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*Research article*

# Entry-exit decisions with output reduction during exit periods

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**Abstract:** In practice, a firm usually reduces the output of a project during exiting the project. We intend to fully analyze the effects of the reduction on the entry-exit decision on the project. To this end, we first obtain the closed expressions of the optimal activating time, optimal start time of the exit, and the maximal expected present value of the project. With these expressions in hand, we completely investigate the effects analytically and numerically. The results show us that the reduction affects the entry-exit decision in different ways due to the different conditions in terms of the parameters involved in the problem. The reduction does not affect the entry-exit decision provided that the firm never exits the project. If the firm exits the project in a finite time, the reduction may postpone or advance the activating time and start time of the exit.

**Keywords:** activating time; start time of the exit; output reduction; optimal stopping problem
**Mathematics Subject Classification:** 60G40, 91B06

## 1. Introduction

The models involving entry-exit decisions apply to many situations, as stated in [1, 2]. Such models may be described simply as follows. A firm decides when to invest in or when to abandon a project that can bring profit. There is plenty of literature on models concerning how to schedule the timing under the assumption that exiting the project does not take time, so we do not list them in detail, but refer to [1–9]. The ideas of entry-exit decisions apply to many concrete issues, for example, relationships of globalization and entrepreneurial entry-exit [10], when to invest or expand a start-up firm [11], when to enter or exit stock markets [12], and how to make a schedule for buying carbon emission rights [13].

In practice, the exiting process may take a long time, so ignoring it is not reasonable. For example, Brexit took more than 3 years, from June 23, 2016 to January 31, 2020 (https://www.britannica.com/topic/Brexit). The models [14, 15] take account of the time, but with equal output rates in the regular production and exit periods. It is an apparent feature that the output rate of the project is usually reduced during the exit period. In this paper, we introduce a parameter to describe the output rate during the exit period and completely discuss the effects of output reduction on entry-exit decisions, under the assumption that the commodity price of the project follows a geometric Brownian motion.

We use the optimal stopping theory to carry out the study and obtain the explicit expressions of the optimal activating time and start time of the exit, which is one of the contributions of the paper. With these explicit expressions in hand, we can carefully analyze the effects of output reduction on entry-exit decisions, which is another contribution of the paper.

If the optimal choice is never to exit the project, the output reduction has no effect on the optimal entry-exit decision.

If the optimal exit is in a finite time, the situation becomes complicated. However, we obtain complete criterions, which determine the effects of output reduction on entry-exit decisions (see Section 4).

We outline the structure of this paper. In Section 2, we describe the model in detail. In Section 3, we determine an optimal entry-exit decision. In Section 4, we discuss the effects of output reduction during exit period on entry-exit decisions. Some conclusions are drawn in Section 5.

## 2. The model

Assume that the price process $P$ of one unit product follows

$$\mathrm{d}P(t) = \mu P(t)\mathrm{d}t + \sigma P(t)\mathrm{d}B(t) \text{ and } P(0) = p, \tag{2.1}$$

where $\mu \in \mathbb{R}$, $\sigma$, $p > 0$, and $B$ is a one dimensional standard Brownian motion, which denotes uncertainty. In this paper, all times are stopping times w.r.t. the filtration generated by the Brownian motion $B$.

Since involving the construction period leads to complicated calculations and distracts us from analyzing the effects of reduction on entry-exit decisions, and the effects of the construction period have been discussed in [3, 15], we assume that there is no construction period (it may happen when the firm buys a project). The firm activates the project at time $\tau_I$ with the entry cost $K_I$ and completes the abandonment of the project during the time interval $[\tau_O, \tau_O + \delta]$, with the exit cost valued at $K_O$ at time $\tau_O + \delta$. Without loss of generality, we assume that the firm produces one unit product per unit time during the period $[\tau_I, \tau_O]$ at the marginal cost $C$ and $\alpha$ $(0 \leq \alpha \leq 1)$ unit products per unit time during the time interval of exiting the project $[\tau_O, \tau_O + \delta]$.

