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Research article

A reliable numerical algorithm based on an operational matrix method for treatment of a fractional order computer virus model

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Abstract: A computer network can detect potential viruses through the use of kill signals, thereby minimizing the risk of virus propagation. In the realm of computer security and defensive strategies, computer viruses play a significant role. Understanding of their spread and extension is a crucial component. To address this issue of computer virus spread, we employ a fractional epidemiological SIRA model by utilizing the Caputo derivative. To solve the fractional-order computer virus model, we employ a computational technique known as the Jacobi collocation operational matrix method. This operational matrix transforms the problem of arbitrary order into a system of nonlinear algebraic equations. To analyze this system of arbitrary order, we derive an approximate solution for the fractional computer virus model, also considering the Vieta Lucas polynomials. Numerical simulations are performed and graphical representations are provided to illustrate the impact of order of the fractional derivative on different profiles.

Keywords: epidemiological model; Jacobi polynomial; SIRA; operational matrix of differentiation; collocation method
Mathematics Subject Classification: 26A33, 33C45, 65L05, 92D30

1. Introduction

Computer viruses are diminutive software programs clandestinely infiltrating computers, operating surreptitiously to dismantle software and hardware components. Their proliferation accelerated, particularly with rapid strains during the 1990s, they marked a notable surge post-mid-1980s. By the close of the 1990s, the viral landscape had metamorphosed dramatically, boasting hundreds of thousands of distinct strains, perpetually adapting and morphing to evade detection and countermeasures. During the 1980s, computer viruses introduced various forms of malicious code, inflicting harm upon small entities within systems unbeknownst to their operators. The detrimental impacts of these agents were relatively mild at the time, and their propagation occurred at a slower pace. In today's interconnected global society, the proliferation of viruses has accelerated significantly. Their unfair activities encompass a range of malicious actions, such as infiltrating email addresses and bank accounts, stealing passwords, manipulating data, causing financial loss and disrupting the basic operation of machines [1–3].

Computer virus epidemiological models utilize computational methods to simulate the propagation of computer viruses within interconnected networks. These models incorporate factors such as network topology, user behavior and virus characteristics to predict how malware spreads. By running simulations, the models aid in understanding virus dynamics, evaluating the impact of cybersecurity measures and guiding strategies for virus containment and prevention. These models are crucial tools for cybersecurity experts, helping them anticipate and respond to evolving cyber threats, safeguarding digital systems and data integrity. Kephart et al. [4] were pioneers in the initial exploration of modeling analogies between computer viruses and biological diseases, incorporating topological aspects. Billings et al. [5] examined the uniform prediction of computer virus propagation within interconnected networks. To obtain more information on virus propagation, various epidemiological models have been examined, such as SLBS [6, 7], SIRA [8, 9], SIRS [10, 11] and SIERS [12]. Han and Tan [1] examined the dynamic behavior of computer viruses on the Internet. In exploring the relationship between epidemiology and computer viruses, Murray [13] appears to be among the early contributors and while he did not delve into specific models, his work focused on drawing analogies between computer virus dynamics and epidemiological defense strategies employed in public health.
Nonlinear equations and systems find diverse applications across science and engineering, from modeling physical phenomena to optimizing complex systems. The richness of approaches for extracting solitary solutions, including numerical methods, analytical techniques and artificial intelligence algorithms, underscores the versatility and importance of nonlinear systems in various fields such as the KdV system [14], resonant wave equation [15], plasma [16], fuzzy fractional PDEs [17] and Schrodinger model [18]. Chen et al. [19, 20] investigated a species interaction model and spatiotemporal SI model.

Fractional calculus and fractional differential equations have garnered significant attention in recent decades, owing to their promising applications in the realms of science and engineering. Fractional derivatives enable the accurate representation of physical phenomena, incorporating both present and past time dependencies. Furthermore, fractional calculus finds practical utility across various scientific and engineering domains, offering versatile solutions and insights for real-world challenges in these fields [21–24]. These fractional operators, characterized by their non-local nature, inherently encapsulate the system's historical information, leading to more realistic mathematical simulations. Additional information on daily life implementation of fractional calculus can be acquired [25–28].

The operational matrix collocation techniques are a numerical scheme employed for solving differential equations, particularly those of fractional order. This scheme offers an efficient approach by transforming fractional differential equations into algebraic systems through the use of operational matrices. These methods are known for their accuracy and versatility in modeling complex phenomena, making it valuable in several areas such as physics, finance and engineering. By discretizing fractional differential equations, the operational matrix collocation method facilitates the solution of problems that involve fractional derivatives, contributing to a better understanding of intricate processes. For a deeper understanding of the operational matrix collocation methods, one can refer to its application with various orthogonal polynomials such as Bernoulli polynomials [29], Genocchi polynomials [30], Laguerre polynomials [31] and Jacobi polynomials [32, 33]. These polynomial functions play a significant role in the operational matrix collocation, helping to solve differential equations efficiently and accurately by transforming them into algebraic systems.

