



Research article

Algorithmic generation of imprecise data from uniform and Weibull distributions

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Abstract: This paper introduced the neutrosophic uniform distribution and innovative simulation methods to generate random numbers from the neutrosophic uniform distribution and the neutrosophic Weibull distribution. We introduced simulation methods and algorithms designed to handle indeterminacy for both of these distributions. We provided random numbers generated from both distributions across a range of parameter values and degrees of indeterminacy. Furthermore, we conducted a comparative analysis between the classical simulation method in classical statistics and the neutrosophic simulation method. Our findings reveal that the proposed neutrosophic simulation method generates random numbers of smaller magnitudes compared to the classical simulation method under classical statistics. This observation forms the basis of our conclusion.

Keywords: classical simulation; neutrosophic simulation; random numbers; uniform distribution; Weibull distribution; algorithms

Mathematics Subject Classification: 62A86

1. Introduction

Random numbers are very important in statistical, probability theory, and mathematical analysis in such complex cases, where the real numbers are difficult to record. The random numbers are generated from the uniform distribution when an interval is defined for their selection of random numbers. The random numbers are generated in sequence and depict the behavior of the real data. In addition, random data can be used for estimation and forecasting purposes. According to [1], “The method is based on running the model many times as in random sampling. For each sample, random

variates are generated on each input variable; computations are run through the model yielding random outcomes on each output variable. Since each input is random, the outcomes are random. In the same way, they generated thousands of such samples and achieved thousands of outcomes for each output variable. In order to carry out this method, a large stream of random numbers was needed". To generate random numbers, a random generator is applied. The random numbers have no specific pattern and are generated from the chance process. Nowadays, the latest computer can be used to generate random numbers using a well-defined algorithm; see [1]. Bang et al. [2] investigated normality using random-number generating. Schulz et al. [3] presented a pattern-based approach. Tanyer [4] generated random numbers from uniform sampling. Kaya and Tuncer [5] proposed a method to generate biological random numbers. Tanackov et al. [6] presented a method to generate random numbers from the exponential distribution. Jacak et al. [7] presented the methods to generate pseudorandom numbers. More methods can be seen in [8–10].

The neutrosophic statistical distributions were found to be more efficient than the distributions under classical statistics. The neutrosophic distributions can be applied to analyze the data that is given in neutrosophic numbers. Sherwani et al. [11] proposed neutrosophic normal distribution. Duan et al. [12] worked on neutrosophic exponential distribution. Aliev et al. [13] generated Z-random numbers from linear programming. Gao and Ralescu [14] studied the convergence of random numbers generated under an uncertain environment. More information on random numbers generators can be seen in [15–18]. In recent works, Aslam [19] introduced a truncated variable algorithm for generating random variates from the neutrosophic DUS-Weibull distribution. Additionally, in another study [20], novel methods incorporating sine-cosine and convolution techniques were introduced to generate random numbers within the framework of neutrosophy. Albassam et al. [21] showcased probability/cumulative density function plots and elucidated the characteristics of the neutrosophic Weibull distribution as introduced by [22]. The estimation and application of the neutrosophic Weibull distribution was also presented by [21].

In [22], the Weibull distribution was introduced within the realm of neutrosophic statistics, offering a more inclusive perspective compared to its traditional counterpart in classical statistics. [21] further examined the properties of the neutrosophic Weibull distribution introduced by [22]. Despite an extensive review of existing literature, no prior research has been identified regarding the development of algorithms for generating random numbers using both the neutrosophic uniform and Weibull distributions. This paper aims to bridge this gap by presenting innovative random number generators tailored specifically for the neutrosophic uniform distribution and the neutrosophic Weibull distribution. The subsequent sections will provide detailed explanations of the algorithms devised to generate random numbers for these distributions. Additionally, the paper will feature multiple tables showcasing sets of random numbers across various degrees of indeterminacy. Upon thorough analysis, the results reveal a noticeable decline in random numbers as the degree of indeterminacy increases.

2. Neutrosophic uniform distribution

Let $x_{NU} = x_{NL} + x_{NU}I_{x_{NU}}$; $I_{x_{NU}} \in [I_{x_{LU}}, I_{x_{UU}}]$ be a neutrosophic random variable that follows the neutrosophic uniform distribution. Note that the first part x_{NL} denotes the determinate part, $x_{NU}I_{x_{NU}}$ the indeterminate part, and $I_{x_{NU}} \in [I_{x_{LU}}, I_{x_{UU}}]$ the degree of indeterminacy. Suppose $f(x_{NU}) = f(x_{LU}) + f(x_{UU})I_{NU}$; $I_{NU} \in [I_{LU}, I_{UU}]$ presents the neutrosophic probability density

function (npdf) of neutrosophic uniform distribution (NUD). Note that the npdf of NUD is based on two parts. The first part x_{NL} , $f(x_{LU})$ denotes the determinate part and presents the probability density function (pdf) of uniform distribution under classical statistics. The second part $x_{NU}I_{x_{NU}}$, $f(x_{UU})I_{NU}$ denotes the indeterminate part and $I_{x_{NU}} \in [I_{x_{LU}}, I_{x_{UU}}]$, $I_{NU} \in [I_{LU}, I_{UU}]$ are the measures of indeterminacy associated with neutrosophic random variable and the uniform distribution. The npdf of the uniform distribution by following [22] is given as

$$f(x_{NU}) = \left(\frac{1}{(b_L - a_L)}\right) + \left(\frac{1}{(b_U - a_U)}\right) I_{x_{NU}}; I_{x_{NU}} \in [I_{x_{LU}}, I_{x_{UU}}], a_N \leq x_{NU} \leq b_N, \quad (1)$$

where $b_N \in [b_L, b_U]$ and $a_N \in [a_L, a_U]$ are neutrosophic parameters of the NUD. The simplified form when $L = U = S_U$ of Eq (1) can be written as

$$f(x_{NS_U}) = \left(\frac{1}{(b_{NS} - a_{NS})}\right) (1 + I_{x_{NS}}); I_{x_{NS}} \in [I_{x_{LS}}, I_{x_{US}}], a_N \leq x_{NU} \leq b_N. \quad (2)$$

