



Research article

Stability analysis of systems with additive time-varying delays via new bivariate quadratic reciprocally convex inequality

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Abstract: This paper focuses on the stability analysis of additive time-varying delay systems. First, a bivariate quadratic reciprocally convex matrix inequality is derived, which serves as a generalization of traditional reciprocally convex inequalities. By applying the Lyapunov–Krasovskii functional method, this matrix inequality is incorporated to form a new stability criterion applicable to systems with additive time-varying delays. Finally, some numerical examples are presented to demonstrate the effectiveness of the theoretical results obtained.

Keywords: stability; reciprocally convex inequality; Lyapunov–Krasovskii functional method; additive time-varying delay

Mathematics Subject Classification: 34K20, 93C05

1. Introduction

The study of systems with time delays poses a significant challenge for control researchers and engineers, primarily because stability and performance are often compromised. While most existing research focuses on systems with a single time-varying delay, many practical control applications, particularly in networked systems, necessitate the consideration of multiple time-varying delays [1–4]. This is due to the fact that signals transmitted from one point to another frequently encounter successive delays with varying characteristics. Such systems are commonly found in practical applications, including network control systems, mechanical systems, and biological systems [5–7]. Consequently, investigating the stability of systems with multiple time delays is essential. Specifically, the practical significance of linear systems with two-time delays has attracted considerable attention in recent years [8–11].

The Lyapunov stability theory is a classical method for analyzing the stability of delay systems [12–14]. By constructing an appropriate Lyapunov, the changes in the system’s energy or state over

time can be examined to determine the stability of the system [15–18]. Additionally, the linear matrix inequalities (LMIs) method is a commonly used technique in modern control theory [19]. This method transforms the stability conditions of delay systems into solvable matrix inequalities, allowing numerical optimization tools to be employed to obtain stability criteria for the system. Numerical simulation is also an effective tool for studying the stability of delay systems. By simulating the dynamic behavior of the system on a computer, the response of the system under different parameters and initial conditions can be observed, thereby analyzing its stability. In practical analyses, multiple methods are often combined, such as integrating the Lyapunov method with matrix inequality approaches, which can yield stronger stability results, especially in complex or high-dimensional delay systems. It is noteworthy that the time delays are assumed to be differentiable, which is a more stringent condition than that in some existing studies [20,21]. This limitation may affect the general applicability of our results. Future research could consider relaxing this assumption to include non-differentiable time delays, thereby broadening the scope of the proposed method and enhancing its relevance to practical applications. Addressing this issue could significantly enrich the understanding of the stability of systems with more complex delay structures.

Recently, many studies have constructed enhanced Lyapunov–Krasovskii functions (LKFs) by introducing time-weighted or space-weighted terms and have incorporated various integral inequalities to obtain less conservative stability criteria [22–24]. For example, the Wirtinger inequality [25], the Bessel–Legendre inequality [26], and the reciprocally convex inequality (RCI) [17] are popular integral inequalities that have been widely applied. Consequently, an increasing number of studies are beginning to improve upon these inequalities to better apply them in different integrals. It is noteworthy that the quadratic function associated with time delay frequently arises in the derivative of the LKFs. Determining the negative definiteness condition (NDC) of this quadratic function has been a crucial topic in developing tractable LMIs over the years. Numerous studies have been conducted on the conditions for ensuring the negative definiteness of quadratic functions; their work has established conditions for constructing the quadratic form reciprocally convex inequality [27, 28].

The RCI effectively addresses these nonlinear characteristics, enhancing the accuracy of stability analysis. Thus, extending the concept of RCI becomes essential. In light of the recent extensive research on negative determination conditions for quadratic functions, their work has provided the necessary conditions for applying quadratic reciprocally convex inequalities in additive delay systems. The proposed bivariate quadratic reciprocally convex matrix inequality in this paper is inspired by their results. We have derived a bivariate quadratic reciprocally convex matrix inequality (BQRCI) which generalizes the original RCI, facilitating its application to bivariate quadratic functions [27]. By utilizing this inequality, we have successfully reduced the conservativeness of the stability criteria, leading to improved stability conditions. In addition, the application of the generalized BQRCI may be diverse, and future research could explore the integration of T-S fuzzy models with the proposed BQRCI framework. This may involve developing stability analysis methods that utilize T-S fuzzy representations to more effectively address complex nonlinear systems [29, 30]. Furthermore, investigating the robustness of the BQRCI under various uncertainties within T-S fuzzy models could provide new insights and enhance the applicability of our findings in reality scenarios. Such extensions could make significant contributions to the field, offering a more comprehensive understanding of stability for systems characterized by time-varying delays and T-S fuzzy dynamics.

The main contributions of this paper can be summarized as follows:

- 1) This paper generalized a new bivariate quadratic reciprocally convex inequality and applied this inequality to additive time-varying delay systems to reduce the conservativeness of system stability.
- 2) Using the Lyapunov–Krasovskii functional method and the new bivariate quadratic reciprocally convex inequality, we obtained a new stability criterion and validated the effectiveness of the proposed method through some numerical examples.

The structure of this paper is as follows: In the second section, we introduce a new inequality related to bivariate quadratic functions, referred to as the improved RCI, called BQRCI. In the next section, we demonstrate the application of this new inequality in LKFs, leading to enhanced stability results and presenting the main theoretical findings. In the final section, we provide four numerical experiments to validate the new stability criterion.

Notations: Let \mathbb{R}^n and $\mathbb{R}^{n \times m}$ denote the sets of n -dimensional Euclidean real vectors space and $n \times m$ real matrices. The symbols \mathbb{S}^n and \mathbb{S}_+^n represent the collections of $n \times n$ symmetric matrices and symmetric positive-definite matrices. The notation *diag* refers to diagonal matrices, while $\text{zeros}(m, n)$ indicates the $m \times n$ matrices in \mathbb{R} where all entries are zero. The identity matrix in $\mathbb{R}^{n \times n}$ is denoted by I_n ; let $*$ be the symmetric part of a matrix. Lastly, we define $\text{sym}(A) = A + A^T$, where T denotes the transpose of a matrix, and we set $\mu_{21} = \mu_2 - \mu_1$ and $h_{21} = h_2 - h_1$.

2. Preliminaries

$$\begin{cases} \dot{y}(t) = \mathcal{A}y(t) + \mathcal{B}y(t - k_1(t) - k_2(t)), & t \geq 0, \\ y(t) = \phi(t), & t \in [-k_1 - k_2, 0], \end{cases} \quad (2.1)$$

where $y(t) \in \mathbb{R}^n$ represents the state vector, the initial conditions of time delay are $\phi(t) \in \mathbb{R}^n$, and $\mathcal{A}, \mathcal{B} \in \mathbb{R}^{n \times n}$ are the given matrices. The time-varying delays $k_1(t)$ and $k_2(t)$ are differentiable functions that satisfy the following condition:

$$0 \leq k_i(t) \leq k_i, \quad 0 \leq \dot{k}_i(t) \leq \mu_i, \quad (2.2)$$

where k_i and μ_i ($i = 1, 2$) are given positive constants.

