda}\left\{E\left\|\int_{-\infty}^t e^{-\int_s^t a_{ij}^0(u)du}(-a_{ij}^c(s)\varphi_{ij}(s)+a_{ij}^c(s)\psi_{ij}(s))ds\right\|_{\flat}^p\right\}\\&+4^{p-1}\max_{ij\in\Lambda}\left\{E\left\|\int_{-\infty}^t e^{-\int_s^t a_{ij}^0(u)du}\sum_{C_{kl}\in N_{h_1}(i,j)}C_{ij}^{kl}(s)\Big[f(\varphi_{kl}(s-\tau_{kl}(s)))\varphi_{ij}(s)\right.\right.\\&\left.\left.-f(\psi_{kl}(s-\tau_{kl}(s)))\psi_{ij}(s)\Big]ds\right\|_{\flat}^p\right\}+4^{p-1}\max_{ij\in\Lambda}\left\{E\left\|\int_{-\infty}^t e^{-\int_s^t a_{ij}^0(u)du}\sum_{C_{kl}\in N_{h_2}(i,j)}B_{ij}^{kl}(s)\right.\right.\\&\left.\left.\times\Big[\int_0^\infty K_{ij}(u)g(\varphi_{kl}(s-u))du\varphi_{ij}(s)-\int_0^\infty K_{ij}(u)g(\psi_{kl}(s-u))du\psi_{ij}(u)\Big]ds\right\|_{\flat}^p\right\}\\&+4^{p-1}\max_{ij\in\Lambda}\left\{E\left\|\int_{-\infty}^t e^{-\int_s^t a_{ij}^0(u)du}\sum_{C_{kl}\in N_{h_3}(i,j)}E_{ij}^{kl}(s)\Big[\delta_{ij}(\varphi_{ij}(s-\sigma_{ij}(s)))\right.\right.\\&\left.\left.-\delta_{ij}(\psi_{ij}(s-\sigma_{ij}(s)))\Big]d\omega_{ij}(s)\right\|_{\flat}^p\right\}\\\leq&4^{p-1}\max_{ij\in\Lambda}\left\{\left(\frac{p}{q\underline{a}_{ij}^0}\right)^{\frac{p}{q}}\frac{q}{p\underline{a}_{ij}^0}\left[(\bar{a}_{ij}^c)^p+\left(\sum_{C_{kl}\in N_{h_1}(i,j)}(C_{ij}^{kl^+})^q\right)^{\frac{p}{q}}(2\kappa L_f+M_f)^p\right.\right.\end{aligned}$$

$$+\Big(\sum_{C_{kl}\in N_{h_2}(i,j)}(B_{ij}^{kl^+})^q\Big)^{\frac{p}{q}}\Big((2\kappa L_g+M_g)\int_0^\infty|K_{ij}(u)|du\Big)^p\Big]+C_p\Big(\frac{p-2}{2\underline{a}_{ij}^0}\Big)^{\frac{p-2}{2}}\frac{q}{p\underline{a}_{ij}^0}$$
$$\times\Big(\sum_{C_{kl}\in N_{h_3}(i,j)}(E_{ij}^{kl^+})^q\Big)^{\frac{p}{q}}(L_{ij}^\delta)^p\Big\}\|\varphi-\psi\|_{\mathbb{X}}^p. \tag{3.12}$$

Similarly, for $p=2$, we can get

$$E\|(\Phi\varphi)(t)-(\Phi\psi)(t)\|_0^2$$
$$\leq 4\max_{ij\in\Lambda}\Big\{\frac{1}{(\underline{a}_{ij}^0)^2}\Big[(\bar{a}_{ij}^c)^2+\sum_{C_{kl}\in N_{h_1}(i,j)}(C_{ij}^{kl^+})^2(2\kappa L_f+M_f)^2+\sum_{C_{kl}\in N_{h_2}(i,j)}(B_{ij}^{kl^+})^2$$
$$\times\Big((2\kappa L_g+M_g)\int_0^\infty|K_{ij}(u)|du\Big)^2\Big]+\frac{1}{2\underline{a}_{ij}^0}\sum_{C_{kl}\in N_{h_3}(i,j)}(E_{ij}^{kl^+})^2(L_{ij}^\delta)^2\Big\}\|\varphi-\psi\|_{\mathbb{X}}^2. \tag{3.13}$$

From (3.12) and (3.13) it follows that

$$\|(\Phi\varphi)(t)-(\Phi\psi)(t)\|_{\mathbb{X}}\leq\sqrt[p]{r^1}\|\varphi-\psi\|_{\mathbb{X}},(p>2),$$
$$\|(\Phi\varphi)(t)-(\Phi\psi)(t)\|_{\mathbb{X}}\leq\sqrt{r^2}\|\varphi-\psi\|_{\mathbb{X}},(p=2).$$

Hence, by virtue of $(A3)$, Ψ is a contraction mapping. So, Ψ has a unique fixed point x in $\mathbb{X}^*$, i.e., (2.1) has a unique solution x in $\mathbb{X}^*$. □

Theorem 3.2. *Assume that* $(A1)$–$(A5)$ *hold. Then the system* (2.1) *has a unique p-th Stepanov-like almost periodic solution in the distribution sense in* $\mathbb{X}^*=\{\phi\in\mathbb{X}:\|\phi-\phi^0\|_{\mathbb{X}}\leq\kappa\}$, *where* κ *is a constant satisfying* $\kappa\geq\|\phi^0\|_{\mathbb{X}}$.

Proof. From Theorem 3.1, we know that (2.1) has a unique solution x in $\mathbb{X}^*$. Now, let us show that x is Stepanov-like almost periodic in distribution. Since $x\in\mathbb{X}^*$, it is $\mathcal{L}^p$-uniformly continuous and satisfies $\|x\|\leq 2\kappa$. So, for any $\varepsilon>0$, there exists $\delta\in(0,\varepsilon)$, when $|h|<\delta$, we have $\sup_{t\in\mathbb{R}}E\|x(t+h)-x(t)\|_0^p<\varepsilon$. Hence, we derive that

$$\sup_{\xi\in\mathbb{R}}\int_\xi^{\xi+1}E\|x(t+h)-x(t)\|_0^p dt<\varepsilon. \tag{3.14}$$

For the δ above, according to $(A2)$, we have, for $ij\in\Lambda$,

$$|a_{ij}^0(t+\tau)-a_{ij}^0(t)|<\delta,\ \ \|a_{ij}^c(t+\tau)-a_{ij}^c(t)\|_\flat^p<\delta,\ \ \|C_{ij}^{kl}(t+\tau)-C_{ij}^{kl}(t)\|_\flat^p<\delta,$$
$$|\tau_{ij}(t+\tau)-\tau_{ij}(t)|<\delta,\ \ \|B_{ij}^{kl}(t+\tau)-B_{ij}^{kl}(t)\|_\flat^p<\delta,\ \ \|E_{ij}^{kl}(t+\tau)-E_{ij}^{kl}(t)\|_\flat^p<\delta,$$
$$|\sigma_{ij}(t+\tau)-\sigma_{ij}(t)|<\delta,\ \ \sup_{\xi\in\mathbb{R}}\int_\xi^{\xi+1}\|L_{ij}(t+\tau)-L_{ij}(t)\|_\flat^p dt<\delta.$$

As $|\tau_{ij}(t+\tau)-\tau_{ij}(t)|<\delta$, by (3.14), there holds

$$\sup_{\xi\in\mathbb{R}}\int_\xi^{\xi+1}E\|x(s-\tau_{ij}(s+\tau))-x(s-\tau_{ij}(s))\|_0^p ds<\varepsilon.$$

