AIMS Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 9(2): 4990–5014.
DOI: 10.3934/math.2024242
Received: 08 December 2023
Revised: 08 January 2024
Accepted: 12 January 2024
Published: 24 January 2024

Research article

A new similarity function for Pythagorean fuzzy sets with application in football analysis

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Abstract: The idea of Pythagorean fuzzy sets (PFSs) has been extensively applied in various decision-making scenarios. Many of the applications of PFSs were carried out based on similarity functions. Some methods of similarity functions for PFSs (SFPFSs) cannot be trusted for a reliable interpretations in practical cases due to some of their setbacks. In this work, a new method of SFPFSs is developed with the capacity to outsmart the efficiency of the extant SFPFSs in terms of precise results and appropriately satisfying the rules of SFs. The new method is described with some results to validate the properties of SFs. In terms of practical application, we use the newly developed method of SFPFSs to discuss the relationship between the players of the Liverpool Football Club (FC) in the 2022/2023 English Premier League (EPL) season to assess their performances in their resurgent moments within the season. Using data from BBC Sport analysis (BBCSA) on the players' rating per match in a Pythagorean fuzzy setting, we establish the players' interactions, communications, passing, contributions, and performances to ascertain the high ranking players based on performances. Similarly, a comparative analyses are presented in tables to undoubtedly express the superiority of the newly developed method of SFPFSs. Due to the flexibility of the newly developed method of SFPFSs, it can be used for clustering analysis. In addition, the new method of SFPFSs can be extended to other uncertain environments other than PFSs.

Keywords: football analysis; Pythagorean fuzzy sets; similarity function; multi-criteria decision-

making approach
Mathematics Subject Classification: 03E72, 20N20, 47S40

1. Introduction

Football analysis is a decision-making aspect that uses data and video to analyze the performance of players and teams by using football analytics metrics like goal threat metrics (i.e., expected goals), creativity metrics (i.e., expected assists, shot creating actions, and goal creating actions), and possession metrics (i.e., passes per defensive action, progressive distance, team sequences). In a way, football analysis is a decision-making problem. Many human decision-making problems are predominantly carried out with the aid of a decision-making method like multi-criteria decision-making (MCDM), especially, in tasks concerning systems involving large-scale. In most cases, a decision-making process has imprecisions and uncertainties. The fuzzy set, represented as $\mathfrak{F}$, was developed by Zadeh [1] in terms of membership degree (MD), denoted by σ to offer some relief to the solution of decision-making under uncertain conditions. But, $\mathfrak{F}$ lacks the ability to tackle imprecision because it does not consider non-membership degree (NMD), denoted by δ, and, hesitation parameter denoted by η.

By incorporating δ (i.e., $\delta \neq 1-\sigma$) and η (i.e., $1-\sigma-\delta$), Atanassov [2] developed a structure called intuitionistic fuzzy sets (IFSs) to perfect the limitation of $\mathfrak{F}$, and enable the tackling of decision-making imprecision. IFS has been industrious in many real-world problems of decision-making diagnosis of disease [3–6], pattern and clustering analysis [7–13], and using different approaches like distance operators, similarity operators, correlation operators, and aggregation operators to discuss various real-life problems [14–16].

Though IFS is very applicable, there are some cases where IFSs cannot be utilized. To be specific, when the aggregate of σ and δ exceeds unity, IFS loses its usefulness. To resolve this issue, Atanassov [17] developed a structure called IFS of the second type, popularly known as Pythagorean fuzzy sets (PFSs), as noted in [18]. In PFS, the aggregate of σ and δ can exceed unity with $\sigma^2+\delta^2 \leq 1$. Some decision problems via the concepts of correlation operators and partial correlation operator, under PFSs have been studied with decision-making applications [19–21]. Certain applications of PFSs have been discussed based on similarity/distance operators for decision-making [22–25], disease diagnosis [26], and pattern recognition [27–31]. Because of the flexibility of PFSs in the discussion of complex real-life problems, the idea has been applied in pattern classification [32], Frank power aggregation operators [33], predicting maternal outcomes [34], disaster control [35], and selection process [36–38], etc.

The idea of PFSs has been applied to discuss various MCDM methods. The study of MCDM under PFSs was initiated, and certain real-world problems were discussed via the approach used in [39]. In addition, some decision problems via MCDM were explicated under PFSs in [40, 41]. In [42], decision making was executed under PFSs using multiobjective optimization on the basis of ratio analysis (MOORA) for MCDM, and Huang et al. [43] used distance measure and score function under PFSs to discuss MOORA in MCDM. Wang et al. [44] discussed a Pythagorean fuzzy (PF) MCDM with a MOORA-Borda method in the evaluation of ecological governance. Gocer and Buyukozkan [45] discussed an extension of PF MULTIMOORA and used it to discuss new product development. Akram et al. [46] also extended the PF MULTIMOORA approach via 2-tuple linguistic PFSs to discuss multi-attribute group decision-making (MAGDM). Other PF MCDM methods were discussed in [47–49].

Moreover, SFPFSs is quite flexible and has been applied in so many areas. Wei et al. [50] developed a method of SFPFS, based on the cosine metric and applied it to decision-making. The approach violated the metric condition of similarity if the PFSs are indistinguishable, because instead of the similarity being 1, the approach yields 0.3333 for $n = 3$, where n is the cardinality of the underlying set. Hussain and Yang [51] constructed a similarity operator for PFSs established on the Hausdorff metric and applied it to discuss fuzzy TOPSIS. The approach in [51] satisfies similarity metric conditions but, does not include the hesitation margin, and hence renders the approach inappropriate. In [52], four approaches of measuring SFPFSs were developed and while the first two do not consider the hesitation margin, the other two considered the all of the parameters of PFSs to circumvent any error that may stems from omission. However, the approaches yield an indistinguishable similarity value (i.e., 0.5 for $n = 2$) in the case of identical PFSs, which is a violation of the metric axioms. In [53], an approach for measuring SFPFSs, which generalized the approach in [50], was developed and applied to MCDM problem. But, the approach yields 0.5 for $n = 2$ in the case of equal PFSs, which is a violation of the metric axioms. Similarly, an approach for similarity measure between PFSs was developed in [54] using the tangent function, but yields an inappropriate similarity value (i.e., 0.5 for $n = 2$) if the PFSs are indistinguishable, which is a violation of the metric axioms and thus renders the approach ineffective.

The itemized approaches of SFPFSs [50–54] are defective with respects to the metric conditions of SF. The two approaches in [52] left out Pythagorean fuzzy hesitation margin (PFHM) from the computations. The approaches in [50, 52–54] absolutely violated the metric conditions for similarity function if the considered PFSs are identical. Although the approach in [51] satisfied the metric conditions satisfactorily, it excludes the PFHM of the PFSs under consideration. The setbacks in these extant methods of SFPFSs constitute the motivation for the development of a new method of SFPFS.

Oftentimes, football analysis is provided on each player immediately after a match is played, and this analysis is mostly challenged by the coaches and football fans due to some uncertainties and imprecisions beyond the control of the analysts. Because of the flexibility and reliability of PFSs in curbing uncertainties and imprecisions, it is expedient to apply SFPFS to discuss football analysis. In addition, a careful study of the applications of PFSs shows that PFSs have not been applied for the purpose of football analysis. To this end, this paper constructs a new similarity function for PFSs with application to football analysis using the case of the Liverpool FC in the 2022/2023 EPL season. The study uses data from BBC Sport analysis of each of the players in some of the matches played by Liverpool FC. The contributions of the work consist of the following;

- Construction of a new similarity function for measuring similarity between PFSs,
- Description of the new similarity function for PFSs in alliance with similarity metric conditions,
- Development of a new application area for PFSs in the analysis of the performance of Liverpool FC in the 2022/2023 EPL season based on the MCDM method via the new similarity function, and
- Comparative analysis of the new similarity function under PFSs with extant similarity approaches [50–54] to authenticate the new similarity approach.

The data for the work is collected from the BBC Sport analyses for the considered number of matches played by the Liverpool team in the 2022/2023 EPL season. After collection, the data is converted to PF data to enhance the encapsulation of uncertainties and imprecisions of the analysts. For the conversion, each MD is the allocated value by the analysts, each NMD is 1 − MD from the corresponding MD, and each HM is computed using HM = $(1 - \mathrm{MD}^2 - \mathrm{NMD}^2)^{0.5}$. The structure of the rest of the paper is as follows: Section 2 discusses some properties of PFSs, outlines some existing similarity functions under PFSs, and identifies their setbacks; Section 3 presents the new similarity function for PFSs and outlines its properties; Section 4 presents the new application of PFSs in football analysis based on the new similarity function to determine the contributions of the eleven frequently used players and, equally, presents comparative studies to showcase the preeminence of the new similarity function over the extant similarity functions; and Section 5 recaps the paper and provides some recommendations.

2. Preliminaries

We reiterate the idea of PFSs and some extant similarity functions between PFSs.

2.1. Pythagorean fuzzy sets

We take S to be the universe of discourse in the work.

Definition 2.1. Consider the structure

$$\aleph = \{\langle s_j, \sigma_{\aleph}(s_j), \delta_{\aleph}(s_j)\rangle \mid s_j \in S\},$$

in which case, $\sigma_{\aleph},\ \delta_{\aleph} \colon X \to [0,1]$ signify MD and NMD of $s_j \in S$.

i) $\aleph$ is called an IFS in S if $\left(\sigma_{\aleph}(s_j)+\delta_{\aleph}(s_j)\right) \in [0,1]$, and $\eta_{\aleph}(s_j) = 1-\sigma_{\aleph}(s_j)-\delta_{\aleph}(s_j)$ is the hesitation margin of $\aleph$ [2].
ii) $\aleph$ is called a PFS in S if $\left(\sigma^2_{\aleph}(s_j) + \delta^2_{\aleph}(s_j)\right) \in [0,1]$, and $\eta_{\aleph}(s_j) = \left(1 - \sigma^2_{\aleph}(s_j) - \delta^2_{\aleph}(s_j)\right)^{0.5}$ is the hesitation margin of $\aleph$ [18].

PFS $\aleph$ can also be represented by $\aleph = \left(\sigma_{\aleph}(s_j), \delta_{\aleph}(s_j)\right)$, called the Pythagorean fuzzy number (PFN).