Note here that the npdf of uniform distribution is a generalization of pdf of the uniform distribution. The neutrosophic uniform distribution reduces to the classical uniform distribution when $I_{x_{UU}} = 0$. The neutrosophic cumulative distribution function (ncdf) of the neutrosophic uniform distribution is given by

$$F(x_{NU}) = \left(\frac{x_{NL} - a_L}{(b_L - a_L)}\right) + \left(\frac{x_{NU} - a_U}{(b_U - a_U)}\right) I_{x_{NU}}; I_{x_{NU}} \in [I_{x_{LU}}, I_{x_{UU}}], a_N \leq x_{NU} \leq b_N. \quad (3)$$

Note that the first part presents the cumulative distribution function (cdf) of the uniform distribution under classical statistics, and the second part is the indeterminate part associated with ncdf. The ncdf reduces to cdf when $I_{x_{UU}} = 0$. The simplified form of ncdf of the Uniform distribution when $L = U = S$ can be written as

$$F(x_{NS_U}) = \left(\frac{x_{NS} - a_{NS}}{(b_{NS} - a_{NS})}\right) (1 + I_{NS}); I_{NS} \in [I_{LS}, I_{US}], a_N \leq x_{NU} \leq b_N. \quad (4)$$

3. Neutrosophic Weibull distribution

Aslam [22] introduced the neutrosophic Weibull distribution (NWD) originally. The neutrosophic form of the Weibull distribution is expressed by

$$f(x_{NW}) = f(x_{LW}) + f(x_{UW})I_{NW}; I_{NW} \in [I_{LW}, I_{UW}]. \quad (5)$$

The following npdf of the Weibull distribution is taken from [22] and reported as

$$f(x_{NW}) = \left\{ \left(\frac{\beta}{\alpha}\right) \left(\frac{x_L}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x_L}{\alpha}\right)^\beta} \right\} + \left\{ \left(\frac{\beta}{\alpha}\right) \left(\frac{x_U}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x_U}{\alpha}\right)^\beta} \right\} I_{NW}; I_{NW} \in [I_{LW}, I_{UW}]. \quad (6)$$

The simplified form of the npdf of the Weibull distribution when $L = U = S_W$ is expressed by

$$f(x_{NS_W}) = \left\{ \left(\frac{\beta}{\alpha}\right) \left(\frac{x_S}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x_S}{\alpha}\right)^\beta} \right\} (1 + I_{NS}); I_{NS} \in [I_{LS}, I_{US}], \quad (7)$$

where α and β are the scale and shape parameters of the Weibull distribution. The npdf of the Weibull distribution reduces to pdf of the Weibull distribution when $I_{NS} = 0$. The ncdf of the Weibull distribution is expressed by

$$F(x_{NSW}) = 1 - \left\{ e^{-\left(\frac{x_{NSW}}{\alpha}\right)^\beta} (1 + I_{NW}) \right\} + I_{NW}; I_{NW} \in [I_{LW}, I_{UW}]. \quad (8)$$

The ncdf of the Weibull distribution reduces to cdf of the Weibull distribution under classical statistics when $I_{NW}=0$. The neutrosophic mean of the Weibull distribution is given as [22]

$$\mu_{NW} = \alpha \Gamma(1 + 1/\beta)(1 + I_{NW}); I_{NW} \in [I_{LW}, I_{UW}]. \quad (9)$$

The neutrosophic median of the Weibull distribution is given by

$$\tilde{\mu}_{NW} = \alpha (\ln(2))^{1/\beta} (1 + I_{NW}); I_{NW} \in [I_{LW}, I_{UW}]. \quad (10)$$

4. Simulation methodology

This section presents the methodology to generate random variates from the proposed neutrosophic uniform distribution and the neutrosophic Weibull distribution. Let $u_N \in [u_L, u_U]$ be a neutrosophic random uniform from $u_N \sim U_N([0,0], [1,1])$. The neutrosophic random numbers from NUD and NWD will be obtained as follows:

Let

$$u_N = F(x_{NU}) = \left(\frac{x_{NL} - a_L}{(b_L - a_L)} \right) + \left(\frac{x_{NU} - a_U}{(b_U - a_U)} \right) I_{x_{NU}}; I_{x_{NU}} \in [I_{x_{LU}}, I_{x_{UU}}], a_N \leq x_{NU} \leq b_N,$$

or

$$u_N = F(x_{NU}) = \left(\frac{x_{NS} - a_{NS}}{(b_{NS} - a_{NS})} \right) (1 + I_{NS}); I_{NS} \in [I_{LS}, I_{US}], a_N \leq x_{NU} \leq b_N.$$

The neutrosophic random numbers x_{NSU} from NWD can be obtained using the following Eq (11)

$$x_{NSU} = a_{NS} + \left(\frac{u_N}{(1 + I_{NS})} \right) (b_{NS} - a_{NS}); u_N \in [u_L, u_U], I_{NS} \in [I_{LS}, I_{US}]. \quad (11)$$

The random number from the Weibull distribution using classical statistics can be obtained when $I_{NS}=0$ using the following Eq (12)

$$x = a + u(b - a); a \leq x \leq b. \quad (12)$$

The neutrosophic random numbers from the NWD will be obtained using the following methodology.