Based on (3.2), we can infer that

$$x_{ij}(t+\tau)=\int_{-\infty}^{t}e^{-\int_s^t a_{ij}^0(u+\tau)du}\Big[-a_{ij}^c(s+\tau)x_{ij}(s+\tau)+\sum_{C_{kl}\in N_{h_1}(i,j)}C_{ij}^{kl}(s+\tau)$$
$$\times f(x_{kl}(s+\tau-\tau_{kl}(s+\tau)))x_{ij}(s+\tau)+\sum_{C_{kl}\in N_{h_2}(i,j)}B_{ij}^{kl}(s+\tau)\int_0^\infty K_{ij}(u)$$
$$\times g(x_{kl}(s+\tau-u))dux_{ij}(s+\tau)+L_{ij}(s+\tau)\Big]ds+\int_{-\infty}^{t}e^{-\int_s^t a_{ij}(u+\tau)du}$$
$$\times\sum_{C_{kl}\in N_{h_3}(i,j)}E_{ij}^{kl}(s+\tau)\delta_{ij}(x_{ij}(s+\tau-\sigma_{ij}(s+\tau)))d[\omega_{ij}(s+\tau)-\omega_{ij}(\tau)],$$

in which $ij\in\Lambda,\omega_{ij}(s+\tau)-\omega_{ij}(\tau)$ is a Brownian motion having the same distribution as $\omega_{ij}(s)$.

Let us consider the process

$$x_{ij}(t+\tau)=\int_{-\infty}^{t}e^{-\int_s^t a_{ij}^0(u+\tau)du}\Big[-a_{ij}^c(s+\tau)x_{ij}(s+\tau)+\sum_{C_{kl}\in N_{h_1}(i,j)}C_{ij}^{kl}(s+\tau)$$
$$\times f(x_{kl}(s+\tau-\tau_{kl}(s+\tau)))x_{ij}(s+\tau)+\sum_{C_{kl}\in N_{h_2}(i,j)}B_{ij}^{kl}(s+\tau)\int_0^\infty K_{ij}(u)$$
$$\times g(x_{kl}(s+\tau-u))dux_{ij}(s+\tau)+L_{ij}(s+\tau)\Big]ds+\int_{-\infty}^{t}e^{-\int_s^t a_{ij}(u+\tau)du}$$
$$\times\sum_{C_{kl}\in N_{h_3}(i,j)}E_{ij}^{kl}(s+\tau)\delta_{ij}(x_{ij}(s+\tau-\sigma_{ij}(s+\tau)))d\omega_{ij}(s). \tag{3.15}$$

From (3.2) and (3.15), we deduce that

$$\int_\xi^{\xi+1}E\|x(t+\tau)-x(t)\|_0^p dt$$
$$\leq 16^{p-1}\max_{ij\in\Lambda}\Big\{\int_\xi^{\xi+1}E\Big\|\int_{-\infty}^{t}e^{-\int_s^t a_{ij}^0(u+\tau)du}\sum_{C_{kl}\in N_{h_1}(i,j)}C_{ij}^{kl}(s+\tau)$$
$$\times\Big[f(x_{kl}(s+\tau-\tau_{kl}(s+\tau)))x_{ij}(s+\tau)-f(x_{kl}(s-\tau_{kl}(s)))x_{ij}(s+\tau)\Big]ds\Big\|_\flat^p dt\Big\}$$
$$+16^{p-1}\max_{ij\in\Lambda}\Big\{\int_\xi^{\xi+1}E\Big\|\int_{-\infty}^{t}e^{-\int_s^t a_{ij}^0(u+\tau)du}\sum_{C_{kl}\in N_{h_1}(i,j)}(C_{ij}^{kl}(s+\tau)-C_{ij}^{kl}(s))$$
$$\times f(x_{kl}(s-\tau_{kl}(s)))x_{ij}(s+\tau)ds\Big\|_{\mathscr{A}}^p dt\Big\}+16^{p-1}\max_{ij\in\Lambda}\Big\{\int_\xi^{\xi+1}E\Big\|\int_{-\infty}^{t}e^{-\int_s^t a_{ij}^0(u+\tau)du}$$
$$\sum_{C_{kl}\in N_{h_1}(i,j)}C_{ij}^{kl}(s)\times f(x_{kl}(s-\tau_{kl}(s)))(x_{ij}(s+\tau)-x_{ij}(s))ds\Big\|_\flat^p dt\Big\}$$
$$+16^{p-1}\max_{ij\in\Lambda}\Big\{\int_\xi^{\xi+1}E\Big\|\int_{-\infty}^{t}(e^{-\int_s^t a_{ij}^0(u+\tau)du}-e^{-\int_s^t a_{ij}^0(u)du})\sum_{C_{kl}\in N_{h_1}(i,j)}C_{ij}^{kl}(t)$$

$$\times f(x_{kl}(s-\tau_{kl}(s)))x_{ij}(s)ds\Big\|_{\flat}^{p}dt\Big\} + 16^{p-1}\max_{ij\in\Lambda}\Big\{\int_{\xi}^{\xi+1}E\Big\|\int_{-\infty}^{t}e^{-\int_s^t a_{ij}^0(u+\tau)du}$$

$$\sum_{C_{kl}\in N_{h_2}(i,j)}B_{ij}^{kl}(s+\tau)\Big[\int_0^{\infty}K_{ij}(u)g(x_{kl}(s+\tau-u))dux_{ij}(s+\tau)-\int_0^{\infty}K_{ij}(u)$$

$$\times g(x_{kl}(s-u))dux_{ij}(s+\tau)\Big]ds\Big\|_{\flat}^{p}dt\Big\} + 16^{p-1}\max_{ij\in\Lambda}\Big\{\int_{\xi}^{\xi+1}E\Big\|\int_{-\infty}^{t}e^{-\int_s^t a_{ij}^0(u+\tau)du}$$

$$\sum_{C_{kl}\in N_{h_2}(i,j)}(B_{ij}^{kl}(s+\tau)-B_{ij}^{kl}(s))\int_0^{\infty}K_{ij}(u)g(x_{kl}(s-u))dux_{ij}(s+\tau)ds\Big\|_{\flat}^{p}dt\Big\}$$

$$+16^{p-1}\max_{ij\in\Lambda}\Big\{\int_{\xi}^{\xi+1}E\Big\|\int_{-\infty}^{t}e^{-\int_s^t a_{ij}^0(u+\tau)du}\sum_{C_{kl}\in N_{h_2}(i,j)}B_{ij}^{kl}(s)\int_0^{\infty}K_{ij}(u)g(x_{kl}(s-u))du$$

$$\times(x_{ij}(s+\tau)-x_{ij}(s))ds\Big\|_{\flat}^{p}dt\Big\} + 16^{p-1}\max_{ij\in\Lambda}\Big\{\int_{\xi}^{\xi+1}E\Big\|\int_{-\infty}^{t}(e^{-\int_s^t a_{ij}^0(u+\tau)du}$$

$$-e^{-\int_s^t a_{ij}^0(u)du})\sum_{C_{kl}\in N_{h_2}(i,j)}B_{ij}^{kl}(s)\int_0^{\infty}K_{ij}(u)g(x_{kl}(s-u))dux_{ij}(s)ds\Big\|_{\flat}^{p}dt\Big\}$$

