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Research article

Stability analysis of switching systems with all modes unstable based on a Φ-dependent max-minimum dwell time method

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Abstract: This paper applies stable and unstable switching instants to study switching systems with unstable modes under time-dependent switching signals. A new Lyapunov function is proposed to study the properties of switching instant using different Φ-dependent max-minimum dwell time methods. After that, the global exponential stability conditions for nonlinear and linear switching systems are derived and stable switching is used to compensate for unstable switching and unstable modes. The relevant results are extended to the controller design of linear switching systems. Finally, a simulation verifies the effectiveness of the results of this paper.

Keywords: switching systems; stable and unstable subsystems; Φ-dependent max-minimum dwell time; Lyapunov function; globally exponentially stable
Mathematics Subject Classification: 34D20, 93D15

1. Introduction

Switching systems are a special class of hybrid systems consisting of finite or infinite subsystems and are adjusted by switching signals. Compared to general systems, switching systems can better describe many practical situations with multi-mode interactions, such as flight control systems [1], communication networks [2], power electronic systems [3], mobile robots [4], and multi-agent systems [5]. In the last decades, there have been many results on the stability of switching systems. In most of them, the system stability is related to the switching signals that can be divided into time-dependent or/and state-dependent switching. Compared to state-dependent switching, time-dependent ones can avoid partitioning the state space and prevent the Zeno phenomenon. This reduces the design difficulty and makes the conclusions more general. Therefore, this article investigates the stability of switching systems with time-dependent switching.

As we know, in the reference [6], the dwell time (DT) method was proposed to design time-dependent switching signals in the stability analysis of switching systems, requiring a long enough time interval between two consecutive switching instants to counteract the instability caused by the switching behavior. In the references [7–9], the DT method played a very important role in the stability analysis of systems for all stable models. Later in the reference [10], the DT method was extended to the average dwell time (ADT) method, which relaxed the requirements of DT. Then, the generalization of ADT, named mode-dependent average dwell time (MDADT), was proposed in the reference [11], which is, to some extent, more applicable than ADT because it takes into account the differences between subsystems and allows different subsystems to have different ADT. Recently, the paper [12] proposed a new concept of Φ-dependent average dwell time (ΦDADT) for switching systems, which groups different subsystems and is a unified form of ADT and MDADT. This is because they can both be considered as two corollaries of the new method. Moreover, the switching designs under different Φ are very different, and each has its advantages. In general, we can take every possible Φ to obtain stable signals for all kinds of dwell time methods, which is more flexible and less conservative than the existing results.

As for the case with stable and unstable modes in the reference [13], by limiting the total dwell time and activation times of unstable modes, the ADT was used to achieve the stability of switching systems. In the reference [14], the improved stability criterion for discrete-time switching systems with unstable modes was obtained by combining the multiple Lyapunov function method with the limit inferior switching strategy for dealing with stable modes and the limit optimal switching strategy for dealing with unstable modes. In the reference [15], the ΦDADT method was extended to impulse switching singular systems with stable and unstable subsystems and proposed a new conclusion on the stability of switching systems. However, the above results all consider switching behavior as a factor leading to system instability while ignoring the contribution of switching behavior to system stability.

For the case where all modes are unstable, in the reference [16], the maximum-minimum dwell time (MMDT) switching was provided for switching systems with all modes unstable (SSUS). After that, for the positive linear switching system, the contribution of the discretized Lyapunov function to the SSUS was further boosted in the reference [17] by constraining the proportion of unstable switching instant. Next, the stability of partially unstable switching behaviors in SSUS was studied in the reference [18]. Based on the reference [18], the reference [19] utilized singular perturbation parameters to control the ratio of stable switching instant to achieve exponential stabilization of the odd perturbation switching system.

In addition, a new mean dwell time called bounded maximum average dwell time (BMADT) was presented in the reference [20], and the exponential stability of SSUS was obtained by BMADT. The switching behavior with stable nature is beneficial for the system's stability, but the existing results do not consider the information on the switching instant when constructing the Lyapunov function. By introducing the DT information into the piecewise Lyapunov function, the stability criteria for switching linear system relative to MMDT was established in the reference [21]. In the reference [22], the stabilizing switching dependent average dwell time (SSDADT) method was proposed to ensure the system stability by introducing switching instant information and constructing a Lyapunov function that decays at stable switching instant. In the reference [23], from the coordination performance of switching signals, the stability criterion was established by the mode-partition-dependent average dwell time (MPDADT) method and the piecewise Lyapunov function approach. However, there are fewer published studies on the unstable switching behavior caused by the ΦDADT approach, which inspires this paper.

In this paper, the stability of SSUS is investigated by the coordinated performance of the switching signals. A new stability criterion is constructed by a new switching frame named Φ-dependent max-minimum dwell time (ΦDMDT) and a segmented Lyapunov function approach. The contributions of this paper are as follows: First, a new frame called ΦDMDT is proposed, which considers the unstable switching behavior caused by Φ-dependent DT and unstable modes. Second, a new segmented piecewise Lyapunov function approach is provided for the stability problem of SSUSs, then the global exponential stability conditions for linear and nonlinear switching systems are given by the new switching frame and the new approach. Here the state divergence generated by the unstable model and unstable switching instant is balanced by the stable switching instant. Third, the parameters are divided and the DT for different periods is studied separately, which increases the flexibility of unstable switching behavior and reduces conservativeness.

The rest is organized as follows. Section 2 gives the preliminary knowledge. Section 3 introduces the stability criteria of nonlinear and linear switching systems with all modes unstable by using the ΦDMDT method. Section 4 gives the simulation experiment of the main results, and Section 5 is the conclusion of this paper.

The symbols of this paper are stated in Table 1.

Table 1. Symbols used in the paper.

Symbol	The denotation of the symbol
$\mathbb{R}$ ($\mathbb{R}_+$)	the set of real numbers (positive real numbers)
$\mathbb{N}$ ($\mathbb{N}_+$)	the set of natural numbers (positive integers)
$\mathbb{R}^n$	the space of n dimensional real vectors
$\mathbb{R}^n_+$	the set of n dimensional positive vectors
$\mathbb{R}^{n\times n}$	the set of $n \times n$ real matrices
$\|\cdot\|$	Euclidean norm
max (min)	the maximum (minimum) value
T	transposition
υ_i	the i^{th} component of the vector υ
$\upsilon > 0$ (≥ 0)	$\upsilon_i > 0$ (≥ 0), $\forall i = 1, \cdots, n$
$\triangleq$	equivalent to
$\Longrightarrow$	imply

2. Preliminaries

Consider the following switching nonlinear system described by

$$\dot{x}(t) = f_{\delta(t)}(x(t)) \tag{2.1}$$

where $f_{\delta(t)}(\cdot)$ are smooth nonlinear functions from $\mathbb{R}^n$ to $\mathbb{R}^n$ and $f_{\delta(t)}(0) = 0$, $x(t) \in \mathbb{R}^n$ is the system state. A continuous from the right piecewise constant function of time $\delta(t) : [0, +\infty) \to \Im_N = \{1, 2, \ldots, N\}$, $N \in \mathbb{N}$ is the number of modes. For a switching sequence $t_0 < t_1 < \cdots < t_k < \cdots$, while t_k is the k th switching instant. When $t \in [t_k, t_k + \tau_{M\Phi_i}]$, the $\delta(t_k)$ subsystem is activated. $\tau_k \triangleq t_k - t_{k-1}$ is the DT between switching instants t_{k-1} and t_k, $k = 1, 2 \ldots.$ This paper assumes that no state jumps occur at the switching instants and that a finite number of switches occur on every bounded time interval. Let $\mathfrak{S} = \{1, 2, \cdots, s\}$, where $s \in \mathbb{N}$ and $s \leq N$, and $\Phi : \mathfrak{I}_N \mapsto \mathfrak{S}$ is a surjection operator. Set $\Phi_i = \{p \in \mathfrak{I}_N \mid \Phi(p) = i \in \mathfrak{S}\}$.

Definition 1. In the time span $[t_0, t]$, let $\tau_j \triangleq t_j - t_{j-1}$ be the corresponding DT of the j th switching, $t_j \in [t_0, t]$, and $\tau_{j\Phi_i}$ denotes the DT of switching signal $\delta(t)$ with $\delta\left(t_j\right) \in \Phi_i$, $t_j \in [t_0, t]$. There exists two positive integers $\tau_{M\Phi_i}$ and $\tau_{m\Phi_i}$, such that

$$\tau_{M\Phi_i} = \sup_{j\in\Phi_i} \left\{\tau_j\right\}, \tag{2.2}$$

$$\tau_{m\Phi_i} = \inf_{j\in\Phi_i} \left\{\tau_j\right\}. \tag{2.3}$$

Let $\mathcal{S}_{[\tau_{m\Phi_i}, \tau_{M\Phi_i}]}$ denote the set of switching signals satisfying

$$\tau_{m\Phi_i} \leqslant \tau_j \leqslant \tau_{M\Phi_i} \tag{2.4}$$

$T_{\Phi_i}(t_0, t)$ denotes the total running time of the Φ_i subsystems family over the interval $[t_0, t]$. For $\tau_{v\Phi_i} > \tau_{u\Phi_i} > 0$, as shown in Figure 1, let $N_{\tau_{u\Phi_i}}(t_0, t)$ be the number of switching instants t_j belonging to the Φ_i subsystems family in the time span $[t_0, t]$ such that $\tau_{m\Phi_i} < t_j - t_{j-1} < \tau_{u\Phi_i}$, and let $N_{\tau_{v\Phi_i}}(t_0, t)$ be the number of switching instants t_j belonging to the Φ_i subsystems family in the time span $[t_0, t]$ such that $\tau_{u\Phi_i} < t_j - t_{j-1} < \tau_{v\Phi_i}$. Further, let $\mathcal{S}^{\{\tau_{u\Phi_i}, \tau_{v\Phi_i}, c\}}_{[\tau_{m\Phi_i}, \tau_{M\Phi_i}]}$ stand for the switching signals satisfying (2.4) and the following condition

$$N_{\tau_{u\Phi_i}}(t_0, t) \leqslant c\,(t - t_0)\,, 0 \leqslant c \leqslant \frac{1}{\tau_{M\Phi_i}} \tag{2.5}$$

and

$$N_{\tau_{v\Phi_i}}(t_0, t) \leqslant c\,(t - t_0)\,, 0 \leqslant c \leqslant \frac{1}{\tau_{M\Phi_i}}. \tag{2.6}$$

Additionally, $N_{\Phi_i}(t_0, t)$ is the total number of the switching instants belonging to the Φ_i subsystems family in the time span $[t_0, t]$. Two constants $\tau_{M\Phi_i}$ and $\tau_{m\Phi_i}$ are called to be the ΦDMDT for $\delta\left(t_j\right)$.

