Mathematics

## Research article

# Performance of the Walrus Optimizer for solving an economic load dispatch problem 

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#### Abstract

A new metaheuristic called the Walrus Optimizer (WO) is inspired by the ways in which walruses move, roost, feed, spawn, gather, and flee in response to important cues (safety and danger signals). In this work, the WO was used to address the economic load dispatch (ELD) issue, which is one of the essential parts of a power system. One type of ELD was designed to reduce fuel consumption expenses. A variety of methodologies were used to compare the WO's performance in order to determine its reliability. These methods included rime-ice algorithm (RIME), moth search algorithm (MSA), the snow ablation algorithm (SAO), and chimp optimization algorithm (ChOA) for the identical case study. We employed six scenarios: Six generators operating at two loads of 700 and 1000 MW each were employed in the first two cases for the ELD problem. For the ELD problem, the second two scenarios involved ten generators operating at two loads of 2000 MW and 1000 MW. Twenty generators operating at a 3000 MW load were the five cases for the ELD issue. Thirty generators operating at a 5000 MW load were the six cases for the ELD issue. The power mismatch factor was the main cause of ELD problems. The ideal value of this component should be close to zero. Using the WO approach, the ideal power mismatch values of $4.1922 \mathrm{E}-13$ and $4.5119 \mathrm{E}-13$ were found for six generator units at demand loads of 700 MW and 1000 MW , respectively. Using metrics for the minimum, mean, maximum, and standard deviation of fitness function, the procedures were evaluated over thirty


separate runs. The WO outperformed all other algorithms, as seen by the results generated for the six ELD case studies.

Keywords: Walrus optimizer; economic load dispatch; power system Mathematics Subject Classification: 68R99

## 1. Introduction

In power systems, economic load dispatch, or ELD, the aim is to distribute power extracted from producing units as economically as possible while meeting operational demands, maintaining supplydemand equilibrium, and figuring out how to cut down on emissions and power generation costs to help address global warming. There is a lack of coal despite an increase in the need for electricity [1,2]. It is important to note that the fuel consumption curve has a wavy shape due to the valve-point effects. The economic load dispatch problem is therefore a large-scale, highly nonlinear, and constrained optimization problem. Considerable cost reductions can be obtained by optimizing the unit output schedule. With fuel prices rising daily, maximizing the output power from each producing unit is necessary to reduce total fuel expenditures. Mathematical and metaheuristic optimization techniques can be used to achieve this [3].

The linear programming approach was used to determine the electrical producing system's actual and reactive power; however, these methods require a large amount of calculation time and often cannot provide a global answer for large data sets. A number of optimization strategies have been created in this application or another problem with the aim of improving the efficacy of solving the ELD issue [4-8]. The outline search method was proposed as a way to find the best solution for the ELD problem, taking into consideration the impacts of valve loading. A range of test data were employed to evaluate the strategy and compare it to existing optimization techniques in order to bolster the findings [9]. This method was applied to four distinct ELD test systems, ranging in size from small too big and with different degrees of complexity, utilizing the BBO (biogeography-based optimization) method [10]. By solving them using the modified differential evolution approach, several test cases of the ELD were discovered [11]. The authors used the search and rescue optimization technique (SAR) to determine the best strategy for the ELD. According to the study findings, the SAR was the best course of action in every instance of ELD [12].

Six generation units employed the Harris Hawks optimizer technique [13] to address ELD concerns, and the heat transfer search algorithm [14] was utilized to explain the difficult ELD problem after wind energy was added. To address ELD problems, the authors suggested using a multi-strategy ensemble BBO (MSEBBO). The no free lunch theory is used by the MEEBBO to support the three BBO pillars. To comply with the many ELD problem restrictions, a robust repair procedure is also advised [15]. Six real-world generator examples had the ELD problem resolved using the memetic sine-cosine approach [16]. However, as a remedy for ELD problems, the greedy sine-cosine nonhierarchical grey wolf optimizer (G-SCNHGWO) was presented by the authors. In total, there are $40,15,10$, and 140 power generators in these four power systems, and each has a distinct valuation time [17]. The ant lion optimization algorithm (ALO) was used to fix problems with the ideal ELD. Applying the ALO method to all three circumstances yielded superior results than alternative solutions for the problem, convergence velocity, and stability [18]. The ED problem can be resolved very well
using a fully decentralized approach (DA) technique that appropriately accounts for transmission losses in a fully decentralized way. We examined three case studies [19].

In ELD circumstances, the exchange market algorithm (EMA) is a dependable and effective way to identify the best choice for worldwide optimization. Furthermore, it was created utilizing four test systems with convex and non-convex cost functions in four distinct dimensional units [20]. The nonconvex ELD problem was solved using the modified crow search algorithm (MCSA), and the results were applied to five popular test systems [21]. In contrast, four ELD with generator counts of 15, 6, 80 , and 40 were examined using the hybrid grey wolf optimizer (HGWO) [22]. The modified symbiotic organisms search algorithm (MSOS) was tested on five systems with varying features, restrictions, and dimensions [23]. To address the non-convex ELD issue with valve-point effects and emissions, the enhanced moth-flame optimizer (EMFO) technique was applied to three sample test systems with 6, 40 , and a large-scale 80 producing units that had non-convex fuel cost functions [24]. The method of using the one rank cuckoo search algorithm (ORCSA) to solve ELD problems proved effective [25]. By employing the adaptive charged system search (ACSS) technique for both large- and small -scale issue [26]. The artificial cooperative search algorithm (ACS) was introduced with the complicated ELD problem [27]. To extract the best solution for the ELD problem, the efficient distributed auction optimization algorithm (DAOA) was applied [28]. The ELD problems were resolved by a new firefly algorithm (FA) [29].

The authors solved an ELD problem using modified krill herd algorithm (MKH). When compared to other metaheuristics, the MKH was found to function fairly well, and adjusting its settings was not too difficult [30]. The ELD problem was solved using the oppositional pigeon-inspired optimizer (OPIO) algorithm [31]. On five valve-point affected generating systems, the evolutionary simplex adaptive Hooke-Jeeves algorithm's (ESAHJ) performance was evaluated. The suggested technique's test results showed good convergence properties and low generating costs, which made them incredibly appealing and successful [32]. Teaching-learning-based optimization (TLBO) was applied to address ELD problems while accounting for gearbox losses. This method explores the solution space around the global optimum point [33]. The traditional IEEE 30 bus was tested for non-convex CEED concerns [34]. With a number of restrictions, the hybrid Nelder-Mead approach can manage non-convex ED problems with ease. Several traditional test systems were simulated, each with a variable number of generating units [35]. The non-convex ELD problem, which has many limitations such as the valve-point loading impact, a broad range of fuel alternatives, and restricted operating zones, was addressed using the distributed auction-based technique [36]. To solve the ELD and CEED issues, the writers created the turbulent flow of water optimization (TFWO) method [37]. Intelligent optimization techniques known as metaheuristic algorithms guide the search process by employing exploitation and exploration. The development of increasingly metaheuristic algorithms has been spurred by the growing complexity of real-world optimization problems. The actions of walruses, which decide to migrate, roost, breed, gather, feed, and flee in response to critical cues (danger signals and safety signals), served as the model for the Walrus Optimizer (WO) [38].

The following illustrations show the primary goals and contributions of this work:

- The ELD issue is covered in four network studies, one for each of the following generator unit counts: thirty, twenty, ten, and six generators.
- The Walrus Optimizer (WO), a novel metaheuristic technique, is used to resolve the ELD case study.
- For the case studies of six units, ten units, twenty and thirty units, the suggested WO method
is assessed using chimp optimization algorithm (ChOA), moth search algorithm (MSA), snow ablation optimization (SAO), and the rime-ice algorithm (RIME).
- Based on the convergence and robustness statistics, all algorithms are evaluated over thirty runs.
- The disparity in power between the unit's generated power and the load demand determines how well the WO and all other methods are evaluated.
The following is the manuscript's order: In Section 2, the ELD analysis is discussed. Section 3 provides clarification on the WO technique. Section 4 provides an explanation of the findings. Section five provides a description of the conclusions and future work.


## 2. Analysis of ELD problem

ELD is one of the issues with power systems' functionality. The main obstacle is fixing the ELD problem and maximizing the financial benefit for power plants is reducing fuel consumption expenses. The resource distribution vector in the ELD issue that maximizes extracted power is defined by the primary variable. Below is an explanation of ELD analysis.

