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Research article

Soft rough fuzzy sets based on covering

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Abstract: Soft rough fuzzy sets ($SRFS\,s$) represent a powerful paradigm that integrates soft computing, rough set theory, and fuzzy logic. This research aimed to comprehensively investigate the various dimensions of $SRFS\,s$ within the domain of approximation structures. The study encompassed a wide spectrum of concepts, ranging from covering approximation structures and soft rough coverings to soft neighborhoods, fuzzy covering approximation operators, and soft fuzzy covering approximation operators. We introduced three models of $SRFS\,s$ based on covering via the core of soft neighborhood. We discussed and analyzed our models' characteristics and properties. The relations between our models for soft fuzzy covering sets and Zhan's model for soft rough fuzzy covering were presented.

Keywords: core soft neighborhood; soft rough; covering; fuzzy logic; mathematical modeling
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1. Introduction

Pawlak introduced the rough set theory [1, 2] as a conceptual framework designed to address the challenges posed by vagueness and uncertainty inherent in data analysis and information systems. Atef et al. [3] discussed the generalization of three types of rough set models based on j-neighborhood structure and explored some of their basic properties. Alcantud et al. [4] introduced a new model that combines multi-granularity, soft set, and rough set-based overlays. It strives to provide a hybrid model that captures the strengths of each theory and can be applied to a variety of industries. Another approximation space based on topological near open sets and the properties of these spaces is presented by Mareay et al. [5]. It also includes an algorithm to detect the side effects of the COVID-19 infection. Azzam et al. [6] discussed a proposed reduction method based on similarity relations and pretopology concepts, as well as new pretopological structures for creating information systems. Hu et al. [7] discussed the combination of kernel methods and rough sets in machine learning. It also proposes a fuzzy rough set model and a Gaussian kernel approximation algorithm for feature ranking and reduction. Rough sets and their applications have attracted many researchers in different fields. Stefania Boffa discussed "Sequences of Refinements of Rough Sets". The idea discussed in the thesis is known as "sequences of orthopairs" within the generalized hard set theory. This notion aims to establish operations between sequences of orthopairs and explore methods for generating them based on operations related to common rough sets [8]. The writer demonstrates multiple representation theorems for the class of finite centered Kleene algebras [9].

Covering based rough set involves the utilization of sets to approximate other sets [10–12]. Bonikowski et al. [13] put forth a new model of covering sets via minimal description concepts. Many other models of covering rough set depending on the neighborhood and the complementary neighborhood are proposed in [14–16]. An intuitionistic fuzzy set (IFS) on the structure of rough sets based on covering is introduced by using the notion of the neighborhood in [17]. It defines three models of IFS approximation structure based on covering. Interestingly, any covering can be linked to a tolerance relation, and vice versa. This technique leaves the upper approximations in rough set theory unchanged. An axiomatic description of the second type of covering higher approximations is also given. Tolerance relations and coverings are powerful tools for comprehending the structure and interactions within sets. Partitions and overlapping covers are mathematical constructions that assist in understanding similarity and discernment [18].

In 1990, Dubois and Prade [19] introduced rough fuzzy sets and fuzzy rough sets, which marked a significant development in the field. Scholars have extensively explored these concepts, as evidenced by studies conducted by various researchers [20–25]. Deng et al. [22] introduced fuzzy covering based on fuzzy relations in 2007, while Ma [23] devised two categories of fuzzy covering rough sets in 2016 using fuzzy β-neighborhood. In 2017, Yang and Hu [24] established various types of fuzzy covering-based rough sets through fuzzy β-neighborhood. Hu [26] conducted a comprehensive study in 2019, investigating four types of fuzzy neighborhood operators and their properties by introducing the concept of fuzzy β-minimal description. Deer et al. [27] delved into fuzzy neighborhoods based on fuzzy covering.

In the realm of soft computing, the synergy of soft sets, rough sets, and fuzzy sets has emerged as a pivotal area of research, providing a nuanced framework for handling uncertainties in diverse domains. The foundational principles of soft set theory were initially formulated by Molodtsov [28] as a versatile mathematical framework tailored to address vagueness and uncertainty. Subsequently, an expanding body of research has explored the properties and advantages of soft set theory [29, 30].

$SRFS\,s$ incorporate elements from three distinct mathematical frameworks: Rough sets, fuzzy sets, and soft sets. In this hybrid model, we examine uncertainty, ambiguity, and indiscernibility at the same time. SRFS enables us to manage imprecise data and make more flexible decisions. Researchers investigated many algebraic structures and operations within this paradigm [31]. The SRFS model builds on classical rough set theory by including fuzzy membership degrees. It addresses ambiguity and gentle transitions between different granularities. By combining rough set approximations with fuzzy membership functions, this paradigm improves our capacity to analyze complex data. Applications include data mining, pattern recognition, and decision-making [32]. Zhan [33] introduced the concept of soft fuzzy rough set-based covering through the notion of soft neighborhoods. Zhan's model for soft rough fuzzy coverings stands at the forefront of this convergence, offering a distinctive perspective on the interplay between soft, rough, and fuzzy characteristics. The relations between models for soft rough fuzzy covering sets and Zhan's model for SRF covering must be investigated in order to comprehend their synergy. These relations shed light on how various mathematical constructs can be combined or related. Researchers explore the implications of these connections and their practical applications in [34].

Throughout in this paper, we analyze and enhance Zhan's model for soft rough fuzzy coverings, emphasizing the interplay between soft, rough, and fuzzy characteristics and developing mathematical formulations and algorithms for accurate representation and manipulation. For Zhan's model, we increase the lower approximation while simultaneously reducing the upper approximation to make it more accurate. Furthermore, we introduce three novel models of $\mathcal{SRFS}s$, strategically grounded in the concept of covering via the core of the neighborhood concept. These additions are crafted to address specific challenges and intricacies encountered in real-world applications, where the management of uncertainties is paramount.

First, we present the basic concepts of rough sets and soft sets. Second, a new model of $\mathcal{SRFS}s$ based on covering is introduced in Section 3 by using the core of the neighborhood concept. We put forth new other two models of soft rough fuzzy covering $\mathcal{SRFC}$ based on neighborhood in Section 4 and established the relations between our models and Zhan's model. The conclusions are presented in Section 5.

2. Basic concepts of rough sets and soft set theory

Along this section, consider $\mathcal{R}$ as an equivalence relation on a nonempty set $\mathcal{U}$. Hence, $\mathcal{U}/\mathcal{R} = \{\mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}_3, ..., \mathcal{Y}_m\}$ is a partition on $\mathcal{U}$, where $\mathcal{R}$ is an equivalence relation that generates the classes of equivalence $\mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}_3, ..., \mathcal{Y}_m$. For the soft set, consider $\mathcal{U}$ is a universe set, $\mathcal{A}$ is a set of parameters on $\mathcal{U}$, $\mathcal{P}(\mathcal{U})$ is the power set of $\mathcal{U}$, and we fix a soft set $\mho_G = (\mathcal{F}, \mathcal{A})$ over $\mathcal{U}$.