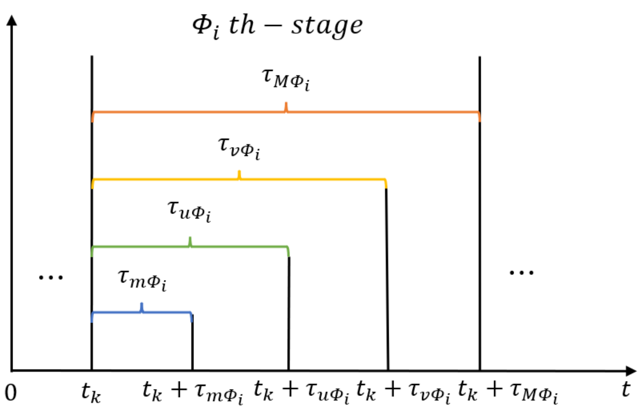


Figure 1. The relationship between stages $\tau_{m_{\Phi_i}}$, $\tau_{u_{\Phi_i}}$, $\tau_{v_{\Phi_i}}$, and $\tau_{M_{\Phi_i}}$ in the Φ_i th-stage.

Definition 2. [18] If a function $f : \mathbb{R}_+ \to \mathbb{R}_+$ is continuous, zero at the origin, and strictly increasing, then it is called a class $\mathcal{K}$ function. If a class $\mathcal{K}$ function is also unbounded, it is called the class $\mathcal{K}_\infty$ function.

Definition 3. [21] A switching system (2.1) under certain switching signal $\delta(t)$ is said to be globally exponentially stable, if the trajectory of the system means

$$\|x(t)\| \leqslant \kappa \|x(t_0)\| e^{-\lambda^*(t-t_0)}, \forall t \geqslant t_0, x(t_0) \in \mathbb{R}^n \tag{2.7}$$

for positive scalars κ, λ^*.

3. Stability analysis

In this section, we will discuss the stability problem of the switching system with all subsystems unstable.

Theorem 1. For given positive constants $\lambda_i > 0, 0 < \mu_{1i} < 1, \mu_{1i} < \mu_{2i} < \mu_{3i}, \mu_{3i} \geqslant 1, 0 \leqslant \tau_{m\Phi_i} \leqslant \tau_{u\Phi_i} \leqslant \tau_{v\Phi_i} \leqslant \tau_{M\Phi_i}$, $0 \leqslant c < \frac{1}{\tau_{M\Phi_i}}$, $\Phi_i = \{p \in \Im_N \mid \Phi(p) = i \in \mathfrak{S}\}$, if there exists positive-definite functions $V_{1p}(t,x), V_{2p}(t,x), V_{3p}(t,x), V_{4p}(t,x)$, and functions $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$, such that for $\forall p, q \in \Im_N, p \neq q$,

$$\alpha_1\|x\|^2 \leqslant V_p(t,x) \leqslant \alpha_2\|x\|^2, \forall t \geqslant t_0, \tag{3.1}$$

$$\frac{\mathrm{d}V_p(t,x)}{\mathrm{d}t} \leqslant \lambda_i V_p(t,x), \forall t \in [t_{n-1}, t_n), \tag{3.2}$$

$$\begin{aligned} V_{1q}(t_n^+, x) \leqslant \mu_{1i}\Big[&V_{1p}(t_n^-, x) + \tau_{u\Phi_i} V_{2p}(t_n^-, x) + (\tau_{v\Phi_i} - \tau_{u\Phi_i}) V_{3p}(t_n^-, x) \\ &+ (t - t_k - \tau_{v\Phi_i}) V_{4p}(t_n^-, x)\Big], t \in [t_k + \tau_{v\Phi_i}, t_k + \tau_{M\Phi_i}], \end{aligned} \tag{3.3}$$

$$V_{1q}(t_n^+, x) \leqslant \mu_{2i}\Big[V_{1p}(t_n^-, x) + \tau_{u\Phi_i} V_{2p}(t_n^-, x) + (t - t_k - \tau_{u\Phi_i}) V_{3p}(t_n^-, x)\Big], t \in [t_k + \tau_{u\Phi_i}, t_k + \tau_{v\Phi_i}), \tag{3.4}$$

$$V_{1q}(t_n^+, x) \leqslant \mu_{3i}\Big[V_{1p}(t_n^-, x) + (t - t_k) V_{2p}(t_n^-, x)\Big], t \in [t_k, t_k + \tau_{u\Phi_i}), \tag{3.5}$$

$$\ln\mu_{1i} + c\tau_{M\Phi_i}(\ln\mu_{2i} + \ln\mu_{3i} - 2\ln\mu_{1i}) + \lambda_{\max}\tau_{M\Phi_i} < 0 \tag{3.6}$$

where

$$V_p(t,x) = \begin{cases} V_{1p}(t_n^-, x) + \tau_{u\Phi_i} V_{2p}(t_n^-, x) + (\tau_{v\Phi_i} - \tau_{u\Phi_i}) V_{3p}(t_n^-, x) \\ \quad + (t - t_k - \tau_{v\Phi_i}) V_{4p}(t_n^-, x), t \in [t_k + \tau_{v\Phi_i}, t_k + \tau_{M\Phi_i}], \\ V_{1p}(t_n^-, x) + \tau_{u\Phi_i} V_{2p}(t_n^-, x) + (t - t_k - \tau_{u\Phi_i}) V_{3p}(t_n^-, x), t \in [t_k + \tau_{u\Phi_i}, t_k + \tau_{v\Phi_i}), \\ V_{1p}(t_n^-, x) + (t - t_k) V_{2p}(t_n^-, x), t \in [t_k, t_k + \tau_{u\Phi_i}) \end{cases} \tag{3.7}$$

and $\lambda_{\max} = \max\limits_{i \in \mathfrak{S}} \lambda_i$, then the switching system (2.1) is globally exponentially stable for any switching signal $\delta(t) \in \mathcal{S}^{\{\tau_{u\Phi_i}, \tau_{v\Phi_i}, c\}}_{[\tau_{m\Phi_i}, \tau_{M\Phi_i}]}$.

Proof. Choose the piece-wise Lyapunov function as $V(t,x) = V_{\delta(t)}(t,x)$. Suppose that $t_k + \tau_{v\Phi_i} \leqslant t \leqslant t_k + \tau_{M\Phi_i}$. It can be obtained from (3.2)–(3.5) that

$$V(t,x) \leqslant e^{\lambda_i(t-t_{n-1})} V_{\delta(t_{n-1}^+)}(t_{n-1}^+, x(t_{n-1}^+)) \leqslant \mu_{1i} e^{\lambda_i(t-t_{n-1})} V_{\delta(t_{n-1}^-)}(t_{n-1}^-, x(t_{n-1}^-)). \tag{3.8}$$

Similarly, for $t_k+\tau_{u\Phi_i} \leqslant t < t_k+\tau_{v\Phi_i}$ and $t_k \leqslant t < t_k+\tau_{u\Phi_i}$, it can be respectively obtained that

$$V(t,x) \leqslant e^{\lambda_i(t-t_{n-1})} V_{\delta(t_{n-1}^+)}\left(t_{n-1}^+, x\left(t_{n-1}^+\right)\right) \leqslant \mu_{2i} e^{\lambda_i(t-t_{n-1})} V_{\delta(t_{n-1}^-)}\left(t_{n-1}^-, x\left(t_{n-1}^-\right)\right) \tag{3.9}$$

and

$$V(t,x) \leqslant e^{\lambda_i(t-t_{n-1})} V_{\delta(t_{n-1}^+)}\left(t_{n-1}^+, x\left(t_{n-1}^+\right)\right) \leqslant \mu_{3i} e^{\lambda_i(t-t_{n-1})} V_{\delta(t_{n-1}^-)}\left(t_{n-1}^-, x\left(t_{n-1}^-\right)\right). \tag{3.10}$$

Therefore, when $t_k \leqslant t \leqslant t_k+\tau_{M\Phi_i}$, it can be inferred from (3.8)–(3.10) that

$$\begin{aligned}
&V(t,x) \\
\leqslant &\prod_{p=1}^{\Im_N} \mu_{1i}^{N_{\Phi_i}(t_0,t)-N_{\tau_{v\Phi_i}}(t_0,t)-N_{\tau_{u\Phi_i}}(t_0,t)} \times \mu_{2i}^{N_{\tau_{v\Phi_i}}(t_0,t)} \mu_{3i}^{N_{\tau_{u\Phi_i}}(t_0,t)} e^{\lambda_i T_{\Phi_i}(t_0,t)} V(t_0, x(t_0)) \\
\leqslant &\prod_{p=1}^{\Im_N} exp(\ln \mu_{1i}^{N_{\Phi_i}(t_0,t)-N_{\tau_{v\Phi_i}}(t_0,t)-N_{\tau_{u\Phi_i}}(t_0,t)} + \ln \mu_{2i}^{N_{\tau_{v\Phi_i}}(t_0,t)} + \ln \mu_{3i}^{N_{\tau_{u\Phi_i}}(t_0,t)} + \lambda_i T_{\Phi_i}(t_0,t)) V(t_0, x(t_0)) \\
\leqslant &exp\left(\sum_{p=1}^{\Im_N}\left(N_{\Phi_i}(t_0,t) - N_{\tau_{v\Phi_i}}(t_0,t) - N_{\tau_{u\Phi_i}}(t_0,t)\right) \times \ln \mu_{1i} + N_{\tau_{v\Phi_i}}(t_0,t) \ln \mu_{2i}\right. \\
&\left. + N_{\tau_{u\Phi_i}}(t_0,t) \ln \mu_{3i} + \lambda_i T_{\Phi_i}(t_0,t)\right) V(t_0, x(t_0)) \\
\leqslant &exp\left(\sum_{p=1}^{\Im_N} N_{\Phi_i}(t_0,t) \ln \mu_{1i} + N_{\tau_{v\Phi_i}}(t_0,t) \times (\ln \mu_{2i} - \ln \mu_{1i}) + N_{\tau_{u\Phi_i}}(t_0,t)(\ln \mu_{3i} - \ln \mu_{1i})\right. \\
&\left. + \lambda_i T_{\Phi_i}(t_0,t)\right) V(t_0, x(t_0)).
\end{aligned} \tag{3.11}$$