Let

$$u_N = F(x_{NSW}) = 1 - \left\{ e^{-\left(\frac{x_{NSW}}{\alpha}\right)^\beta} (1 + I_{NW}) \right\} + I_{NW}; I_{NW} \in [I_{LW}, I_{UW}], u_N \in [u_L, u_U].$$

The neutrosophic random numbers from NWD can be obtained through the following expression

$$x_{NSW} = \alpha \left[-\ln \left(\frac{1-(u_N-I_{NW})}{1+I_{NW}} \right) \right]^{\frac{1}{\beta}}; I_{NW} \in [I_{LW}, I_{UW}], u_N \in [u_L, u_U]. \quad (13)$$

The NWD reduces to neutrosophic exponential distribution (NED) when $\beta = 1$. The neutrosophic random numbers from the NED can be obtained as follows:

$$x_{NSE} = -\alpha \ln \left(\frac{1-(u_N-I_{NW})}{1+I_{NW}} \right); I_{NW} \in [I_{LW}, I_{UW}], u_N \in [u_L, u_U]. \quad (14)$$

The random numbers from the Weibull distribution using classical statistics can be obtained as

$$x_{NSW} = -\alpha \ln(1-u)^{\frac{1}{\beta}}. \quad (15)$$

The random numbers from the exponential distribution using classical statistics can be obtained as

$$x_{NSW} = -\alpha \ln(1-u). \quad (16)$$

The following routine can be run to generate n random numbers from the NUD.

Step-1: Generate a uniform random number u_N from $u_N \sim U_N([0,0], [1,1])$.

Step-2: Fix the values of I_{NS} .

Step-3: Generate values of x_{NSU} using the expression

$$x_{NSU} = a_{NS} + \left(\frac{u_N}{(1+I_{NS})} \right) (b_{NS} - a_{NS}); u_N \in [u_L, u_U], I_{NS} \in [I_{LS}, I_{US}].$$

Step-4: From the routine, the first value of x_{NSU} will be generated.

Step-5: Repeat the routine k times to generate k random numbers from NUD.

The following routine can be run to generate n random numbers from the NUD.

Step-1: Generate a uniform random number u_N from $u_N \sim U_N([0,0], [1,1])$.

Step-2: Fix the values of I_{NS} , α and β .

Step-3: Generate values of x_{NSW} using the expression

$$x_{NSW} = \alpha \left[-\ln \left(\frac{1-(u_N-I_{NW})}{1+I_{NW}} \right) \right]^{\frac{1}{\beta}}; I_{NW} \in [I_{LW}, I_{UW}], u_N \in [u_L, u_U].$$

Step-4: From the routine, the first value of x_{NSW} will be generated.

Step-5: Repeat the routine k times to generate k random numbers from NWD.

4.1. Examples

To illustrate the proposed simulation methods, two examples will be discussed in this section.

4.1.1. Example 1

Suppose that x_{NSU} is a neutrosophic uniform random variable with parameters

([20,20],[30,30]) and a random variate x_{NSU} under indeterminacy is needed. To generate a random number from NUD, the following steps have been carried out.

Step-1: Generate a uniform random number $u_N = 0.05$ from $u_N \sim U_N([0,0], [1,1])$.

Step-2: Fix the values of $I_{NS} = 0.1$.

Step-3: Generate values of x_{NSU} using the expression $x_{NSU} = 20 + \left(\frac{0.05}{(1+0.1)}\right)(30 - 20) = 20.5$.

Step-4: From the routine, the first value of $x_{NSU} = 20.5$ will be generated.

Step-5: Repeat the routine k times to generate k random numbers from NUD.

4.1.2. Example 2

Step-1: Generate a uniform random number $u_N = 0.30$ from $u_N \sim U_N([0,0], [1,1])$.

Step-2: Fix the values of $I_{NS} = 0.20$, $\alpha = 5$, and $\beta = 0.5$.

Step-3: Generate values of x_{NSW} using the expression $x_{NSW} = 5 \left[-\ln \left(\frac{1-(u_N-I_{NW})}{1+I_{NW}} \right) \right]^{0.5} = 0.04$.

Step-4: From the routine, the first value of $x_{NSW} = 0.04$ will be generated.

Step-5: Repeat the routine k times to generate k random numbers from NWD.

5. Simulation study

In this section, random numbers are generated by simulation using the above-mentioned algorithms for NUD and NWD. To generate random numbers from NUD, several uniform numbers are generated from $u_N \sim U_N([0,0], [1,1])$ and placed in Tables 1 and 2. In Tables 1 and 2, several values of I_{NS} are considered to generate random numbers from the NUD. Table 1 is depicted by assuming that NUD has the parameters $a_{NS}=10$ and $b_{NS}=20$ and Table 2 is shown by assuming that NUD has the parameters $a_{NS}=20$ and $b_{NS}=30$. From Tables 1 and 2, the following trends can be noted in random numbers generated from NUD.

- 1) For fixed I_{NS} , $a_{NS}=10$ and $b_{NS}=20$, as the values of u increase from 0.05 to 0.95, there is an increasing trend in random numbers.
- 2) For fixed u , $a_{NS}=10$ and $b_{NS}=20$, as the values of I_{NS} increase from 0 to 1.1, there is a decreasing trend in random numbers.
- 3) For fixed values of u and I_{NS} , as the values of a_{NS} and b_{NS} increases, there is an increasing trend in random numbers.