Definition 2.2 ([18]). Assume that $\aleph$, $\aleph_1$, $\aleph_2$, and $\aleph_3$ are PFSs in S. Then,

i) equality
$$\aleph_1 = \aleph_2 \text{ iff } \sigma_{\aleph_1}(s_j) = \sigma_{\aleph_2}(s_j) \text{ and } \delta_{\aleph_1}(s_j) = \delta_{\aleph_2}(s_j), \forall s_j \in S.$$
ii) inclusion
$$\aleph_1 \subseteq \aleph_2 \text{ iff } \sigma_{\aleph_1}(s_j) \le \sigma_{\aleph_2}(s_j) \text{ and } \delta_{\aleph_1}(s_j) \ge \delta_{\aleph_2}(s_j), \forall s_j \in S.$$
iii) complement
$$\overline{\aleph} = \{\langle s_j, \delta_{\aleph}(s_j), \sigma_{\aleph}(s_j)\rangle | s_j \in S\}.$$
iv) union
$$\aleph_1 \cup \aleph_2 = \{\langle s_j, \max\{\sigma_{\aleph_1}(s_j), \sigma_{\aleph_2}(s_j)\}, \min\{\delta_{\aleph_1}(s_j), \delta_{\aleph_2}(s_j)\}\rangle | s_j \in S\}.$$
v) intersection
$$\aleph_1 \cap \aleph_2 = \{\langle s_j, \min\{\sigma_{\aleph_1}(s_j), \sigma_{\aleph_2}(s_j)\}, \max\{\delta_{\aleph_1}(s_j), \delta_{\aleph_2}(s_j)\}\rangle | s_j \in S\}.$$

Definition 2.3 ([23]). If $\aleph$, $\aleph_1$, and $\aleph_2$ are PFSs in S, then the SFPFS represented by $\Upsilon(\aleph_1, \aleph_2)$ is Υ: $PFS \times PFS \to [0, 1]$, satisfying:

i) $\Upsilon(\aleph_1, \aleph_2) \in [0, 1]$.
ii) $\Upsilon(\aleph_1, \aleph_2) = 1 \Leftrightarrow \aleph_1 = \aleph_2$.
iii) $\Upsilon(\aleph_1, \aleph_2) = \Upsilon(\aleph_2, \aleph_1)$.
iv) $\Upsilon(\aleph_1, \aleph) \leq \Upsilon(\aleph_1, \aleph_2) + \Upsilon(\aleph_2, \aleph)$.

Table 1 explains the nature of the similarity function.

Table 1. Nature of $\Upsilon(\aleph_1, \aleph_2)$.

Nature	Interpretations
$\Upsilon(\aleph_1, \aleph_2) = 0$	$\aleph_1$ and $\aleph_2$ have no similarity
$\Upsilon(\aleph_1, \aleph_2) = 1$	$\aleph_1$ and $\aleph_2$ have perfect similarity
$\Upsilon(\aleph_1, \aleph_2) \approx 0$	$\aleph_1$ and $\aleph_2$ have no significant similarity
$\Upsilon(\aleph_1, \aleph_2) \approx 1$	$\aleph_1$ and $\aleph_2$ have significant similarity

2.2. *Extant methods of SFPFSs*

Assume we have two PFSs

$$\aleph_1 = \{\langle s_j, \sigma_{\aleph_1}(s_j), \delta_{\aleph_1}(s_j)\rangle | s_j \in S\}$$

and

$$\aleph_2 = \{\langle s_j, \sigma_{\aleph_2}(s_j), \delta_{\aleph_2}(s_j)\rangle | s_j \in S\}$$

for $S = \{s_1, s_2, \cdots, s_k\}$. Let us assume:

$$\wp_1 = \sigma_{\aleph_1}(s_j) - \sigma_{\aleph_2}(s_j),\ \wp_2 = \delta_{\aleph_1}(s_j) - \delta_{\aleph_2}(s_j), \wp_3 = \eta_{\aleph_1}(s_j) - \eta_{\aleph_2}(s_j),$$

$$\tilde{\wp}_1 = \sigma^2_{\aleph_1}(s_j) - \sigma^2_{\aleph_2}(s_j),\ \tilde{\wp}_2 = \delta^2_{\aleph_1}(s_j) - \delta^2_{\aleph_2}(s_j), \tilde{\wp}_3 = \eta^2_{\aleph_1}(s_j) - \eta^2_{\aleph_2}(s_j).$$

The following are some extant methods of finding similarity for PFSs:

1) Similarity function in [50]

$$\Upsilon_1(\aleph_1, \aleph_2) = \frac{1}{k}\Sigma_{j=1}^{k} \cos\Big[\frac{\pi}{4}(|\tilde{\wp}_1| + |\tilde{\wp}_2| + |\tilde{\wp}_3|)\Big]. \tag{2.1}$$

This method violates the rule of similarity function. For example, while computing the similarity between $\aleph_1$ and $\aleph_2$ in $S = \{s_1, s_2, s_3\}$, if $\aleph_1 = \aleph_2$, then we see that

$$\Upsilon_1(\aleph_1, \aleph_2) = \frac{\cos 0}{3} = 0.3333,$$

which disagrees with $\Upsilon(\aleph_1, \aleph_2) = 1 \Leftrightarrow \aleph_1 = \aleph_2$. Thus, the method [50] is not a reliable similarity measure.

2) Similarity function in [51]

$$\Upsilon_2(\aleph_1, \aleph_2) = \frac{1 - \Delta(\aleph_1, \aleph_2)}{1 + \Delta(\aleph_1, \aleph_2)}, \tag{2.2}$$

where

$$\Delta(\aleph_1,\aleph_2) = \frac{1}{k}\Sigma_{j=1}^{k} \max\{|\tilde{\wp}_1|,|\tilde{\wp}_2|\}.$$

Though this approach fulfills the rules of the similarity function, it does not into take account the hesitation margins. Thus, its results cannot be trusted.

3) Similarity functions in [52]

$$\Upsilon_3(\aleph_1,\aleph_2) = \frac{1}{k}\Sigma_{j=1}^{k}\Big[2^{1-\frac{1}{2}\left(|\tilde{\wp}_1|+|\tilde{\wp}_2|\right)} - 1\Big], \tag{2.3}$$

$$\Upsilon_4(\aleph_1,\aleph_2) = \frac{1}{k}\Sigma_{j=1}^{k}\Big[2^{1-\max\{|\tilde{\wp}_1|,|\tilde{\wp}_2|\}} - 1\Big], \tag{2.4}$$

$$\Upsilon_5(\aleph_1,\aleph_2) = \frac{1}{k}\Sigma_{j=1}^{k}\Big[2^{1-\frac{1}{2}\left(|\tilde{\wp}_1|+|\tilde{B\wp}_2|+|\tilde{\wp}_3|\right)} - 1\Big], \tag{2.5}$$

$$\Upsilon_6(\aleph_1,\aleph_2) = \frac{1}{k}\Sigma_{j=1}^{k}\Big[2^{1-\max\{|\tilde{\wp}_1|,|\tilde{\wp}_2|,|\tilde{\wp}_3|\}} - 1\Big]. \tag{2.6}$$

Methods (2.3) and (2.5) are approximately the same if the values of the hesitation margins are negligible. It is likewise for (2.4) and (2.6). The similarity approaches in (2.3) and (2.4) do not take into account the hesitation margins, and so the approaches are not appropriate. In addition, if $\aleph_1$ and $\aleph_2$ are PFSs in $S = \{s_1, s_2\}$, and $\aleph_1 = \aleph_2$, then (2.3)–(2.6) yield

$$\left.\begin{array}{l} \Upsilon_3(\aleph_1,\aleph_2) = \Upsilon_4(\aleph_1,\aleph_2) = \Upsilon_5(\aleph_1,\aleph_2) \\ \Upsilon_6(\aleph_1,\aleph_2) = 0.5 \end{array}\right\},$$

which contradict the similarity maxim (i.e., $\Upsilon(\aleph_1,\aleph_2) = 1 \Leftrightarrow \aleph_1 = \aleph_2$). Hence, the approaches [52] are not reliable.

4) Similarity function in [53]

$$\Upsilon_7(\aleph_1,\aleph_2) = \frac{1}{k}\Sigma_{j=1}^{k} \cos\Big[\frac{\pi}{2}\Big(\frac{|\tilde{\wp}_1|^q + |\tilde{\wp}_2|^q + |\tilde{\wp}_3|^q}{2}\Big)^{\frac{1}{q}}\Big], \tag{2.7}$$

where $q \geq 1$ is the L_q norm. To verify the appropriateness of this function, we assume there are two equal PFSs $\aleph_1$ and $\aleph_2$ in $S = \{s_1, s_2, s_3\}$, and so (2.7) yields

$$\Upsilon_7(\aleph_1,\aleph_2) = \frac{\cos 0}{3} = 0.3333, \tag{2.8}$$

which contradicts the similarity maxim of separability (i.e., $\Upsilon(\aleph_1,\aleph_2) = 1 \Leftrightarrow \aleph_1 = \aleph_2$). Hence, the results from this approach cannot be reliable.

5) Similarity function in [54]

$$\Upsilon_8(\aleph_1,\aleph_2) = \frac{1}{k}\Sigma_{j=1}^{k}\Big[1 - \tan\frac{\pi}{8}(|\tilde{\wp}_1| + |\tilde{\wp}_2|)\Big]. \tag{2.9}$$

The output from this approach [54] is not reliable. To see this, assume there are two equal PFSs $\aleph_1$ and $\aleph_2$ in $S = \{s_1, s_2\}$, and so we have

$$\Upsilon_8(\aleph_1,\aleph_2) = \frac{1}{2} = 0.5,$$

and so $\Upsilon_8(\aleph_1,\aleph_2) \neq 1$. In addition, the approach also omits the hesitation margins.

3. New similarity function for PFSs

Due to the setbacks in the discussed extant methods of SFPFSs, we are motivated to develop a new method of SFPFSs which is well constructed without the exclusion of any parameters, satisfies the similarity maxims, and possesses better accuracy.

Definition 3.1. Given we have two PFSs

$$\aleph_1 = \{\langle s_j, \sigma_{\aleph_1}(s_j), \delta_{\aleph_1}(s_j)\rangle | s_j \in S\} \text{ and}$$

$$\aleph_2 = \{\langle s_j, \sigma_{\aleph_2}(s_j), \delta_{\aleph_2}(s_j)\rangle | s_j \in S\}$$

for feature space $S = \{s_1, s_2, \cdots, s_k\}$, we define the new similarity function for $\aleph_1$ and $\aleph_2$ as follows;

$$\Upsilon(\aleph_1, \aleph_2) = \frac{3k - \left(|\sigma^2_{\aleph_1} - \sigma^2_{\aleph_2}| + |\delta^2_{\aleph_1} - \delta^2_{\aleph_2}| + |\eta^2_{\aleph_1} - \eta^2_{\aleph_2}|\right)}{3k + \left(|\sigma^2_{\aleph_1} - \sigma^2_{\aleph_2}| + |\delta^2_{\aleph_1} - \delta^2_{\aleph_2}| + |\eta^2_{\aleph_1} - \eta^2_{\aleph_2}|\right)}, \tag{3.1}$$

where

$$\left(|\sigma^2_{\aleph_1} - \sigma^2_{\aleph_2}| + |\delta^2_{\aleph_1} - \delta^2_{\aleph_2}| + |\eta^2_{\aleph_1} - \eta^2_{\aleph_2}|\right) = \sum_{j=1}^{k} \left(|\sigma^2_{\aleph_1}(s_j) - \sigma^2_{\aleph_2}(s_j)| + |\delta^2_{\aleph_1}(s_j) - \delta^2_{\aleph_2}(s_j)| + |\eta^2_{\aleph_1}(s_j) - \eta^2_{\aleph_2}(s_j)|\right).$$

Now, we find the similarities between three PFSs to ascertain the superiority of the new similarity function over the other similarity functions [50–54].