To answer the two questions, what time is optimal to activate the project and what time is optimal to start the abandonment procedure, we solve the optimization problem

$$\begin{aligned} J(p) = \sup_{\tau_I \leq \tau_O} \mathbb{E}^p \Big[ \int_{\tau_I}^{\tau_O} \exp(-rt)(P(t) - C)\mathrm{d}t + \alpha \int_{\tau_O}^{\tau_O+\delta} \exp(-rt)(P(t) - C)\mathrm{d}t \\ - \exp(-r\tau_I)K_I - \exp(-r(\tau_O + \delta))K_O \Big]. \end{aligned} \tag{2.2}$$

We call stopping times $\tau_I$ and $\tau_O$ the activating times and start times of the exit, respectively, and we call the function $J$ the maximal expected present value of the project.

## 3. An optimal entry-exit decision

If $r \leq \mu$, some straight calculations show us that

$$\mathbb{E}^p\left[\int_0^{+\infty} \exp(-rt)(P(t)-C)\mathrm{d}t\right] = +\infty.$$

Thus, we obtain the following result.

**Theorem 3.1.** *Assume that $r \leq \mu$, then $\tau_I^* := 0$ is an optimal activating time and $\tau_O^* := +\infty$ is an optimal start time of the exit, i.e., the firm should never exit the project. In addition, the function $J$ in* (2.2) *is given by $J \equiv +\infty$.*

In the rest of this section, we assume $r > \mu$.
Taking $\tau_I = \tau_O := 0$ in (2.2), we have

$$J(p) \geq -K_I + \frac{\alpha(1-\exp((\mu-r)\delta))}{r-\mu}p - \exp(-r\delta)K_O - \alpha\frac{C}{r}(1-\exp(-r\delta)),$$

thus, in the remains of this section, we always assume that

$$rK_I + \exp(-r\delta)rK_O + \alpha(1-\exp(-r\delta))C \geq 0$$

to avoid arbitrage opportunities.
Let $\lambda_1$ and $\lambda_2$ be the solutions to the equation

$$r - \mu\lambda - \frac{1}{2}\sigma^2\lambda(\lambda-1) = 0$$

with $\lambda_1 < \lambda_2$, then we have $\lambda_1 < 0$ and $\lambda_2 > 1$.

**Theorem 3.2.** *Assume that $r > \mu$. The following are true:*

(i) *If*

$$(1-\alpha)C + (\alpha C - rK_O)\exp(-r\delta) \leq 0,$$

*then $(\tau_I^*, \tau_O^*)$ is a solution to* (2.2)*, where*

$$\tau_I^* = \inf\{t : t > 0, P(t) \geq p_I\}$$

*and $\tau_O^* = +\infty$. Here,*

$$p_I = \frac{\lambda_2}{\lambda_2-1}(r-\mu)\left(\frac{C}{r}+K_I\right).$$

*In addition,*

$$J(p) = \begin{cases} Bp^{\lambda_2}, & \text{if } p < p_I, \\ \dfrac{p}{r-\mu} - \dfrac{C}{r} - K_I, & \text{if } p \geq p_I, \end{cases}$$

*where*

$$B = \frac{{p_I}^{1-\lambda_2}}{\lambda_2(r-\mu)}.$$

(ii) *If*

$$(1-\alpha)C + (\alpha C - rK_O)\exp(-r\delta) > 0$$

*and*

$$\alpha(1-\exp((\mu-r)\delta))p_O - \alpha C(1-\exp(-r\delta)) - \exp(-r\delta)rK_O - rK_I \le 0, \tag{3.1}$$

*where*

$$p_O = \frac{\lambda_1}{\lambda_1-1}\frac{r-\mu}{1-\alpha+\alpha\exp((\mu-r)\delta)}\left((1-\alpha)\frac{C}{r} + \exp(-r\delta)\left(\alpha\frac{C}{r} - K_O\right)\right),$$