The mathematical analysis for ELD can be described using the following notations. The following phrase will be used to compute the cost of fuel used to run n generators:

$$
\begin{equation*}
\operatorname{Min}(\mathrm{F})=\mathrm{F}_{1}\left(\mathrm{P}_{1}\right)+\cdots \mathrm{F}_{\mathrm{n}}\left(\mathrm{P}_{\mathrm{n}}\right) \tag{1}
\end{equation*}
$$

$F$ is the net cost, $F_{n}$ is the cost of the nth generator, and $F_{1}$ is the cost of the first generator. The following techniques will be used to obtain the petrol cost function in quadratic equation:

$$
\begin{equation*}
\operatorname{Min}(\mathrm{F})=\sum_{\mathrm{k}=1}^{\mathrm{n}} \mathrm{~F}_{\mathrm{i}}\left(\mathrm{P}_{\mathrm{i}}\right)=\sum_{\mathrm{k}=1}^{\mathrm{n}} \mathrm{a}_{\mathrm{k}} \mathrm{P}_{\mathrm{k}}^{2}+\mathrm{b}_{\mathrm{k}} \mathrm{P}_{\mathrm{k}}+\mathrm{c}_{\mathrm{k}} \tag{2}
\end{equation*}
$$

where the weight constants are $\mathrm{a}, \mathrm{b}$, and c . Moreover, Eqs (3) and (5) can be used to modify the generator limits.

$$
\begin{equation*}
\sum_{k=1}^{n} P_{k}-P_{D}-P_{L}=0 \tag{3}
\end{equation*}
$$

If $P_{L}$ represents the losses of networks, which are calculated as follows, and $P_{D}$ is the demand networks.

$$
\begin{equation*}
P_{L}=\sum_{i=1}^{n} \sum_{j=1}^{n} P_{i} B_{i j} P_{j}, \tag{4}
\end{equation*}
$$

where the power extracted at the ith generator is indicated by $\mathrm{P}_{i}$, the power extracted at the $j$ th generator by $P_{j}$, and the factor of loss is indicated by $B_{i j}$.

$$
\begin{equation*}
P_{k}^{\min } \leq P_{k} \leq P_{k}^{\max } . \tag{5}
\end{equation*}
$$

## 3. Walrus optimizer

The Walrus Optimizer (WO) mathematical framework will be addressed in this section [38].

### 3.1. Initialization

Equation (6) shows how a set of randomly generated candidate keys (X) serves as the starting
point for the optimization process.

$$
\begin{equation*}
X=L B+\operatorname{rand}(U B-L B), \tag{6}
\end{equation*}
$$

where LB and UB represent the lower and upper bounds of the problem parameters, and rand is a uniform random vector in the range of 0 to 1 .

The term "walrus" refers to the agents that perform the optimization process. They change their positions continuously during iterations.

$$
\mathrm{X}=\left[\begin{array}{c}
X_{1,1} X_{1,2} \cdots X_{1, d}  \tag{7}\\
X_{2,1} X_{2,2} \cdots X_{2, d} \\
\vdots: \vdots \\
\vdots: \vdots \\
X_{n, 1} X_{n, 2} \cdots X_{n, d}
\end{array}\right]_{n \times d},
$$

where $n$ is the size of population and $d$ is the variables dimension.
The matched fitness values of each search agent are retained as follows:

$$
\mathrm{F}=\left[\begin{array}{c}
\left(f_{1,1} f_{1,2} \cdots f_{1, d}\right)  \tag{8}\\
\left(f_{2,1} f_{2,2} \cdots f_{2, d}\right) \\
\vdots: \vdots \\
\vdots: \\
\left(f_{n, 1} f_{n, 2} \cdots f_{n d}\right)
\end{array}\right]_{n \times d} .
$$

$90 \%$ and $10 \%$ of the overall walrus population is made up of adult and juvenile populations, respectively. In adult walruses, the male to female ratio is 1:1.

### 3.2. Safety and danger signals

Foraging and roosting need walruses to be highly watchful. As protectors, a walrus or two will patrol the area, sounding warning signals as soon as they notice any unexpected activity. The meaning of the danger and safety signals in WO is as follows:

$$
\begin{align*}
& \text { Danger signal }=A * R,  \tag{9}\\
& \qquad \begin{array}{l}
\alpha=1-t / T, \\
A=2 \times \alpha, \\
R=2 \times r_{1}-1,
\end{array} \tag{10}
\end{align*}
$$

where $\alpha$ decreases from 1 to 0 with the number of iterations $\mathrm{t}, \mathrm{T}$ is the maximum iteration, and A and R are danger factors.

The safety signal in WO that correlates to the danger signal is defined as follows:

$$
\begin{equation*}
\text { Safety signal }=r_{2} \tag{13}
\end{equation*}
$$

where, $r_{2}$ and $r_{1}$ are random values that fall between $(0,1)$.

### 3.3. Migration

When risks become too large, walrus herds will relocate to areas more conducive to population survival. In this phase, the position of the walrus is updated as follows:

$$
\begin{gather*}
X_{i, j}^{t+1}=X_{i, j}^{t}+\text { migratin step }  \tag{14}\\
\text { migration step }=\left(X_{m}^{t}-X_{n}^{t}\right) \cdot \beta \cdot r_{3}^{2} \\
\beta=1-\frac{1}{1+\exp \left(-\frac{t-\frac{T}{2}}{T} \times 10\right)}, \tag{16}
\end{gather*}
$$

where $X_{i, j}^{t}$ represents the i th walrus's current location on the j thension, and $X_{i, j}^{t+1}$ represents its new position. The step size of the walrus movement is called migration_step, two vigilantes are randomly selected from the population so that their positions match $X_{m}^{t}$ and $X_{n}^{t}$, the control factor for migration steps is called $\beta$, it evolves iteratively as a smooth curve, and $r_{3}$ is a random value between 0 and 1 .

### 3.4. Reproduction

Walrus herds usually do not migrate, instead, they reproduce in currents when danger factors are minimal. The position update of female walruses indicates that the lead walrus ( $X_{\text {best }}^{t}$ ) and the male walrus ( Male $_{i, j}^{t}$ ) impact the female walrus during reproduction. As the iteration progresses, the female walrus starts to rely more on the leader and less on her mate.

$$
\operatorname{Female}_{i, j}^{t+1}=\operatorname{Female}_{i, j}^{t}+\alpha \cdot\left(\operatorname{Male}_{i, j}^{t}-\text { Female }_{i, j}^{t}\right)+(1-\alpha) \cdot\left(X_{\text {best }}^{t}-\text { Female }_{i, j}^{t}\right),
$$

where Male $_{i, j}^{t}$ and Female ${ }_{i, j}^{t}$ are the positions of the $i$ th male and female walruses on the $j$ th dimension, and Female $e_{i, j}^{t+1}$ is the new position for the $i$ th female walrus on the $j$ th dimension.

Then, juvenile walruses are often hunted by polar bears and killer whales close to the population's edge. Because of this, young walruses have to get used to their new location in order to avoid predators.

$$
\begin{gather*}
\text { Juvenile }_{i, j}^{t+1}=(O \text {-Juvenile }  \tag{18}\\
i, j  \tag{19}\\
O=X_{\text {best }}^{t}+\text { Juvenile }_{i, j}^{t} \cdot L F
\end{gather*}
$$

where O is the reference safety position, Juvenile ${ }_{i, j}^{t+1}$ is the new position for the i th juvenile walrus on the j th dimension, and P is the juvenile walrus's distress coefficient, a random number between 0 and 1. Based on the Lévy distribution, LF is a vector of random values that represent Lévy movement.

$$
\begin{equation*}
\operatorname{Levy}(a)=0.05 \times \frac{x}{|y|^{\frac{1}{a}}}, \tag{20}
\end{equation*}
$$

where $y$ and $x$ are two normally distributed parameters, $x \mathrm{~N}\left(0, \sigma_{x}^{2}\right), y \mathrm{~N}\left(0, \sigma_{y}^{2}\right)$.

$$
\begin{equation*}
\sigma_{x}=\left[\frac{\Gamma(1+\alpha) \sin \left(\frac{\pi \alpha}{2}\right)}{\Gamma\left(\frac{1+\alpha}{2}\right) \alpha 2^{\frac{(\alpha-1)}{2}}}\right]^{\frac{1}{\alpha}}, \sigma_{y}=1, \alpha=1.5, \tag{21}
\end{equation*}
$$

where, $\sigma_{y}$ and $\sigma_{x}$ are the standard deviations, $\Gamma(x)=(x+1)$ !.
When walruses dive for food, they are also a target for natural predators, and when their companions alert them to danger, the animals will leave the area where they are now active. The late WO iteration demonstrates this behavior, and some population disturbance helps walruses in their quest for global exploration.

$$
\begin{equation*}
\sigma_{x}=\left[\frac{\Gamma(1+\alpha) \sin \left(\frac{\pi \alpha}{2}\right)}{\Gamma\left(\frac{1+\alpha}{2}\right) \alpha 2^{\frac{(\alpha-1)}{2}}}\right]^{\frac{1}{\alpha}}, \sigma_{y}=1, \alpha=1.5, \tag{22}
\end{equation*}
$$

where the distance between the best and current walrus is shown by the symbol $\left|X_{\text {best }}^{t}-X_{i, j}^{t}\right|$, and $r_{4}$ is a random value between 1 and 0 .

