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Research article

Numerical method for a compound Poisson risk model with liquid reserves and proportional investment

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Abstract: In this paper, a classical risk model with liquid reserves and proportional investment is considered, and the expected total discounted dividend before ruin of insurance companies under the threshold dividend strategy is studied. First, the integral differential equations of the expected total discounted dividend before ruin satisfying certain boundary conditions is derived. Second, since the explicit solutions of the equations cannot be obtained, the numerical approximation solutions are obtained by the sinc approximation method. Finally, we discuss the effects of parameters such as risk capital ratio and liquid reserve on the expected total discounted dividend before ruin by some examples.

Keywords: liquid reserves; threshold strategy; proportional investment; dividends paid up to ruin; sinc numerical method
Mathematics Subject Classification: 65C30, 91B05, 91G05

1. Introduction

Since the Lundberg-Cramer classical risk model [1] was proposed, many researchers have paid attention to the ruin problem and dividend payments in various risk models. Then, many researchers focused on investment and dividend issues in various risk models, expecting to get the optimal investment strategy so that the company's shareholders can get more dividends [2–5].

The issue of investment can be traced back to 1995, when Sundt and Teugels [6] proposed a classical risk model with interest and considered the ruin probability of the model. On investment, Fang and Wu [7] introduced the assumption of risk-less asset investment based on the classical risk model. However, companies do not invest all of their positive assets in practice, but keep some of it as liquid reserve to protect against contingencies. Hence, Cai et al. [8] introduced the level of liquid reserves into the model and discussed the effects of parameters such as the level of liquid reserves on the ruin probability through some examples. Yang and He [9] further introduced loan interest into the model and discussed its absolute ruin probability. In order to obtain higher returns, insurance companies not only invest their assets in risk-less markets, but also in risk markets in reality. Chen et al. [10] proposed an improved risk model with proportional investment under the threshold strategy.

In addition, researchers pay more attention to the dividend problem under different risk models and different dividend strategies [11–13]. In the actual situation, shareholders are more concerned about what strategy can maximize the dividend, that is, to find the optimal dividend strategy. The problem of optimal dividend can be traced back to 1957, when it was discussed by De Finetti [14] with the goal of maximizing dividends to a company's shareholders. Subsequently, there are many studies around the issue of optimal dividends, and a large number of papers and academic monographs have been published to study risk models under investment or dividend strategies [15–19].

At present, there are few relevant studies on proportional investment, which are all about investing all positive assets, and there is no situation of combining proportional investment with liquid reserve. Therefore, this paper starts from the actual situation, and considers the classical risk model with liquid reserves and proportional investment to study the expected total discounted dividend before ruin.

In order to clearly demonstrate the innovation of our work, it is compared with some existing literature in Table 1.

Table 1. Compared with previous literature.

Existing literature	model				sinc
	liquid reserve	risk investment	risk-less investment	dividend strategy	
Fang and Wu [7]			✓	✓	
Cai et al. [8]	✓		✓	✓	
Chen and Ou [10]		✓	✓	✓	✓
Zhang and Han [17]				✓	
Zhang et al. [18]	✓		✓	✓	
Yu et al. [3]	✓			✓	
Peng et al. [19]	✓		✓	✓	
Lin and Pavlova [12]				✓	
wan [13]				✓	
Yin and Yuen [11]				✓	
Lin and Sendova [16]				✓	
This paper	✓	✓	✓	✓	✓

2. The model

First, the expression of the classical risk model is

$$C_t = u + ct - Z_t, \quad t \geq 0, \tag{2.1}$$

where $\{C_t\}_{t\geq 0}$ represents the surplus at time t, and $c > 0$ is the premium rate. $Z_t = \sum_{i=1}^{M(t)} Y_i$ is a compound Poisson process, which represents the cumulative claimed size at moment t. $M(t) = \sup\{k : S_1 + S_2 + \cdots + S_k \leq t\}$ is a homogeneous Poisson process with parameter $\theta \geq 0$, where the claim interval $\{S_i\}_{i=1}^{\infty}$

has a common exponential distribution with the parameter θ. Y_i is the i-th claim of $\{Y_i\}_{i=1}^{\infty}$, which is a sequence of independent and identically distributed (*i.i.d*) non-negative random variables with c.d.f. F_Y and p.d.f. f_Y, and the common mean is μ. $\{M(t)\}_{t\geq 0}$ and $\{Y_i\}_{i=1}^{\infty}$ are independent each other. Besides, the Eq (2.1) should satisfies the safe load condition $c > \theta\mu$.

In order to improve profitability and reduce risks, insurance companies usually invest their assets in a portfolio. There are two types of assets in the financial market: risk-less assets and risk assets. By making a portfolio investment, insurers can pursue higher returns while protecting their interests by moderately diversifying risk. The risk-less asset $\{P_t\}_{t\geq 0}$ satisfies the equation

$$\mathrm{d}P_t = r_0 P_t \mathrm{d}t, \tag{2.2}$$

where $r_0 > 0$ is the interest rate under risk-less investment. Risk asset $\{Q_t\}_{t\geq 0}$ obeys a geometric Lévy process, and we have

$$Q_t = e^{\epsilon t + \sigma_r B_t + \sum_{i=1}^{N(t)} X_i}, \tag{2.3}$$

where $\epsilon(\epsilon > 0)$ is the expected instantaneous rate of return of the risk asset, and σ_r is the price volatility of the risk asset. $\{B_t\}_{t\geq 0}$ is a standard Brownian motion, and $\{X_i\}_{i=1}^{\infty}$ is an i.i.d random sequence with c.d.f. F_X and p.d.f. f_X. $N(t) = \sup\{k : G_1 + G_2 + \cdots + G_k \leq t\}$ is a homogeneous Poisson process with parameter $\eta \geq 0$, where $\{G_i\}_{i=1}^{\infty}$ is the jump interval time of Q_t with a common exponential distribution. $\{Y_i\}_{i=1}^{\infty}$, $\{X_i\}_{i=1}^{\infty}$, $\{M(t)\}_{t\geq 0}$ and $\{N(t)\}_{t\geq 0}$ are independent each other.