Example 3.1. Suppose there are three PFSs $\aleph_1$, $\aleph_2$, and $\aleph_3$ defined in $S = \{s_1, s_2, s_3\}$ as follows;

$$\aleph_1 = \{\langle s_1, 0, 1\rangle, \langle s_2, 1, 0\rangle, \langle s_3, 0.5, 0.7\rangle\},$$

$$\aleph_2 = \{\langle s_1, 1, 0\rangle, \langle s_2, 0, 1\rangle, \langle s_3, 0.45, 0.68\rangle\},$$

$$\aleph_3 = \{\langle s_1, 0.99, 0\rangle, \langle s_2, 0.98, 0\rangle, \langle s_3, 0.55, 0.69\rangle\}.$$

By using the new similarity method and the similarity methods in [50–54], we obtain Table 2.

Table 2. Results of similarity methods.

Similarity Methods	$(\aleph_1, \aleph_1)$	$(\aleph_2, \aleph_2)$	$(\aleph_3, \aleph_3)$	$(\aleph_1, \aleph_2)$	$(\aleph_1, \aleph_3)$	$(\aleph_2, \aleph_3)$
Υ_1 [50]	0.3333	0.3333	0.3333	−0.0550	−0.2073	0.0259
Υ_2 [51]	1	1	1	0.1887	0.4662	0.4563
Υ_3 [52]	0.3333	0.3333	0.3333	−0.1709	−0.0098	−0.0107
Υ_4 [52]	0.3333	0.3333	0.3333	−0.1721	−0.0206	−0.0266
Υ_5 [52]	0.3333	0.3333	0.3333	−0.1751	−0.0206	−0.0295
Υ_6 [52]	0.3333	0.3333	0.3333	−0.1721	−0.0163	−0.0180
Υ_7 [53]	0.3333	0.3333	0.3333	−0.0605	−0.0810	0.2190
Υ_8 [54]	0.3333	0.3333	0.3333	0.1132	0.3963	0.3363
Υ	1	1	1	0.3688	0.6094	0.5976

The results in Table 2 show that the similarity methods in [50,52–54] could not satisfy the similarity metric condition in the sense that, $\Upsilon(\aleph_1, \aleph_1) \neq 1$, $\Upsilon(\aleph_2, \aleph_2) \neq 1$, and $\Upsilon(\aleph_3, \aleph_3) \neq 1$, though the PFSs are equal. On the contrary, the new similarity method and the method in [51] fulfill this condition. In addition, the similarity methods in [50, 52–54] yield similarity values outside the unit interval [0, 1], which is again a violation of the similarity metric condition. In this case, we conclude that:

- the similarity methods in [50, 52–54] are not appropriate similarity methods, and
- the new similarity method yields the most accurate results by comparison to the similarity methods in [50–54].

Next, we consider some of the properties of the new similarity function for PFSs to show its alignment with the similarity metric conditions.

Theorem 3.1. *The similarity function $\Upsilon(\aleph_1, \aleph_2)$ of PFSs $\aleph_1$ and $\aleph_2$ in $S = \{s_1, s_2, \cdots, s_k\}$ are symmetric and separable.*

Proof. To verify the symmetric nature of $\Upsilon(\aleph_1, \aleph_2)$, we show that $\Upsilon(\aleph_1, \aleph_2) = \Upsilon(\aleph_2, \aleph_1)$. Thus,

$$\begin{aligned}\Upsilon(\aleph_1, \aleph_2) &= \frac{3k - \left(|\sigma^2_{\aleph_1} - \sigma^2_{\aleph_2}| + |\delta^2_{\aleph_1} - \delta^2_{\aleph_2}| + |\eta^2_{\aleph_1} - \eta^2_{\aleph_2}|\right)}{3k + \left(|\sigma^2_{\aleph_1} - \sigma^2_{\aleph_2}| + |\delta^2_{\aleph_1} - \delta^2_{\aleph_2}| + |\eta^2_{\aleph_1} - \eta^2_{\aleph_2}|\right)} \\ &= \frac{3k - \left(|\sigma^2_{\aleph_2} - \sigma^2_{\aleph_1}| + |\delta^2_{\aleph_2} - \delta^2_{\aleph_1}| + |\eta^2_{\aleph_2} - \eta^2_{\aleph_1}|\right)}{3k + \left(|\sigma^2_{\aleph_2} - \sigma^2_{\aleph_1}| + |\delta^2_{\aleph_2} - \delta^2_{\aleph_1}| + |\eta^2_{\aleph_2} - \eta^2_{\aleph_1}|\right)},\end{aligned}$$

i.e., $\Upsilon(\aleph_1, \aleph_2) = \Upsilon(\aleph_2, \aleph_1)$ since

$$\begin{aligned}|\sigma^2_{\aleph_2} - \sigma^2_{\aleph_1}| &= |-(\sigma^2_{\aleph_2} - \sigma^2_{\aleph_1})|, \\ |\delta^2_{\aleph_2} - \delta^2_{\aleph_1}| &= |-(\delta^2_{\aleph_2} - \delta^2_{\aleph_1})|, \\ |\eta^2_{\aleph_2} - \eta^2_{\aleph_1}| &= |-(\delta^2_{\aleph_2} - \delta^2_{\aleph_1})|.\end{aligned}$$

Next, we verify separability, i.e., we show that $\Upsilon(\aleph_1, \aleph_2) = 1$ iff $\aleph_1 = \aleph_2$. Suppose that $\Upsilon(\aleph_1, \aleph_2) = 1$. Then, we have

$$3k - \left(|\sigma^2_{\aleph_1} - \sigma^2_{\aleph_2}| + |\delta^2_{\aleph_1} - \delta^2_{\aleph_2}| + |\eta^2_{\aleph_1} - \eta^2_{\aleph_2}|\right) = 3k + \left(|\sigma^2_{\aleph_1} - \sigma^2_{\aleph_2}| + |\delta^2_{\aleph_1} - \delta^2_{\aleph_2}| + |\eta^2_{\aleph_1} - \eta^2_{\aleph_2}|\right),$$

i.e.,

$$2\left(|\sigma^2_{\aleph_1} - \sigma^2_{\aleph_2}| + |\delta^2_{\aleph_1} - \delta^2_{\aleph_2}| + |\eta^2_{\aleph_1} - \eta^2_{\aleph_2}|\right) = 0.$$

Then,

$$\left(|\sigma^2_{\aleph_1} - \sigma^2_{\aleph_2}| + |\delta^2_{\aleph_1} - \delta^2_{\aleph_2}| + |\eta^2_{\aleph_1} - \eta^2_{\aleph_2}|\right) = 0,$$

which imples that $\sigma^2_{\aleph_1} = \sigma^2_{\aleph_2}$, $\delta^2_{\aleph_1} = \delta^2_{\aleph_2}$, and $\eta^2_{\aleph_1} = \eta^2_{\aleph_2}$. Hence, $\aleph_1 = \aleph_2$.

Conversely, if $\aleph_1 = \aleph_2$, then

$$|\sigma^2_{\aleph_1} - \sigma^2_{\aleph_2}| = 0, |\delta^2_{\aleph_1} - \delta^2_{\aleph_2}| = 0, |\eta^2_{\aleph_1} - \eta^2_{\aleph_2}| = 0.$$

Thus,

$$\Upsilon(\aleph_1, \aleph_2) = \frac{3k}{3k} = 1.$$

Theorem 3.2. *The similarity function* $\Upsilon(\aleph_1, \aleph_2)$ *is bounded, where* $\aleph_1$ *and* $\aleph_2$ *are PFSs in* $S = \{s_1, s_2, \cdots, s_k\}$.

Proof. To prove boundedness, we show that $\Upsilon(\aleph_1, \aleph_2)$ is the subset of a finite interval, [0, 1]. To prove this, we verify $\Upsilon(\aleph_1, \aleph_2) \geq 0$ and $\Upsilon(\aleph_1, \aleph_2) \leq 1$. It is easy to see that $\Upsilon(\aleph_1, \aleph_2) \geq 0$, because

$$|\sigma^2_{\aleph_1} - \sigma^2_{\aleph_2}| \geq 0, |\delta^2_{\aleph_1} - \delta^2_{\aleph_2}| \geq 0, |\eta^2_{\aleph_1} - \eta^2_{\aleph_2}| \geq 0.$$

Next, we investigate $\Upsilon(\aleph_1, \aleph_2) \leq 1$. In

$$\Upsilon(\aleph_1, \aleph_2) = \frac{3k - \left(|\sigma^2_{\aleph_1} - \sigma^2_{\aleph_2}| + |\delta^2_{\aleph_1} - \delta^2_{\aleph_2}| + |\eta^2_{\aleph_1} - \eta^2_{\aleph_2}|\right)}{3k + \left(|\sigma^2_{\aleph_1} - \sigma^2_{\aleph_2}| + |\delta^2_{\aleph_1} - \delta^2_{\aleph_2}| + |\eta^2_{\aleph_1} - \eta^2_{\aleph_2}|\right)},$$

by letting

$$|\sigma^2_{\aleph_1} - \sigma^2_{\aleph_2}| = F_x, |\delta^2_{\aleph_1} - \delta^2_{\aleph_2}| = F_y, |\eta^2_{\aleph_1} - \eta^2_{\aleph_2}| = F_z,$$

we get

$$\Upsilon(\aleph_1, \aleph_2) = \frac{3k - \left(F_x + F_y + F_z\right)}{3k + \left(F_x + F_y + F_z\right)}.$$

Then,

$$\begin{aligned}
\Upsilon(\aleph_1, \aleph_2) - 1 &= \frac{3k - \left(F_x + F_y + F_z\right)}{3k + \left(F_x + F_y + F_z\right)} - 1 \\
&= \frac{3k - \left(F_x + F_y + F_z\right) - 3k - \left(F_x + F_y + F_z\right)}{3k + \left(F_x + F_y + F_z\right)} \\
&= -\frac{2\left(F_x + F_y + F_z\right)}{3k + \left(F_x + F_y + F_z\right)} \\
&\leq 0.
\end{aligned}$$

Thus, $\Upsilon(\aleph_1, \aleph_2) \leq 1$.