*then* $(\tau_I^*, \tau_O^*)$ *is a solution to* (2.2)*, where*

$$\tau_I^* = \inf\{t : t > 0, P(t) \ge p_I\}$$

*and*

$$\tau_O^* = \inf\{t : t > \tau_I^*, P(t) \le p_O\}.$$

*Here,* $p_I$ *is the largest solution of the algebraic equation*

$$A(\lambda_2-\lambda_1)p_I^{\lambda_1} + \frac{(\lambda_2-1)}{r-\mu}p_I - \lambda_2\left(\frac{C}{r} + K_I\right) = 0.$$

*In addition,*

$$J(p) = \begin{cases} Bp^{\lambda_2}, & \text{if } p < p_I, \\ Ap^{\lambda_1} + \dfrac{p}{r-\mu} - \dfrac{C}{r} - K_I, & \text{if } p \ge p_I, \end{cases}$$

*where*

$$A = \frac{1-\alpha+\alpha\exp((\mu-r)\delta)}{\lambda_1(\mu-r)}{p_O}^{1-\lambda_1}$$

*and*

$$B = \lambda_1\lambda_2^{-1}Ap_I^{\lambda_1-\lambda_2} + \frac{{p_I}^{1-\lambda_2}}{\lambda_2(r-\mu)}.$$

**Remark 3.3.** We propose condition (3.1) to eliminate the possibility that the firm enters the project at a trigger price lower than the optimal trigger price of the exit, i.e., the firm enters the project and then immediately decides to exit the project.

In light of [14, Theorems 3.1 and 5.2], we have the following Lemma 3.4, which serves as preparation for the proof of Theorem 3.2.

**Lemma 3.4.** *If* $(\tau_1^*, \tau_2^*)$ *is a solution to the optimization problem*

$$\widetilde{J}(p) := \sup_{\tau_I \le \tau_O} \mathbb{E}^p\left[\int_{\tau_I}^{\tau_O} \exp(-rt)(P(t)-C)\mathrm{d}t - \exp(-r\tau_I)K_I - \exp(-r\tau_O)(l_1P(\tau_O)+l_0)\right], \tag{3.2}$$

*it is also a solution to* (2.2) *and* $J(x) = \widetilde{J}(x)$*, where*

$$l_1 := \frac{\alpha(1-\exp((\mu-r)\delta))}{\mu-r}$$

*and*

$$l_0 := \alpha\frac{C}{r}(1-\exp(-r\delta)) + \exp(-r\delta)K_O.$$

*Proof.* By the strong Markov property of the process $\{(s+t, P(t)), t \geq 0\}$, where $s \in \mathbb{R}$, we have

$$\mathbb{E}^p\Big[\int_{\tau_I}^{\tau_O} \exp(-rt)(P(t)-C)\mathrm{d}t + \alpha\int_{\tau_O}^{\tau_O+\delta} \exp(-rt)(P(t)-C)\mathrm{d}t - \exp(-r\tau_I)K_I - \exp(-r(\tau_O+\delta))K_O\Big]$$
$$= \mathbb{E}^p\Big[\int_{\tau_I}^{\tau_O} \exp(-rt)(P(t)-C)\mathrm{d}t + \alpha\exp(-r\tau_O)\mathbb{E}^{P(\tau_O)}\Big[\int_0^{\delta} \exp(-rt)(P(t)-C)\mathrm{d}t\Big]$$
$$- \exp(-r\tau_I)K_I - \exp(-r(\tau_O+\delta))K_O\Big].$$

Thus, we need to calculate

$$\begin{aligned}
&\alpha\mathbb{E}^{P(\tau_O)}\Big[\int_0^{\delta} \exp(-rt)(P(t)-C)\mathrm{d}t\Big] - \exp(-r\delta)K_O \\
&= \alpha\int_0^{\delta} \exp(-rt)(\exp(\mu t)P(\tau_O)-C)\mathrm{d}t - \exp(-r\delta)K_O \\
&= -l_1P(\tau_O) - l_0.
\end{aligned}$$

The proof is complete. □

**Remark 3.5.** The proof of Lemma 3.4 has an economic meaning as follows. We first discount the benefit during the abandonment period to time $\tau_O$, then discount this value to time zero.