The novelty of this article lies in the transformation of the conventional ordinary differential equations into a fractional differential equation of a computer virus model and finds the solution of governing model with the help of the Jacobi operational matrix with the collocation method. The utilization of the operational matrix collocation method for solving these fractional differential equations represents a significant enhancement to the existing SIRA model [8, 34], especially in the context of countering computer virus propagation. This innovative approach opens new avenues for understanding and addressing the dynamics of virus spread, contributing to the field of computer security and virus containment strategies. In this paper, we use the advantages of the aforementioned numerical algorithm to approximate solutions for our fractional SIRA model. By doing so, we aim to analyze the dynamics of the model with the utmost precision, minimizing errors in our analysis. The recommended approach combines the collocation method with the operational matrix of differentiation method for Jacobi polynomials. We obtain a system of nonlinear algebraic equations (NLAEs) whose approximate solutions provide results to combining the collocation approach with the operational matrix of fractional differentiations. Behavior of the solution is presented for the distinct fractional orders for the SIRA fractional model. We provide a comparative study with solutions obtained by the Vieta Lucas polynomial. The organization of this study is as follows: In Section 1, we describe the antidotal computer virus model. The basic definition of the calculus of fractional order and properties of Jacobi and Vieta Lucas polynomial are discussed in Section 3. In Section 4, we introduce the fractional SIRA model. In Section 5, a numerical scheme is discussed. we demonstrate the Jacobi collocation method to find a solution of the arbitrary order SIRA model in Section 6. Section 7 is dedicated to error and convergence analysis of the scheme. Numerical results are discussed in Section 8. Concluding remarks are given in Section 9.

2. The SIRA computer virus model

In this section, we introduce a key component designed to prevent the propagation of computer viruses across networks. Within our considered model, the total population N consists of four distinct types of computers at time ξ, $S(\xi)$ defines the numbers of susceptible computers but not those infected, the numbers of infected computers and those removed from a network at time ξ, represented by $I(\xi)$,

and $R(\xi)$ is the number of removed computers but not those infected at ξ in the network and computers that equipped with antivirus programs denoted by $A(\xi)$.
Our system modeling is predicated on the following assumptions [34]:

. The network expands by Ξ through the addition of new computers.

. The mortality rate for each group, excluding those attributed to virus causes, is represented as μ.

. Susceptible individuals represented by $S(\xi)$ become infected as determined by the probability of infection during interactions with infected individuals and the rate is proportional to $S(\xi)I(\xi)$, by a factor of β.

. At a rate α, the transfer of $S(\xi)$ to antidotes occurs proportional to $S(\xi)A(\xi)$. This implies that susceptible computers establish effective communication with the antivirus, which subsequently installs its protective software on the susceptible machine.

. An infected computer equipped with an antivirus program, effective against known viruses, undergoes a transformation process. It either transitions to an antidotal state at a rate γ proportional to $A(\xi)I(\xi)$ or it becomes susceptible to additional infections at a rate δ.

. Because viruses are periodically detected and eliminated, infected computers are constantly reverting to a susceptible state at a consistent rate denoted as c. However, it is important to note that certain antivirus softwares may not possess the capability to completely eradicate all forms of malware.

. At a rate of ε, machines that have been removed from the network can be restored and transformed into a vulnerable ones.

Khanh [34] introduced the conventional SIRA computer virus propagation model with four types of computers for a total population N expressed as follows:

$$\frac{dS}{d\xi} = \Xi - \alpha S(\xi)A(\xi) - \beta S(\xi)I(\xi) + cI(\xi) + \varepsilon R(\xi) - \mu S(\xi), \tag{2.1}$$

$$\frac{dI}{d\xi} = \beta S(\xi)I(\xi) - \gamma A(\xi)I(\xi) - (c + \delta - \mu)I(\xi), \tag{2.2}$$

$$\frac{dR}{d\xi} = \delta I(\xi) - (\varepsilon + \mu)R(\xi), \tag{2.3}$$

$$\frac{dA}{d\xi} = \alpha S(\xi)A(\xi) + \gamma A(\xi)I(\xi) - \mu A(\xi). \tag{2.4}$$

Also, at time ξ, $N(\xi) = S(\xi) + I(\xi) + R(\xi) + A(\xi)$ and $N(\xi)$ is constant and equal to N.

3. Preliminaries

We applied the Caputo type derivative of arbitrary order in this research paper. The Caputo derivative of fractional order $\rho \geq 0$ is provided [27] as

$$(D^{\rho} g(\xi)) = \begin{cases} \frac{1}{\Gamma(l-\rho)} \int_0^{\xi} (\xi - t)^{l-\rho-1} \frac{d^l}{dt^l} g(t)dt, & l - 1 < \rho < l, \\ \frac{d^l}{d\xi^l} g(\xi), & \rho = l \in N. \end{cases} \tag{3.1}$$

3.1. Jacobi polynomials

The analytical form of the shifted Jacobi polynomials on [0, 1] [35] is given as

$$M_k(z) = \sum_{l=0}^{k}(-1)^{k-l}\frac{\Gamma(k+b+1)\Gamma(1+k+a+l+b)}{\Gamma(l+b+1)\Gamma(k+a+b+1)(k-l)!l!}z^l,$$

where a and b are Jacobi polynomial parameters [35].