Table 1. Random numbers from NUD when $a_{NS}=10$ and $b_{NS}=20$.

u	I_{NS}											
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1
0.05	10.5	10.45	10.42	10.38	10.36	10.33	10.31	10.29	10.28	10.26	10.3	10.2
0.1	11	10.91	10.83	10.77	10.71	10.67	10.63	10.59	10.56	10.53	10.5	10.5
0.15	11.5	11.36	11.25	11.15	11.07	11.00	10.94	10.88	10.83	10.79	10.8	10.7
0.2	12	11.82	11.67	11.54	11.43	11.33	11.25	11.18	11.11	11.05	11.0	11.0
0.25	12.5	12.27	12.08	11.92	11.79	11.67	11.56	11.47	11.39	11.32	11.3	11.2
0.3	13	12.73	12.50	12.31	12.14	12.00	11.88	11.76	11.67	11.58	11.5	11.4
0.35	13.5	13.18	12.92	12.69	12.50	12.33	12.19	12.06	11.94	11.84	11.8	11.7

Continued on next page

u	I_{NS}											
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1
0.4	14	13.64	13.33	13.08	12.86	12.67	12.50	12.35	12.22	12.11	12.0	11.9
0.45	14.5	14.09	13.75	13.46	13.21	13.00	12.81	12.65	12.50	12.37	12.3	12.1
0.5	15	14.55	14.17	13.85	13.57	13.33	13.13	12.94	12.78	12.63	12.5	12.4
0.55	15.5	15.00	14.58	14.23	13.93	13.67	13.44	13.24	13.06	12.89	12.8	12.6
0.6	16	15.45	15.00	14.62	14.29	14.00	13.75	13.53	13.33	13.16	13.0	12.9
0.65	16.5	15.91	15.42	15.00	14.64	14.33	14.06	13.82	13.61	13.42	13.3	13.1
0.7	17	16.36	15.83	15.38	15.00	14.67	14.38	14.12	13.89	13.68	13.5	13.3
0.75	17.5	16.82	16.25	15.77	15.36	15.00	14.69	14.41	14.17	13.95	13.8	13.6
0.8	18	17.27	16.67	16.15	15.71	15.33	15.00	14.71	14.44	14.21	14.0	13.8
0.9	19	18.18	17.50	16.92	16.43	16.00	15.63	15.29	15.00	14.74	14.5	14.3
0.95	19.5	18.64	17.92	17.31	16.79	16.33	15.94	15.59	15.28	15.00	14.8	14.5

Table 2. Random numbers from NUD when $a_{NS}=20$ and $b_{NS}=30$.

u	I_{NS}											
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1
0.05	20.5	20.5	20.4	20.4	20.4	20.3	20.3	20.3	20.3	20.3	20.3	20.2
0.1	21.0	20.9	20.8	20.8	20.7	20.7	20.6	20.6	20.6	20.5	20.5	20.5
0.15	21.5	21.4	21.3	21.2	21.1	21.0	20.9	20.9	20.8	20.8	20.8	20.7
0.2	22.0	21.8	21.7	21.5	21.4	21.3	21.3	21.2	21.1	21.1	21.0	21.0
0.25	22.5	22.3	22.1	21.9	21.8	21.7	21.6	21.5	21.4	21.3	21.3	21.2
0.3	23.0	22.7	22.5	22.3	22.1	22.0	21.9	21.8	21.7	21.6	21.5	21.4
0.35	23.5	23.2	22.9	22.7	22.5	22.3	22.2	22.1	21.9	21.8	21.8	21.7
0.4	24.0	23.6	23.3	23.1	22.9	22.7	22.5	22.4	22.2	22.1	22.0	21.9
0.45	24.5	24.1	23.8	23.5	23.2	23.0	22.8	22.6	22.5	22.4	22.3	22.1
0.5	25.0	24.5	24.2	23.8	23.6	23.3	23.1	22.9	22.8	22.6	22.5	22.4
0.55	25.5	25.0	24.6	24.2	23.9	23.7	23.4	23.2	23.1	22.9	22.8	22.6
0.6	26.0	25.5	25.0	24.6	24.3	24.0	23.8	23.5	23.3	23.2	23.0	22.9
0.65	26.5	25.9	25.4	25.0	24.6	24.3	24.1	23.8	23.6	23.4	23.3	23.1
0.7	27.0	26.4	25.8	25.4	25.0	24.7	24.4	24.1	23.9	23.7	23.5	23.3
0.75	27.5	26.8	26.3	25.8	25.4	25.0	24.7	24.4	24.2	23.9	23.8	23.6
0.8	28.0	27.3	26.7	26.2	25.7	25.3	25.0	24.7	24.4	24.2	24.0	23.8
0.9	29.0	28.2	27.5	26.9	26.4	26.0	25.6	25.3	25.0	24.7	24.5	24.3
0.95	29.5	28.6	27.9	27.3	26.8	26.3	25.9	25.6	25.3	25.0	24.8	24.5

The random numbers for NWD are generated using the algorithm discussed in the last section. The random numbers for various values of u , I_{NS} , α , and β are considered. The random numbers when $\alpha = 5$ and $\beta = 0$ are shown in Table 3. The random numbers when $\alpha = 5$ and $\beta = 1$ are shown in Table 4. The random numbers when $\alpha = 5$ and $\beta = 2$ are shown in Table 5.