With the help of Lemma 3.4, we can prove Theorem 3.2.

*Proof of Theorem 3.2.* (1) By Lemma 3.4, we solve problem (3.2),

$$\begin{aligned}
&\sup_{\tau_I\leq\tau_O} \mathbb{E}^p\Big[\int_{\tau_I}^{\tau_O} \exp(-rt)(P(t)-C)\mathrm{d}t - \exp(-r\tau_I)K_I - \exp(-r\tau_O)(l_1P(\tau_O)+l_0)\Big] \\
&= \sup_{\tau_I\leq\tau_O} \mathbb{E}^p\Big[\exp(-r\tau_I)\int_0^{\tau_O-\tau_I} \exp(-rt)(P(t+\tau_I)-C)\mathrm{d}t \\
&\quad - \exp(-r\tau_I)K_I - \exp(-r\tau_O)(l_1P(\tau_O)+l_0)\Big] \\
&= \sup_{\tau_I} \mathbb{E}^p\Big[\exp(-r\tau_I)(G(P(\tau_I))-K_I)\Big] \\
&=: H(p),
\end{aligned} \tag{3.3}$$

where

$$G(p) := \sup_{\tau_O} \mathbb{E}^p\Big[\int_0^{\tau_O} \exp(-rt)(P(t)-C)\mathrm{d}t - \exp(-r\tau_O)(l_1P(\tau_O)+l_0)\Big]. \tag{3.4}$$

(2) Assume

$$(1-\alpha)C + (\alpha C - rK_O)\exp(-r\delta) \leq 0.$$

We first solve problem (3.4) and then problem (3.3).

For problem (3.4), noting

$$\mathbb{E}^p\Big[\int_0^{\infty} \exp(-rt)(P(t)-C)\mathrm{d}t\Big] = \frac{p}{r-\mu} - \frac{C}{r},$$

we see that

$$\mathbb{E}^p\Big[\int_0^\infty \exp(-rt)(P(t)-C)\mathrm{d}t\Big] \geq -l_1 p - l_0,$$

which implies

$$\tau_O^* = +\infty$$

and

$$G(p) = \frac{p}{r-\mu} - \frac{C}{r}.$$

Set

$$h(p) := \frac{p}{r-\mu} - \frac{C}{r} - K_I.$$

Since

$$\{p : p > 0 \text{ and } rh(p) - \mu p h'(p) \geq 0\} = \{p : p \geq C + rK_I\},$$

the exercise region of problem (3.3) takes the form $[p_I, +\infty)$ for some

$$p_I \geq C + rK_I.$$

The function $H$ satisfies

$$rH - \mu p H' - \frac{1}{2}\sigma^2 p^2 H'' = 0$$

on the continuation region $(0, p_I)$ and is Lipschitz continuous on $(0, +\infty)$ and $C^1$ continuous at $p_I$. Thus, we get

$$H(p) = Bp^{\lambda_2}$$

and $B$ and $p_I$ solve

$$\begin{cases} Bp_I^{\lambda_2} = \dfrac{p_I}{r-\mu} - \dfrac{C}{r} - K_I, \\ \lambda_2 B p_I^{\lambda_2 - 1} = \dfrac{1}{r-\mu}, \end{cases}$$

by which we finish the proof of (i).

(3) Assume

$$(1-\alpha)C + (\alpha C - rK_O)\exp(-r\delta) > 0$$

and (3.1) hold. We again first solve problem (3.4) and then problem (3.3).