Definition 2.1. *[35] If $X_1 \subseteq \mathcal{U}$ and $\mathcal{U} \neq \phi$, then the set of approximation operators lower (upper) is defined as : $\underline{\mathcal{R}}(X_1) = \bigcup\{\mathcal{Y}_i \in \mathcal{U}/\mathcal{R} : \mathcal{Y}_i \subseteq X_1\}$. $\overline{\mathcal{R}}(X_1) = \bigcup\{\mathcal{Y}_i \in \mathcal{U}/\mathcal{R} : \mathcal{Y}_i \cap X_1 \neq \emptyset\}$, respectively.*

Proposition 2.1. *[35] If $\mathcal{K} = (\mathcal{U}, \mathcal{R})$ is an approximation structure, then the following axioms hold for $Q_1, Q_2 \subseteq \mathcal{U}$:*

(I) $\underline{\mathcal{R}}(Q_1) = Q_1$, $\overline{\mathcal{R}}(Q_1) = Q_1$;
(II) $\underline{\mathcal{R}}(\emptyset) = \emptyset$, $\overline{\mathcal{R}}(\emptyset) = \emptyset$;
(III) $\underline{\mathcal{R}}(Q_1) \subseteq Q_1 \subseteq \overline{\mathcal{R}}(Q_1)$;
(IV) $\underline{\mathcal{R}}(Q_1 \cap Q_2) = \underline{\mathcal{R}}(Q_1) \cap \underline{\mathcal{R}}(Q_2)$;
(V) $\overline{\mathcal{R}}(Q_1 \cup Q_2) = \overline{\mathcal{R}}(Q_1) \cup \overline{\mathcal{R}}(Q_2)$;
(VI) $\underline{\mathcal{R}}(Q_1^c) = [\overline{\mathcal{R}}(Q_1)]^c$, *where* (Q_1^c) *is the complement of* Q_1;
(VII) $\underline{\mathcal{R}}(\underline{\mathcal{R}}(Q_1)) = \underline{\mathcal{R}}(Q_1)$;
(VIII) $\overline{\mathcal{R}}(\overline{\mathcal{R}}(Q_1)) = \overline{\mathcal{R}}(Q_1)$;
(IX) $Q_1 \subseteq Q_2 \Rightarrow \underline{\mathcal{R}}(Q_1) \subseteq \underline{\mathcal{R}}(Q_2)$ *and* $\overline{\mathcal{R}}(Q_1) \subseteq \overline{\mathcal{R}}(Q_2)$;
(X) $\underline{\mathcal{R}}(\underline{\mathcal{R}}(Q_1))^c = (\underline{\mathcal{R}}(Q_1))^c$, $\overline{\mathcal{R}}(\overline{\mathcal{R}}(Q_1))^c = (\overline{\mathcal{R}}(Q_1))^c$;
(XI) $\underline{\mathcal{R}}(Q_1) \cup \underline{\mathcal{R}}(Q_2) \subseteq \underline{\mathcal{R}}(Q_1 \cup Q_2)$;

(XII) $\overline{\mathcal{R}}(Q_1 \cap Q_2) \subseteq \overline{\mathcal{R}}(\mathcal{Y}) \cap \overline{\mathcal{R}}(Q_2)$.

Definition 2.2. *[13] Consider C is a family of subsets of the universe $\mathcal{U}$. We call C a covering of $\mathcal{U}$ if $\cup C = \mathcal{U}$, where no subset in C is empty.*

Definition 2.3. *[13,36] Assume that C is a covering of the nonempty set $\mathcal{U}$, so the structure $< \mathcal{U}, C >$ is a rough approximation structure based on covering.*

Definition 2.4. *[13, 36] Suppose that $< \mathcal{U}, C >$ is covering rough approximation structure and let $\mho_1 \in \mathcal{U}$. Hence, the set family $Md(\mho_1)$ is called the minimal description of $\mho_1$, since $Md(\mho_1) = \{\omega \in C : \mho_1 \in \omega \wedge (\forall \mathcal{S} \in C \wedge \mho_1 \in \mathcal{S} \wedge \mathcal{S} \subseteq \omega \Rightarrow \omega = \mathcal{S})\}$.*

Definition 2.5. *[37] Let $\mathcal{F} : \mathcal{A} \to \mathcal{P}(U)$, so the structure $\mho_G =< \mathcal{F}, \mathcal{A} >$ is called a soft set on $\mathcal{U}$. If $\bigcup_{e \in \mathcal{A}} \mathcal{F}(e) = \mathcal{U}$, then the soft set is full soft set.*

Definition 2.6. *[38] Let $\mho_G = (\mathcal{F}, \mathcal{A})$ be a soft set over $\mathcal{U}$. The structure $\mathcal{S} = (\mathcal{U}, \mho_G)$ is called a soft covering approximation structure ($\mathcal{SCAS}$) based on $\mathcal{S}$.*

Definition 2.7. *[33] If $\mathcal{S} = (\mathcal{U}, \mho_G)$ is a soft covering approximation structure, then the soft neighborhood of $x \in \mathcal{U}$ is defined as follows:*
$\mathscr{N}_s(x) = \cap\{\mathcal{F}(e) : x \in \mathcal{F}(e)\}$.

Definition 2.8. *[33] If $\mathcal{S}$ is a soft covering approximation structure and $\mathscr{A} \in \mathscr{F}(\mathcal{U})$, then the two operators:*
$\aleph^{-0}(\mathscr{A})(x) = \wedge\{\mathscr{A}(y) : y \in \mathscr{N}_s(x)\}$,
$\aleph^{+0}(\mathscr{A})(x) = \vee\{\mathscr{A}(y) : y \in \mathscr{N}_s(x)\}$, *for all* $x \in \mathcal{U}$
are called the soft fuzzy covering lower (upper) approximation structure $\mathcal{SFCLA} - 0$ ($\mathcal{SFCUA} - 0$), respectively.

Clearly, the set $\mathscr{A}$ is called a soft rough covering-based fuzzy set ($\mathcal{SRCF} - 0$) if $\aleph^{-0}(\mathscr{A}) \neq \aleph^{+0}(\mathscr{A})$. Otherwise, the set $\mathscr{A}$ is definable.

During this research, we will express that $\aleph^{-i}(\mathscr{A})$ ($\aleph^{+i}(\mathscr{A})$) is the i type of $\mathcal{SFCLA}$ ($\mathcal{SFCUA}$) as $\mathcal{SFCLA} - i$ ($\mathcal{SFCUA} - i$), and if $\aleph^{-i}(\mathscr{A}) \neq \aleph^{+i}(\mathscr{A})$, then the set $\mathscr{A}$ is called ($\mathcal{SRCF} - i$). Otherwise, the set $\mathscr{A}$ is definable.

3. A new model of $\mathcal{SRFS}s$ based on covering

In this section, we introduce new models of $\mathcal{SRFS}s$ based on covering by the core of soft neighborhood. We present the properties of the new models along with some illustrative examples.

Definition 3.1. *Consider that $(\mathcal{U}, \mho_G)$ is a soft rough covering approximation structure ($\mathcal{SRCAS}$) where we fix the soft set $\mho_G =< \mathcal{F}, \mathcal{A} >$, then $\forall x \in \mathcal{U}$ the core of the soft neighborhood is defined as* $\mathscr{C}\mathscr{N}_s(x) = \{y \in \mathcal{U} : \mathscr{N}_s(x) = \mathscr{N}_s(y)\}$.

Example 3.1. *Consider that $(U, \mho_G)$ is a $\mathcal{SRCAS}$ where $\mathcal{U} = \{\mho_1, \mho_2, \mho_3, \mho_4, \mho_5, \mho_6\}$, $C = \{\{\mho_1, \mho_2\}, \{\mho_1, \mho_2, \mho_3\}, \{\mho_1, \mho_2, \mho_4, \mho_5\}, \{\mho_3, \mho_4, \mho_5, \mho_6\}, , \{\mho_3, \mho_5, \mho_6\}\}$, and $\mho_G =< \mathcal{F}, \mathcal{A} >$is a soft set defined in Table 1.*

Table 1. Tabular representation of the soft set $\mho_G =\prec \mathcal{F}, \mathcal{A} \succ$.