$$+16^{p-1}\max_{ij\in\Lambda}\Big\{\int_{\xi}^{\xi+1}E\Big\|\int_{-\infty}^{t}e^{-\int_s^t a_{ij}^0(u+\tau)du}(L_{ij}(s+\tau)-L_{ij}(s))ds\Big\|_{\flat}^{p}dt\Big\}$$

$$+16^{p-1}\max_{ij\in\Lambda}\Big\{\int_{\xi}^{\xi+1}E\Big\|\int_{-\infty}^{t}(e^{-\int_s^t a_{ij}^0(u+\tau)du}-e^{-\int_s^t a_{ij}^0(u)du})L_{ij}(s)ds\Big\|_{\flat}^{p}dt\Big\}$$

$$+16^{p-1}\max_{ij\in\Lambda}\Big\{\int_{\xi}^{\xi+1}E\Big\|\int_{-\infty}^{t}e^{-\int_s^t a_{ij}(u+\tau)du}\sum_{C_{kl}\in N_{h_3}(i,j)}E_{ij}^{kl}(s+\tau)$$

$$\times\Big[\delta_{ij}(x_{ij}(s+\tau-\sigma_{ij}(s+\tau)))-\delta_{ij}(x_{ij}(s-\sigma_{ij}(s)))\Big]d\omega_{ij}(s)\Big\|_{\mathscr{A}}^{p}dt\Big\}$$

$$+16^{p-1}\max_{ij\in\Lambda}\Big\{\int_{\xi}^{\xi+1}E\Big\|\int_{-\infty}^{t}e^{-\int_s^t a_{ij}^0(u+\tau)du}\sum_{C_{kl}\in N_{h_3}(i,j)}(E_{ij}^{kl}(s+\tau)-E_{ij}^{kl}(s))$$

$$\times\delta_{ij}(x_{ij}(s-\sigma_{ij}(s)))d\omega_{ij}(s)\Big\|_{\flat}^{p}dt\Big\} + 16^{p-1}\max_{ij\in\Lambda}\Big\{\int_{\xi}^{\xi+1}E\Big\|\int_{-\infty}^{t}(e^{-\int_s^t a_{ij}^0(u+\tau)du}$$

$$-e^{-\int_s^t a_{ij}^0(u)du})\sum_{C_{kl}\in N_{h_3}(i,j)}E_{ij}^{kl}(t)\delta_{ij}(x_{ij}(s-\sigma_{ij}(s)))d\omega_{ij}(s)\Big\|_{\flat}^{p}dt\Big\}$$

$$+16^{p-1}\max_{ij\in\Lambda}\Big\{\int_{\xi}^{\xi+1}E\Big\|\int_{-\infty}^{t}e^{-\int_s^t a_{ij}^0(u+\tau)du}(a_{ij}^c(s)-a_{ij}^c(s+\tau))x_{ij}(s+\tau)ds\Big\|_{\flat}^{p}dt\Big\}$$

$$+16^{p-1}\max_{ij\in\Lambda}\Big\{\int_{\xi}^{\xi+1}E\Big\|\int_{-\infty}^{t}e^{-\int_s^t a_{ij}(u+\tau)du}a_{ij}^c(s)(x_{ij}(s)-x_{ij}(s+\tau))ds\Big\|_{\flat}^{p}dt\Big\}$$

$$+16^{p-1}\max_{ij\in\Lambda}\Big\{\int_{\xi}^{\xi+1}E\Big\|\int_{-\infty}^{t}(e^{-\int_s^t a_{ij}(u+\tau)du}-e^{-\int_s^t a_{ij}(u)du})(-a_{ij}^c(s))x_{ij}(s)ds\Big\|_{\flat}^{p}dt\Big\}$$

$$:= \sum_{i=1}^{16} H_i. \tag{3.16}$$

Employing the Hölder inequality, we can obtain

$$\begin{aligned} H_1 \leq & 32^{p-1} \max_{ij\in\Lambda} \Big\{ \Big(\frac{p}{q\underline{a}_{ij}^0}\Big)^{\frac{p}{q}} \Big(\sum_{C_{kl}\in N_{h_1}(i,j)} (C_{ij}^{kl^+})^q \Big)^{\frac{p}{q}} (L_f)^p \sum_{C_{kl}\in N_{h_1}(i,j)} \int_{\xi}^{\xi+1} \Big[\int_{-\infty}^{t} e^{-\frac{p}{q}(t-s)\underline{a}_{ij}^0} \\ & \times E\Big\|[x(s+\tau-\tau_{kl}(s+\tau)) - x(s-\tau_{kl}(s+\tau))]x(t+\tau)\Big\|_0^p ds \\ & + \int_{-\infty}^{t} e^{-\frac{p}{q}(t-s)\underline{a}_{ij}^0} E\Big\|[x(s-\tau_{kl}(s+\tau)) - x(s-\tau_{kl}(s))]x(t+\tau)\Big\|_0^p ds \Big] dt \Big\}. \end{aligned}$$

By a change of variables and Fubini's theorem, we infer that

$$\begin{aligned} H_1 \leq & 32^{p-1} \max_{ij\in\Lambda} \Big\{ \Big(\frac{p}{q\underline{a}_{ij}^0}\Big)^{\frac{p}{q}} \Big(\sum_{C_{kl}\in N_{h_1}(i,j)} (C_{ij}^{kl^+})^q \Big)^{\frac{p}{q}} (L_f)^p \\ & \times \sum_{C_{kl}\in N_{h_1}(i,j)} \int_{\xi}^{\xi+1} \Big[\int_{-\infty}^{t-\tau_{kl}(t+\tau)} \frac{1}{1-\dot{\tau}_{kl}(s+\tau)} e^{-\frac{p}{q}} \underline{a}_{ij}^0 (t-u-\tau_{kl}(s+\tau)) \\ & \times E\Big\|[x(u+\tau) - x(u)]x(t+\tau)\Big\|_0^p du + \frac{q\varepsilon^p(2\kappa)^p}{p\underline{a}_{ij}^0} \Big] dt \Big\} \\ \leq & 32^{p-1} \max_{ij\in\Lambda} \Big\{ \Big(\frac{p}{q\underline{a}_{ij}^0}\Big)^{\frac{p}{q}} \Big(\sum_{C_{kl}\in N_{h_1}(i,j)} (C_{ij}^{kl^+})^q \Big)^{\frac{p}{q}} (L_f)^p \sum_{C_{kl}\in N_{h_1}(i,j)} \frac{e^{\frac{p}{q}} \underline{a}_{ij}^0 \tau_{kl}{}^+ (2\kappa)^p}{1-\dot{\tau}_{kl}^+} \\ & \times \int_{-\infty}^{\xi} e^{-\frac{p}{q}(\xi-s)\underline{a}_{ij}^0} \Big(\int_s^{s+1} E\Big\|x(t+\tau) - x(t)\Big\|_0^p dt \Big) ds \Big\} + \triangle_{H_1}\varepsilon, \end{aligned} \tag{3.17}$$

where

$$\triangle_{H_1} = 32^{p-1} \max_{ij\in\Lambda} \Big\{ \frac{q(2\kappa)^p}{p\underline{a}_{ij}^0} \Big(\frac{p}{q\underline{a}_{ij}^0}\Big)^{\frac{p}{q}} \Big(\sum_{C_{kl}\in N_{h_1}(i,j)} (C_{ij}^{kl^+})^q \Big)^{\frac{p}{q}} (L_f)^p \varepsilon^{p-1} \Big\}.$$