Next, some important lemmas are introduced as follows:

Lemma 2.1. [25] For a matrix $R \in \mathbb{S}_+^n$, real scalars a and b , if there exists a continuous differentiable function $x : [a, b] \rightarrow \mathbb{R}^n$, the following integral inequality holds:

$$\int_a^b \dot{y}^T(s) R \dot{y}(s) ds \geq \frac{1}{b-a} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}^T \begin{bmatrix} R & 0 \\ 0 & 3R \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}, \quad (2.3)$$

where

$$\begin{aligned} \xi_1 &= y(b) - y(a), \\ \xi_2 &= y(b) + y(a) - \frac{2}{b-a} \int_a^b y(s) ds. \end{aligned}$$

Lemma 2.2. [27] For a bivariate quadratic function $f(x, y) = p_2x^2 + p_1x + q_2y^2 + q_1y + r_2xy + r_1$, where $p_2, p_1, q_2, q_1, r_2, r_1 \in \mathbb{R}$, and $0 \leq h_1 \leq x \leq h_2, \mu_1 \leq y \leq \mu_2$, if $f(x, y) < 0$ for $\forall (x, y) \in [h_1, h_2] \times [\mu_1, \mu_2]$, the following inequalities hold:

$$\begin{cases} g_1(h_1) = p_2h_1^2 + (p_1 + r_2\mu_1)h_1 + (q_2\mu_1^2 + q_1\mu_1 + r_1) < 0, \\ g_1(h_2) = p_2h_2^2 + (p_1 + r_2\mu_1)h_2 + (q_2\mu_1^2 + q_1\mu_1 + r_1) < 0, \\ \frac{h_{21}}{2}(2p_2h_1 + p_1 + r_2\mu_1) + g_1(h_1) < 0, \\ g_2(h_1) = p_2h_1^2 + (p_1 + r_2\mu_2)h_1 + (q_2\mu_2^2 + q_1\mu_2 + r_1) < 0, \\ g_2(h_2) = p_2h_2^2 + (p_1 + r_2\mu_2)h_2 + (q_2\mu_2^2 + q_1\mu_2 + r_1) < 0, \\ \frac{h_{21}}{2}(2p_2h_1 + p_1 + r_2\mu_2) + g_2(h_1) < 0, \\ g_3(h_1) = p_2h_1^2 + (p_1 + r_2\mu_1 + \frac{\mu_{21}}{2}r_2)h_1 + [q_2(\mu_{21}\mu_1 + \mu_1^2) + q_1(\frac{\mu_{21}}{2} + \mu_1) + r_1] < 0, \\ g_3(h_2) = p_2h_2^2 + (p_1 + r_2\mu_1 + \frac{\mu_{21}}{2}r_2)h_2 + [q_2(\mu_{21}\mu_1 + \mu_1^2) + q_1(\frac{\mu_{21}}{2} + \mu_1) + r_1] < 0, \\ \frac{h_{21}}{2}(2p_2h_1 + p_1 + r_2\mu_1 + \frac{\mu_{21}}{2}r_2) + g_3(h_1) < 0. \end{cases}$$

Lemma 2.3. [17] Let $h_1, h_2, \dots, h_N : \mathbb{R}^n \rightarrow \mathbb{R}$ be functions that take positive values in an open subset H of \mathbb{R}^n . Then, the reciprocally convex combination of h_i defined over H satisfies the following equations:

$$\min_{\{\alpha_i | \alpha_i > 0, \sum_i \alpha_i = 1\}} \sum_i \frac{1}{\alpha_i} h_i(t) = \sum_i h_i(t) + \max_{b_{i,j}(t)} \sum_{i \neq j} b_{i,j}(t),$$

subject to $\left\{ b_{i,j} : \mathbb{R}^n \rightarrow \mathbb{R}, b_{j,i}(t) = b_{i,j}(t), \begin{bmatrix} h_i(t) & b_{i,j}(t) \\ b_{j,i}(t) & h_j(t) \end{bmatrix} \geq 0 \right\}$.

Corollary 2.4. For any real positive scalars $\alpha, \beta, \gamma, \delta$, and holds $\alpha + \beta + \gamma + \delta = 1$, for given $R_i \in \mathbb{S}_+^n (i = 1, 2, \dots, 4)$, and free matrices $H_{12}, H_{13}, H_{14}, H_{23}, H_{24}, H_{34}$, if the size of the matrix is 4×4 , the reciprocally convex inequality shows:

$$\begin{bmatrix} \frac{1}{\alpha}R_1 & 0 & 0 & 0 \\ * & \frac{1}{\beta}R_2 & 0 & 0 \\ * & * & \frac{1}{\gamma}R_3 & 0 \\ * & * & * & \frac{1}{\delta}R_4 \end{bmatrix} \geq \begin{bmatrix} R_1 & H_{12} & H_{13} & H_{14} \\ * & R_2 & H_{23} & H_{24} \\ * & * & R_3 & H_{34} \\ * & * & * & R_4 \end{bmatrix}, \quad (2.4)$$

where the following restrictive conditions hold:

$$\begin{bmatrix} R_1 & H_{12} \\ * & R_2 \end{bmatrix} \geq 0, \quad \begin{bmatrix} R_1 & H_{13} \\ * & R_3 \end{bmatrix} \geq 0, \quad \begin{bmatrix} R_1 & H_{14} \\ * & R_4 \end{bmatrix} \geq 0, \\ \begin{bmatrix} R_1 & H_{12} \\ * & R_2 \end{bmatrix} \geq 0, \quad \begin{bmatrix} R_1 & H_{13} \\ * & R_3 \end{bmatrix} \geq 0, \quad \begin{bmatrix} R_1 & H_{14} \\ * & R_4 \end{bmatrix} \geq 0.$$

Remark 2.5. The reciprocally convex inequality can be efficiently applied in the stability of systems with time-varying delays; if the size of the matrix is $n \times n$, the number of restrictive conditions of the matrix inequality is $\frac{n(n-1)}{2}$.

Next, we introduce the bivariate quadratic reciprocally convex inequality based on Lemma 2.3 and Corollary 2.4, and also provide a detailed proof for this novel inequality.

Lemma 2.6. *Bivariate quadratic reciprocally convex inequality (BQRCI).* For any real positive scalars $\alpha, \beta, \gamma, \delta$, and holds $\alpha + \beta + \gamma + \delta = 1$, for given $R_i \in \mathbb{S}_+^n (i = 1, 2, \dots, 4)$, and free matrices $S_{1j}, S_{2j}, S_{3j}, S_{4j}, S_{5j}, S_{6j}, U_{1j}, U_{2j}, U_{3j}, U_{4j}, U_{5j}, U_{6j}, U_{7j}, U_{8j}, U_{9j}, U_{10j}, U_{11j}, U_{12j} \in \mathbb{R}^{n \times n} (j = 1, 2)$, such that the following inequality holds:

$$\begin{bmatrix} \frac{1}{\alpha}R_1 & 0 & 0 & 0 \\ * & \frac{1}{\beta}R_2 & 0 & 0 \\ * & * & \frac{1}{\gamma}R_3 & 0 \\ * & * & * & \frac{1}{\delta}R_4 \end{bmatrix} \geq \begin{bmatrix} R_1 & 0 & 0 & 0 \\ * & R_2 & 0 & 0 \\ * & * & R_3 & 0 \\ * & * & * & R_4 \end{bmatrix} + \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ * & M_{22} & M_{23} & M_{24} \\ * & * & M_{33} & M_{34} \\ * & * & * & M_{44} \end{bmatrix}. \quad (2.5)$$