The orthogonal property of Jacobi polynomials:

$$\int_0^1 M_u^{(a,b)}(z)\mu^{(a,b)}(z)M_v^{(a,b)}(z)dz = \delta_{uv}\phi_u^{a,b},$$

where δ_{uv} is the Kronecker delta function and $\mu^{(a,b)}(z)$ is the weight function defined as

$$\mu^{(a,b)}(z) = (1-z)^a z^b,$$

and

$$\phi_u^{a,b} = \frac{\Gamma(u+a+1)\Gamma(u+b+1)}{(2u+a+b+1)u!\Gamma(u+a+b+1)}.$$

The function g defined in $L^2[0,1]$, having $|g''(\xi)| \leq K$, can be expanded as an infinite sum of the shifted Jacobi polynomials:

$$g(\xi) = \lim_{q\to\infty}\sum_{i=0}^{q} c_i M_i(\xi), \tag{3.2}$$

where

$$c_i = \frac{1}{\phi_i^{(a,b)}}\int_0^1 g(\xi)M_i(\xi)\mu_i^{(a,b)}d\xi; \quad i = 0,1,2,\ldots,. \tag{3.3}$$

Taking finite dimension approximations in Eq (3.2), we find

$$g(\xi) \cong \sum_{i=0}^{n} c_i M_i(\xi) = \Pi^T M_n(\xi), \tag{3.4}$$

where Π and $M_n(\xi)$ are $(n+1)\times 1$ matrices represented by

$$\Pi = [\Pi_0, \Pi_1, \ldots., \Pi_n]^T \text{ and } M_n(\xi) = \left[M_0(\xi), M_1(\xi), \ldots . M_n(\xi)\right]^T. \tag{3.5}$$

Theorem 3.1. If $M_n(\xi) = \left[M_0(\xi), M_1(\xi), \ldots\ldots., M_n(\xi)\right]^T$ is Jacobi polynomials vector and $\rho > 0$, then

$$D^{\rho}M_i(\xi) = D^{(\rho)}M_n(\xi), \tag{3.6}$$

where $D^{(\rho)}$ is $(n+1)\times(n+1)$ operational matrix of Caputo derivative of fractional order ρ and is specified as [33, 35, 36]:

$$D^{(\rho)} = \sum_{l=[\rho]}^{k}(-1)^{k-l}\frac{\Gamma(1+k+b)\Gamma(1+a+k+b+l)}{(k-l)!\Gamma(b+1+l)\Gamma(a+k+b+1)\Gamma(1-\rho+l)}$$

$$*\sum_{m=0}^{i}(-1)^{i-m}\frac{i!\Gamma(1+a)\Gamma(a+i+b+m+1)\Gamma(1+l-\rho++mb)(2i+a+b+1)}{(i-m)!\Gamma(1+i+a)\Gamma(a+m+1)(m)!\Gamma(2+l-\rho+a+b+m)}.$$

Proof. Please see [33, 35, 36].

3.2. Vieta-Lucas polynomials

The analytical form of the shifted Vieta Lucas polynomials on [0, 1] [37] is given as

$$Y_n(\tau) = 2n \sum_{J=0}^{n} \frac{(-1)^J 4^{n-J} (2n-J-1)!}{J!(2n-2J)!} \tau^{n-J}; n \geq 1, \tag{3.7}$$

with $Y_0(\tau) = 2$.

The function f defined in $L^2[0,1]$, having $|f''(\tau)| \leq A$, can be expanded as an infinite sum of the shifted Vieta Lucas polynomials:

$$f(\tau) = \lim_{r\to\infty} \sum_{i=0}^{r} p_i Y_i(\tau), \tag{3.8}$$

where

$$p_i = \frac{1}{\theta_i \pi} \int_0^1 f(\tau) y_i(\tau) u(\tau) d\tau; \quad i = 0, 1, 2, \ldots, u(\tau) = \frac{1}{\sqrt{\tau - \tau^2}}, \quad \theta_0 = 4 \ \textit{and} \ \theta_i = 2 (i \geq 1). \tag{3.9}$$

Taking finite dimension approximations in Eq (3.8), we find

$$f(\tau) \cong \sum_{i=0}^{n} a_i Y_i(\tau) = Q^T Y_n(\tau), \tag{3.10}$$

where Q and $Y_n(\tau)$ are $(n+1) \times 1$ matrices represented by

$$Q = [q_0, q_1, \ldots, q_n]^T \text{ and } Y_n(\tau) = [Y_0(\tau), Y_1(\tau), \ldots . Y_n(\tau)]^T . \tag{3.11}$$

Theorem 3.2. If $Y_n(\tau) = [Y_0(\tau), Y_1(\tau), \ldots \ldots ., Y_n(\tau)]^T$ is Vieta Lucas polynomials vector and $\gamma > 0$, then

$$D^\gamma Y_i(\tau) = D^{(\gamma)} Y_n(\tau), \tag{3.12}$$

where $D^{(\gamma)}$ is $(n+1) \times (n+1)$ operational matrix of Caputo derivative of fractional order γ and is specified as [37]:

$$D^{(\gamma)} = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 \\ \sum_{k=0}^{i-\lceil\gamma\rceil} \Omega_{i,0,k} & \sum_{k=0}^{i-\lceil\gamma\rceil} \Omega_{i,1,k} & \cdots & \sum_{k=0}^{i-\lceil\gamma\rceil} \Omega_{i,m,k} \\ \vdots & \vdots & \cdots & \vdots \\ \sum_{k=0}^{m-\lceil\gamma\rceil} \Omega_{m,0,k} & \sum_{k=0}^{m-\lceil\gamma\rceil} \Omega_{m,1,k} & \cdots & \sum_{k=0}^{m-\lceil\gamma\rceil} \Omega_{m,m,k} \end{pmatrix} \text{ and } \Omega_{i,j,k} \text{ is given } \textit{by}$$