Similarly, when $p > 2$, one can obtain

$$\begin{aligned} H_{11} \leq & 16^{p-1} C_p \max_{ij\in\Lambda} \Big\{ \int_{\xi}^{\xi+1} E\Big[\int_{-\infty}^{t} e^{-2\int_s^t a_{ij}^0(u+\tau)du} E\Big\| \sum_{C_{kl}\in N_{h_2}(i,j)} E_{ij}^{kl}(s+\tau) \\ & \times (\delta_{ij}(x_{ij}(s+\tau-\sigma_{ij}(s+\tau))) - \delta_{ij}(x_{ij}(s-\sigma_{ij}(s))))\Big\|_{\flat}^2 ds \Big]^{\frac{p}{2}} dt \Big\} \\ \leq & 32^{p-1} C_p \max_{ij\in\Lambda} \Big\{ (\frac{p-2}{2\underline{a}_{ij}^0})^{\frac{p-2}{2}} (\sum_{C_{kl}\in N_{h_2}(i,j)} (E_{ij}^{kl^+})^q)^{\frac{p}{q}} (L_{ij}^{\delta})^p \frac{e^{\frac{p}{q}\underline{a}_{ij}^0 \sigma_{ij}^+}}{1-\dot{\sigma}_{ij}^+} \int_{-\infty}^{\xi} e^{-\frac{p}{q}(\xi-s)\underline{a}_{ij}^0} \\ & \times \Big(\int_s^{s+1} E\Big\|x(t+\tau) - x(t)\Big\|_0^p dt \Big) ds \Big\} + \triangle_{H_{11}^1}\varepsilon, \end{aligned} \tag{3.18}$$

where

$$\triangle_{H_{11}^1} = 32^{p-1}C_p \max_{ij\in\Lambda}\Big\{(\frac{p-2}{2\underline{a}_{ij}^0})^{\frac{p-2}{2}}(\sum_{C_{kl}\in N_{h_3}(i,j)}(E_{ij}^{kl^+})^q)^{\frac{p}{q}}\sum_{C_{kl}\in N_{h_3}(i,j)}(L_{ij}^{\delta})^p\frac{q}{p\underline{a}_{ij}^0}\varepsilon^{p-1}\Big\},$$

$$\begin{aligned}H_{12} \leq& 16^{p-1}C_p \max_{ij\in\Lambda}\Big\{\int_{\xi}^{\xi+1}E\Big[\int_{-\infty}^{t}e^{-2\int_s^t a_{ij}^0(u+\tau)du}E\Big\|\sum_{C_{kl}\in N_{h_3}(i,j)}(E_{ij}^{kl}(s+\tau)-E_{ij}^{kl}(s))\\ &\times\delta_{ij}(x_{ij}(s-\sigma_{ij}(s)))\Big\|_{\mathscr{A}}^2 ds\Big]^{\frac{p}{2}}dt\Big\}\\ \leq& 16^{p-1}C_p \max_{ij\in\Lambda}\Big\{(\frac{p}{q\underline{a}_{ij}^0})^{\frac{p}{q}}\frac{q}{p\underline{a}_{ij}^0}(L_{ij}^{\delta})^p\frac{e^{\frac{p}{q}\underline{a}_{ij}^0\sigma_{ij}^+}}{1-\dot{\sigma}_{ij}^+}(2\kappa)^p\Big\}\varepsilon^{\frac{p}{q}} := \triangle_{H_{12}^1}\varepsilon,\end{aligned}\tag{3.19}$$

and when $p = 2$, we have

$$\begin{aligned}H_{11} \leq& 16\max_{ij\in\Lambda}\Big\{\int_{\xi}^{\xi+1}E\Big[\int_{-\infty}^{t}e^{-2\int_s^t a_{ij}^0(u+\tau)du}\sum_{C_{kl}\in N_{h_3}(i,j)}(E_{ij}^{kl^+})^2\\ &\times\sum_{C_{kl}\in N_{h_3}(i,j)}(L_{ij}^{\delta})^2\|x_{ij}(s+\tau-\sigma_{ij}(s+\tau))-x_{ij}(s-\sigma_{ij}(s))\|_{\mathfrak{b}}^2 ds\Big]dt\Big\}\\ \leq& 32\max_{ij\in\Lambda}\Big\{\sum_{C_{kl}\in N_{h_3}(i,j)}(E_{ij}^{kl^+})^2(L_{ij}^{\delta})^2\frac{e^{2\underline{a}_{ij}^0\sigma_{ij}^+}}{1-\dot{\sigma}_{ij}^+}\int_{-\infty}^{\xi}e^{-2(\xi-s)\underline{a}_{ij}^0}\\ &\times\Big(\int_s^{s+1}E\|x(t+\tau)-x(t)\|_0^2dt\Big)ds\Big\}+\triangle_{H_{11}^2}\varepsilon,\end{aligned}\tag{3.20}$$

where

$$\triangle_{H_{11}^2} = \frac{32}{\underline{a}_{ij}^0}\max_{ij\in\Lambda}\Big\{\sum_{C_{kl}\in N_{h_3}(i,j)}(E_{ij}^{kl^+})^2\sum_{C_{kl}\in N_{h_3}(i,j)}(L_{ij}^{\delta})^2\varepsilon\Big\},$$

$$H_{12} \leq 16\max_{ij\in\Lambda}\Big\{\frac{1}{(\underline{a}_{ij}^0)^2}(L_{ij}^{\delta})^2\frac{e^{\underline{a}_{ij}^0\sigma_{ij}^+}}{1-\dot{\sigma}_{ij}^+}4\kappa^2\Big\}\varepsilon := \triangle_{H_{12}^2}\varepsilon.\tag{3.21}$$

In the same way, we can get

$$H_2 \leq 16^{p-1}\max_{ij\in\Lambda}\Big\{(\frac{mnp}{q\underline{a}_{ij}^0})^{\frac{p}{q}}\frac{q}{p\underline{a}_{ij}^0}(2\kappa M_f)^p\Big\}\varepsilon^{\frac{p}{q}} := \triangle_{H_2}\varepsilon,\tag{3.22}$$

$$\begin{aligned}H_3 \leq& 16^{p-1}\max_{ij\in\Lambda}\Big\{(\frac{p}{q\underline{a}_{ij}^0})^{\frac{p}{q}}\Big(\sum_{C_{kl}\in N_{h_1}(i,j)}(C_{ij}^{kl^+})^q\Big)^{\frac{p}{q}}(M_f)^p\int_{-\infty}^{\xi}e^{-\frac{p}{q}(\xi-s)\underline{a}_{ij}^0}\\ &\times\Big(\int_s^{s+1}E\|x(t+\tau)-x(t)\|_0^p dt\Big)ds\Big\},\end{aligned}\tag{3.23}$$

$$H_5 \leq 16^{p-1}\max_{ij\in\Lambda}\Big\{\Big(\frac{p}{q\underline{a}_{ij}^0}\Big)^{\frac{p}{q}}\Big(\sum_{C_{kl}\in N_{h_2}(i,j)}(B_{ij}^{kl^+})^q\Big)^{\frac{p}{q}}\Big(2\kappa L_g\int_0^\infty|K_{ij}(u)|du\Big)^p\int_{-\infty}^{\xi}e^{-\frac{p}{q}(\xi-s)\underline{a}_{ij}^0}$$