Because of (2.5), (2.6), and

$$N_{\Phi_i}(t_0,t) \geqslant \frac{t-t_0}{\tau_{M\Phi_i}} - 1 \tag{3.12}$$

the following holds

$$\begin{aligned}
&N_{\Phi_i}(t_0,t) \ln \mu_{1i} + N_{\tau_{v\Phi_i}}(t_0,t)(\ln \mu_{2i} - \ln \mu_{1i}) + N_{\tau_{u\Phi_i}}(t_0,t)(\ln \mu_{3i} - \ln \mu_{1i}) + \lambda_i T_{\Phi_i}(t_0,t) \\
\leqslant &\left(\frac{\ln \mu_{1i}}{\tau_{M\Phi_i}} + c(\ln \mu_{2i} + \ln \mu_{3i} - 2\ln \mu_{1i})\right)(t-t_0) + \lambda_i T_{\Phi_i}(t_0,t) - \ln \mu_{1i} \\
\leqslant &\left(\frac{\ln \mu_{1i}}{\tau_{M\Phi_i}} + c(\ln \mu_{2i} + \ln \mu_{3i} - 2\ln \mu_{1i}) + \lambda_{\max}\right) \times (t-t_0) - \ln \mu_{1i}.
\end{aligned} \tag{3.13}$$

Combining Eqs (3.1), (3.11), and (3.13), we can obtain

$$\|x\| \leqslant \kappa \|x(t_0)\| e^{-\lambda(t-t_0)} \tag{3.14}$$

where $\kappa = \sqrt{\frac{\alpha_2}{\alpha_1 \mu_{1i}}}$, $\lambda = -\sum_{p=1}^{\Im_N} \frac{\ln \mu_{1i} + c\tau_{M\Phi_i}(\ln \mu_{2i} + \ln \mu_{3i} - 2\ln \mu_{1i}) + \lambda_{\max}\tau_{M\Phi_i}}{2\tau_{M\Phi_i}}$. □

Remark 1. When $V_{2p}=V_{3p}=V_{4p}$, Lyapunov functions can be uniformly written in the form of $V_p(t,x) = V_{1p}(t_n^-, x) + (t-t_k) V_{2p}(t_n^-, x), t \in [t_k, t_k+\tau_{M\Phi_i}]$. On the basis of $V_{2p}=V_{3p}=V_{4p}$, if we further take $\mathfrak{S} = \{1\}$, the result of the reference [21] can be obtained, which shows it can be seen as a corollary of this article.

Remark 2. The relationships between this article and the literature [12] and [15] can be summarized as follows. On the one hand, they all introduce the idea of subsystem classification Φ into the study of switching system stability. On the other hand, both [12] and [15] are based on the ADT method to propose the ΦDADT method that is only applicable to the subsystems being stable or partially stable, while this paper is based on the MMDT method to put forward the ΦDMDT method, which can handle the situations where all subsystems are unstable.

Consider the linear case of the system (2.1)

$$\dot{x}(t) = A_{\delta(t)}x \tag{3.15}$$

where A_p, $p \in \Im_N$, is a matrix with appropriate dimension, and the globally exponentially stable is stated as the following theorem.

Theorem 2. For given positive constants $\lambda_i > 0$, $\alpha \geq 0$, $0 < \mu_{1i} < 1$, $\mu_{1i} < \mu_{2i} < \mu_{3i}$, $\mu_{3i} \geqslant 1$, $0 \leqslant \tau_{m\Phi_i} \leqslant \tau_{u\Phi_i} \leqslant \tau_{v\Phi_i} \leqslant \tau_{M\Phi_i}$, $0 \leq c < \frac{1}{\tau_{M\Phi_i}}$, $\Phi_i = \{p \in \Im_N \mid \Phi(p) = i \in \mathfrak{S}\}$, if there exists positive definite matrices P_p, Q_p , R_p and S_p, $\forall p, q \in \Im_N, p \neq q$, such that (3.6) holds and

$$\phi_p < 0, \tag{3.16}$$

$$e^{\alpha\tau_{m\Phi p}}\phi_p + h_n\psi_p + h_n\omega_p + h_n\xi_p < 0, h_n \in [\tau_{m\Phi_i}, \tau_{M\Phi_i}], \tag{3.17}$$

$$\begin{aligned} P_q \leqslant \mu_{1i} \Big[& P_p + \tau_{u\Phi_i}e^{-\alpha\tau_{u\Phi_i}}Q_p + (\tau_{v\Phi_i} - \tau_{u\Phi_i})\, e^{-\alpha\left(\tau_{v\Phi_i} - \tau_{u\Phi_i}\right)}R_p \\ & + (t - t_k - \tau_{v\Phi_i})\, e^{-\alpha\left(t - t_k - \tau_{v\Phi_i}\right)}S_p \Big], t \in [t_k + \tau_{v\Phi_i}, t_k + \tau_{M\Phi_i}], \end{aligned} \tag{3.18}$$

$$P_q \leqslant \mu_{2i} \Big[P_p + \tau_{u\Phi_i}e^{-\alpha\tau_{u\Phi_i}}Q_p + (t - t_k - \tau_{u\Phi_i})\, e^{-\alpha\left(t - t_k - \tau_{u\Phi_i}\right)}R_p \Big], t \in [t_k + \tau_{u\Phi_i}, t_k + \tau_{v\Phi_i}), \tag{3.19}$$

$$P_q \leqslant \mu_{3i} \Big[P_p + (t - t_k)\, e^{-\alpha(t - t_k)}Q_p \Big], t \in [t_k, t_k + \tau_{u\Phi_i}) \tag{3.20}$$

for $\forall p \in \Im_N$, where

$$\phi_p = P_pA_p + A_p^{\mathrm{T}}P_p + e^{-\alpha\tau_{m\Phi_i}}Q_p + e^{-\alpha\tau_{m\Phi_i}}R_p + e^{-\alpha\tau_{m\Phi_i}}S_p - \lambda_iP_p, \tag{3.21}$$

$$\psi_p = Q_pA_p + A_p^{\mathrm{T}}Q_p - (\alpha + \lambda_i)Q_p, \tag{3.22}$$

$$\omega_p = R_pA_p + A_p^{\mathrm{T}}R_p - (\alpha + \lambda_i)R_p, \tag{3.23}$$

$$\xi_p = S_pA_p + A_p^{\mathrm{T}}S_p - (\alpha + \lambda_i)S_p, \tag{3.24}$$

$$h_n = t_{n+1} - t_n \tag{3.25}$$

then the switching linear system (3.15) is globally exponentially stable for any switching signal $\delta(t) \in \mathcal{S}_{[\tau_{m\Phi_i}, \tau_{M\Phi_i}]}^{\{\tau_{u\Phi_i}, \tau_{v\Phi_i}, c\}}$.

Proof. For $t \in [t_k, t_k + \tau_{M\Phi_i}]$, $p \in \Im_N$, construct the piece-wise Lyapunov function of the switching linear system (3.15) as

$$V_p(t, x) = \begin{cases} x^{\mathrm{T}}P_px + \tau_{u\Phi_i}e^{-\alpha\tau_{u\Phi_i}}x^{\mathrm{T}}Q_px + (\tau_{v\Phi_i} - \tau_{u\Phi_i})\, e^{-\alpha\left(\tau_{v\Phi_i} - \tau_{u\Phi_i}\right)}x^{\mathrm{T}}R_px \\ \quad + (t - t_k - \tau_{v\Phi_i})\, e^{-\alpha\left(t - t_k - \tau_{v\Phi_i}\right)}x^{\mathrm{T}}S_px, t \in [t_k + \tau_{v\Phi_i}, t_k + \tau_{M\Phi_i}], \\ x^{\mathrm{T}}P_px + \tau_{u\Phi_i}e^{-\alpha\tau_{u\Phi_i}}x^{\mathrm{T}}Q_px + (t - t_k - \tau_{u\Phi_i})\, e^{-\alpha\left(t - t_k - \tau_{u\Phi_i}\right)}x^{\mathrm{T}}R_px, \\ \qquad t \in [t_k + \tau_{u\Phi_i}, t_k + \tau_{v\Phi_i}), \\ x^{\mathrm{T}}P_px + (t - t_k)\, e^{-\alpha(t - t_k)}x^{\mathrm{T}}Q_px, t \in [t_k, t_k + \tau_{u\Phi_i}). \end{cases} \tag{3.26}$$

When $t_k+\tau_{v\Phi_i} \leqslant t \leqslant t_k+\tau_{M\Phi_i}$, the derivative of $V_p(t,x)$ is

$$\begin{aligned}
&\frac{\mathrm{d}V_p(t,x)}{\mathrm{d}t}\\
&= 2x^{\mathrm{T}}P_p\dot{x} + e^{-\alpha\tau_{u\Phi_i}}x^{\mathrm{T}}Q_px + 2\tau_{u\Phi_i}e^{-\alpha\tau_{u\Phi_i}}x^{\mathrm{T}}Q_p\dot{x} - \alpha\tau_{u\Phi_i}e^{-\alpha\tau_{u\Phi_i}}x^{\mathrm{T}}Q_px + e^{-\alpha\left(\tau_{v\Phi_i}-\tau_{u\Phi_i}\right)}x^{\mathrm{T}}R_px\\
&\quad + 2\left(\tau_{v\Phi_i}-\tau_{u\Phi_i}\right)e^{-\alpha\left(\tau_{v\Phi_i}-\tau_{u\Phi_i}\right)}x^{\mathrm{T}}R_p\dot{x} - \alpha\left(\tau_{v\Phi_i}-\tau_{u\Phi_i}\right)e^{-\alpha\left(\tau_{v\Phi_i}-\tau_{u\Phi_i}\right)}x^{\mathrm{T}}R_px + e^{-\alpha\left(t-t_k-\tau_{v\Phi_i}\right)}x^{\mathrm{T}}S_px\\
&\quad + 2\left(t-t_k-\tau_{v\Phi_i}\right)e^{-\alpha\left(t-t_k-\tau_{v\Phi_i}\right)}x^{\mathrm{T}}S_p\dot{x} - \alpha\left(t-t_k-\tau_{v\Phi_i}\right)e^{-\alpha\left(t-t_k-\tau_{v\Phi_i}\right)}x^{\mathrm{T}}S_px.
\end{aligned} \tag{3.27}$$