Therefore, the risk asset $\{Q_t\}_{t\geq 0}$ becomes

$$\frac{\mathrm{d}Q_t}{Q_t} = (\epsilon + \frac{1}{2}{\sigma_r}^2)\mathrm{d}t + \sigma_r \mathrm{d}B_t + \mathrm{d}\sum_{i=1}^{N(t)}(e^{X_i} - 1). \tag{2.4}$$

In addition, q is used to represent the proportion of investment in risk asset, and p is used to represent the proportion of investment in risk-less asset, and apparently there is $0 < q < 1, p + q = 1$. Therefore, the surplus process satisfying the above investment strategy is

$$\mathrm{d}C_t = qC_{t-}\frac{\mathrm{d}Q_t}{Q_t} + (1-q)C_{t-}\frac{\mathrm{d}P_t}{P_t} + c\mathrm{d}t - \mathrm{d}Z_t. \tag{2.5}$$

In fact, the insurance company will not invest all of its assets, but will keep a portion of the funds as liquid reserves, marking the level of liquid reserves as $\Delta(\Delta > 0)$ [8]. On this basis, we consider the dividend payment problem of model 2.5 under the threshold strategy, and label the threshold level as $b(b \geq \Delta)$. If the company's positive assets are lower than Δ, it will neither invest nor pay dividends to shareholders; if the surplus is higher than Δ, the surplus exceeding Δ is invested in a portfolio; if the surplus continues to exceed b, the surplus exceeding b will be paid to the shareholder at a fixed dividend rate $\vartheta(0 < \vartheta < c)$. Therefore, the surplus process $\{C_t\}_{t\geq 0}$ is

$$\mathrm{d}C_t = \begin{cases} c\mathrm{d}t - \mathrm{d}Z_t, & 0 \leq C_{t-} < \Delta, \\ \\ q\hat{C}_t\frac{\mathrm{d}Q_t}{Q_t} + (1-q)\hat{C}_t\frac{\mathrm{d}P_t}{P_t} + c\mathrm{d}t - \mathrm{d}Z_t, & \Delta \leq C_{t-} < b, \\ \\ q\hat{C}_t\frac{\mathrm{d}Q_t}{Q_t} + (1-q)\hat{C}_t\frac{\mathrm{d}P_t}{P_t} + (c-\vartheta)\mathrm{d}t - \mathrm{d}Z_t, & b \leq C_{t-} < \infty, \end{cases} \tag{2.6}$$

where $\hat{C}_t = C_{t-} - \Delta$, and, according to Eqs (2.2)–(2.6), it can be obtained that

$$\mathrm{d}C_t = \begin{cases} c\mathrm{d}t - \mathrm{d}Z_t, & 0 \le C_{t-} < \Delta, \\ q\sigma_r \hat{C}_t \mathrm{d}B_t - \mathrm{d}\sum_{i=1}^{M(t)} Y_i + (\hat{r}\hat{C}_t + c)\mathrm{d}t + q\hat{C}_t \mathrm{d}\sum_{i=1}^{N(t)} (e^{X_i} - 1), & \Delta \le C_{t-} < b, \\ q\sigma_r \hat{C}_t \mathrm{d}B_t - \mathrm{d}\sum_{i=1}^{M(t)} Y_i + (\hat{r}\hat{C}_t + c - \vartheta)\mathrm{d}t + q\hat{C}_t \mathrm{d}\sum_{i=1}^{N(t)} (e^{X_i} - 1), & b \le C_{t-} < \infty, \end{cases} \tag{2.7}$$

where $\hat{r} = q(\epsilon + \frac{1}{2}{\sigma_r}^2) + (1-q)r_0$, and the security loading condition is $c - \vartheta > \theta E[Y_1]$.

In this paper, we will consider the expected total discounted dividend up to ruin, expressed as $V_\Delta(u;b)$ and briefly recorded as V_Δ, which is

$$V_\Delta = \vartheta E\Big[\int_0^{T_u} e^{-\varepsilon t} I(C_t > b)\mathrm{d}t\Big], \tag{2.8}$$

where ε is the interest force and $T_u = \inf\{t : C_t \le 0\}$ is the time of ruin.

The remainder is organized as follows: In Section 3, we derive a system of integral differential equations (IDEs) in which the expected total discounted dividend V_Δ before ruin satisfies certain boundary conditions. In Section 4, we get an approximate solution of V_Δ by means of sinc numerical approximation. In Section 5, we go through numerical examples to discuss the effects of parameters such as the risk capital ratio q and liquid reserves Δ on the expected total discounted dividend before ruin.

3. Integral differential equation for V_Δ

When the initial capital of the insurance company is different, the expected discounted dividend payment V_Δ satisfies different expressions, so we let

$$V_\Delta(u;b) = \begin{cases} V_1(u;b), & 0 \le u < \Delta; \\ V_2(u;b), & \Delta \le u < b; \\ V_3(u;b), & b \le u < +\infty. \end{cases}$$

For convenience, we will simply write $V_i(u;b)$ as $V_i, i = 1,2,3$. Then, we get the following theorem:

Theorem 3.1. *For $0 \le u < \Delta$, we have*

$$cV_1' - (\theta + \eta + \varepsilon)V_1 + \theta\int_0^u V_1(u-y;b)\mathrm{d}F_Y(y) = 0, \tag{3.1}$$

for $\Delta \le u < b$, we have

$$\frac{1}{2}q^2{\sigma_r}^2(u-\Delta)^2V_2'' + [\hat{r}(u-\Delta)+c]V_2' - (\theta+\eta+\varepsilon)V_2$$

$$
\begin{aligned}
&+\theta\left[\int_0^{u-\Delta} V_2(u-y;b)\mathrm{d}F_Y(y)+\int_{u-\Delta}^{u} V_1(u-y;b)\mathrm{d}F_Y(y)\right]\\
&+\eta\left[\int_{-\infty}^{R} V_2(u_q;b)\mathrm{d}F_X(x)+\int_{R}^{+\infty} V_3(u_q;b)\mathrm{d}F_X(x)\right]=0, \qquad (3.2)
\end{aligned}
$$

and for $b \le u < +\infty$, we have

$$
\begin{aligned}
&\frac{1}{2}q^2\sigma_r^2(u-\Delta)^2V_3'' + [\hat{r}(u-\Delta)+c-\vartheta]V_3' - (\theta+\eta+\varepsilon)V_3\\
&+\theta\left[\int_0^{u-b} V_3(u-y;b)dF_Y(y)+\int_{u-b}^{u-\Delta} V_2(u-y;b)dF_Y(y)+\int_{u-\Delta}^{u} V_1(u-y;b)dF_Y(y)\right]\\
&+\eta I(u<B)\left[\int_{-\infty}^{R} V_2(u_q;b)dF_X(x)+\int_{R}^{+\infty} V_3(u_q;b)dF_X(x)\right]\\
&+\eta I(u\ge B)\int_{-\infty}^{+\infty} V_3(u_q;b)dF_X(x)+\vartheta=0, \qquad (3.3)
\end{aligned}
$$

with boundary conditions

$$V_\Delta(0;b)=0, \qquad (3.4)$$

$$\lim_{u\to\infty} V_\Delta(u;b)=\frac{\vartheta}{\varepsilon}, \qquad (3.5)$$

where

$$
\begin{aligned}
B&=\frac{b-q\Delta}{1-q},\\
u_q&=u+q(u-\Delta)(e^x-1),\\
R&=\ln\frac{q(u-\Delta)+b-u}{q(u-\Delta)}.
\end{aligned}
$$

Proof. We refer to the method in reference [13, 20]. Consider the infinitesimal interval $[0, dt]$ and apply the total probability formula according to whether there is a jump in claims and risk capital in the interval.