Furthermore, walruses can cooperate to move and forage dependent on the whereabouts of other walruses in the group as part of their social gathering behavior. Walruses have the ability to help the entire herd find regions of the sea with more food by exchanging location data.

$$
\begin{gather*}
X_{i, j}^{t+1}=\left(X_{1}+X_{2}\right) / 2,  \tag{23}\\
\left\{\begin{array}{c}
X_{1}=X_{\text {best }}^{t}-a_{1} \times b_{1} \times\left|X_{\text {best }}^{t}-X_{i, j}^{t}\right| \\
X_{2}=X_{\text {second }}^{t}-a_{2} \times b_{2} \times\left|X_{\text {second }}^{t}-X_{i, j}^{t}\right| \\
a=\beta \times r_{5}-\beta, \\
b=\tan (\theta)
\end{array}\right. \tag{24}
\end{gather*}
$$

where $a$ and $b$ are the gathering coefficients, $X_{1}$ and $X_{2}$ are two weights influencing the walrus's gathering behaviour, and $X_{\text {second }}^{t}$ reflects the position of the second walrus in the current iteration. $\left|X_{\text {second }}^{t}-X_{i, j}^{t}\right|$ indicates the distance between the current walrus and the second walrus. Whereas the values of $\theta$ range from 0 to $\pi$, the random integer $r_{5}$ is between 0 and 1 .

### 3.5. WO flow chart

The flowchart of WO is described in detail in Figure 1.


Figure 1. Flow chart of WO algorithm.

## 4. Results of ELD cases

The ELD receives a donation of the WO performance. The moth search algorithm (MSA) [39], the snow ablation optimization (SAO) [40], the chimp optimization algorithm (ChOA) [41], and the rime-ice algorithm (RIME) [42] were used to evaluate the suggested WO method. The ELD issue was used in the following case studies:

- Six generators operating at two distinct loads (1000 and 700 MW ) comprise the first case study.
- There are 10 generators in the second case study with two different loads ( 1000 and 2000 MW ).
- The five-case study has 20 generators operating at 3000 MW loads.
- The six-case study included 30 generators operating at 5000 MW loads.


### 4.1. Results of six generators

In order to test the ELD issue, a case study with six generators operating at two loads is donated. A variety of techniques were used, including the WO, RIME, SAO, MSA, and ChOA algorithms. Thirty different runs were used to assess each competing strategy's efficacy. The mean, maximum, minimum, and standard deviation values were documented as statistical information for each load using these runs, as shown in Table 1. Using this data, the WO determines the optimal standard deviation and the best objective function. For ELD, the WO algorithm is therefore the most accurate and reliable. Table 2 displays the ideal fuel cost for each situation. Table 3, which was developed using the best objective function among all approaches, displays the ideal power provided by each unit for a load requirement of 700 MW . Table 4 which was generated on the best objective function across all techniques, displays the optimal power provided by each unit to recover load requirement of 1000 MW . The robustness curve determines the fitness function value for each run based on the results of all techniques recorded over the course of the 30 runs.

Table 1. Statistical data of all methods in $(\$ / h)$ for six generators.

| Load (MW) | Algorithm | Minimum | SD | Mean | Maximum |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 700 | WO | 8400.997136 | 196.5147708 | 8662.170541 | 9139.212021 |
|  | SAO | 8465.975761 | 197.2010827 | 8753.891609 | 9155.235854 |
|  | ChOA | 39475.15105 | 1415165.954 | 1594567.940 | 4960872.827 |
|  | MSA | 8427.541921 | 163.3349358 | 8716.009085 | 9032.640980 |
|  | RIME | 154035.3045 | 78735556.03 | 64584338.77 | 348563546.7 |
| 1000 | WO | 12120.25084 | 131.2152977 | 12270.39366 | 12615.36259 |
|  | SAO | 12141.46728 | 128.4638379 | 12318.57722 | 12643.00452 |
|  | ChOA | 35568.69974 | 859586.3590 | 789710.3604 | 3975199.582 |
|  | MSA | 12196.59431 | 89.66107262 | 12335.59750 | 12552.83025 |
|  | RIME | 2696415.881 | 84144505.04 | 76815454.04 | 279959787.1 |

Table 2. The optimal fuel usage costs $(\$ / \mathrm{h})$ for six generators.

| Method | 700 MW | 1000 MW |
| :--- | :--- | :--- |
| WO | 8400.996922 | 12120.24203 |
| SAO | 8465.925401 | 12136.61857 |
| ChOA | 8995.251114 | 12330.14858 |
| MSA | 9794.135519 | 14034.62198 |
| RIME | 8648.886480 | 12206.03121 |

Table 3. The best distribution of power (MW) among six generators for a demand of 700 MW .

| WO | SAO | ChOA | MSA | RIME |
| :--- | :--- | :--- | :--- | :--- |
| 281.8419643 | 259.8161060 | 100 | 56.03971489 | 186.8762755 |
| 50.50407930 | 54.58630261 | 52.35878389 | 69.97467027 | 62.88659606 |
| 192.6399883 | 159.3362502 | 257.1559643 | 72.80852851 | 279.7127278 |
| 50.02267017 | 58.20871339 | 50 | 103.0726077 | 51.47179647 |
| 78.64437068 | 94.60440179 | 200 | 103.7752893 | 70.08507476 |
| 58.16395368 | 85.58268340 | 56.51473367 | 305.2492779 | 62.78030728 |

Table 4. The best distribution of power (MW) among six generators for a demand of 1000 MW .

| WO | SAO | ChOA | MSA | RIME |
| :--- | :--- | :--- | :--- | :--- |
| 418.8719894 | 374.5651735 | 500 | 54.48410890 | 462.2467195 |
| 118.4807842 | 173.0458359 | 176.9449449 | 79.19002879 | 142.3428705 |
| 207.6997509 | 210.9436951 | 116.0761643 | 124.3800484 | 226.2392259 |
| 87.95314532 | 77.57138608 | 86.42470161 | 155.7910172 | 50.00204391 |
| 139.8294433 | 132.5356769 | 62.15523186 | 246.3641047 | 50 |
| 50.59678231 | 55.14492812 | 79.69065610 | 365.0459344 | 91.25167800 |

The robustness curve properties for each case on the system with six units are displayed in Figures 2 and 3. Based on the recorded outcomes from each of the top 30 runs that yield the best fitness function, the convergence curve describes the quickest approach that meets the goal function. The characteristics of the convergence curve for the system with six units at each load level are shown
in Figures 4 and 5. Using the convergence and robustness properties as a guide, the WO finds the optimal global solution. Tables 5 and 6 clarify the estimated sharing power from all units over the thirty runs for six units based on the WO method.


Figure 2. Six generators' robustness curves at a 700 MW load.



Figure 3. Six generators' robustness curves at a 1000 MW load.


Figure 4. Six generators' convergence curves at a 700 MW load.


Figure 5. Six generators' convergence curves at a 1000 MW load.
Table 5. The estimated sharing power from all units over the thirty runs for six units based on the WO method at 700 MW .

| Unit 1 | Unit 2 | Unit 3 | Unit 4 | Unit 5 | Unit 6 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 102.4803134 | 98.90858263 | 146.5974124 | 57.05988267 | 199.9998093 | 109.9802472 |
| 252.0493973 | 88.14888891 | 144.6226086 | 86.19141850 | 78.69709537 | 61.97854274 |
| 217.1549197 | 98.80989745 | 143.4855378 | 75.38821598 | 116.6628773 | 60.83235654 |
| 100 | 200 | 80 | 58.07392804 | 199.7337076 | 76.81948039 |
| 127.1120581 | 52.58933520 | 178.9286417 | 100.8793515 | 178.6835637 | 76.58050636 |
| 171.2244733 | 193.7244687 | 134.8106730 | 52.13909507 | 99.90726427 | 60.85348710 |
| 303.3576528 | 89.67266612 | 110.0581270 | 80.08162413 | 61.72363427 | 66.07303450 |
| 152.3377111 | 135.9783145 | 146.2957426 | 132.4204045 | 57.71403532 | 88.13135753 |


| Unit 1 | Unit 2 | Unit 3 | Unit 4 | Unit 5 | Unit 6 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 272.1414685 | 127.9969590 | 127.5023324 | 75.40811217 | 50 | 58.06615553 |
| 242.3004371 | 76.5465688 | 155.8124437 | 50.43508027 | 129.8969541 | 57.36082528 |
| 158.0997819 | 68.39168686 | 136.5241416 | 120.3722600 | 171.8359666 | 58.78901335 |
| 188.1262448 | 115.8998675 | 113.4359860 | 87.19074785 | 118.9889349 | 89.14564073 |
| 198.5167438 | 105.0685979 | 240.4188648 | 54.73838761 | 50.89307970 | 63.11961272 |
| 207.4749216 | 121.7413134 | 149.1441977 | 77.26002026 | 103.6364154 | 52.96163304 |
| 124.0668650 | 90.92339930 | 249.1107843 | 54.05206681 | 90.69455487 | 105.4528787 |
| 158.8493266 | 150.8833468 | 80 | 85.19020735 | 183.8349391 | 54.90617341 |
| 245.2819280 | 83.65885527 | 149.1188361 | 52.40031462 | 107.4175349 | 74.22420733 |
| 250.5707815 | 132.2436871 | 107.0744415 | 93.33921683 | 75.77756745 | 52.51220603 |
| 100.2306225 | 75.89233881 | 261.0982595 | 126.1334462 | 79.42216520 | 71.86777543 |
| 154.0799986 | 59.35882146 | 187.5369360 | 106.2150155 | 134.8700152 | 71.79567099 |
| 237.8466632 | 91.53005291 | 159.3975559 | 56.00072280 | 80.97575690 | 86.27944518 |
| 177.6241311 | 65.73269041 | 294.1448464 | 55.61023583 | 67.27121818 | 53.70427278 |
| 140.7040655 | 123.5236114 | 97.96158459 | 85.70199318 | 196.2508439 | 69.98467789 |
| 281.8419643 | 50.50407930 | 192.6399883 | 50.02267017 | 78.64437068 | 58.16395368 |
| 138.2306531 | 92.01201083 | 137.1201323 | 99.63543302 | 185.3094505 | 61.83791201 |
| 120.1891075 | 165.0697700 | 176.0335154 | 149.5190604 | 50.95775454 | 51.47374498 |
| 162.7158423 | 131.8547034 | 166.5554811 | 90.71884640 | 103.3414511 | 57.59958377 |
| 239.5906960 | 147.6249374 | 99.59586416 | 71.95502938 | 90.85440986 | 62.14291823 |
| 222.4746072 | 96.56678798 | 114.6609715 | 78.22236349 | 114.0120391 | 86.46675339 |
| 217.8622484 | 95.40899320 | 147.2814524 | 82.06577941 | 95.45064912 | 74.16733930 |

