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Research article

The new soliton solution types to the Myrzakulov-Lakshmanan-XXXII-equation

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Abstract: Our attention concenters on deriving diverse forms of the soliton arising from the Myrzakulov-Lakshmanan XXXII (M-XXXII) that describes the generalized Heisenberg ferromagnetic equation. This model has been solved numerically only using the N-fold Darboux Transformation method, not solved analytically before. We will derive new types of the analytical soliton solutions that will be constructed for the first time in the framework of three impressive schemas that are prepared for this target. These three techniques are the Generalized Kudryashov scheme (GKS), the (G'/G)-expansion scheme and the extended direct algebraic scheme (EDAS). Moreover, we will establish the 2D, 3D graphical simulations that clear the new dynamic properties of our achieved solutions.

Keywords: the Myrzakulov-Lakshmanan XXXII-equation; generalized Kudryashov scheme; the (G'/G)-expansion scheme; the extended direct algebraic scheme; the soliton solutions
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1. Introduction

The NLPDEs play important roles in different fields of science such as [1], who introduced a book for the Nonlinear Partial Differential Equations for Scientists and Engineers. The dispersion-less equations include the integrable hydrodynamic equations that are a constitute branch of the integrable partial differential equations which are crucial to applications for its extensive application in various branches as condensed matter theory, string theory, quantum arena theories, and conformal field theory; particularly, [2] proposed CTE solvability, nonlocal symmetry, and interaction solutions of coupled integrable dispersion-less system; complexity, [3] who investigated the Dispersion-less Toda hierarchy and two-dimensional string theory; and [4], who introduced the Topological conformal field theory with a rational W potential and the dispersion less KP hierarchy. Here, we will focus on one of the famous models that plays significant role in the magnetism theory. The suggested model is a general form of the well-known Heisenberg ferromagnetic equation, which is considered one of the fundamental integrable systems in soliton theory and has been studied by some authors. The study of the (1+1)-dimensions: Integrable generalized Heisenberg ferromagnetic equations: Reductions and gauge equivalence [5]. Furthermore, Ratbay Myrzakulov [6] introduced this model as a sigma model with potentials, which is one of the coupled integrable dispersion-less equations that covers generalized Heisenberg ferromagnetic equations and [7] who studied the Coupled dispersion less and generalized Heisenberg ferromagnetic equations with self-consistent sources: Geometry and equivalence. Moreover, there are many authors that have extracted studies of high quality and are qualitative studying the soliton solutions for the (2+1)-dimensional HFSC equation, which has a significant role in the phenomena and processes in various fields that relate to the ferromagnetic materials, nonlinear optics and optical fibers [8–12]. In the same way, various magnetic interactions of the (2+1) dimensional HFSC equation that have integral behaviors and split into classical and semi-classical limit have been studied in [13–17]. Furthermore, the soliton solutions for various nonlinear problems in different fields have been published [18–24].

The suggested model has been introduced in reference [12] to be

$$
\begin{aligned}
&Q_{xt} + 2P_x - 4mQ = 0,\\
&m_x = \sigma\left(\frac{1}{2}(|Q|^2)_x + PQ^* + QP^*\right),\\
&n_x = -\sigma\left(PQ^* + QP^*\right),\\
&P_x = -2i\tau P - 2nQ,
\end{aligned}
\tag{1}
$$

where $P(x,y,t), Q(x,y,t)$ are complex functions in the normalized spatial variables x and y and temporal variable t that are appropriate continuum approximation of the coherent magnetism amplitude to the bosonic operators at spin-lattice sites, while m,n are real functions, τ is real parameter and $\sigma = \pm 1$. The main target is transforming this system to one equation that we will name the modified equation of the Myrzakulov-Lakshmanan XXXII, by differentiation the first and the last part of Eq (1) with respect to x we get

$$
\begin{aligned}
&Q_{xxt} + 2P_{xx} - 4m_x Q - 4mQ_x = 0.\\
&P_{xx} = -2i\tau P_x - 2n_x Q - 2nQ_x.
\end{aligned}
\tag{2}
$$

By inserting the second part of Eq (2) into the first part of Eq (2) we get

$$
Q_{xxt} - 4i\tau P_x - 4n_x Q - 4nQ_x - 4m_x Q - 4mQ_x = 0. \tag{3}
$$

By inserting the second, third parts of Eq (1) into Eq (3) we get

$$Q_{xxt} - 4i\,\tau P_x + 4Q\sigma\left(PQ^* + QP^*\right) - 4nQ_x - 4Q\sigma\left(\frac{1}{2}(|Q|^2)_x + PQ^* + QP^*\right) - 4mQ_x = 0. \tag{4}$$

The above equation can be reduced to

$$Q_{xxt} - 4i\,\tau P_x - 4Q\sigma\left(\frac{1}{2}(|Q|^2)_x\right) - 4(m+n)Q_x = 0. \tag{5}$$

Now, let us suppose these complex transformations $Q(\zeta) = R_1(\zeta_1)e^{i\psi(x,t)}$, $\zeta_1 = x + w_1 t$, $P(\zeta) = R_2(\zeta_2)e^{i\psi(x,t)}$, $\zeta_2 = x + w_2 t$, where R_1, R_2 denote to the wave amplitudes, while w_1, w_2 are the soliton speeds and $\psi = x + \delta t + \theta_0$ denotes to the phase portion, δ is the wave number, θ_0 is the phase constant.