When $0 \le u < \Delta$, it is relatively simple: there is no investment in the interval $[0, dt]$, and the premium income cannot make the surplus higher than Δ. Under such conditions, only two situations occur: (1) neither the jump in risk investment process nor the claim occurs, with a probability of $P(G_1 > \mathrm{d}t, S_1 > \mathrm{d}t)$; (2) no jump in risk investment process has occurred but the claim has occurred, with a probability of $P(G_1 > \mathrm{d}t, S_1 \le \mathrm{d}t)$. Therefore, according to model (2.7), using the total probability formula, the expression for V_Δ is

$$V_1 = e^{-\varepsilon \mathrm{d}t}\Big\{P_1V_1(u+c\mathrm{d}t;b)+P_2V_1(u+c\mathrm{d}t-Y_1;b)\Big\}, \qquad (3.6)$$

where

$$P_1 = P(G_1 > \mathrm{d}t, S_1 > \mathrm{d}t) = 1-(\eta+\theta)\mathrm{d}t + o(\mathrm{d}t), \qquad (3.7)$$

$$P_2 = P(G_1 > \mathrm{d}t, S_1 \le \mathrm{d}t) = \theta\mathrm{d}t + o(\mathrm{d}t). \qquad (3.8)$$

By Taylor's formula, we can get

$$V_1(u + c\mathrm{d}t; b) = V_1(u; b) + cV_1{}'(u; b)\mathrm{d}t + o(\mathrm{d}t). \tag{3.9}$$

Then, substituting (3.7)–(3.9) into (3.6), rearranging the equation, and letting $\mathrm{d}t \to 0$, we can get Eq (3.1).

When $\Delta \le u < b$, the insurance company makes a portfolio investment. Under this condition, there are three situations: (1) neither the jump in risk investment process nor the claim occurs; (2) no jump in risk investment process has occurred but the claim has occurred, in which case the size of the claim needs to be considered; (3) no claim but the jump in risk investment process has occurred, with probability$P(G_1 \le \mathrm{d}t, S_1 > \mathrm{d}t)$, and also needs to consider the size of the jump in risk investment process. Because the occurrence of both the claims and the jump in risk investment process is a small probability event, we can get that the expression V_Δ is

$$\begin{aligned} V_2 =& e^{-\varepsilon \mathrm{d}t}\Big\{P_1E[V_2(A_1; b)] + P_2E\Big[E[V_2(A_1 - Y_1; b)|Y_1 \in (0, A_1 - \Delta))] \\ &+ E[V_1(A_1 - Y_1; b)|Y_1 \in (A_1 - \Delta, +\infty))]\Big] \\ &+ P_3E\Big[E[V_2(A_1 + q(u - \Delta)(e^{X_1} - 1); b)|X_1 \in (-\infty, h_1)] \\ &+ E[V_3(A_1 + q(u - \Delta)(e^{X_1} - 1); b)|X_1 \in (h_1, +\infty)]\Big]\Big\}, \end{aligned} \tag{3.10}$$

where

$$A_1 = u + q\sigma_r(u - \Delta)\mathrm{d}B_t + [\hat{r}(u - \Delta) + c]\mathrm{d}t, \tag{3.11}$$

$$P_3 = P(G_1 \le \mathrm{d}t, S_1 > \mathrm{d}t) = \eta\mathrm{d}t + o(\mathrm{d}t), \tag{3.12}$$

$$h_1 = \ln\frac{q(u - \Delta) + b - A_1}{q(u - \Delta)}. \tag{3.13}$$

By the *Itô* formula, we can get

$$\begin{aligned} E[V_2(A_1; b)] &= E[V_2(u + q\sigma_r(u - \Delta)\mathrm{d}B_t + [\hat{r}(u - \Delta) + c]\mathrm{d}t; b)] \\ &= V_2(u; b) + [\hat{r}(u - \Delta) + c]V_2{}'(u; b)\mathrm{d}t + \frac{1}{2}q^2\sigma_r{}^2(u - \Delta)^2V_2{}''(u; b)\mathrm{d}t, \end{aligned} \tag{3.14}$$

and then substituting (3.11)–(3.14) into (3.10) and doing the same thing with $0 \le u < \Delta$, we can get Eq (3.2).

When $b \le u < +\infty$, using the same treatment, we can get

$$\begin{aligned} V_3 =& e^{-\varepsilon \mathrm{d}t}\Big\{\vartheta\mathrm{d}t + P_1E[V_3(A_2; b)] + P_2E\Big[E[V_3(A_2 - Y_1; b)|Y_1 \in (0, A_2 - b)] \\ &+ E[V_2(A_2 - Y_1; b)|Y_1 \in (A_2 - b, A_2 - \Delta)] \\ &+ E[V_1(A_2 - Y_1; b)|Y_1 \in (A_2 - \Delta, +\infty)]\Big] \\ &+ P_3E\Big[E[V_2(A_2 + q(U - \Delta)(e^{X_1} - 1); b)|X_1 \in (-\infty, h_2)] \end{aligned}$$

$$+ E[V_3(A_2 + q(b-\Delta)(e^{X_1}-1);b)|X_1 \in (h_2,+\infty)]\bigg]\bigg\}, \tag{3.15}$$

where

$$A_2 = u + q\sigma_r(u-\Delta)\mathrm{d}B_t + [\hat{r}(u-\Delta)+c-\vartheta]\mathrm{d}t, \tag{3.16}$$
$$P_3 = P(G_1 \le \mathrm{d}t, S_1 > \mathrm{d}t) = \eta\mathrm{d}t + o(\mathrm{d}t), \tag{3.17}$$
$$h_2 = \ln\frac{q(u-\Delta)+b-A_2}{q(u-\Delta)}. \tag{3.18}$$

By the Itô formula, we can get

$$\begin{aligned} E[V_3(A_2;b)] &= E[V_3(u+q\sigma_r(u-\Delta)\mathrm{d}B_t + [\hat{r}(u-\Delta)+c-\vartheta]\mathrm{d}t;b)] \\ &= V_3(u;b) + [\hat{r}(b-\Delta)+c-\vartheta]{V_3}'(u;b)\mathrm{d}t + \frac{1}{2}q^2{\sigma_r}^2(u-\Delta)^2{V_3}''(u;b)\mathrm{d}t, \end{aligned} \tag{3.19}$$

and then substituting (3.16)–(3.19) into (3.15) and doing the same as above, we can get Eq (3.3).