Table 6. The estimated sharing power from all units over the thirty runs for six units based on the WO method at 1000 MW.

| Unit 1 | Unit 2 | Unit 3 | Unit 4 | Unit 5 | Unit 6 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 341.3981624 | 140.9413161 | 198.2286577 | 110.6886852 | 140.6813018 | 92.81221650 |
| 284.1819988 | 143.7225151 | 210.2178285 | 97.30437360 | 179.8132314 | 111.1492323 |
| 309.7621619 | 117.0385621 | 246.3548164 | 107.4187872 | 152.8137221 | 92.46722036 |
| 344.2115321 | 131.0580592 | 211.3389364 | 100.1877574 | 148.0912579 | 89.99270845 |
| 266.5532907 | 173.4169070 | 186.7969442 | 139.8422806 | 190.8918946 | 68.91202388 |
| 366.1897574 | 115.5512483 | 219.4230681 | 95.09084517 | 122.7059296 | 105.5195606 |
| 476.9243509 | 54.21667175 | 187.2306249 | 68.02355672 | 125.2463485 | 111.6764112 |
| 336.5581280 | 138.7634309 | 221.5640531 | 104.2301337 | 142.8800824 | 80.87432334 |
| 311.6684933 | 157.2394350 | 220.4660037 | 113.0076546 | 128.1239647 | 94.57957665 |
| 357.4839710 | 152.4713473 | 230.4426551 | 62.25645741 | 125.7710309 | 96.01873016 |
| 326.8077345 | 149.1901841 | 235.2728939 | 73.89699066 | 135.9788826 | 103.9965005 |
| 375.4374685 | 197.9903023 | 97.54151456 | 54.84573108 | 179.6313230 | 119.6729541 |
| 310.5525430 | 200 | 228.4033442 | 131.5740260 | 64.71385590 | 89.25542297 |
| 198.3478713 | 121.0773544 | 296.9073980 | 100.1264281 | 200 | 112.5644537 |
| 364.5289296 | 116.8251593 | 215.8054688 | 61.26854929 | 184.5750177 | 82.22239739 |
| 343.2550109 | 138.7541221 | 183.5203179 | 113.6092577 | 158.6660371 | 87.11194786 |
| 231.2846259 | 171.2273378 | 299.9999969 | 65.36650287 | 181.4682842 | 78.30160867 |

Continued on next page

| Unit 1 | Unit 2 | Unit 3 | Unit 4 | Unit 5 | Unit 6 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 334.0385473 | 140.8650324 | 217.1734714 | 105.0756383 | 144.9430579 | 82.81971939 |
| 344.0653060 | 138.7780745 | 208.6015833 | 107.2240187 | 139.1283197 | 86.88806222 |
| 493.0435454 | 169.1438687 | 82.45797790 | 131.6592578 | 52.40150981 | 93.07448376 |
| 347.6552919 | 123.7009673 | 290.5128504 | 73.74211351 | 95.65015628 | 93.71152034 |
| 500 | 182.2766504 | 80 | 149.9880007 | 59.31734212 | 50 |
| 418.8719894 | 118.4807842 | 207.6997509 | 87.95314532 | 139.8294433 | 50.59678231 |
| 415.2861783 | 160.6555421 | 218.3004690 | 81.13067886 | 93.20998978 | 54.21886616 |
| 315.4930839 | 187.8881017 | 236.6649001 | 107.9465190 | 59.40055137 | 117.2313561 |
| 346.2844095 | 159.9314812 | 139.6791036 | 126.9515242 | 136.3862798 | 115.5694083 |
| 435.8349834 | 146.8766561 | 186.0508011 | 69.62593103 | 102.9392750 | 81.32075210 |
| 382.4916920 | 182.5880896 | 174.2718067 | 50 | 140.8667595 | 93.75208220 |
| 366.9104058 | 126.1658610 | 248.4103510 | 63.15425596 | 139.9832928 | 80.01889125 |
| 308.1807194 | 122.1205565 | 298.0516559 | 50.56191778 | 155.8335116 | 91.75587284 |

### 4.2. Results of ten generators

In order to test the ELD issue, a case study with ten generators operating at two loads is donated. A variety of techniques were used, including the WO, RIME, SAO, MSA, and ChOA algorithms. Thirty different runs were used to assess each competing strategy's efficacy. The mean, maximum, minimum, and standard deviation values were documented as statistical information for each load using these runs, as shown in Table 7. Using this data, the WO determines the optimal standard deviation and the best objective function. For ELD, the WO algorithm is therefore the most accurate and reliable. Table 8 displays the ideal fuel cost for each situation. Table 9 , which was developed using the best objective function among all approaches, displays the ideal power provided by each unit for a load requirement of 1000 MW . Table 10, which was generated on the best objective function across all techniques, displays the optimal power provided by each unit to recover load requirement of 2000 MW .

Table 7. Statistics data of all methods in ( $\$ / \mathrm{h}$ ) for ten generators.

| Load (MW) | Algorithm | Minimum | SD | Mean | Maximum |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1000 | WO | 99172455.07 | 22083746.37 | 125028621.9 | 204189415.1 |
|  | SAO | 91025531.85 | 47461035.30 | 135774959.0 | 278926971.6 |
|  | ChOA | 96169315.81 | 9092193.300 | 110754717.8 | 134377790.8 |
|  | MSA | 90206604.16 | 23733842.24 | 119742282.7 | 171657419.9 |
|  | RIME | 96981916.42 | 77022779.49 | 187450044.4 | 394817673.1 |
| 2000 | WO | 475068075.3 | 40792448.60 | 533415296.7 | 617012683.9 |
|  | SAO | 459517895.5 | 43408269.26 | 575882737.8 | 635659941.4 |
|  | ChOA | 432244579.1 | 39158317.85 | 501777836.9 | 583106906.4 |
|  | MSA | 472373206.1 | 40398057.93 | 592236498.7 | 639339146.9 |
|  | RIME | 465555201.9 | 71077574.92 | 576425864.7 | 777638652.5 |

Table 8. The optimal fuel usage costs $(\$ / \mathrm{h})$ for ten generators.

| Method | 1000 MW | 2000 MW |
| :--- | :--- | :--- |
| WO | 99172455.07 | 475068075.2 |
| SAO | 88057595.74 | 459419310.3 |
| ChOA | 94849826.08 | 416417404.5 |
| MSA | 71535847.32 | 349536183.6 |
| RIME | 94886247.04 | 459186431.8 |

Table 9. The best distribution of power (MW) among ten generators for a demand of 1000 MW .

| WO | SAO | ChOA | MSA | RIME |
| :--- | :--- | :--- | :--- | :--- |
| 189.9585506 | 162.2053566 | 154.3329761 | 12.33748823 | 174.1680447 |
| 137.8622921 | 135.2619336 | 135 | 44.01184975 | 135 |
| 105.6144183 | 137.8238444 | 73 | 66.25826943 | 115.6444194 |
| 60.46282183 | 125.1864930 | 97.81644810 | 68.77385114 | 128.2000558 |
| 120.3049668 | 79.00989904 | 198.1933143 | 117.8098698 | 153.3188340 |
| 148.0244135 | 82.28033061 | 140.7465156 | 126.1331838 | 117.8700572 |
| 38.22264078 | 91.22832092 | 79.42576608 | 135.0403635 | 25.56604190 |
| 105.1471066 | 91.97426679 | 96.67066137 | 143.4200038 | 68.71055217 |
| 49.81657885 | 77.85222711 | 26.78146665 | 146.6648195 | 80 |
| 54.62738077 | 26.43244080 | 10 | 150.0006410 | 12.19912388 |

Table 10. The best distribution of power (MW) among ten generators for a demand of 2000 MW .

| WO | SAO | ChOA | MSA | RIME |
| :--- | :--- | :--- | :--- | :--- |
| 412.2313307 | 416.1371402 | 395.8661795 | 26.65588505 | 418.1936554 |
| 354.1194035 | 317.6418237 | 269.9925437 | 45.93059863 | 322.4049869 |
| 297.3361751 | 330.0870262 | 340 | 119.9951250 | 332.5336200 |
| 299.4981562 | 290.3535755 | 300 | 124.2309670 | 277.6069160 |
| 209.7173057 | 224.5346635 | 243 | 159.9953976 | 220.4361023 |
| 159.9999989 | 160 | 111.8960325 | 242.9999999 | 129.0094100 |
| 81.48722139 | 120.7940973 | 130 | 256.3870356 | 128.5908911 |
| 112.9551673 | 99.61729215 | 120 | 299.9996923 | 103.1062452 |
| 68.12411680 | 42.42283454 | 80 | 302.2501288 | 70.52565350 |
| 47.17968826 | 40.10417154 | 47.55800085 | 465.4640164 | 39.03613209 |

The robustness curve determines the fitness function value for each run based on the results of all techniques recorded over the course of the 30 runs. The robustness curve properties for each case on the system with ten units are displayed in Figures 6 and 7. Based on the recorded outcomes from each of the top 30 runs that yield the best fitness function, the convergence curve describes the quickest approach that meets the goal function. The characteristics of the convergence curve for the system with ten units at each load level are shown in Figures 8 and 9. Using the convergence and robustness properties as a guide, the WO finds the optimal global solution. Tables 11 and 12 clarify the estimated sharing power from all units over the thirty runs for ten units based on the WO method.