By simple calculation, it is easy to show that

$$\begin{aligned}
&\frac{\mathrm{d}V_p(t,x)}{\mathrm{d}t} - \lambda_iV_p(t,x)\\
&= x^{\mathrm{T}}\Big\{P_pA_p + A_p^{\mathrm{T}}P_p + e^{-\alpha\tau_{u\Phi_i}}Q_p + e^{-\alpha\left(\tau_{v\Phi_i}-\tau_{u\Phi_i}\right)}R_p + e^{-\alpha\left(t-t_k-\tau_{v\Phi_i}\right)}S_p - \lambda_iP_p\\
&\quad +\tau_{u\Phi_i}e^{-\alpha\tau_{u\Phi_i}}\times\left[Q_pA_p + A_p^{\mathrm{T}}Q_p - (\alpha+\lambda_i)Q_p\right]\\
&\quad +\left(\tau_{v\Phi_i}-\tau_{u\Phi_i}\right)e^{-\alpha\left(\tau_{v\Phi_i}-\tau_{u\Phi_i}\right)}\times\left[R_pA_p + A_p^{\mathrm{T}}R_p - (\alpha+\lambda_i)R_p\right]\\
&\quad +\left(t-t_k-\tau_{v\Phi_i}\right)e^{-\alpha\left(t-t_k-\tau_{v\Phi_i}\right)}\times\left[S_pA_p + A_p^{\mathrm{T}}S_p - (\alpha+\lambda_i)S_p\right]\Big\}x\\
&\leqslant \frac{(t_{n+1}-t)}{h_n}x^{\mathrm{T}}\phi_px + \frac{(t-t_n)\,e^{-\alpha(t-t_n)}}{h_n}\times x^{\mathrm{T}}\left(e^{\alpha\tau_{m\Phi_i}}\phi_p + h_n\psi_p + h_n\omega_p + h_n\xi_p\right)x.
\end{aligned} \tag{3.28}$$

Similarly, when $t_k+\tau_{u\Phi_i} \leqslant t < t_k+\tau_{v\Phi_i}$ and $t_k \leqslant t < t_k+\tau_{u\Phi_i}$, it can be respectively obtained that

$$\begin{aligned}
&\frac{\mathrm{d}V_p(t,x)}{\mathrm{d}t} - \lambda_iV_p(t,x)\\
&= x^{\mathrm{T}}\Big\{P_pA_p + A_p^{\mathrm{T}}P_p + e^{-\alpha\tau_{u\Phi_i}}Q_p + e^{-\alpha\left(t-t_k-\tau_{u\Phi_i}\right)}R_p - \lambda_iP_p\\
&\quad +\tau_{u\Phi_i}e^{-\alpha\tau_{u\Phi_i}}\times\left[Q_pA_p + A_p^{\mathrm{T}}Q_p - (\alpha+\lambda_i)Q_p\right]\\
&\quad +\left(t-t_k-\tau_{u\Phi_i}\right)e^{-\alpha\left(t-t_k-\tau_{u\Phi_i}\right)}\times\left[R_pA_p + A_p^{\mathrm{T}}R_p - (\alpha+\lambda_i)R_p\right]\Big\}x\\
&\leqslant \frac{(t_{n+1}-t)}{h_n}x^{\mathrm{T}}\phi_px + \frac{(t-t_n)\,e^{-\alpha(t-t_n)}}{h_n}\times x^{\mathrm{T}}\left(e^{\alpha\tau_{m\Phi_i}}\phi_p + h_n\psi_p + h_n\omega_p + h_n\xi_p\right)x
\end{aligned} \tag{3.29}$$

and

$$\begin{aligned}
&\frac{\mathrm{d}V_p(t,x)}{\mathrm{d}t} - \lambda_iV_p(t,x)\\
&= x^{\mathrm{T}}\left\{P_pA_p + A_p^{\mathrm{T}}P_p + e^{-\alpha(t-t_k)}Q_p - \lambda_iP_p + (t-t_k)\,e^{-\alpha(t-t_k)}\left[Q_pA_p + A_p^{\mathrm{T}}Q_p - (\alpha+\lambda_i)Q_p\right]\right\}x\\
&\leqslant \frac{(t_{n+1}-t)}{h_n}x^{\mathrm{T}}\phi_px + \frac{(t-t_n)\,e^{-\alpha(t-t_n)}}{h_n}\times x^{\mathrm{T}}\left(e^{\alpha\tau_{m\Phi_i}}\phi_p + h_n\psi_p + h_n\omega_p + h_n\xi_p\right)x.
\end{aligned} \tag{3.30}$$

It is worth noting that

$$\frac{\mathrm{d}V_p(t,x)}{\mathrm{d}t} - \lambda_iV_p(t,x) < 0 \tag{3.31}$$

holds for any $t \in [t_k, t_k+\tau_{M\Phi_i}]$, if (3.16) and (3.17) hold. On the other hand, it is easy to obtain (3.3)–(3.5) from (3.18)–(3.20), then the proof ends according to Theorem 1. □

As a special case of $0 < \tau_{m\Phi_i} \leqslant \tau_{u\Phi_i} \leqslant \tau_{v\Phi_i} \leqslant \tau_{M\Phi_i}$, it is easy to prove the following corollary when $\tau_{m\Phi_i} = \tau_{u\Phi_i} = \tau_{v\Phi_i}$.

Corollary 1. For given positive constants $\lambda_i > 0$, $\alpha \geqslant 0$, $0 < \mu_{1i} < 1$, $0 < \tau_{m\Phi_i} \leqslant \tau_{M\Phi_i}$, $\Phi_i = \{p \in \Im_N \mid \Phi(p) = i \in \mathfrak{S}\}$, if there exists positive definite matrices P_p and Q_p, $\forall p, q \in \Im_N, p \neq q$, such that (3.16) and (3.17) hold and

$$P_q \leqslant \mu_{1i}\left(P_p + \tau_{m\Phi_i} e^{-\alpha\tau_{M\Phi_i}} Q_p\right), \tag{3.32}$$

$$\ln \mu_{1i} + \lambda_i \tau_{M\Phi_i} < 0 \tag{3.33}$$

then the switching linear system (3.15) is globally exponentially stable for any switching signal $\delta(t) \in \mathcal{S}_{[\tau_{m\Phi_i}, \tau_{M\Phi_i}]}$.

Remark 3. The condition (3.33) shows that $0 < \lambda_i < -\frac{\ln\mu_{1i}}{\tau_{M\Phi_i}}$. Introduce a lower bound of λ_i such that

$$-\frac{\beta \ln\mu_{1i}}{\tau_{M\Phi_i}} < \lambda_i < -\frac{\ln\mu_{1i}}{\tau_{M\Phi_i}} \tag{3.34}$$

for a given scalar $\beta \in (0, 1)$, then the condition of Corollary 3.1 is improved as that: For given constants $\alpha \geqslant 0, 0 < \beta < 1, 0 < \mu_{1i} < 1, 0 < \tau_{m\Phi_i} \leqslant \tau_{M\Phi_i}$, if there exists positive definite matrices P_p and Q_p, $\forall p \in \Im_N$, such that Eqs (3.32)–(3.34) hold with

$$\phi_{rp} = P_p A_p + A_p^{\mathrm{T}} P_p + e^{-\alpha\tau_{m\Phi_i}} Q_p + \frac{\beta \ln \mu_{1i}}{\tau_{M\Phi_i}} P_p < 0, \tag{3.35}$$

$$e^{\alpha\tau_{m\Phi_i}} \phi_{rp} + h_n \left(Q_p A_p + A_p^{\mathrm{T}} Q_p - \alpha Q_p + \frac{\beta \ln \mu_{1i}}{\tau_{M\Phi_i}} Q_p\right) < 0. \tag{3.36}$$

This operation provides a new degree of freedom to the parameter λ_i while avoiding the nonlinear coming from $\lambda_i P_p$. For the given α, β, μ_{1i}, $\tau_{M\Phi_i}$, the admissible minimal dwell time $\underline{\tau_{m\Phi_i}}$ can be estimated by

$$\min_{\tau_{m\Phi_i} \leqslant \tau_{M\Phi_i}} \underline{\tau_{m\Phi_i}}, \quad \text{s.t. (3.32)–(3.36).} \tag{3.37}$$

This inference derives linear condition by introducing the information of switching instants into the Lyapunov function in form as $(t - t_k)\, e^{-\alpha(t-t_k)} Q_p$.

4. Simulation

Example 1. In this section, simulations are performed with the highly maneuverable aircraft technology (HiMAT) as an example, thus verifying the effectiveness and practicality of the method in this paper. Considering the short-period motion characteristics of the aircraft, a longitudinal short-period linear model is used to construct the switching system and the corresponding data are obtained from [24,25]. For more clarity, three operating points are selected within the flight envelope. Therefore, it is reasonable to assume that three linear modes can describe the dynamic behavior of the HiMAT vehicle at the three operating points, as shown in Table 2.

Table 2. Three operating points of HiMAT vehicle.

Operating point	Mach	Altitude	Angle of attack
1	0.6	20000 ft	4.55 deg
2	0.9	35000 ft	4.89 deg
3	1.3	27000 ft	5.36 deg

Based on this data, the no-fault delta operator switching linear system can be described as

$$\dot{x}(t) = A_{\delta(t)}x + B_{\delta(t)}u. \tag{4.1}$$

The state of the linearized HiMAT dynamics is $x(t) = (g, h)^T$, with g and h denoting the angle of attack and pitch rate, respectively. The control input is $u(t) = (\Theta_e, \Theta_c)^T$, where Θ_e and Θ_c are the elevon and carnard input, respectively. Assuming the control input $u(t) = (0, 0)$, the switching law determines the alternating operation of the three operating points of the vehicle. Thus, the problem can be simplified by considering the linear switching systems (3.15) with the following parameters: $\Im_N = \{1, 2, 3\}$, $A_1 = \begin{bmatrix} -1.9 & 0.6 \\ 0.6 & -0.1 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0.1 & -0.9 \\ 0.1 & -1.4 \end{bmatrix}$, $A_3 = \begin{bmatrix} 0.3 & -0.2 \\ 2.1 & -1.7 \end{bmatrix}$, and $x(t_0) = \begin{bmatrix} 5 & -3 \end{bmatrix}^{\mathrm{T}}$. It is clear that all subsystems are unstable. The state response of each mode is shown in Figure 2. Letting $\alpha = 0.1$, a feasible solution of Eqs (3.33) and (3.16)–(3.20) is obtained by applying the Matlab linear matrix inequality (LMI) toolbox. By dividing different subsystem families, the following cases are obtained.