From Tables 3–5, the following trends can be noted in random numbers generated from NUD.

- 1) For fixed I_{NS} , $\alpha = 5$ and $\beta = 0.5$, as the values of u increase from 0.05 to 0.95, there is an increasing trend in random numbers generated from NWD.

- 2) For fixed u , $\alpha = 5$, and $\beta = 0.5$, as the values of I_{NS} increase from 0 to 1.1, there is an increasing trend in random numbers.
- 3) For fixed values of I_{NS} and α , as the values of β increase, there is an increasing trend in random numbers.

Table 3. Random numbers from NUD when $\alpha = 5$ and $\beta = 0.5$.

u	I_{NS}											
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1
0.05	0.01	0.01	0.07	0.15	0.23	0.31	0.38	0.43	0.48	0.52	0.56	0.58
0.1	0.06	0.00	0.03	0.10	0.18	0.25	0.32	0.38	0.43	0.48	0.51	0.54
0.15	0.13	0.01	0.01	0.06	0.13	0.20	0.27	0.33	0.39	0.43	0.47	0.51
0.2	0.25	0.05	0.00	0.03	0.08	0.15	0.22	0.28	0.34	0.39	0.43	0.47
0.25	0.41	0.11	0.01	0.01	0.05	0.11	0.18	0.24	0.30	0.35	0.39	0.43
0.3	0.64	0.21	0.04	0.00	0.02	0.07	0.13	0.20	0.25	0.31	0.35	0.39
0.35	0.93	0.34	0.09	0.01	0.01	0.04	0.10	0.16	0.21	0.27	0.31	0.36
0.4	1.30	0.53	0.17	0.03	0.00	0.02	0.06	0.12	0.17	0.23	0.28	0.32
0.45	1.79	0.77	0.29	0.08	0.01	0.01	0.04	0.09	0.14	0.19	0.24	0.28
0.5	2.40	1.08	0.44	0.15	0.03	0.00	0.02	0.06	0.11	0.16	0.21	0.25
0.55	3.19	1.48	0.64	0.24	0.07	0.01	0.00	0.03	0.08	0.12	0.17	0.22
0.6	4.20	1.99	0.91	0.38	0.13	0.02	0.00	0.02	0.05	0.10	0.14	0.19
0.65	5.51	2.63	1.24	0.55	0.21	0.06	0.01	0.00	0.03	0.07	0.11	0.16
0.7	7.25	3.47	1.67	0.77	0.32	0.11	0.02	0.00	0.01	0.05	0.09	0.13
0.75	9.61	4.55	2.21	1.06	0.47	0.18	0.05	0.00	0.00	0.03	0.06	0.10
0.8	12.95	5.99	2.92	1.42	0.67	0.28	0.10	0.02	0.00	0.01	0.04	0.08
0.9	26.51	10.70	5.03	2.48	1.23	0.58	0.25	0.09	0.02	0.00	0.01	0.04
0.95	44.87	14.87	6.67	3.26	1.63	0.79	0.36	0.14	0.04	0.00	0.00	0.02

Table 4. Random numbers from NUD when $\alpha = 5$ and $\beta = 1$.

u	I_{NS}											
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1
0.05	0.26	-	-	-	-	-	-	-	-	-	-	-
0.1	0.53	0.00	-	-	-	-	-	-	-	-	-	-
0.15	0.81	0.23	-	-	-	-	-	-	-	-	-	-
0.2	1.12	0.48	0.00	-	-	-	-	-	-	-	-	-
0.25	1.44	0.74	0.21	-	-	-	-	-	-	-	-	-
0.3	1.78	1.01	0.44	0.00	-	-	-	-	-	-	-	-
0.35	2.15	1.31	0.68	0.20	-	-	-	-	-	-	-	-
0.4	2.55	1.62	0.93	0.41	0.00	-	-	-	-	-	-	-
0.45	2.99	1.96	1.20	0.63	0.18	-	-	-	-	-	-	-
0.5	3.47	2.32	1.49	0.86	0.38	0.00	-	-	-	-	-	-
0.55	3.99	2.72	1.79	1.11	0.58	0.17	-	-	-	-	-	-
0.6	4.58	3.15	2.13	1.37	0.80	0.35	0.00	-	-	-	-	-

Continued on next page

u	I_{NS}											
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1
0.65	5.25	3.63	2.49	1.66	1.03	0.54	0.16	-	-	-	-	-
0.7	6.02	4.16	2.89	1.96	1.27	0.74	0.33	0.00	-	-	-	-
0.75	6.93	4.77	3.33	2.30	1.54	0.96	0.51	0.15	-	-	-	-
0.8	8.05	5.47	3.82	2.67	1.82	1.19	0.70	0.31	0.00	-	-	-
0.9	11.51	7.32	5.02	3.52	2.48	1.70	1.11	0.66	0.29	0.00	-	-
0.95	14.98	8.62	5.78	4.04	2.85	1.99	1.35	0.85	0.45	0.13	-	-

Table 5. Random numbers from NUD when $\alpha = 5$ and $\beta = 2$.