Figure 6. Ten generators' robustness curves at a 1000 MW load.


Figure 7. Ten generators' robustness curves at a 2000 MW load.


Figure 8. Ten generators' convergence curves at a 1000 MW load.


Figure 9. Ten generators' convergence curves at a 2000 MW load.

Table 11. The estimated sharing power from all units over the thirty runs for ten units based on the WO method at 1000 MW .
Unit $1 \quad$ Unit $2 \quad$ Unit 3 $\quad$ Unit 4 $\quad$ Unit $5 \quad$ Unit $6 \quad$ Unit 7 $\quad$ Unit 8 $\quad$ Unit 9 $\quad$ Unit 10
$159.2159925205 .154262777 .35307172175 .9486537166 .592644992 .3537639234 .4088337247 .5618075 \quad 21.0476298733 .99557635$ 160.1030938162 .0074058182 .693650164 .8472269296 .9095634675 .2867325597 .8511103356 .3764912359 .7835778655 $231.6370454172 .9971196124 .0327942138 .1810285120 .443839976 .8360923343 .2149801854 .0797019 \quad 27.5047181222 .34660415$ 297.0510775138 .7638412111 .2012504107 .317820577 .78582904104 .401547546 .3774838458 .5267065320 .1087607550 .40279529 $154.1313930135 \quad 214.196954592 .19856823165 .986842966 .1856173486 .3082599855 .9262042 \quad 28.6651816112 .22339933$ 222.348739139 .4559695100 .8188328186 .705246779 .1497880976 .1865487981 .0323941648 .4863853961 .2588953715 .88444985 $175.4413944135 .022601889 .64707499267 .772813873 \quad 127.284284030 .1219572978 .9325933625 .2918438810 .00000005$ 235.2670748158 .3604452176 .921430477 .9478101975 .05940957159 .904348133 .2679872847 .0348022935 .4323519112 .61601629 222.2455874198 .9585264133 .2846076114 .6020956106 .214098575 .198161546 .2624906660 .9649452631 .7147594621 .85463807 $222.7335791185 .3936952168 .1266204114 .470871 \quad 75.5569208464 .0912453387 .6056862147 .7945344324 .5041578120 .85087625$ 241.2563633168 .4369199172 .915630460 .8055856478 .0338663257 .5467168420 .00685788103 .602460177 .6578439228 .23306747 188.1525628194 .451753116 .752925387 .14449367118 .0462450111 .930345436 .1126094966 .9749334774 .4393220117 .23178485 180.2933015148 .2087336182 .579071898 .9003020496 .7266705776 .2653845345 .8746969174 .6605618974 .6652895230 .87468895 $213.6771494172 .0073669110 .430799060 \quad 129.064921990 .4659924422 .83272537120 \quad 78.1446417413 .00750412$ $209.5248331162 .358337982 .39817875180 .7584185145 .714440957 \quad 64.3881232765 .5405476133 .0194012 \quad 11.37312312$ $154.9925692173 .6751037112 .0207399211 .3494988121 .3415918110 .25215 \quad 24.1583455558 .4803331922 .5136409323 .11060133$ 236.400897165 .3637865100 .5876852143 .0854971101 .104806598 .258387428 .3224006174 .3901285922 .473701141 .45500504 $290.9903704180 .1161077111 .193952789 .44872095124 .063247873 .682961334 .3507097 \quad 56.2331576737 .1480760215 .51345443$ 212.5603041208 .9283893134 .4543464107 .680623294 .9523051175 .5813743337 .1065499766 .1677394555 .1399933418 .44545653 $158.2374826345 .626373173 .1647521571 .45397246152 .265639659 .8508950762 .8970225 \quad 53.7669536927 .4996417214 .07082341$ 189.9585506137 .8622921105 .614418360 .46282183120 .3049668148 .024413538 .22264078105 .147106649 .8165788554 .62738077 $197.6620039186 .9874661135 .826066107 .545056987 .00044521109 .816946 \quad 71.7976446765 .4133760634 .5266580014 .51310919$ 235.8701998138 .9810028129 .333068169 .23232825223 .284818461 .0435902928 .6158014569 .1050002223 .7868210532 .85745335 181.1324307184 .9282264153 .1808598108 .5734387114 .846516966 .1100242585 .3487915563 .5131289937 .4968106915 .75990624 233.6393711139 .523693384 .2260604160 .5875545115 .8070188158 .928970279 .2483221376 .2960152229 .207687734 .39384127 $322.6035983140 .3409282131 .236310077 .0150188 \quad 73.2849912283 .9447189166 .8724010347 .1032357 \quad 37.7525361832 .45997671$

Continued on next page

| Unit 1 | Unit 2 | Unit 3 | Unit 4 | Unit 5 | Unit 6 | Unit 7 | Unit 8 | Unit 9 | Unit 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

180.2520248185 .420592794 .11881173264 .001145875 .432711157
69.8401421849 .451401623 .8529578713 .07985644 $267.5710745135 \quad 93.19580244153 .987771278 .5779560160 .6247170562 .7821201859 .3732133879 .9994415920 .57353603$ 150.0500502280 .288356899 .0465793492 .9210720494 .65560650140 .911658029 .9428884974 .0010976927 .4217922925 .32489640 $\underline{150.0440645284 .879997673 .00000058223 .507177674 .6958129562 .2564456146 .5537137747}$ 20.1688813633 .88947346

Table 12. The estimated sharing power from all units over the thirty runs for ten units based on the WO method at 2000 MW.

| Unit 1 | Unit 2 | Unit 3 | Unit 4 | Unit 5 | Unit 6 | Unit 7 | Unit 8 | Unit 9 | Unit 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

428.0982723417 .7643835290 .7395516285 .9557676170 .8925859134 .5408063121 .278425994 .5939120153 .5499727549 .80925878 388.1017765446 .1179611269 .6210476265 .3121883194 .5587579147 .1267506107 .5622969105 .797131875 .6055308848 .57343027 419.9645588458 .1691315275 .7242539252 .0763891190 .0235745152 .6207006122 .3828213100 .470963334 .8970304343 .96784911 $470 \quad 446.72697 \quad 339.8623129299 .980666882 .99905601159 .495634929 .54956098119 .826527252 .4209300144 .14627465$ 450.1496196335 .2756905304 .3891891294 .4576684229 .4492262114 .333833995 .61309569103 .695150775 .5252795439 .65895347 372.71844398 .4222273308 .3259975262 .9682916193 .9403648158 .8206066119 .5484443112 .769619270 .7248321546 .28807342 $444.5795722419 .9636507238 .966268 \quad 285.881978 \quad 224.2082906125 .314206497 .3935127782 .6195349876 .5025992354 .8422977$ 384.5500323413 .8709687265 .3486526299 .8081442231 .9730852154 .1573168104 .094908376 .8347911770 .521336647 .2983988 406.1802318459 .8603496333 .4903322242 .8711336231 .6858442151 .698135585 .3345290992 .7110696720 .0863778424 .39942245 467.9679976428 .3309168296 .3827596299 .5874475101 .2616463143 .0174477129 .900161110 .771722620 .4707382750 .1455135 414.5919729408 .2329251297 .3225483262 .5842117205 .0380753133 .2135416111 .6110689103 .556503366 .1690226643 .53730963 465.3926078455 .0110264330 .6164285298 .921721883 .0229062160 .19747039129 .9709871119 .893476775 .8121530928 .03921375 457.3085445341 .6409802324 .1213944299 .7258376195 .3717413141 .295350949 .3740634199 .4153393279 .9997338153 .44677173 $387.6028351403 .7838295301 .1756945251 .572342 \quad 239.1790535140 .31152 \quad 110.1653136108 .141653467 .326893136 .27017624$ 394.6879332423 .9236727289 .2115421252 .6339103224 .8741427143 .773207190 .88320764109 .697858373 .1623150643 .43877876 $468.4662924427 .911449306 .6047926201 .7387548242 .644638 \quad 82.1622170397 .3784371103 .433995868 .5312633149 .5647736$ 426.6172538406 .7945802262 .0453947272 .5524587206 .1571276147 .4071999102 .6374548104 .289813171 .5086000747 .1176062 424.884686343 .5060783314 .1299114297 .3862956240 .5331876104 .965890782 .22764731117 .759380763 .7028741552 .38247512 $382.6193821450 .9885973293 .6944731300 \quad 209.3489807159 .967829247 .8044620797 .9904529475 .9501015328 .52894728$ $383.6260834442 .241986 \quad 275.3078995288 .3220013216 .4277126151 .5372682105 .497244172 .7062060174 .2142791439 .5823027$ 437.9366298416 .0085116318 .8706489228 .2631104173 .905161154 .3336904109 .0310292105 .846197561 .862399539 .76218428 $469.4466025459 .9999637340 \quad 150.2114857208 .261423591 .5986107130 \quad 108.364580442 .0187718250 .79978166$ 432.8995631404 .4758705327 .7598376277 .7123065185 .4334227156 .471307286 .30860545111 .660848528 .6814053532 .71694383 403.5274754405 .6548252308 .1246595253 .4877107203 .2876437138 .3456277110 .475784197 .9041114771 .8598028752 .96546268 412.2313307354 .1194035297 .3361751299 .4981562209 .7173057159 .999998981 .48722139112 .955167368 .124116847 .17968826 431.268416396 .176757307 .5683835256 .6920566185 .7613956133 .519391112 .4859199101 .179712271 .5435103248 .91764929 $401.5443966424 .3925779319 .1914116253 .0436952228 .9338026151 .206190537 .8231001798 .1229227480 \quad 50.92031943$ 371.0873022382 .6125425338 .884372 242.4323102230.8253159145.3676986107.1052479103.248286768.1668628454.03182796 419.1885618390 .0135451286 .854260260 .6994848212 .2401164148 .6285556114 .4061915103 .235678668 .8754878641 .46820991 | $376.6740049459 .9832529313 .6472886263 .762858 ~$ | 237.0071809147 .171602 | 20 | 110.924450767 .4636679649 .75691359 |
| :--- | :--- | :--- | :--- | :--- | :--- |