Set

$$g(p) := -l_1 p - l_0.$$

A straight calculation shows that

$$\{p : p > 0 \text{ and } rg - \mu p g'(p) - p + C \geq 0\} = \Big(0, \frac{(1-\alpha)C + (\alpha C - rK_O)\exp(-r\delta)}{1 - \alpha(1 - \exp((\mu - r)\delta))}\Big),$$

which means the exercise region of problem (3.4) takes the form $(0, p_O]$ for some

$$p_O \leq \frac{(1-\alpha)C + (\alpha C - rK_O)\exp(-r\delta)}{1 - \alpha(1 - \exp((\mu - r)\delta))}.$$

The function $G$ satisfies

$$rG - \mu pG' - \frac{1}{2}\sigma^2 p^2 G'' - p + C = 0$$

on the continuation region $(p_O, +\infty)$ and is Lipschitz continuous on $(0, +\infty)$ and $C^1$ continuous at $p_O$. Thus, we get

$$G(p) = Ap^{\lambda_1} + \frac{p}{r-\mu} - \frac{C}{r},$$

and $A$ and $p_O$ solve

$$\begin{cases} Ap_O^{\lambda_1} + \dfrac{p_O}{r-\mu} - \dfrac{C}{r} = -l_1 p_O - l_0, \\ \lambda_1 A p_O^{\lambda_1 - 1} + \dfrac{1}{r-\mu} = -l_1, \end{cases}$$

which implies coefficient $A$ and the optimal exit trigger pricer of (ii).

To solve problem (3.4), we define

$$h(p) := G(p) - K_I.$$

In light of (3.1),

$$\{p : p > 0 \text{ and } rh(p) - \mu ph'(p) \geq 0\} = \{p : p \geq C + rK_I\},$$

thus, the exercise region of problem (3.3) takes the form $[p_I, +\infty)$ for some

$$p_I \geq C + rK_I.$$

The function $H$ satisfies

$$rH - \mu pH' - \frac{1}{2}\sigma^2 p^2 H'' = 0$$

on the continuation region $(0, p_I)$ and is Lipschitz continuous on $(0, +\infty)$ and $C^1$ continuous at $p_I$. Thus, we get

$$H(p) = Bp^{\lambda_2},$$

and $B$ and $p_I$ solve

$$\begin{cases} Bp_I^{\lambda_2} = Ap_I^{\lambda_1} + \dfrac{p_I}{r-\mu} - \dfrac{C}{r} - K_I, \\ \lambda_2 B p_I^{\lambda_2 - 1} = \lambda_1 A p_I^{\lambda_1 - 1} + \dfrac{1}{r-\mu}, \end{cases}$$

by which we finish the proof of (ii). □

## 4. Discussions

We analyze the effects of output reduction during the exit period on entry-exit decisions.

If $r \leq \mu$, the firm has an optimal time $\tau_I^* = 0$ to activate the project and should never exit the project. Thus, the reduction does not affect entry-exit decisions.

If $r > \mu$ and

$$(1-\alpha)C + (\alpha C - rK_O)\exp(-r\delta) \leq 0,$$

the optimal time to activate the project is given by

$$\tau_I^* = \inf\{t : t > 0, P(t) \geq p_I\},$$

where

$$p_I = \frac{\lambda_2}{\lambda_2 - 1}(r - \mu)\left(\frac{C}{r} + K_I\right).$$

Thus, the reduction does not affect the optimal activating time. As same as the case of $r \leq \mu$, the firm should never exit the project and the reduction does not affect exit decisions.

Assume that $r > \mu$ and

$$(1 - \alpha)C + (\alpha C - rK_O)\exp(-r\delta) > 0.$$

We first analyze the effects of reduction on the optimal start time of the exit. By (ii) of Theorem 3.2, the optimal trigger price is an increasing function of $\alpha$ if

$$\exp(-\mu\delta)(C - rK_O) > C - \exp(-r\delta)rK_O,$$

a decreasing function if

$$\exp(-\mu\delta)(C - rK_O) < C - \exp(-r\delta)rK_O,$$

and a constant function if

$$\exp(-\mu\delta)(C - rK_O) = C - \exp(-r\delta)rK_O.$$

We list some examples to illustrate the analysis. Taking

$$r = 0.2, \quad \mu = -0.1, \quad \sigma = 0.3, \quad \delta = 2, \quad C = 5, \quad K_I = 20, \quad \text{and} \quad K_O = -10,$$

we have

$$\exp(-\mu\delta)(C - rK_O) > C - \exp(-r\delta)rK_O,$$

then the optimal trigger price is an increasing function of $\alpha$. See Figure 1. If we replace $\mu = -0.1$ with $\mu = 0.1$, we have a decreasing function. See Figure 2.