Case 1: $\mathfrak{S} = \{1\}$, $\Phi_1 = \{1, 2, 3\}$;
Case 2: $\mathfrak{S} = \{1, 2\}$, $\Phi_1 = \{1, 2\}$, $\Phi_2 = \{3\}$;
Case 3: $\mathfrak{S} = \{1, 2\}$, $\Phi_1 = \{1, 3\}$, $\Phi_2 = \{2\}$;
Case 4: $\mathfrak{S} = \{1, 2\}$, $\Phi_1 = \{1\}$, $\Phi_2 = \{2, 3\}$;
Case 5: $\mathfrak{S} = \{1, 2, 3\}$, $\Phi_1 = \{1\}$, $\Phi_2 = \{2\}$; $\Phi_3 = \{3\}$.
Among them, Cases 1 and 5 correspond to the results of the DT and mode-dependent DT methods, respectively.

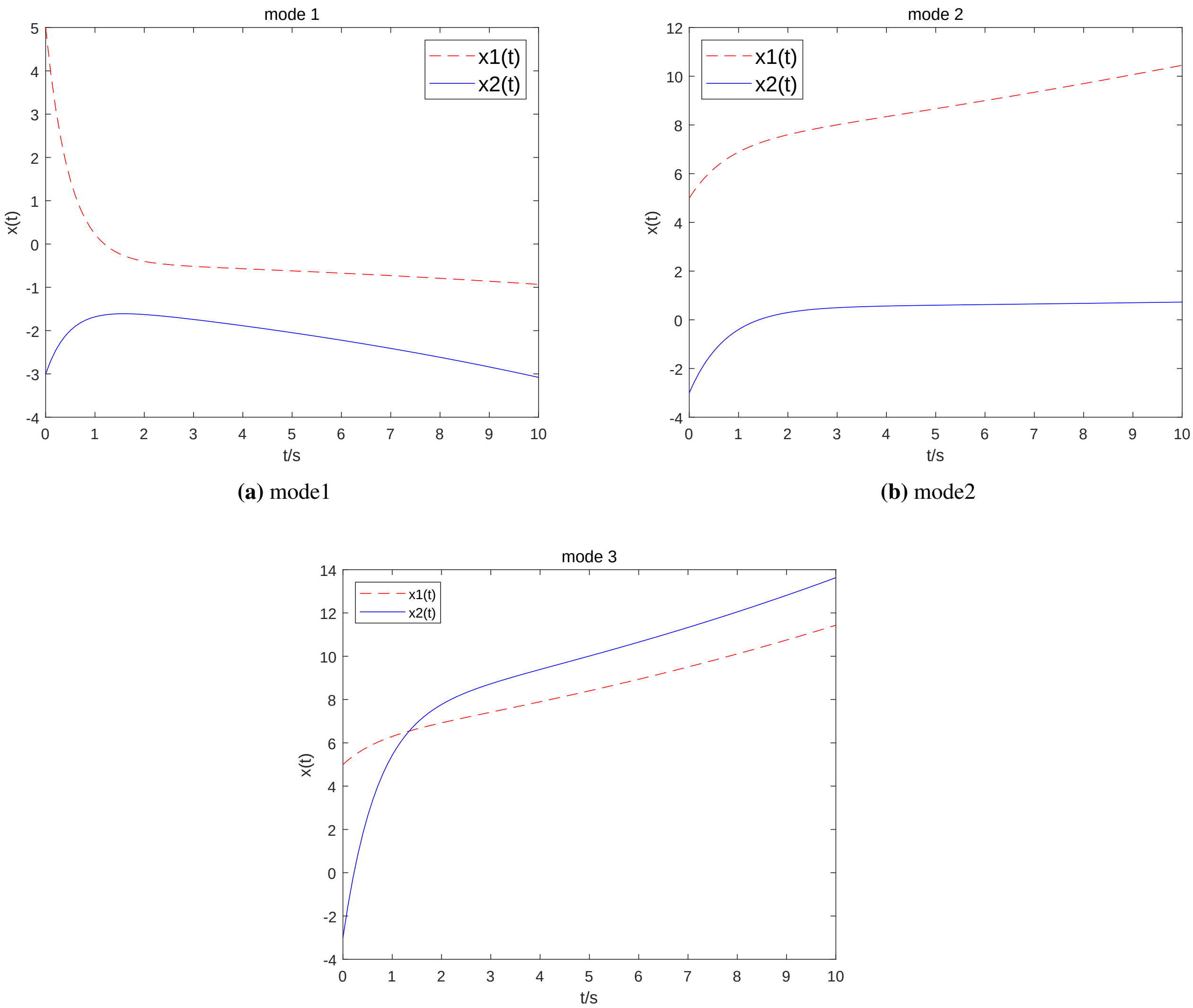
Next, the following conclusions can be drawn from Figures 3–8 and Tables 3–7:

(I) In Case 1, by comparing Figures 3(a) and 3(b), it can be seen that compared with the DT method, the MMDT method allows for shorter DTs, thereby reducing conservatism and increasing design flexibility, and the results corresponding to the MMDT method are shown in Table 3.

(II) In Cases 2–5, due to the superiority of the MMDT method, we have uniformly used this method. The results are shown in Tables 4–7 and the corresponding system's state response under switching signals is shown in Figures 4–7. The convergence curve explains the validity of our results.

(III) In the reference [21], when $\alpha = 0$, its allowed DT region gradually increases with the increase of β value. Based on this, according to (3.37), Figure 8 illustrates the minimum DT when β=0.5, 0.8, 0.9, and 0.99 are taken under Case 1. Taking β=0.5 as an example, take the appropriate τ_u and τ_v between τ_m and τ_M, and different Lyapunov functions are selected under different DTs to further reduce conservatism. Compared to reference [21], this paper not only considers the case under the Φ-dependent DT method but also makes the method more flexible and convenient by constructing a new segmented Lyapunov function.

(IV) The examples in Tables 3–7 provide some switching signals that can only be obtained through the criteria in the corresponding Tables. That is to say, the stability criteria for each table is different, so they are not comparable to each other.

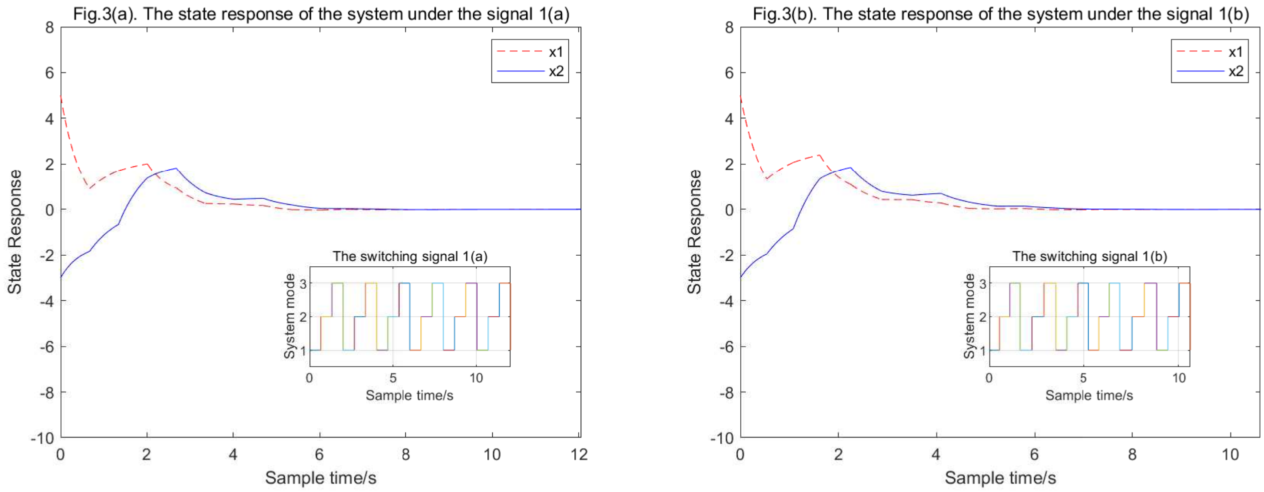


(a) mode1

(b) mode2

(c) mode3

Figure 2. State response of each mode.



(a) $\tau_{m\Phi_1} = \tau_{M\Phi_1} = 0.67$ **(b)** $\tau_{m\Phi_1} = 0.49, \tau_{M\Phi_1} = 0.67$

Figure 3. The state response of the system under the signals 1(a) and 1(b), respectively.

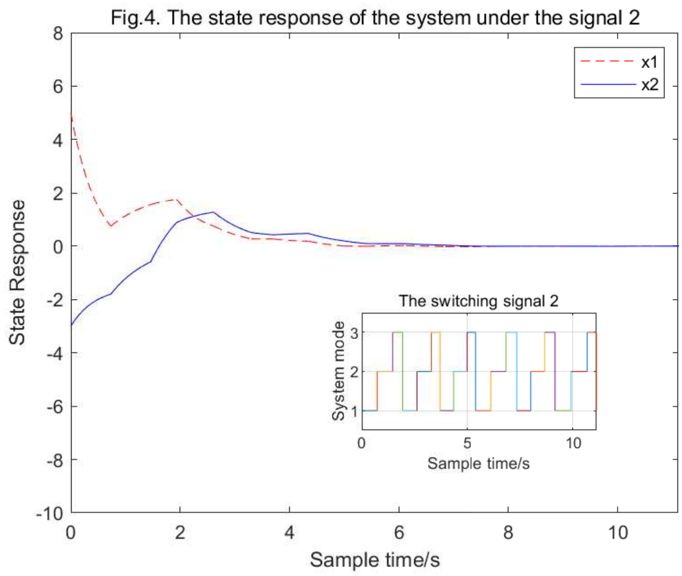


Figure 4. The state response of the system under the signal 2.