$$\times\Big(\int_s^{s+1}E\|x(t+\tau)-x(t)\|_0^p dt\Big)ds\Big\}, \tag{3.24}$$

$$H_6 \leq 16^{p-1}\max_{ij\in\Lambda}\Big\{(\frac{mnp}{q\underline{a}_{ij}^0})^{\frac{p}{q}}\frac{q}{p\underline{a}_{ij}^0}\Big((2\kappa M_g)\int_0^\infty|K_{ij}(u)|du\Big)^p\Big\}\varepsilon^{\frac{p}{q}} := \triangle_{H_6}\varepsilon, \tag{3.25}$$

$$H_7 \leq 16^{p-1}\max_{ij\in\Lambda}\Big\{(\frac{p}{q\underline{a}_{ij}^0})^{\frac{p}{q}}\Big(\sum_{C_{kl}\in N_{h_2}(i,j)}(B_{ij}^{kl^+})^q\Big)^{\frac{p}{q}}(M_g\int_0^\infty|K_{ij}(u)|du)^p\int_{-\infty}^{\xi}e^{-\frac{p}{q}(\xi-s)\underline{a}_{ij}^0}$$

$$\times\Big(\int_s^{s+1}E\|x(t+\tau)-x(t)\|_0^p dt\Big)ds\Big\}, \tag{3.26}$$

$$H_9 \leq 16^{p-1}\max_{ij\in\Lambda}\Big\{(\frac{p}{q\underline{a}_{ij}^0})^{\frac{p}{q}}\frac{q}{p\underline{a}_{ij}^0}\Big\}\varepsilon^p := \triangle_{H_9}\varepsilon, \tag{3.27}$$

$$H_{14} \leq 16^{p-1}\max_{ij\in\Lambda}\Big\{(\frac{p}{q\underline{a}_{ij}^0})^{\frac{p}{q}}\frac{q}{p\underline{a}_{ij}^0}(2\kappa)^p\Big\}\varepsilon^p := \triangle_{H_{14}}\varepsilon, \tag{3.28}$$

$$H_{15} \leq 16^{p-1}\max_{ij\in\Lambda}\Big\{(\frac{p}{q\underline{a}_{ij}^0})^{\frac{p}{q}}(\bar{a}_{ij}^c)^p\int_{-\infty}^{\xi}e^{-\frac{p}{q}(\xi-s)\underline{a}_{ij}^0}\Big(\int_s^{s+1}E\|x(t+\tau)-x(t)\|_0^p dt\Big)ds\Big\}. \tag{3.29}$$

Noting that

$$\Big[\int_{-\infty}^t\Big|e^{-\int_s^t a_{ij}^0(u+\tau)du}-e^{-\int_s^t a_{ij}^0(u)du}\Big|^{\frac{q}{p}}ds\Big]^{\frac{p}{q}}$$

$$\leq\Big[\int_{-\infty}^t e^{-\frac{q}{p}\underline{a}_{ij}^0(t-s)}\Big(\int_s^t|a_{ij}^0(u+\tau)-a_{ij}^0(u)|du\Big)^{\frac{q}{p}}ds\Big]^{\frac{p}{q}} \leq \Big(\Gamma(\frac{q+p}{p})\Big)^{\frac{p}{q}}\Big(\frac{p}{q\underline{a}_{ij}^0}\Big)^{\frac{p+q}{q}}\varepsilon. \tag{3.30}$$

We can gain

$$H_4 \leq 16^{p-1}\max_{ij\in\Lambda}\Big\{\Big(\Gamma(\frac{q+p}{p})\Big)^{\frac{p}{q}}\Gamma(\frac{q+p}{p})(\frac{1}{\underline{a}_{ij}^0})^{\frac{2(p+q)}{q}}$$

$$\times\Big(\sum_{C_{kl}\in N_{h_1}(i,j)}(C_{ij}^{kl^+})^q\Big)^{\frac{p}{q}}(2\kappa M_f)^p\Big\}\varepsilon^{\frac{p}{q}+1} := \triangle_{H_4}\varepsilon, \tag{3.31}$$

$$H_8 \leq 16^{p-1}\max_{ij\in\Lambda}\Big\{\Big(\Gamma(\frac{q+p}{p})\Big)^{\frac{p}{q}}\Gamma(\frac{q+p}{p})(\frac{1}{\underline{a}_{ij}^0})^{\frac{2(p+q)}{q}}$$

$$\times\Big(\sum_{C_{kl}\in N_{h_2}(i,j)}(B_{ij}^{kl^+})^q\Big)^{\frac{p}{q}}(2\kappa M_g\int_0^\infty|K_{ij}(u)|du)^p\Big\}\varepsilon^{\frac{p}{q}+1} := \triangle_{H_8}\varepsilon, \tag{3.32}$$

$$H_{10} \leq 16^{p-1}\max_{ij\in\Lambda}\Big\{\Big(\Gamma(\frac{q+p}{p})\Big)^{\frac{p}{q}}\Gamma(\frac{q+p}{p})(\frac{1}{\underline{a}_{ij}^0})^{\frac{2(p+q)}{q}}(M_L)^p\Big\}\varepsilon^{\frac{p}{q}+1} := \triangle_{H_{10}}\varepsilon, \tag{3.33}$$

$$H_{16} \leq 16^{p-1}\max_{ij\in\Lambda}\Big\{\Big(\Gamma(\frac{q+p}{p})\Big)^{\frac{p}{q}}\Gamma(\frac{q+p}{p})(\frac{1}{\underline{a}_{ij}^0})^{\frac{2(p+q)}{q}}(2\kappa\bar{a}_{ij}^c)^p\Big\}\varepsilon^{\frac{p}{q}+1} := \triangle_{H_{16}}\varepsilon, \tag{3.34}$$

when $p > 2$, we have

$$
\begin{aligned}
H_{13} \leq & 16^{p-1} C_p \max_{ij\in\Lambda} \bigg\{ \int_{\xi}^{\xi+1} E\Big[\int_{-\infty}^{t} \Big(e^{-\int_s^t a_{ij}^0(u+\tau)du} - e^{-\int_s^t a_{ij}^0(u)du}\Big)^2 \\
& \times \sum_{C_{kl}\in N_{h_3}(i,j)} E_{ij}^{kl}(s) \|\delta_{ij}(x_{ij}(s-\sigma_{ij}(s)))\|_{\mathfrak{b}}^2 ds\Big]^{\frac{p}{2}} dt \bigg\} \\
\leq & 16^{p-1} C_p \max_{ij\in\Lambda} \bigg\{ \Big(\Gamma(\frac{p}{p-2})\Big)^{\frac{p-2}{2}} (\frac{p-2}{2\underline{a}_{ij}^0})^{\frac{p}{2}} \\
& \times \frac{q}{p\underline{a}_{ij}^0} \Big(\sum_{C_{kl}\in N_{h_3}(i,j)} (E_{ij}^{kl^+})^q \Big)^{\frac{p}{q}} (L_{ij}^{\delta})^p \frac{e^{\frac{p}{q}\underline{a}_{ij}^0 \sigma_{ij}^+}}{1-\dot{\sigma}_{ij}^+} (2\kappa)^p \bigg\} \varepsilon := \triangle_{H_{13}^1} \varepsilon,
\end{aligned}
\tag{3.35}
$$

for $p = 2$, we get

$$
H_{13} \leq 16 \max_{ij\in\Lambda} \bigg\{ \sum_{C_{kl}\in N_{h_3}(i,j)} (E_{ij}^{kl^+})^2 (L_{ij}^{\delta})^2 \frac{e^{2\underline{a}_{ij}^0 \sigma_{ij}^+}}{1-\dot{\sigma}_{ij}^+} \frac{\Gamma(3)}{8(\underline{a}_{ij}^0)^3} (2\kappa)^2 \bigg\} \varepsilon := \triangle_{H_{13}^2} \varepsilon. \tag{3.36}
$$