When the condition below is satisfied:

$$\mathbb{A}_1 = \begin{bmatrix} \alpha U_{11} + \alpha^2 U_{12} - R_1 & (\alpha^2 + \beta^2)S_{12} + S_{11} & 0 & 0 \\ * & \beta U_{21} + \beta^2 U_{22} - R_2 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} \leq 0, \quad (2.6)$$

$$\mathbb{A}_2 = \begin{bmatrix} \alpha U_{31} + \alpha^2 U_{32} - R_1 & 0 & (\alpha^2 + \beta^2)S_{22} + S_{21} & 0 \\ * & 0 & 0 & 0 \\ * & * & \gamma U_{41} + \gamma^2 U_{42} - R_3 & 0 \\ * & * & * & 0 \end{bmatrix} \leq 0, \quad (2.7)$$

$$\mathbb{A}_3 = \begin{bmatrix} \alpha U_{51} + \alpha^2 U_{52} - R_1 & 0 & 0 & (\alpha^2 + \beta^2)S_{32} + S_{31} \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & \delta U_{61} + \delta^2 U_{62} - R_4 \end{bmatrix} \leq 0, \quad (2.8)$$

$$\mathbb{A}_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & \beta U_{71} + \beta^2 U_{72} - R_2 & (\alpha^2 + \beta^2)S_{42} + S_{41} & 0 \\ * & * & \gamma U_{81} + \gamma^2 U_{82} - R_3 & 0 \\ * & * & * & 0 \end{bmatrix} \leq 0, \quad (2.9)$$

$$\mathbb{A}_5 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & \beta U_{91} + \beta^2 U_{92} - R_2 & 0 & (\alpha^2 + \beta^2)S_{52} + S_{51} \\ * & * & 0 & 0 \\ * & * & * & \delta U_{101} + \delta^2 U_{102} - R_4 \end{bmatrix} \leq 0, \quad (2.10)$$

$$\mathbb{A}_6 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & \beta U_{111} + \beta^2 U_{112} - R_3 & (\alpha^2 + \beta^2)S_{62} + S_{61} \\ * & * & * & \delta U_{121} + \delta^2 U_{122} - R_4 \end{bmatrix} \leq 0, \quad (2.11)$$

where

$$\begin{aligned}
 M_{12} &= (\alpha^2 + \beta^2)S_{12} + S_{11}, & M_{13} &= (\alpha^2 + \beta^2)S_{22} + S_{21}, \\
 M_{14} &= (\alpha^2 + \beta^2)S_{32} + S_{31}, & M_{23} &= (\alpha^2 + \beta^2)S_{42} + S_{41}, \\
 M_{24} &= (\alpha^2 + \beta^2)S_{52} + S_{51}, & M_{34} &= (\alpha^2 + \beta^2)S_{62} + S_{61}, \\
 M_{11} &= \beta U_{11} + \alpha\beta U_{12} + \gamma U_{31} + \alpha\gamma U_{32} + \delta U_{51} + \alpha\delta U_{52}, \\
 M_{22} &= \alpha U_{21} + \alpha\beta U_{22} + \gamma U_{71} + \beta\gamma U_{72} + \delta U_{91} + \beta\delta U_{92}, \\
 M_{33} &= \alpha U_{41} + \alpha\gamma U_{42} + \beta U_{81} + \beta\gamma U_{82} + \delta U_{111} + \gamma\delta U_{112}, \\
 M_{44} &= \alpha U_{61} + \alpha\delta U_{62} + \beta U_{101} + \beta\delta U_{102} + \gamma U_{121} + \gamma\delta U_{122}.
 \end{aligned}$$

Proof: We first write the difference of inequality in six parts,

$$\begin{aligned}
 & \begin{bmatrix} \frac{1}{\alpha}R_1 & 0 & 0 & 0 \\ * & \frac{1}{\beta}R_2 & 0 & 0 \\ * & * & \frac{1}{\gamma}R_3 & 0 \\ * & * & * & \frac{1}{\delta}R_4 \end{bmatrix} - \begin{bmatrix} R_1 & S_{11} & S_{21} & S_{31} \\ * & R_2 & S_{41} & S_{51} \\ * & * & R_3 & S_{61} \\ * & * & * & R_4 \end{bmatrix} = \begin{bmatrix} \frac{\beta}{\alpha}R_1 & -S_{11} & 0 & 0 \\ * & \frac{\alpha}{\beta}R_2 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} + \begin{bmatrix} \frac{\gamma}{\alpha}R_1 & 0 & -S_{21} & 0 \\ * & 0 & 0 & 0 \\ * & * & \frac{\alpha}{\gamma}R_3 & 0 \\ * & * & * & 0 \end{bmatrix} \\
 & + \begin{bmatrix} \frac{\delta}{\alpha}R_1 & 0 & 0 & -S_{31} \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & \frac{\alpha}{\delta}R_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & \frac{\gamma}{\beta}R_2 & -S_{41} & 0 \\ * & * & \frac{\beta}{\gamma}R_3 & 0 \\ * & * & * & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & \frac{\delta}{\beta}R_2 & 0 & -S_{51} \\ * & * & 0 & 0 \\ * & * & * & \frac{\beta}{\delta}R_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & \frac{\delta}{\gamma}R_3 & -S_{61} \\ * & * & * & \frac{\gamma}{\delta}R_4 \end{bmatrix}.
 \end{aligned}$$

For the first part,

$$\begin{aligned}
 & \begin{bmatrix} \frac{\beta}{\alpha}R_1 & -S_{11} & 0 & 0 \\ * & \frac{\alpha}{\beta}R_2 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} = \begin{bmatrix} \beta U_{11} + \alpha\beta U_{12} & (\alpha^2 + \beta^2)S_{12} & 0 & 0 \\ * & \alpha U_{21} + \alpha\beta U_{22} & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} \\
 & -D_{12}(\alpha \begin{bmatrix} U_{11} & S_{11} & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} + \beta \begin{bmatrix} 0 & S_{11} & 0 & 0 \\ * & U_{21} & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} + \gamma \begin{bmatrix} 0 & S_{11} & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} + \delta \begin{bmatrix} 0 & S_{11} & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} \\
 & + \alpha^2 \begin{bmatrix} U_{12} & S_{12} & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} + \beta^2 \begin{bmatrix} 0 & S_{12} & 0 & 0 \\ * & U_{22} & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} - \begin{bmatrix} R_1 & 0 & 0 & 0 \\ * & R_2 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix})D_{12},
 \end{aligned}$$

where

$$D_{12} = \begin{bmatrix} \sqrt{\frac{\beta}{\alpha}}I & 0 & 0 & 0 \\ * & \sqrt{\frac{\alpha}{\beta}}I & 0 & 0 \\ * & * & I & 0 \\ * & * & * & I \end{bmatrix}.$$

In other words,

$$\begin{bmatrix} \frac{\beta}{\alpha}R_1 & -S_{11} & 0 & 0 \\ * & \frac{\alpha}{\beta}R_2 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} = \begin{bmatrix} \beta U_{11} + \alpha\beta U_{12} & (\alpha^2 + \beta^2)S_{12} & 0 & 0 \\ * & \alpha U_{21} + \alpha\beta U_{22} & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} - D_{12}^T \mathbb{A}_1 D_{12}.$$

When Eq (2.6) holds,

$$\begin{bmatrix} \frac{\beta}{\alpha}R_1 & -S_{11} & 0 & 0 \\ * & \frac{\alpha}{\beta}R_2 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} \geq \begin{bmatrix} \beta U_{11} + \alpha\beta U_{12} & (\alpha^2 + \beta^2)S_{12} & 0 & 0 \\ * & \alpha U_{21} + \alpha\beta U_{22} & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}.$$

Similarly, the other five parts can be described by

$$\begin{bmatrix} \frac{\gamma}{\alpha}R_1 & 0 & -S_{21} & 0 \\ * & 0 & 0 & 0 \\ * & * & \frac{\alpha}{\gamma}R_3 & 0 \\ * & * & * & 0 \end{bmatrix} = \begin{bmatrix} \gamma U_{31} + \alpha\gamma U_{32} & 0 & (\alpha^2 + \beta^2)S_{22} & 0 \\ * & 0 & 0 & 0 \\ * & * & \alpha U_{41} + \alpha\gamma U_{42} & 0 \\ * & * & * & 0 \end{bmatrix} - D_{13}^T \mathbb{A}_2 D_{13},$$