Finally, if $u = 0$, the insurance company will immediately ruin, and therefore dividends will not be paid and condition (3.4) can be obtained. If $u \to \infty$, ruin does not always occur, so $T_u = \infty$ and then the boundary conditions (3.5) can be obtained by calculating (2.8), proving the theorem. □

Remark 3.1. *It should be noted that, when $u > \Delta$, there is $q(u-\Delta)+\Delta-u < 0$, which means $\ln\frac{q(u-\Delta)+\Delta-u}{q(u-\Delta)}$ is meaningless. Therefore, it is impossible to jump to a value less than Δ in* (3.10) *and* (3.15).

Remark 3.2. *The fact that V_Δ and $(V_\Delta)'$ are continuous at Δ and b has been discussed in reference [13].*

4. Numerical approximate solution of V_Δ

The explicit solutions of Theorem 3.1 cannot be obtained, so a method of sinc approximation to find the approximate solution is provided in this section. Since E.T. Whiaker first proposed the idea of the sinc numerical method [21], the sinc numerical method has attracted a lot of scholars to explore and study it [22–25]. Compared with traditional methods, the sinc method not only converges faster and can reach exponential order convergence, but also has stronger applicability to problems with singularity, finite, infinite, and semi-finite integral regions [26].

We construct a one-to-one mapping of $\mathbb{R}^+ \to \mathbb{R}$, $\varphi(x) = \log x$. Hence, the sinc grid point is defined as

$$x_k = \varphi^{-1}(kh) = e^{kh}, \tag{4.1}$$

where $h > 0$ is the interval of grid points, and k is a positive integer.

To facilitate the use of the sinc method, rearranging the integral differential equations (3.1)–(3.3), we have

$$\begin{aligned} &\frac{1}{2}q^2{\sigma_r}^2(u-\Delta)^2 I_{u\ge\Delta}(V_\Delta)'' + [\hat{r}(u-\Delta)I_{u\ge\Delta} + c - \vartheta I_{u\ge b}](V_\Delta)' \\ &-(\varepsilon+\theta+\eta)V_\Delta + \theta\int_0^u V_\Delta(u-y;b)f_Y(y)\mathrm{d}y \end{aligned}$$

$$+\eta I_{u\geq\Delta}\int_{-\infty}^{+\infty}V_{\varDelta}(u+q(u-\varDelta)(e^{x}-1);b)f_{X}(x)\mathrm{d}x+\vartheta I_{u\geq b}=0. \tag{4.2}$$

Next, letting $z=u+q(u-\Delta)(e^{x}-1)$ and transforming Eq (4.2), we get

$$\begin{aligned}&\frac{1}{2}q^{2}{\sigma_{r}}^{2}(u-\Delta)^{2}I_{u\geq\Delta}(V_{\varDelta})''+[\hat{r}(u-\varDelta)I_{u\geq\varDelta}+c-\vartheta I_{u\geq b}](V_{\varDelta})'\\&-(\varepsilon+\theta+\eta)V_{\Delta}+\theta\int_{0}^{u}V_{\Delta}(y;b)f_{Y}(u-y)\mathrm{d}y+\vartheta I_{u\geq b}\\&+\eta I_{u\geq\Delta}\int_{(1-q)u+q\Delta}^{+\infty}V_{\Delta}(z;b)f_{X}\Big(\ln\frac{z-u+q(u-\Delta)}{q(u-\Delta)}\Big)\frac{1}{z-u+q(u-\Delta)}\mathrm{d}z=0,\end{aligned} \tag{4.3}$$

with boundary conditions

$$\begin{aligned}V_{\Delta}(0;b)&=0,\\\lim_{u\to\infty}V_{\varDelta}(u;b)&=\frac{\vartheta}{\varepsilon}.\end{aligned}$$

According to reference [27] (p. 73), let $K(u)=V_{\Delta}-\frac{V_{\Delta}(0)+e^{\varphi(u)}V_{\Delta}(s)}{1+e^{\varphi(u)}}$, and when $s\to\infty$, we have

$$K(u)=V_{\Delta}-\frac{u}{1+u}\frac{\vartheta}{\varepsilon}. \tag{4.4}$$

Substituting (4.4) into (4.3),we have

$$\begin{aligned}&\kappa_{1}(u)K''(u)+\kappa_{2}(u)K'(u)+\kappa_{3}(u)K(u)+\theta\int_{0}^{u}K(y)L_{1}(u-y)\mathrm{d}y\\&+\kappa_{4}(u)\int_{(1-q)u+q\Delta}^{\infty}K(z)L_{2}(u,z)\mathrm{d}z+H(u)=0,\end{aligned} \tag{4.5}$$

with boundary conditions

$$\begin{aligned}K(0)&=0,\\\lim_{u\to\infty}K(u)&=0,\end{aligned}$$

where

$$\begin{aligned}\kappa_{1}(u)&=\frac{1}{2}q^{2}{\sigma_{r}}^{2}(u-\Delta)^{2}I_{u\geq\Delta},\quad\kappa_{2}(u)=\hat{r}(u-\Delta)I_{u\geq\Delta}+c-\vartheta I_{u\geq b},\\\kappa_{3}(u)&=-(\varepsilon+\theta+\eta),\quad\kappa_{4}(u)=\eta I_{u\geq\Delta},\quad L_{1}(u-y)=f_{Y}(u-y),\\L_{2}(u,z)&=\frac{f_{X}\Big(\ln\frac{z-u+q(u-\Delta)}{q(u-\Delta)}\Big)}{z-u+q(u-\Delta)},\\H(u)&=-\frac{2}{(1+u)^{3}}\frac{\vartheta}{\varepsilon}\kappa_{1}(u)+\frac{1}{(1+u)^{2}}\frac{\vartheta}{\varepsilon}\kappa_{2}(u)+\frac{u}{(1+u)}\frac{\vartheta}{\varepsilon}\kappa_{3}(u)+\vartheta I_{u\geq\Delta}\\&\quad+\theta\int_{0}^{u}\frac{y}{1+y}\frac{\vartheta}{\varepsilon}L_{1}(u-y)\mathrm{d}y+\kappa_{4}(u)\int_{(1-q)u+q\Delta}^{\infty}\frac{z}{1+z}\frac{\vartheta}{\varepsilon}L_{2}(u,z)\mathrm{d}z.\end{aligned}$$

