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Research article

New soft operators related to supra soft δ_i-open sets and applications

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Abstract: This project aimed to introduce the notion of supra soft δ_i-open sets in supra soft topological spaces. Also, we declared the differences between the new concept and other old generalizations. We presented new operators such as supra soft δ_i-interior, supra soft δ_i-closure, supra soft δ_i-boundary and supra soft δ_i-cluster. We found out many deviations to our new operators; to name a few: If $int^s_{\delta_i}(F, E) = (F, E)$, then it doesn't imply that $(F, E) \in SOS_{\delta_i}(X)$. Furthermore, we applied this notion to define new kinds of mappings, like supra soft δ_i-continuous mappings, supra soft δ_i-irresolute mappings, supra soft δ_i-open mappings and supra soft δ_i-closed mappings. We studied their main properties in special to distinguish between our new notions and the previous generalizations. It has been pointed out in this work that many famous previous studies have been investigated here; in fact, I believe that this is an extra justification for the work included in this manuscript.

Keywords: supra soft topological space; supra soft δ_i-open sets; supra soft δ_i-interior operator; supra soft δ_i-closure operator; supra soft δ_i-continuous mappings
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1. Introduction

Sometimes weaker conditions suffice to ensure some valid properties. Based on this rule, in 1983 Mashhour et al. [1] defined the notion of supra topological spaces as a generalization to the topological spaces. In 2002, Alpers [2] showed that these spaces were easier in the application. In 2016, Kozae et al. [3] applied these spaces in real-life problems. Al-shami and Alshammari [4] used these spaces to present new rough set operators and models.

Also, El-Sheikh and Abd El-latif [5] in 2014 defined the notion of supra soft topological space as a wider collection of soft topological spaces [6]. This manuscript gained the attention of many researchers. So, some basic notions related to supra soft topologies such as supra soft compactness [7], supra soft connectedness [8–10], soft supra (strongly-regular) generalized closed sets [11–14] and supra soft separation axioms [15–22] have been introduced and investigated.

Generalizations of open sets play an important role in topology through their use to improve on some famous results or to open the way to reintroduce and generalize some of the topological notions, such as separation axioms, connectedness, compactness and paracompactness. So, various types of weaker forms of (supra) soft open sets [5, 15, 23, 24] have been introduced. The combination between supra soft topological space and a partial order relation was introduced in [25]. Recently, the notion of soft i-open sets was discussed in [26] as a generalization to previous weaker forms of soft open sets [27–29]. Recently, new classes of generalized soft open sets were presented in [30, 31].

This paper is organized as follows: In the preliminaries section, we presented some basic definitions and results, which makes the paper more easier for the reader. In Section 3, we go forward to provide a weaker form of supra soft open set named the supra soft δ_i-open set and discuss its properties. The relationships with other previous generalizations have been studied. In Section 4, we investigated some operators like interior, closure, boundary and cluster by using the new class of weaker supra soft subsets. In Section 5, we applied the new concepts to the soft continuity. We found out many interested results and deviations on the other generalized notions. This deviation emphasized the importance to our results, especially with the examples and counterexamples that were introduced to support our claims. So, with these interested and important results, I expect that this paper will have many extensions in the future.

2. Preliminaries

In this paper, we follow the notions and terminologies as mentioned in [5, 6, 23, 32–34]. Let (F, E) be a soft set over X. We denote the family of all soft sets by $S(X)_E$. Throughout this paper, soft topological space will be denoted by STS and supra soft topological space will be denoted by SSTS. Let (X, τ, E) be an STS, and we denote the set of all soft open (respectively, closed) sets over X by $OS(X)$ (respectively, $CS(X)$). Let (X, μ, E) be an SSTS, and we denote the set of all supra soft open (respectively, closed) sets over X by $OS^s(X)$ (respectively, $CS^s(X)$). The collections of supra soft semi- (respectively, regular-, β-, α-, pre-, b-) open sets will denoted by $SOS^s(X)$ (respectively, $ROS^s(X)$, $\beta OS^s(X)$, $\alpha OS^s(X)$, $POS^s(X)$, $BOS^s(X)$).

Definition 2.1. *[33] Let X be an initial universe and E be a set of parameters. A pair (F, E) denoted by F_E is called a soft set over X, and is a parameterized family of subsets of the universe X, i.e., $F_E = \{F(e) : e \in E,\ F : E \to P(X)\}$.*

Definition 2.2. *[6] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq S(X)_E$ is called a soft topology on X if:*

(1) $\tilde{X}, \tilde{\varphi} \in \tau$.

(2) *The soft union of any number of soft sets in τ belongs to τ.*

(3) *The soft intersection of any two soft sets in τ belongs to τ.*

The triplet (X, τ, E) is called an STS over X.

Definition 2.3. *[6] Let (X, τ, E) be an STS and $(F, E) \in S(X)_E$. The soft closure of (F, E), denoted by $cl(F, E)$, is the soft intersection of all soft closed super sets of (F, E).*

Definition 2.4. *[34] Let (X, τ, E) be an STS and $(G, E) \in S(X)_E$. The soft interior of (G, E), denoted by $int(G, E)$, is the soft union of all soft open subsets of (G, E).*

Definition 2.5. *[6, 35] The soft set $(F, E) \in S(X)_E$ is called a soft point in $\tilde{X}$, denoted by x_e, if there exists $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \varphi$ for each $e' \in E - \{e\}$. Also, $x_e \tilde{\in} (G, E)$, if for the element $e \in E$, $F(e) \subseteq G(e)$.*

Definition 2.6. *[26] Let (X, τ, E) be an STS and $(H, E) \in S(X)_E$, then (H, E) is called a soft i-open set if there is a soft open set $\tilde{\varphi} \neq (G, E) \neq \tilde{X}$ such that $(H, E) \tilde{\subseteq} cl((H, E) \tilde{\cap} (G, E))$.*

Definition 2.7. *[34] A soft function $f_{pu} : (X, \tau_1, A) \rightarrow (Y, \tau_2, B)$ is said to be:*

(1) *Soft continuous if $f_{pu}^{-1}(G, B) \in \tau_1 \ \forall \ (G, B) \in \tau_2$.*

(2) *Soft open if $f_{pu}(G, A) \in \tau_2 \ \forall \ (G, A) \in \tau_1$.*

(3) *Soft closed if $f_{pu}(G, A) \in \tau_2^c \ \forall \ (G, A) \in \tau_1^c$.*

Theorem 2.8. *[32] For the soft function $f_{pu} : S(X)_A \rightarrow S(Y)_B$, the following statements hold:*

(a) *$f_{pu}^{-1}((G, B)^c) = (f_{pu}^{-1}(G, B))^c \ \forall \ (G, B) \in S(Y)_B$.*

(b) *$f_{pu}(f_{pu}^{-1}((G, B))) \tilde{\subseteq} (G, B) \ \forall \ (G, B) \in S(Y)_B$. If f_{pu} is surjective, then the equality holds.*

(c) *$(F, A) \tilde{\subseteq} f_{pu}^{-1}(f_{pu}((F, A))) \ \forall \ (F, A) \in S(X)_A$. If f_{pu} is injective, then the equality holds.*

(d) *$f_{pu}(\tilde{X}) \tilde{\subseteq} \tilde{Y}$. If f_{pu} is surjective, then the equality holds.*

Definition 2.9. *[5] The collection $\mu \subseteq S(X)_E$ is called SSTS on X if:*

(1) *$\tilde{X}, \tilde{\varphi} \in \mu$,*

(2) *The soft union of any number of soft sets in μ belongs to μ.*

Definition 2.10. *[5] Let (X, τ, E) be an STS and (X, μ, E) be an SSTS. We say that, μ is an SSTS associated with τ if $\tau \subset \mu$.*

Definition 2.11. *[5] Let (X, μ, E) be an SSTS over X and $(F, E) \in S(X)_E$, then the supra soft interior of (G, E), denoted by $int^s(G, E)$, is the soft union of all supra soft open subsets of (G, E). Also, the supra soft closure of (F, E), denoted by $cl^s(F, E)$, is the soft intersection of all supra soft closed supersets of (F, E).*

3. Supra soft δ_i-open sets in supra soft topological spaces

Here, we introduce a new class of soft sets, named supra soft δ_i-open sets. We discuss its basic properties. Also, we declare the differences between our concept and other generalizations. In special, we found that it fails to be closed under soft union and soft intersection, in general. In addition, the relationships between our concept and other previous generalized structures have been discussed. Specifically, we found that it is a generalization to the notions of supra soft open sets, supra soft semi open sets and supra soft α-open sets. In addition, we found that there is no priori relation between our new collection $SOS_{\delta_i}(X)$ and the collections $\beta OS^s(X)$, $POS^s(X)$ and $BOS^s(X)$.