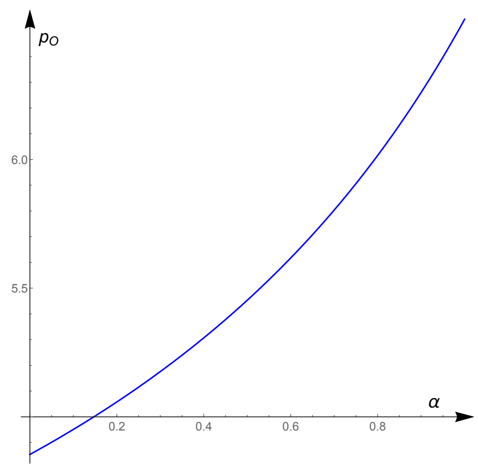


**Figure 1.** $\alpha$-$p_O$-increasing.

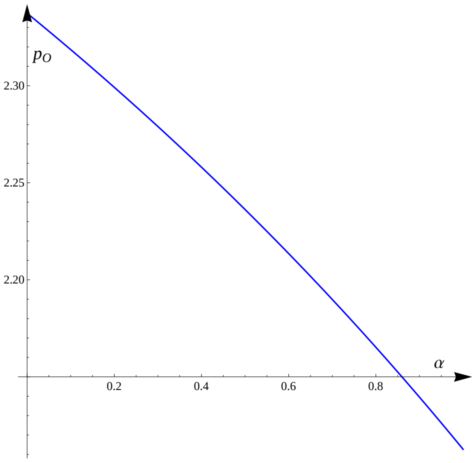


**Figure 2.** $\alpha$-$p_O$-decreasing.

We conclude that:

(1) If
$$\exp(-\mu\delta)(C - rK_O) > C - \exp(-r\delta)rK_O,$$
as the reduction increases (i.e., $\alpha$ decreases), the firm will postpone the exit.

(2) If
$$\exp(-\mu\delta)(C - rK_O) < C - \exp(-r\delta)rK_O,$$
as the reduction increases, the firm will advance the exit.

(3) If
$$\exp(-\mu\delta)(C - rK_O) = C - \exp(-r\delta)rK_O,$$
the reduction does not affect the exit.

To analyze the effects of reduction on the optimal time to activate the project, we define two functions as follows:
$$f(p) := A(\lambda_2 - \lambda_1)p^{\lambda_1}$$
and
$$g(p) := -\frac{(\lambda_2 - 1)}{r - \mu}p + \lambda_2\left(\frac{C}{r} + K_I\right)$$
according to (ii) of Theorem 3.2. Thus, the functions $f$ and $g$ intersect at two points, say, $(p_1, f(p_1))$ and $(p_2, f(p_2))$ with $p_1 < p_2$, and the optimal trigger price $p_I$ of activating the project is given by $p_I = p_2$.

To capture the behavior of $p_I$ as $\alpha$ varies, we only need to examine the behavior of the coefficient $A$ as $\alpha$ varies. Setting
$$\begin{aligned} M :=& (C - \exp(-r\delta)rK_O)(1 - \exp((\mu - r)\delta)) \\ &+ (1 - \lambda_1)\exp(-r\delta)(C - rK_O - C\exp(\mu\delta) + \exp((\mu - r)\delta)rK_O) \end{aligned}$$

and

$$N := -C(1 - \exp(-r\delta))(1 - \exp((\mu - r)\delta)),$$

after some calculations, we find that:

(1) If $M/N \leq 0$, $A$ is increasing on $[0, 1]$ and $p_I$ is decreasing on $[0, 1]$.

(2) If $M/N \geq 1$, $A$ is decreasing on $[0, 1]$ and $p_I$ is increasing on $[0, 1]$.

(3) If $0 < M/N < 1$, $A$ is decreasing on $[0, M/N]$ and increasing on $[M/N, 1]$ and $p_I$ is increasing on $[0, M/N]$ and decreasing on $[M/N, 1]$.