u	I_{NS}											
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1
0.05	1.13	-	-	-	-	-	-	-	-	-	-	-
0.1	1.62	0.00	-	-	-	-	-	-	-	-	-	-
0.15	2.02	1.08	-	-	-	-	-	-	-	-	-	-
0.2	2.36	1.55	0.00	-	-	-	-	-	-	-	-	-
0.25	2.68	1.92	1.03	-	-	-	-	-	-	-	-	-
0.3	2.99	2.25	1.48	0.00	-	-	-	-	-	-	-	-
0.35	3.28	2.56	1.84	0.99	-	-	-	-	-	-	-	-
0.4	3.57	2.85	2.16	1.42	0.00	-	-	-	-	-	-	-
0.45	3.87	3.13	2.45	1.77	0.96	-	-	-	-	-	-	-
0.5	4.16	3.41	2.73	2.07	1.37	0.00	-	-	-	-	-	-
0.55	4.47	3.69	3.00	2.35	1.70	0.92	-	-	-	-	-	-
0.6	4.79	3.97	3.26	2.62	2.00	1.33	0.00	-	-	-	-	-
0.65	5.12	4.26	3.53	2.88	2.27	1.65	0.90	-	-	-	-	-
0.7	5.49	4.56	3.80	3.13	2.52	1.93	1.28	0.00	-	-	-	-
0.75	5.89	4.88	4.08	3.39	2.77	2.19	1.59	0.87	-	-	-	-
0.8	6.34	5.23	4.37	3.65	3.02	2.44	1.87	1.24	0.00	-	-	-
0.9	7.59	6.05	5.01	4.20	3.52	2.92	2.36	1.81	1.21	0.00	-	-
0.95	8.65	6.57	5.37	4.49	3.78	3.16	2.59	2.06	1.50	0.82	-	-

The algorithms to generate the random variables from NUD and NWD are depicted in Figures 1 and 2.

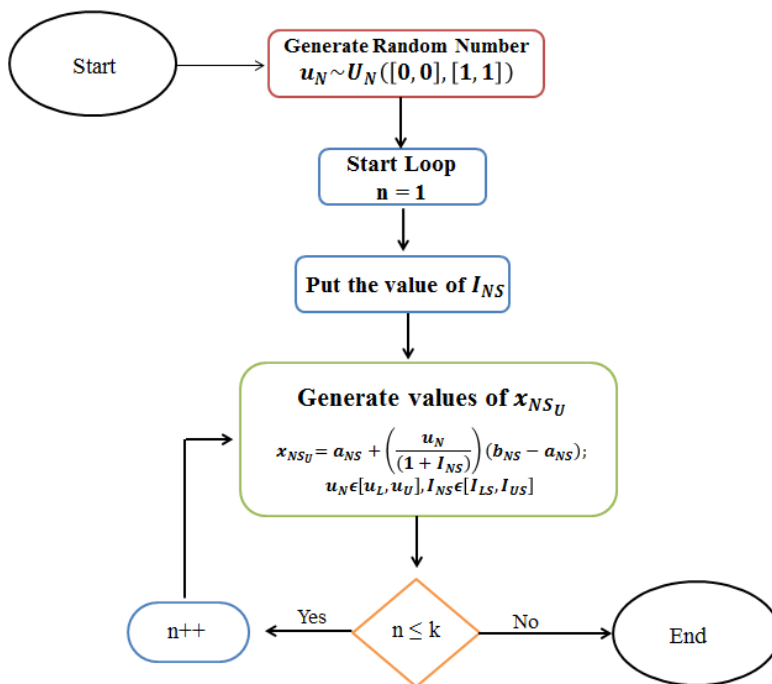


Figure 1. Algorithm to generate random numbers from NUD.

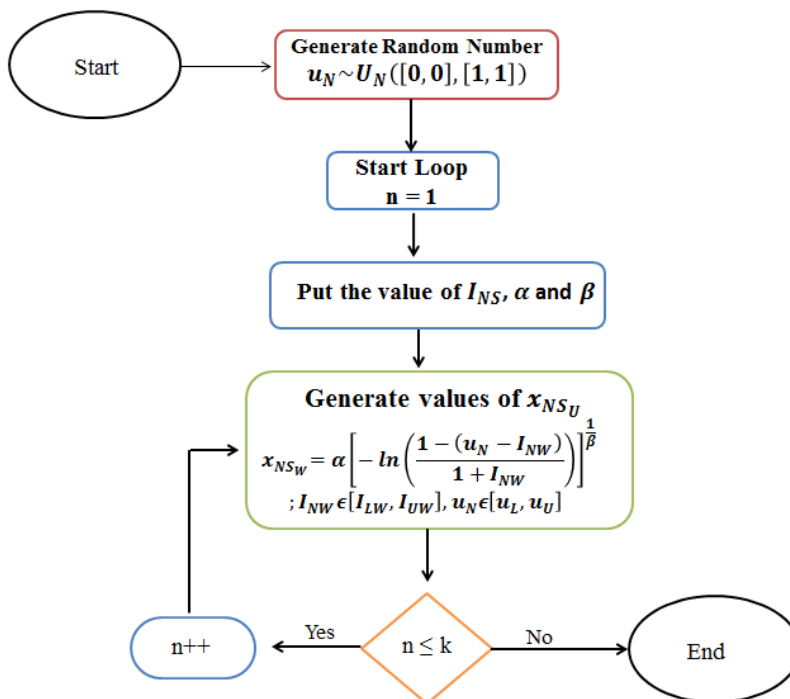


Figure 2. Algorithm to generate random numbers from NWD.