### 4.3. Results of twenty generators

In order to test the ELD issue, a case study with twenty generators operating at 3000 MW load is donated. A variety of techniques were used including the WO, RIME, SAO, MSA, and ChOA algorithms. Thirty different runs were used to assess each competing strategy's efficacy. The mean,
maximum, minimum, and standard deviation values were documented as statistical information for each load using these runs, as shown in Table 13. Using this data, the WO determines the optimal standard deviation and the best objective function. For ELD, the WO algorithm is therefore the most accurate and reliable. Table 14 displays the ideal fuel cost. Table 15, which was developed using the best objective function among all approaches, displays the ideal power provided by each unit for a load requirement of 3000 MW . The robustness curve determines the fitness function value for each run based on the results of all techniques recorded over the course of the 30 runs. The robustness curve properties for the system with twenty units is displayed in Figure 10. Based on the recorded outcomes from each of the top 30 runs that yield the best fitness function, the convergence curve describes the quickest approach that meets the goal function. The characteristics of the convergence curve for the system with twenty units is shown in Figure 11. Using the convergence and robustness properties as a guide, the WO finds the optimal global solution.

Table 13. Statistics data of all methods in ( $\$ / \mathrm{h}$ ) for twenty generators.

| Load (MW) | Method | Minimum | SD | Mean | Maximum |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 3000 | WO | 464390784.6 | 80385278.05 | 601882926.3 | 781664638.0 |
|  | SAO | 386134352.9 | 93352458.10 | 647520789.4 | 837732536.6 |
|  | ChOA | 378479313.7 | 62470909.71 | 508890772.8 | 674400824.9 |
|  | MSA | 474920850.4 | 62784168.97 | 587293620.1 | 706048241.3 |
|  | RIME | 492737490 | 112461311.7 | 703071978.6 | 999229765.2 |

Table 14. The optimal fuel usage costs ( $\$ / \mathrm{h}$ ) for twenty generators.

| Method | 3000 MW |
| :--- | :--- |
| WO | 464390784.6 |
| SAO | 382573772.5 |
| ChOA | 377610655.8 |
| MSA | 382886269.5 |
| RIME | 453827249.1 |

Table 15. The best distribution of power (MW) among twenty generators for a demand of 3000 MW .

| WO | SAO | ChOA | MSA | RIME |
| :--- | :--- | :--- | :--- | :--- |
| 395.8114381 | 151.6528541 | 189.1359163 | 27.67575842 | 289.6249510 |
| 135 | 184.2648197 | 151.4504418 | 36.49556758 | 189.5337518 |
| 273.1416448 | 190.7183576 | 209.1384204 | 38.00643724 | 209.1991579 |
| 194.9170343 | 300 | 273.7818598 | 48.00263656 | 296.5966165 |
| 215.7882232 | 221.9169631 | 188.7549064 | 49.00183238 | 113.8978953 |
| 121.2980218 | 156.1339232 | 160 | 69.00346450 | 160 |
| 71.60958816 | 130 | 94.57254207 | 86.99422477 | 50.97508515 |
| 84.48652671 | 47.29866951 | 112.7781839 | 115.0102537 | 47.02325440 |
| 68.96129708 | 80 | 20 | 143.6024661 | 57.67211139 |
| 29.73786810 | 34.40590622 | 41.68599962 | 153.1579086 | 13.74646693 |
| 225.5746355 | 211.8911419 | 163.8029810 | 174.9828705 | 246.0582500 |

Continued on next page

| WO | SAO | ChOA | MSA | RIME |
| :--- | :--- | :--- | :--- | :--- |
| 203.6821211 | 179.8235087 | 136.9346654 | 180.0031293 | 233.0110822 |
| 188.0154112 | 283.2667993 | 335.6229908 | 180.9930170 | 174.9402437 |
| 240.0733159 | 223.3191179 | 300 | 184.0201986 | 277.3906292 |
| 195.9195348 | 137.2674686 | 177.1829979 | 193.0178273 | 198.0835046 |
| 104.0455988 | 159.8454509 | 160 | 205.0149350 | 156.9012038 |
| 72.67502373 | 119.0941065 | 64.86963812 | 212.0163116 | 78.02459914 |
| 76.59300054 | 114.2839665 | 112.3046859 | 253.0011000 | 118.9234432 |
| 64.06909787 | 64.58733512 | 65.54825073 | 319.9601154 | 40.53005757 |
| 38.60061834 | 10.22996721 | 42.44420644 | 330.0399455 | 47.87158727 |




Figure 10. Twenty generators' robustness curves at a 3000 MW load.


Figure 11. Twenty generators' convergence curves at a 3000 MW load.

### 4.4. Results of thirty generators

In order to test the ELD issue, a case study with thirty generators operating at 3000 MW load is donated. A variety of techniques were used including the WO, RIME, SAO, MSA, and ChOA
algorithms. Thirty different runs were used to assess each competing strategy's efficacy. The mean, maximum, minimum, and standard deviation values were documented as statistical information for each load using these runs, as shown in Table 16. Using this data, the WO determines the optimal standard deviation and the best objective function. For ELD, the WO algorithm is therefore the most accurate and reliable. Table 17 displays the ideal fuel cost. Table 18, which was developed using the best objective function among all approaches, displays the ideal power provided by each unit for a load requirement of 5000 MW . The robustness curve determines the fitness function value for each run based on the results of all techniques recorded over the course of the 30 runs. The robustness curve properties for the system with thirty units is displayed in Figure 12. Based on the recorded outcomes from each of the top 30 runs that yield the best fitness function, the convergence curve describes the quickest approach that meets the goal function. The characteristics of the convergence curve for the system with thirty units is shown in Figure 13. Using the convergence and robustness properties as a guide, the WO finds the optimal global solution.

Table 16. Statistics data of all methods in $(\$ / h)$ for thirty generators.

| Load (MW) | Method | Minimum | SD | Mean | Maximum |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 5000 | WO | 982540947.5 | 91938497.93 | 1156733522 | 1336706848 |
|  | SAO | 908558160.8 | 143278429.4 | 1211191850 | 1513082334 |
|  | ChOA | 801191500.7 | 99936778.88 | 1007797884 | 1207996618 |
|  | RIME | 1037852896 | 159765617.6 | 1253663161 | 1617847664 |

Table 17. The optimal fuel usage costs ( $\$ / \mathrm{h}$ ) for thirty generators.

| Method | 3000 MW |
| :--- | :--- |
| WO | 982529390 |
| SAO | 904975533.8 |
| ChOA | 787860293.7 |
| RIME | 1012965738 |

Table 18. The best distribution of power (MW) among twenty generators for a demand of 3000 MW .