$$\Omega_{i,j,k} = \begin{cases} i \sum_{k=0}^{i-\lceil\gamma\rceil} (-1)^k \frac{4^{i-k}\Gamma(2i-k)\Gamma(i-k+1)\Gamma(i-k-\gamma+1/2)}{\sqrt{\pi}\Gamma(k+1)\Gamma(2i-2k+1)\Gamma(i-k-\gamma+1)^2}, & j = 0, \\ 2i \sum_{k=0}^{i-\lceil\gamma\rceil} \sum_{r=0}^{j} \frac{(-1)^{k+r}}{\sqrt{\pi}} \frac{4^{i-k}\Gamma(2i-k)\Gamma(i-k+1)}{\Gamma(k+1)\Gamma(2i-2k+1)\Gamma(i-k-\gamma+1)} & \\ \times \frac{4^{j-r}\Gamma(2j-r)\Gamma(i+j-k-r-\gamma+1/2)}{\Gamma(r+1)\Gamma(2j-2r+1)\Gamma(i+j+k-r-\gamma+1)}, & j = 1, 2, 3, \ldots \end{cases}$$

Proof. Please see [37].

4. Fractional SIRA computer virus model

The fractional SIRA model in the Caputo sense [8, 34] with the arbitrary order ρ is expressed as

$$D^{\rho}S(\xi) = \Xi - \alpha S(\xi)A(\xi) - \beta S(\xi)I(\xi) + cI(\xi) + \varepsilon R(\xi) - \mu S(\xi), \tag{4.1}$$

$$D^{\rho}I(\xi) = \beta S(\xi)I(\xi) - \gamma A(\xi)I(\xi) - (c + \delta - \mu)I(\xi), \tag{4.2}$$

$$D^{\rho}R(\xi) = \delta I(\xi) - (\varepsilon + \mu)R(\xi), \tag{4.3}$$

$$D^{\rho}A(\xi) = \alpha S(\xi)A(\xi) + \gamma A(\xi)I(\xi) - \mu A(\xi). \tag{4.4}$$

5. Numerical scheme

Let $S(\xi)$, $I(\xi)$, $R(\xi)$ and $A(\xi)$ be defined as square summable function in $L^2[0, 1]$. Thus, by Eq (3.4), they can be approximated subsequently:

$$S(\xi) = \sum_{i=0}^{n} c_i M_i(\xi) = \Pi_1^T M_n(\xi), \tag{5.1}$$

$$I(\xi) = \sum_{i=0}^{n} c_i M_i(\xi) = \Pi_2^T M_n(\xi), \tag{5.2}$$

$$R(\xi) = \sum_{i=0}^{n} c_i M_i(\xi) = \Pi_3^T M_n(\xi), \tag{5.3}$$

$$A(\xi) = \sum_{i=0}^{n} c_i M_i(\xi) = \Pi_4^T M_n(\xi). \tag{5.4}$$

Now, by applying ρ order Caputo derivative in Eqs (5.1)–(5.4), we get

$$D^{\rho}S(\xi) = \Pi_1^T D^{\rho} M_n(\xi) \cong \Pi_1^T D^{(\rho)} M_n(\xi), \tag{5.5}$$

$$D^{\rho}I(\xi) = \Pi_2^T D^{\rho} M_n(\xi) \cong \Pi_2^T D^{(\rho)} M_n(\xi), \tag{5.6}$$

$$D^{\rho}R(\xi) = \Pi_3^T D^{\rho} M_n(\xi) \cong \Pi_3^T D^{(\rho)} M_n(\xi), \tag{5.7}$$

$$D^{\rho}A(\xi) = \Pi_4^T D^{\rho} M_n(\xi) \cong \Pi_4^T D^{(\rho)} M_n(\xi), \tag{5.8}$$

where $D^{(\rho)}$ is an operational matrix of the Caputo derivative for the Jacobi polynomial of arbitrary order ρ.

Also, from Eqs (5.1) to (5.4), we obtain

$$S(0) = \Pi_1^T M_n(0), \tag{5.9}$$

$$I(0) = \Pi_2^T M_n(0), \tag{5.10}$$

$$R(0) = \Pi_3^T M_n(0), \tag{5.11}$$

$$A(0) = \Pi_4^T M_n(0). \tag{5.12}$$

6. Numerical simulation of the arbitrary order SIRA model

Grouping Eqs (4.1) and (5.5) with use of Eqs (5.1)–(5.4), we obtain

$$\Pi_1^T D^{(\rho)} M_n(\xi) - \Xi + \alpha[(\Pi_1^T M_n(\xi))(\Pi_4^T M_n(\xi))] + \beta[(\Pi_1^T M_n(\xi))\Pi_2^T M_n(\xi)] - c(\Pi_2^T M_n(\xi)) - \varepsilon(\Pi_4^T M_n(\xi)) + \mu(\Pi_1^T M_n(\xi)) = 0. \tag{6.1}$$