Numerical examples help us understand the analysis above.

Figure 3 demonstrates the decreasing of $p_I$ ($r = 0.2$, $\mu = -0.1$, $\sigma = 0.3$, $\delta = 0.6$, $C = 5$, $K_I = 20$, and $K_O = -10$). Figure 4 demonstrates the increasing of $p_I$ ($r = 0.2$, $\mu = 0.1$, $\sigma = 0.3$, $\delta = 2$, $C = 5$, $K_I = 20$, and $K_O = -10$), and Figure 5 demonstrates the non-monotonicity of $p_I$ ($r = 0.2$, $\mu = -0.1$, $\sigma = 0.3$, $\delta = 0.6$, $C = 5$, $K_I = 20$, and $K_O = -10$).

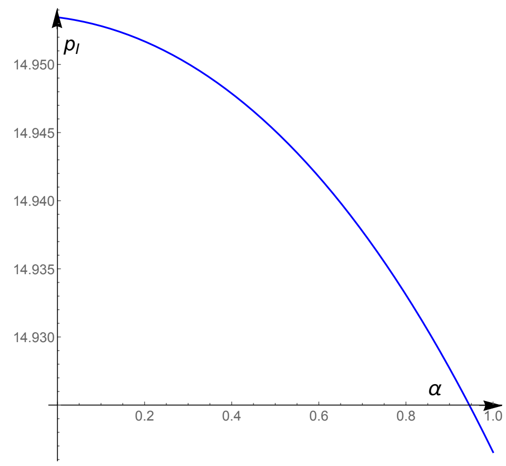


**Figure 3.** $\alpha$-$p_I$-decreasing.

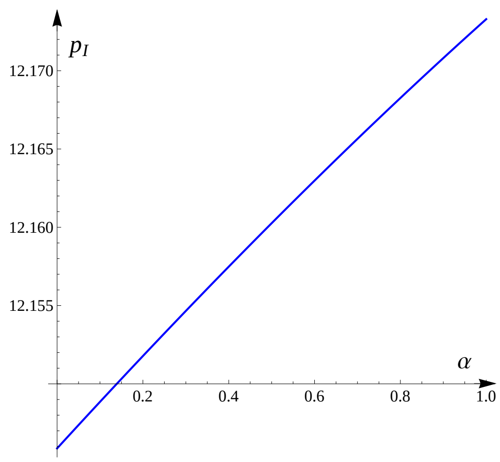


**Figure 4.** $\alpha$-$p_I$-increasing.

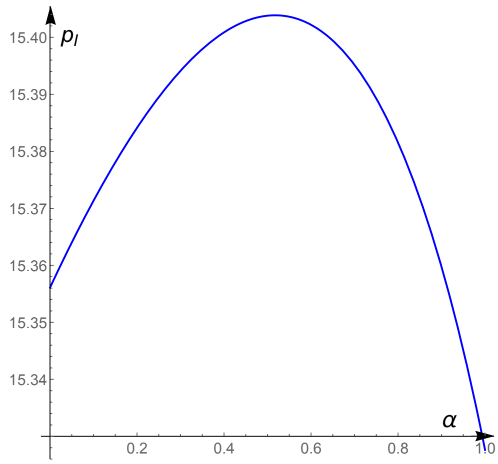


**Figure 5.** $\alpha$-$p_I$-non-monotonicity.

In other words, here are the effects of the reduction on the optimal time of activating the project:

(1) If $M/N \leq 0$, as the reduction increases (i.e., $\alpha$ decreases), the firm will postpone activating the project.

(2) If $M/N \geq 0$, as the reduction increases, the firm will advance activating the project.

(3) If $0 < M/N < 1$, as the reduction increases, the firm will postpone activating the project then advance activating the project.

## 5. Conclusions

There is an apparent phenomenon that firms usually reduce their output rate during stopping the regular production of a project. We intend to analyze the effects of reduction on the optimal entry-exit decision. Since the papers [3, 15] have investigated the effects of the construction period on entry-exit decisions, we assume that the project has been constructed and the production immediately starts for concentrating on the study of the effects of reduction.