$$Q_x = \left(R_1' + i\,R_1\right)e^{i\,\psi(x,t)}. \tag{6}$$

$$P_x = \left(R_2' + i\,R_2\right)e^{i\,\psi(x,t)}. \tag{7}$$

$$Q_{xx} = \left(R_1'' + 2i\,R_1' - R_1\right)e^{i\,\psi(x,t)}. \tag{8}$$

$$Q_{xxt} = \left(w_1 R_1''' + i\,(\delta + 2w_1)\,R_1'' - (w_1 + 2\delta)R_1' - i\,\delta R_1\right)e^{i\,\psi(x,t)}. \tag{9}$$

$$(|Q|^2)_x = 2R_1 R_1'. \tag{10}$$

By inserting the relations (6)–(10) into Eq (5) we get

$$\begin{aligned}&\left(w_1 R_1''' + i(\delta + 2w_1)\,R_1'' - (w_1 + 2\delta)R_1' - i\delta R_1\right)e^{i\psi(x,t)} - 4i\tau\left(R_2' + i\,R_2\right)e^{i\psi(x,t)}\\&-4(m+n)\left(R_1' + i\,R_1\right)e^{i\psi(x,t)} - 4\sigma R_1^2 R_1' e^{i\psi(x,t)} = 0.\end{aligned} \tag{11}$$

That can be divided into the following real and imaginary parts:

$$w_1 R_1''' - \left(w_1 + 2\delta + 4(m+n)\right)R_1' - 4\sigma R_1^2 R_1' + 4\tau R_2 = 0. \tag{12}$$

$$(\delta + 2w_1)\,R_1'' - 4\tau R_2' - \left(4(m+n) + \delta\right)R_1 = 0. \tag{13}$$

By integrating Eq (13) with respect to ζ we get

$$(\delta + 2w_1)\,R_1' - 4\tau R_2 - \left(2(m+n) + \frac{\delta}{2}\right)R_1^2 = 0. \tag{14}$$

By emerging Eqs (12) and (14) we obtain

$$w_1 R_1''' + \left(w_1 - \delta - 4(m+n)\right) R_1' - 4\sigma R_1^2 R_1' - \left(2(m+n) + \frac{\delta}{2}\right) R_1^2 = 0. \tag{15}$$

Now, our aim is to extract the exact solutions of the Eq (15) in the framework of three various schemas namely the GKS [25–29], the (G'/G)-expansion scheme [30,31] and the EDAS [32–34].

2. The concept of the GKS

According to the GKS, any NLPDE which is

$$\Phi(R, R_x, R_t, R_{xx}, R_{tt}, ...) = 0. \tag{16}$$

With the transformation $R(x,t) = R(\zeta),\ \zeta = kx \pm wt, where\ (k\ \&w)$ are the wave number and traveling wave speed can be transformed to the ODE

$$\mathrm{E}(R, R', R'', R''', ...) = 0. \tag{17}$$

The Kudryashov scheme assumed the solution of Eq (17) in the form

$$R(\zeta) = \frac{\sum_{i=1}^{N} A_i S^i(\zeta)}{\sum_{j=1}^{M} B_j S^j(\zeta)} = \frac{A_0 + A_1 S(\zeta) + A_2 S^2(\zeta) + ...}{B_0 + B_1 S(\zeta) + B_2 S^2(\zeta) + ...}, \tag{18}$$

where $A_i (i = 0,1,..., \mathrm{N})\ \& \mathrm{B}_j (j = 0,1,..., M)$ are constants which will be determined later such that $A_N \neq 0 \& B_M \neq 0$ and the function $S(\zeta)$ is the solution of the nonlinear equation

$$\frac{dS(\zeta)}{d\zeta} = S^2(\zeta) - \mathrm{S}(\zeta). \tag{19}$$

Integrating Eq (19) then solution takes the form

$$S(\zeta) = \frac{1}{1 + Ce^{\zeta}}, \tag{20}$$

here C is the integration constsnt.

The next step in the Kudryashov scheme is determining the positive numbers N and M in solution Eq (18) by implementing the balance rule for the suggested model Eq (15) it implies $M = 1$, hence according to the GKS $N = M + 1 = 2$ thus the solution is

$$R_1(\zeta) = \frac{A_0 + A_1 S(\zeta) + A_2 S^2(\zeta)}{B_0 + B_1 S(\zeta)}. \tag{21}$$

By evaluation R_1, R_1', R_1'', R_1''' and substituting into Eq (15), collecting and equating the coefficients of like powers of $S^i(\zeta)$ to zero; one can obtain a system of equations from which the following two results will be appeared.

$$(1) A_0 \to \frac{\left(-1+\sqrt{3}\right)A_1}{12-8\sqrt{3}}, \quad \mathrm{A}_2 \to \frac{1}{2}\left(-3+\sqrt{3}\right)A_1, \quad \delta \to -4\left(m+n\right),$$
$$\mathrm{B}_1 \to 2\left(3-2\sqrt{3}\right)B_0, \qquad w_1 \to \frac{\left(2+\sqrt{3}\right)\sigma A_1^2}{12 B_0^2}. \tag{22}$$

$$(2) A_0 \to -\frac{\left(1+\sqrt{3}\right)A_1}{12+8\sqrt{3}}, \mathrm{A}_2 \to -\frac{1}{2}\left(3+\sqrt{3}\right)A_1,$$
$$\delta \to -4\left(m+n\right), B_1 \to 2\left(3+2\sqrt{3}\right)B_0, w_1 \to \frac{\left(-3+2\sqrt{3}\right)\sigma A_1^2}{12\sqrt{3}B_0^2}. \tag{23}$$