Substituting (3.17)–(3.36) into (3.16), we have the following two cases:
Case 1. When $p > 2$, we have

$$
\begin{aligned}
& \int_{\xi}^{\xi+1} E\|x(t+\tau) - x(t)\|_0^p dt \\
\leq & H^1 \varepsilon + \rho^1 \int_{-\infty}^{\xi} e^{-(\xi-s)\frac{p}{q}\underline{a}^0} \Big(\int_s^{s+1} E\|x(t+\tau) - x(t)\|_0^p dt \Big) ds \\
\leq & H^1 \varepsilon + \rho^1 \sup_{s\in\mathbb{R}} \Big(\int_s^{s+1} E\|x(t+\tau) - x(t)\|_0^p dt \Big) \int_{-\infty}^{\xi} e^{-(\xi-s)\frac{p}{q}\underline{a}^0} ds \\
\leq & H^1 \varepsilon + \rho^1 \frac{q}{p\underline{a}^0} \sup_{s\in\mathbb{R}} \Big(\int_s^{s+1} E\|x(t+\tau) - x(t)\|_0^p dt \Big),
\end{aligned}
$$

where ρ^1 is the same as that in $(A3)$ and $H^1 = \triangle_{H_1} + \triangle_{H_2} + \triangle_{H_4} + \triangle_{H_6} + \triangle_{H_8} + \triangle_{H_9} + \triangle_{H_{10}} + \triangle_{H_{14}} + \triangle_{H_{16}} + \triangle_{H_{11}^1} + \triangle_{H_{12}^1} + \triangle_{H_{13}^1}$.

By $(A4)$, we know $\rho^1 < \frac{p\underline{a}^0}{q}$. Hence, we derive that

$$
\sup_{\xi\in\mathbb{R}} \int_{\xi}^{\xi+1} E\|x(t+\tau) - x(t)\|_0^p dt \; < \; \frac{p\underline{a}^0 H^1}{p\underline{a}^0 - \rho^1 q} \varepsilon. \tag{3.37}
$$

Case 2. When $p = 2$, we can obtain

$$
\begin{aligned}
& \int_{\xi}^{\xi+1} E\|x(t+\tau) - x(t)\|_0^2 dt \\
\leq & H^2 \varepsilon + \rho^2 \int_{-\infty}^{\xi} e^{-(\xi-s)\frac{p}{q}\underline{a}^0} \Big(\int_s^{s+1} E\|x(t+\tau) - x(t)\|_0^2 dt \Big) ds,
\end{aligned}
$$

where ρ^2 is defined in $(A4)$ and

$$
\begin{aligned}
H^2 =&32 \max_{ij\in\Lambda}\left\{\left(\frac{2\kappa}{\underline{a}_{ij}^0}\right)^2 \sum_{C_{kl}\in N_{h_1}(i,j)} (C_{ij}^{kl+})^2(L_f)^2\right\}\varepsilon + 16\max_{ij\in\Lambda}\left\{\frac{m^2n^2}{(\underline{a}_{ij}^0)^2}\left[(2\kappa M_f)^2 + \left(2\kappa M_g\right.\right.\right. \\
&\times \left.\left.\int_0^\infty |K_{ij}(u)|du\right)^2\right] + (\Gamma(2))^2\left(\frac{1}{\underline{a}_{ij}^0}\right)^4\left[\sum_{C_{kl}\in N_{h_1}(i,j)} (C_{ij}^{kl+})^2(2\kappa M_f)^2 + \sum_{C_{kl}\in N_{h_2}(i,j)} (B_{ij}^{kl+})^2\right. \\
&\times \left.\left(2\kappa M_g\int_0^\infty |K_{ij}(u)|du\right)^2 + (M_L)^2 + (2\kappa\bar{a}_{ij}^c)^2\right] + \frac{1}{(\underline{a}_{ij}^0)^2}\left[\sum_{C_{kl}\in N_{h_2}(i,j)} (B_{ij}^{kl+})^2\right. \\
&\times \left.\left.\left(2\kappa M_g\int_0^\infty |K_{ij}(u)|du\right)^2 + (2\kappa)^2 + 1\right]\right\}\varepsilon + \triangle_{H_{11}^2} + \triangle_{H_{12}^2} + \triangle_{H_{13}^2}.
\end{aligned}
$$

Similar to the previous case, by $(A4)$, we know $\rho^2 < \underline{a}^0$ and hence, we can get that

$$
\int_\xi^{\xi+1} E\|x(t+\tau) - x(t)\|_0^2 dt \ < \ \frac{\underline{a}^0 H^2}{\underline{a}^0 - \rho^2}\varepsilon. \tag{3.38}
$$

Noting that

$$
\begin{aligned}
&d_{BL}(P\circ[x(t+\tau)]^{-1}, P\circ[x(t)]^{-1})\Big) \\
\leq& \sup_{\|f\|_{BL}\leq 1}\left|\int_{\mathscr{A}^{m\times n}} f d(P\circ[x(t+\tau)]^{-1} - P\circ[x(t)]^{-1})\right| \\
\leq& \sup_{\|f\|_{BL}\leq 1}\left|\int_\Omega f(x(t+\tau)) - f(x(t))\right| dP \\
\leq& \int_\Omega \|x(t+\tau) - x(t)\|_0 dP \\
\leq& (E\|x(t+\tau) - x(t)\|_0^p)^{\frac{1}{p}}.
\end{aligned}
$$

Hence, we have

$$
\begin{aligned}
&\sup_{\xi\in\mathbb{R}}\left(\int_\xi^{\xi+1} d_{BL}^p(P\circ[x(t+\tau)]^{-1}, P\circ[x(t)]^{-1})dt\right)^{\frac{1}{p}} \\
\leq&\left(\sup_{\xi\in\mathbb{R}}\int_\xi^{\xi+1} E\|x(t+\tau) - x(t)\|_0^p dt\right)^{\frac{1}{p}}.
\end{aligned} \tag{3.39}
$$

Combining (3.37)–(3.39), we can conclude that $x(t)$ is p-th Stepanov almost periodic in the distribution sense. The proof is complete. □

Similar to the proof of Theorem 3.7 in [21], one can easily show that.

Theorem 3.3. *Suppose that* $(A1)$–$(A5)$ *are fulfilled and let x be the Stepanov almost periodic solution in the distribution sense of system* (2.1) *with initial value* φ*. Then there exist constants* $\lambda > 0$ *and* $M > 0$ *such that for an arbitrary solution y with initial value* ψ *satisfies*

$$
E\|y(t) - x(t)\|_0^p \leq M\|\varphi - \psi\|_1 e^{-\frac{p}{q}\lambda t}, \ \ t > 0,
$$

where $\|\varphi - \psi\|_1 = \sup_{a\in[-\theta,0]} E\|\varphi(s) - \psi(s)\|_0^p$*, i.e., the solution x is globally exponentially stable.*

4. Numerical example

The purpose of this section is to demonstrate the effectiveness of the results obtained in this paper through a numerical example.