$$\begin{bmatrix} \frac{\delta}{\alpha}R_1 & 0 & 0 & -S_{31} \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & \frac{\alpha}{\delta}R_4 \end{bmatrix} = \begin{bmatrix} \delta U_{51} + \alpha\delta U_{52} & 0 & 0 & (\alpha^2 + \beta^2)S_{32} \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & \alpha U_{61} + \alpha\delta U_{62} \end{bmatrix} - D_{14}^T \mathbb{A}_3 D_{14},$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ * & \frac{\gamma}{\beta}R_2 & -S_{41} & 0 \\ * & * & \frac{\beta}{\gamma}R_3 & 0 \\ * & * & * & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & \gamma U_{71} + \beta\gamma U_{72} & (\alpha^2 + \beta^2)S_{42} & 0 \\ * & * & \beta U_{81} + \beta\gamma U_{82} & 0 \\ * & * & * & 0 \end{bmatrix} - D_{23}^T \mathbb{A}_4 D_{23},$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ * & \frac{\delta}{\beta}R_2 & 0 & -S_{51} \\ * & * & 0 & 0 \\ * & * & * & \frac{\beta}{\delta}R_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & \delta U_{91} + \beta\delta U_{92} & 0 & (\alpha^2 + \beta^2)S_{52} \\ * & * & 0 & 0 \\ * & * & * & \beta U_{101} + \beta\delta U_{102} \end{bmatrix} - D_{24}^T \mathbb{A}_5 D_{24},$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & \frac{\delta}{\gamma}R_3 & -S_{61} \\ * & * & * & \frac{\gamma}{\delta}R_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & \delta U_{111} + \gamma\delta U_{112} & (\alpha^2 + \beta^2)S_{62} \\ * & * & * & \gamma U_{121} + \gamma\delta U_{122} \end{bmatrix} - D_{34}^T \mathbb{A}_6 D_{34},$$

where

$$D_{13} = \begin{bmatrix} \sqrt{\frac{\gamma}{\alpha}}I & 0 & 0 & 0 \\ * & I & 0 & 0 \\ * & * & \sqrt{\frac{\alpha}{\gamma}}I & 0 \\ * & * & * & I \end{bmatrix}, D_{14} = \begin{bmatrix} \sqrt{\frac{\delta}{\alpha}}I & 0 & 0 & 0 \\ * & I & 0 & 0 \\ * & * & I & 0 \\ * & * & * & \sqrt{\frac{\alpha}{\delta}}I \end{bmatrix}, D_{23} = \begin{bmatrix} I & 0 & 0 & 0 \\ * & \sqrt{\frac{\gamma}{\beta}}I & 0 & 0 \\ * & * & \sqrt{\frac{\beta}{\gamma}}I & 0 \\ * & * & * & I \end{bmatrix},$$

$$D_{24} = \begin{bmatrix} I & 0 & 0 & 0 \\ * & \sqrt{\frac{\delta}{\beta}}I & 0 & 0 \\ * & * & I & 0 \\ * & * & * & \sqrt{\frac{\beta}{\delta}}I \end{bmatrix}, D_{34} = \begin{bmatrix} I & 0 & 0 & 0 \\ * & I & 0 & 0 \\ * & * & \sqrt{\frac{\delta}{\gamma}}I & 0 \\ * & * & * & \sqrt{\frac{\gamma}{\delta}}I \end{bmatrix}.$$

Hence, the lemma is proved whenever (2.7)–(2.11) hold.

Remark 2.7. If the free matrix $U_{1j}, U_{2j}, U_{3j}, U_{4j}, U_{5j}, U_{6j}, U_{7j}, U_{8j}, U_{9j}, U_{10j}, U_{11j}, U_{12j}$, for $j = 1, 2$ is the zero matrix and S_{i2} is also the zero matrix for $i = 1, 2, \dots, 6$, the BQRCI will become the traditional reciprocally convex inequality.

Remark 2.8. As far as we know, a bivariate quadratic reciprocally convex matrix inequality is proposed for the first time, and the new lemma effectively addresses the bivariate quadratic terms generated in the system. Lemma 2.6 has broader applications and achieves less conservativeness.

3. Stability analysis based on the bivariate quadratic reciprocally convex inequality and the improved Lyapunov–Krasovskii functional

For brevity, the following symbols are defined:

$$\begin{aligned} \sigma_1(t) &= \left[\begin{array}{c} y(t) - y(t - k_1(t)) \\ y(t) + y(t - k_1(t)) - \frac{2}{k_1(t)} \int_{t-k_1(t)}^t y(s) ds \end{array} \right], \\ \sigma_2(t) &= \left[\begin{array}{c} y(t - k_1(t)) - y(t - k_1(t) - k_2(t)) \\ y(t - k_1(t)) + y(t - k_1(t) - k_2(t)) - \frac{2}{k_2(t)} \int_{t-k_1(t)-k_2(t)}^{t-k_1(t)} y(s) ds \end{array} \right], \\ \sigma_3(t) &= \left[\begin{array}{c} y(t - k_1(t) - k_2(t)) - y(t - k_1(t) - k_2) \\ y(t - k_1(t) - k_2(t)) + y(t - k_1(t) - k_2) - \frac{2}{k_2 - k_2(t)} \int_{t-k_1(t)-k_2}^{t-k_1(t)-k_2(t)} y(s) ds \end{array} \right], \\ \sigma_4(t) &= \left[\begin{array}{c} y(t - k_1(t) - k_2) - y(t - k_1 - k_2) \\ y(t - k_1(t) - k_2) + y(t - k_1 - k_2) - \frac{2}{k_1 - k_1(t)} \int_{t-k_1(t)-k_2}^{t-k_1-k_2} y(s) ds \end{array} \right], \\ \xi(t) &= \left[\begin{array}{c} y^T(t), y^T(t - k_1(t)), y^T(t - k_1(t) - k_2(t)), y^T(t - k_1(t) - k_2), \\ y^T(t - k_1 - k_2), \frac{1}{k_1(t)} \int_{t-k_1(t)}^t y^T(s) ds, \frac{1}{k_2(t)} \int_{t-k_1(t)-k_2(t)}^{t-k_1(t)} y^T(s) ds, \\ \frac{1}{k_2 - k_2(t)} \int_{t-k_1(t)-k_2}^{t-k_1(t)-k_2(t)} y^T(s) ds, \frac{1}{k_1 - k_1(t)} \int_{t-k_1-k_2}^{t-k_1(t)-k_2} y^T(s) ds \end{array} \right]^T \\ e_i &= [\text{zeros}(n, (i - 1)n) \quad I_n \quad \text{zeros}(n, (9 - i)n)], \quad i = 1, 2, \dots, 9. \end{aligned}$$

The important notations are denoted as

$$E_i = \begin{bmatrix} e_i - e_{i+1} \\ e_i + e_{i+1} - 2e_{i+5} \end{bmatrix} \quad (i = 1, 2, 3, 4), \quad \tilde{R} = \begin{bmatrix} R & 0 \\ 0 & 3R \end{bmatrix}, \quad E = [E_1^T, E_2^T, E_3^T, E_4^T]^T,$$