6. Comparative studies

In this section, the performance of simulations using classical simulation and neutrosophic

simulation will be discussed using the random numbers from the NUD and the NWD distribution. As explained earlier, the proposed simulation method under neutrosophy will be reduced to the classical simulation method under classical statistics when no uncertainty is found in the data. To study the behavior of random numbers, random numbers from NUD when $I_{NS} = 1.1$, $a_{NS}=20$, and $b_{NS}=30$ are considered and depicted in Figure 3. In Figure 3, it can be seen that the curve of random numbers from the classical simulation is higher than the curve of random numbers from the neutrosophic simulation. From Figure 3, it is clear that the proposed neutrosophic simulation method gives smaller values of random numbers than the random numbers generated by the neutrosophic simulation method. The random numbers from NWD when $I_{NS} = 0.9$, $\alpha = 5$, and $\beta = 0.5$ are considered and their curves are shown in Figure 4. From Figure 4, it can be seen that random numbers generated by neutrosophic simulation are smaller than the random numbers generated by the classical simulation method under classical statistics. The random numbers generated by the neutrosophic simulation are close to zero. The random numbers from NWD when $I_{NS} = 0.1$, $\alpha = 5$, and $\beta = 1$ (exponential distribution) are considered and their curves are shown in Figure 5. From Figure 5, it can be seen that the curve of random numbers generated by neutrosophic simulation is lower than the curve of random numbers generated by the classical simulation method under classical statistics. The random numbers from NWD when $I_{NS} = 0.1$, $\alpha = 5$, and $\beta = 2$ are considered and their curves are shown in Figure 6. From Figure 6, it can be seen that the curve of random numbers generated by neutrosophic simulation is lower than the curve of random numbers generated by the classical simulation method under classical statistics. From Figures 4–6, it can be concluded that the proposed simulation gives smaller values of random numbers as compared to the classical simulation method under classical statistics.

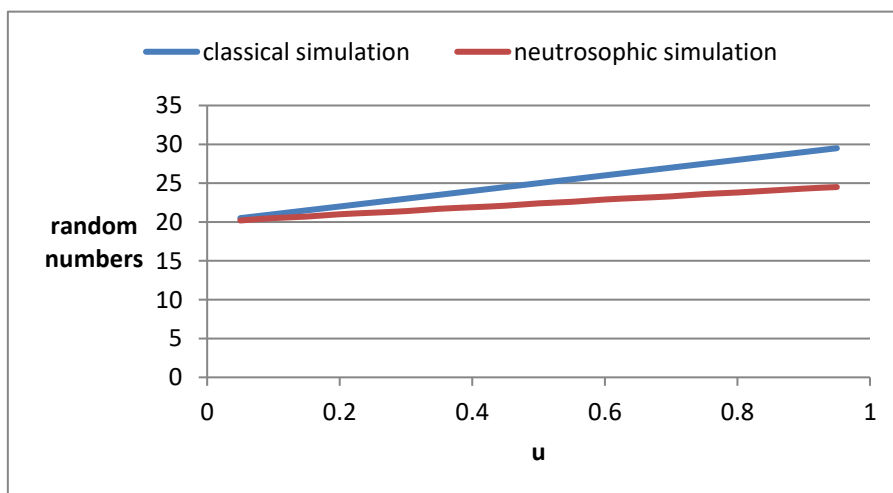


Figure 3. Random numbers behavior from NUD when $I_{NS} = 1.1$, $a_{NS}=20$, and $b_{NS}=30$.

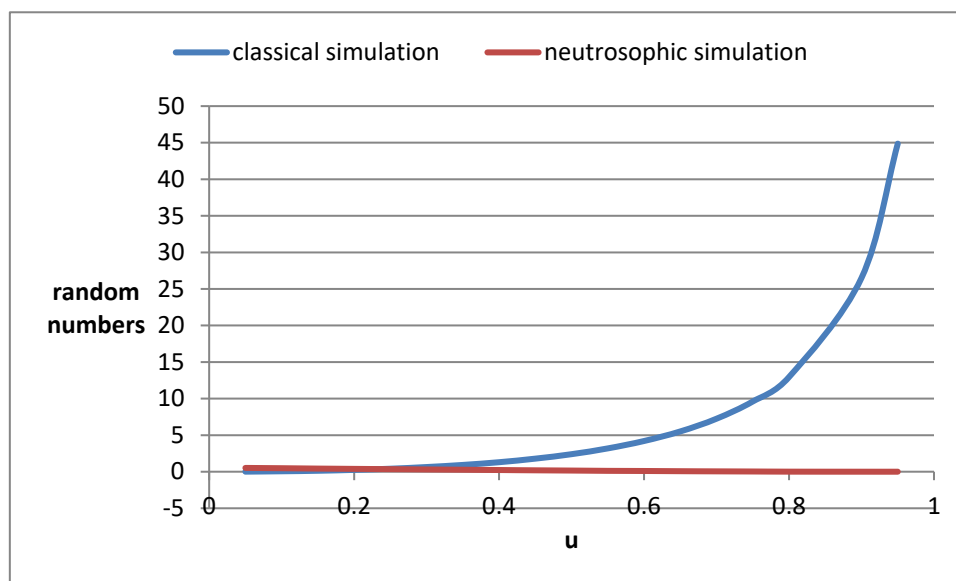


Figure 4. Random numbers behavior from NWD when $I_{NS} = 0.9$, and when $\alpha = 5$, and $\beta = 0.5$.