Theorem 3.3. *Suppose* $\aleph_1$, $\aleph_2$, *and* $\aleph_3$ *are PFSs in* $S = \{s_1, s_2, \cdots, s_k\}$ *with the inclusion* $\aleph_1 \subseteq \aleph_2 \subseteq \aleph_3$. *Then, the new similarity function satisfies the following properties:*

i) $\Upsilon(\aleph_1, \aleph_3) \geq \Upsilon(\aleph_1, \aleph_2)$ and $\Upsilon(\aleph_1, \aleph_3) \geq \Upsilon(\aleph_2, \aleph_3)$,
ii) $\Upsilon(\aleph_1, \aleph_3) \geq \max\{\Upsilon(\aleph_1, \aleph_2), \Upsilon(\aleph_2, \aleph_3)\}$,
iii) $\Upsilon(\aleph_1, \aleph_3) \leq \Upsilon(\aleph_1, \aleph_2) + \Upsilon(\aleph_2, \aleph_3)$.

Proof. i) In the light of the inclusion, we have

$$|\sigma^2_{\aleph_1} - \sigma^2_{\aleph_3}| \geq |\sigma^2_{\aleph_1} - \sigma^2_{\aleph_2}|,\ |\sigma^2_{\aleph_1} - \sigma^2_{\aleph_3}| \geq |\sigma^2_{\aleph_2} - \sigma^2_{\aleph_3}|,$$
$$|\delta^2_{\aleph_1} - \delta^2_{\aleph_3}| \geq |\delta^2_{\aleph_1} - \delta^2_{\aleph_2}|,\ |\delta^2_{\aleph_1} - \delta^2_{\aleph_3}| \geq |\delta^2_{\aleph_2} - \delta^2_{\aleph_3}|,$$
$$|\eta^2_{\aleph_1} - \eta^2_{\aleph_3}| \geq |\eta^2_{\aleph_1} - \eta^2_{\aleph_2}|,\ |\eta^2_{\aleph_1} - \eta^2_{\aleph_3}| \geq |\eta^2_{\aleph_2} - \eta^2_{\aleph_3}|.$$

Thus,

$$\frac{3k - \left(|\sigma^2_{\aleph_1} - \sigma^2_{\aleph_3}| + |\delta^2_{\aleph_1} - \delta^2_{\aleph_3}| + |\eta^2_{\aleph_1} - \eta^2_{\aleph_3}|\right)}{3k + \left(|\sigma^2_{\aleph_1} - \sigma^2_{\aleph_3}| + |\delta^2_{\aleph_1} - \delta^2_{\aleph_3}| + |\eta^2_{\aleph_1} - \eta^2_{\aleph_3}|\right)} \geq \frac{3k - \left(|\sigma^2_{\aleph_1} - \sigma^2_{\aleph_2}| + |\delta^2_{\aleph_1} - \delta^2_{\aleph_2}| + |\eta^2_{\aleph_1} - \eta^2_{\aleph_2}|\right)}{3k + \left(|\sigma^2_{\aleph_1} - \sigma^2_{\aleph_2}| + |\delta^2_{\aleph_1} - \delta^2_{\aleph_2}| + |\eta^2_{\aleph_1} - \eta^2_{\aleph_2}|\right)}$$

and

$$\frac{3k - \left(|\sigma^2_{\aleph_1} - \sigma^2_{\aleph_3}| + |\delta^2_{\aleph_1} - \delta^2_{\aleph_3}| + |\eta^2_{\aleph_1} - \eta^2_{\aleph_3}|\right)}{3k + \left(|\sigma^2_{\aleph_1} - \sigma^2_{\aleph_3}| + |\delta^2_{\aleph_1} - \delta^2_{\aleph_3}| + |\eta^2_{\aleph_1} - \eta^2_{\aleph_3}|\right)} \geq \frac{3k - \left(|\sigma^2_{\aleph_2} - \sigma^2_{\aleph_3}| + |\delta^2_{\aleph_2} - \delta^2_{\aleph_3}| + |\eta^2_{\aleph_2} - \eta^2_{\aleph_3}|\right)}{3k + \left(|\sigma^2_{\aleph_2} - \sigma^2_{\aleph_3}| + |\delta^2_{\aleph_2} - \delta^2_{\aleph_3}| + |\eta^2_{\aleph_2} - \eta^2_{\aleph_3}|\right)}.$$

Hence, $\Upsilon(\aleph_1, \aleph_3) \geq \Upsilon(\aleph_1, \aleph_2)$ and $\Upsilon(\aleph_1, \aleph_3) \geq \Upsilon(\aleph_2, \aleph_3)$.

ii) From i), it is certain that ii) holds.

iii) Given the fact that $\Upsilon(\aleph_1, \aleph_3) \geq \max\{\Upsilon(\aleph_1, \aleph_2), \Upsilon(\aleph_2, \aleph_3)\}$, it follows that

$$\Upsilon(\aleph_1, \aleph_3) \geq \min\{\Upsilon(\aleph_1, \aleph_2), \Upsilon(\aleph_2, \aleph_3)\},$$

where $\max\{\Upsilon(\aleph_1, \aleph_2), \Upsilon(\aleph_2, \aleph_3)\}$ and $\min\{\Upsilon(\aleph_1, \aleph_2), \Upsilon(\aleph_2, \aleph_3)\}$ are either $\Upsilon(\aleph_1, \aleph_2)$ or $\Upsilon(\aleph_2, \aleph_3)$, respectively. Hence, it is certain that,

$$\Upsilon(\aleph_1, \aleph_3) \leq \Upsilon(\aleph_1, \aleph_2) + \Upsilon(\aleph_2, \aleph_3),$$

which implies that the similarity function satisfies the triangle inequality.

4. Analysis of Liverpool FC during the 2022/2023 EPL season based on SFPFSs

Liverpool FC is a prominent football club situated in Liverpool, England. Liverpool FC was founded in 1892 and played its home matches at Anfield. Liverpool FC plays in EPL, the highest tier of the English football divisions. Liverpool FC is presently managed by a German football manager called Jurgen Klopp. The club has won several domestic titles, namely: nineteen League/EPL titles, eight FA Cups, nine League Cups, and 16 FA Community Shields. In addition, the club has won international titles namely: six European Cups/UEFA Leagues, three UEFA Cups, four UEFA Super Cups, and one FIFA Club World Cup. Liverpool FC has one of the widest fans bases across the whole world in comparison to other prominent football clubs.

4.1. Liverpool matches for the analysis

Liverpool FC performances in the 2022/2023 EPL season were inconsistent, especially during the first half of the season due to several uncertainties, like the issue of adaptability for new players, loss of forms, injuries, and the inability of the club to adequately replace some departing players. These issues dampened the players' performances a great deal. However, Liverpool FC regained form in the middle of the second half of the season, starting from the match Liverpool FC played with Arsenal FC on 09/04/2023 to the match played with Aston Villa FC on 21/05/2023, covering nine matches. The results of the matches can be seen in Table 3.

Table 3. Liverpool's matches and scores.

Match Day	Matches	Fixture Place	Scores	Remarks
09/04/2023	Liverpool Vs Arsenal	Home	2:2	Draw
17/04/2023	Leeds United Vs Liverpool	Away	1:6	Win
22/04/2023	Liverpool Vs Nottingham Forest	Home	3:2	Win
26/04/2023	Westham United Liverpool	Away	1:2	Win
30/04/2023	Liverpool Vs Tottenham	Home	4:3	Win
03/05/2023	Liverpool Vs Fulham	Home	1:0	Win
06/05/2023	Liverpool Vs Brentford	Home	1:0	Win
15/05/2023	Leicester City Vs Liverpool	Away	0:3	Win
21/05/2023	Liverpool Vs Aston Villa	Home	1:1	Draw

By denoting the matches as $\mathcal{M}_i$ for $i = 1, 2, \cdots, 9$, the performance ratings of eleven frequently used players according to BBC Sport analysis are presented in Table 4.

Table 4. Liverpool players's ratings.

Match ratings										
Players	Positions	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$
Alisson	Goalie	$\frac{5.92}{10}$	$\frac{7.27}{10}$	$\frac{6.55}{10}$	$\frac{7.12}{10}$	$\frac{6.51}{10}$	$\frac{7.17}{10}$	$\frac{7.46}{10}$	$\frac{8.08}{10}$	$\frac{5.82}{10}$
Arnold	Defender	$\frac{5.63}{10}$	$\frac{7.76}{10}$	$\frac{6.77}{10}$	$\frac{7.27}{10}$	$\frac{7.00}{10}$	$\frac{7.13}{10}$	$\frac{6.10}{10}$	$\frac{8.45}{10}$	$\frac{5.45}{10}$
Konate	Defender	$\frac{6.09}{10}$	$\frac{6.79}{10}$	$\frac{6.44}{10}$	ABS	$\frac{6.38}{10}$	$\frac{6.89}{10}$	$\frac{7.38}{10}$	$\frac{7.87}{10}$	$\frac{5.35}{10}$
Van Dijk	Defender	$\frac{5.25}{10}$	$\frac{7.07}{10}$	$\frac{6.41}{10}$	$\frac{6.88}{10}$	$\frac{6.31}{10}$	$\frac{6.79}{10}$	$\frac{8.40}{10}$	$\frac{7.83}{10}$	$\frac{5.47}{10}$
Robertson	Defender	$\frac{5.77}{10}$	$\frac{7.49}{10}$	$\frac{6.08}{10}$	$\frac{6.85}{10}$	$\frac{6.49}{10}$	$\frac{6.93}{10}$	$\frac{7.45}{10}$	$\frac{8.01}{10}$	$\frac{5.45}{10}$
Fabinho	Midfielder	$\frac{5.36}{10}$	$\frac{6.83}{10}$	$\frac{6.37}{10}$	$\frac{6.85}{10}$	$\frac{6.37}{10}$	$\frac{6.63}{10}$	$\frac{7.36}{10}$	$\frac{7.73}{10}$	$\frac{5.33}{10}$
Henderson	Midfielder	$\frac{5.66}{10}$	$\frac{7.28}{10}$	$\frac{6.37}{10}$	$\frac{6.76}{10}$	$\frac{6.11}{10}$	$\frac{6.58}{10}$	$\frac{6.62}{10}$	$\frac{7.66}{10}$	$\frac{5.29}{10}$
Jones	Midfielder	$\frac{5.16}{10}$	$\frac{7.16}{10}$	$\frac{6.22}{10}$	$\frac{6.80}{10}$	$\frac{7.0}{10}$	$\frac{6.73}{10}$	$\frac{7.37}{10}$	$\frac{8.62}{10}$	$\frac{5.35}{10}$
Salah	Striker	$\frac{5.51}{10}$	$\frac{8.18}{10}$	$\frac{7.21}{10}$	$\frac{7.01}{10}$	$\frac{7.16}{10}$	$\frac{7.20}{10}$	$\frac{7.72}{10}$	$\frac{8.16}{10}$	$\frac{5.71}{10}$
Gakpo	Striker	$\frac{5.65}{10}$	$\frac{7.84}{10}$	$\frac{6.73}{10}$	$\frac{7.56}{10}$	$\frac{7.24}{10}$	$\frac{6.96}{10}$	$\frac{7.29}{10}$	$\frac{7.59}{10}$	$\frac{5.68}{10}$
Jota	Striker	$\frac{5.29}{10}$	$\frac{7.94}{10}$	$\frac{7.51}{10}$	$\frac{7.11}{10}$	$\frac{7.42}{10}$	$\frac{6.72}{10}$	$\frac{7.39}{10}$	$\frac{6.99}{10}$	$\frac{5.61}{10}$