$$\begin{aligned}
a_2 &= \begin{bmatrix} 0 & \frac{-S_{12}}{(k_1+k_2)^2} & \frac{-S_{22}}{(k_1+k_2)^2} & \frac{-S_{32}}{(k_1+k_2)^2} \\ * & \frac{U_{72}}{(k_1+k_2)^2} & \frac{-S_{42}}{(k_1+k_2)^2} & \frac{-S_{52}}{(k_1+k_2)^2} \\ * & * & \frac{U_{82}}{(k_1+k_2)^2} & \frac{-S_{62}}{(k_1+k_2)^2} \\ * & * & * & 0 \end{bmatrix}, b_2 = \begin{bmatrix} \frac{U_{52}}{(k_1+k_2)^2} & \frac{-S_{12}}{(k_1+k_2)^2} & \frac{-S_{22}}{(k_1+k_2)^2} & \frac{-S_{32}}{(k_1+k_2)^2} \\ * & 0 & \frac{-S_{42}}{(k_1+k_2)^2} & \frac{-S_{52}}{(k_1+k_2)^2} \\ * & * & 0 & \frac{-S_{62}}{(k_1+k_2)^2} \\ * & * & * & \frac{U_{62}}{(k_1+k_2)^2} \end{bmatrix}, \\
c_2 &= \begin{bmatrix} \frac{-U_{12}+U_{32}}{(k_1+k_2)^2} & 0 & 0 & 0 \\ * & \frac{-U_{22}+U_{92}}{(k_1+k_2)^2} & 0 & 0 \\ * & * & \frac{U_{42}-U_{112}}{k_1+k_2} & 0 \\ * & * & * & \frac{U_{102}-U_{122}}{(k_1+k_2)} \end{bmatrix}, \\
a_{11} &= \frac{-U_{11} + U_{31}}{(k_1 + k_2)}, \quad a_{22} = \frac{U_{71}}{k_1 + k_2} - \frac{k_2 U_{72} + k_1 U_{92}}{(k_1 + k_2)^2}, \\
a_{33} &= \frac{-U_{81}}{k_1 + k_2} - \frac{k_2 U_{82} - k_1 U_{112}}{(k_1 + k_2)^2}, \quad a_{44} = \frac{-U_{101} + U_{121}}{(k_1 + k_2)} - \frac{k_1 U_{102} - k_1 U_{122}}{(k_1 + k_2)^2}, \\
b_{11} &= \frac{U_{51}}{(k_1 + k_2)} - \frac{k_2 U_{32} + k_1 U_{52}}{(k_1 + k_2)^2}, \quad b_{22} = \frac{-U_{21} + U_{91}}{k_1 + k_2}, \\
b_{33} &= \frac{-U_{41} + U_{111}}{k_1 + k_2} - \frac{k_2 U_{42} - k_2 U_{112}}{(k_1 + k_2)^2}, \quad b_{44} = \frac{-U_{61}}{k_1 + k_2} - \frac{k_1 U_{62} - k_2 U_{122}}{(k_1 + k_2)^2}, \\
c_{11} &= \frac{-k_2 U_{31} - k_1 U_{51}}{(k_1 + k_2)} - \tilde{R}, \quad c_{22} = \frac{-k_2 U_{71} - k_1 U_{91}}{(k_1 + k_2)} - \tilde{R}, \\
c_{33} &= \frac{-k_1 U_{111}}{k_1 + k_2} + \frac{-k_1 k_2 U_{112}}{(k_1 + k_2)^2} - \tilde{R}, \quad c_{44} = \frac{-k_2 U_{121}}{k_1 + k_2} + \frac{-k_1 k_2 U_{122}}{(k_1 + k_2)^2} - \tilde{R}, \\
a_1 &= \text{diag}(a_{11}, a_{22}, a_{33}, a_{44}), \quad b_1 = \text{diag}(b_{11}, b_{22}, b_{33}, b_{44}), \quad c_1 = \text{diag}(c_{11}, c_{22}, c_{33}, c_{44}), \\
\alpha_2^1 &= \begin{bmatrix} 0 & \frac{S_{12}}{(k_1+k_2)^2} & 0 & 0 \\ * & \frac{U_{22}}{(k_1+k_2)^2} & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}, \beta_2^1 = \begin{bmatrix} \frac{U_{12}}{(k_1+k_2)^2} & \frac{S_{12}}{(k_1+k_2)^2} & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}, \alpha_1^1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & \frac{U_{21}}{k_1+k_2} & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}, \\
\beta_1^1 &= \begin{bmatrix} \frac{U_{11}}{k_1+k_2} & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}, \gamma_1^1 = \begin{bmatrix} -\tilde{R} & S_{11} & 0 & 0 \\ * & -\tilde{R} & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}, \alpha_2^2 = \begin{bmatrix} 0 & 0 & \frac{S_{22}}{(k_1+k_2)^2} & 0 \\ * & 0 & 0 & 0 \\ * & * & \frac{U_{42}}{(k_1+k_2)^2} & 0 \\ * & * & * & 0 \end{bmatrix}, \\
\beta_2^2 &= \begin{bmatrix} \frac{U_{32}}{(k_1+k_2)^2} & 0 & \frac{S_{22}}{(k_1+k_2)^2} & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}, \alpha_1^2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & \frac{-U_{41}}{(k_1+k_2)} + \frac{-2k_2 U_{42}}{(k_1+k_2)^2} & 0 \\ * & * & * & 0 \end{bmatrix}, \\
\beta_1^2 &= \begin{bmatrix} \frac{U_{31}}{k_1+k_2} & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}, \gamma_1^2 = \begin{bmatrix} -\tilde{R} & 0 & S_{21} & 0 \\ * & 0 & 0 & 0 \\ * & * & \frac{k_2 U_{41}}{(k_1+k_2)} + \frac{k_2^2 U_{42}}{(k_1+k_2)^2} - \tilde{R} & 0 \\ * & * & * & 0 \end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
\alpha_2^3 &= \begin{bmatrix} 0 & 0 & 0 & \frac{S_{32}}{(k_1+k_2)^2} \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}, \beta_2^3 = \begin{bmatrix} \frac{U_{52}}{(k_1+k_2)^2} & 0 & 0 & \frac{S_{32}}{(k_1+k_2)^2} \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & \frac{U_{62}}{(k_1+k_2)^2} \end{bmatrix}, \alpha_1^3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}, \\
\beta_1^3 &= \begin{bmatrix} \frac{-U_{51}}{k_1+k_2} & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & \frac{-U_{61}}{(k_1+k_2)} + \frac{-2k_1U_{62}}{(k_1+k_2)^2} \end{bmatrix}, \gamma_1^3 = \begin{bmatrix} -\tilde{R} & 0 & 0 & S_{31} \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & \frac{k_1U_{61}}{(k_1+k_2)} + \frac{k_1^2U_{62}}{(k_1+k_2)^2} - \tilde{R} \end{bmatrix}, \\
\alpha_2^4 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & \frac{U_{72}}{(k_1+k_2)^2} & \frac{S_{42}}{(k_1+k_2)^2} & 0 \\ * & * & \frac{U_{82}}{(k_1+k_2)^2} & 0 \\ * & * & * & 0 \end{bmatrix}, \beta_2^4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & 0 & \frac{S_{42}}{(k_1+k_2)^2} & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}, \beta_1^4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}, \\
\alpha_1^4 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & \frac{U_{71}}{(k_1+k_2)} & 0 & 0 \\ * & * & \frac{-U_{81}}{(k_1+k_2)} + \frac{-2k_2U_{82}}{(k_1+k_2)^2} & 0 \\ * & * & * & 0 \end{bmatrix}, \gamma_1^4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & -\tilde{R} & S_{41} & 0 \\ * & * & \frac{k_2U_{81}}{(k_1+k_2)} + \frac{k_1^2U_{82}}{(k_1+k_2)^2} - \tilde{R} & 0 \\ * & * & * & 0 \end{bmatrix}, \\
\alpha_2^5 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & \frac{U_{92}}{(k_1+k_2)^2} & 0 & \frac{S_{52}}{(k_1+k_2)^2} \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}, \beta_2^5 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & \frac{S_{52}}{(k_1+k_2)^2} \\ * & * & 0 & 0 \\ * & * & * & \frac{U_{102}}{(k_1+k_2)^2} \end{bmatrix}, \\
\alpha_1^5 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & \frac{U_{91}}{(k_1+k_2)} & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}, \beta_1^5 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & \frac{-U_{101}}{(k_1+k_2)} + \frac{-2k_1U_{102}}{(k_1+k_2)^2} \end{bmatrix}, \\
\gamma_1^5 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & -\tilde{R} & 0 & S_{51} \\ * & * & 0 & 0 \\ * & * & * & \frac{k_1U_{101}}{(k_1+k_2)} + \frac{k_1^2U_{102}}{(k_1+k_2)^2} - \tilde{R} \end{bmatrix}, \alpha_2^6 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & \frac{U_{112}}{(k_1+k_2)^2} & \frac{S_{62}}{(k_1+k_2)^2} \\ * & * & * & 0 \end{bmatrix}, \\
\beta_2^6 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & \frac{S_{62}}{(k_1+k_2)^2} \\ * & * & * & \frac{U_{122}}{(k_1+k_2)^2} \end{bmatrix}, \alpha_1^6 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & \frac{-U_{111}}{(k_1+k_2)} + \frac{-2k_2U_{112}}{(k_1+k_2)^2} & 0 \\ * & * & * & 0 \end{bmatrix}, \\
\beta_1^6 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & \frac{-U_{121}}{(k_1+k_2)} + \frac{-2k_1U_{122}}{(k_1+k_2)^2} \end{bmatrix},
\end{aligned}$$

$$\gamma_1^6 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & \frac{k_2 U_{111}}{(k_1+k_2)} + \frac{k_1^2 U_{112}}{(k_1+k_2)^2} - \tilde{R} & S_{61} \\ * & * & * & \frac{k_1 U_{121}}{(k_1+k_2)} + \frac{k_1^2 U_{122}}{(k_1+k_2)^2} - \tilde{R} \end{bmatrix}.$$

Then, we propose a new delay-dependent stability criterion for the system based on the bivariate quadratic reciprocally convex matrix inequality and the improved Lyapunov–Krasovskii functional.