Now, let us deduce the corresponding solutions to the first result (22) that can be simplified to be

$$A_1 = B_0 = \sigma = \delta = n = 1, A_0 = -0.4, A_2 = -0.634, B_1 = -0.928, w_1 = 0.3, m = -1.25. \tag{24}$$

Substituting from (24) into (21) the solution will be

$$R_1(\zeta) = \frac{-0.4 + S(\zeta) - 0.634 S^2(\zeta)}{1 - 0.928 S(\zeta)}. \tag{25}$$

Using Eq (20) with $C = 1$ the solution after simplifying becomes

$$R_1(\zeta) = \frac{-0.034 + 0.2\mathrm{e}^{\xi} - 0.4\mathrm{e}^{2\xi}}{\left(0.072 + \mathrm{e}^{\xi}\right)\left(1. + \mathrm{e}^{\xi}\right)}. \tag{26}$$

Since $Q(\mathrm{x}, \mathrm{t}) = R_1(\zeta_1) e^{i\psi(x,t)}, \zeta_1 = x + y + w_1 t = x + y + 0.3t, \psi = x + y + \delta t + \theta_0$, then

$$Q(\mathrm{x}, \mathrm{t}) = \frac{-0.034 + 0.2\mathrm{e}^{(x+0.3t)} - 0.4\mathrm{e}^{2^{(x+0.3t)}}}{\left(0.072 + \mathrm{e}^{(x+0.3t)}\right)\left(1 + \mathrm{e}^{(x+0.3t)}\right)} \mathrm{e}^{i(x+t+0.1)}.$$

$$Q(\mathrm{x}, \mathrm{t}) = \frac{-0.034 + 0.2\mathrm{e}^{(x+0.3t)} - 0.4\mathrm{e}^{2^{(x+0.3t)}}}{\left(0.072 + \mathrm{e}^{(x+0.3t)}\right)\left(1 + \mathrm{e}^{(x+0.3t)}\right)} (\cos(\mathrm{x} + \mathrm{t} + 0.1) + \mathrm{i}\sin(\mathrm{x} + \mathrm{t} + 0.1)). \tag{27}$$

$$\operatorname{Re} Q(\mathrm{x}, \mathrm{t}) = \frac{-0.034 + 0.2\mathrm{e}^{(x+0.3t)} - 0.4\mathrm{e}^{2^{(x+0.3t)}}}{\left(0.072 + \mathrm{e}^{(x+0.3t)}\right)\left(1 + \mathrm{e}^{(x+0.3t)}\right)} \cos(\mathrm{x} + \mathrm{t} + 0.1). \tag{28}$$

$$\operatorname{Im} Q(\mathrm{x}, \mathrm{t}) = \frac{-0.034 + 0.2\mathrm{e}^{(x+0.3t)} - 0.4\mathrm{e}^{2^{(x+0.3t)}}}{\left(0.072 + \mathrm{e}^{(x+0.3t)}\right)\left(1 + \mathrm{e}^{(x+0.3t)}\right)} \sin(\mathrm{x} + \mathrm{t} + 0.1). \tag{29}$$

By the same way for the second solution (23)

$$A_1 = B_0 = \sigma = \delta = n = 1, A_0 = -0.1, A_2 = -2.366, B_1 = 12.93, w_1 = 0.02, m = -1.25. \tag{30}$$

Substituting from (30) into (21) the solution will be

$$R_1(\zeta) = \frac{-0.1 + S(\zeta) - 2.366 S^2(\zeta)}{1 + 12.93 S(\zeta)}. \tag{31}$$

Using Eq (20) with $C = 1$ the solution after simplifying becomes

$$R_1(\zeta) = \frac{-1.466 + 0.8 e^{\xi} - 0.1 e^{2\xi}}{\left(1 + e^{\xi}\right)\left(13.93 + e^{\xi}\right)}. \tag{32}$$

Then

$$Q(x,t) = \frac{-1.466 + 0.8 e^{(x+0.02t)} - 0.1 e^{2(x+0.02t)}}{\left(1 + e^{(x+0.02t)}\right)\left(13.93 + e^{(x+0.02t)}\right)} e^{i(x+t+0.1)}.$$

$$Q(x,t) = \frac{-1.466 + 0.8 e^{(x+0.02t)} - 0.1 e^{2(x+0.02t)}}{\left(1 + e^{(x+0.02t)}\right)\left(13.93 + e^{(x+0.02t)}\right)} (\cos(x+t+0.1) + i \sin(x+t+0.1). \tag{33}$$

$$\operatorname{Re} Q(x,t) = \frac{-1.466 + 0.8 e^{(x+0.02t)} - 0.1 e^{2(x+0.02t)}}{\left(1 + e^{(x+0.02t)}\right)\left(13.93 + e^{(x+0.02t)}\right)} \cos(x+t+0.1). \tag{34}$$

$$\operatorname{Im} Q(x,t) = \frac{-1.466 + 0.8 e^{(x+0.02t)} - 0.1 e^{2(x+0.02t)}}{\left(1 + e^{(x+0.02t)}\right)\left(13.93 + e^{(x+0.02t)}\right)} \sin(x+t+0.1). \tag{35}$$