Definition 3.1. *Let (X, μ, E) be an SSTS and $(A, E) \in S(X)_E$, then (A, E) is called a supra soft δ_i-open set if $\exists$ a proper supra soft open set (G, E) such that $(A, E)\tilde{\subseteq}cl^s((A, E)\tilde{\cap}(G, E))$. The complement of a supra soft δ_i-open set is a supra soft δ_i-closed. The collection of all supra soft δ_i-open sets will denoted by $SOS_{\delta_i}(X)$ and the collection of all supra soft δ_i-closed sets will denoted by $SCS_{\delta_i}(X)$.*

Proposition 3.2. *Let (X, μ, E) be an SSTS and $(K, E) \in S(X)_E$.*

(1) *If $(K, E) \in SOS_{\delta_i}(X)$, then $\exists$ a proper supra soft open set (G, E) such that $(K, E)\tilde{\subseteq}cl^s(G, E)$.*

(2) *If $(K, E) \in SCS_{\delta_i}(X)$, then $\exists$ a proper supra soft closed set (H, E) such that $int^s(H, E)\tilde{\subseteq}(K, E)$.*

Proof. (1) Direct from Definition 3.1.

(2) Assume that $(K, E) \in SCS_{\delta_i}(X)$, then $(K^{\tilde{c}}, E) \in SOS_{\delta_i}(X)$. It follows that $\exists$ a proper supra soft open set (G, E) such that $(K^{\tilde{c}}, E)\tilde{\subseteq}cl^s(G, E)$ from (1). Hence, $int^s(G^{\tilde{c}}, E) = [cl^s((G, E)]^{\tilde{c}}\tilde{\subseteq}(K, E)$, $(G^{\tilde{c}}, E)$ is a proper supra soft closed subset of $\tilde{X}$.

Remark 3.3. *A finite soft intersection (respectively, union) of supra soft δ_i-open sets need not be supra soft δ_i-open, as shown in the following examples.*

Examples 3.4. **(1)** *Assume that $U = \{u_1, u_2, u_3\}$. Let $D = \{d_1, d_2\}$ be the set of parameters. Let $(C_1, D), (C_2, D)$ be soft sets over the universe U, where:*

$$C_1(d_1) = \{u_1, u_2\}, \quad C_1(d_2) = \{u_2, u_3\},$$

$$C_2(d_1) = \{u_2, u_3\}, \quad C_2(d_2) = \{u_1, u_2\},$$

then $\mu = \{\tilde{U}, \tilde{\varphi}, (C_1, E), (C_2, E)\}$ defines an SSTS on U. Hence, the soft sets (A, D) and (B, D), where:

$$A(d_1) = \{u_1, u_3\}, \quad A(d_2) = \{u_2, u_3\},$$

$$B(d_1) = \{u_2, u_3\}, \quad B(d_2) = \{u_1, u_3\}$$

are supra soft δ_i-open sets on U, but their soft intersection is $(A, D)\tilde{\cap}(B, D) = (M, D)$, where: $M(d_1) = \{u_3\}$, $M(d_2) = \{u_3\}$, which is not supra soft δ_i-open.

(2) *Assume that $U = \{u_1, u_2, u_3\}$. Let $D = \{p_1, p_2\}$ be the set of parameters. Let $(A, D), (B, D)$ be soft sets over the universe U, where:*

$$A(p_1) = \{u_1, u_3\}, \quad A(p_2) = \{u_2, u_3\},$$

$$B(p_1) = \{u_2, u_3\}, \quad B(p_2) = \{u_1, u_3\},$$

then $\mu = \{\tilde{U}, \tilde{\varphi}, (C_1, E), (C_2, E)\}$ defines an SSTS on U. Hence, the soft sets (L, D) and (M, D), where:

$$L(p_1) = \{u_1\}, \quad L(p_2) = \varphi,$$

$$M(p_1) = \varphi, \quad M(p_2) = \{u_1\}$$

are supra soft δ_i-open sets on U, but their soft union is $(L, D)\tilde{\cup}(M, D) = (N, D)$, where $N(p_1) = \{u_1\}$, $N(p_2) = \{u_1\}$, which is not supra soft δ_i-open.

Definition 3.5. *[5, 13, 23] Let (X,μ,E) be a supra soft topological space and $(F,E)\in S(X)_E$, then (F,E) is said to be*

(1) *Supra soft pre open set if $(F,E)\tilde{\subseteq} int^s(cl^s(F,E))$,*

(2) *Supra soft semi open set if $(F,E)\tilde{\subseteq} cl^s(int^s(F,E))$,*

(3) *Supra soft α-open set if $(F,E)\tilde{\subseteq} int^s(cl^s(int^s(F,E)))$,*

(4) *Supra soft β-open set if $(F,E)\tilde{\subseteq} cl^s(int^s(cl^s(F,E)))$,*

(5) *Supra soft b-open set if $(F,E)\tilde{\subseteq} cl^s(int^s(F,E))\tilde{\cup} int^s(cl^s(F,E))$,*

(6) *Supra soft regular-open set if $int^s(cl^s(F,E)) = (F,E)$.*

The set of all supra soft pre- (respectively, regular, semi, α-, β-, b-) open sets is denoted by $POS^s(X)$ (respectively, $ROS^s(X)$, $SOS^s(X)$, $\alpha OS^s(X)$, $\beta OS^s(X)$, $BOS^s(X)$).

In the next theorem, we introduce the relationships between our new notions and some special other notions mentioned in [5, 23].

Theorem 3.6. *In an SSTS (X,μ,E), the following statements hold:*

(1) *Every supra soft open (respectively, supra soft closed) set is a supra soft δ_i-open (respectively, supra soft δ_i-closed).*

(2) *Every supra soft semi open (respectively, supra soft semi closed) set is a supra soft δ_i-open (respectively, supra soft δ_i-closed).*

(3) *Every supra soft α-open (respectively, supra soft α-closed) set is a supra soft δ_i-open (respectively, supra soft δ_i-closed).*

(4) *Every supra soft regular-open (respectively, supra soft regular-closed) set is a supra soft δ_i-open (respectively, supra soft δ_i-closed).*

Proof. (1) Clear from Definition 3.1.

(2) Let $(A,E)\in SOS^s(X)$, then $\exists$ a supra soft open set (S,E) such that $(S,E)\tilde{\subseteq}(A,E)\tilde{\subseteq} cl^s(S,E)$, and so $(S,E)\tilde{\cap}(A,E) = (S,E)$. Hence, $(A,E)\tilde{\subseteq} cl^s((A,E)\tilde{\cap}(S,E))$. Therefore, (A,E) is a supra soft δ_i-open set.

(3) Let $(G,E)\in \alpha OS^s(X)$, then

$$(G,E)\tilde{\subseteq} int^s(cl^s(int^s(G,E)))\tilde{\subseteq} cl^s(int^s(G,E)) = cl^s[(int^s(G,E))\tilde{\cap}(G,E)].$$

Thus, (G,E) is a supra soft δ_i-open set.

(4) Let $(K,E)\in ROS^s(X)$, then $int^s(cl^s(K,E)) = (K,E)$. It follows that

$$(K,E) = int^s(cl^s(K,E))\tilde{\cap}(K,E)\tilde{\subseteq} cl^s[int^s(cl^s(K,E))\tilde{\cap}(K,E)],\quad int^s(cl^s(K,E))\in\mu.$$

Hence, (G,E) is a supra soft δ_i-open set.