Theorem 3.1. For known $k_i > 0$, $\mu_i > 0$ ($i = 1, 2$), the time delays of the additive system: $k_1(t)$ and $k_2(t)$ follow conditions (2.2); the additive system (2.1) is asymptotically stable if there exists a matrix, $\begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix} \in \mathbb{S}_{2n}^+$, $Z_i \in \mathbb{S}_n^+$ ($i = 1, 2, 3, 4$), $R_l \in \mathbb{S}^+$, for any $l = 1, 2, 3, 4, 5, 6$, $S_{1j}, S_{2j}, S_{3j}, S_{4j}, S_{5j}, S_{6j}, U_{1j}, U_{2j}, U_{3j}, U_{4j}, U_{5j}, U_{6j}, U_{7j}, U_{8j}, U_{9j}, U_{10j}, U_{11j}, U_{12j} \in \mathbb{R}^{n \times n}$ ($j = 1, 2$) such that LMIs (3.1)–(3.6) hold.

$$\Psi_1(0) < 0, \quad \Psi_1(k_2) < 0, \quad \frac{k_2}{2} \hat{a}_1 + \Psi_1(0) < 0, \quad (3.1)$$

$$\Psi_2(0) < 0, \quad \Psi_2(k_2) < 0, \quad \frac{k_2}{2} (\hat{a}_1 + \hat{c}_2 k_1) + \Psi_2(0) < 0, \quad (3.2)$$

$$\Psi_3(0) < 0, \quad \Psi_3(k_2) < 0, \quad \frac{k_2}{2} (\hat{a}_1 + \frac{k_1}{2} \hat{c}_2) + \Psi_3(0) < 0, \quad (3.3)$$

$$\Omega_1^j(0) < 0, \quad \Omega_1^j(k_2) < 0, \quad \frac{k_2}{2} \alpha_1^j + \Omega_1^j(0) < 0, \quad (3.4)$$

$$\Omega_2^j(0) < 0, \quad \Omega_2^j(k_2) < 0, \quad \frac{k_2}{2} \alpha_1^j + \Omega_2^j(0) < 0, \quad (3.5)$$

$$\Omega_3^j(0) < 0, \quad \Omega_3^j(k_2) < 0, \quad \frac{k_2}{2} \alpha_1^j + \Omega_3^j(0) < 0. \quad (3.6)$$

The important notations are defined:

$$\Psi_1(h) = \hat{a}_2 h^2 + \hat{a}_1 h + \hat{c}_1, \quad \Psi_2(h) = \hat{a}_2 h^2 + (\hat{a}_1 + \hat{c}_2 k_1) h + (\hat{b}_2 k_1^2 + \hat{b}_1 k_1 + \hat{c}_1) < 0,$$

$$\Psi_3(h) = \hat{a}_2 h^2 + (\hat{a}_1 + \frac{k_1}{2} \hat{c}_2) h + (\hat{b}_1 \frac{k_1}{2} + \hat{c}_1) < 0,$$

$$\Omega_1^j(h) = \alpha_2^j h^2 + \alpha_1^j h + \gamma_1^j < 0, \quad \Omega_2^j(h) = \alpha_2^j h^2 + \alpha_1^j h + (\beta_2^j k_1^2 + \beta_1^j k_1 + \gamma_1^j) < 0,$$

$$\Omega_3^j(h) = \alpha_2^j h^2 + \alpha_1^j h + (\beta_1^j \frac{k_1}{2} + \gamma_1^j) < 0,$$

$$\Theta_1 = \text{Sym}\{e_1^T (P_1 A + P_2) e_1 + e_1^T P_1 B e_3 - e_1^T P_2 e_5\},$$

$$\Theta_2 = e_1^T (Z_1 + Z_2 + Z_3 + Z_4) e_1 - (1 - \mu_1) e_2^T Z_1 e_2 - (1 - \mu_1 - \mu_2) e_3^T Z_2 e_3 \\ - (1 - \mu_1) e_4^T Z_3 e_4 - e_5^T Z_4 e_5,$$

$$\Theta_3 = (k_1 + k_2) (e_1^T A^T R A e_1 + e_3^T B^T R B e_3 + \text{Sym}\{e_1^T A^T R B e_3\}),$$

$$\Upsilon_i = \text{Sym}\{e_1^T (A^T P_2 + P_3) e_{i+5} + e_3^T B^T P_2 e_{i+5} - e_5^T P_3 e_{i+5}\} \quad (i = 1, 2, 3, 4),$$

$$\hat{a}_2 = \frac{E^T a_2 E}{k_1 + k_2}, \quad \hat{b}_2 = \frac{E^T b_2 E}{k_1 + k_2}, \quad \hat{c}_2 = \frac{E^T c_2 E}{k_1 + k_2}, \quad \hat{a}_1 = \Upsilon_2 - \Upsilon_3 + \frac{E^T a_1 E}{k_1 + k_2},$$

$$\hat{b}_1 = \Upsilon_1 - \Upsilon_4 + \frac{E^T b_1 E}{k_1 + k_2}, \quad \hat{c}_1 = \Theta_1 + \Theta_2 + \Theta_3 + k_2 \Upsilon_3 + k_1 \Upsilon_4 + \frac{E^T c_1 E}{k_1 + k_2}.$$

Let us prove this theorem:

Proof: Choose some LKFs candidate first,

$$V(y_t) = V_1(y_t) + V_2(y_t) + V_3(y_t), \quad (3.7)$$

where

$$\begin{aligned} V_1(y_t) &= \begin{bmatrix} y(t) \\ \int_{t-k_1-k_2}^t y(s) ds \end{bmatrix}^T \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix} \begin{bmatrix} y(t) \\ \int_{t-k_1-k_2}^t y(s) ds \end{bmatrix}, \\ V_2(y_t) &= \int_{t-k_1(t)}^t y^T(s) Z_1 y(s) ds + \int_{t-k_1(t)-k_2(t)}^t y^T(s) Z_2 y(s) ds \\ &\quad + \int_{t-k_1(t)-k_2}^t y^T(s) Z_3 y(s) ds + \int_{t-k_1-k_2}^t y^T(s) Z_4 y(s) ds, \\ V_3(y_t) &= \int_{-k_1-k_2}^0 \int_{t+\theta}^t \dot{y}^T(s) R \dot{y}(s) ds d\theta. \end{aligned}$$