3. The (G'/G)-expansion scheme

The (G'/G)-expansion scheme introduces the solution of Eq (17) to be in the form

$$R(\zeta) = A_0 + \sum_{k=1}^{M} A_k \left[\frac{G'}{G}\right]^k, A_M \neq 0, \tag{36}$$

where the function $G(\zeta)$ achieves the 2nd-order differential equation $G'' + \mu G' + \lambda G = 0,$ that admits the following forms of solutions according to the discriminate of this equation which is either any one of these inequalities $\mu^2 - 4\lambda \succ 0$, $\mu^2 - 4\lambda \prec 0$, and $\mu^2 - 4\lambda = 0$.

(I) When $\mu^2 - 4\lambda \succ 0$, the solution is

$$\left(\frac{G'}{G}\right) = \frac{\sqrt{\mu^2 - 4\lambda}}{2} \left(\frac{l_1 \sinh(\frac{\sqrt{\mu^2 - 4\lambda}}{2})\zeta + l_2 \cosh(\frac{\sqrt{\mu^2 - 4\lambda}}{2})\zeta}{l_1 \cosh(\frac{\sqrt{\mu^2 - 4\lambda}}{2})\zeta + l_2 \sinh(\frac{\sqrt{\mu^2 - 4\lambda}}{2})\zeta} \right) - \frac{\mu}{2}. \tag{37}$$

(II) When $\mu^2 - 4\lambda \prec 0$, the solution is

$$(\frac{G'}{G})=\frac{\sqrt{\mu^2-4\lambda}}{2}\left(\frac{-l_1\sin(\frac{\sqrt{\mu^2-4\lambda}}{2})\zeta+l_2\cos(\frac{\sqrt{\mu^2-4\lambda}}{2})\zeta}{l_1\cos(\frac{\sqrt{\mu^2-4\lambda}}{2})\zeta+l_2\sin(\frac{\sqrt{\mu^2-4\lambda}}{2})\zeta}\right)-\frac{\mu}{2}. \tag{38}$$

(III) When $\mu^2-4\lambda=0$, the solution is

$$(\frac{G'}{G})=(\frac{l_2}{l_1+l_2\zeta})-\frac{\mu}{2}, \tag{39}$$

where M appearing in Eq (36) has been calculated before to be $M=1$, hence the solution is

$$R_1(\zeta)=A_0+A_1(\frac{G'}{G}). \tag{40}$$

By substituting $R_1''', R_1^2R_1', R_1$ into Eq (15), collecting and equating the coefficients of various powers of $(\frac{G'}{G})^i$ to zero, this leads to a system of equations from which the following two results will be emerged

$$(1)A_0=0,\mu=0,w=\frac{2\delta}{-3+2\lambda},A_1=\frac{1.7\sqrt{\delta}}{\sqrt{2\lambda\sigma-3\sigma}},$$
$$m=0.01\left\{\begin{array}{l}-72n-54\delta-\frac{81\delta^2\sigma}{(-3\sigma+2\lambda\sigma)^2}+\frac{54\delta^2\lambda\sigma}{(-3\sigma+2\lambda\sigma)^2}+\frac{432\delta\lambda\sigma^2}{(-3\sigma+2\lambda\sigma)^2}\\-\frac{288\delta\lambda^2\sigma^2}{(-3\sigma+2\lambda\sigma)^2}+\frac{94\delta^{\frac{3}{2}}\sigma}{(-3\sigma+2\lambda\sigma)^{\frac{3}{2}}}-\frac{62\delta^{\frac{3}{2}}\lambda\sigma}{(-3\sigma+2\lambda\sigma)^{\frac{3}{2}}}-\frac{27\delta^2}{(-3\sigma+2\lambda\sigma)}\\-\frac{108\delta\sigma}{(-3\sigma+2\lambda\sigma)}+\frac{216\delta\sigma\lambda}{(-3\sigma+2\lambda\sigma)}-\frac{94\delta^{\frac{3}{2}}}{\sqrt{-3\sigma+2\lambda\sigma}}\end{array}\right\}. \tag{41}$$

$$(2)A_0=0,\mu=0,w=\frac{2\delta}{-3+2\lambda},A_1=\frac{-1.7\sqrt{\delta}}{\sqrt{2\lambda\sigma-3\sigma}},$$
$$m=0.01\left\{\begin{array}{l}-72n-54\delta-\frac{81\delta^2\sigma}{(-3\sigma+2\lambda\sigma)^2}+\frac{54\delta^2\lambda\sigma}{(-3\sigma+2\lambda\sigma)^2}+\frac{432\delta\lambda\sigma^2}{(-3\sigma+2\lambda\sigma)^2}\\-\frac{288\delta\lambda^2\sigma^2}{(-3\sigma+2\lambda\sigma)^2}+\frac{94\delta^{\frac{3}{2}}\sigma}{(-3\sigma+2\lambda\sigma)^{\frac{3}{2}}}-\frac{62\delta^{\frac{3}{2}}\lambda\sigma}{(-3\sigma+2\lambda\sigma)^{\frac{3}{2}}}-\frac{27\delta^2}{(-3\sigma+2\lambda\sigma)}\\-\frac{108\delta\sigma}{(-3\sigma+2\lambda\sigma)}+\frac{216\delta\sigma\lambda}{(-3\sigma+2\lambda\sigma)}-\frac{94\delta^{\frac{3}{2}}}{\sqrt{-3\sigma+2\lambda\sigma}}\end{array}\right\}. \tag{42}$$