4.2. Players' ratings under Pythagorean fuzzy environment

Due to indecision in everyday events, the BBC Sport analysts would have definitely encountered imprecisions while rating players. Following this, we transform the players' ratings into PFSs. BBC Sport analysts give analysis of every EPL match immediately after the match is played. After collecting the data, it is converted to PF data to enhance the encapsulation of uncertainties and imprecisions of the analysts. For the conversion, each MD is the allocated value by the analysts, each NMD is $1 - \text{MD}$ from the corresponding MD, and each HM is computed using $\text{HM} = (1 - \text{MD}^2 - \text{NMD}^2)^{0.5}$. By letting the players be denoted by $\mathcal{P}_j$ for $j = 1, 2, \cdots, 11$, their ratings in the Pythagorean fuzzy setting can be seen in Table 5.

Table 5. Pythagorean fuzzy rtings.

Match ratings									
Players	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$
$\mathcal{P}_1$	$(\frac{5.92}{10}, \frac{4.08}{10})$	$(\frac{7.27}{10}, \frac{2.73}{10})$	$(\frac{6.55}{10}, \frac{3.45}{10})$	$(\frac{7.12}{10}, \frac{2.88}{10})$	$(\frac{6.51}{10}, \frac{3.49}{10})$	$(\frac{7.17}{10}, \frac{2.83}{10})$	$(\frac{7.46}{10}, \frac{2.54}{10})$	$(\frac{8.08}{10}, \frac{1.92}{10})$	$(\frac{5.82}{10}, \frac{4.18}{10})$
$\mathcal{P}_2$	$(\frac{5.63}{10}, \frac{4.37}{10})$	$(\frac{7.76}{10}, \frac{2.24}{10})$	$(\frac{6.77}{10}, \frac{3.23}{10})$	$(\frac{7.27}{10}, \frac{2.73}{10})$	$(\frac{7.0}{10}, \frac{3.0}{10})$	$(\frac{7.13}{10}, \frac{2.87}{10})$	$(\frac{6.1}{10}, \frac{3.9}{10})$	$(\frac{8.45}{10}, \frac{1.55}{10})$	$(\frac{5.45}{10}, \frac{4.55}{10})$
$\mathcal{P}_3$	$(\frac{6.09}{10}, \frac{3.91}{10})$	$(\frac{6.79}{10}, \frac{3.21}{10})$	$(\frac{6.44}{10}, \frac{3.56}{10})$	$(\frac{0}{10}, \frac{10}{10})$	$(\frac{6.38}{10}, \frac{3.62}{10})$	$(\frac{6.89}{10}, \frac{3.11}{10})$	$(\frac{7.38}{10}, \frac{2.62}{10})$	$(\frac{7.87}{10}, \frac{2.13}{10})$	$(\frac{5.35}{10}, \frac{4.65}{10})$
$\mathcal{P}_4$	$(\frac{5.25}{10}, \frac{4.75}{10})$	$(\frac{7.07}{10}, \frac{2.93}{10})$	$(\frac{6.41}{10}, \frac{3.59}{10})$	$(\frac{6.88}{10}, \frac{3.12}{10})$	$(\frac{6.31}{10}, \frac{3.69}{10})$	$(\frac{6.79}{10}, \frac{3.21}{10})$	$(\frac{8.40}{10}, \frac{1.60}{10})$	$(\frac{7.83}{10}, \frac{2.17}{10})$	$(\frac{5.47}{10}, \frac{4.53}{10})$
$\mathcal{P}_5$	$(\frac{5.77}{10}, \frac{4.23}{10})$	$(\frac{7.49}{10}, \frac{2.51}{10})$	$(\frac{6.08}{10}, \frac{3.92}{10})$	$(\frac{6.85}{10}, \frac{3.15}{10})$	$(\frac{6.49}{10}, \frac{3.51}{10})$	$(\frac{6.93}{10}, \frac{3.07}{10})$	$(\frac{7.45}{10}, \frac{2.55}{10})$	$(\frac{8.01}{10}, \frac{1.99}{10})$	$(\frac{5.45}{10}, \frac{4.55}{10})$
$\mathcal{P}_6$	$(\frac{5.36}{10}, \frac{4.64}{10})$	$(\frac{6.83}{10}, \frac{3.17}{10})$	$(\frac{6.37}{10}, \frac{3.63}{10})$	$(\frac{6.85}{10}, \frac{3.15}{10})$	$(\frac{6.37}{10}, \frac{3.63}{10})$	$(\frac{6.63}{10}, \frac{3.37}{10})$	$(\frac{7.36}{10}, \frac{2.64}{10})$	$(\frac{7.73}{10}, \frac{2.27}{10})$	$(\frac{5.33}{10}, \frac{4.67}{10})$
$\mathcal{P}_7$	$(\frac{5.66}{10}, \frac{4.34}{10})$	$(\frac{7.28}{10}, \frac{2.72}{10})$	$(\frac{6.37}{10}, \frac{3.63}{10})$	$(\frac{6.76}{10}, \frac{3.24}{10})$	$(\frac{6.11}{10}, \frac{3.89}{10})$	$(\frac{6.58}{10}, \frac{3.42}{10})$	$(\frac{6.62}{10}, \frac{3.38}{10})$	$(\frac{7.66}{10}, \frac{2.34}{10})$	$(\frac{5.29}{10}, \frac{4.71}{10})$
$\mathcal{P}_8$	$(\frac{5.16}{10}, \frac{4.84}{10})$	$(\frac{7.16}{10}, \frac{2.84}{10})$	$(\frac{6.22}{10}, \frac{3.78}{10})$	$(\frac{6.80}{10}, \frac{3.20}{10})$	$(\frac{7.0}{10}, \frac{3.0}{10})$	$(\frac{6.73}{10}, \frac{3.27}{10})$	$(\frac{7.37}{10}, \frac{2.63}{10})$	$(\frac{8.62}{10}, \frac{1.38}{10})$	$(\frac{5.35}{10}, \frac{4.65}{10})$
$\mathcal{P}_9$	$(\frac{5.51}{10}, \frac{4.49}{10})$	$(\frac{8.18}{10}, \frac{1.82}{10})$	$(\frac{7.21}{10}, \frac{2.79}{10})$	$(\frac{7.01}{10}, \frac{2.99}{10})$	$(\frac{7.16}{10}, \frac{2.84}{10})$	$(\frac{7.20}{10}, \frac{2.80}{10})$	$(\frac{7.72}{10}, \frac{2.28}{10})$	$(\frac{8.16}{10}, \frac{1.84}{10})$	$(\frac{5.71}{10}, \frac{4.29}{10})$
$\mathcal{P}_{10}$	$(\frac{5.65}{10}, \frac{4.35}{10})$	$(\frac{7.84}{10}, \frac{2.16}{10})$	$(\frac{6.73}{10}, \frac{3.27}{10})$	$(\frac{7.56}{10}, \frac{2.44}{10})$	$(\frac{7.24}{10}, \frac{2.76}{10})$	$(\frac{6.96}{10}, \frac{3.04}{10})$	$(\frac{7.29}{10}, \frac{2.71}{10})$	$(\frac{7.59}{10}, \frac{2.41}{10})$	$(\frac{5.68}{10}, \frac{4.32}{10})$
$\mathcal{P}_{11}$	$(\frac{5.29}{10}, \frac{4.71}{10})$	$(\frac{7.94}{10}, \frac{2.06}{10})$	$(\frac{7.51}{10}, \frac{2.49}{10})$	$(\frac{7.11}{10}, \frac{2.89}{10})$	$(\frac{7.42}{10}, \frac{2.58}{10})$	$(\frac{6.72}{10}, \frac{3.28}{10})$	$(\frac{7.39}{10}, \frac{2.61}{10})$	$(\frac{6.99}{10}, \frac{3.01}{10})$	$(\frac{5.61}{10}, \frac{4.39}{10})$

In Table 5, $\mathcal{P}_1$ represents the goalkeeper, $\mathcal{P}_2$, $\mathcal{P}_3$, $\mathcal{P}_4$, and $\mathcal{P}_5$ represent the defenders, $\mathcal{P}_6$, $\mathcal{P}_7$, and $\mathcal{P}_8$ represent the midfielders, and $\mathcal{P}_9$, $\mathcal{P}_{10}$, and $\mathcal{P}_{11}$ represent the strikers/attackers, respectively. Now, we establish the relationship between the players using the similarity functions (2.1)–(2.9) [50–54] and the new similarity function (3.1).

4.2.1. Determination of relationships between goalkeeper and defenders

Here, we compute the similarities between the goalkeeper and the defenders to establish their relationships. By computation, we get the results in Table 6.

Table 6. Goalkeeper vs. Defenders.

SF	$(\mathcal{P}_1, \mathcal{P}_2)$	$(\mathcal{P}_1, \mathcal{P}_3)$	$(\mathcal{P}_1, \mathcal{P}_4)$	$(\mathcal{P}_1, \mathcal{P}_5)$
Υ_1 [50]	−0.0933	0.0379	−0.0799	−0.0934
Υ_2 [51]	0.8913	0.7693	0.9023	0.9485
Υ_3 [52]	0.0442	−0.0126	0.0502	0.0773
Υ_4 [52]	0.0604	0.0076	0.0648	0.0848
Υ_5 [52]	0.0442	−0.0036	0.0502	0.0773
Υ_6 [52]	0.0442	−0.0126	0.0502	0.0773
Υ_7 [53]	0.0704	−0.1097	−0.1078	0.0233
Υ_8 [54]	−0.1026	0.124	0.1782	0.4292
Υ	0.9262	0.8401	0.9338	0.9654

The results show that the new similarity function yields the most precise outputs compare to the other methods. Some of the extant methods [50, 52–54] yield negative results which are not defined within the closed interval (i.e., $[0, 1]$), and hence they violate the rule of similarity function. From Table 6, we see that Alisson relates more with Robertson than the rest of the defenders. The sequence of the relationships between the goalkeeper and the defenders range from Robertson, Van Dijk, and Alexander-Arnold to Konate.