$\mathcal{U}$	a_1	a_2	a_3	a_4	a_5
$\mho_1$	1	1	1	0	0
$\mho_2$	1	1	1	0	0
$\mho_3$	0	1	0	1	1
$\mho_4$	0	0	1	1	0
$\mho_5$	0	0	1	1	1
$\mho_6$	0	0	0	1	1

From Table 1, the soft neighborhood and the core of the soft neighborhood are computed as follows: $\mathscr{N}_s(\mho_1) = \{\mho_1, \mho_2\}$, $\mathscr{N}_s(\mho_2) = \{\mho_1, \mho_2\}$, $\mathscr{N}_s(\mho_3) = \{\mho_3\}$, $\mathscr{N}_s(\mho_4) = \{\mho_4, \mho_5\}$, $\mathscr{N}_s(\mho_5) = \{\mho_5\}$,$\mathscr{N}_s(\mho_6) = \{\mho_3, \mho_5, \mho_6\}$. Therefore, $\mathscr{C}\mathscr{N}_s(\mho_1) = \{\mho_1, \mho_2\}$,$\mathscr{C}\mathscr{N}_s(\mho_2) = \{\mho_1, \mho_2\}$, $\mathscr{C}\mathscr{N}_s(\mho_3) = \{\mho_3\}$,$\mathscr{C}\mathscr{N}_s(\mho_4) = \{\mho_4\}$,$\mathscr{C}\mathscr{N}_s(\mho_5) = \{\mho_5\}$,$\mathscr{C}\mathscr{N}_s(\mho_6) = \{\mho_6\}$.

Definition 3.2. *Assume that $\mathcal{S} = (\mathcal{U}, \mho_G)$ is $\mathcal{S}CAS$ and $\mathscr{A}_1 \in \mathscr{F}(\mathcal{U})$. The two operators:*
$\aleph^{-1}(\mathscr{A}_1)(x) = \wedge\{\mathscr{A}_1(y) : y \in \mathscr{C}\mathscr{N}_s(x)\}$ *for all* $x \in \mathcal{U}\}$ *is called* $\mathcal{S}FCLA - 1$,
$\aleph^{+1}(\mathscr{A}_1)(x) = \vee\{\mathscr{A}_1(y) : y \in \mathscr{C}\mathscr{N}_s(x)\}$, *for all* $x \in \mathcal{U}\}$ *is called* $\mathcal{S}FCUA - 1$.

Clearly, the set $\mathscr{A}_1$ is called $(\mathcal{S}RCF - 1)$ if $\aleph^{-1}(\mathscr{A}_1) \neq \aleph^{+1}(\mathscr{A}_1)$. Otherwise the set $\mathscr{A}_1$ is definable.

Example 3.2. *If* $\mathscr{A}_1 = \{(\mho_1, 0.1), (\mho_2, 0.3), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$. *By using Example 3.1, we get the following:*
$\aleph^{-0}(\mathscr{A}_1) = \{(\mho_1, 0.1), (\mho_2, 0.1), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.5)\}$,
$\aleph^{+0}(\mathscr{A}_1) = \{(\mho_1, 0.3), (\mho_2, 0.3), (\mho_3, 0.8), (\mho_4, 0.5), (\mho_5, 0.5), (\mho_6, 0.8)\}$,
$\aleph^{-1}(\mathscr{A}_1) = \{(\mho_1, 0.1), (\mho_2, 0.1), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$,
$\aleph^{+1}(\mathscr{A}_1) = \{(\mho_1, 0.3), (\mho_2, 0.3), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$.
Obviously, $\aleph^{-0}(\mathscr{A}_1) \subseteq \aleph^{-1}(\mathscr{A}_1)$ *and* $\aleph^{+1}(\mathscr{A}_1 \subseteq \aleph^{+0}(\mathscr{A}_1)$.

Theorem 3.1. *Assume that $\mathcal{S} = (\mathcal{U}, \mho_G)$ is ($\mathcal{S}RCAS$) and $\mathscr{A}_1, \mathscr{A}_2 \in \mathscr{F}(\mathcal{U})$, then $\forall \mho_1, \mho_2, \mho_3 \in \mathcal{U}$ the following properties are satisfied:*

(iL) *If* $\mathscr{A}_1 \subseteq \mathscr{A}_2$, *then* $\aleph^{-1}(\mathscr{A}_1) \subseteq \aleph^{-1}(\mathscr{A}_2)$;
(iH) *If* $\mathscr{A}_1 \subseteq \mathscr{A}_2$, *then* $\aleph^{+1}(\mathscr{A}_1) \subseteq \aleph^{+1}(\mathscr{A}_2)$;
(iiL) $\aleph^{-1}(\mathscr{A}_1 \cap \mathscr{A}_2) = \aleph^{-1}(\mathscr{A}_1) \cap \aleph^{-1}(\mathscr{A}_2)$;
(iiH) $\aleph^{+1}(\mathscr{A}_1 \cap \mathscr{A}_2) \subseteq \aleph^{+1}(\mathscr{A}_1) \cap \aleph^{+1}(\mathscr{A}_2)$;
(iiiL) $\aleph^{-1}(\mathscr{A}_1) \cup \aleph^{-1}(\mathscr{A}_2) \subseteq \aleph^{-1}(\mathscr{A}_1 \cup \mathscr{A}_2)$;
(iiiH) $\aleph^{+1}(\mathscr{A}_1) \cup \aleph^{+1}(\mathscr{A}_2) = \aleph^{+1}(\mathscr{A}_1 \cup \mathscr{A}_2)$;
(ivL) $\aleph^{-1}(\mathscr{A}_1^c) = (\aleph^{+1}(\mathscr{A}_1))^c$;
(ivH) $\aleph^{+1}(\mathscr{A}_1^c) = (\aleph^{-1}(\mathscr{A}_1))^c$;
(vL) $\aleph^{-1}(\mathscr{A}_1) = \aleph^{-1}(\aleph^{-1}(\mathscr{A}_1))$;
(vH) $\aleph^{+1}(\mathscr{A}_1) = \aleph^{+1}(\aleph^{+1}(\mathscr{A}_1))$;
(viLH) $\aleph^{-1}(\mathscr{A}_1) \subseteq \mathscr{A}_1 \subseteq \aleph^{+1}(\mathscr{A}_1)$.

Proof. *We will prove only iL, iiL, iiiL, ivL and ivL items. The proof of other items is similar:*

(iL) *If* $\mathscr{A}_1 \subseteq \mathscr{A}_2$ *where* $\mathscr{A}_1, \mathscr{A}_2 \in \mathscr{F}(\mathcal{U})$ *and* $\mho_1, \mho_2 \in \mathcal{U}$*, hence, we get* $\aleph^{-1}(\mathscr{A}_1)(\mho_1) = \wedge\{\mathscr{A}_1(\mho_2) : \mho_2 \in \mathscr{C}\mathscr{N}_s(\mho_1)\} \leq \wedge\{\mathscr{A}_2(\mho_2) : \mho_2 \in \mathscr{C}\mathscr{N}_s(\mho_1)\} = \aleph^{-1}(\mathscr{A}_2)(\mho_1)$*;*

(iiL) $\aleph^{-1}(\mathscr{A}_1 \cap \mathscr{A}_2)(\mho_1) = \wedge\{(\mathscr{A}_1 \cap \mathscr{A}_2)(\mho_2) : \mho_2 \in \mathscr{C}\mathscr{N}_s(\mho_1)\} = \wedge\{\mathscr{A}_1(\mho_2) : \mho_2 \in \mathscr{C}\mathscr{N}_s(\mho_1)\} \cap \wedge\{\mathscr{A}_2(\mho_2) : \mho_2 \in \mathscr{C}\mathscr{N}_s(\mho_1)\} = \aleph^{-1}(\mathscr{A}_1)(\mho_1) \cap \aleph^{-1}(\mathscr{A}_2)(\mho_1)$*;*

(iiiL) *Since* $\mathscr{A}_1 \subseteq \mathscr{A}_1 \cup \mathscr{A}_2$*,* $\mathscr{A}_2 \subseteq \mathscr{A}_1 \cup \mathscr{A}_2$*, then* $\aleph^{-1}(\mathscr{A}_1) \subseteq \aleph^{-1}(\mathscr{A}_1 \cup \mathscr{A}_2)$ *and* $\aleph^{-1}(\mathscr{A}_2) \subseteq \aleph^{-1}(\mathscr{A}_1 \cup \mathscr{A}_2)$*. Therefore,* $\aleph^{-1}(\mathscr{A}_1) \cup \aleph^{-1}(\mathscr{A}_2) \subseteq \aleph^{-1}(\mathscr{A}_1 \cup \mathscr{A}_2)$*;*