By using Theorems 1.5.13, 1.5.19, and 1.5.20 of reference [27], let

$$M = \left[\frac{\hat{\beta}N}{\hat{\vartheta}}\right],\ h = \left(\frac{\pi d}{\hat{\beta}N}\right)^{\frac{1}{2}},$$

where $0 < \hat{\vartheta}, \hat{\beta} \leq 1$ and $0 < d < \pi$. Then, we have

$$\int_0^u K(y)L_1(u-y)\mathrm{d}y \approx \sum_{j=-M}^{N}\sum_{i=-M}^{N} \omega_i A_{ij} K_j, \tag{4.6}$$

$$\int_{(1-q)u+q\Delta}^{\infty} L_2(u,z)K(z)\mathrm{d}z \approx h\sum_{j=-M}^{N}\sum_{i=-M}^{N} \omega_i \delta_{ji}^{(-1)} \frac{L_2(u,u_j)}{\varphi'(u_j)} K_j, \tag{4.7}$$

$$K(u) \approx \hat{K}(u) = \sum_{j=-M}^{N} K_j [S(j,h)\circ\varphi(u)], \tag{4.8}$$

$$\omega_i = \omega_i(u) = S(i,h)\circ\varphi(u) = sinc\left(\frac{\varphi(u) - ih}{h}\right), i = -M, -M+1, ..., N, \tag{4.9}$$

$$A = hI^{(-1)}D_m\left(\frac{1}{\varphi'}\right), \tag{4.10}$$

where

$$I^{(-1)} = [\delta_{kj}^{(-1)}],\ \ \delta_{kj}^{(-1)} = 0.5 + \int_0^{k-j} \frac{sin(\pi t)}{\pi t} dt,\ \ sinc(x) = \begin{cases} \frac{sinc(\pi x)}{\pi x}, & x \neq 0, \\ 1, & else, \end{cases}$$

K_j denotes the approximate value of $K(u_j)$, N is a positive integer, and A_{ij} represents the elements of matrix A.

Let $u_k = e^{kh}, k = \pm 1, \pm 2, ...$, which is defined by (4.1) as the sinc grid points. According to reference [28] (p. 106), we have

$$\hat{K}(u_k) = \sum_{j=-M}^{N} K_j[S(j,h)\circ\varphi(u_k)] = \sum_{j=-M}^{N} K_j \delta_{jk}^{(0)}, \tag{4.11}$$

$$\hat{K}'(u_k) = \sum_{j=-M}^{N} K_j[S(j,h)\circ\varphi(u_k)]' = \sum_{j=-M}^{N} K_j \varphi'(u_k)\delta_{jk}^{(1)}, \tag{4.12}$$

$$\hat{K}''(u_k) = \sum_{j=-M}^{N} K_j[S(j,h)\circ\varphi(u_k)]'' = \sum_{j=-M}^{N} K_j[\varphi''(u_k)h^{-1}\delta_{jk}^{(1)} + (\varphi'(u_k))^2 h^{-2}\delta_{jk}^{(2)}], \tag{4.13}$$

where

$$\delta_{jk}^{(0)} = \begin{cases} 0, & j \neq k, \\ 1, & else. \end{cases} \quad \delta_{jk}^{(1)} = \begin{cases} \frac{(-1)^{(k-j)}}{k-j}, & j \neq k; \\ 0, & else. \end{cases} \quad \delta_{jk}^{(2)} = \begin{cases} \frac{-2(-1)^{(k-j)}}{(k-j)^2}, & j \neq k, \\ -\frac{\pi^2}{3}, & else. \end{cases}$$

Substituting (4.6)–(4.8) and (4.11)–(4.13) into Eq (4.5), we obtain

$$\sum_{j=-M}^{N}\Bigg\{\kappa_1(u_k)\varphi''(u_k)\frac{\delta_{jk}^{(1)}}{h}+\kappa_1(u_k)(\varphi'(u_k))^2\frac{\delta_{jk}^{(2)}}{h^2}+\kappa_2(u_k)\varphi'(u_k)\frac{\delta_{jk}^{(1)}}{h}+\kappa_3(u_k)\delta_{jk}^{(0)}$$
$$+\theta\sum_{i=-M}^{N}\omega_i(u_k)A_{ij}+h\kappa_4(u_k)\sum_{i=-M}^{N}\omega_i(u_k)\delta_{ji}^{(-1)}\frac{L_2(u_k,u_j)}{\varphi'(u_j)}\Bigg\}K_j=-H(u_k). \tag{4.14}$$

Let us multiply both sides of the Eq (4.14) by $\frac{h^2}{(\varphi'(u_k))^2}$. Then we have

$$\sum_{j=-M}^{N}\Bigg\{\kappa_1(u_k)\delta_{jk}^{(2)}+h\left[\kappa_1(u_k)\frac{\varphi''(u_k)}{(\varphi'(u_k))^2}+\frac{\kappa_2(u_k)}{\varphi'(u_k)}\right]\delta_{jk}^{(1)}+h^2\frac{\kappa_3(u_k)}{(\varphi'(u_k))^2}\delta_{jk}^{(0)}$$
$$+\theta\frac{h^2}{(\varphi'(u_k))^2}\sum_{i=-M}^{N}\omega_i(u_k)A_{ij}+\frac{\kappa_4(u_k)h^3}{(\varphi'(u_k))^2\varphi'(u_j)}\sum_{i=-M}^{N}\omega_i(u_k)\delta_{ji}^{(-1)}L_2(u_k,u_j)\Bigg\}K_j=-\frac{h^2H(u_k)}{(\varphi'(u_k))^2}. \tag{4.15}$$

Since

$$\delta_{jk}^{(0)}=\delta_{kj}^{(0)},\quad \delta_{jk}^{(1)}=-\delta_{kj}^{(1)},\quad \delta_{jk}^{(2)}=\delta_{kj}^{(2)},\quad \frac{\varphi''(u_k)}{(\varphi'(u_k))^2}=-\left(\frac{1}{\varphi'(u_k)}\right)',$$

Eq (4.15) can be rewritten as

$$\sum_{j=-M}^{N}\Bigg\{\kappa_1(u_k)\delta_{kj}^{(2)}+h\left[\kappa_1(u_k)\left(\frac{1}{\varphi'(u_k)}\right)'-\frac{\kappa_2(u_k)}{\varphi'(u_k)}\right]\delta_{kj}^{(1)}+h^2\frac{\kappa_3(u_k)}{(\varphi'(u_k))^2}\delta_{kj}^{(0)}$$
$$+\theta\frac{h^2}{(\varphi'(u_k))^2}\sum_{i=-M}^{N}\omega_i(u_k)A_{ij}+\frac{\kappa_4(u_k)h^3}{(\varphi'(u_k))^2\varphi'(u_j)}\sum_{i=-M}^{N}\omega_i(u_k)\delta_{ji}^{(-1)}L_2(u_k,u_j)\Bigg\}K_j$$
$$=-\frac{h^2H(u_k)}{(\varphi'(u_k))^2},\ k=-M,\dots,N. \tag{4.16}$$