$$(3)\ w=-0.2(3\delta+2\sigma A_0^2), \lambda=0, m=0.3(-4n-\delta),$$
$$\mu=\frac{3.5A_0\sqrt{\sigma}\sqrt{-3\delta-2\sigma A_0^2}}{3\delta+2\sigma A_0^2}, A_1=\frac{-0.6\sqrt{-3\delta-2\sigma A_0^2}}{\sqrt{\sigma}}. \tag{43}$$

$$(4)\ w=-0.2(3\delta+2\sigma A_0^2), \lambda=0, m=0.3(-4n-\delta),$$
$$\mu=\frac{3.5A_0\sqrt{\sigma}}{\sqrt{-3\delta-2\sigma A_0^2}}, A_1=\frac{0.6\sqrt{-3\delta-2\sigma A_0^2}}{\sqrt{\sigma}}. \tag{44}$$

Let us now derive the corresponding solutions to the first result that can be simplified to be

$$A_0=0, A_1=1.7, \mu=0, w=1, \sigma=n=\delta=1, \lambda=2, m=-2.14, l_1=1, l_2=2. \tag{45}$$

The solution in the framework of this result can be extracted as follow:

$$(\frac{G'}{G})=\frac{\sqrt{\mu^2-4\lambda}}{2}\left(\frac{-l_1\sin(\frac{\sqrt{\mu^2-4\lambda}}{2})\zeta+l_2\cos(\frac{\sqrt{\mu^2-4\lambda}}{2})\zeta}{l_1\cos(\frac{\sqrt{\mu^2-4\lambda}}{2})\zeta+l_2\sin(\frac{\sqrt{\mu^2-4\lambda}}{2})\zeta}\right)-\frac{\mu}{2}.$$

$$(\frac{G'}{G})=2i\left(\frac{-\sin 2i\,\zeta+2\cos 2i\,\zeta}{\cos 2i\,\zeta+2\sin 2i\,\zeta}\right). \tag{46}$$

$$R_1(\zeta)=A_0+A_1(\frac{G'}{G}).$$

$$R_1(\zeta)=8i\left(\frac{-\sin 2i\,\zeta+2\cos 2i\,\zeta}{\cos 2i\,\zeta+2\sin 2i\,\zeta}\right).$$

$$R_1(\zeta)=8i\left(\frac{-i\sinh 2\zeta+2\cosh 2\zeta}{\cosh 2\zeta+2i\sinh 2\zeta}\right).$$

$$R_1(\zeta)=\left(\frac{8\sinh 2\zeta+16i\cosh 2\zeta}{\cosh 2\zeta+2i\sinh 2\zeta}\right). \tag{47}$$

Hence, $Q(\zeta)=R_1(\zeta_1)e^{i\psi(x,t)}, \zeta_1=x+w_1t,, \psi=x+\delta t+\theta_0.$

$$Q(x,t)=\left(\frac{40\sinh 2\zeta\cosh 2\zeta+16i}{\cosh^2 2\zeta+4\sinh^2 2\zeta}\right)e^{i(x+t+0.1)}. \tag{48}$$

$$Q(x,t)=\left(\frac{40\sinh 2\zeta\cosh 2\zeta+16i}{\cosh^2 2\zeta+4\sinh^2 2\zeta}\right)\left(\cos(x+t+0.1)+i\sin(x+t+0.1)\right). \tag{49}$$

$$\mathrm{Re}Q(x,t)=\left(\frac{\left(40\sinh 2\zeta\cosh 2\zeta\cos(x+t+0.1)-16\sin(x+t+0.1)\right)}{\cosh^2 2\zeta+4\sinh^2 2\zeta}\right). \tag{50}$$

$$\operatorname{Im} Q(x,t) = \left(\frac{\left(40 \sinh 2\zeta \cosh 2\zeta \sin(x+t+0.1) + 16 \cos(x+t+0.1)\right)}{\cosh^2 2\zeta + 4 \sinh^2 2\zeta} \right). \tag{51}$$

4. The EDAS

This technique proposes the solution of Eq (17) to be

$$R_1(\zeta) = \sum_{i=0}^{M} b_i\ \varphi^i(\zeta),\ \ \varphi'^2 = \alpha\varphi^2 + \beta\varphi^3 + \gamma\varphi^4. \tag{52}$$