(ivL) $\aleph^{-1}(\mathscr{A}_1^c) = \wedge\{\mathscr{A}_1^c(\mho_2) : \mho_2 \in \mathscr{C}\mathscr{N}_s(\mho_1)\} = \wedge\{1 - \mathscr{A}_1(\mho_2) : \mho_2 \in \mathscr{C}\mathscr{N}_s(\mho_1)\} = 1 - \vee\{\mathscr{A}_1(\mho_2) : \mho_2 \in \mathscr{C}\mathscr{N}_s(\mho_1)\} = (\aleph^{+1}(\mathscr{A}_1))^c$*;*

(vL) $\aleph^{-1}(\aleph^{-1}(\mathscr{A}_1))(\mho_1) = \wedge\{\aleph^{-1}(\mathscr{A}_1^c(\mho_2)) : \mho_2 \in \mathscr{C}\mathscr{N}_s(\mho_1)\} = \wedge\{\wedge\{\mathscr{A}_1^c(\mho_3) : \mho_3 \in (\mathscr{C}\mathscr{N}_s(\mho_2))\} : \mho_2 \in \mathscr{C}\mathscr{N}_s(\mho_1)\} = \wedge\{\mathscr{A}_1(\mho_3) : \mho_3 \in \mathscr{C}\mathscr{N}_s(\mho_2) \wedge \mho_2 \in \mathscr{C}\mathscr{N}_s(\mho_1)\} = \wedge\{\mathscr{A}_1(\mho_3) : \mho_3 \in \mathscr{C}\mathscr{N}_s(\mho_2) \subseteq \mathscr{C}\mathscr{N}_s(\mho_1)\} = \wedge\{\mathscr{A}_1(\mho_3) : \mho_3 \in \mathscr{C}\mathscr{N}_s(\mho_1)\} = \aleph^{-1}(\mathscr{A}_1)(\mho_1)$.

We define the first type of soft measure degree $(\mathcal{S}MD - 1)$ as follows.

Definition 3.3. *Assume that* $\mathcal{S} = (\mathcal{U}, \mho_G)$ *is* $(\mathcal{S}CAS)$ *and* $\mho_1, \mho_2 \in \mathcal{U}$*, then* $(\mathcal{S}MD - 1)$ *is defined as:*

$$\mathscr{D}_s^1(\mho_1, \mho_2) = \left| \frac{\mathscr{C}\mathscr{N}_s(\mho_1) \cap \mathscr{C}\mathscr{N}_s(\mho_2)}{\mathscr{C}\mathscr{N}_s(\mho_1) \cup \mathscr{C}\mathscr{N}_s(\mho_2)} \right|.$$

Clearly, $0 \leq \mathscr{D}_s^1(\mho_1, \mho_2) \leq 1$*,* $\mathscr{D}_s^1(\mho_1, \mho_2) = \mathscr{D}_s^1(\mho_2, \mho_1)$*, and* $\mathscr{D}_s^1(\mho_1, \mho_1) = 1$.

Example 3.3. *Let us consider Example 3.1. The* $(\mathcal{S}MD - 1)$ *between each two elements* $\mho_i, \mho_j \in \mathcal{U}, i, j = 1, 2, ..., 6$ *is calculated in Table 2.*

Table 2. Tabular for $\mathscr{D}_s^1(\mho_i, \mho_j) \forall i, j \in \{1, 2, ..., 6\}$.

$\mathcal{U}$	$\mho_1$	$\mho_2$	$\mho_3$	$\mho_4$	$\mho_5$	$\mho_6$
$\mho_1$	1	1	0	0	0	0
$\mho_2$	1	1	0	0	0	0
$\mho_3$	0	0	1	0	0	0
$\mho_4$	0	0	0	1	0	0
$\mho_5$	0	0	0	0	1	0
$\mho_6$	0	0	0	0	0	1

Based on Definition 3.3, we define the first type of $\mathcal{S}CRF$ based on λ-lower (upper) approximation $\{\lambda - \mathcal{S}FCLA - 1, (\lambda - \mathcal{S}FCUA - 1)\}$ as follows.

Definition 3.4. *If* $\mathcal{S} = (\mathcal{U}, \mho_G)$ *is* $(\mathcal{S}CAS)$ *and* $\mathscr{D}_s^1(\mho_1, \mho_2)$ *is* $(\mathcal{S}MD - 1)$ *for* $\mho_1, \mho_2 \in \mathcal{U}$*, for* $\mathscr{A}_1 \in \mathscr{F}(\mathcal{U})$*, the* $\lambda - \mathcal{S}FCLA - 1$*,* $(\lambda - \mathcal{S}FCUA - 1)$ *is defined as follows, respectively:*
$\aleph_\lambda^{-1}(\mathscr{A}_1)(\mho_1) = \wedge\{\mathscr{A}_1)(\mho_2) : \mathscr{D}_s^1(\mho_1, \mho_2) > \lambda\}$,
$\aleph_\lambda^{+1}(\mathscr{A}_1)(\mho_1) = \vee\{\mathscr{A}_1)(\mho_2) : \mathscr{D}_s^1(\mho_1, \mho_2) > \lambda\}$, $\forall \mho_1, \mho_2 \in \mathcal{U}$.

If $\aleph_\lambda^{-1}(\mathscr{A}_1) \neq \aleph_\lambda^{+1}(\mathscr{A}_1)$, then $\mathscr{A}_1$ is called $\lambda - \mathcal{S}CRF - 1$; otherwise $\mathscr{A}_1$ is called definable.

Example 3.4. *Continued from Example 3.3 and* $\mathscr{A}_1 = \{(\mho_1, 0.1), (\mho_2, 0.3), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$*, for* $\lambda = 0.5$*, we have the following approximations operators:*
$\aleph_\lambda^{-1}(\mathscr{A}_1) = \{(\mho_1, 0.1), (\mho_2, 0.1), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$,
$\aleph_\lambda^{+1}(\mathscr{A}_1) = \{(\mho_1, 0.3), (\mho_2, 0.3), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$.

Theorem 3.2. *If $\mathcal{S} = (\mathcal{U}, \mho_G)$ is ($\mathcal{SCAS}$), then for $\mathscr{A}_1, A_2 \in \mathscr{F}(\mathcal{U})$, the following properties hold:*

(iL) *If $\mathscr{A}_1 \subseteq \mathscr{A}_2$, then $\aleph_\lambda^{-1}(\mathscr{A}_1) \subseteq \aleph_\lambda^{-1}(\mathscr{A}_2)$;*
(iH) *If $\mathscr{A}_1 \subseteq \mathscr{A}_2$, then $\aleph_\lambda^{+1}(\mathscr{A}_1) \subseteq \aleph_\lambda^{+1}(\mathscr{A}_2)$;*
(iiL) $\aleph_\lambda^{-1}(\mathscr{A}_1 \cap \mathscr{A}_2) = \aleph_\lambda^{-1}(\mathscr{A}_1) \cap \aleph_\lambda^{-1}(\mathscr{A}_2)$;
(iiH) $\aleph_\lambda^{+1}(\mathscr{A}_1 \cap \mathscr{A}_2) \subseteq \aleph_\lambda^{+1}(\mathscr{A}_1) \cap \aleph_\lambda^{+1}(\mathscr{A}_2)$;
(iiiL) $\aleph_\lambda^{-1}(\mathscr{A}_1) \cup \aleph_\lambda^{-1}(\mathscr{A}_2) \subseteq \aleph_\lambda^{-1}(\mathscr{A}_1 \cup \mathscr{A}_2)$;
(iiiH) $\aleph_\lambda^{+1}(\mathscr{A}_1) \cup \aleph_\lambda^{+1}(\mathscr{A}_2) = \aleph_\lambda^{+1}(\mathscr{A}_1 \cup \mathscr{A}_2)$;
(ivL) $\aleph_\lambda^{-1}(\mathscr{A}_1^c) = (\aleph_\lambda^{+1}(\mathscr{A}_1))^c$;
(ivH) $\aleph_\lambda^{+1}(\mathscr{A}_1^c) = (\aleph_\lambda^{-1}(\mathscr{A}_1))^c$;
(vL) *If $\alpha_1 \leq \alpha_2$, then $\aleph_\lambda^{-1}(\mathscr{A}_1) \subseteq \aleph_\lambda^{-1}(\aleph_\lambda^{-1}(\mathscr{A}_2))$;*
(vH) *If $\alpha_1 \leq \alpha_2$, then $\aleph_\lambda^{+1}(\mathscr{A}_1) \subseteq \aleph_\lambda^{+1}(\aleph_\lambda^{+1}(\mathscr{A}_2))$;*
(viLH) $\aleph_\lambda^{-1}(\mathscr{A}_1) \subseteq \mathscr{A}_1 \subseteq \aleph_\lambda^{+1}(\mathscr{A}_1)$.