| WO | SAO | ChOA | RIME |
| :--- | :--- | :--- | :--- |
| 361.7630873 | 168.0622420 | 205.4777664 | 399.2227256 |
| 286.4259419 | 305.2577836 | 239.8677456 | 248.9833745 |
| 217.4959595 | 274.3045256 | 340 | 181.6597908 |
| 201.1152074 | 259.3996384 | 208.6892241 | 277.8493217 |
| 179.0307835 | 242.9886794 | 118.1445066 | 183.6734269 |
| 119.9338007 | 103.2976540 | 160 | 118.7190545 |
| 81.03038860 | 23.58266429 | 78.83205900 | 35.43137407 |
| 97.99971207 | 119.0761287 | 120 | 107.3780597 |
| 52.00113619 | 72.59531050 | 62.56425338 | 33.81402317 |
| 34.57432282 | 47.68354245 | 55 | 49.59621356 |
| 306.8486138 | 187.2472111 | 175.9067214 | 393.0652103 |
| 338.7023182 | 252.4448604 | 261.7622078 | 302.1274649 |
|  |  |  | Continued on next page |


| WO | SAO | ChOA | RIME |
| :--- | :--- | :--- | :--- |
| 238.4998547 | 316.2628216 | 268.3381195 | 280.9736853 |
| 223.3491903 | 299.3864879 | 207.1881123 | 85.59393119 |
| 180.5191017 | 218.6818710 | 243 | 230.2698858 |
| 126.9556699 | 154.6733846 | 94.46223648 | 137.8008438 |
| 109.8918576 | 129.9483846 | 97.66863888 | 60.35154872 |
| 87.51357368 | 59.94573212 | 120 | 120 |
| 63.94496690 | 22.39995053 | 55.02037904 | 36.08305683 |
| 38.23076929 | 10 | 50.45341015 | 46.60655867 |
| 357.4468090 | 449.0518667 | 332.6299565 | 234.7062162 |
| 296.4047702 | 139.3053811 | 162.8862309 | 337.5808507 |
| 234.5334060 | 317.9662357 | 340 | 274.3350697 |
| 219.2973389 | 299.9910290 | 300 | 238.5976231 |
| 185.8148553 | 241.8390582 | 222.6389246 | 242.1331374 |
| 114.0038275 | 58.80668289 | 160 | 107.6366599 |
| 64.37558862 | 102.8953648 | 130 | 68.95720026 |
| 92.47625889 | 47.00006457 | 80.31172714 | 68.00115191 |
| 53.81758571 | 21.03713721 | 80 | 73.10745947 |
| 36.00330252 | 54.86794885 | 29.29109226 | 25.74259255 |



Figure 12. Thirty generators' robustness curves at a 5000 MW load.

### 4.5. Discussion

The main factor contributing to ELD problems is the power mismatch value, the precise difference between the units of power generated and the entire demand plus transmission losses. The power mismatch value is nearly zero, hence the high-performance methods is employed to get it. Table 19 explains the importance of this component for ELD. In addition to the five methods used in the run, the proposed WO algorithm is contrasted with other methods found in the literature, such as the monarch butterfly optimization (MBO), the grey wolf optimization (GWO), Earth Worm Algorithm (EWA), Elephant herding optimization (EHO), and Sine cosine algorithm (SCA) [9,43,44].


Figure 13. Thirty generators' convergence curves at a 5000 MW load.

Table 19. the power mismatch for all unit generators based on all algorithms.

| Case | Technique | $\mathbf{7 0 0} \mathbf{~ M W}$ | $\mathbf{1 0 0 0} \mathbf{~ M W}$ |
| :--- | :--- | :--- | :--- |
| 6 generators | WO | $\mathbf{4 . 1 9 2 2 E - 1 3}$ | $\mathbf{4 . 5 1 1 9 5 E - 1 3}$ |
|  | SAO | $5.03597 \mathrm{E}-12$ | $4.85414 \mathrm{E}-10$ |
|  | ChOA | 0.000304799 | 0.000232386 |
|  | MSA | 7.850121146 | 12.80174784 |
|  | RIME | 0.0000145386 | 0.000268421 |
|  | MBO [44] | 8.624894662 | 10.11850299 |
|  | GWO [44] | 0.000067 | 0.0000136 |
|  | SCA [43] | 0.00076719 | 0.000182 |
|  | EWA [43] | 5.71 | 20.1 |
|  | EHO [9] | 2.239431602 | 9.904979361 |
|  | SMA [9] | $5.61 \mathrm{E}-9$ | $4.18 \mathrm{E}-9$ |
| 20 generators | Technique | 3000 MW |  |
|  | WO | $\mathbf{9 . 0 9 4 9 5 E - 1 3}$ |  |
|  | SAO | 0.000356058 |  |
|  | ChOA | 0.00868658 |  |
|  | MSA | $4.47608 \mathrm{E}-9$ |  |
|  | RIME | 0.003891024 |  |
| 30 generators | Technique | 4000 MW |  |
|  | WO | $\mathbf{1 . 1 5 5 7 5 E}-\mathbf{6}$ |  |
|  | SAO | 0.000358263 |  |
|  | ChOA | 0.133312071 |  |
|  | MSA | 0.002488716 |  |
|  | RIME | $1.15575 \mathrm{E}-6$ |  |

## 5. Conclusions

The walrus optimizer (WO), a novel metaheuristic method, mimics how walruses migrate, roost,
feed, spawn, gather, and run away in response to crucial cues (safety and danger signals). Moreover, the effectiveness of four distinct algorithms was compared with that of the WO. Economic load dispatch (ELD) is a crucial problem that this work employs the WO to solve. ELD specifically helps to lower the cost of petrol. The fuel cost is the main factor to consider when optimizing the ELD problem. The WO aims to minimize this cost while maximizing the economic value of the power system. The primary variable of ELD issue reflects the unit-specific allocation vector that determines the best outcome for each system. The WO's performance was compared to other algorithms, including the rime-ice algorithm (RIME), moth search algorithm (MSA), snow ablation optimization (SAO), and chimp optimization algorithm (ChOA). Using the WO approach, the ideal power mismatch values of 4.1922E-13 and $4.5119 \mathrm{E}-13$ are found for six generator units at demand loads of 700 MW and 1000 MW , respectively. Using the WO approach, the ideal power mismatch values of $4.5474 \mathrm{E}-13$ and $1.05729 \mathrm{E}-11$ are found for ten generator units at demand loads of 1000 MW and 2000 MW , respectively. Using the WO approach, the ideal power mismatch values of $9.09495 \mathrm{E}-13$ is found for twenty generator units at demand load of 4000 MW . Using the WO approach, the ideal power mismatch values of $1.15575 \mathrm{E}-6$ is found for thirty generator units at demand load of 5000 MW . Ultimately, results verified that the WO was effective in lowering the fuel expenses for every ELD cases when compared to the alternatives. Additional major, practical optimization issues relating to solar energy, ELD of renewable energy sources, and power systems may be resolved in the future using the WO technique.

## Use of AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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## Conflict of interest

The authors declare that there are no conflicts of interest.