Remark 3.7. *The following examples shall show that the converse of Theorem 3.6 is not true in general.*

Examples 3.8. **(1)** *In Examples 3.4 (1), the soft sets* (A, D) *and* (B, D) *are supra soft* δ_i*-open sets, but not supra soft open.*

(2) *Suppose that* $X = \{h_1, h_2, h_3\}$*, and consider* $E = \{e_1, e_2\}$ *is the set of parameters. Let* $(F_1, E), (F_2, E), (F_3, E), (F_4, E)$ *be four soft sets over the common universe X, where:*

$$F_1(e_1) = \{h_1\}, \quad F_1(e_2) = \{h_1, h_2\},$$
$$F_2(e_1) = \{h_1, h_2\}, \quad F_2(e_2) = \{h_1\},$$
$$F_3(e_1) = \{h_1, h_2\}, \quad F_3(e_2) = \{h_1, h_2\},$$
$$F_4(e_1) = \{h_2, h_3\}, \quad F_4(e_2) = \{h_2, h_3\},$$

then $\mu = \{\tilde{X}, \tilde{\varphi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$ *defines an SSTS on X. Hence, the soft set* (G, E)*, which is defined by* $G(e_1) = \{h_2\}$*,* $G(e_2) = \{h_1, h_3\}$ *is a supra soft* δ_i*-open set, but it is not supra soft semi-open.*

(3) *Suppose that* $X = \{h_1, h_2, h_3\}$*, and consider* $E = \{e_1, e_2\}$ *is two parameters. Let* $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)$ *be five soft sets over the common universe X, where:*

$$F_1(e_1) = \{h_1\}, \quad F_1(e_2) = \{h_1\},$$
$$F_2(e_1) = \{h_1, h_3\}, \quad F_2(e_2) = \{h_2, h_3\},$$
$$F_3(e_1) = \{h_2, h_3\}, \quad F_3(e_2) = \{h_1, h_3\},$$
$$F_4(e_1) = X, \quad F_4(e_2) = \{h_1, h_3\},$$
$$F_5(e_1) = \{h_1, h_3\}, \quad F_5(e_2) = X,$$

then $\mu = \{\tilde{X}, \tilde{\varphi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)\}$ *defines an SSTS on X. Hence, the soft set* (G, E)*, which is defined as follows:* $G(e_1) = \{h_1, h_2\}$*,* $G(e_2) = \{h_2, h_3\}$ *is a supra soft* δ_i*-open set, but it is not supra soft* α*-open.*

(4) *In (3), the soft set* (G, E) *is a supra soft* δ_i*-open set, but it is not supra soft regular-open.*

Remark 3.9. *There is no priori relation between our new collection* $SOS_{\delta_i}(X)$ *and the collections* $\beta OS^s(X)$ *(respectively,* $POS^s(X)$*,* $BOS^s(X)$*), in general. The following examples shall support our claim.*

Examples 3.10. **(1)** *In Examples 3.4 (1), the soft set* (M, D)*, where:* $M(d_1) = \{u_3\}$*,* $M(d_2) = \{u_3\}$*, is supra soft* β*-open, but it is not supra soft* δ_i*-open.*

(2) *In Examples 3.8 (3), the soft set* (S, E)*, which is defined as* $S(e_1) = \{h_1\}$*,* $S(e_2) = \{h_2\}$*, is a supra soft* δ_i*-open set, but it is not supra soft* β*-open.*

(3) *In Examples 3.4 (1), the soft set* (M, D)*, where* $M(d_1) = \{u_3\}$*,* $M(d_2) = \{u_3\}$*, is supra soft pre-open, but it is not supra soft* δ_i*-open.*

(4) *In Example 3.8 (2), the soft set* (T, E)*, which defined as* $T(e_1) = \{h_3\}$*,* $T(e_2) = \{h_2, h_3\}$*, is a supra soft* δ_i*-open set, but it is not supra soft pre-open.*

(5) *In Example 3.8 (2), the soft set* (Q, E)*, which defined as* $Q(e_1) = \{h_1, h_3\}$*,* $Q(e_2) = \{h_1, h_3\}$*, is a supra soft b-open set, but it is not supra soft* δ_i*-open.*

(6) *Suppose that* $X = \{h_1, h_2, h_3\}$ *and consider* $E = \{e_1, e_2\}$ *is the set of parameters. Let* $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)$ *be five soft sets over X, where:*

$$F_1(e_1) = \{h_1\}, \;\; F_1(e_2) = \{h_2, h_3\},$$
$$F_2(e_1) = \{h_2, h_3\}, \;\; F_2(e_2) = \{h_1\},$$
$$F_3(e_1) = \{h_1, h_2\}, \;\; F_3(e_2) = \{h_1, h_2\},$$
$$F_4(e_1) = \{h_1, h_2\}, \;\; F_4(e_2) = X,$$
$$F_5(e_1) = X, \;\; F_5(e_2) = \{h_1, h_2\},$$

then $\mu = \{\tilde{X}, \tilde{\varphi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)\}$ *defines an SSTS on X. Hence, the soft set* (G, E)*, which is defined by* $G(e_1) = \{h_1, h_2\}$*,* $G(e_2) = \{h_1\}$*, is supra soft* δ_i*-open set, but it is not supra soft b-open.*

Corollary 3.11. *The following implications hold from Theorem 3.6 and [23, Corollary 4.1] for an SSTS* (X, μ, E)*. These implications are not reversible.*

$$\begin{array}{ccccccccc} ROS^s(X) & \longrightarrow OS^s(X) \longrightarrow & \alpha OS^s(X) & \longrightarrow & SOS^s(X) & \longrightarrow & SOS_{\delta_i}(X) & \nleftrightarrow & \beta OS^s(X) \\ & \downarrow & & & & \searrow & \not\Updownarrow & \nearrow & \\ & POS^s(X) & \longrightarrow & & & & BOS^s(X) & & \end{array}$$

4. Supra δ_i-interior and supra δ_i-closure operators

In this section we introduce new operators named supra soft δ_i-interior, supra soft δ_i-closure, supra soft δ_i-boundary and supra soft δ_i-cluster. We found out many deviations to our new operators, such as if $int^s_{\delta_i}(F, E) = (F, E)$, then it doesn't imply $(F, E) \in SOS_{\delta_i}(X)$. Also, if $cl^s_{\delta_i}(F, E) = (F, E)$, then it doesn't imply $(F, E) \in SCS_{\delta_i}(X)$. We gave many examples and counterexamples to support our claims.

Definition 4.1. *Let* (X, μ, E) *be an SSTS,* $(F, E) \in S(X)_E$ *and* $x_e \in S(X)_E$*, then*

(1) x_e *is called a supra soft* δ_i*-interior point of* (F, E) *if* $\exists\, (G, E) \in SOS_{\delta_i}(X)$ *such that* $x_e \in (G, E) \tilde{\subseteq} (F, E)$*. The set of all supra soft* δ_i*-interior points of* (F, E) *is called the supra soft* δ_i*-interior of* (F, E) *and is denoted by* $int^s_{\delta_i}(F, E))$*.*

(2) x_e *is called a supra soft* δ_i*-closure point of* (F, E) *if* $(F, E) \tilde{\cap} (H, E) \neq \tilde{\varphi}\ \forall\ (H, E) \in SOS_{\delta_i}(X)$ *and* $x_e \in (H, E)$*. The set of all supra soft* δ_i*-closure points of* (F, E) *is called supra soft* δ_i*-closure of* (F, E) *and is denoted by* $cl^s_{\delta_i}(F, E)$*.*

The proof of the following two propositions follows directly from Definition 4.1.

Proposition 4.2. *Let* (X, μ, E) *be an SSTS and* $(F, E) \in S(X)_E$*, then*

(1) $int^s_{\delta_i}(F, E)) = \tilde{\bigcup}\{(G, E) : (G, E) \tilde{\subseteq} (F, E),\ (G, E) \in SOS_{\delta_i}(X)\}$.

(2) $cl^s_{\delta_i}(F, E)) = \tilde{\bigcap}\{(H, E) : (F, E) \tilde{\subseteq} (H, E),\ (H, E) \in SCS_{\delta_i}(X)\}$.

Proposition 4.3. *Let* (X, μ, E) *be an SSTS and* $(F, E) \in S(X)_E$*, then*

(1) $int^s_{\delta_i}(\tilde{X}) = \tilde{X}$ *and* $int^s_{\delta_i}(\tilde{\varphi}) = \tilde{\varphi}$*.*

(2) *$int^s_{\delta_i}(F,E)$ is the largest supra soft δ_i-open set contained in (F,E).*

(3) $int^s_{\delta_i}(int^s_{\delta_i}(F,E)) = int^s_{\delta_i}(F,E)$.

(4) $int^s_{\delta_i}[(F,E)\tilde{\cap}(G,E)]\tilde{\subseteq}int^s_{\delta_i}(F,E)\tilde{\cap}int^s_{\delta_i}(G,E)$.

In the next proposition, we illustrate the deviation between the supra soft δ_i-interior operator $int^s_{\delta_i}(F,E)$) and its corresponding in previous studies.

Proposition 4.4. *Let (X,μ,E) be an SSTS and $(F,E)\in S(X)_E$, then*

(1) *If $(F,E)\in SOS_{\delta_i}(X)$, then $int^s_{\delta_i}(F,E) = (F,E)$.*

2) $int^s_{\delta_i}(F,E)\tilde{\subseteq}(F,E)$.

(3) *If $(F,E)\tilde{\subseteq}(G,E)$, then $int^s_{\delta_i}(F,E)\tilde{\subseteq}int^s_{\delta_i}(G,E)$.*

(4) $int^s_{\delta_i}(F,E)\tilde{\cup}int^s_{\delta_i}(G,E)\tilde{\subseteq}int^s_{\delta_i}[(F,E)\tilde{\cup}(G,E)]$.