Proof. *Similar to Theorem 3.1.*

Definition 3.5. *If $\mathcal{S} = (\mathcal{U}, \mho_G)$ is ($\mathcal{SCAS}$) and $\mathscr{D}_s^1(\mho_1, \mho_2)$ is ($\mathcal{SMD} - 1$) for $\mho_1, \mho_2 \in \mathcal{U}$, then for $\mathscr{A}_1 \in \mathscr{F}(\mathcal{U})$, the first type of SFC $\mathscr{D}$-lower (upper) approximation ($\mathscr{D}-\mathcal{SFCLA}-1$), ($\mathscr{D}-\mathcal{SFCUA}-1$) is defined as follows, respectively:*

$$\aleph_{\mathscr{D}}^{-1}(\mathscr{A}_1)(\mho_1) = \bigwedge_{\mho_2 \in \mathcal{U}} \{(1 - \mathscr{D}_s^1)(\mho_1, \mho_2) \vee \mathscr{A}_1(\mho_2)\},$$
$$\aleph_{\mathscr{D}}^{+1}(\mathscr{A}_1)(\mho_1) = \bigvee_{\mho_2 \in \mathcal{U}} \{\mathscr{D}_s^1(\mho_1, \mho_2) \wedge \mathscr{A}_1(\mho_2)\}, \forall \mho_1 \in \mathcal{U}.$$

If $\aleph_{\mathscr{D}}^{-1}(\mathscr{A}_1) \neq \aleph_{\mathscr{D}}^{+1}(\mathscr{A}_1)$, then $\mathscr{A}_1$ is called $\mathscr{D} - \mathcal{SCRF} - 1$; otherwise, $\mathscr{A}_1$ is called definable.

Example 3.5. *Continued from Example 3.3 and* $\mathscr{A}_1 = \{(\mho_1, 0.1), (\mho_2, 0.3), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$, *then we get the following approximations operators:*
$\aleph_{\mathscr{D}}^{-1}(\mathscr{A}_1) = \{(\mho_1, 0.1), (\mho_2, 0.1), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$,
$\aleph_{\mathscr{D}}^{+1}(\mathscr{A}_1) = \{(\mho_1, 0.3), (\mho_2, 0.3), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$.

Theorem 3.3. *Assume that $\mathcal{S} = (\mathcal{U}, \mho_G)$ is ($\mathcal{SCAS}$) and $\mathscr{A}_1, \mathscr{A}_2 \in \mathscr{F}(\mathcal{U})$, then $\forall \mho_1, \mho_2, \mho_3 \in \mathcal{U}$, and the following properties are satisfied:*

(iL) *If $\mathscr{A}_1 \subseteq \mathscr{A}_2$, then $\aleph_{\mathscr{D}}^{-1}(\mathscr{A}_1) \subseteq \aleph_{\mathscr{D}}^{-1}(\mathscr{A}_2)$;*
(iH) *If $\mathscr{A}_1 \subseteq \mathscr{A}_2$, then $\aleph_{\mathscr{D}}^{+1}(\mathscr{A}_1) \subseteq \aleph_{\mathscr{D}}^{+1}(\mathscr{A}_2)$;*
(iiL) $\aleph_{\mathscr{D}}^{-1}(\mathscr{A}_1 \cap \mathscr{A}_2) = \aleph_{\mathscr{D}}^{-1}(\mathscr{A}_1) \cap \aleph_{\mathscr{D}}^{-1}(\mathscr{A}_2)$;
(iiH) $\aleph^{+1}(\mathscr{A}_1 \cap \mathscr{A}_2) \subseteq \aleph^{+1}(\mathscr{A}_1) \cap \aleph^{+1}(\mathscr{A}_2)$;
(iiiL) $\aleph_{\mathscr{D}}^{-1}(\mathscr{A}_1) \cup \aleph_{\mathscr{D}}^{-1}(\mathscr{A}_2) \subseteq \aleph_{\mathscr{D}}^{-1}(\mathscr{A}_1 \cup \mathscr{A}_2)$;
(iiiH) $\aleph_{\mathscr{D}}^{+1}(\mathscr{A}_1) \cup \aleph_{\mathscr{D}}^{+1}(\mathscr{A}_2) = \aleph_{\mathscr{D}}^{+1}(\mathscr{A}_1 \cup \mathscr{A}_2)$;
(ivL) $\aleph_{\mathscr{D}}^{-1}(\mathscr{A}_1^c) = (\aleph_{\mathscr{D}}^{+1}(\mathscr{A}_1))^c$;
(ivH) $\aleph_{\mathscr{D}}^{+1}(\mathscr{A}_1^c) = (\aleph_{\mathscr{D}}^{-1}(\mathscr{A}_1))^c$;
(vL) $\aleph_{\mathscr{D}}^{-1}(\mathscr{A}_1) = \aleph_{\mathscr{D}}^{-1}(\aleph_{\mathscr{D}}^{-1}(\mathscr{A}_1))$;
(vH) $\aleph_{\mathscr{D}}^{+1}(\mathscr{A}_1) = \aleph_{\mathscr{D}}^{+1}(\aleph_{\mathscr{D}}^{+1}(\mathscr{A}_1))$;
(viLH) $\aleph_{\mathscr{D}}^{-1}(\mathscr{A}_1) \subseteq \mathscr{A}_1 \subseteq \aleph_{\mathscr{D}}^{+1}(\mathscr{A}_1)$.

Proof. *Similar to Theorem 3.1.*

3.1. A simulation experiment for our model

Suppose that $\mathcal{U} = \{\mho_1, \mho_2, \mho_3, \mho_4, \mho_5, \mho_6, \mho_7, \mho_8, \mho_9, \mho_{10}\}$ is a set of pilots. They are trained with respect to five attributes $\mathcal{A} = \{e_1, e_2, e_3, e_4, e_5\}$. They had been evaluated by an expert to determine whether they are sufficiently well trained according to these attributes or not, as shown in Table 3.

Table 3. Tabular representation of the soft set $\mho_G$.