4.2.2. Determination of relationships among defenders

The similarities among the defenders are calculated to establish their relationships. The relationships among the defenders can be seen in Table 7.

Table 7. Defenders vs. Defenders.

SF	$(\mathcal{P}_2, \mathcal{P}_3)$	$(\mathcal{P}_2, \mathcal{P}_4)$	$(\mathcal{P}_2, \mathcal{P}_5)$	$(\mathcal{P}_3, \mathcal{P}_4)$	$(\mathcal{P}_3, \mathcal{P}_5)$	$(\mathcal{P}_4, \mathcal{P}_5)$
Υ_1 [50]	−0.0517	0.0178	0.1105	0.0408	−0.0998	−0.1061
Υ_2 [51]	0.7048	0.8326	0.8834	0.7572	0.7739	0.9175
Υ_3 [52]	−0.0357	0.0146	0.0399	−0.0172	−0.0108	≈ 0
Υ_4 [52]	−0.0131	0.0381	0.0576	0.0049	0.0049	≈ 0
Υ_5 [52]	−0.0143	0.0146	0.0399	−0.0047	−0.0017	≈ 0
Υ_6 [52]	−0.0357	0.0146	0.0399	−0.0172	−0.0108	≈ 0
Υ_7 [53]	0.1111	0.0739	0.1111	−0.1093	0.0683	0.1093
Υ_8 [54]	0.1446	0.0153	0.2733	−0.4328	−0.5565	0.1224
Υ	0.793	0.8852	0.9207	0.8313	0.8434	0.9442

The results in Table 7 show that the new similarity function yields satisfactory results with better precision compared to the other methods. Again, the methods in [50, 52–54] yield negative results which are not defined within the closed interval (i.e., $[0, 1]$). From Table 7, we see that Robertson has a very good performance among the defenders. Robertson and Van Dijk have the best relationship between themselves, followed by Robertson and Alexander-Arnold. In addition, Alexander-Arnold and Van Dijk also have a good relationship. The least relationship among the defenders is the relationship between Alexander-Arnold and Konate.

4.2.3. Determination of relationships between defenders and midfielders

Here, we compute the similarities between defenders and midfielders to establish their relationships. The relationships between the defenders and the midfielders are presented in Tables 8 and 9, respectively.

Table 8. Defenders vs. Midfielders I.

SF	$(\mathcal{P}_2, \mathcal{P}_6)$	$(\mathcal{P}_2, \mathcal{P}_7)$	$(\mathcal{P}_2, \mathcal{P}_8)$	$(\mathcal{P}_3, \mathcal{P}_6)$	$(\mathcal{P}_3, \mathcal{P}_7)$	$(\mathcal{P}_3, \mathcal{P}_8)$
Υ_1 [50]	−0.1048	−0.0769	0.0304	0.025	0.0497	0.0257
Υ_2 [51]	0.8501	0.8745	0.886	0.7889	0.7571	0.7456
Υ_3 [52]	0.0229	0.0353	0.0413	−0.0047	−0.0173	−0.0216
Υ_4 [52]	0.0438	0.054	0.0575	0.0106	0.0153	0.005
Υ_5 [52]	0.0229	0.0353	0.0413	0.0002	−0.0049	−0.0071
Υ_6 [52]	0.0229	0.0353	0.0413	−0.0108	−0.0047	−0.0173
Υ_7 [53]	−0.1035	0.0981	0.0909	0.1093	0.0951	0.1111
Υ_8 [54]	−0.6365	0.0608	0.2586	0.3137	−0.2592	0.0657
Υ	0.8975	0.9146	0.9225	0.8541	0.8312	0.8229

Table 9. Defenders vs. Midfielders II.

SF	$(\mathcal{P}_4, \mathcal{P}_6)$	$(\mathcal{P}_4, \mathcal{P}_7)$	$(\mathcal{P}_4, \mathcal{P}_8)$	$(\mathcal{P}_5, \mathcal{P}_6)$	$(\mathcal{P}_5, \mathcal{P}_7)$	$(\mathcal{P}_5, \mathcal{P}_8)$
Υ_1 [50]	0.1103	−0.0799	−0.0791	−0.0571	−0.0832	0.0674
Υ_2 [51]	0.9401	0.9023	0.9023	0.9349	0.9198	0.9232
Υ_3 [52]	0.0722	0.0502	0.0502	0.691	0.0601	0.0621
Υ_4 [52]	0.0834	0.0654	0.0676	0.0788	0.0723	0.0741
Υ_5 [52]	0.0722	0.0502	0.0502	0.0891	0.0601	0.0621
Υ_6 [52]	0.0722	0.0502	0.0502	0.0691	0.0601	0.0621
Υ_7 [53]	0.0327	−0.007	0.0513	0.0514	0.0132	0.0683
Υ_8 [54]	0.2221	0.2184	−0.3919	−0.0011	0.1224	0.2108
Υ	0.9597	0.9338	0.9338	0.9561	0.9458	0.9482

The results in Tables 8 and 9 show that the report on the performances of the methods in [50, 52–54] is similar to the reports in Tables 6 and 7. From Tables 8 and 9, we see that Van Dijk and Fabinho have the best relationship in terms of passing and communications between each other. In addition, Robertson has a good relationship with Fabinho, Henderson, and Jones in that order. The least relationship between a defender and a midfielder is that between Konate and Jones. From the analysis, the best contributing defenders are Robertson and Van Dijk, in that order.

4.2.4. Determination of relationships among midfielders

The relationships among the midfielders are presented in Table 10 as determined by similarity methods.

Table 10. Midfielders vs. Midfielders.

SF	$(\mathcal{P}_6, \mathcal{P}_7)$	$(\mathcal{P}_6, \mathcal{P}_8)$	$(\mathcal{P}_7, \mathcal{P}_8)$
Υ_1 [50]	0.0441	0.0734	0.0586
Υ_2 [51]	0.9425	0.9273	0.8947
Υ_3 [52]	0.0737	0.0645	0.046
Υ_4 [52]	0.0823	0.0773	0.0618
Υ_5 [52]	0.0737	0.0645	0.046
Υ_6 [52]	0.0737	0.0645	0.046
Υ_7 [53]	0.1039	0.1093	0.0951
Υ_8 [54]	0.1614	−0.2673	0.038
Υ	0.9613	0.9509	0.9286

From Table 10, we see that the new similarity function is sufficiently reliable with precise results compared to the existing methods. It is observed that the midfielders have better relationships among themselves. Clearly, Fabinho and Henderson have the best relationship, followed by the relationship between Fabinho and Jones. Finally, the least relationship among the midfielders is that between Henderson and Jones, which is also good.

4.2.5. Determination of relationships between midfielders and attackers

Here, we present the relationships between midfielders and attackers to ascertain the fluidity of the team via similarity functions. The results are presented in Table 11.

Table 11. Midfielders vs. Attackers.

SF	$(\mathcal{P}_6,\mathcal{P}_9)$	$(\mathcal{P}_6,\mathcal{P}_{10})$	$(\mathcal{P}_6,\mathcal{P}_{11})$	$(\mathcal{P}_7,\mathcal{P}_9)$	$(\mathcal{P}_7,\mathcal{P}_{10})$	$(\mathcal{P}_7,\mathcal{P}_{11})$	$(\mathcal{P}_8,\mathcal{P}_9)$	$(\mathcal{P}_8,\mathcal{P}_{10})$	$(\mathcal{P}_8,\mathcal{P}_{11})$
Υ_1 [50]	0.0877	0.1019	−0.1056	−0.074	−0.1069	0.0088	0.0282	0.00004	0.1104
Υ_2 [51]	0.8546	0.8822	0.8621	0.8345	0.8757	0.8396	0.8723	0.8729	0.8558
Υ_3 [52]	0.0252	0.0393	0.0289	0.0154	0.0359	0.0179	0.0341	0.0344	0.0257
Υ_4 [52]	0.0457	0.0558	0.0485	0.0372	0.053	0.0383	0.053	0.0533	0.0477
Υ_5 [52]	0.0252	0.0393	0.0289	0.0154	0.0359	0.0179	0.0341	0.0344	0.0257
Υ_6 [52]	0.0252	0.0393	0.0289	0.0154	0.0359	0.0179	0.0341	0.0344	0.0257
Υ_7 [53]	−0.105	−0.1109	−0.0924	−0.0854	−0.1016	−0.1083	−0.1098	−0.1102	−0.0798
Υ_8 [54]	0.0277	0.1412	0.1756	−0.1084	−0.0069	0.0262	−0.0069	0.0126	0.1297
Υ	0.9007	0.9199	0.9059	0.8865	0.9154	0.8902	0.913	0.9134	0.9015

From the results in Table 11, we see that the midfielders contribute immensely towards the winning streak of the Liverpool FC in the EPL 2022/2023 season. However, it is necessary to note that Fabinho is an exceptional among the midfielders in terms of contribution.

4.2.6. Determination of relationships among attackers

The relationships among the attackers are shown in Table 12 to determine the most effective attackers.

Table 12. Attackers vs. Attackers.

SF	$(\mathcal{P}_9,\mathcal{P}_{11})$	$(\mathcal{P}_6,\mathcal{P}_{10})$	$(\mathcal{P}_6,\mathcal{P}_{11})$
Υ_1 [50]	0.1101	−0.0682	0.0011
Υ_2 [51]	0.9108	0.902	0.9146
Υ_3 [52]	0.0549	0.05	0.0571
Υ_4 [52]	0.0711	0.0669	0.0709
Υ_5 [52]	0.0549	0.05	0.0571
Υ_6 [52]	0.0549	0.05	0.0571
Υ_7 [53]	0.1069	0.067	0.0888
Υ_8 [54]	0.0763	0.5241	0.065
Υ	0.9396	0.9336	0.9422

From Table 12, we see that the attackers have a good number of goals shared among them. In fact, they make a good use of the contributions of the midfielders. Though the attackers related well among themselves, the relationship between Gakpo and Jota is the best.

4.2.7. Determination of the most valuable players

Here, we want to determine the most valuable players among the eleven frequently used players by their manager. The MCDM approach is adopted for the determination process.

Algorithm for the MCDM

The following steps will be followed for the MCDM approach.