The solution according to this technique for the suggested model whose balance $M = 1$ is

$$R_1 = b_0 + b_1\varphi. \tag{53}$$

Consequently

$$R_1' = b_1\varphi' = b_1\sqrt{\alpha\varphi^2 + \beta\varphi^3 + \gamma\varphi^4}. \tag{54}$$

$$R_1'' = b_1\varphi'' = b_1(\alpha\varphi + 1.5\beta\varphi^2 + 2\gamma\varphi^3). \tag{55}$$

$$R_1''' = b_1\varphi''' = b_1(\alpha + 3\beta\varphi + 6\gamma\varphi^2)\sqrt{\alpha\varphi^2 + \beta\varphi^3 + \gamma\varphi^4}. \tag{56}$$

By inserting the relations (53)–(56) into Eq (15) we get

$$w_1 R_1'(\alpha + 3\beta\varphi + 6\gamma\varphi^2) + \left(w_1 - \delta - 4(m+n)\right)R_1' - 4\sigma(b_0 + b_1\varphi)^2 R_1' - \left(2(m+n) + \frac{\delta}{2}\right)R_1^2 = 0.$$

Collecting and equating the coefficients of various powers of φ^i to zero we get a system whose unique solution is:

$$\begin{aligned} \alpha &= \frac{(-4\sigma b_1^2 + 6w\gamma)}{w b_1^2(0.5\delta + 2\eta + 2\mu)}\left\{ b_0^2(0.5\delta + 2\eta + 2\mu) + \frac{b_1^2(0.5\delta + 2\eta + 2\mu)}{-4\sigma b_1^2 + 6w\gamma}\left(3w + 3\delta + 4\mu + 4\eta + 4\sigma b_0^2\right)\right\}, \\ \beta &= \frac{0.3(-4\sigma b_1^2 + 6w\gamma)}{w b_1^2(0.5\delta + 2\eta + 2\mu)}\left\{ b_0 b_1(\delta + 4\eta + 4\mu) + \frac{8\sigma b_0 b_1^3(0.5\delta + 2\eta + 2\mu)}{-4\sigma b_1^2 + 6w\gamma}\right\}. \end{aligned} \tag{57}$$

This result can be simplified to be

$$\alpha = 20, \beta = 3, \gamma = 1, \sigma = w = \delta = \mu = \eta = b_0 = b_1 = 1. \tag{58}$$

Thus the solution is $R_1 = b_0 + b_1\varphi$, where φ can be derived from the relation

$$\varphi' = \sqrt{20\varphi^2 + 3\varphi^3 + \gamma\varphi^4}.$$

To be

$$\varphi=\frac{2\sqrt{20}\left(e^{-\sqrt{20}\zeta}-\frac{1.5}{\sqrt{20}}\right)}{1-\left(e^{-\sqrt{20}\zeta}-\frac{1.5}{\sqrt{20}}\right)^{2}}. \tag{59}$$

$$R_{1}=1+\frac{2\sqrt{20}\left(e^{-\sqrt{20}\zeta}-\frac{1.5}{\sqrt{20}}\right)}{1-\left(e^{-\sqrt{20}\zeta}-\frac{1.5}{\sqrt{20}}\right)^{2}}. \tag{60}$$

This result generates two results; we will implement the identical solution for one of them which is:

$$R_{1}=1+\frac{1+\sqrt{1+2\left(e^{-2\sqrt{2}\zeta}-\sqrt{2}e^{-\sqrt{2}\zeta}-1.5\right)}}{\left(e^{-2\sqrt{2}\zeta}-\sqrt{2}e^{-\sqrt{2}\zeta}-1.5\right)}.$$

$$Q(x,t)=\left\{1+\frac{2\sqrt{20}\left(e^{-\sqrt{20}\zeta}-\frac{1.5}{\sqrt{20}}\right)}{1-\left(e^{-\sqrt{20}\zeta}-\frac{1.5}{\sqrt{20}}\right)^{2}}\right\}e^{i(x+t+0.1)}. \tag{61}$$

$$\operatorname{Re}Q(x,t)=\left\{1+\frac{2\sqrt{20}\left(e^{-\sqrt{20}\zeta}-\frac{1.5}{\sqrt{20}}\right)}{1-\left(e^{-\sqrt{20}\zeta}-\frac{1.5}{\sqrt{20}}\right)^{2}}\right\}\cos(x+t+0.1). \tag{62}$$

$$\operatorname{Im}Q(x,t)=\left\{1+\frac{2\sqrt{20}\left(e^{-\sqrt{20}\zeta}-\frac{1.5}{\sqrt{20}}\right)}{1-\left(e^{-\sqrt{20}\zeta}-\frac{1.5}{\sqrt{20}}\right)^{2}}\right\}\sin(x+t+0.1). \tag{63}$$

The other solutions corresponding to the other results can be extracted in the same connection.

5. Conclusions

Throughout this work, three various schemas have been introduced to obtain new perceptions of the soliton solutions for the Myrzakulov-Lakshmanan XXXII-equation. These three schemas are the GKS, the (G'/G)-expansion schema and the EDAS. These suggested schemas have been implemented in the same way and are parallel. All these schemas have been used for the first time for this target. Each schema archives many forms of results from which we choose only one result and design the corresponding solution. The designed solutions using these three schemas have been established. The obtained solutions appear in many forms as M-shaped soliton solutions, W-shaped soliton solutions

Figures 1–4, kink soliton solution Figure 5, bright soliton solution Figure 6 and hyperbolic function Figures 7 and 8. Our achieved soliton solutions in the framework of any other methods have not been achieved before. The realized soliton solutions are new compared with [12], who solved the suggested model numerically using the N-fold Darboux Transformation method; hence, the novelty of our obtained solutions is clear. The 2D, 3D soliton behaviors that describe the dynamic properties for all achieved soliton solutions that have emerged from the suggested model have been configured.