$\mathcal{U}$	e_1	e_2	e_3	e_4	e_5
$\mho_1$	1	0	0	0	1
$\mho_2$	0	1	1	0	1
$\mho_3$	0	1	1	0	1
$\mho_4$	1	0	1	0	0
$\mho_5$	1	0	0	1	1
$\mho_6$	0	0	0	1	1
$\mho_7$	0	1	1	0	0
$\mho_8$	1	0	0	1	1
$\mho_9$	0	1	1	0	1
$\mho_{10}$	1	0	0	0	1

The core of soft neighborhood is: $\mathscr{C}\mathscr{N}_s(\mho_1) = \mathscr{C}\mathscr{N}_s(\mho_{10}) = \{\mho_1, \mho_{10}\}, \mathscr{C}\mathscr{N}_s(\mho_2)$.
$= \mathscr{C}\mathscr{N}_s(\mho_3) = \mathscr{C}\mathscr{N}_s(\mho_9) = \{\mho_2, \mho_3, \mho_9\}$, $\mathscr{C}\mathscr{N}_s(\mho_4) = \{\mho_4\}$, $\mathscr{C}\mathscr{N}_s(\mho_5) = \mathscr{C}\mathscr{N}_s(\mho_8) = \{\mho_5, \mho_8\}$, $\mathscr{C}\mathscr{N}_s(\mho_6) = \{\mho_6\}$, $\mathscr{C}\mathscr{N}_s(\mho_7) = \{\mho_7\}$.
Suppose $\mathscr{A}_1 = \{(\mho_1, 0.1), (\mho_2, 0.3), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7), (\mho_7, 0.9), (\mho_8, 0.3), (\mho_9, 0.9), (\mho_{10}, 0.4)\}$
represents evaluation's degrees which are given by the expert. We can check the accuracy of this evaluation by our model which helps in decision making as follows:
$\aleph^{-1}(\mathscr{A}_1) = \{(\mho_1, 0.1), (\mho_2, 0.3), (\mho_3, 0.3), (\mho_4, 0.2), (\mho_5, 0.3), (\mho_6, 0.7), (\mho_7, 0.9), (\mho_8, 0.3), (\mho_9, 0.9), (\mho_{10}, 0.1)\}$,
$\aleph^{+1}(\mathscr{A}_1) = \{(\mho_1, 0.4), (\mho_2, 0.9), (\mho_3, 0.9), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7), (\mho_7, 0.9), (\mho_8, 0.5), (\mho_9, 0.9), (\mho_{10}, 0.4)\}$.

4. New other two models of $\mathcal{SCRF}$ based on neighborhoods

We introduce new two models of $\mathcal{SCRF}$ based on merging core soft neighborhoods and soft neighborhoods. The second model of $\mathcal{SCRF}$ is denoted by $\mathcal{SCRF}-2$ and the third model is denoted by $\mathcal{SCRF}-3$.

4.1. $\mathcal{SCRF}-2$-model

Definition 4.1. *If $\mathcal{S} = (\mathcal{U}, \mho_G)$ is $(\mathcal{SCAS})$, then for $\mathscr{A}_1 \in \mathscr{F}(\mathcal{U})$*
$\aleph^{-2}(\mathscr{A}_1)(\mho_1) = \wedge\{\mathscr{A}_1(\mho_2) : \mho_2 \in (\mathscr{N}_s \cap \mathscr{C}\mathscr{N}_s)(\mho_1)\}$, *$\forall \mho_1, \mho_2 \in \mathcal{U}$ is called $\mathcal{SFCLA}-2$,*
$\aleph^{+2}(\mathscr{A}_1)(\mho_1) = \vee\{\mathscr{A}_1(\mho_2) : \mho_2 \in (\mathscr{N}_s \cap \mathscr{C}\mathscr{N}_s)(\mho_1)\}$, *$\forall \mho_1, \mho_2 \in \mathcal{U}$ is called $\mathcal{SFCUA}-2$.*

If $\aleph^{-2}(\mathscr{A}_1) \neq \aleph^{+1}(\mathscr{A}_1)$, then $\mathscr{A}_1$ is called $\mathcal{S}CRF-2$; otherwise, $\mathscr{A}_1$ is called definable.

Example 4.1. *Continued from Example 3.1,* $\mathscr{CN}_s(\mho_1) \cap \mathscr{N}_s(\mho_1) = \{\mho_1, \mho_2\}$, $\mathscr{CN}_s(\mho_2) \cap \mathscr{N}_s(\mho_2) = \{\mho_1, \mho_2\}$, $\mathscr{CN}_s(\mho_3) \cap \mathscr{N}_s(\mho_3) = \{\mho_3\}$, $\mathscr{CN}_s(\mho_4) \cap \mathscr{N}_s(\mho_4) = \{\mho_4\}$, $\mathscr{CN}_s(\mho_5) \cap \mathscr{N}_s(\mho_5) = \{\mho_5\}$, $\mathscr{CN}_s(\mho_6) \cap \mathscr{N}_s(\mho_6) = \{\mho_6\}$, $\mathscr{CN}_s(\mho_1) \cup \mathscr{N}_s(\mho_1) = \{\mho_1, \mho_2\}$, $\mathscr{CN}_s(\mho_2) \cup \mathscr{N}_s(\mho_2) = \{\mho_1, \mho_2\}$, $\mathscr{CN}_s(\mho_3) \cup \mathscr{N}_s(\mho_3) = \{\mho_3\}$, $\mathscr{CN}_s(\mho_4) \cup \mathscr{N}_s(\mho_4) = \{\mho_4, \mho_5\}$, $\mathscr{CN}_s(\mho_5) \cup \mathscr{N}_s(\mho_5) = \{\mho_5\}$, $\mathscr{CN}_s(\mho_6) \cup \mathscr{N}_s(\mho_6) = \{\mho_3, \mho_5, \mho_6\}$. *Therefore,* $\aleph^{-2}(\mathscr{A}_1) = \{(\mho_1, 0.1), (\mho_2, 0.1), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$, *and* $\aleph^{+2}(\mathscr{A}_1) = \{(\mho_1, 0.3), (\mho_2, 0.3), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$.

Let us define the second type of soft measure degree ($\mathcal{S}MD-2$) as follows.

Definition 4.2. *Assume that* $\mathcal{S} = (\mathcal{U}, \mho_G)$ *is* ($\mathcal{S}CAS$) *and* $\mho_1, \mho_2 \in \mathcal{U}$, *then the* ($\mathcal{S}MD-2$) *is defined as:*

$$\mathscr{D}_s^2(\mho_1, \mho_2) = \left| \frac{(\mathscr{CN}_s \cap \mathscr{N}_s)(\mho_1) \cap (\mathscr{CN}_s \cap \mathscr{N}_s)(\mho_2)}{(\mathscr{CN}_s \cap \mathscr{N}_s)(\mho_1) \cup (\mathscr{CN}_s \cap \mathscr{N}_s)(\mho_2)} \right|.$$

Clearly, $0 \leq \mathscr{D}_s^2(\mho_1, \mho_2) \leq 1$, $\mathscr{D}_s^2(\mho_1, \mho_2) = \mathscr{D}_s^2(\mho_2, \mho_1)$, *and* $\mathscr{D}_s^1(\mho_1, \mho_1) = 1$.

Example 4.2. *According to Example 4.1, the values of* $\mathcal{S}MD-2$ *are shown in the following Table* 4

Table 4. Tabular for $\mathscr{D}_s^2(\mho_i, \mho_j) \forall i, j \in \{1, 2, ..., 6\}$.

$\mathcal{U}$	$\mho_1$	$\mho_2$	$\mho_3$	$\mho_4$	$\mho_5$	$\mho_6$
$\mho_1$	1	1	0	0	0	0
$\mho_2$	1	1	0	0	0	0
$\mho_3$	0	0	1	0	0	0
$\mho_4$	0	0	0	1	0	0
$\mho_5$	0	0	0	0	1	0
$\mho_6$	0	0	0	0	0	1

Based on Definition 4.1, we define the second type of $SCRF$ based on λ-lower (upper) approximation ($\lambda-\mathcal{S}FCLA-2$ ($\lambda-\mathcal{S}FCUA-2$)) as follows.

Definition 4.3. *If* $\mathcal{S} = (\mathcal{U}, \mho_G)$ *is* ($\mathcal{S}CAS$) *and* $\mathscr{D}_s^2(\mho_1, \mho_2)$ *is* ($\mathcal{S}MD-2$) *for* $\mho_1, \mho_2 \in \mathcal{U}$, *then for* $\mathscr{A}_1 \in \mathscr{F}(\mathcal{U})$, *the* $\lambda-\mathcal{S}FCLA-2$ ($\lambda-\mathcal{S}FCUA-2$) *is defined as follows, respectively:*
$\aleph_\lambda^{-2}(\mathscr{A}_1)(\mho_1) = \wedge\{\mathscr{A}_1)(\mho_2) : \mathscr{D}_s^2(\mho_1, \mho_2) > \lambda\}$,
$\aleph_\lambda^{+2}(\mathscr{A}_1)(\mho_1) = \vee\{\mathscr{A}_1)(\mho_2) : \mathscr{D}_s^2(\mho_1, \mho_2) > \lambda\}$, $\forall \mho_1, \mho_2 \in \mathcal{U}$.