Set $\Omega_M=(\omega_{-M}(u_k),...,\omega_N(u_k))$, $L_2=[\frac{L_2(u_k,u_j)}{\varphi'(u_k)}]$. $D_m(.)$ represents a diagonal matrix and $I^{(m)}=[\delta_{kj}^{(m)}]$, $m=-1,0,1,2$, is a square matrix of order $M+N+1$, where $\delta_{kj}^{(m)}$ represents the elements of matrix $I^{(m)}$. We rewrite Eq (4.16) as

$$CK=H, \tag{4.17}$$

where $K=[K_{-M},K_{-M+1},\cdots,K_N]^T$,

$$H=\left[-h^2\frac{H(u_{-M})}{(\varphi'(u_{-M}))^2},\dots,-h^2\frac{H(u_N)}{(\varphi'(u_N))^2}\right]^T,$$
$$C=\kappa_1I^{(2)}+hD_m\left(\kappa_1\left(\frac{1}{\varphi'}\right)'-\frac{\kappa_2}{\varphi'}\right)I^{(1)}+h^2D_m\left(\frac{\kappa_3}{(\varphi')^2}\right)I^{(0)}+h^2\theta D_m\left(\frac{1}{(\varphi')^2}\right)\Omega_MA$$
$$+h^3D_m\left(\frac{\kappa_4}{(\varphi')^2}\right)\Omega_M[I^{(-1)}]^TL_2.$$

By solving the matrix equation (4.17) we can get K_j, and according to Eqs (4.4) and (4.8), we obtain

$$V_\Delta(u;b) \approx \sum_{j=-M}^{N} K_j S(j,h) \circ \varphi(u) + \frac{1}{1+u}\frac{\vartheta}{\varepsilon}. \tag{4.18}$$

Therefore, we can get an approximate solution to V_Δ by substituting the obtained K_j into Eq (4.18) and assigning the parameters.

5. Numerical example

In this section, we study the influence of some arguments on $V_\Delta(u;b)$ by some examples. The p.d.f. $f_X(x)$ of the random returns for all examples is assumed to be

$$f_X(x) = p_1\gamma_1 e^{-\gamma_1 x} I_{x\geq 0} + q_1\gamma_2 e^{\gamma_2 x} I_{x<0}, \tag{5.1}$$

where $p_1 + q_1 = 1$, $0 < p_1, q_1 < 1$, $\gamma_1 \geq 1$, and $\gamma_2 > 0$. $f_Y(y)$ is assumed to be the exponential and lognormal distributions commonly used in actuarial research.

5.1. The exponential distribution case

In this subsection, the p.d.f. of the random variable Y is an exponential distribution, defined as

$$f_Y(y) = \mu e^{-\mu y}, 0 < y < \infty.$$

Then

$$L_1(u-y) = \begin{cases} \mu e^{-\mu(u-y)}, & 0 < y \leq u; \\ 0, & u < y < \infty. \end{cases}$$

Next, we discuss the effects of parameter q, p_1, σ_r, and Δ on V_Δ and the optimal b. The basic parameter of this subsection are $\theta = 1, \varepsilon = 0.06, r = 0.06, \epsilon = 0.5, c = 0.5, \vartheta = 0.2, \gamma_1 = 1, \gamma_2 = 2, \mu = 5$, and the parameters involved in the sinc numerical method are set to $N = 10$, $\hat{\vartheta} = \frac{\pi}{4}, \hat{\beta} = \frac{1}{4}$, and $\hat{d} = \frac{\pi}{10}$.

Example 5.1. *For different initial surplus, we first observe the impact of different dividend boundaries b on V_Δ. The basic parameter settings are the same as above, and then we fix $q = 0.9$, $p_1 = 0.2$, $\sigma_r = 0.2$, $\eta = 1$, and $\Delta = 0.1$. When the initial surplus is different, the optimal b is different, and some of the data are listed in Table 2.*

Table 2. The values of V_Δ for different b with an exponential distribution.

u	0.5	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0
$b = 0.5$	0.4412	1.8274	2.2429	2.5167	2.7387	2.8447	2.8897	2.9170	2.9447
$b = 1.0$	0.6950	1.8095	2.2408	2.4925	2.7458	2.8599	2.8910	2.9004	2.9164
$b = 2.0$	0.7069	1.8082	2.2377	2.4921	2.7479	2.8618	2.8917	2.8998	2.9151
$b = 3.0$	0.7069	1.8082	2.2377	2.4921	2.7479	2.8618	2.8917	2.8998	2.9151
$b = 4.0$	0.7066	1.8098	2.2363	2.4902	2.7471	2.8621	2.8924	2.9005	2.9157
$b = 5.0$	0.7066	1.8098	2.2363	2.4902	2.7471	2.8621	2.8924	2.9005	2.9157

Example 5.2. *We consider the impact of p_1 and investment ratio q on the expected discounted dividend payments V_Δ. As shown in Figure 1, when p_1 is large, the value of V_Δ is more affected by q. On the whole, the curve of V_Δ shows a fluctuating upward trend.*

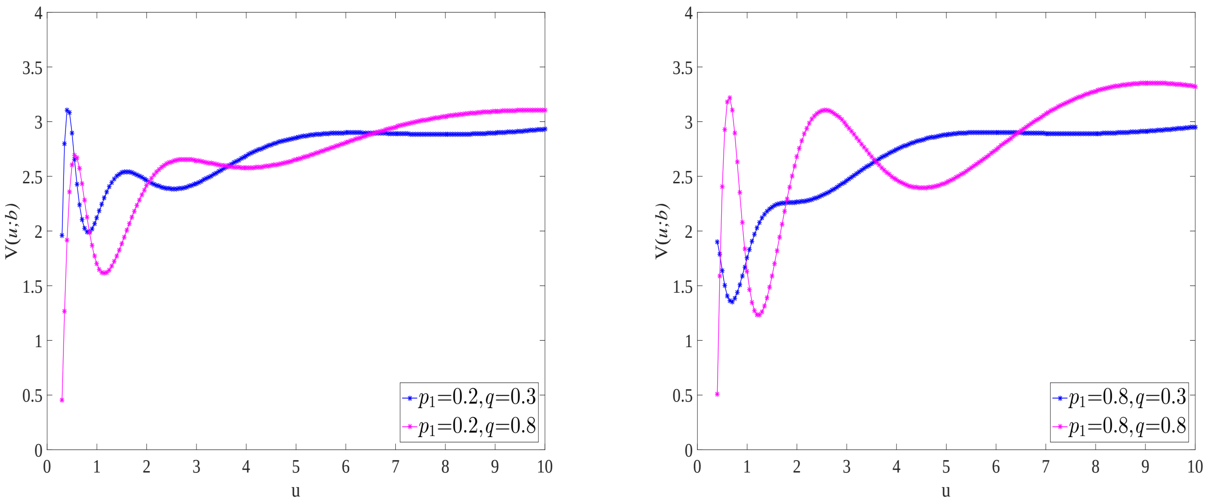

Figure 1. Curves of V_Δ when $b = 1$, $\sigma_r = 0.2$, $\eta = 1$, $\Delta = 0.1$.