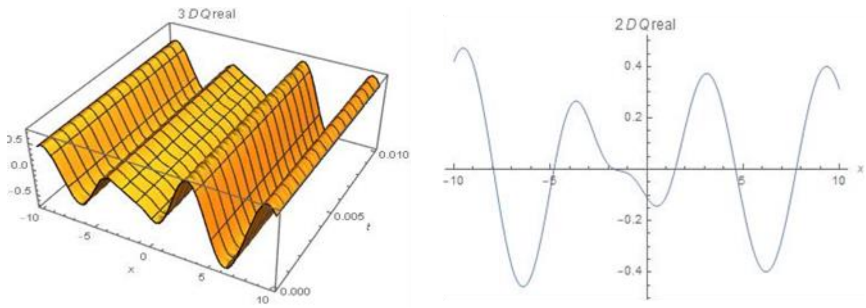

Figure 1. The soliton behavior of the Re. Part Eq (28) in 2D and 3D with values: $A_1 = B_0 = \sigma = \delta = n = 1, A_0 = -0.4, A_2 = -0.634, B_1 = -0.928, w_1 = 0.3, m = -1.25.$

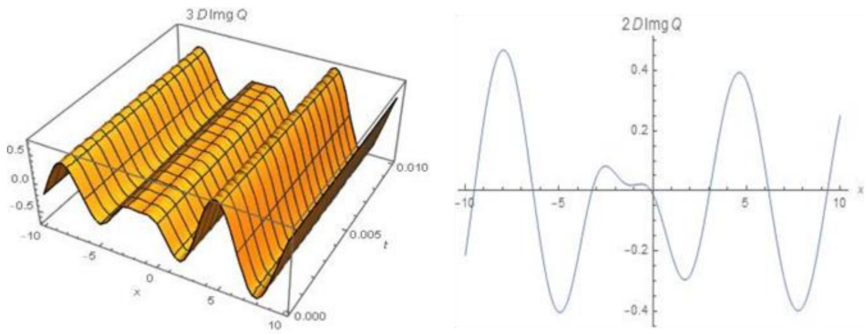

Figure 2. The soliton behavior of the Im. Part Eq (29) in 2D and 3D with values: $A_1 = B_0 = \sigma = \delta = n = 1, A_0 = -0.4, A_2 = -0.634, B_1 = -0.928, w_1 = 0.3, m = -1.25.$

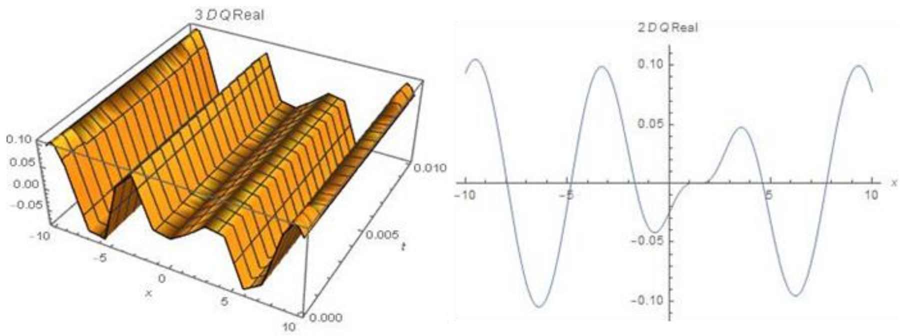

Figure 3. The soliton behavior of the Re. Part Eq (34) in 2D and 3D with values: $A_1 = B_0 = \sigma = \delta = n = 1, A_0 = -0.1, A_2 = -2.366, B_1 = 12.93, w_1 = 0.02, m = -1.25.$

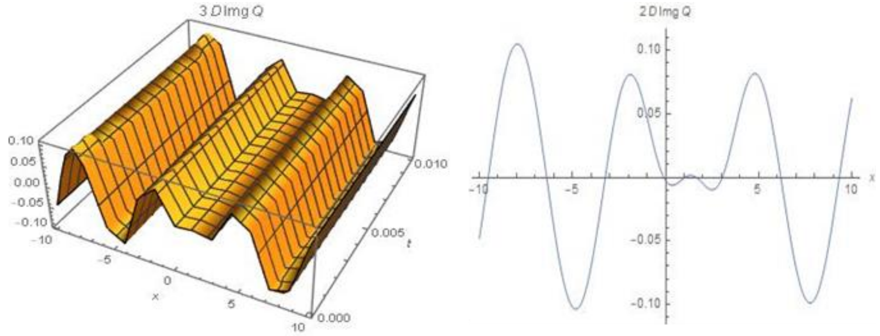

Figure 4. The soliton behavior of the Im. Part Eq (35) in 2D and 3D with values: $A_1 = B_0 = \sigma = \delta = n = 1, A_0 = -0.1, A_2 = -2.366, B_1 = 12.93, w_1 = 0.02, m = -1.25.$