Grouping Eqs (4.2) and (5.6) with use of Eqs (5.1)–(5.4), we get

$$\Pi_2^T D^{(\rho)} M_n(\xi) - \beta[(\Pi_1^T M_n(\xi))(\Pi_2^T M_n(\xi))] + \gamma[(\Pi_4^T M_n(\xi))(\Pi_2^T M_n(\xi))] + (c + \delta - \mu)\Pi_2^T M_n(\xi) = 0. \tag{6.2}$$

Grouping Eqs (4.3) and (5.7) with use of Eqs (5.1)–(5.4), we obtain

$$\Pi_3^T D^{(\rho)} M_n(\xi) - \delta(\Pi_2^T M_n(\xi)) + (\varepsilon + \mu)\Pi_3^T M_n(\xi) = 0. \tag{6.3}$$

Grouping Eqs (4.4) and (5.8) with use of Eqs (5.1)–(5.4), we obtain

$$\Pi_4^T D^{(\rho)} M_n(\xi) - \alpha[(\Pi_1^T M_n(\xi))(\Pi_4^T M_n(\xi))] - \gamma[(\Pi_4^T M_n(\xi))(\Pi_2^T M_n(\xi))] + \mu(\Pi_4^T M_n(\xi)) = 0. \tag{6.4}$$

The residual for Eqs (6.1)–(6.4) are given as

$$R_{1n}(\xi) = \Pi_1^T D^{(\rho)} M_n(\xi) - \Xi + \alpha[(\Pi_1^T M_n(\xi))(\Pi_4^T M_n(\xi))] + \beta[(\Pi_1^T M_n(\xi))\Pi_2^T M_n(\xi)] - c(\Pi_2^T M_n(\xi)) - \varepsilon(\Pi_4^T M_n(\xi)) + \mu(\Pi_1^T M_n(\xi)), \tag{6.5}$$

$$R_{2n}(\xi) = \Pi_2^T D^{(\rho)} M_n(\xi) - \beta[(\Pi_1^T M_n(\xi))(\Pi_2^T M_n(\xi))] + \gamma[(\Pi_4^T M_n(\xi))(\Pi_2^T M_n(\xi))] + (c+\delta-\mu)\Pi_2^T M_n(\xi), \tag{6.6}$$

$$R_{3n}(\xi) = \Pi_3^T D^{(\rho)} M_n(\xi) - \delta(\Pi_2^T M_n(\xi)) + (\varepsilon + \mu)\Pi_3^T M_n(\xi), \tag{6.7}$$

$$R_{4n}(\xi) = \Pi_4^T D^{(\rho)} M_n(\xi) - \alpha[(\Pi_1^T M_n(\xi))(\Pi_4^T M_n(\xi))] - \gamma[(\Pi_4^T M_n(\xi))(\Pi_2^T M_n(\xi))] + \mu(\Pi_4^T M_n(\xi)). \tag{6.8}$$

Now, we collocate at $n - 1$ points presented as $\xi_i = \frac{i}{n}$, $i = 0, 1, ..., n - 2$; in Eqs (6.5)–(6.8), we obtain

$$R_{1n}(\xi_i) = \Pi_1^T D^{(\rho)} M_n(\xi_i) - \Xi + \alpha[(\Pi_1^T M_n(\xi_i))(\Pi_4^T M_n(\xi_i))] + \beta[(\Pi_1^T M_n(\xi_i))\Pi_2^T M_n(\xi_i)] - c(\Pi_2^T M_n(\xi_i)) - \varepsilon(\Pi_4^T M_n(\xi_i)) + \mu(\Pi_1^T M_n(\xi_i)), \tag{6.9}$$

$$R_{2n}(\xi_i) = \Pi_2^T D^{(\rho)} M_n(\xi_i) - \beta[(\Pi_1^T M_n(\xi_i))(\Pi_2^T M_n(\xi_i))] + \gamma[(\Pi_4^T M_n(\xi_i))(\Pi_2^T M_n(\xi_i))] + (c + \delta - \mu)\Pi_2^T M_n(\xi_i), \tag{6.10}$$

$$R_{3n}(\xi_i) = \Pi_3^T D^{(\rho)} M_n(\xi_i) - \delta(\Pi_2^T M_n(\xi_i)) + (\varepsilon + \mu)\Pi_3^T M_n(\xi_i), \tag{6.11}$$

$$R_{4n}(\xi_i) = \Pi_4^T D^{(\rho)} M_n(\xi_i) - \alpha[(\Pi_1^T M_n(\xi_i))(\Pi_4^T M_n(\xi_i))] - \gamma[(\Pi_4^T M_n(\xi_i))(\Pi_2^T M_n(\xi_i))] + \mu(\Pi_4^T M_n(\xi_i)). \tag{6.12}$$

Furthermore, we can write from Eqs (5.1) to (5.4)

$$\Pi_1^T M_n(0) - S(0) = 0, \tag{6.13}$$

$$\Pi_2^T M_n(0) - I(0) = 0, \tag{6.14}$$

$$\Pi_3^T M_n(0) - R(0) = 0, \tag{6.15}$$

$$\Pi_4^T M_n(0) - A(0) = 0. \tag{6.16}$$

Using the collocation points in Eqs (6.9)–(6.12) along with Eqs (6.13)–(6.16), we obtain a set of equations with a similar number of unknowns. The approximated solution of the fractional SIRA model is obtained by solving this system.