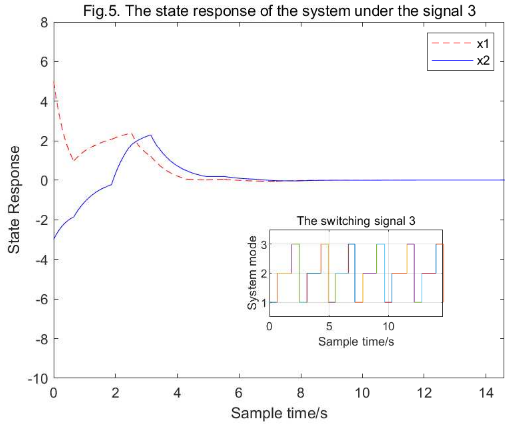


Figure 5. The state response of the system under the signal 3.

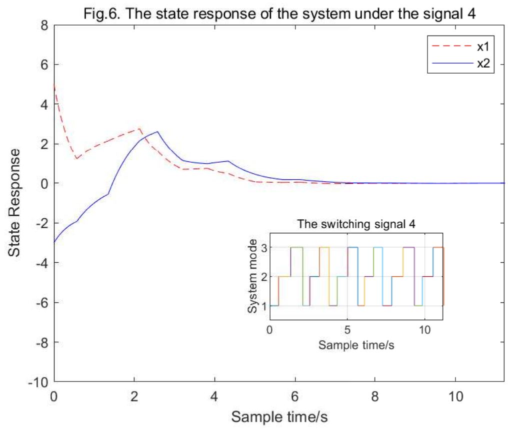


Figure 6. The state response of the system under the signal 4.

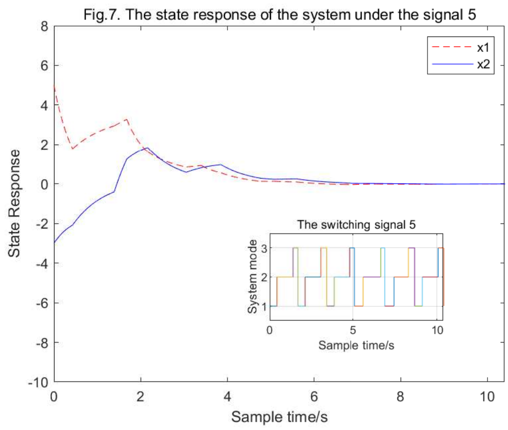


Figure 7. The state response of the system under the signal 5.

Table 3. $\mathfrak{S} = \{1\}$, $\Phi_1 = \{1, 2, 3\}$.

δ	$\tau_{m\Phi_1} = 0.49$, $\tau_{u\Phi_1} = 0.57$, $\tau_{v\Phi_1} = 0.62$, $\tau_{M\Phi_1} = 0.67$		
τ	$[\tau_{m\Phi_i}, \tau_{u\Phi_i})$	$[\tau_{u\Phi_i}, \tau_{v\Phi_i})$	$[\tau_{v\Phi_i}, \tau_{M\Phi_i}]$
μ	$\mu_{11} = 0.68$	$\mu_{21} = 0.95$	$\mu_{31} = 1.42$
λ		$\lambda_1 = 0.57$	
P_1	$\begin{bmatrix} 23.1455 & -1.0184 \\ * & 24.6775 \end{bmatrix}$	$\begin{bmatrix} 18.0000 & -3.4430 \\ * & 15.4327 \end{bmatrix}$	$\begin{bmatrix} 12.2816 & -4.7228 \\ * & 4.5930 \end{bmatrix}$
P_2	$\begin{bmatrix} 47.4731 & -15.6408 \\ * & 26.6013 \end{bmatrix}$	$\begin{bmatrix} 33.2016 & -11.5852 \\ * & 16.4465 \end{bmatrix}$	$\begin{bmatrix} 22.6673 & -10.2381 \\ * & 6.3310 \end{bmatrix}$
P_3	$\begin{bmatrix} 50.9908 & -7.9662 \\ * & 13.9737 \end{bmatrix}$	$\begin{bmatrix} 32.9636 & -6.1425 \\ * & 10.3295 \end{bmatrix}$	$\begin{bmatrix} 19.1526 & -6.2937 \\ * & 4.1589 \end{bmatrix}$
Q_1	$\begin{bmatrix} 43.7268 & -14.1297 \\ * & 6.9276 \end{bmatrix}$	$\begin{bmatrix} 44.7904 & -15.2723 \\ * & 7.4147 \end{bmatrix}$	$\begin{bmatrix} 60.7108 & -22.6981 \\ * & 9.3635 \end{bmatrix}$
Q_2	$\begin{bmatrix} 4.9875 & 4.0591 \\ * & 16.8078 \end{bmatrix}$	$\begin{bmatrix} 5.7717 & 5.1066 \\ * & 14.3445 \end{bmatrix}$	$\begin{bmatrix} 10.3258 & 1.3298 \\ * & 2.0830 \end{bmatrix}$
Q_3	$\begin{bmatrix} 7.6333 & -12.8279 \\ * & 22.9404 \end{bmatrix}$	$\begin{bmatrix} 10.1925 & -14.0171 \\ * & 22.3963 \end{bmatrix}$	$\begin{bmatrix} 27.0090 & -18.0787 \\ * & 14.6216 \end{bmatrix}$
R_1	$\begin{bmatrix} 11.2592 & -2.8717 \\ * & 3.3432 \end{bmatrix}$	$\begin{bmatrix} 8.0780 & -2.5442 \\ * & 1.9918 \end{bmatrix}$	$\begin{bmatrix} 0.5563 & -0.2376 \\ * & 0.1062 \end{bmatrix}$
R_2	$\begin{bmatrix} 4.3474 & 0.9650 \\ * & 11.1927 \end{bmatrix}$	$\begin{bmatrix} 2.5397 & 0.3567 \\ * & 6.1008 \end{bmatrix}$	$\begin{bmatrix} 0.0414 & -0.0334 \\ * & 0.1254 \end{bmatrix}$
R_3	$\begin{bmatrix} 6.6895 & -5.2996 \\ * & 6.8285 \end{bmatrix}$	$\begin{bmatrix} 4.2319 & -3.7296 \\ * & 4.6057 \end{bmatrix}$	$\begin{bmatrix} 0.0580 & -0.0363 \\ * & 0.0289 \end{bmatrix}$
S_1	$\begin{bmatrix} 11.2592 & -2.8717 \\ * & 3.3432 \end{bmatrix}$	$\begin{bmatrix} 7.8090 & -2.4543 \\ * & 1.9411 \end{bmatrix}$	$\begin{bmatrix} 0.5246 & -0.2241 \\ * & 0.1002 \end{bmatrix}$
S_2	$\begin{bmatrix} 7.0833 & 3.5152 \\ * & 20.0495 \end{bmatrix}$	$\begin{bmatrix} 4.0294 & 0.5394 \\ * & 8.4653 \end{bmatrix}$	$\begin{bmatrix} 0.5454 & -0.6141 \\ * & 0.7593 \end{bmatrix}$
S_3	$\begin{bmatrix} 6.3717 & -3.5740 \\ * & 3.7721 \end{bmatrix}$	$\begin{bmatrix} 4.3078 & -2.3080 \\ * & 1.8567 \end{bmatrix}$	$\begin{bmatrix} 0.0377 & -0.0225 \\ * & 0.0192 \end{bmatrix}$

Table 4. $\mathfrak{S} = \{1, 2\}$, $\Phi_1 = \{1, 2\}$, $\Phi_2 = \{3\}$.

δ	$\tau_{m\Phi_1} = 0.65$, $\tau_{u\Phi_1} = 0.71$, $\tau_{v\Phi_1} = 0.75$, $\tau_{M\Phi_1} = 0.78$ $\tau_{m\Phi_2} = 0.39$, $\tau_{u\Phi_2} = 0.44$, $\tau_{v\Phi_2} = 0.52$, $\tau_{M\Phi_2} = 0.55$		
τ	$[\tau_{m\Phi_i}, \tau_{u\Phi_i})$	$[\tau_{u\Phi_i}, \tau_{v\Phi_i})$	$[\tau_{v\Phi_i}, \tau_{M\Phi_i}]$
μ	$\mu_{11} = 0.6$, $\mu_{12} = 0.7$	$\mu_{21} = 1.1$, $\mu_{22} = 1.3$	$\mu_{31} = 1.5$, $\mu_{32} = 1.4$
λ		$\lambda_1 = 0.65$, $\lambda_2 = 0.64$	
P_1	$\begin{bmatrix} 1.8030 & 0.0519 \\ * & 1.9026 \end{bmatrix}$	$\begin{bmatrix} 35.4538 & -0.9615 \\ * & 31.2897 \end{bmatrix}$	$\begin{bmatrix} 32.9410 & -10.7097 \\ * & 9.5317 \end{bmatrix}$
P_2	$\begin{bmatrix} 4.3456 & -1.6147 \\ * & 2.2093 \end{bmatrix}$	$\begin{bmatrix} 75.6495 & -27.1008 \\ * & 39.2251 \end{bmatrix}$	$\begin{bmatrix} 83.3425 & -38.1635 \\ * & 24.0705 \end{bmatrix}$
P_3	$\begin{bmatrix} 3.4673 & -0.5049 \\ * & 0.9682 \end{bmatrix}$	$\begin{bmatrix} 72.6041 & -10.2586 \\ * & 19.8556 \end{bmatrix}$	$\begin{bmatrix} 70.5578 & -23.2847 \\ * & 12.7411 \end{bmatrix}$
Q_1	$\begin{bmatrix} 2.5740 & -0.6251 \\ * & 0.4084 \end{bmatrix}$	$\begin{bmatrix} 60.7027 & -13.7958 \\ * & 7.1664 \end{bmatrix}$	$\begin{bmatrix} 162.6794 & -54.2278 \\ * & 20.8942 \end{bmatrix}$
Q_2	$\begin{bmatrix} 0.6059 & 0.0989 \\ * & 0.9366 \end{bmatrix}$	$\begin{bmatrix} 11.2718 & 3.1598 \\ * & 18.7407 \end{bmatrix}$	$\begin{bmatrix} 45.2944 & 3.4210 \\ * & 3.9775 \end{bmatrix}$
Q_3	$\begin{bmatrix} 0.6441 & -1.0339 \\ * & 1.8021 \end{bmatrix}$	$\begin{bmatrix} 15.7046 & -24.2426 \\ * & 41.4485 \end{bmatrix}$	$\begin{bmatrix} 104.1536 & -62.2539 \\ * & 43.6225 \end{bmatrix}$
R_1	$\begin{bmatrix} 0.7493 & -0.1605 \\ * & 0.2963 \end{bmatrix}$	$\begin{bmatrix} 15.0934 & -3.7218 \\ * & 5.1203 \end{bmatrix}$	$\begin{bmatrix} 2.5914 & -1.1278 \\ * & 0.5088 \end{bmatrix}$
R_2	$\begin{bmatrix} 0.5724 & 0.0004 \\ * & 0.8522 \end{bmatrix}$	$\begin{bmatrix} 9.4656 & 0.4454 \\ * & 15.7757 \end{bmatrix}$	$\begin{bmatrix} 0.7519 & -1.3915 \\ * & 3.0789 \end{bmatrix}$
R_3	$\begin{bmatrix} 0.4958 & -0.3777 \\ * & 0.5543 \end{bmatrix}$	$\begin{bmatrix} 9.5314 & -7.2156 \\ * & 10.1955 \end{bmatrix}$	$\begin{bmatrix} 0.1622 & -0.0990 \\ * & 0.0747 \end{bmatrix}$
S_1	$\begin{bmatrix} 0.7493 & -0.1605 \\ * & 0.2963 \end{bmatrix}$	$\begin{bmatrix} 13.6225 & -3.3477 \\ * & 5.0301 \end{bmatrix}$	$\begin{bmatrix} 2.2158 & -0.9644 \\ * & 0.4354 \end{bmatrix}$
S_2	$\begin{bmatrix} 0.6275 & 0.2973 \\ * & 1.7646 \end{bmatrix}$	$\begin{bmatrix} 11.0030 & 5.0564 \\ * & 31.3379 \end{bmatrix}$	$\begin{bmatrix} 1.5357 & -2.9132 \\ * & 5.9531 \end{bmatrix}$
S_3	$\begin{bmatrix} 0.5196 & -0.3495 \\ * & 0.4397 \end{bmatrix}$	$\begin{bmatrix} 9.6860 & -5.7207 \\ * & 8.1551 \end{bmatrix}$	$\begin{bmatrix} 0.1505 & -0.0912 \\ * & 0.0690 \end{bmatrix}$