Example 5.3. *Next, we consider the impact of fluctuations in risk assets σ_r on V_Δ. As shown in Figure 2, the larger σ_r is, the more the curve of V_Δ fluctuates.*

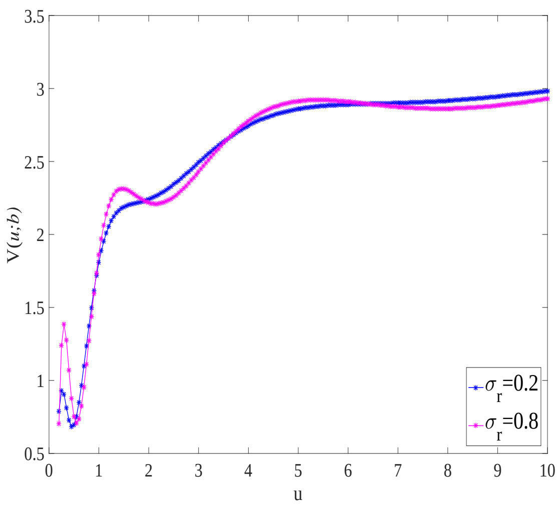

Figure 2. Curves of V_Δ when $b = 1$, $q = 0.9$, $p_1 = 0.2$, $\eta = 1$, $\Delta = 0.1$.

Example 5.4. *Then, we consider the impact of the value of the jump in risky assets η on V_Δ. As shown in Figure 3, as the value of η increases, the curve of V_Δ becomes more stable.*

Example 5.5. *Finally, we consider the impact of liquid reserves Δ on V_Δ. As shown in Figure 4, when the liquid reserve is large, with the increase of initial surplus, the dividend of insurance companies is more stable and the curve of V_Δ shows an upward trend.*

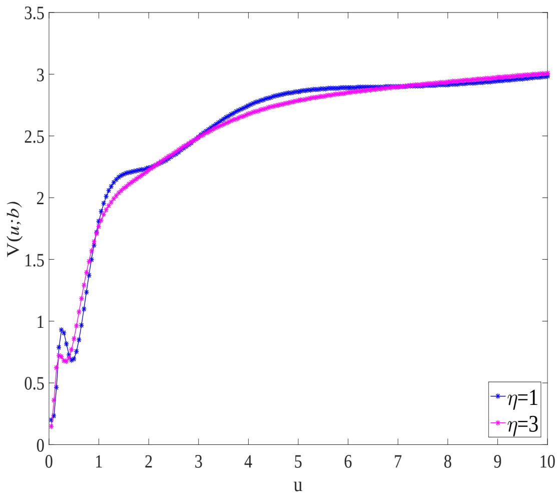

Figure 3. Curves of V_Δ when $b = 1$, $q = 0.9$, $p_1 = 0.2$, $\sigma_r = 0.2$, $\Delta = 0.1$.

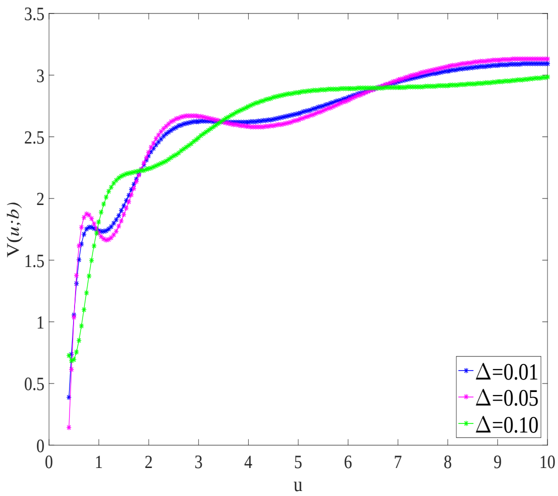

Figure 4. Curves of V_Δ when $b = 1$, $q = 0.9$, $p_1 = 0.2$, $\eta = 1$, $\sigma_r = 0.2$.

5.2. The lognormal distribution case

In this subsection, we assume that $f_Y(y)$ obeys the lognormal distribution with the parameter $(\mu_0, 2\nu^2)$, where μ_0 is the mean and $2\nu^2$ is the variance. The expression for $f_Y(y)$ is

$$f_Y(y) = \begin{cases} \frac{1}{2\pi\nu y} e^{-\frac{(\ln y - \mu_0)^2}{4\nu^2}}, & 0 < y < +\infty; \\ 0, & -\infty < y \leq 0. \end{cases}$$

Then

$$L_1(u-y) = \begin{cases} \frac{1}{2\pi\nu(u-y))} e^{-\frac{(\ln(u-y)) - \mu_0)^2}{4\nu^2}}, & 0 < y < +\infty; \\ 0, & -\infty < y \leq 0. \end{cases}$$

Next, we discuss the effects of parameter q, p_1, and σ_r on V_Δ and the optimal b. The basic parameter of this subsection are $\theta = 1$, $\varepsilon = 0.06$, $r = 0.06$, $\epsilon = 0.6$, $c = 0.6$, $\vartheta = 0.2$, $\gamma_1 = 1$, $\gamma_2 = 2$, $\nu = 0.03$, and $\mu_0 = 0.05$, and the parameters involved in the sinc numerical method are $N = 10$, $\hat{\vartheta} = \frac{\pi}{4}$, $\hat{\beta} = \frac{1}{4}$, and $\hat{d} = \frac{\pi}{10}$.

Example 5.6. *The basic parameter setting is the same as above, and we fix $q = 0.9$, $p_1 = 0.2$, $\sigma_r = 0.2$, $\eta = 1$, and $\Delta = 0.1$. For different initial surplus, the optimal b is different, and some of the data are listed in Table 3.*

Table 3. The values of V_Δ for different b with a lognormal distribution.

u	0.5	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0
b=0.5	2.1493	1.6224	2.1652	2.4479	2.6599	2.7804	2.8380	2.8701	2.8981
b=1.0	1.9732	1.7393	2.1281	2.4094	2.6885	2.8161	2.8458	2.8480	2.8587
b=2.0	1.9612	1.7400	2.1320	2.4102	2.6860	2.8136	2.8449	2.8487	2.8603
b=3.0	1.9612	1.7400	2.1320	2.4102	2.6860	2.8136	2.8449	2.8487	2.8603
b=4.0	1.9614	1.7392	2.1327	2.4112	2.6865	2.8136	2.8445	2.8483	2.8599
b=5.0	1.9614	1.7392	2.1327	2.4112	2.6865	2.8136	2.8445	2.8483	2.8599

Example 5.7. *We consider the effects of p_1 and q on V_Δ. As shown in Figure 5, when q increases, the curve fluctuates greatly, and when p_1 is larger, the curve fluctuation amplitude of V_Δ is also larger.*