7. Analysis of the scheme

Theorem 7.1. Consider the functions $\Psi : [0,1] \to R$ and $\Psi \in C^{(n+1)}[0,1]$, where $\Psi_n(t)$ is the n^{th} approximation acquired by utilizing Jacobi polynomials. Then, we have

$$F^h_{\Psi,n} = \|\Psi - \Psi_n\|_{L^2_\delta[0,1]}, \tag{7.1}$$

and the error vector in Eq (7.1) tending to 0 as $n \to \infty$.

Proof. Please refer the work [38, 39].

Theorem 7.2. If $F^{\rho,h}_{D,n}$ is the error vector for the ρ order operational matrix of differentiation, that is acquired by utilizing $(n+1)$ Jacobi polynomials. in this situation, we have

$$F^{\rho,h}_{D,n} = D^{(\rho)} M_n(\xi) - D^\rho M_n(\xi), \tag{7.2}$$

tending to 0 as $n \to \infty$.
Proof. Please see [40].

Suppose that V_n is the n-dimensional subspace generated by $(M_i)_{0 \le i \le n}$ for $L^2_h[0,1]$. Consider ψ_n as infimum of the functional on the space V_n. Then, it can be written as

$$V_n \subset V_{n+1} \text{ and } \psi_{n+1} \ge \psi_n.$$

Theorem 7.3. Suppose the functional L. Then,

$$\lim_{n\to\infty} \psi_n(\xi) = \psi(\xi) = \inf_{\xi\in[0,1]} L(\xi).$$

Proof. See [41].

Functional for Eq (4.1) is

$$L(\xi) = D^\rho S(\xi) - \Xi + \alpha S(\xi)A(\xi) + \beta S(\xi)I(\xi) - cI(\xi) - \varepsilon R(\xi) + \mu S(\xi) = 0. \tag{7.3}$$

using Eqs (5.1) and (5.4), we get

$$L^{(E)}(\xi) = \Pi_1^T D^{(\rho)}(M_n(\xi) + F^{\rho,h}_{D,n}) - \Xi + \alpha[(\Pi_1^T(M_n(\xi) + F^h_{D,n}))(\Pi_4^T(M_n(\xi) + F^h_{D,n}0)]+$$

$$\beta[(\Pi_1^T(M_n(\xi)+F^h_{D,n})\Pi_2^T(M_n(\xi)+F^h_{D,n})-c(\Pi_2^T M_n(\xi)+F^h_{D,n})-\varepsilon(\Pi_4^T M_n(\xi)+F^h_{D,n})+\mu(\Pi_1^T M_n(\xi)+F^h_{D,n}), \tag{7.4}$$

where

$$F^h_{D,n} = \Pi^T M(\xi) - \Pi^T M_n(\xi), \tag{7.5}$$

$$F^{\rho,h}_{D,n} = D^{(\rho)} M_n(\xi) - D^\rho S_n(\xi). \tag{7.6}$$

Residual for Eq (7.4)

$$R_1 n^{(E)}(\xi) = \Pi_1^T D^{(\rho)}(M_n(\xi) + F^{\rho,h}_{D,n}) - \Xi + \alpha[(\Pi_1^T(M_n(\xi) + F^h_{D,n}))(\Pi_4^T(M_n(\xi) + F^h_{D,n}0)]$$

$$+\beta[(\Pi_1^T(M_n(\xi)+F_{D,n}^h)\Pi_2^T(M_n(\xi)+F_{D,n}^h)-c(\Pi_2^T M_n(\xi)+F_{D,n}^h)-\varepsilon(\Pi_4^T M_n(\xi)+F_{D,n}^h)+\mu(\Pi_1^T M_n(\xi)+F_{D,n}^h). \tag{7.7}$$

Now, colocating $n-1$ points in Eq (7.7) by $\xi_i = \frac{i}{n}, \quad i = 0, 1, 2, \ldots, n-2$, we find

$$R_n^{(E)}(\xi_i) = 0. \tag{7.8}$$

Combining Eqs (6.13) to (6.16) and the collocation points in Eq (7.8), we attain a system of NLAEs. The result for SIRA of the fractional order is provided by this system's solution. Consider this solution to be presented by $\psi_n^*(\xi)$. With the help of results 7.1, 7.2 and $n \to \infty$,

$$\psi_n^*(\xi) \to \psi_n(\xi). \tag{7.9}$$

From result 7.3 and Eq (7.9), we obtain

$$\lim_{n\to\infty} \psi_n^*(\xi) = \psi(\xi).$$

The same proof can be formed for Eqs (4.2)–(4.4).