Table 5. $\mathfrak{S} = \{1, 2\}$, $\Phi_1 = \{1, 3\}$, $\Phi_2 = \{2\}$.

δ	$\tau_{m\Phi_1} = 0.58, \tau_{u\Phi_1} = 0.63, \tau_{v\Phi_1} = 0.66, \tau_{M\Phi_1} = 0.69$ $\tau_{m\Phi_2} = 1.05, \tau_{u\Phi_2} = 1.17, \tau_{v\Phi_2} = 1.25, \tau_{M\Phi_2} = 1.33$		
τ	$[\tau_{m\Phi_i}, \tau_{u\Phi_i})$	$[\tau_{u\Phi_i}, \tau_{v\Phi_i})$	$[\tau_{v\Phi_i}, \tau_{M\Phi_i}]$
μ	$\mu_{11} = 0.73,$ $\mu_{12} = 0.5$	$\mu_{21} = 1.1,$ $\mu_{22} = 1.3$	$\mu_{31} = 1.5,$ $\mu_{32} = 1.46$
λ		$\lambda_1 = 0.45, \lambda_2 = 0.52$	
P_1	$\begin{bmatrix} 25.1826 & 5.0234 \\ \ast & 46.4133 \end{bmatrix}$	$\begin{bmatrix} 0.7806 & 0.0539 \\ \ast & 1.1895 \end{bmatrix}$	$\begin{bmatrix} 92.8432 & -34.6199 \\ \ast & 31.7638 \end{bmatrix}$
P_2	$\begin{bmatrix} 72.0757 & -24.2516 \\ \ast & 39.5019 \end{bmatrix}$	$\begin{bmatrix} 2.5068 & -0.8700 \\ \ast & 1.2439 \end{bmatrix}$	$\begin{bmatrix} 110.9377 & -52.8871 \\ \ast & 34.2876 \end{bmatrix}$
P_3	$\begin{bmatrix} 86.6261 & -13.5677 \\ \ast & 14.9473 \end{bmatrix}$	$\begin{bmatrix} 2.4189 & -0.3971 \\ \ast & 0.4561 \end{bmatrix}$	$\begin{bmatrix} 97.3575 & -29.5522 \\ \ast & 15.5354 \end{bmatrix}$
Q_1	$\begin{bmatrix} 41.6405 & -14.3029 \\ \ast & 7.9802 \end{bmatrix}$	$\begin{bmatrix} 1.5272 & -0.5695 \\ \ast & 0.2678 \end{bmatrix}$	$\begin{bmatrix} 256.1535 & -106.5029 \\ \ast & 46.3996 \end{bmatrix}$
Q_2	$\begin{bmatrix} 7.6862 & 4.6802 \\ \ast & 18.3632 \end{bmatrix}$	$\begin{bmatrix} 0.3157 & 0.2196 \\ \ast & 0.6849 \end{bmatrix}$	$\begin{bmatrix} 47.6677 & 1.0352 \\ \ast & 18.9710 \end{bmatrix}$
Q_3	$\begin{bmatrix} 10.5528 & -13.8201 \\ \ast & 21.2454 \end{bmatrix}$	$\begin{bmatrix} 0.3258 & -0.4712 \\ \ast & 0.7143 \end{bmatrix}$	$\begin{bmatrix} 120.1687 & -73.7954 \\ \ast & 51.5851 \end{bmatrix}$
R_1	$\begin{bmatrix} 13.6659 & -3.3204 \\ \ast & 4.4454 \end{bmatrix}$	$\begin{bmatrix} 0.4175 & -0.1063 \\ \ast & 0.1208 \end{bmatrix}$	$\begin{bmatrix} 90.0255 & -26.1669 \\ \ast & 7.6379 \end{bmatrix}$
R_2	$\begin{bmatrix} 5.9166 & -0.0680 \\ \ast & 9.4563 \end{bmatrix}$	$\begin{bmatrix} 0.2020 & -0.0048 \\ \ast & 0.2804 \end{bmatrix}$	$\begin{bmatrix} 0.1141 & -0.0802 \\ \ast & 0.1034 \end{bmatrix}$
R_3	$\begin{bmatrix} 10.4183 & -7.1803 \\ \ast & 8.8115 \end{bmatrix}$	$\begin{bmatrix} 0.3034 & -0.2159 \\ \ast & 0.2686 \end{bmatrix}$	$\begin{bmatrix} 0.6266 & -0.6588 \\ \ast & 0.7310 \end{bmatrix}$
S_1	$\begin{bmatrix} 13.6659 & -3.3204 \\ \ast & 4.4454 \end{bmatrix}$	$\begin{bmatrix} 0.4040 & -0.1004 \\ \ast & 0.1195 \end{bmatrix}$	$\begin{bmatrix} 90.0962 & -26.1791 \\ \ast & 7.6375 \end{bmatrix}$
S_2	$\begin{bmatrix} 8.7216 & 7.9675 \\ \ast & 35.0883 \end{bmatrix}$	$\begin{bmatrix} 0.2739 & 0.2216 \\ \ast & 0.9458 \end{bmatrix}$	$\begin{bmatrix} 0.9355 & -0.7122 \\ \ast & 0.5871 \end{bmatrix}$
S_3	$\begin{bmatrix} 9.6164 & -5.1749 \\ \ast & 5.9313 \end{bmatrix}$	$\begin{bmatrix} 0.2925 & -0.1577 \\ \ast & 0.1613 \end{bmatrix}$	$\begin{bmatrix} 0.2516 & -0.2539 \\ \ast & 0.2926 \end{bmatrix}$

Table 6. $\mathfrak{S} = \{1, 2\}$, $\Phi_1 = \{1\}$, $\Phi_2 = \{2, 3\}$.