In neural network (2.1), choose $f(x) = \frac{1}{25}\sin x^0 e_0 + \frac{1}{30}\sin(x^2 + x^0)e_1 + \frac{1}{35}\sin^2 x^{12} e_2 + \frac{1}{40}\tanh^2 x^2 e_{12}$, $g(x) = \frac{1}{40}\sin x^0 e_0 + \frac{1}{35}\sin(x^2 + x^1)e_1 + \frac{1}{30}\sin^2 x^{12} e_2 + \frac{1}{25}\tanh^2 x^1 e_{12}$, $K_{ij}(t) = \frac{1}{e^t}$ and

$$\begin{bmatrix} a_{11}(t) \\ a_{12}(t) \\ a_{21}(t) \\ a_{22}(t) \end{bmatrix} = \begin{bmatrix} (7 + 2\sin t)\ e_0 + \cos t\ e_1 + \cos\sqrt{5}t\ e_2 + \sin\sqrt{3}t\ e_{12} \\ (6 - \cos t)e_0 + \sin\sqrt{3}te_2 + \cos\sqrt{3}te_{12} \\ (8 + \cos\sqrt{3}t)\ e_0 + \sin\sqrt{5}t\ e_1 + \sin t\ e_{12} \\ (10 - 2\sin\sqrt{5}t)\ e_0 + \cos\sqrt{3}t\ e_1 + \cos t\ e_2 + \tanh t\ e_{12} \end{bmatrix},$$

$$\begin{bmatrix} C_{11}(t) & C_{12}(t) \\ C_{21}(t) & C_{22}(t) \end{bmatrix} = \begin{bmatrix} B_{11}(t) & B_{12}(t) \\ B_{21}(t) & B_{22}(t) \end{bmatrix} = \begin{bmatrix} E_{11}(t) & E_{12}(t) \\ E_{21}(t) & E_{22}(t) \end{bmatrix}$$
$$= \begin{bmatrix} 0.02 + 0.01\text{cost} & 0.03 + 0.2\sin\sqrt{3}\text{t} \\ 0.05 + 0.07\sin\sqrt{5}\text{t} & 0.02 + 0.05\cos\sqrt{3}\text{t} \end{bmatrix} e_0,$$

$$\begin{bmatrix} L_{11}(t) \\ L_{12}(t) \\ L_{21}(t) \\ L_{22}(t) \end{bmatrix} = \begin{bmatrix} (0.2|\cos|t)\ e_0 + (0.2\cos\sqrt{3}t)\ e_1 + 0.3\sin(\sqrt{3}t)\ e_2 + (0.08\sin t + 0.04e^{-t})\ e_{12} \\ (0.3\sin(\sqrt{2}t) + e^{-t})\ e_0 + (0.1\cos\sqrt{5}\text{t} + 0.04e^{-t})\ e_1 + 0.2e^{-t}\ e_2 + 0.2\sin t\ e_{12} \\ 0.02\sin t\ e_0 + 0.05\sin\sqrt{5}t\ e_1 + (0.03\cos t + 0.01e^{-t})\ e_2 + 0.02\cos\sqrt{3}t\ e_{12} \\ 0.08\sin t\ e_0 + (0.04\cos\sqrt{5}t + 0.04e^{-t})\ e_1 + 0.03\sin\sqrt{5}t\ e_{12} \end{bmatrix},$$

$$\begin{bmatrix} \tau_{11}(t) & \tau_{12}(t) \\ \tau_{21}(t) & \tau_{22}(t) \end{bmatrix} = \begin{bmatrix} \sigma_{11}(t) & \sigma_{12}(t) \\ \sigma_{21}(t) & \sigma_{22}(t) \end{bmatrix} = \begin{bmatrix} 0.03 + 0.009\sin 0.6\text{t} & 0.05 + 0.05\cos 1.2\text{t} \\ 0.02 - 0.008\sin 1.1\text{t} & 0.09 + 0.04\sin 1.7\text{t} \end{bmatrix},$$

$$\begin{bmatrix} \delta_{11}(x) \\ \delta_{12}(x) \\ \delta_{21}(x) \\ \delta_{22}(x) \end{bmatrix} = \begin{bmatrix} \frac{1}{15}\sin\ \sqrt{3}x^0 e_0 + \frac{1}{20}\sin x^2 e_1 + \frac{1}{30}\tanh^2 x^1 e_{12} \\ 0.04\sin x^0 e_1 + 0.03\sin^2 x^{12} e_2 + 0.05\sin^2 x^2 e_{12} \\ 0.02\tanh x^2 e_0 + 0.06\sin x^1 e_1 + 0.015\sin^2 x^0 e_2 \\ \frac{1}{20}\sin x^1 e_0 + \frac{1}{15}\tanh x^2 e_1 + \frac{1}{25}\sin x^{12} e_2 + \frac{1}{40}\sin x^0 e_{12} \end{bmatrix},$$

and let $h_1 = h_2 = 1, h_3 = 0, m = n = 2$. Then we get

$$L_f = L_g = M_g = M_f = 0.04,\ \underline{a}^0 = 5,\ \bar{a}^c = 1,\ M_L = 1,\ M = u = 1,\ L_{11}^{\delta} = \frac{1}{15},\ L_{12}^{\delta} = 0.05,$$
$$L_{21}^{\delta} = 0.06,\ L_{22}^{\delta} = \frac{1}{15},\ \tau_{11}^+ = \sigma_{11}^+ = 0.039,\ \tau_{12}^+ = \sigma_{12}^+ = 0.1,\ \tau_{21}^+ = \sigma_{21}^+ = 0.028,\ \tau_{22}^+ = \sigma_{22}^+$$
$$= 0.13,\ \dot{\tau}_{11}^+ = \dot{\sigma}_{11}^+ = 0.0054,\ \dot{\tau}_{12}^+ = \dot{\sigma}_{12}^+ = 0.006,\ \dot{\tau}_{21}^+ = \dot{\sigma}_{21}^+ = 0.0088,\ \dot{\tau}_{22}^+ = \dot{\sigma}_{22}^+ = 0.068,$$
$$\sum_{C_{kl}\in N_1(1,1)} {C_{11}^{kl}}^+ = \sum_{C_{kl}\in N_1(1,1)} {B_{11}^{kl}}^+ = \sum_{C_{kl}\in N_1(1,2)} {C_{12}^{kl}}^+ = \sum_{C_{kl}\in N_1(1,2)} {B_{12}^{kl}}^+ = \sum_{C_{kl}\in N_1(2,1)} {C_{21}^{kl}}^+$$
$$= \sum_{C_{kl}\in N_1(2,1)} {B_{21}^{kl}}^+ = \sum_{C_{kl}\in N_1(2,2)} {C_{22}^{kl}}^+ = \sum_{C_{kl}\in N_1(2,2)} {B_{22}^{kl}}^+ = 0.45,\quad \sum_{C_{kl}\in N_0(1,1)} {E_{11}^{kl}}^+ = 0.03$$

$$\sum_{C_{kl}\in N_0(1,2)} E_{12}^{kl\,+} = 0.23, \quad \sum_{C_{kl}\in N_0(2,1)} E_{21}^{kl\,+} = 0.12, \quad \sum_{C_{kl}\in N_0(2,2)} E_{22}^{kl\,+} = 0.07.$$

Take $\kappa = 1$, $p = \frac{21}{10}, q = \frac{21}{11}$, then we have

$$r^1 < 0.6812 < 1, \quad \frac{q}{p\underline{a}^0}\rho^1 < 0.6259 < 1.$$

And when $p = 2$, we have

$$r^2 < 0.6408 < 1, \ \frac{\rho^2}{\underline{a}^0} < 0.6786 < 1.$$

Thus, all assumptions in Theorems 3.2 and 3.3 are fulfilled. So we can conclude that the system (2.1) has a unique S^p-almost periodic solution in the distribution sense which is globally exponentially stable.