Proof. Immediate.

Remark 4.5. *The inclusion relations in Proposition 4.4 are proper, as shown in the following examples.*

Examples 4.6. **(1)** *In Examples 3.4 (2), $int^s_{\delta_i}(N,D) = (N,D)$, whereas $(N,D)\notin SOS_{\delta_i}(X)$.*

(2) *In Examples 3.4 (2), for the soft set (R,D) where: $R(p_1) = \{u_1,u_2\}$, $R(p_2) = \{u_2\}$, we have*

$$(R,D)\tilde{\nsubseteq}int^s_{\delta_i}(R,D) = \{(p_1,\{u_1\}),(p_2,\{u_2\})\}.$$

(3) *In Examples 3.4 (2), for the soft set (R,D) where: $R(p_1) = \{u_1,u_2\}$, $R(p_2) = \{u_2\}$, we have $(R,D)\tilde{\nsubseteq}(A,D)$, whereas*

$$\{(p_1,\{u_1\}),(p_2,\{u_2\})\} = int^s_{\delta_i}(R,D)\tilde{\subseteq}int^s_{\delta_i}(A,D) = \{(p_1,\{u_1,u_3\}),(p_2,\{u_2,u_3\})\}.$$

(4) *In Examples 3.4 (2), for the soft sets (R,E) and (Q,E), where:*

$$R(p_1) = \{u_1,u_2\},\quad R(p_2) = \{u_2\},$$
$$Q(p_1) = \{u_3\},\quad Q(p_2) = \{u_1,u_3\},$$

we have

$$int^s_{\delta_i}(R,E)\tilde{\cup}int^s_{\delta_i}(Q,E) = \{(e_1,\{u_1,u_3\}),(e_2,X)\}\tilde{\nsupseteq}int^s_{\delta_i}[(R,E)\tilde{\cup}(Q,E)] = \tilde{X}.$$

In the next proposition, we illustrate the deviation to the supra soft δ_i-closure operator $cl^s_{\delta_i}(F,E)$) and its corresponding in previous studies.

Proposition 4.7. *Let (X,μ,E) be an SSTS and $(F,E)\in S(X)_E$, then*

(1) *If $(F,E)\in SCS_{\delta_i}(X)$, then $cl^s_{\delta_i}(F,E) = (F,E)$.*

(2) $(F,E)\tilde{\subseteq}cl^s_{\delta_i}(F,E)$.

(3) *if* $(F,E)\tilde{\subseteq}(G,E)$, *then* $cl^s_{\delta_i}(F,E)\tilde{\subseteq}cl^s_{\delta_i}(G,E)$.

(4) $cl^s_{\delta_i}[(F,E)\tilde{\cap}(G,E)]\tilde{\subseteq}cl^s_{\delta_i}(F,E)\tilde{\cap}cl^s_{\delta_i}(G,E)$.

Remark 4.8. *The inclusion relations in Proposition 4.7 are proper, as shown in the following examples.*

Examples 4.9. (1) *In Examples 3.4 (2), for the soft set* (Y,D) *where:*

$$Y(p_1)=\{u_2,u_3\},\ \ Y(p_2)=\{u_2,u_3\},$$

we have $cl^s_{\delta_i}(Y,D)=(Y,D)$, *whereas* $(Y,D)\notin SCS_{\delta_i}(X)$.

(2) *In Examples 3.4 (2), for the soft set* (C,D) *where:*

$$C(p_1)=\{u_3\}, C(p_2)=\{u_1,u_3\},$$

we have $\{(p_1,\{u_2,u_3\}),(p_2,\{u_1,u_3\})\}=cl^s_{\delta_i}(C,E)\tilde{\nsubseteq}(C,E)$.

(3) *In Examples 3.4 (2), for the soft sets* $(V,D),(Z,E)$ *where:*

$$V(p_1)=\{u_3\},\ \ V(p_2)=\{u_1,u_3\},$$

$$Z(p_1)=\{u_2\},\ \ Z(p_2)=\{u_1\},$$

we have $(Z,D)\tilde{\nsubseteq}(V,D)$, *whereas*

$$\{(p_1,\{u_2\}),(p_2,\{u_1\})\}=cl^s_{\delta_i}(Z,D)\tilde{\subseteq}cl^s_{\delta_i}(V,D)=\{(p_1,\{u_2,u_3\}),(p_2,\{u_1,u_3\})\}.$$

(4) *In Examples 3.4 (2), for the soft sets* (M,E) *and* (N,E) *where:*

$$M(p_1)=\{u_3\},\ \ M(p_2)=\{u_1,u_3\},$$

$$N(p_1)=\{u_1,u_2\},\ \ N(p_2)=\{u_2\},$$

we have $cl^s_{\delta_i}(M,E)\tilde{\cap}cl^s_{\delta_i}(N,E)=\{(p_1,\{u_2\},(p_2,\varphi)\}\tilde{\nsubseteq}cl^s_{\delta_i}[(M,E)\tilde{\cap}(N,E)]=\tilde{\varphi}$.

Proposition 4.10. *Let* (X,μ,E) *be an SSTS and* $(F,E)\in S(X)_E$, *then*

(1) $cl^s_{\delta_i}(\tilde{X})=\tilde{X}$ *and* $cl^s_{\delta_i}(\tilde{\varphi})=\tilde{\varphi}$.

(2) $cl^s_{\delta_i}(F,E)$ *is the smallest supra soft* δ_i*-closed set contains* (F,E).

(3) $cl^s_{\delta_i}(cl^s_{\delta_i}(F,E))=cl^s_{\delta_i}(F,E)$.

(4) $cl^s_{\delta_i}(F,E)\tilde{\cup}cl^s_{\delta_i}(G,E)\tilde{\subseteq}cl^s_{\delta_i}[(F,E)\tilde{\cup}(G,E)]$.

Proof. Immediate.

Theorem 4.11. *Let* (X,μ,D) *be an SSTS and* $(H,D)\in S(X)_E$, *then* $x_e\in cl^s_{\delta_i}(H,D)$ *if, and only if,* $\forall$ *supra soft* δ_i*-open set* (G,D) *contains* x_e, $(H,D)\tilde{\cap}(G,D)\neq\tilde{\varphi}$.

Proof. Assume conversely, whereas $x_e \in cl^s_{\delta_i}(H, D)$ $\exists$ a supra soft δ_i-open set (G, D) contains x_e such that $(H, D)\tilde{\cap}(G, D) = \tilde{\varphi}$, which follows $(H, D)\tilde{\subseteq}(G^{\tilde{c}}, D)$. From Preposition 4.7 (1), $cl^s_{\delta_i}(H, D)\tilde{\subseteq}(G^{\tilde{c}}, D)$. Therefore, $x_e \notin cl^s_{\delta_i}(H, D)$, which is a contradiction.

For the reverse inclusion, conversely assume that $x_e \notin cl^s_{\delta_i}(H, D)$. This means $\exists$ a supra soft δ_i-closed set (K, D) with $x_e \notin (K, D)$ and $(H, D)\tilde{\subseteq}(K, D)$. This follows that $x_e \in (K^{\tilde{c}}, D)$, $(K^{\tilde{c}}, D)\tilde{\cap}(H, D) = \tilde{\varphi}$ and $K^{\tilde{c}}, D)$ is a supra soft δ_i-open set, which is a contradiction.

Theorem 4.12. *Let (X, μ, E) be an SSTS and $(F, E) \in S(X)_E$, then the following hold:*

(1) $cl^s_{\delta_i}(F^{\tilde{c}}, E) = [int^s_{\delta_i}(F, E)]^{\tilde{c}}$.

(2) $int^s_{\delta_i}(F^{\tilde{c}}, E) = [cl^s_{\delta_i}(F, E)]^{\tilde{c}}$.

Proof. (1) First, we prove that $cl^s_{\delta_i}(F^{\tilde{c}}, E)\tilde{\subseteq}[int^s_{\delta_i}(F, E)]^{\tilde{c}}$. Assume that $x_e \notin [int^s_{\delta_i}(F, E)]^{\tilde{c}}$, then $x_e \in int^s_{\delta_i}(F, E)$; hence, $\exists$ $(G, E) \in SOS_{\delta_i}(X)$ such that $x_e \in (G, E)\tilde{\subseteq}(F, E)$. It follows that $(F^{\tilde{c}}, E)\tilde{\cap}(G, E) = \tilde{\varphi}$, thus $x_e \notin cl^s_{\delta_i}(F^{\tilde{c}}, E)$ from Theorem 4.11.