We introduce the output rate into the models [14, 15] and describe the problem using the optimal stopping theory, obtaining explicit solutions in Theorems 3.1 and 3.2. These explicit solutions help us in discovering the effects of reduction on the optimal entry-exit decision.

We carefully examine the effects of reduction. According the analysis listed in Section 4, we come to the effects of reduction as follows. If the firm should never exit the project to obtain the maximal profit, the reduction does not affect the optimal entry-exit decision. However, if the firm exits the project in finite time, the effects show in a complicated manner. We study the effects analytically and numerically, and provide the relations of the parameters involved in the model that can determine the effects.

## Use of AI tools declaration

The author declares he has not used Artificial Intelligence (AI) tools in the creation of this article.

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## Conflict of interest

No potential conflict of interest exists regarding the publication of this paper.

## References

1 A. Dixit, Entry and exit decisions under uncertainty, *J. Political Econ.*, **97** (1989), 620–638. https://doi.org/10.1086/261619

2 A. E. Tsekrekos, The effect of mean reversion on entry and exit decisions under uncertainty, *J. Econ. Dyn. Control*, **34** (2010), 725–742. https://doi.org/10.1016/j.jedc.2009.10.015

3 A. B. Ilan, W. C. Strange, Investment lags, *Amer. Econ. Rev.*, **86** (1996), 610–622.

4 A. L. Bronstein, M. Zervos, Sequential entry and exit decisions with an ergodic performance criterion, *Stochastics*, **78** (2006), 99–121. https://doi.org/10.1080/17442500600715840

5 J. K. Duckworth, M. Zervos, An investment model with entry and exit decisions, *J. Appl. Probab.*, **37** (2000), 547–559. https://doi.org/10.1239/jap/1014842558

6 R. R. Lumley, M. Zervos, A model for investments in the natural resource industry with switching costs, *Math. Oper. Res.*, **26** (2001), 637–653. https://doi.org/10.1287/moor.26.4.637.10008

7 H. Wang, A sequential entry problem with forced exits, *Math. Oper. Res.*, **30** (2005), 501–520. https://doi.org/10.1287/moor.1040.0141

8 Y. Zhang, Entry-exit decisions with underlying processes following geometric Lévy processes, *J. Optim. Theory Appl.*, **172** (2017), 309–327. https://doi.org/10.1007/s10957-016-1026-7

9 Y. Zhang, Entry and exit decisions with linear costs under uncertainty, *Stochastics*, **87** (2015), 209–234. https://doi.org/10.1080/17442508.2014.939976

10 H. Aslana, P. Kumar, Globalization and entrepreneurial entry and exit: evidence from U.S. households, *J. Monetary Econ.*, **120** (2021), 83–100. https://doi.org/10.1016/j.jmoneco.2021.05.001

11 R. M. Ferreira, P. J. Pereira, A dynamic model for venture capitalists' entry-exit investment decisions, *Eur. J. Oper. Res.*, **290** (2021), 779–789. https://doi.org/10.1016/j.ejor.2020.08.014

12 S. Lleo, M. Zhitlukhin, W. T. Ziemba, Using a mean-changing stochastic processes exit–entry model for stock market long-short prediction, *J. Portf. Manage.*, **49** (2022), 172–197. https://doi.org/10.3905/jpm.2022.1.429

13 Y. Liu, H. Sun, B. Meng, S. Jin, B. Chen, How to purchase carbon emission right optimally for energy-consuming enterprises? Analysis based on optimal stopping model, *Energy Econ.*, **124** (2023), 106758. https://doi.org/10.1016/j.eneco.2023.106758

14 Y. Zhang, Entry-exit decisions with implementation delay under uncertainty, *Appl. Math.*, **63** (2018), 399–422. https://doi.org/10.21136/AM.2018.0205-17

15 Y. Zhang, Y. Zhang, An entry-exit model involving construction and abandonment periods, *Math. Methods Appl. Sci.*, **43** (2020), 2735–2746. https://doi.org/10.1002/mma.6080