If $\aleph_\lambda^{-2}(\mathscr{A}_1) \neq \aleph_\lambda^{+2}(\mathscr{A}_1)$, then $\mathscr{A}_1$ is called $\lambda-\mathcal{S}CRF-2$; otherwise $\mathscr{A}_1$ is called definable.

Example 4.3. *Consider Example 4.2 and* $\mathscr{A}_1 = \{(\mho_1, 0.3), (\mho_2, 0.4), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$, *then for* $\lambda = 0.5$, *we have the following approximations operators:*
$\aleph_\lambda^{-2}(\mathscr{A}_1) = \{(\mho_1, 0.3), (\mho_2, 0.3), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$,
$\aleph_\lambda^{+2}(\mathscr{A}_1) = \{(\mho_1, 0.4), (\mho_2, 0.4), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$.

Definition 4.4. *If* $\mathcal{S} = (\mathcal{U}, \mho_G)$ *is* ($\mathcal{S}CAS$) *and* $\mathscr{D}_s^2(\mho_1, \mho_2)$ *is* ($\mathcal{S}MD-2$) *for* $\mho_1, \mho_2 \in \mathcal{U}$, *then for* $\mathscr{A}_1 \in \mathscr{F}(\mathcal{U})$, *the second type of SFC* $\mathscr{D}$*-lower (upper) approximation* $\mathscr{D}-\mathcal{S}FCLA-2$ ($\mathscr{D}-\mathcal{S}FCUA-2$)

is defined as follows, respectively:
$\aleph^{-2}_{\mathscr{D}}(\mathscr{A}_1)(\mho_1) = \bigwedge_{\mho_2 \in \mathcal{U}} \{(1 - \mathscr{D}^2_s)(\mho_1, \mho_2) \vee \mathscr{A}_1(\mho_2)\}$,
$\aleph^{+2}_{\mathscr{D}}(\mathscr{A}_1)(\mho_1) = \bigvee_{\mho_2 \in \mathcal{U}} \{\mathscr{D}^2_s(\mho_1, \mho_2) \wedge \mathscr{A}_1(\mho_2)\}, \forall \mho_1 \in \mathcal{U}$.

Example 4.4. *Consider Example 4.2 and* $\mathscr{A}_1 = \{(\mho_1, 0.3), (\mho_2, 0.4), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$, *then we have the following approximations operators:*
$\aleph^{-2}_{\mathscr{D}}(\mathscr{A}_1) = \{(\mho_1, 0.3), (\mho_2, 0.3), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$
$\aleph^{+2}_{\mathscr{D}}(\mathscr{A}_1) = \{(\mho_1, 0.4), (\mho_2, 0.4), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$.

4.2. $\mathcal{SCRF} - 3$-model

Definition 4.5. *If* $\mathcal{S} = (\mathcal{U}, \mho_G)$ *is* ($\mathcal{SCAS}$), *then for* $\mathscr{A}_1 \in \mathscr{F}(\mathcal{U})$:
$\aleph^{-3}(\mathscr{A}_1)(\mho_1) = \wedge\{\mathscr{A}_1(\mho_2) : \mho_2 \in (\mathscr{N}_s \cup \mathscr{C}\mathscr{N}_s)(\mho_1)\}$, $\forall \mho_1, \mho_2 \in \mathcal{U}$ *is called* $\mathcal{SFCLA} - 3$,
$\aleph^{+3}(\mathscr{A}_1)(\mho_1) = \vee\{\mathscr{A}_1(\mho_2) : \mho_2 \in (\mathscr{N}_s \cup \mathscr{C}\mathscr{N}_s)(\mho_1)\}$, $\forall \mho_1, \mho_2 \in \mathcal{U}$ *is called* $\mathcal{SFCUA} - 3$.

If $\aleph^{-3}_{\lambda}(\mathscr{A}_1) \neq \aleph^{+3}_{\lambda}(\mathscr{A}_1)$, then $\mathscr{A}_1$ is called $\lambda - \mathcal{SCRF} - 3$; otherwise, $\mathscr{A}_1$ is called definable.

Example 4.5. *From Example 4.1 and* $\mathscr{A}_1 = \{(\mho_1, 0.1), (\mho_2, 0.3), (\mho_3, 0.8), (\mho_4, 0.2),$
$(\mho_5, 0.5), (\mho_6, 0.7)\}$, *we have the following approximations operators:*
$\aleph^{-3}(\mathscr{A}_1) = \{(\mho_1, 0.1), (\mho_2, 0.1), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.5)\}$,
$\aleph^{-3}(\mathscr{A}_1) = \{(\mho_1, 0.3), (\mho_2, 0.3), (\mho_3, 0.8), (\mho_4, 0.5), (\mho_5, 0.5), (\mho_6, 0.8)\}$.

We define the third type of soft measure degree ($\mathcal{SMD} - 3$) as follows.

Definition 4.6. *Assume that* $\mathcal{S} = (\mathcal{U}, \mho_G)$ *is* $\mathcal{SCAS}$ *and* $\mho_1, \mho_2 \in \mathcal{U}$, *then the* ($\mathcal{SMD} - 3$) *is defined as:*

$$\mathscr{D}^3_s(\mho_1, \mho_2) = \left| \frac{(\mathscr{C}\mathscr{N}_s \cup \mathscr{N}_s)(\mho_1) \cap (\mathscr{C}\mathscr{N}_s \cup \mathscr{N}_s)(\mho_2)}{(\mathscr{C}\mathscr{N}_s \cup \mathscr{N}_s)(\mho_1) \cup (\mathscr{C}\mathscr{N}_s \cup \mathscr{N}_s)(\mho_2)} \right|.$$

Clearly, $0 \leq \mathscr{D}^3_s(\mho_1, \mho_2) \leq 1$, $\mathscr{D}^3_s(\mho_1, \mho_2) = \mathscr{D}^3_s(\mho_2, \mho_1)$, *and* $\mathscr{D}^3_s(\mho_1, \mho_1) = 1$.

Example 4.6. *From Example 4.1, the values of* $\mathcal{SMD} - 3$ *are shown in the following Table 5.*

Table 5. Tabular for $\mathscr{D}^3_s(\mho_i, \mho_j) \forall i, j \in \{1, 2, ..., 6\}$.

$\mathcal{U}$	$\mho_1$	$\mho_2$	$\mho_3$	$\mho_4$	$\mho_5$	$\mho_6$
$\mho_1$	1	1	0	0	0	0
$\mho_2$	1	1	0	0	0	0
$\mho_3$	0	0	1	0	0	0
$\mho_4$	0	0	0	1	$\frac{1}{2}$	$\frac{1}{4}$
$\mho_5$	0	0	0	$\frac{1}{2}$	1	$\frac{1}{3}$
$\mho_6$	0	0	0	$\frac{1}{4}$	$\frac{1}{3}$	1

Based on Definition 4.6, we define the third type of SFC based on λ-lower(upper) approximation{ $\lambda - \mathcal{SFCLA} - 3(\lambda - \mathcal{SFCUA} - 3)\}$ as follows.