δ	$\tau_{m\Phi_1} = 0.42, \tau_{u\Phi_1} = 0.46, \tau_{v\Phi_1} = 0.56, \tau_{M\Phi_1} = 0.61$ $\tau_{m\Phi_2} = 0.54, \tau_{u\Phi_2} = 0.68, \tau_{v\Phi_2} = 0.74, \tau_{M\Phi_2} = 0.82$		
τ	$[\tau_{m\Phi_i}, \tau_{u\Phi_i})$	$[\tau_{u\Phi_i}, \tau_{v\Phi_i})$	$[\tau_{v\Phi_i}, \tau_{M\Phi_i}]$
μ	$\mu_{11} = 0.7,$ $\mu_{12} = 0.6$	$\mu_{21} = 1.1,$ $\mu_{22} = 0.9$	$\mu_{31} = 1.3,$ $\mu_{32} = 1.2$
λ		$\lambda_1 = 0.58, \lambda_2 = 0.62$	
P_1	$\begin{bmatrix} 14.2412 & -0.9687 \\ \ast & 12.7752 \end{bmatrix}$	$\begin{bmatrix} 12.0771 & -1.3657 \\ \ast & 9.7231 \end{bmatrix}$	$\begin{bmatrix} 0.1492 & -0.0140 \\ \ast & 0.0578 \end{bmatrix}$
P_2	$\begin{bmatrix} 25.7962 & -8.3240 \\ \ast & 15.2188 \end{bmatrix}$	$\begin{bmatrix} 20.6336 & -6.5373 \\ \ast & 10.8676 \end{bmatrix}$	$\begin{bmatrix} 0.1965 & -0.0523 \\ \ast & 0.0536 \end{bmatrix}$
P_3	$\begin{bmatrix} 30.8084 & -4.0040 \\ \ast & 8.0691 \end{bmatrix}$	$\begin{bmatrix} 23.6927 & -3.3149 \\ \ast & 6.6102 \end{bmatrix}$	$\begin{bmatrix} 0.2553 & -0.0292 \\ \ast & 0.0405 \end{bmatrix}$
Q_1	$\begin{bmatrix} 30.4620 & -9.5505 \\ \ast & 4.6947 \end{bmatrix}$	$\begin{bmatrix} 30.2020 & -9.3917 \\ \ast & 5.1282 \end{bmatrix}$	$\begin{bmatrix} 0.6383 & -0.1429 \\ \ast & 0.0538 \end{bmatrix}$
Q_2	$\begin{bmatrix} 2.9149 & 1.4889 \\ \ast & 8.8800 \end{bmatrix}$	$\begin{bmatrix} 2.3365 & 1.6878 \\ \ast & 7.5598 \end{bmatrix}$	$\begin{bmatrix} 0.0582 & 0.0632 \\ \ast & 0.0770 \end{bmatrix}$
Q_3	$\begin{bmatrix} 3.1889 & -4.5924 \\ \ast & 9.5792 \end{bmatrix}$	$\begin{bmatrix} 2.6112 & -3.9970 \\ \ast & 8.4352 \end{bmatrix}$	$\begin{bmatrix} 0.0446 & -0.0517 \\ \ast & 0.1337 \end{bmatrix}$
R_1	$\begin{bmatrix} 7.2379 & -1.8822 \\ \ast & 1.6869 \end{bmatrix}$	$\begin{bmatrix} 5.7849 & -1.6328 \\ \ast & 1.0970 \end{bmatrix}$	$\begin{bmatrix} 0.0191 & -0.0076 \\ \ast & 0.0034 \end{bmatrix}$
R_2	$\begin{bmatrix} 2.8166 & 0.4395 \\ \ast & 6.3467 \end{bmatrix}$	$\begin{bmatrix} 2.1749 & 0.5789 \\ \ast & 4.7941 \end{bmatrix}$	$\begin{bmatrix} 0.0060 & 0.0027 \\ \ast & 0.0060 \end{bmatrix}$
R_3	$\begin{bmatrix} 3.3693 & -2.7784 \\ \ast & 3.7029 \end{bmatrix}$	$\begin{bmatrix} 2.7576 & -2.4861 \\ \ast & 3.2929 \end{bmatrix}$	$\begin{bmatrix} 0.0324 & -0.0170 \\ \ast & 0.0097 \end{bmatrix}$
S_1	$\begin{bmatrix} 7.2379 & -1.8822 \\ \ast & 1.6869 \end{bmatrix}$	$\begin{bmatrix} 5.7105 & -1.6113 \\ \ast & 1.0821 \end{bmatrix}$	$\begin{bmatrix} 0.0179 & -0.0071 \\ \ast & 0.0032 \end{bmatrix}$
S_2	$\begin{bmatrix} 4.1120 & 2.2650 \\ \ast & 12.8642 \end{bmatrix}$	$\begin{bmatrix} 3.3561 & 1.5102 \\ \ast & 7.6711 \end{bmatrix}$	$\begin{bmatrix} 0.0264 & 0.0061 \\ \ast & 0.0059 \end{bmatrix}$
S_3	$\begin{bmatrix} 3.5775 & -2.0192 \\ \ast & 2.8210 \end{bmatrix}$	$\begin{bmatrix} 2.9734 & -1.6497 \\ \ast & 2.0698 \end{bmatrix}$	$\begin{bmatrix} 0.0176 & -0.0085 \\ \ast & 0.0047 \end{bmatrix}$

Table 7. $\mathfrak{S} = \{1, 2, 3\}$, $\Phi_1 = \{1\}$, $\Phi_2 = \{2\}$; $\Phi_3 = \{3\}$.

δ	$\tau_{m\Phi_1} = 0.39, \tau_{u\Phi_1} = 0.43, \tau_{v\Phi_1} = 0.51, \tau_{M\Phi_1} = 0.57$ $\tau_{m\Phi_2} = 0.84, \tau_{u\Phi_2} = 0.92, \tau_{v\Phi_2} = 1.01, \tau_{M\Phi_2} = 1.11$ $\tau_{m\Phi_3} = 0.23, \tau_{u\Phi_3} = 0.29, \tau_{v\Phi_3} = 0.35, \tau_{M\Phi_3} = 0.38$		
τ	$[\tau_{m\Phi_i}, \tau_{u\Phi_i})$	$[\tau_{u\Phi_i}, \tau_{v\Phi_i})$	$[\tau_{v\Phi_i}, \tau_{M\Phi_i}]$
μ	$\mu_{11} = 0.7,$ $\mu_{12} = 0.6,$ $\mu_{13} = 0.8$	$\mu_{21} = 0.9,$ $\mu_{22} = 1.1,$ $\mu_{23} = 0.9$	$\mu_{31} = 1.2,$ $\mu_{32} = 1.4,$ $\mu_{33} = 1.1$
λ	$\lambda_1 = 0.62, \lambda_2 = 0.46, \lambda_3 = 0.58$		
P_1	$\begin{bmatrix} 170.2179 & -26.9700 \\ * & 114.8395 \end{bmatrix}$	$\begin{bmatrix} 81.9472 & -18.0570 \\ * & 49.1218 \end{bmatrix}$	$\begin{bmatrix} 0.5667 & -0.2133 \\ * & 0.2024 \end{bmatrix}$
P_2	$\begin{bmatrix} 339.0560 & -129.4043 \\ * & 171.8188 \end{bmatrix}$	$\begin{bmatrix} 161.2581 & -63.8093 \\ * & 73.4982 \end{bmatrix}$	$\begin{bmatrix} 1.0658 & -0.4673 \\ * & 0.2870 \end{bmatrix}$
P_3	$\begin{bmatrix} 248.5446 & -45.5923 \\ * & 76.7228 \end{bmatrix}$	$\begin{bmatrix} 121.5331 & -26.1927 \\ * & 40.0762 \end{bmatrix}$	$\begin{bmatrix} 0.9123 & -0.3153 \\ * & 0.2129 \end{bmatrix}$
Q_1	$\begin{bmatrix} 304.5953 & -96.3485 \\ * & 46.1145 \end{bmatrix}$	$\begin{bmatrix} 212.5213 & -68.0405 \\ * & 30.9250 \end{bmatrix}$	$\begin{bmatrix} 2.7818 & -1.0215 \\ * & 0.4200 \end{bmatrix}$
Q_2	$\begin{bmatrix} 30.3074 & 13.9767 \\ * & 67.1472 \end{bmatrix}$	$\begin{bmatrix} 19.6843 & 15.5687 \\ * & 46.7836 \end{bmatrix}$	$\begin{bmatrix} 0.3409 & 0.1537 \\ * & 0.0751 \end{bmatrix}$
Q_3	$\begin{bmatrix} 72.6087 & -96.0867 \\ * & 144.0311 \end{bmatrix}$	$\begin{bmatrix} 52.2238 & -61.5195 \\ * & 85.7181 \end{bmatrix}$	$\begin{bmatrix} 1.3211 & -0.8988 \\ * & 0.7293 \end{bmatrix}$
R_1	$\begin{bmatrix} 74.6952 & -20.5649 \\ * & 17.3008 \end{bmatrix}$	$\begin{bmatrix} 39.6596 & -12.2920 \\ * & 7.8771 \end{bmatrix}$	$\begin{bmatrix} 0.0303 & -0.0133 \\ * & 0.0062 \end{bmatrix}$
R_2	$\begin{bmatrix} 25.2725 & 0.9295 \\ * & 43.0587 \end{bmatrix}$	$\begin{bmatrix} 11.5189 & 0.6762 \\ * & 18.6364 \end{bmatrix}$	$\begin{bmatrix} 0.0034 & -0.0014 \\ * & 0.0030 \end{bmatrix}$
R_3	$\begin{bmatrix} 37.5371 & -35.2715 \\ * & 45.1790 \end{bmatrix}$	$\begin{bmatrix} 19.1271 & -17.6947 \\ * & 21.5690 \end{bmatrix}$	$\begin{bmatrix} 0.0061 & -0.0044 \\ * & 0.0036 \end{bmatrix}$
S_1	$\begin{bmatrix} 74.6952 & -20.5649 \\ * & 17.3008 \end{bmatrix}$	$\begin{bmatrix} 32.9758 & -10.5545 \\ * & 7.2031 \end{bmatrix}$	$\begin{bmatrix} 0.0276 & -0.0122 \\ * & 0.0058 \end{bmatrix}$
S_2	$\begin{bmatrix} 38.1771 & 31.6006 \\ * & 137.2085 \end{bmatrix}$	$\begin{bmatrix} 15.7266 & 6.7316 \\ * & 37.4554 \end{bmatrix}$	$\begin{bmatrix} 0.0552 & -0.0333 \\ * & 0.0222 \end{bmatrix}$
S_3	$\begin{bmatrix} 35.3095 & -26.7339 \\ * & 36.0276 \end{bmatrix}$	$\begin{bmatrix} 17.1981 & -12.6850 \\ * & 15.0303 \end{bmatrix}$	$\begin{bmatrix} 0.0039 & -0.0027 \\ * & 0.0022 \end{bmatrix}$

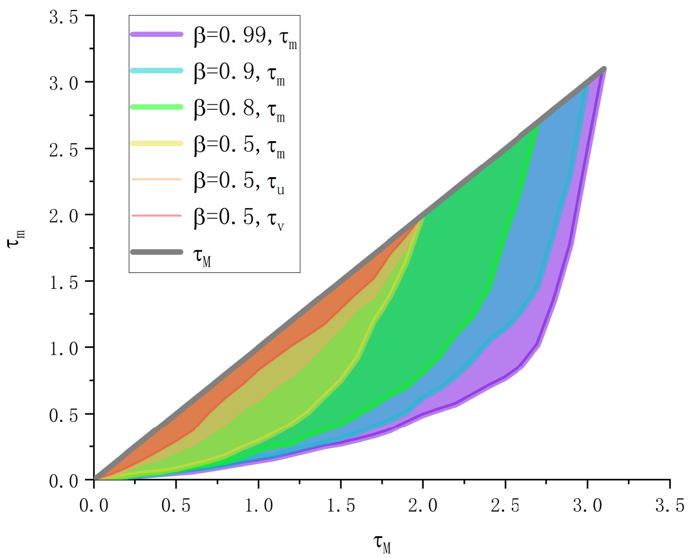


Figure 8. Stability regions under different β and τ conditions.

5. Conclusions

Under the assumption that all modes are unstable, this paper studies the stabilization of switching systems using time-dependent switching signals. Several stability conditions are proposed in the framework of DT switching, using the stability characteristics of switching behavior to compensate for unstable modes and state divergence caused by unstable switching instant and providing a new solution to stability control problems. Finally, the effectiveness of the proposed method is verified by simulation.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflict of interest.

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