8. Numerical results and discussion

In this part, we applied our suggested technique to a numerical simulation of the arbitrary order antidotal computer virus model with a starting value as $S(0) = 0.15$, $I(0) = 0.25$, $A(0) = 0.5$ and $R(0) = 0.5$ with value of various parameters $\Xi = 0.5$, $\alpha = 0.1$, $\gamma = 0.01$, $\beta = 0.5$, $\delta = 0.1$, $\mu = 0.035$ and $\varepsilon = 0.009$. We obtained results for the Jacobi Collocation method (JCM) as well as the Vieta Lucas collocation method (VLCM) for arbitrary order $\rho = 0.7$, $\rho = 0.8$, $\rho = 0.9$ and $\rho = 1$. From Figures 1–4, it is observed that the solution changes regularly from arbitrary order to classical order. For $\rho = 1$, obtained results for $S(\xi)$, $I(\xi)$, $R(\xi)$ and (ξ) are presented by tabular form for various values of ξ. The behavior of the solution $S(\xi)$ is shown in Figure 1 at various values of different parameters and ρ . From Figure 1, we observe that the suspected machine $S(\xi)$ decreases as the time derivative order ρ increases and as time ξ increases, suspected computer increases. The comparative analysis of the obtained results for $S(\xi)$ by utilizing distinct techniques is shown in Table 1. From Table 1, we can observe that outcomes from the presented techniques are in a good agreement for practical implementations. The response of the solution $I(\xi)$ is shown in Figure 2 at various values of the parameters and ρ . From Figure 2, we observe that infected machines $I(\xi)$ enhance as the time derivative order ρ increases and as time ξ increases, the infected computers decrease. The comparative analysis of the acquired results for $I(\xi)$ by utilizing both techniques is shown in Table 2, which are in good agreement. The behavior of the solution $R(\xi)$ is shown in Figure 3 at distinct value of ρ. From Figure 3, we observe that the removed machine $R(\xi)$ increases as the time derivative order ρ increases and when time ξ increases, the removed computers decreases. The comparative analysis of the obtained results for $R(\xi)$ by utilizing JCM and VLCM is shown in Table 3. The response of the solution $A(\xi)$ is shown in Figure 4 at distinct values of various parameters and ρ. From Figure 4, we observe that $A(\xi)$ decreases as the time derivative order ρ increases. As time ξ increases, $A(\xi)$ decreases initially and increases after some time. The comparative analysis of the acquired results for $A(\xi)$ by utilizing distinct techniques is shown in Table 4.

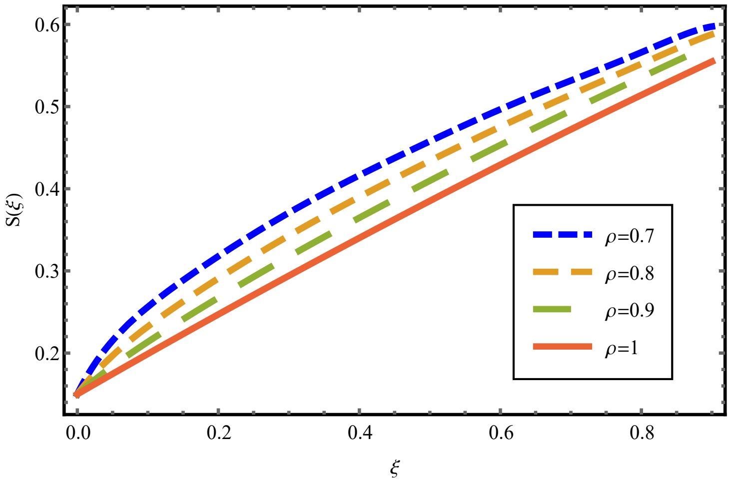

Figure 1. Plot of $S(\xi)$ with respect to ξ for distinct value of ρ.

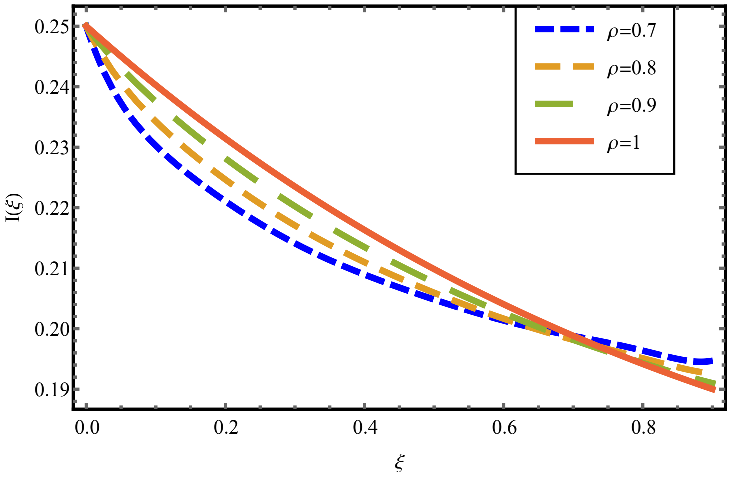

Figure 2. The response of $I(\xi)$ with respect to time ξ for distinct value of ρ.

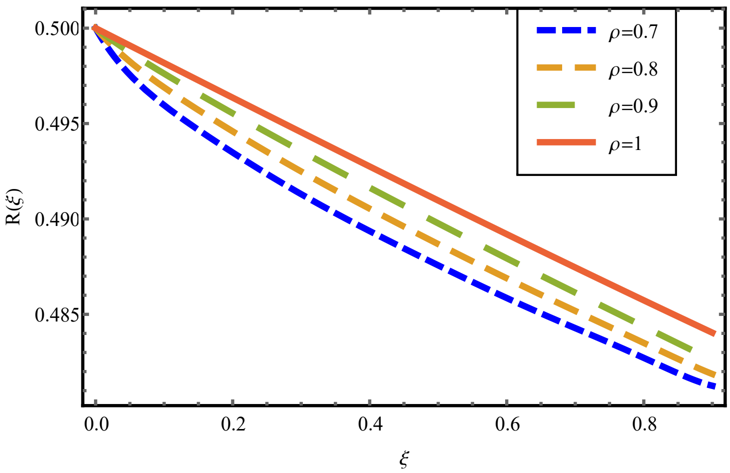

Figure 3. The response of solution $R(\xi)$ with respect to ξ for distinct value of ρ.