Definition 4.7. *If* $\mathcal{S} = (\mathcal{U}, \mho_G)$ *is* ($\mathcal{SCAS}$) *and* $\mathscr{D}^3_s(\mho_1, \mho_2)$ *is* ($\mathcal{SMD} - 3$) *for* $\mho_1, \mho_2 \in \mathcal{U}$, *then for* $\mathscr{A}_1 \in \mathscr{F}(\mathcal{U})$, *the* $\{\lambda - \mathcal{SFCLA} - 3(\lambda - \mathcal{SFCUA} - 3)\}$ *is defined as:*
$\aleph^{-3}_{\lambda}(\mathscr{A}_1)(\mho_1) = \wedge\{\mathscr{A}_1)(\mho_2) : \mathscr{D}^3_s(\mho_1, \mho_2) > \lambda\}$,
$\aleph^{+3}_{\lambda}(\mathscr{A}_1)(\mho_1) = \vee\{\mathscr{A}_1)(\mho_2) : \mathscr{D}^3_s(\mho_1, \mho_2) > \lambda\}$, $\forall \mho_1, \mho_2 \in \mathcal{U}$.

If $\aleph^{-3}_{\lambda}(\mathscr{A}_1) \neq \aleph^{+3}_{\lambda}(\mathscr{A}_1)$, then $\mathscr{A}_1$ is called $\lambda - \mathcal{SCRF} - 3$; otherwise, $\mathscr{A}_1$ is called definable.

Example 4.7. *From Example 4.6 and* $\mathscr{A}_1 = \{(\mho_1, 0.1), (\mho_2, 0.3), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$, *then for* $\lambda = 0.2$, *we have the following approximations operators:*
$\aleph^{-2}_{\lambda}(\mathscr{A}_1) = \{(\mho_1, 0.1), (\mho_2, 0.1), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.2), (\mho_6, 0.2)\}$,
$\aleph^{+2}_{\lambda}(\mathscr{A}_1) = \{(\mho_1, 0.3), (\mho_2, 0.3), (\mho_3, 0.8), (\mho_4, 0.7), (\mho_5, 0.7), (\mho_6, 0.7)\}$.

Definition 4.8. *If* $\mathcal{S} = (\mathcal{U}, \mho_G)$ *is* $(\mathcal{SCAS})$ *and* $\mathscr{D}^3_s(\mho_1, \mho_2)$ *is* $(\mathcal{SMD}-3)$ *for* $\mho_1, \mho_2 \in \mathcal{U}$, *then for* $\mathscr{A}_1 \in \mathscr{F}(\mathcal{U})$, *the third type of SFC* $\mathscr{D}$*-lower (upper) approximation* $((\mathscr{D} - \mathcal{SFCLA} - 3), (\mathscr{D} - \mathcal{SFCUA} - 3))$ *is defined as follows, respectively:*
$\aleph^{-3}_{\mathscr{D}}(\mathscr{A}_1)(\mho_1) = \bigwedge_{\mho_2 \in \mathcal{U}} \{(1 - \mathscr{D}^3_s)(\mho_1, \mho_2) \vee \mathscr{A}_1(\mho_2)\}$,
$\aleph^{+3}_{\mathscr{D}}(\mathscr{A}_1)(\mho_1) = \bigvee_{\mho_2 \in \mathcal{U}} \{\mathscr{D}^3_s(\mho_1, \mho_2) \wedge \mathscr{A}_1(\mho_2)\}, \forall \mho_1 \in \mathcal{U}$.

Example 4.8. *Consider Example 4.6 and* $\mathscr{A}_1 = \{(\mho_1, 0.1), (\mho_2, 0.3), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$, *then, we have the following approximations operators:*
$\aleph^{-3}_{\mathscr{D}}(\mathscr{A}_1) = \{(\mho_1, 0.1), (\mho_2, 0.1), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$,
$\aleph^{+3}_{\mathscr{D}}(\mathscr{A}_1) = \{(\mho_1, 0.3), (\mho_2, 0.3), (\mho_3, 0.8), (\mho_4, 0.7), (\mho_5, 0.5), (\mho_6, 0.7)\}$.

4.3. Comparison between our $\mathcal{SCRF}$-models and Zhan's model

We set forth the relationship between our proposed $\mathcal{SCRF}$-models and Zhan's model for soft rough fuzzy approximation structure.

Theorem 4.1. *If* $\mathcal{S} = (\mathcal{U}, \mho_G)$ *is* $(\mathcal{SCAS})$ *and* $\mathscr{A}_1 \in \mathscr{F}(\mathcal{U})$, *then the following axioms are satisfied:*

(iL) $\aleph^{-3}(\mathscr{A}_1) \subseteq \aleph^{-1}(\mathscr{A}_1) \subseteq \aleph^{-2}(\mathscr{A}_1)$;
(iH) $\aleph^{-3}(\mathscr{A}_1) \subseteq \aleph^{-0}(\mathscr{A}_1) \subseteq \aleph^{-2}(\mathscr{A}_1)$;
(iiL) $\aleph^{+2}(\mathscr{A}_1) \subseteq \aleph^{+1}(\mathscr{A}_1) \subseteq \aleph^{+3}(\mathscr{A}_1)$;
(iiH) $\aleph^{+2}(\mathscr{A}_1) \subseteq \aleph^{+0}(\mathscr{A}_1) \subseteq \aleph^{+3}(\mathscr{A}_1)$.

Proof. The proof comes from Definitions 3.2, 4.1, and 4.5.

Remark 4.1. *From the previous theorem, the lower approximation of our model* $\aleph^{-2}$ *is bigger than Zhan's model* $\aleph^{-0}$ *while the upper approximation of our model* $\aleph^{+2}$ *is less than Zhan's model* $\aleph^{+0}$. *This leads to decreasing the boundary region and makes the soft rough fuzzy set more accurate in solving the uncertainty issues.*

Example 4.9. *If* $\mathscr{A}_1 = \{(\mho_1, 0.1), (\mho_2, 0.3), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$, *then by Example 3.1,* $\aleph^{-0}(\mathscr{A}_1) = \{(\mho_1, 0.1), (\mho_2, 0.1), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.5)\}$,
$\aleph^{+0}(\mathscr{A}_1) = \{(\mho_1, 0.3), (\mho_2, 0.3),$
$(\mho_3, 0.8), (\mho_4, 0.5), (\mho_5, 0.5), (\mho_6, 0.8)\}$. $\aleph^{-2}(\mathscr{A}_1) = \{(\mho_1, 0.1), (\mho_2, 0.1), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$ *and* $\aleph^{+2}(\mathscr{A}_1) = \{(\mho_1, 0.3), (\mho_2, 0.3), (\mho_3, 0.8), (\mho_4, 0.2), (\mho_5, 0.5), (\mho_6, 0.7)\}$.

Theorem 4.2. *If* $\mathcal{S} = (\mathcal{U}, \mho_G)$ *is* $(\mathcal{SCAS})$ *and* $\mathscr{A}_1 \in \mathscr{F}(\mathcal{U})$, *then the following properties are satisfied:*

(iL) $\aleph^{-2}(\mathscr{A}_1) = \aleph^{-0}(\mathscr{A}_1) \cup \aleph^{-1}(\mathscr{A}_1)$;
(iH) $\aleph^{+2}(\mathscr{A}_1) = \aleph^{-0}(\mathscr{A}_1) \cap \aleph^{-1}(\mathscr{A}_1)$;
(iiL) $\aleph^{-3}(\mathscr{A}_1) = \aleph^{-0}(\mathscr{A}_1) \cap \aleph^{-1}(\mathscr{A}_1)$
(iiH) $\aleph^{+3}(\mathscr{A}_1) = \aleph^{-0}(\mathscr{A}_1) \cup \aleph^{-1}(\mathscr{A}_1)$.

Proof. The proof is straightforward.

5. Conclusions

Zhan's model for soft rough fuzzy covering stands as an innovative approach, though further exploration and refinement are warranted. Similarly, the exploration of soft covering-based rough fuzzy sets opens avenues for the integration of various mathematical structures. We introduced a combination of soft sets, fuzzy sets and rough sets. Three models of the approximation of $\mathcal{SRFS}$ - based covering are presented. We deduced that our approximation is more refined than Zhan's model as we decreased the boundary region. The integration of soft sets with fuzzy logic in soft fuzzy covering and the discernment of upper approximation provide additional layers to the $\mathcal{SCRF}$ framework, offering a comprehensive solution to complex problem structures.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflict of interest.

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