The results are also verified by the numerical simulations in Figures 1–4.

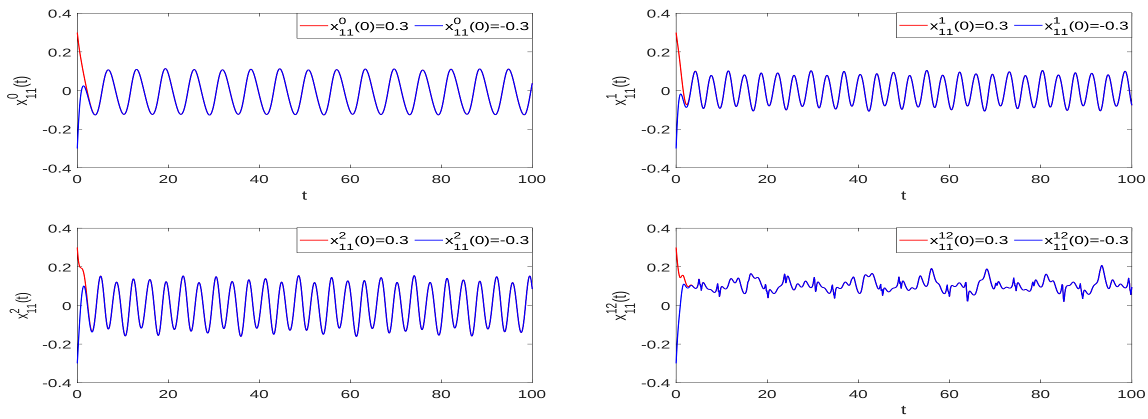

Figure 1. Global exponential stability of states $x_{11}^0, x_{11}^1, x_{11}^2$ and x_{11}^{12} of (2.1).

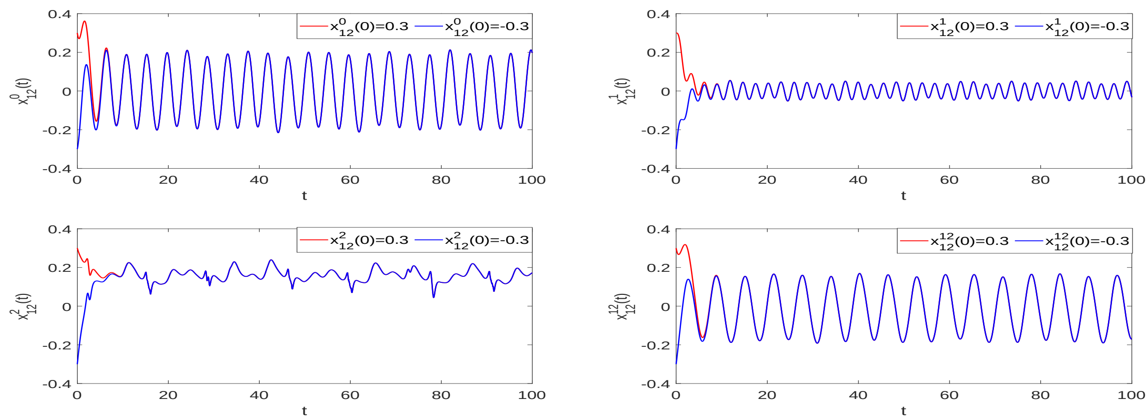

Figure 2. Global exponential stability of states $x_{12}^0, x_{12}^1, x_{12}^2$ and x_{12}^{12} of (2.1).

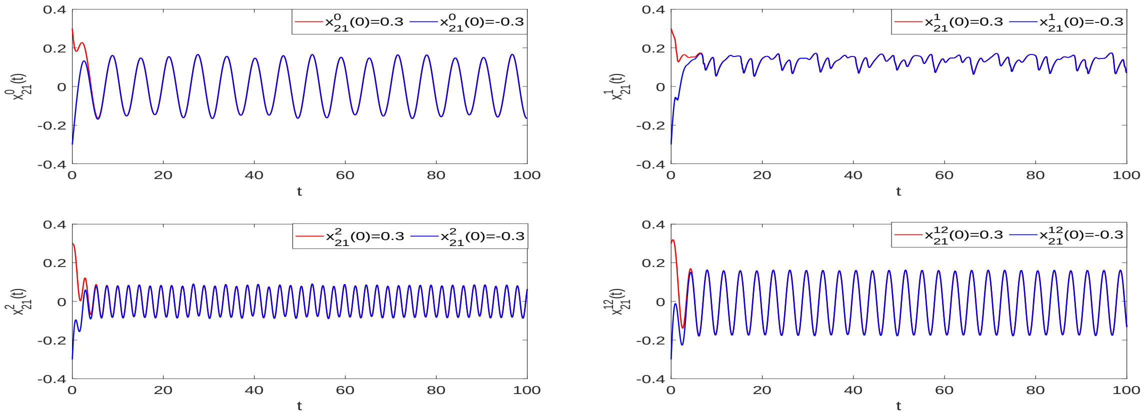

Figure 3. Global exponential stability of states $x_{21}^0, x_{21}^1, x_{21}^2$ and x_{21}^{12} of (2.1).

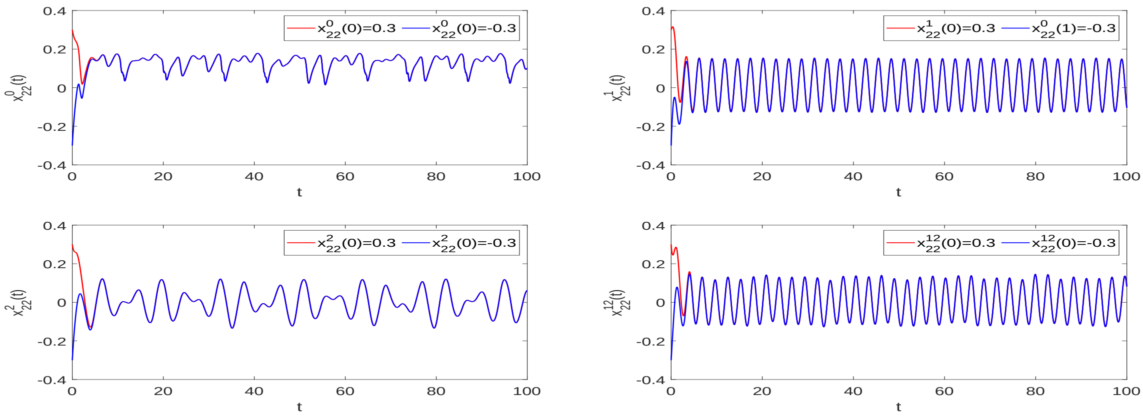

Figure 4. Global exponential stability of states $x_{22}^0, x_{22}^1, x_{22}^2$ and x_{22}^{12} of (2.1).

From these figures, we can observe that when the four primitive components of each solution of this system take different initial values, they eventually tend to stabilize. It can be seen that these solutions that meet the above conditions do exist and are exponentially stable.

5. Conclusions

In this article, we establish the existence and global exponential stability of Stepanov almost periodic solutions in the distribution sense for a class of stochastic Clifford-valued SICNNs with mixed delays. Even when network (2.1) degenerates into a real-valued NN, the results of this paper are new. In fact, uncertainty, namely fuzziness, is also a problem that needs to be considered in real system modeling. However, we consider only the disturbance of random factors and do not consider the issue of fuzziness. In a NN, considering the effects of both random perturbations and fuzziness is our future direction of effort.

Use of AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that they have no conflicts of interest.

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