Step 1. Formulate the PFDM (Pythagorean fuzzy decision matrix) $\tilde{P}_j = \{M_i(\tilde{P}_j)\}_{(m\times n)}$, where $i =$

$1, 2, \cdots, k$, $j = 1, 2, \cdots, l$, M_i, and $\tilde{P}_j$ represent matches and players, respectively.
Step 2. Normalize the PFDM to get the normalized PFDM denoted by

$$\tilde{P} = \langle \sigma_{\tilde{P}^*_j}(M_i), \delta_{\tilde{P}^*_j}(M_i) \rangle_{k \times l},$$

where $\langle \sigma_{\tilde{P}^*_j}(M_i), \delta_{\tilde{P}^*_j}(M_i) \rangle$ are PFNs, and $\tilde{P}$ is

$$\langle \sigma_{\tilde{P}^*_j}(M_i), \delta_{\tilde{P}^*_j}(M_i) \rangle = \begin{cases} \langle \sigma_{\tilde{P}_j}(M_i), \delta_{\tilde{P}_j}(M_i) \rangle & \text{for BC of } \tilde{P} \\ \langle \delta_{\tilde{P}_j}(M_i), \sigma_{\tilde{P}_j}(M_i) \rangle & \text{for CC of } \tilde{P} \end{cases} \tag{4.1}$$

where BC and CC are the benefit criterion and cost criterion, respectively.
Step 3. Compute PIS and NIS using

$$\tilde{P}^+ = \{\tilde{P}^+_1, \tilde{P}^+_2, \cdots, \tilde{P}^+_k\}, \ \tilde{P}^- = \{\tilde{P}^-_1, \tilde{P}^-_2, \cdots, \tilde{P}^-_k\}, \tag{4.2}$$

where

$$\tilde{P}^+ = \begin{cases} \langle \max\{\sigma_{\tilde{P}_j}(M_i)\}, \min\{\delta_{\tilde{P}_j}(M_i)\} \rangle, & \text{if } M_i \text{ is a BC} \\ \langle \min\{\sigma_{\tilde{P}_j}(M_i)\}, \max\{\delta_{\tilde{P}_j}(M_i)\} \rangle, & \text{if } M_i \text{ is a CC} \end{cases} \tag{4.3}$$

and

$$\tilde{P}^- = \begin{cases} \langle \min\{\sigma_{\tilde{P}_j}(M_i)\}, \max\{\delta_{\tilde{P}_j}(M_i)\} \rangle, & \text{if } M_i \text{ is a BC} \\ \langle \max\{\sigma_{\tilde{P}_j}(M_i)\}, \min\{\delta_{\tilde{P}_j}(M_i)\} \rangle, & \text{if } M_i \text{ is a CC} \end{cases} \tag{4.4}$$

Note that, PIS is the positive ideal solution and NIS is the negative ideal solution, respectively.
Step 4. Find the similarities $\Upsilon(\tilde{P}_j, \tilde{P}^-)$ and $\Upsilon(\tilde{P}_j, \tilde{P}^+)$ based on (3.1).
Step 5. Compute the closeness coefficients $\Theta(\tilde{P}_j)$ using (4.5),

$$\Theta(\tilde{P}_j) = \frac{\Upsilon(\tilde{P}_j, \tilde{P}^+)}{\Upsilon(\tilde{P}_j, \tilde{P}^+) + \Upsilon(\tilde{P}_j, \tilde{P}^-)}, \tag{4.5}$$

for $j = 1, 2, \cdots, k$.
Step 6. Decide the maximum closeness coefficient for the analysis. The flowchart for the algorithm is presented in Figure 1.

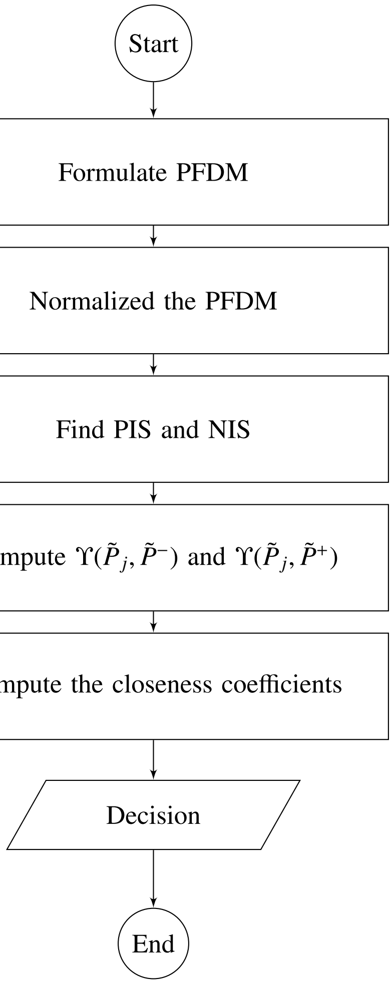

Figure 1. Flowchart for the MCDM.

Implementation

The PFDM has been presented in Table 5. The CC is M_9 since it is the match where the players have the lowest performance ratings according to BBC Sport analysts (i.e., the match played with Aston Villa on 21/05/2023). The normalized PFDM is presented in Table 13, and the PIS and NIS are in Table 14.

Table 13. Normalized Pythagorean fuzzy ratings.

Match ratings									
Players	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$
$\tilde{P}_1$	$(\frac{5.92}{10}, \frac{4.08}{10})$	$(\frac{7.27}{10}, \frac{2.73}{10})$	$(\frac{6.55}{10}, \frac{3.45}{10})$	$(\frac{7.12}{10}, \frac{2.88}{10})$	$(\frac{6.51}{10}, \frac{3.49}{10})$	$(\frac{7.17}{10}, \frac{2.83}{10})$	$(\frac{7.46}{10}, \frac{2.54}{10})$	$(\frac{8.08}{10}, \frac{1.92}{10})$	$(\frac{4.18}{10}, \frac{5.82}{10})$
$\tilde{P}_2$	$(\frac{5.63}{10}, \frac{4.37}{10})$	$(\frac{7.76}{10}, \frac{2.24}{10})$	$(\frac{6.77}{10}, \frac{3.23}{10})$	$(\frac{7.27}{10}, \frac{2.73}{10})$	$(\frac{7.0}{10}, \frac{3.0}{10})$	$(\frac{7.13}{10}, \frac{2.87}{10})$	$(\frac{6.1}{10}, \frac{3.9}{10})$	$(\frac{8.45}{10}, \frac{1.55}{10})$	$(\frac{4.55}{10}, \frac{5.45}{10})$
$\tilde{P}_3$	$(\frac{6.09}{10}, \frac{3.91}{10})$	$(\frac{6.79}{10}, \frac{3.21}{10})$	$(\frac{6.44}{10}, \frac{3.56}{10})$	$(\frac{0}{10}, \frac{10}{10})$	$(\frac{6.38}{10}, \frac{3.62}{10})$	$(\frac{6.89}{10}, \frac{3.11}{10})$	$(\frac{7.38}{10}, \frac{2.62}{10})$	$(\frac{7.87}{10}, \frac{2.13}{10})$	$(\frac{4.65}{10}, \frac{5.35}{10})$
$\tilde{P}_4$	$(\frac{5.25}{10}, \frac{4.75}{10})$	$(\frac{7.07}{10}, \frac{2.93}{10})$	$(\frac{6.41}{10}, \frac{3.59}{10})$	$(\frac{6.88}{10}, \frac{3.12}{10})$	$(\frac{6.31}{10}, \frac{3.69}{10})$	$(\frac{6.79}{10}, \frac{3.21}{10})$	$(\frac{8.40}{10}, \frac{1.60}{10})$	$(\frac{7.83}{10}, \frac{2.17}{10})$	$(\frac{4.53}{10}, \frac{5.47}{10})$
$\tilde{P}_5$	$(\frac{5.77}{10}, \frac{4.23}{10})$	$(\frac{7.49}{10}, \frac{2.51}{10})$	$(\frac{6.08}{10}, \frac{3.92}{10})$	$(\frac{6.85}{10}, \frac{3.15}{10})$	$(\frac{6.49}{10}, \frac{3.51}{10})$	$(\frac{6.93}{10}, \frac{3.07}{10})$	$(\frac{7.45}{10}, \frac{2.55}{10})$	$(\frac{8.01}{10}, \frac{1.99}{10})$	$(\frac{4.55}{10}, \frac{5.45}{10})$
$\tilde{P}_6$	$(\frac{5.36}{10}, \frac{4.64}{10})$	$(\frac{6.83}{10}, \frac{3.17}{10})$	$(\frac{6.37}{10}, \frac{3.63}{10})$	$(\frac{6.85}{10}, \frac{3.15}{10})$	$(\frac{6.37}{10}, \frac{3.63}{10})$	$(\frac{6.63}{10}, \frac{3.37}{10})$	$(\frac{7.36}{10}, \frac{2.64}{10})$	$(\frac{7.73}{10}, \frac{2.27}{10})$	$(\frac{4.67}{10}, \frac{5.33}{10})$
$\tilde{P}_7$	$(\frac{5.66}{10}, \frac{4.34}{10})$	$(\frac{7.28}{10}, \frac{2.72}{10})$	$(\frac{6.37}{10}, \frac{3.63}{10})$	$(\frac{6.76}{10}, \frac{3.24}{10})$	$(\frac{6.11}{10}, \frac{3.89}{10})$	$(\frac{6.58}{10}, \frac{3.42}{10})$	$(\frac{6.62}{10}, \frac{3.38}{10})$	$(\frac{7.66}{10}, \frac{2.34}{10})$	$(\frac{4.71}{10}, \frac{5.29}{10})$
$\tilde{P}_8$	$(\frac{5.16}{10}, \frac{4.84}{10})$	$(\frac{7.16}{10}, \frac{2.84}{10})$	$(\frac{6.22}{10}, \frac{3.78}{10})$	$(\frac{6.80}{10}, \frac{3.20}{10})$	$(\frac{7.0}{10}, \frac{3.0}{10})$	$(\frac{6.73}{10}, \frac{3.27}{10})$	$(\frac{7.37}{10}, \frac{2.63}{10})$	$(\frac{8.62}{10}, \frac{1.38}{10})$	$(\frac{4.65}{10}, \frac{5.35}{10})$
$\tilde{P}_9$	$(\frac{5.51}{10}, \frac{4.49}{10})$	$(\frac{8.18}{10}, \frac{1.82}{10})$	$(\frac{7.21}{10}, \frac{2.79}{10})$	$(\frac{7.01}{10}, \frac{2.99}{10})$	$(\frac{7.16}{10}, \frac{2.84}{10})$	$(\frac{7.20}{10}, \frac{2.80}{10})$	$(\frac{7.72}{10}, \frac{2.28}{10})$	$(\frac{8.16}{10}, \frac{1.84}{10})$	$(\frac{4.29}{10}, \frac{5.71}{10})$
$\tilde{P}_{10}$	$(\frac{5.65}{10}, \frac{4.35}{10})$	$(\frac{7.84}{10}, \frac{2.16}{10})$	$(\frac{6.73}{10}, \frac{3.27}{10})$	$(\frac{7.56}{10}, \frac{2.44}{10})$	$(\frac{7.24}{10}, \frac{2.76}{10})$	$(\frac{6.96}{10}, \frac{3.04}{10})$	$(\frac{7.29}{10}, \frac{2.71}{10})$	$(\frac{7.59}{10}, \frac{2.41}{10})$	$(\frac{4.32}{10}, \frac{5.68}{10})$
$\tilde{P}_{11}$	$(\frac{5.29}{10}, \frac{4.71}{10})$	$(\frac{7.94}{10}, \frac{2.06}{10})$	$(\frac{7.51}{10}, \frac{2.49}{10})$	$(\frac{7.11}{10}, \frac{2.89}{10})$	$(\frac{7.42}{10}, \frac{2.58}{10})$	$(\frac{6.72}{10}, \frac{3.28}{10})$	$(\frac{7.39}{10}, \frac{2.61}{10})$	$(\frac{6.99}{10}, \frac{3.01}{10})$	$(\frac{4.39}{10}, \frac{5.61}{10})$

Table 14. PIS and NIS.