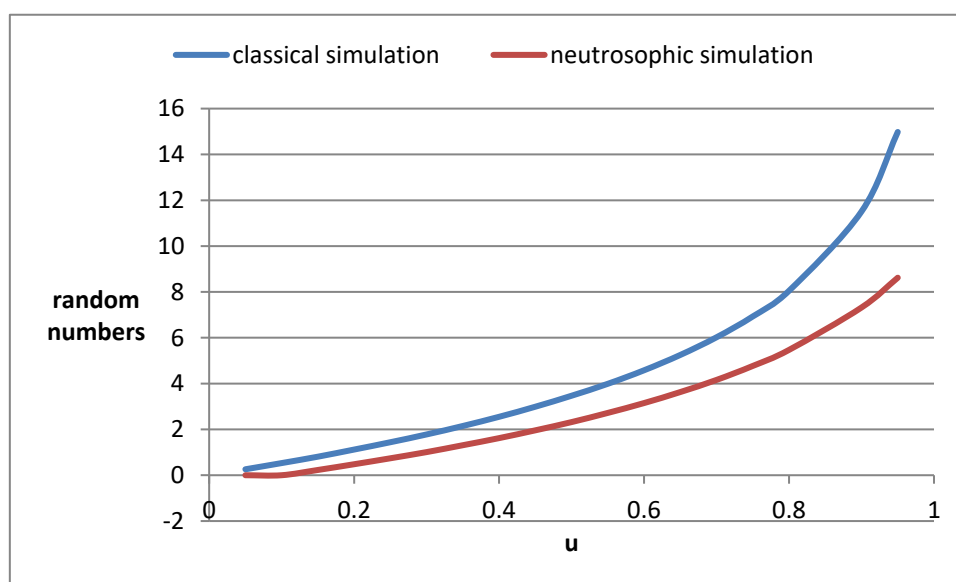


Figure 5. Random numbers behavior from NWD when $I_{NS} = 0.1$, and when $\alpha = 5$, and $\beta = 1$.

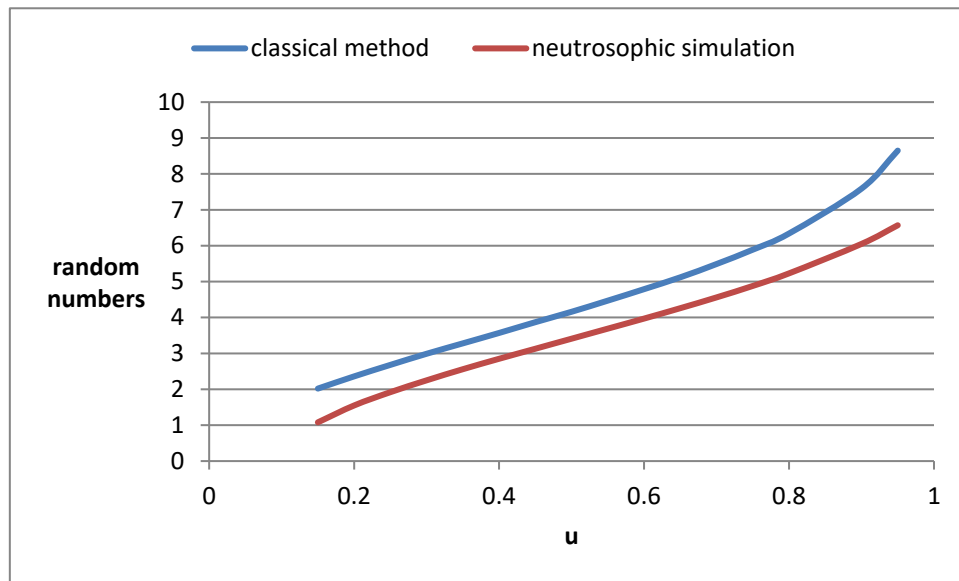


Figure 6. Random numbers behavior from NWD when $I_{NS} = 0.1$, and when $\alpha = 5$, and $\beta = 2$.

7. Discussion

The simulation method under neutrosophic statistics and classical methods was discussed in the last sections. From Tables 1 and 2, it can be seen that random numbers from the NUD can be generated when $I_{NS} < 1$, $I_{NS} = 1$ and $I_{NS} > 1$. On the other hand, the random numbers from the NWD can be generated for $I_{NS} < 1$, $I_{NS} = 1$, and $I_{NS} > 1$ when the shape parameter $\beta < 1$. From Table 4 and 5, it can be noted that for several cases, the NWD generates negative results or random numbers do not exist. Based on the simulation studies, it can be concluded that the NWD generates random numbers $I_{NS} < 1$, $I_{NS} = 1$, and $I_{NS} > 1$ only when $\beta < 1$. To generate random numbers from NWD when $\beta \geq 1$, the following expression will be used

$$x_{NSW} = \alpha \left[-\ln \left(\frac{1-u_N + I_{NW}}{1+I_{NW}} \right) \right]^{\frac{1}{\beta}}; 1 - u_N + I_{NW} \geq 0.$$

8. Concluding remarks

In this paper, we initially introduced the NUD and presented a novel method for generating random numbers from both NUD and the NWD. We also introduced algorithms for generating random numbers within the context of neutrosophy. These algorithms were applied to generate random numbers from both distributions using various parameters. We conducted an extensive discussion on the behavior of these random numbers, observing that random numbers generated under neutrosophy tend to be smaller than those generated under uncertain environments. It is worth noting that generating random numbers from computers is a common practice. Tables 1–5 within this paper offer valuable insights into how the degree of determinacy influences random number generation. Additionally, these tables can be utilized for simulation purposes in fields marked by uncertainty, such as reliability, environmental studies, and medical science. From our study, we conclude that the proposed method for generating random numbers from NUD and NWD can be effectively applied in complex scenarios. In future research, exploring the statistical properties of the

proposed NUD would be advantageous. Additionally, investigating the proposed algorithm utilizing the accept-reject method could be pursued as a future research avenue. Moreover, there is potential to develop algorithms using other statistical distributions for further investigation.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflicts of interest.

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