Now, we prove the other inclusion, assuming that $x_e \notin cl^s_{\delta_i}(F^{\tilde{c}}, E)$. From Theorem 4.11, $\exists$ $(G, E) \in SOS_{\delta_i}(X)$ containing x_e such that $(F^{\tilde{c}}, E)\tilde{\cap}(G, E) = \tilde{\varphi}$, which means $x_e \in (G, E)\tilde{\subseteq}(F, E)$; therefore, $x_e \in int^s_{\delta_i}(F, E)$; hence, $x_e \notin [int^s_{\delta_i}(F, E)]^{\tilde{c}}$. This completes the proof.

(2) By a similar way to (1).

Definition 4.13. *Let (X, μ, E) be an SSTS and $(F, E) \in S(X)_E$, then $x_e \in S(X)_E$ is called a supra soft δ_i-boundary point of (F, E) if $x_e \in [cl^s_{\delta_i}(F, E) - int^s_{\delta_i}(F, E)]$. The set of all supra soft δ_i-boundary points of (F, E) is called supra soft δ_i-boundary set of (F, E) and denoted by $b^s_{\delta_i}(F, E)$.*

Theorem 4.14. *Let (X, μ, E) be an SSTS and $(F, E) \in S(X)_E$, then*

(1) $b^s_{\delta_i}(F, E) = cl^s_{\delta_i}(F, E)\tilde{\cap}[int^s_{\delta_i}(F, E)]^{\tilde{c}} = [int^s_{\delta_i}(F, E)\tilde{\cup}int^s_{\delta_i}(F^{\tilde{c}}, E)]^{\tilde{c}}$.

(2) $b^s_{\delta_i}(F, E) = b^s_{\delta_i}(F^{\tilde{c}}, E)$.

(3) $cl^s_{\delta_i}(F, E) = int^s_{\delta_i}(F, E)\tilde{\cup}b^s_{\delta_i}(F, E)$.

(4) $int^s_{\delta_i}(F, E) = (F, E) - b^s_{\delta_i}(F, E)$.

Proof. (1)

$$\begin{aligned}
[int^s_{\delta_i}(F, E)\tilde{\cup}int^s_{\delta_i}(F^{\tilde{c}}, E)]^{\tilde{c}} &= [int^s_{\delta_i}(F, E)]^{\tilde{c}}\tilde{\cap}[int^s_{\delta_i}(F^{\tilde{c}}, E)]^{\tilde{c}} \\
&= cl^s_{\delta_i}(F, E)\tilde{\cap}[int^s_{\delta_i}(F, E)]^{\tilde{c}} \text{ from Theorem 4.12 (1)} \\
&= cl^s_{\delta_i}(F, E) - int^s_{\delta_i}(F, E) \\
&= b^s_{\delta_i}(F, E).
\end{aligned}$$

(2) Clear from Definition 4.13.

(3)

$$\begin{aligned}
R.H. = int^s_{\delta_i}(F, E)\tilde{\cup}b^s_{\delta_i}(F, E) &= int^s_{\delta_i}(F, E)\tilde{\cup}[cl^s_{\delta_i}(F, E)\tilde{\cap}[int^s_{\delta_i}(F, E)]^{\tilde{c}}] \text{ from (1)} \\
&= [int^s_{\delta_i}(F, E)\tilde{\cup}cl^s_{\delta_i}(F, E)]\tilde{\cap}[int^s_{\delta_i}(F, E)\tilde{\cup}[int^s_{\delta_i}(F, E)]^{\tilde{c}}] \\
&= cl^s_{\delta_i}(F, E)\tilde{\cap}\tilde{X} = cl^s_{\delta_i}(F, E) = L.H.S.
\end{aligned}$$

(4)

$$\begin{aligned}(F,E)-b^s_{\delta_i}(F,E) &= (F,E)\tilde{\cap}[cl^s_{\delta_i}(F,E)\tilde{\cap}[int^s_{\delta_i}(F,E)]^{\tilde{c}}]^{\tilde{c}}\\ &= (F,E)\tilde{\cap}[[cl^s_{\delta_i}(F,E)]^{\tilde{c}}\tilde{\cup}[int^s_{\delta_i}(F,E)]]\\ &= [(F,E)\tilde{\cap}[cl^s_{\delta_i}(F,E)]^{\tilde{c}}]\tilde{\cup}[(F,E)\tilde{\cap}int^s_{\delta_i}(F,E)]\\ &= \tilde{\varphi}\tilde{\cup}int^s_{\delta_i}(F,E)\\ &= int^s_{\delta_i}(F,E).\end{aligned}$$

Proposition 4.15. *Let (X,μ,E) be an SSTS and $(F,E)\in S(X)_E$, then*

(1) $b^s_{\delta_i}[int^s_{\delta_i}(F,E)]\tilde{\subseteq}b^s_{\delta_i}(F,E)$.

(2) $b^s_{\delta_i}[cl^s_{\delta_i}(F,E)]\tilde{\subseteq}b^s_{\delta_i}(F,E)$.

(3) *If $int^s_{\delta_i}(F,E)=(F,E)$, then $b^s_{\delta_i}(F,E)\tilde{\cap}(F,E)=\tilde{\varphi}$.*

(4) *If $cl^s_{\delta_i}(F,E)=(F,E)$, then $b^s_{\delta_i}(F,E)\tilde{\subseteq}(F,E)$.*

(5) $int^s_{\delta_i}(F,E)=(F,E)=cl^s_{\delta_i}(F,E)\Longleftrightarrow b^s_{\delta_i}(F,E)=\tilde{\varphi}$.

Proof. Follows from Theorem 4.14.

Definition 4.16. *Let (X,μ,E) be an SSTS, $(F,E)\in S(X)_E$ and $x_e\in S(X)_E$, then x_e is called a supra soft δ_i-cluster point of (F,E) if $[(H,E)-x_e]\tilde{\cap}(F,E)\neq\tilde{\varphi}\ \forall\ (H,E)\in SOS_{\delta_i}(X)$ and $x_e\in(H,E)$. The set of all supra soft δ_i-cluster points of (F,E) is called supra soft δ_i-derived of (F,E) and is denoted by $d^s_{\delta_i}(F,E)$).*

As follows from Proposition 4.7, the reader can prove the following properties.

Proposition 4.17. *Let (X,μ,E) be an SSTS and $(F,E)\in S(X)_E$, then*

(1) *If $(F,E)\in SCS_{\delta_i}(X)$, then $d^s_{\delta_i}(F,E)\tilde{\subseteq}(F,E)$.*

(2) *If $(F,E)\tilde{\subseteq}(G,E)$, then $d^s_{\delta_i}(F,E)\tilde{\subseteq}d^s_{\delta_i}(G,E)$.*

(3) *If (F,E) is a soft superset to any soft set (A,E), then $d^s_{\delta_i}(A,E)\tilde{\subseteq}d^s_{\delta_i}(F,E)$.*

(4) $d^s_{\delta_i}[(F,E)\tilde{\cap}(G,E)]\tilde{\subseteq}d^s_{\delta_i}(F,E)\tilde{\cap}d^s_{\delta_i}(G,E)$.

(5) $d^s_{\delta_i}(F,E)\tilde{\cup}d^s_{\delta_i}(G,E)\tilde{\subseteq}d^s_{\delta_i}[(F,E)\tilde{\cup}(G,E)]$.

5. Applications of supra soft δ_i-open sets in soft continuity

In this section, we apply the concept of supra soft δ_i-open sets to supra soft continuity. In more detail, we use the notion of supra soft δ_i-open sets to introduce the notion of supra soft δ_i-continuous functions, which is a generalization to such soft continuity discussed in previous studies. We support our discussion with many counterexamples to declare the relationships between our new notion and the old one. Also, we introduced the notions of supra soft δ_i-irresolute, supra soft δ_i-open and supra soft δ_i-closed functions. We studied the main properties in special that differ from our new notion on the other ones.

Definition 5.1. *A soft function $f_{pu} : (X, \tau_1, A) \rightarrow (Y, \tau_2, B)$ with μ_1 as an associated SSTS with τ_1 is said to be a supra soft δ_i-continuous (briefly, SS-δ_i-cts) if $f_{pu}^{-1}(G, B) \in SOS_{\delta_i}(X)$ $\forall$ $(G, B) \in \tau_2$.*

Theorem 5.2. *Let $f_{pu} : (X, \tau_1, A) \rightarrow (Y, \tau_2, B)$ be an SS-δ_i-cts with μ_1 an associated SSTS with τ_1, then the following are equivalent:*

(1) $int^s_{\delta_i}(f_{pu}^{-1}(H, B)) = f_{pu}^{-1}(H, B)$ $\forall$ $(H, B) \in \tau_2$.