PIS/NIS	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$
$\tilde{P}^+$	$(\frac{6.09}{10}, \frac{3.91}{10})$	$(\frac{8.18}{10}, \frac{1.82}{10})$	$(\frac{7.51}{10}, \frac{2.49}{10})$	$(\frac{7.56}{10}, \frac{2.44}{10})$	$(\frac{7.42}{10}, \frac{2.58}{10})$	$(\frac{7.2}{10}, \frac{2.8}{10})$	$(\frac{8.4}{10}, \frac{1.6}{10})$	$(\frac{8.62}{10}, \frac{1.38}{10})$	$(\frac{4.18}{10}, \frac{5.82}{10})$
$\tilde{P}^-$	$(\frac{5.16}{10}, \frac{4.84}{10})$	$(\frac{6.79}{10}, \frac{3.21}{10})$	$(\frac{6.08}{10}, \frac{3.92}{10})$	$(\frac{0}{10}, \frac{1}{10})$	$(\frac{6.11}{10}, \frac{3.89}{10})$	$(\frac{6.58}{10}, \frac{3.42}{10})$	$(\frac{6.1}{10}, \frac{3.9}{10})$	$(\frac{6.99}{10}, \frac{3.01}{10})$	$(\frac{4.71}{10}, \frac{5.29}{10})$

We observe that $\tilde{P}^+$ and $\tilde{P}^-$ represent the best and worst ratings of the players in each matchday, respectively.

Next, we compute the similarities of $(\tilde{P}_j, \tilde{P}^+)$ and $(\tilde{P}_j, \tilde{P}^-)$ based on (3.1), and Table 15 contains the results.

Table 15. Similarities for $(\tilde{P}_j, \tilde{P}^+)$ and $(\tilde{P}_j, \tilde{P}^-)$.

Players	$\Upsilon(\tilde{P}_j, \tilde{P}^+)$	$\Upsilon(\tilde{P}_j, \tilde{P}^-)$
$\tilde{P}_1$	0.8757	0.8135
$\tilde{P}_2$	0.8836	0.8162
$\tilde{P}_3$	0.7541	0.7901
$\tilde{P}_4$	0.8635	0.832
$\tilde{P}_5$	0.8628	0.8328
$\tilde{P}_6$	0.8367	0.862
$\tilde{P}_7$	0.8286	0.8707
$\tilde{P}_8$	0.8674	0.8305
$\tilde{P}_9$	0.9188	0.7767
$\tilde{P}_{10}$	0.892	0.8047
$\tilde{P}_{11}$	0.8851	0.8098

Using the information in Table 15, we get the closeness coefficients in Table 16, which is represented in Figure 2.

Table 16. Closeness coefficients.

Players	$\Theta(\tilde{P}_j)$	Ranking
$\tilde{P}_1$	0.5184	Fifth
$\tilde{P}_2$	0.5198	Fourth
$\tilde{P}_3$	0.4883	Tenth
$\tilde{P}_4$	0.5093	Seventh
$\tilde{P}_5$	0.5088	Eighth
$\tilde{P}_6$	0.4926	Ninth
$\tilde{P}_7$	0.4876	Eleventh
$\tilde{P}_8$	0.5109	Sixth
$\tilde{P}_9$	0.5419	First
$\tilde{P}_{10}$	0.5257	Second
$\tilde{P}_{11}$	0.5222	Third

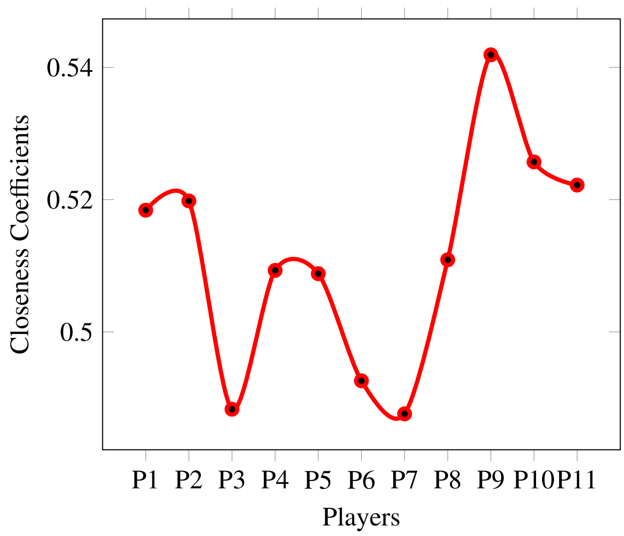

Figure 2. Closeness coefficients of the players.

From the ranking in Table 16 and Figure 2, we see that the player that contributes most to the overall performance of the club in the EPL 2022/2023 season is Salah. He is followed by Gakpo, Jota, Alexander-Arnold, Alisson, Jones, Van Dijk, Robertson, Fabinho, Konate, and Henderson, respectively. Overall, all the players (including the less featured ones due to forms and injuries) contributed immensely to the resurgency of the club towards the end of the EPL season. We recommend that, the club should ensure the high ranked players are given contract renewal/extension to enable them to contribute more in the forthcoming seasons.

4.3. Practical implications of the players' similarity analysis

Table 6 indicates that Alisson and Robertson have a closer relationship than the other defenders. The order of the relationships between the goalie and the defenders range from Robertson, Van Dijk, and Alexander-Arnold to Konate. Table 7 shows that, out of all the defenders, Robertson performs exceptionally well. The strongest bond between Robertson and Van Dijk is followed by that between Robertson and Alexander-Arnold. Furthermore, Van Dijk and Alexander-Arnold get along well. The relationship between Alexander-Arnold and Konate is the least strong among the defenders. Van Dijk and Fabinho have the best relationship when it comes to passing and communication between them, as shown by Tables 8 and 9. Furthermore, Robertson gets along well with Jones, Henderson, and Fabinho, in that order. Konate and Jones have the least relationship of any defender and midfielder. According to the analysis, Van Dijk and Robertson are the two best contributing defenders, in that order.

It has been noted that the relationships amongst the midfielders are better. It is obvious that the relationship between Fabinho and Jones is superior to that of Fabinho and Henderson. Last but not least, Henderson and Jones have the least relationship of any midfield player, which is also positive. According to Table 11, midfield players had a significant impact on Liverpool FC's winning streak in the EPL 2022/2023 season. But, it is important to recognize that Fabinho stands out among the midfield players in terms of his contributions. Table 12 indicates that there is a considerable goal distribution among the attackers. In actuality, they effectively utilize the midfielders' contributions. Even though the attackers get along well with one another, Gakpo and Jota have the best relationship.

From the PF MCDM method based on similarity function, we see that the overall players performances are ranked as follows: Salah, Gakpo, Jota, Alexander-Arnold, Alisson, Jones, Van Dijk, Robertson, Fabinho, Konate, and Henderson, respectively. In this approach, the possibility of uncertainties and imprecisions are reliably curbed. Based on the classical approach, which is obtained by summing the player ratings in the nine matches as provided by the BBC Sport analysts, the overall players performances are ranked as follows: Salah, Gakpo, Jota, Alisson, Alexander-Arnold, Robertson, Van Dijk and Jones (tied), Fabinho, Henderson, and Konate, respectively.

We observe that there are no ties in the PF MCDM-based similarity function, whereas ties exists using the classical approach. Though Konate missed matchday 4, he still ranked better than Henderson using the PF MCDM-based similarity function, but that is not the case with the classical approach. By the PF MCDM-based similarity function, Alexander-Arnold ranked better than Alisson, and Jones and Van Dijk ranked better than Robertson against the rankings via the classical approach. These discrepancies are observed in the classical case because the PF MCDM-based similarity function curbed the uncertainties, indecisions, and imprecisions encountered by the BBC Sport analysts. For a reliable football analysis, we strongly recommend the use of the PF MCDM-based similarity function ahead of the classical approach.

5. Conclusions

In this paper, a new method of SFPFSs is developed and applied in the analysis of football matches played by Liverpool FC in the EPL 2022/2023 season. The motivation for the development of this similarity function is because of the limitations of the extant methods of SFPFSs, which include lack of precision, inability to satisfy similarity conditions, omission of the PFHM, and unreliable interpretations in practical cases. These limitations are justified by presenting comparative analyses of the new similarity function versus the extant similarity functions under the Pythagorean fuzzy domain, from which it is certain that the newly developed function outperforms the existing functions. Some theoretic properties of the newly developed similarity function are discussed to showcase its alignment with the similarity conditions. In addition, the new similarity function is used to discuss the relationships that exist among the players of Liverpool FC in the EPL 2022/2023 season in terms of passing, communications, contributions, and performances based on the recognition principle and the MCDM approach by using the players' rating data from BBC Sport analysts in nine consecutive matches. The analyses of the contributions of the players show that the performances of the players are ranked as follows: Salah, Gakpo, Jota, Alexander-Arnold, Alisson, Jones, Van Dijk, Robertson, Fabinho, Konate, and Henderson, respectively. In addition, it is observed that the MCDM approach yields more reliable results compared to the recognition principle and the classical approach. This application of similarity function for PFSs in football analysis is the first of its kind within the fuzzy domain. The new method of SFPFSs is limited, in the sense that it cannot be directly applicable in other variants of fuzzy set like Fermatean fuzzy sets, q-rung orthopair fuzzy sets, and picture fuzzy sets, etc. without modifications. This is because the new method was not constructed to incorporate the properties of these sets. The newly developed similarity function and the novel application are recommended to be studied in other higher variants of fuzzy sets in the future. Specifically, the new similarity function can be applied to the evaluation of ecological governance [44], attitude and cost-driven consistency optimization models [55], multi-stage consistency optimization algorithms [56], and other real-life problems [57–59].

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgement

This work is supported by the Rural Revitalization Special Project of Chongqing Science and Technology Bureau (No. CSTB2023TIAD-ZXX0017).

Conflict of interest

The authors declare that they have no competing interests.

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