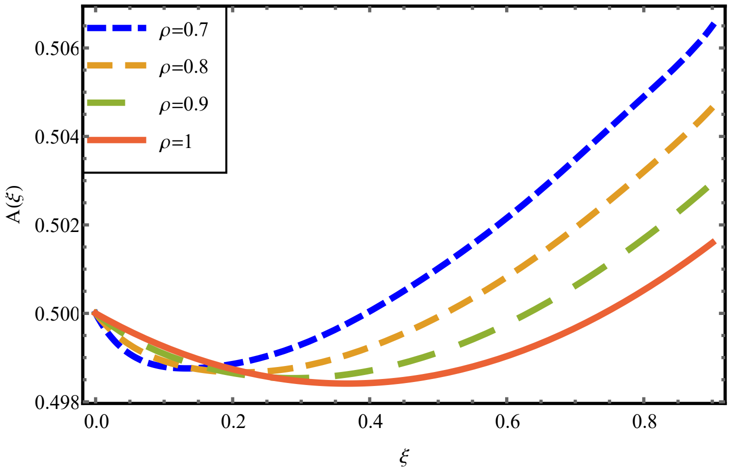

Figure 4. The behavior of $A(\xi)$ with respect to time ξ for distinct value of ρ.

Table 1. Analysis between obtained solutions for $S(\xi)$ by JCM and VLCM when $\rho = 1$.

ξ	JCM	VLCM
0.0	0.15	0.15
0.1	0.1990325248	0.1975906395
0.2	0.2470553771	0.2446003537
0.3	0.2940870100	0.2910559541
0.4	0.3401301272	0.3369814909
0.5	0.3851731501	0.3823973678
0.6	0.4291925157	0.4273203404
0.7	0.4721548196	0.4717635179
0.8	0.5140188299	0.5157363618
0.9	0.5547373901	0.5592446871
1.0	0.5942592167	0.6022906612

Table 2. Analysis between obtained solutions for $I(\xi)$ by JCM and VLCM when $\rho = 1$.

ξ	JCM	VLCM
0.0	0.25	0.25
0.1	0.2402530314	0.2412089787
0.2	0.2314469362	0.2330196999
0.3	0.2234947441	0.2253965391
0.4	0.2163200810	0.2183060971
0.5	0.2098559901	0.2117171136
0.6	0.2040436907	0.2056004677
0.7	0.1988315292	0.1999291775
0.8	0.1941740904	0.1946784005
0.9	0.1900314437	0.1898254336
1.0	0.1863684903	0.1853497128

Table 3. Analysis between obtained solutions for $R(\xi)$ by JCM and VLCM when $\rho = 1$.

ξ	JCM	VLCM
0.0	0.5	0.5
0.1	0.4981759351	0.4981771325
0.2	0.4963571199	0.4963492400
0.3	0.4945469086	0.4945170834
0.4	0.4927486700	0.4926811502
0.5	0.4909655233	0.4908419282
0.6	0.4892003515	0.4889999050
0.7	0.4874558102	0.4871555681
0.8	0.4857343370	0.4853094055
0.9	0.4840381578	0.4834619045
1.0	0.4823692952	0.4816135528

Table 4. Analysis between obtained solutions for $A(\xi)$ by JCM and VLCM when $\rho = 1$.

ξ	JCM	VLCM
0.0	0.5	0.5
0.1	0.4992475202	0.4997353336
0.2	0.4987372610	0.4995822083
0.3	0.4984644860	0.4995398331
0.4	0.4984251515	0.4996084505
0.5	0.4986153142	0.4997892670
0.6	0.4990311170	0.5000844538
0.7	0.4996687841	0.5004971474
0.8	0.5005246269	0.5010314475
0.9	0.5005246269	0.5016924198
1.0	0.5028766090	0.5024860939

9. Conclusions

In this study, our primary objective is to explore the potential advantages of fractional derivatives in enhancing memory and addressing computer virus-related issues and computing the numerical results for a fractional model. Mathematical models serve as pivotal tools in computer network security, offering valuable insights and early detection capabilities for viruses. In this context, the concept of kill signals becomes invaluable as it empowers users to pro actively safeguard their systems against virus threats. We have suggested a computational scheme for the fractional model of a computer virus model. The arbitrary order SIRA model is solved using the collocation operational matrix method of Jacobi polynomials. The obtained solution is also compared by the results computed using Vieta Lucas polynomials. Also, it is noticeable that the implemented techniques are much easier and user friendly in comparison to other methods. The numerical simulation for $S(\xi)$, $I(\xi)$, $R(\xi)$ and $A(\xi)$ are shown graphically. We obtaine the behavior of $S(\xi)$, $I(\xi)$, $R(\xi)$ and $A(\xi)$ for different arbitrary orders $\rho = 0.7$,

$\rho = 0.8$, $\rho = 0.9$ and $\rho = 1$. We observe that at $\rho = 1$, the solutions of the computer virus model by applying both operational matrix techniques with collocation are in a great agreement. The outcomes indicate that proposed techniques are quite suitable and accurate to examine arbitrary order models with the Caputo operator.

Use of AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare no conflicts of interest in this manuscript.

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