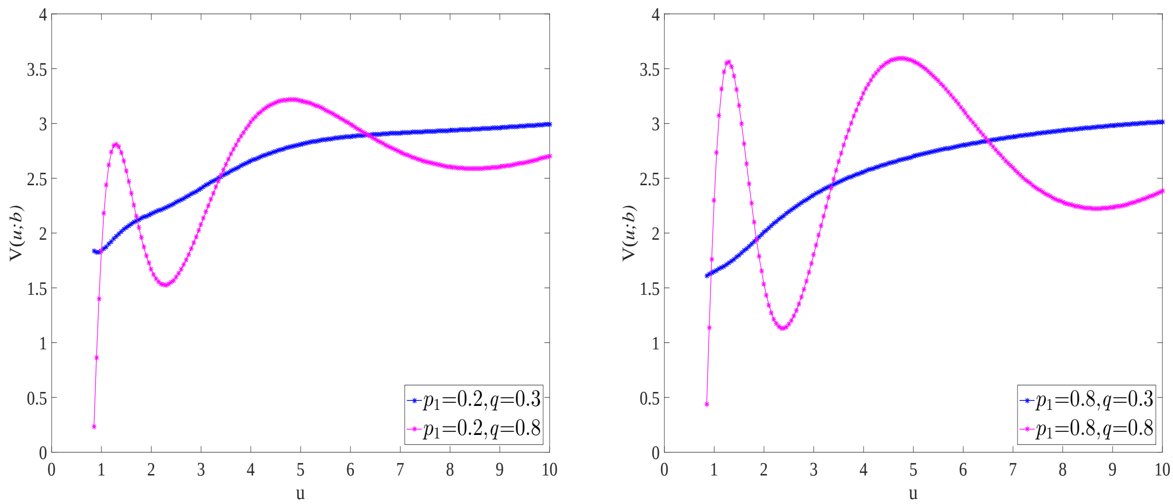

Figure 5. Curves of V_Δ when $b = 1$, $\sigma_r = 0.2$, $\eta = 1$, $\Delta = 0.1$.

Example 5.8. *Next, we consider the impact of σ_r on V_Δ. As shown in Figure 6, the overall curve shows an upward trend, but σ_r has little influence on V_Δ.*

Example 5.9. *Then, we consider the impact of η on V_Δ. As shown in Figure 7, the curve shows an overall upward trend. When $u \in (0, 2)$, the larger η, the larger the curve fluctuation of V_Δ; when $u \in [2, \infty)$, η has almost no effect on V_Δ.*

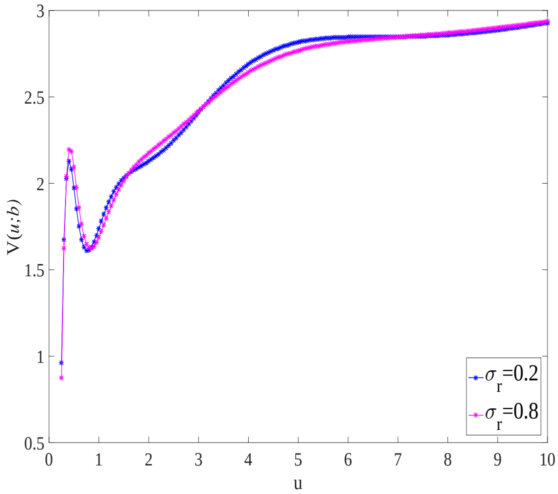

Figure 6. Curves of V_Δ when $b = 1$, $q = 0.9$, $p_1 = 0.2$, $\eta = 1$, $\Delta = 0.1$.

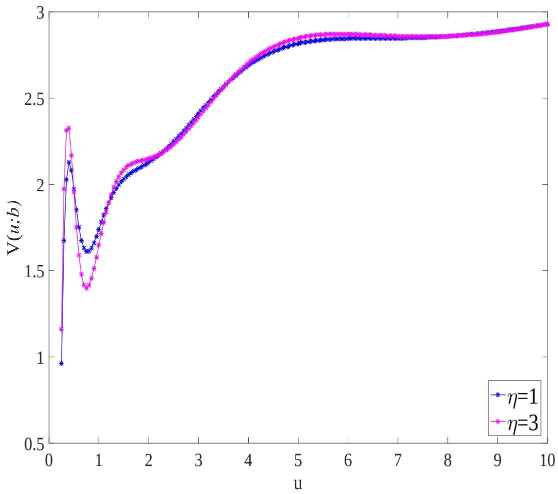

Figure 7. Curves of V_Δ when $b = 1$, $q = 0.9$, $p_1 = 0.2$, $\sigma_r = 0.2$, $\Delta = 0.1$.

Example 5.10. *Finally, we consider the impact of liquid reserve* Δ *on expected discounted dividend payments* V_Δ*. As shown in Figure 8, when* $u \in (0, 1)$*, the curve of* V_Δ *fluctuates greatly. When* $u \in [1, \infty)$*,* $\Delta = 0.10$*, and the curve of* V_Δ *generally shows a steady upward trend; for* $\Delta = 0.01, 0.05$*, the curve of* V_Δ *generally shows a fluctuating upward trend.*

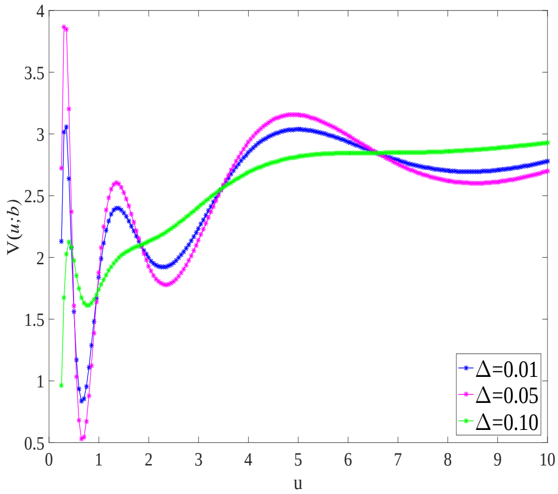

Figure 8. Curves of V_Δ when $b = 1$, $q = 0.9$, $p_1 = 0.2$, $\eta = 1$, $\sigma_r = 0.2$.

6. Conclusions

In this paper, the risk model with liquid reserve and proportional investment is considered. This model is more in line with the actual operation of the company and has practical significance. Because of the complexity of the model, it is impossible to find the exact solution of the expected total discounted dividend before ruin, so the sinc numerical approximation method is used to find the approximate solution. In the end, some examples are given to describe the effects of some parameters on the expected total discounted dividend before ruin.

In addition, we can do further research as follows: (1) We can consider the Gerber-shu function the model; (2) We can consider cooperating with insurance companies to study the optimal dividend strategy based on this risk model and using real data; (3) We can find an optimal solution that is better than the sinc numerical approximation. However, these goals and research methods may be hampered by technical difficulties and practical problems, which are difficult to achieve, and we leave it to future research to solve.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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