Then, we calculate the derivatives of the functionals, which are given by:

$$\dot{V}_1(y_t) = \xi^T(t) (\Theta_1 + \Upsilon_1 k_1(t) + \Upsilon_2 k_2(t) + \Upsilon_3 (k_2 - k_2(t)) + \Upsilon_4 (k_1 - k_1(t))) \xi(t), \quad (3.8)$$

$$\dot{V}_2(y_t) \leq \xi^T(t) \Theta_2 \xi(t), \quad (3.9)$$

$$\begin{aligned} \dot{V}_3(y_t) &= (k_1 + k_2) \dot{y}^T(t) R \dot{y}(t) - \int_{t-k_1-k_2}^t \dot{y}^T(s) R \dot{y}(s) ds \\ &= (k_1 + k_2) \dot{y}^T(t) R \dot{y}(t) - \int_{t-k_1(t)}^t \dot{y}^T(s) R \dot{y}(s) ds - \int_{t-k_1(t)-k_2(t)}^{t-k_1(t)} \dot{y}^T(s) R \dot{y}(s) ds \\ &\quad - \int_{t-k_1(t)-k_2}^{t-k_1(t)-k_2(t)} \dot{y}^T(s) R \dot{y}(s) ds - \int_{t-k_1-k_2}^{t-k_1(t)-k_2} \dot{y}^T(s) R \dot{y}(s) ds. \end{aligned} \quad (3.10)$$

Using the Wirtinger-based integral inequality (2.3) to estimate the upper bounds of the derivative of $V_3(t)$.

$$\begin{aligned} \dot{V}_3(y_t) &\leq (k_1 + k_2) \dot{y}^T(t) R \dot{y}(t) - \frac{\sigma_1^T(t) \tilde{R} \sigma_1(t)}{k_1(t)} - \frac{\sigma_2^T(t) \tilde{R} \sigma_2(t)}{k_2(t)} - \frac{\sigma_3^T(t) \tilde{R} \sigma_3(t)}{k_2 - k_2(t)} - \frac{\sigma_4^T(t) \tilde{R} \sigma_4(t)}{k_1 - k_1(t)} \\ &= (k_1 + k_2) \dot{y}^T(t) R \dot{y}(t) \\ &\quad + \begin{bmatrix} \sigma_1(t) \\ \sigma_2(t) \\ \sigma_3(t) \\ \sigma_4(t) \end{bmatrix}^T \begin{bmatrix} -\frac{1}{k_1(t)} \tilde{R} & 0 & 0 & 0 \\ * & -\frac{1}{k_2(t)} \tilde{R} & 0 & 0 \\ * & * & -\frac{1}{k_2 - k_2(t)} \tilde{R} & 0 \\ * & * & * & -\frac{1}{k_1 - k_1(t)} \tilde{R} \end{bmatrix} \begin{bmatrix} \sigma_1(t) \\ \sigma_2(t) \\ \sigma_3(t) \\ \sigma_4(t) \end{bmatrix}. \end{aligned} \quad (3.11)$$

By using Lemma 2.6,

$$\dot{V}(y_t) = \xi^T(t) \left(\Theta_3 - \frac{1}{k_1 + k_2} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix}^T \begin{bmatrix} \frac{k_1+k_2}{k_1(t)} \tilde{R} & 0 & 0 & 0 \\ * & \frac{k_1+k_2}{k_2(t)} \tilde{R} & 0 & 0 \\ * & * & \frac{k_1+k_2}{k_2 - k_2(t)} \tilde{R} & 0 \\ * & * & * & \frac{k_1+k_2}{k_1 - k_1(t)} \tilde{R} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix} \right) \xi(t)$$

$$\leq \xi^T(t) \left(\Theta_3 + \frac{1}{k_1 + k_2} E^T [a_2 k_2^2(t) + b_2 k_1^2(t) + a_1 k_2(t) + b_1 k_1(t) + c_2 k_1(t) k_2(t) + c_1] E \right) \xi(t).$$

Therefore,

$$\dot{V}(y_t) \leq \xi^T(t) \left(\hat{a}_2 k_2^2(t) + \hat{b}_2 k_1^2(t) + \hat{a}_1 k_2(t) + \hat{b}_1 k_1(t) + \hat{c}_2 k_1(t) k_2(t) + \hat{c}_1 \right) \xi(t). \quad (3.12)$$

Using Lemma 2.2, from the negative determined condition, $\dot{V} < 0$ holds if the LMIs conditions (3.1)–(3.3) hold.

On the other hand, there are some necessary conditions for the new reciprocally convex matrix inequalities (2.6)–(2.11):

$$\mathbb{A}_j \leq 0, \quad \text{for } j = 1, 2, \dots, 6.$$

Similarly, using Lemma 2.2 again,

$$\mathbb{A}_j = \left(\alpha_2^j k_2^2(t) + \beta_2^j k_1^2(t) + \alpha_1^j k_2(t) + \beta_1^j k_1(t) + \gamma_1^j \right) \leq 0. \quad (3.13)$$

This is ensured by the LMIs (3.4)–(3.6) based on Lemma 2.2. Consequently, if the LMIs (3.1) to (3.6) are feasible, there exists a sufficiently small $\varepsilon > 0$ such that $\dot{V}(y_t) < -\varepsilon \|y(t)\|^2$. This condition guarantees that system (2.1) with additive time-varying delays and satisfying condition (2.2) is asymptotically stable. The proof is complete.

Remark 3.2. When using Lemma 2.6, we can let $\alpha = \frac{k_1(t)}{k_1+k_2}$; $\beta = \frac{k_2(t)}{k_1+k_2}$; $\gamma = \frac{k_2-k_2(t)}{k_1+k_2}$; $\delta = \frac{k_1-k_1(t)}{k_1+k_2}$. In this way, the derivative of $V(y_t)$ can become a bivariate quadratic function. In the same way, the necessary condition $\mathbb{A}_j \leq 0$ for the Lemma (2.6) can also be written as a bivariate quadratic function. The only difference is that the coefficients of $k_1(t)$ and $k_2(t)$ are 0.

Remark 3.3. The LMIs presented in Theorem 3.1 are based on the newly derived bivariate quadratic reciprocally convex inequality. This inequality allows for a more comprehensive application of bivariate quadratic negative definiteness conditions, enabling the formulation of LMIs that effectively utilize the delay information of the systems under discussion. Furthermore, the resulting stability criteria exhibit lower conservativeness without significantly increasing the number of decision variables (NoDVs). Thus, the advantage of the LMIs lies in their foundation on this inequality, which enhances the applicability of negative definiteness conditions while maintaining practical relevance in stability analysis.

4. Numerical experiments

Example 4.1. Consider the time-varying delay system (2.1) with the given matrices \mathcal{A} and \mathcal{B} ,

$$\mathcal{A} = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \mathcal{B} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}.$$

The upper bounds for the delay derivatives, μ_1 and μ_2 , are set at 0.1 and 0.8. Our objective is to determine the upper bounds of delays $k_1(t)$ and $k_2(t)$, specifically finding k_1 and k_2 when one is known. Some previous methods use traditional RCI like [31], which do not achieve results as favorable as those obtained by applying BQRCI. Table 1 compares different k_1 values with the results from previous studies, including some recent research [32–34], and Theorem 3.1 under different delay conditions.

As shown in Table 1, the upper bound of k_2 obtained using the current method is greater than that of several recent results, such as those in [32–34], while the NoDVs is smaller than in [33]. Therefore, this method effectively controls the increase in computational complexity without significantly raising the number of decision variables, while also achieving less conservativeness for the system.

Table 1. The time-varying delay upper bound k_2 for various k_1 .

k_1	1.0	1.2	1.5	NoDVs
[32]	0.982	0.782	0.482	-
[33]	0.999	0.972	0.680	$202n^2 + 25n$
[34]	1.163	0.965	0.669	$32n^2 + 10n$
[35]	0.415	0.376	0.248	$12.5n^2 + 4.5n$
[36]	0.512	0.406	0.283	$7.5n^2 + 3.5n$
[37]	0.595	0.462	0.312	-
[38]	0.873	0.673	0.452	$7.5n^2 + 5n$
Theorem 3.1	1.171	0.975	0.715	$100.5n^2 + 27.5n$

Example 4.2. Consider the time-varying delay system (2.1) with the given matrices \mathcal{A} and \mathcal{B} ,

$$\mathcal{A} = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \mathcal{B} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}.$$

We also assess the decay rates for different values of k_2 . From Table 2, it can be observed that the upper bound of k_2 obtained by the current method is greater than that of several recent results, including [31, 33, 34, 39], and NoDVs is smaller than [31, 33, 39]. Therefore, this method controls the increase of computation without excessively increasing NoDVs. The proposed method can yield an improved stability criterion for additive time-varying delay systems.