(2) $cl^s_{\delta_i}(f_{pu}^{-1}(H, B)) = f_{pu}^{-1}(H, B)$ $\forall$ $(H, B) \in \tau_2^c$.

(3) $cl^s_{\delta_i}(f_{pu}^{-1}(H, B))\tilde{\subseteq}f_{pu}^{-1}(cl(H, B))$ $\forall$ $(H, B)\tilde{\subseteq}\tilde{Y}$.

(4) $f_{pu}(cl^s_{\delta_i}(G, A))\tilde{\subseteq}cl(f_{pu}(G, A))$ $\forall$ $(G, A)\tilde{\subseteq}\tilde{X}$.

(5) $f_{pu}^{-1}(int(H, B))\tilde{\subseteq}int^s_{\delta_i}(f_{pu}^{-1}(H, B))$ $\forall$ $(H, B)\tilde{\subseteq}\tilde{Y}$.

Proof.

(1) $\Rightarrow$ (2) Let $(H, B) \in \tau_2^c$, then $(H^{\tilde{c}}, B) \in \tau_2$ and $int^s_{\delta_i}(f_{pu}^{-1}(H^{\tilde{c}}, B)) = f_{pu}^{-1}(H^{\tilde{c}}, B)$ from (1). It follows that

$$[f_{pu}^{-1}(H, B)^{\tilde{c}}]^{\tilde{c}} = [int^s_{\delta_i}(f_{pu}^{-1}(H^{\tilde{c}}, B))]^{\tilde{c}}.$$

Thus,

$$cl^s_{\delta_i}(f_{pu}^{-1}(H, B)) = f_{pu}^{-1}(H, B) \ from \ Theorem \ 4.12 \ (1).$$

(2) $\Rightarrow$ (3) Let $(H, B)\tilde{\subseteq}\tilde{Y}$. From (2),

$$cl^s_{\delta_i}[f_{pu}^{-1}[cl(H, B)] = f_{pu}^{-1}[cl(H, B).$$

Hence,

$$cl^s_{\delta_i}(f_{pu}^{-1}(H, B))\tilde{\subseteq}cl^s_{\delta_i}(f_{pu}^{-1}[cl(H, B)]) = f_{pu}^{-1}(cl(H, B)).$$

(3) $\Rightarrow$ (4) Let $(G, A)\tilde{\subseteq}\tilde{X}$. From Theorem 2.8 (c),

$$(G, A)\tilde{\subseteq}f_{pu}^{-1}(f_{pu}(G, A)).$$

From Proposition 4.7 (3),

$$cl^s_{\delta_i}(G, A)\tilde{\subseteq}cl^s_{\delta_i}[f_{pu}^{-1}(f_{pu}(G, A))].$$

From (3),

$$cl^s_{\delta_i}(G, A)\tilde{\subseteq}cl^s_{\delta_i}[f_{pu}^{-1}(f_{pu}(G, A))]\tilde{\subseteq}f_{pu}^{-1}(cl(f_{pu}(H, B))).$$

Therefore,

$$f_{pu}[cl^s_{\delta_i}(G, A)]\tilde{\subseteq}f_{pu}[f_{pu}^{-1}(cl(f_{pu}(H, B)))]\tilde{\subseteq}cl(f_{pu}(H, B)), \quad \text{from Theorem 2.8 (b,c).}$$

(4) $\Rightarrow$ (5) Let $(H, B)\tilde{\subseteq}\tilde{Y}$. Applying (4) for $f_{pu}^{-1}(H^{\tilde{c}}, B)\tilde{\subseteq}\tilde{X}$,

$$f_{pu}(cl^s_{\delta_i}[f_{pu}^{-1}(H^{\tilde{c}}, B)]\tilde{\subseteq}cl(f_{pu}[f_{pu}^{-1}(H^{\tilde{c}}, B)]\tilde{\subseteq}cl(H^{\tilde{c}}, B), \quad \text{from Theorem 2.8 (b).}$$

Hence,

$$
\begin{aligned}
[int^{s}_{\delta_i}(f^{-1}_{pu}(H,B))]^{\tilde{c}} &= cl^{s}_{\delta_i}[f^{-1}_{pu}(H^{\tilde{c}},B)] \\
&\tilde{\subseteq} f^{-1}_{pu}[f_{pu}(cl^{s}_{\delta_i}[f^{-1}_{pu}(H^{\tilde{c}},B)])]\tilde{\subseteq} f^{-1}_{pu}[cl(H^{\tilde{c}},B)], \text{ from (4)} \\
&= [int(f^{-1}_{pu}(H,B))]^{\tilde{c}}, \text{ from Theorem 4.12.}
\end{aligned}
$$

Thus,

$$int(f^{-1}_{pu}(H,B))\tilde{\subseteq} int^{s}_{\delta_i}(f^{-1}_{pu}(H,B)).$$

(5) $\Rightarrow$ (1) Let $(H,B) \in \tau_2$, then

$$int(H,B) = (H,B) \text{ and } f^{-1}_{pu}(int(H,B)) = f^{-1}_{pu}((H,B))\tilde{\subseteq} int^{s}_{\delta_i}(f^{-1}_{pu}(H,B)), \text{ from (5).}$$

However, we have

$$int^{s}_{\delta_i}(f^{-1}_{pu}(H,B))\tilde{\subseteq} f^{-1}_{pu}(H,B),$$

which means that,

$$int^{s}_{\delta_i}(f^{-1}_{pu}(H,B)) = f^{-1}_{pu}(H,B).$$

Theorem 5.3. *A soft function $f_{pu} : (X,\tau_1,A) \rightarrow (Y,\tau_2,B)$ with μ_1 as an associated SSTS with τ_1 is an SS-δ_i-cts if, and only if, $f^{-1}_{pu}(H,B)) \in SCS_{\delta_i}(X)\ \forall\ (H,B) \in \tau^{c}_{2}$.*

Proof. Let $(H,B) \in \tau^{c}_{2}$, then $(H^{\tilde{c}},B) \in \tau$. Since f_{pu} is SS-δ_i-cts,

$$f^{-1}_{pu}(H^{\tilde{c}},B) = [f^{-1}_{pu}(H,B)]^{\tilde{c}} \in SOS_{\delta_i}(X).$$

Thus,

$$f^{-1}_{pu}(H,B)) \in SCS_{\delta_i}(X).$$

Conversely, let $(H,B) \in \tau_2$, then $(H^{\tilde{c}},B) \in \tau^{c}_{2}$. From the condition,

$$f^{-1}_{pu}(H^{\tilde{c}},B) \in SCS_{\delta_i}(X).$$

Therefore,

$$f^{-1}_{pu}(H,B) \in SOS_{\delta_i}(X).$$

Thus, f_{pu} is an SS-δ_i-cts.

Definition 5.4. *[5, 23] A soft function $f_{pu} : (X,\tau_1,A) \rightarrow (Y,\tau_2,B)$ with μ_1 as an associated SSTS with τ_1 is said to be:*

(1) *Supra soft continuous (briefly, SS-cts) if $f^{-1}_{pu}(G,B) \in \mu_1\ \forall\ (G,B) \in \tau_2$.*

(2) *Supra soft pre-continuous (briefly, SS-pre-cts) if $f^{-1}_{pu}(G,B) \in POS^{s}(X)\ \forall\ (G,B) \in \tau_2$.*

(3) *Supra soft semi-continuous (briefly, SS-semi-cts) if $f^{-1}_{pu}(G,B) \in SOS^{s}(X)\ \forall\ (G,B) \in \tau_2$.*

(4) *Supra soft α-continuous (briefly, SS-α-cts) if $f^{-1}_{pu}(G,B) \in \alpha OS^{s}(X)\ \forall\ (G,B) \in \tau_2$.*

(5) *Supra soft β-continuous (briefly, SS-β-cts) if $f^{-1}_{pu}(G,B) \in \beta OS^{s}(X)\ \forall\ (G,B) \in \tau_2$.*

(6) *Supra soft B-continuous (briefly, SS-b-cts) if* $f_{pu}^{-1}(G, B) \in BOS^s(X)\ \forall\ (G, B) \in \tau_2$.

Proposition 5.5. *Every SS- (respectively, SS-semi-, SS-α-) cts function is an SS-δ_i-cts.*

Proof. Immediate from Theorem 3.6.

Remark 5.6. *The converse of Proposition 5.5 is not true in general, as shown in the following examples.*

Examples 5.7. **(1)** *Let* $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2, y_3\}$, $A = \{a_1, a_2\}$ *and* $B = \{b_1, b_2\}$.