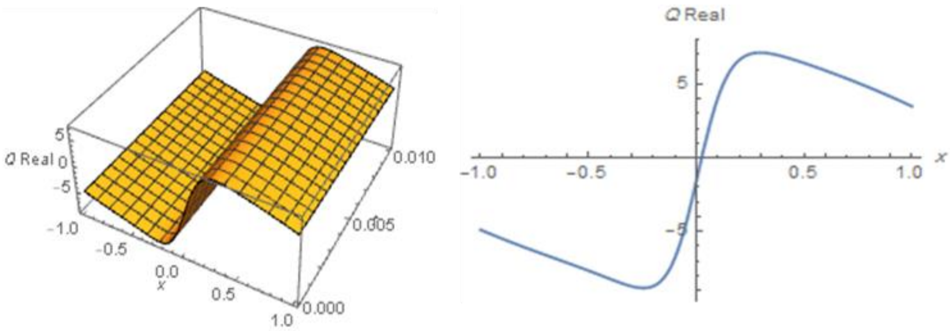

Figure 5. The soliton behavior of the Re. Part Eq (50) in 2D and 3D with values: $A_0 = 0, A_1 = 1.7, \mu = 0, w = 1, \sigma = n = \delta = 1, \lambda = 2, m = -2.14, l_1 = 1, l_2 = 2.$

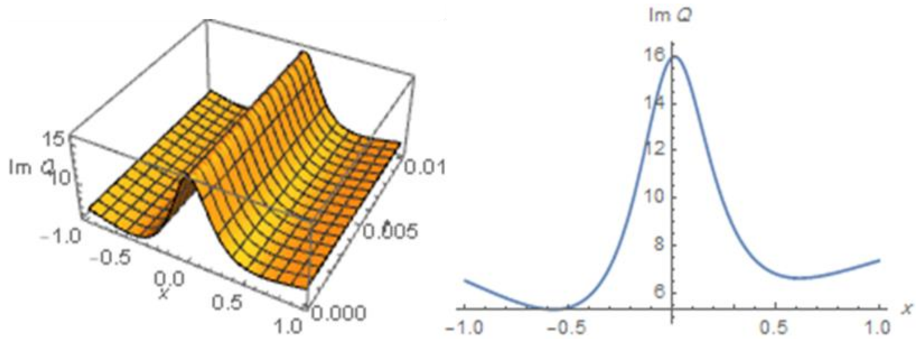

Figure 6. The soliton behavior of the Im. Part Eq (51) in 2D and 3D with values: $A_0 = 0, A_1 = 1.7, \mu = 0, w = 1, \sigma = n = \delta = 1, \lambda = 2, m = -2.14, l_1 = 1, l_2 = 2.$

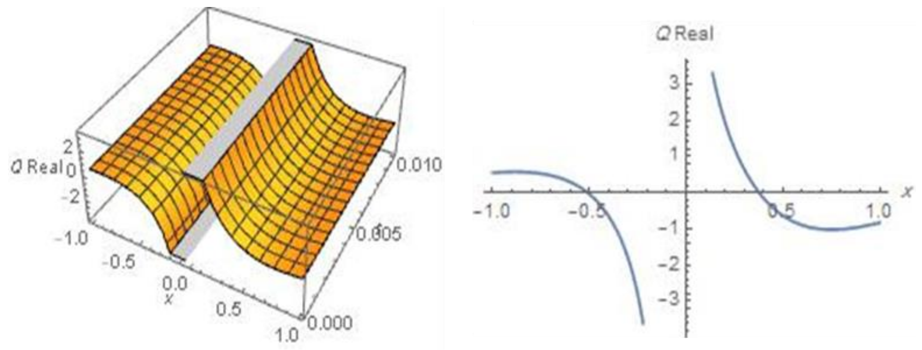

Figure 7. The soliton behavior of the Re. Part Eq (62) in 2D and 3D with values: $\alpha = 20, \beta = 3, \gamma = 1, \sigma = w = \delta = \mu = \eta = b_0 = b_1 = 1.$

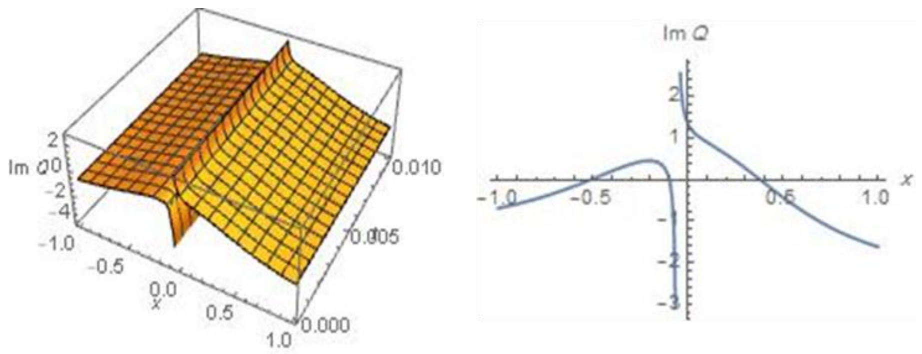

Figure 8. The soliton behavior of the Im. Part Eq (63) in 2D and 3D with values: $\alpha = 20, \beta = 3, \gamma = 1, \sigma = w = \delta = \mu = \eta = b_0 = b_1 = 1.$

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflict of interest.

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