Table 2. The time-varying delay upper bound k_1 for various k_2 .

k_2	0.3	0.4	0.5	NoDVs
[31]	1.967	1.883	1.788	$149n^2 + 25n$
[32]	1.682	1.582	1.482	-
[33]	1.880	1.779	1.675	$202n^2 + 25n$
[34]	1.875	1.773	1.671	$32n^2 + 10n$
[35]	1.324	1.039	0.806	$12.5n^2 + 4.5n$
[36]	1.453	1.214	1.021	$19.5n^2 + 3.5n$
[37]	1.531	1.313	1.140	-
[38]	1.808	1.593	1.424	$7n^2 + 5n$
[39]	1.913	1.813	1.713	$195.5n^2 + 30.5n$
Theorem 3.1	2.280	2.010	1.806	$100.5n^2 + 27.5n$

Example 4.3. Consider the time-varying delay system (2.1) with the given matrices \mathcal{A} and \mathcal{B} ,

$$\mathcal{A} = \begin{bmatrix} -2 & 0 \\ 0 & -9 \end{bmatrix}, \mathcal{B} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}.$$

From Table 3, it can be seen that the upper bound of k_1 and k_2 obtained by the current method is greater than that of several recent results, including [10, 32, 40]. Therefore, this method achieves less conservativeness for the system.

Table 3. The time-varying delay upper bound for different cases.

Method	$k_1 = 1$	$k_1 = 1.2$	$k_1 = 1.5$	$k_2 = 0.3$	$k_2 = 0.4$	$k_2 = 0.5$
[8]	0.415	0.340	0.248	1.324	1.039	0.806
[10]	0.873	0.673	0.373	1.573	1.473	1.373
[32]	0.982	0.782	0.482	1.682	1.582	1.482
[36]	0.512	0.406	0.283	1.453	1.214	1.021
[40]	0.872	0.672	0.371	1.572	1.472	1.372
Theorem 3.1	2.163	1.928	1.598	7.111	5.309	4.230

Example 4.4. By introducing the virtual state and measurement output vectors defined as $y(t) = \left[\Delta f \ \Delta P_m \ \Delta P_v \ \int \mathcal{ACE} \right]^T$ and $z(t) = \left[\mathcal{ACE} \ \int \mathcal{ACE} \right]^T$, the closed-loop LFC system can be represented as the linear system (2.1), which includes two additive time-varying delays. The system parameters are presented in the following format:

$$x(t) = \begin{bmatrix} \Delta f \\ \Delta P_m \\ \Delta P_v \\ \int \mathcal{ACE} \end{bmatrix}, \mathcal{A} = \begin{bmatrix} -\frac{\mathcal{D}}{\mathcal{M}} & \frac{1}{\mathcal{M}} & 0 & 0 \\ 0 & -\frac{1}{\mathcal{T}_{ch}} & \frac{1}{\mathcal{T}_{ch}} & 0 \\ -\frac{1}{\mathcal{RT}_g} & 0 & -\frac{1}{\mathcal{T}_s} & 0 \\ \varpi & 0 & 0 & 0 \end{bmatrix}, \mathcal{B} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{\mathcal{K}_p \varpi}{\mathcal{T}_g} & 0 & 0 & -\frac{\mathcal{K}_I}{\mathcal{T}_g} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

with the parameters given in [31]: $\mathcal{M} = 10$, $\mathcal{D} = 1$, $\mathcal{T}_{ch} = 0.3$, $\mathcal{T}_g = 0.1$, $\mathcal{R} = 0.05$, $\varpi = 21$.

To facilitate comparison with existing results, Figure 1 presents the findings for the case where $\mathcal{K}_I = 0.2$, $\mathcal{K}_p = 0.1$, $|\dot{k}_1(t)| \leq 0.1$, and $|\dot{k}_2(t)| \leq 0.8$. A simple simulation is conducted with the assumption of time-varying delays defined as $k_1(t) = \frac{1}{2} \sin(0.2x(t)) + \frac{1}{2}$ and $k_2(t) = \frac{3.278}{2} \sin\left(\frac{1.6}{3.278}y(t)\right) + \frac{3.278}{2}$. The results of this simulation are illustrated in Figure 1, where the LFC achieves its objectives, and the control system remains stable for MAUB $k = 3.278$. The state responses converge to zero, confirming that system 2.1 is stable as shown in Figure 1 under condition Example 4.4.

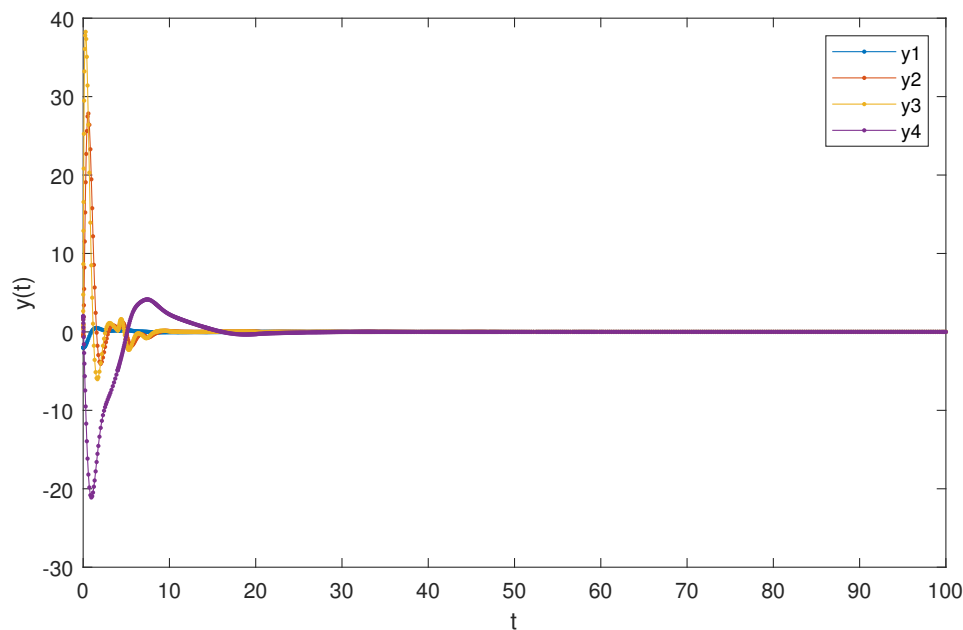


Figure 1. Trajectory of Example 4.3.

5. Conclusions

In this study, we successfully generalize a bivariate quadratic reciprocally convex inequality that is effectively applied to additive time-varying delay systems, significantly reducing the conservativeness typically associated with stability analyses. By employing the Lyapunov–Krasovskii function method in conjunction with this new inequality, we derive a novel stability criterion. Ensure that the conservativeness of the system decreases while controlling the excessive growth of the NoDV's without significantly increasing the computational burden. The effectiveness of the new approach is demonstrated through four numerical examples, underscoring its practical applicability in enhancing the stability analysis of additive time-varying delay systems.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Author contributions

Xiao Ge: conceptualization, methodology, software MATLAB, writing-review and editing; Xinzuo Ma: software MATLAB, writing-review and editing, validation; Yuanyuan Zhang: methodology, writing-original and draft, software MATLAB, validation; Han Xue: software MATLAB, validation; Seakweng Vong: writing-review and editing, validation, funding acquisition, supervision. All authors have read and agreed to the published version of the manuscript.

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Conflict of interest

The authors declare no conflicts of interest.

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