Define $u : X \to Y$ *and* $p : A \to B$ *as follows:*

$$u(x_1) = y_3,\ u(x_2) = y_1,\ u(x_3) = y_2,\ p(a_1) = b_1,\ p(a_2) = b_2.$$

Let $\tau_1 = \{\tilde{X}, \tilde{\varphi}, (S_1, A)\}$ *be an* STS *over* X *and* $\mu_1 = \{\tilde{X}, \tilde{\varphi}, (S_1, A), (S_2, A)\}$ *be an associated* $SSTS$ *with* τ_1, *where:*

$$S_1(a_1) = \{x_1, x_2\}, \quad S_1(a_2) = \{x_2, x_3\}.$$

$$S_2(a_1) = \{x_2, x_3\}, \quad S_2(a_2) = \{x_1, x_2\}.$$

Let $\tau_2 = \{\tilde{Y}, \tilde{\varphi}, (J, B)\}$ *be an* STS *over* Y, *where:*

$$J(b_1) = \{y_2, y_3\}, \quad J(b_2) = \{y_1, y_2\},$$

then

$$f_{pu}^{-1}((J, B)) = \{(a_1, \{x_1, x_3\}), (a_2, \{x_2, x_3\})\}$$

is a supra soft δ_i-open set, but it is not supra soft open. Hence, f_{pu} *is an SS-δ_i-cts function, but it is not SS-cts.*

(2) *Let* $X = \{\alpha_1, \alpha_2, \alpha_3\}$, $Y = \{\beta_1, \beta_2, \beta_3\}$, $A = \{a_1, a_2\}$ *and* $B = \{b_1, b_2\}$.

Define $u : X \to Y$ *and* $p : A \to B$ *as follows:*

$$u(\alpha_1) = \beta_2,\ u(\alpha_2) = \beta_3,\ u(\alpha_3) = \beta_1,\ p(a_1) = b_1,\ p(a_2) = b_2.$$

Let $\tau_1 = \{\tilde{X}, \tilde{\varphi}, (H_2, A)\}$ *be an* STS *over* X *and* $\mu_1 = \{\tilde{X}, \tilde{\varphi}, (H_1, A), (H_2, A), (H_3, A), (H_4, A)\}$ *be an associated* $SSTS$ *with* τ_1, *where:*

$$H_1(a_1) = \{\alpha_1\}, \quad H_1(a_2) = \{\alpha_1, \alpha_2\}.$$

$$H_2(a_1) = \{\alpha_1, \alpha_2\}, \quad H_2(a_2) = \{\alpha_1\}.$$

$$H_3(a_1) = \{\alpha_1, \alpha_2\}, \quad H_3(a_2) = \{\alpha_1, \alpha_2\}.$$

$$H_4(a_1) = \{\alpha_2, \alpha_3\}, \quad H_4(a_2) = \{\alpha_2, \alpha_3\}.$$

Let $\tau_2 = \{\tilde{Y}, \tilde{\varphi}, (M, B)\}$ *be an* STS *over* Y, *where:*

$$M(b_1) = \{\beta_3\}, \quad M(b_2) = \{\beta_1, \beta_2\},$$

then

$$f_{pu}^{-1}((M, B)) = \{(a_1, \{\alpha_2\}), (a_2, \{\alpha_1, \alpha_3\})\},$$

is a supra soft δ_i-open set, but it is not supra soft semi-open. Hence, f_{pu} *is a SS-δ_i-cts function, but it is not SS-semi-cts.*

(3) *Let* $X = \{s_1, s_2, s_3\}$, $Y = \{t_1, t_2, t_3\}$, $A = \{a_1, a_2\}$ *and* $B = \{b_1, b_2\}$.
Define $u : X \to Y$ *and* $p : A \to B$ *as follows:*

$$u(s_1) = t_3,\ u(s_2) = t_2,\ u(s_3) = t_1,\ p(a_1) = b_1,\ p(a_2) = b_2.$$

Let $\tau_1 = \{\tilde{X}, \tilde{\varphi}, (K_3, A)\}$ *be an* STS *over* X *and*

$$\mu_1 = \{\tilde{X}, \tilde{\varphi}, (K_1, A), (K_2, A), (K_3, A), (K_4, A), (K_5, A)\}$$

be an associated $SSTS$ *with* τ_1, *where:*

$$K_1(a_1) = \{s_1\}, \quad K_1(a_2) = \{s_1\}.$$

$$K_2(a_1) = \{s_1, s_3\}, \quad K_2(a_2) = \{s_2, s_3\}.$$

$$K_3(a_1) = \{s_2, s_3\}, \quad K_3(a_2) = \{s_1, s_3\}.$$

$$K_4(a_1) = X, \quad K_4(a_2) = \{s_1, s_3\}.$$

$$K_5(a_1) = \{s_1, s_3\}, \quad K_5(a_2) = X.$$

Let $\tau_2 = \{\tilde{Y}, \tilde{\varphi}, (Q, B)\}$ *be an* STS *over* Y, *where:*

$$Q(b_1) = \{t_2, t_3\}, \quad Q(b_2) = \{t_1, t_2\},$$

then

$$f_{pu}^{-1}((Q, B)) = \{(a_1, \{s_1, s_2\}), (a_2, \{s_2, s_3\})\},$$

is a supra soft δ_i*-open set, but it is not supra soft* α*-open. Hence,* f_{pu} *is an SS-*δ_i*-cts function, but it is not SS-*α*-cts.*

Remark 5.8. *There is no priori relation between our new collection SS-*δ_i*-cts functions and the collections of SS-*β*- (respectively, pre-, b-) cts functions discussed in [5, 23] in general. The following examples shall support our claim.*

Examples 5.9. **(1)** *In Examples 5.7 (1), consider* $\tau_2 = \{\tilde{Y}, \tilde{\varphi}, (J, B)\}$ *as an* STS *over* Y, *where:*

$$J(b_1) = \{y_2\}, \quad J(b_2) = \{y_2\},$$

then

$$f_{pu}^{-1}((J, B)) = \{(a_1, \{x_3\}), (a_2, \{x_3\})\}$$

is a supra soft β*- (respectively, pre-, b-) open, but it is not supra soft* δ_i*-open. Hence,* f_{pu} *is an SS-*β*- (respectively, pre-, b-) cts function, but it is not SS-*δ_i*-cts.*

(2) *In Examples 5.7 (3), consider* $\tau_2 = \{\tilde{Y}, \tilde{\varphi}, (W, B)\}$ *as an* STS *over* Y, *where:*

$$W(b_1) = \{t_3\}, \quad W(b_2) = \{t_2\},$$

then

$$f_{pu}^{-1}((W, B)) = \{(a_1, \{s_1\}), (a_2, \{s_2\})\}$$

is a supra soft δ_i*-open set, but it is not supra soft* β*-open. Hence,* f_{pu} *is an SS-*δ_i*-cts, but it is not SS-*β*-cts.*

(3) *In Examples 5.7 (2), consider* $\tau_2 = \{\tilde{Y}, \tilde{\varphi}, (L, B)\}$ *as an* STS *over* Y*, where:*

$$L(b_1) = \{\beta_1\}, \quad L(b_2) = \{\beta_1, \beta_3\},$$

then

$$f_{pu}^{-1}((L, B)) = \{(a_1, \{\alpha_3\}), (a_2, \{\alpha_2, \alpha_3\})\}$$

is a supra soft δ_i*-open set, but it is not supra soft pre-open. Hence,* f_{pu} *is an SS-*δ_i*-cts, but it is not SS-pre-cts.*

(4) *Let* $X = \{g_1, g_2, g_3\}$*,* $Y = \{n_1, n_2, n_3\}$*,* $A = \{a_1, a_2\}$ *and* $B = \{b_1, b_2\}$*.*