## References

1. W. K. Hao, Y. P. Li, J. S. Wang, Q. Zhu, Solving economic load dispatch problem of power system based on differential evolution algorithm with different mutation strategies, IAENG Int. J. Comput. Sci., 49 (2022), 156-165.
2. N. Singh, T. Chakrabarti, P. Chakrabarti, M. Margala, A. Gupta, S. P. Praveen, et al., Novel heuristic optimization technique to solve economic load dispatch and economic emission load dispatch problems, Electronics, 12 (2023), 2921. https://doi.org/10.3390/electronics 12132921
3. G. Abbas, I. A. Khan, N. Ashraf, M. T. Raza, M. Rashad, R. Muzzammel, On employing a constrained nonlinear optimizer to constrained economic dispatch problems, Sustainability, 15 (2023), 9924. https://doi.org/10.3390/su15139924
4. D. S. AbdElminaam, E. H. Houssein, M. Said, D. Oliva, A. Nabil, An efficient heap-based optimizer for parameters identification of modified photovoltaic models, Ain Shams Eng. J., 13 (2022), 101728. https://doi.org/10.1016/j.asej.2022.101728
5. A. A. K. Ismaeel, E. H. Houssein, D. Oliva, M. Said, Gradient-based optimizer for parameter extraction in photovoltaic models, IEEE Access, 9 (2021), 13403-13416. https://doi.org/10.1109/ACCESS.2021.3052153
6. E. H. Houssein, S. Deb, D. Oliva, H. Rezk, H. Alhumade, M. Said, Performance of gradient-based optimizer on charging station placement problem, Mathematics, 9 (2021), 2821. https://doi.org/10.3390/math9212821
7. D. S. Abdelminaam, M. Said, E. H. Houssein, Turbulent flow of water-based optimization using new objective function for parameter extraction of six photovoltaic models, IEEE Access, 9 (2021), 35382-35398. https://doi.org/10.1109/ACCESS.2021.3061529
8. M. Said, E. H. Houssein, S. Deb, A. A. Alhussan, R. M. Ghoniem, A novel gradient-based optimizer for solving unit commitment problem, IEEE Access, 10 (2022), 18081-18092. https://doi.org/10.1109/ACCESS.2022.3150857
9. A. A. K. Ismaeel, E. H. Houssein, D. S. Khafaga, E. A. Aldakheel, A. S. AbdElrazek, M. Said, Performance of osprey optimization algorithm for solving economic load dispatch problem, Mathematics, 11 (2023), 4107. https://doi.org/10.3390/math11194107
10. A. Bhattacharya, P. K. Chattopadhyay, Biogeography-based optimization for different economic load dispatch problems, IEEE T. Power Syst., 25 (2010), 1064-1077. https://doi.org/10.1109/TPWRS.2009.2034525
11. G. L. Andrade, C. Schepke, N. Lucca, J. P. J. Neto, Modified differential evolution algorithm applied to economic load dispatch problems, In: Computational science and its applicationsICCSA 2023, 2023. https://doi.org/10.1007/978-3-031-36805-9_2
12. M. Said, E. H. Houssein, S. Deb, R. M. Ghoniem, A. G. Elsayed, Economic load dispatch problem based on search and rescue optimization algorithm, IEEE Access, 10 (2022), 47109-47123. https://doi.org/10.1109/ACCESS.2022.3168653
13. M. A. Al-Betar, M. A. Awadallah, S. N. Makhadmeh, I. A. Doush, R. A. Zitar, S. Alshathri, et al., A hybrid Harris Hawks optimizer for economic load dispatch problems, Alex. Eng. J., 64 (2023), 365-389. https://doi.org/10.1016/j.aej.2022.09.010
14. A. Hazra, S. Das, A. Laddha, M. Basu, Economic power generation strategy for wind integrated large power network using heat transfer search algorithm, J. Inst. Eng. Ser. B, 101 (2020), 15-21. $\mathrm{https}: / /$ link.springer.com/article/10.1007/s40031-020-00427-y
15. G. Xiong, D. Shi, X. Duan, Multi-strategy ensemble biogeography based optimization for economic dispatch problems. Appl. Energy, 111 (2013), 801-811. https://doi.org/10.1016/j.apenergy.2013.04.095
16. M. A. Al-Betar, M. A. Awadallah, R. A. Zitar, K. Assaleh, Economic load dispatch using memetic sine cosine algorithm, J. Ambient. Intell. Humaniz. Comput., 14 (2022), 11685-11713. https://link.springer.com/article/10.1007/s12652-022-03731-1
17. A. S. Alghamdi, Greedy sine-cosine non-hierarchical grey wolf optimizer for solving non-convex economic load dispatch problems, Energies, 15 (2022), 3904. https://doi.org/10.3390/en15113904
18. T. P. Van, V. Snasel, T. T. Nguyen, Antlion optimization algorithm for optimal non-smooth economic load dispatch, Int. J. Elec. Comput. Eng., 10 (2020), 1187-1199. http://doi.org/10.11591/ijece.v10i2.pp1187-1199
19. W. T. Elsayed, E. F. El-Saadany, A fully decentralized approach for solving the economic dispatch problem, IEEE T. Power Syst., 30 (2015), 2179-2189. https://doi.org/10.1109/TPWRS.2014.2360369
20. N. Ghorbani, E. Babaei, Exchange market algorithm for economic load dispatch, Int. J. Elec. Power Energy Syst., 75 (2016), 19-27. https://doi.org/10.1016/j.ijepes.2015.08.013
21. F. Mohammadi, H. Abdi, A modified crow search algorithm (MCSA) for solving economic load dispatch problem, Appl. Soft Comput., 71 (2018), 51-65. https://doi.org/10.1016/j.asoc.2018.06.040
22. T. Jayabarathi, T. Raghunathan, B. R. Adarsh, P. N. Suganthan, Economic dispatch using hybrid grey wolf optimizer, Energy, 111 (2016), 630-641. https://doi.org/10.1016/j.energy.2016.05.105
23. D. C. Secui, A modified symbiotic organisms search algorithm for large scale economic dispatch problem with valve-point effects, Energy, 113 (2016), 366-384. https://doi.org/10.1016/j.energy.2016.07.056
24. A. A. Elsakaan, R. A. El-Sehiemy, S. S. Kaddah, M. I. Elsaid, An enhanced moth-flame optimizer for solving non-smooth economic dispatch problems with emissions, Energy, 157 (2018), 10631078. https://doi.org/10.1016/j.energy.2018.06.088
25. T. T. Nguyen, D. N. Vo, The application of one rank cuckoo search algorithm for solving economic load dispatch problems, Appl. Soft Comput., 37 (2015), 763-773. https://doi.org/10.1016/j.asoc.2015.09.010
26. P. Zakian, A. Kaveh, Economic dispatch of power systems using an adaptive charged system search algorithm, Appl. Soft Comput., 73 (2018), 607-622. https://doi.org/10.1016/j.asoc.2018.09.008
27. S. H. A. Kaboli, A. K. Alqallaf, Solving non-convex economic load dispatch problem via artificial cooperative search algorithm, Expert Syst. Appl., 128 (2019), 14-27. https://doi.org/10.1016/j.eswa.2019.02.002
28. S. Cui, Y. W, Wang, X. Lin, X. K. Liu, Distributed auction optimization algorithm for the nonconvex economic dispatch problem based on the gossip communication mechanism, Int. J. Elec. Power Energy Syst., 95 (2018), 417-426. https://doi.org/10.1016/j.ijepes.2017.09.012
29. K. Kapelinski, J. P. J. Neto, E. M. dos Santos, Firefly Algorithm with non-homogeneous population: A case study in economic load dispatch problem., J. Oper. Res. Soc., 72 (2021), 519534. https://doi.org/10.1080/01605682.2019.1700184
30. A. Kaur, L. Singh, J. S. Dhillon, Modified krill herd algorithm for constrained economic load dispatch problem, Int. J. Ambient Energy, 43 (2022), 4332-4342. https://doi.org/10.1080/01430750.2021.1888798
31. R. Ramalingam, D. Karunanidy, S. S. Alshamrani, M. Rashid, S. Mathumohan, A. Dumka, Oppositional pigeon-inspired optimizer for solving the non-convex economic load dispatch problem in power systems, Mathematics, 10 (2022), 3315. https://doi.org/10.3390/math10183315
32. M. F. Tabassum, M. Saeed, N. A. Chaudhry, J. Ali, M. Farman, S. Akram, Evolutionary simplex adaptive HookeJeeves algorithm for economic load dispatch problem considering valve point loading effects., Ain Shams Eng. J., 12 (2021), 1001-1015. https://doi.org/10.1016/j.asej.2020.04.006
33. S. Banerjee, D. Maity, C. K. Chanda, Teaching learning based optimization for economic load dispatch problem considering valve point loading effect. Int. J. Elec. Power Energy Syst., 73 (2015), 456-464. https://doi.org/10.1016/j.ijepes.2015.05.036
34. H. Shayeghi, A. Ghasemi, A modified artificial bee colony based on chaos theory for solving nonconvex emission/economic dispatch., Energy. Convers. Manag., 79 (2014), 344-354. https://doi.org/10.1016/j.enconman.2013.12.028
35. B. K. Panigrahi, V. R. Pandi, Bacterial foraging optimisation: Nelder-Mead hybrid algorithm for economic load dispatch., IET Gener. Transm. Dis., 2 (2008), 556-565. https://doi.org/10.1049/ietgtd:20070422
36. G. B inetti, A. Davoudi, D. Naso, B. Turchiano, F. L. Lewis, A distributed auction-based algorithm for the nonconvex economic dispatch problem, IEEE T. Ind. Inform., 10 (2014), 1124-1132. https://doi.org/10.1109/TII.2013.2287807
37. S. Deb, E. H. Houssein, M. Said, D. S. Abdelminaam, Performance of turbulent flow of water optimization on economic load dispatch problem, IEEE Access, 9 (2021), 77882-77893. https://doi.org/10.1109/ACCESS.2021.3083531
38. M. Han, Z. Du, Y. K. Yuan, H. Zhu, Y. Li, Q. Yuan, Walrus optimizer: A novel nature-inspired metaheuristic algorithm., Expert Syst. Appl., 239 (2024), 122413. https://doi.org/10.1016/j.eswa.2023.122413
39. G. G. Wang, Moth search algorithm: A bio-inspired metaheuristic algorithm for global optimization problems, Memetic Comp., 10 (2018), 151-164. https://doi.org/10.1007/s12293-016-0212-3
40. L. Deng, S. Liu, Snow ablation optimization: A novel metaheuristic technique for numerical optimization and engineering design, Expert Syst. Appl., 225 (2023), 120069. https://doi.org/10.1016/j.eswa.2023.120069
41. M. Khishe, M. R. Mosavi, Chimp optimization algorithm, Expert Syst. Appl., 149 (2020), 113338. https://doi.org/10.1016/j.eswa.2020.113338
42. H. Su, D. Zhao, A. A. Heidari, L. Liu, X. Zhang, M. Mafarja, et al., RIME: A physics-based optimization, Neurocomputing, 532 (2023), 183-214. https://doi.org/10.1016/j.neucom.2023.02.010
43. M. Said, A. M. El-Rifaie, M. A. Tolba, E. H. Houssein, S. Deb, An efficient chameleon swarm algorithm for economic load dispatch problem, Mathematics, 9 (2021), 2770. https://doi.org/10.3390/math9212770
44. A. A. K. Ismaeel, E. H. Houssein, D. S. Khafaga, E. A. Aldakheel, A. S. AbdElrazek, M. Said, Performance of snow ablation optimization for solving optimum allocation of generator units., IEEE Access, 12 (2024), 17690-17707. https://doi.org/10.1109/ACCESS.2024.3357489
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