Define $u : X \to Y$ *and* $p : A \to B$ *as follows:*

$$u(g_1) = n_2,\ u(g_2) = n_3,\ u(g_3) = n_1,\ p(a_1) = b_1,\ p(a_2) = b_2.$$

Let $\tau_1 = \{\tilde{X}, \tilde{\varphi}, (C_2, A)\}$ *be an* STS *over* X *and*

$$\mu_1 = \{\tilde{X}, \tilde{\varphi}, (C_1, A), (C_2, A), (C_3, A), (C_4, A), (C_5, A)\}$$

be an associated $SSTS$ *with* τ_1*, where:*

$$C_1(a_1) = \{g_1\}, \quad C_1(a_2) = \{g_2, g_3\}.$$

$$C_2(a_1) = \{g_2, g_3\}, \quad C_2(a_2) = \{g_1\}.$$

$$C_3(a_1) = \{g_1, g_2\}, \quad C_3(a_2) = \{g_1, g_2\}.$$

$$C_4(a_1) = \{g_1, g_2\}, \quad C_4(a_2) = X.$$

$$C_5(a_1) = X, \quad C_5(a_2) = \{g_1, g_2\}.$$

Let $\tau_2 = \{\tilde{Y}, \tilde{\varphi}, (U, B)\}$ *be an* STS *over* Y*, where:*

$$U(b_1) = \{n_2, n_3\}, \quad U(b_2) = \{n_2\},$$

then

$$f_{pu}^{-1}((U, B)) = \{(a_1, \{g_1, g_2\}), (a_2, \{g_1\})\}$$

is a supra soft δ_i*-open set, but it is not supra soft b-open. Hence,* f_{pu} *is an SS-*δ_i*-cts function, but it is not SS-b-cts.*

Definition 5.10. *A soft function* $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ *with* μ_1, μ_2 *associated supra soft topologies with* τ_1, τ_2 *respectively, is said to be supra soft* δ_i*-irresolute (briefly, SS-*δ_i*-irresolute) if* $f_{pu}^{-1}(G, B) \in SOS_{\delta_i}(X)\ \forall\ (G, B) \in SOS_{\delta_i}(Y)$*.*

Proposition 5.11. *Let* $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ *be an SS-*δ_i*-irresolute with* μ_1, μ_2 *associated supra soft topologies with* τ_1, τ_2*, respectively, then the following are equivalent:*

(1) $int^s_{\delta_i}(f_{pu}^{-1}(H, B)) = f_{pu}^{-1}(H, B)\ \forall\ (H, B) \in SOS_{\delta_i}(Y)$*.*

(2) $cl^s_{\delta_i}(f_{pu}^{-1}(H, B)) = f_{pu}^{-1}(H, B)\ \forall\ (H, B) \in SCS_{\delta_i}(Y)$*.*

(3) $cl^s_{\delta_i}(f^{-1}_{pu}(H,B))\tilde{\subseteq}f^{-1}_{pu}(cl^s_{\delta_i}(H,B))\ \forall\ (H,B)\tilde{\subseteq}\tilde{Y}$.

(4) $f_{pu}(cl^s_{\delta_i}(G,A))\tilde{\subseteq}cl^s_{\delta_i}(f_{pu}(G,A))\ \forall\ (G,A)\tilde{\subseteq}\tilde{X}$.

(5) $f^{-1}_{pu}(int^s_{\delta_i}(H,B))\tilde{\subseteq}int^s_{\delta_i}(f^{-1}_{pu}(H,B))\ \forall\ (H,B)\tilde{\subseteq}\tilde{Y}$.

Proof. Follows from Theorem 5.2.

Theorem 5.12. *Let $f_{pu} : (X,\tau_1,A) \rightarrow (Y,\tau_2,B)$ be an SS-δ_i-irresolute with μ_1,μ_2 associated supra soft topologies with τ_1,τ_2, respectively, and $g_{pu} : (Y,\tau_2,B) \rightarrow (Z,\tau_3,C)$ be an SS-δ_i-cts with μ_2 an associated SSTS with τ_2, then the composition $g_{pu}of_{pu} : (X,\tau_1,A) \rightarrow (Z,\tau_3,C)$ is SS-δ_i-cts.*

Proof. Let $(K,C) \in \tau_3$. Since g_{pu} is SS-δ_i-cts, $g^{-1}_{pu}(K,C) \in SOS_{\delta_i}(Y)$. Since f_{pu} is SS-δ_i-irresolute, $[g_{pu}of_{pu}]^{-1}(K,C) = f^{-1}_{pu}[g^{-1}_{pu}(K,C)] \in SOS_{\delta_i}(X)$. Hence, $g_{pu}of_{pu}$ is SS-δ_i-cts.

Corollary 5.13. *The composition of two SS-δ_i-irresolute functions is also SS-δ_i-irresolute.*

Proof. Follows from Theorem 5.12.

Proposition 5.14. *Every SS-δ_i-irresolute function is an SS-δ_i-cts.*

Proof. Obvious.

Remark 5.15. *The converse of Proposition 5.14 is not true in general, as shown in the following example.*

Example 5.16. *In Examples 5.7 (1), consider $\mu_2 = \{\tilde{X},\tilde{\varphi},(J_1,A),(J_2,A),(J_3,A)\}$ as an associated $SSTS$ with τ_2, where:*

$$J_1(b_1) = \{y_2,y_3\},\quad J_1(b_2) = \{y_1,y_2\}.$$

$$J_2(a_1) = \{y_3\},\quad J_2(a_2) = \{y_1,y_2\}.$$

$$J_3(a_1) = \{y_2\},\quad J_3(a_2) = \{y_2\}.$$

Thus, f_{pu} is an SS-δ_i-cts.

On the other hand, $(J_3,B) \in SOS_{\delta_i}(Y)$, but $f^{-1}_{pu}((J_3,B)) = \{(a_1,\{x_3\}),(a_2,\{x_3\})\} \notin SOS_{\delta_i}(X)$.

Hence, f_{pu} is not SS-δ_i-irresolute.

Corollary 5.17. *The following implications hold from Theorem 5.5 and [23, Corollary 6.1] for an SSTS (X,μ,E). These implications are not reversible.*

$$\begin{array}{ccccccccc}
\textit{SS-cts} & \longrightarrow & \textit{SS-}\alpha\textit{-cts} & \longrightarrow & \textit{SS-semi-cts} & \longrightarrow & \textit{SS-}\delta_i\textit{-cts} & \not\leftrightarrow & \textit{SS-}\beta\textit{-cts} \\
 & \searrow & & & & \searrow & \not\Updownarrow & \nearrow & \\
 & & \textit{SS-pre-cts} & & \longrightarrow & & \textit{SS-b-cts} & &
\end{array}$$

Definition 5.18. *A soft function $f_{pu} : (X,\tau_1,A) \rightarrow (Y,\tau_2,B)$ with μ_2 as an associated SSTS with τ_2, is called:*

(1) *A supra soft δ_i-open if $f_{pu}(G,A) \in SOS_{\delta_i}(Y)\ \forall\ (G,A) \in \tau_1$.*

(2) *A supra soft δ_i-closed if $f_{pu}(H,A) \in SCS_{\delta_i}(Y)\ \forall\ (H,A) \in \tau^c_1$.*

Proposition 5.19. *Let $f_{pu} : (X, \tau_1, A) \rightarrow (Y, \tau_2, B)$ be a soft function with μ_2 be an associated SSTS with τ_2 and $(G, A)\tilde{\subseteq}\tilde{X}$, then*

(1) *If f_{pu} is supra soft δ_i-open, then $f_{pu}(int(G, A))\tilde{\subseteq}int^s_{\delta_i}[f_{pu}(G, A)]$.*

(2) *If f_{pu} is supra soft δ_i-closed, then $cl^s_{\delta_i}(f_{pu}(G, A))\tilde{\subseteq}f_{pu}(cl(G, A))$.*

Proof. (1) Let f_{pu} be a supra soft δ_i-open, then $f_{pu}(int(G, A)) \in SOS_{\delta_i}(Y)$ and it follows that

$$f_{pu}(int(G, A)) = int^s_{\delta_i}[f_{pu}(int(G, A))]\tilde{\subseteq}int^s_{\delta_i}[f_{pu}(G, A)].$$

(2) By a similar way to (1).

Proposition 5.20. *Let $f_{pu} : (X, \tau_1, A) \rightarrow (Y, \tau_2, B)$ be a bijective soft function with μ_2 as an associated SSTS with τ_2, then f_{pu} is supra soft δ_i-open if, and only if, it is supra soft δ_i-closed.*

Proof. Clear.

6. Conclusions

In this paper, we introduced a new generalization to some kinds of supra soft open sets. Specifically, we defined the concept of supra soft δ_i-open sets and discussed their basic properties. A diagram described the relationships between the new concept and other weaker forms of supra soft open sets is introduced. Furthermore, we used this concept to investigate some new operators. With more dissections, we found out that our notions differ on such previous notions. To ensure our results, many counterexamples were studied. Finally, as an expected application to such important results, we introduced the supra soft δ_i-continuous mappings. The relations with other kinds of soft continuity have been studied. In the future, we will use these new concepts with the view of [30] to introduce more topological properties such as soft compactness, soft connectedness and soft separation axioms by using the soft ideal notion [29]. Moreover, we will apply these notions to develop the accuracy measures of subsets in information systems to investigate such notions [36].

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflicts of interest.

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